TWO TAILED T-TEST

MULTIVARIATE ANALYSIS PROJECT

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STUDENT'S T-TEST

- Method developed by William Sealy Gosset
- During his time at the Guinness Brewery he found that existing statistical techniques requiring large samples were not useful for the small sample sizes that he encountered in his work
- His Employer Guinness Brewery feared publishing his statistical findings would jeopardise company secretes.
- Used the pseudo name - Student

Fig.1

MEAN OF THE DATASET DOESN'T FULLY REPRESENT ALL THE DATA

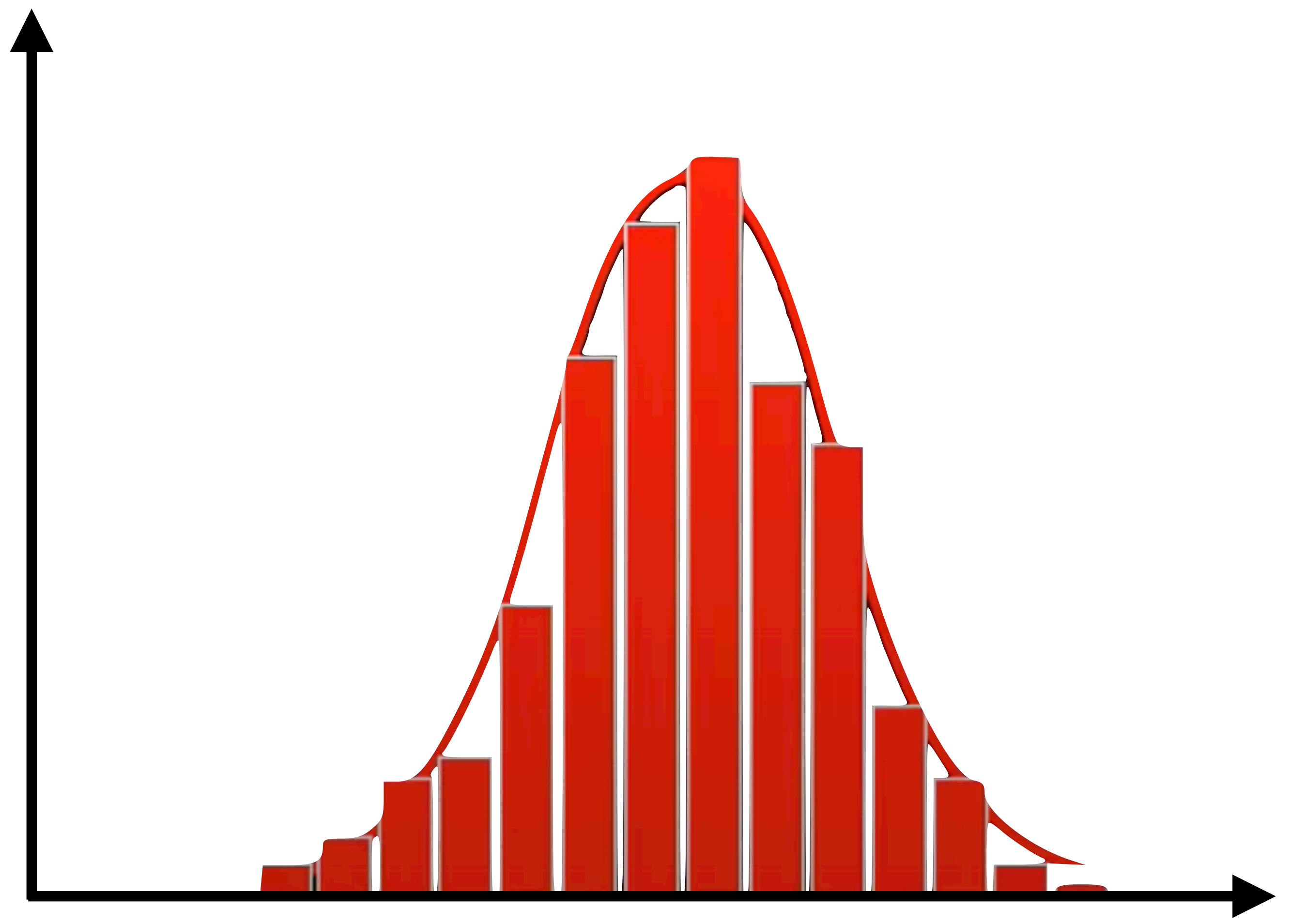


Fig.2 - Let us assume that when some arbitrary dataset is graphed as histogram, it forms a bell curve as shown above

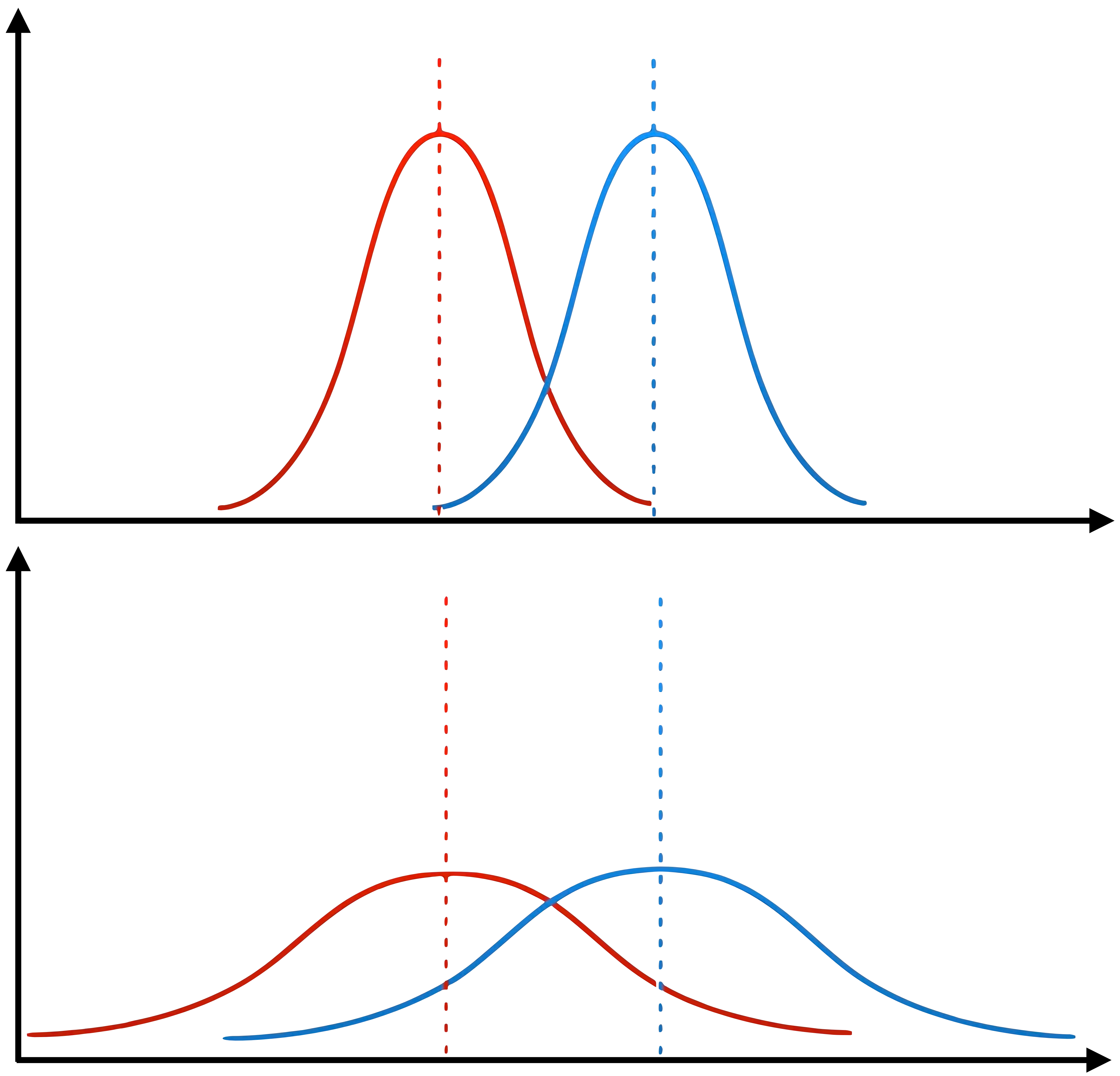
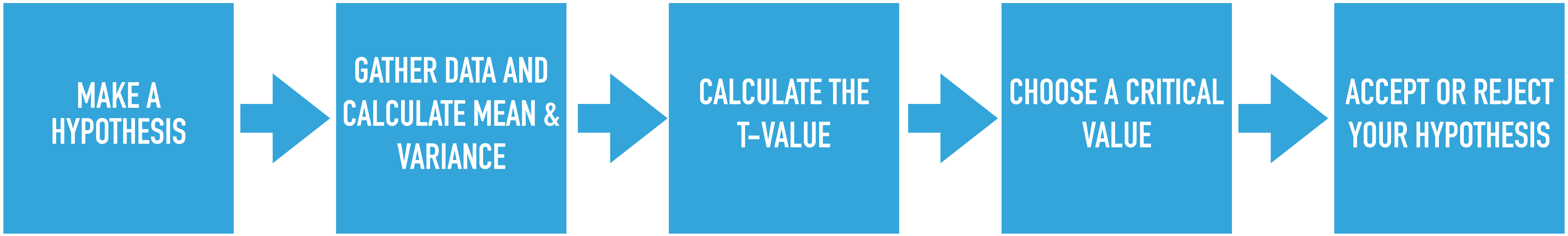


Fig.3

As we can see from the two bell curve distributions, which represent two different datasets, that even though they have the same mean it doesn't mean that the two datasets are highly correlated. The variance within the datasets has not been taken into account.

THE T-TEST

- A t-test is a type of inferential statistic used to determine if there is a significant difference between the means of two groups, which may be related in certain features.
- It is used as a hypothesis testing tool, which allows testing of an assumption applicable to datasets.



Flowchart.1 - T-test pipeline

T – VALUE : SIGNAL TO NOISE RATIO

- The T-value for any given data sets can be thought as a signal to noise ratio where the signal represents the the numbers that differentiate the datasets and the noise represents the numbers that get in the way.

$$T - Value = \frac{\textbf{Signal}}{\textbf{Noise}} = \frac{\textbf{Difference between group means}}{\textbf{Variability of groups}} = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}}}$$

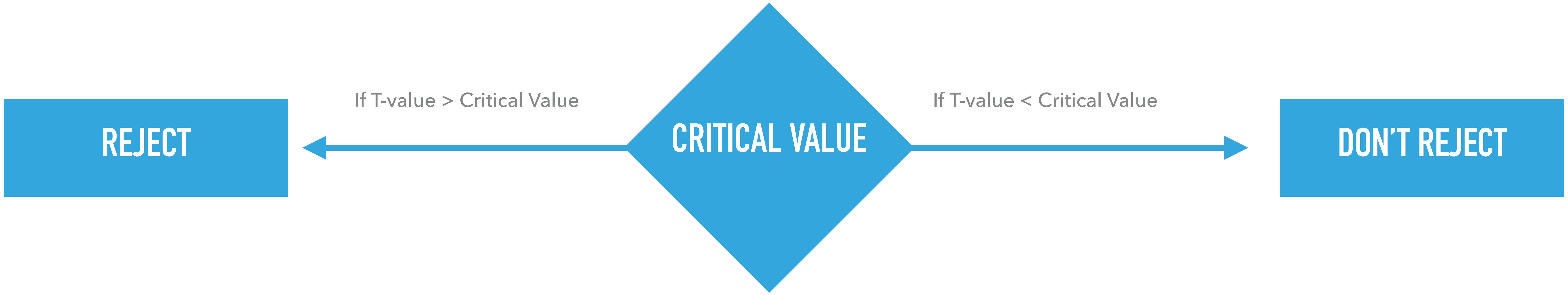
$\bar{x}_1$ and $\bar{x}_2$ are the mean of the two datasets

n_1 and n_2 are the no. of data points in the datasets respectively

${s_1}^2$ and ${s_2}^2$ represents the variance of the datasets

NULL HYPOTHESIS AND CRITICAL VALUE

- **Null Hypothesis: There is no significant difference between the observed and expected assumption.** Using the T-test we attempt to either accept or reject the null hypothesis.
- **Critical Value:** This is the final value on the basis of which a null hypothesis is accepted or rejected



Flowchart.2 - Arriving at a statistical conclusion

PERFORMING THE T-TEST : DATASET USED FOR POSITIVE OUTCOME

- Using t-test to compare the agricultural yield of two barley produce batches. The first batch was produced under natural conditions the second batch was produced in a greenhouse.
- Both fields are 100 hectares large but for performing the t-test we note down yield for a subset of this land ie. 16 hectares each.
- Our Null Hypothesis says that there is no significant difference between the yield from the two fields

Field -1	Field -2
15.2	15.3
15.3	15.4
16.0	15.2
15.8	16.6
15.6	15.2
14.9	15.8
15.0	15.8
15.4	16.2
15.6	15.6
15.7	15.6
15.5	15.8
15.2	15.5
15.5	15.5
15.1	15.5
15.3	14.9
15.0	15.9

Unit for Yield: Quintal/Hectare

PERFORMING THE T-TEST: CODE OUTPUT FOR POSITIVE OUTCOME

```
***********************************************************************
Operation                  Field_1                    Field_2
***********************************************************************
Mean:                    15.381250000000001         15.6125
Standard Deviation:      0.31244999599935985        0.41129875597510246
Variance:                0.09762499999999999        0.16916666666666688
T-Value:                 1.7908351198149484
***********************************************************************
We accept the null hypothesis
***********************************************************************
```

Fig.4 - T-test Positive output

- Fig.4 shows a positive output for the t-test python code. By calculating various statistical parameters such as Mean, variance and the t-value and plugging in those values in the t-test pipeline we have reached a statistical conclusion.
- We accepted the hypothesis, ie. we conclude that for the given data, there is no statistical difference between the yield of the two fields.

PERFORMING THE T-TEST : DATASET USED FOR NEGATIVE OUTCOME

- We use the same dataset from the previous example for a positive output but we alter just two values for a negative output.
- The yield for two 1-hectare plots in Field-2 have been changed from 15.3 to15.6 and 15.4 to 15.9 quintal per hectare.
- The null hypothesis remains the same; and it says that there is no significant difference between the yield from the two fields

Field -1	Field -2
15.2	15.6
15.3	15.9
16.0	15.2
15.8	16.6
15.6	15.2
14.9	15.8
15.0	15.8
15.4	16.2
15.6	15.6
15.7	15.6
15.5	15.8
15.2	15.5
15.5	15.5
15.1	15.5
15.3	14.9
15.0	15.9

Unit for Yield: Quintal/Hectare

PERFORMING THE T-TEST: CODE OUTPUT FOR NEGATIVE OUTCOME

```
**********************************************************************
Operation                  Field_1                   Field_2
**********************************************************************
Mean:                 15.381250000000001         15.662500000000001
Standard Deviation:   0.31244999599935985        0.4031128874149278
Variance:             0.09762499999999999        0.16250000000000026
T-Value:              2.2057763493200304
**********************************************************************
We reject the null hypothesis
**********************************************************************
```

Fig.5 - T-test Negative output

- This time Fig.5 shows a negative output for the t-test python code. By calculating various statistical parameters such as Mean, variance and the t-value and plugging in those values in the t-test pipeline we have reached a statistical conclusion.
- We reject the hypothesis, ie. we conclude that for the given data, there is a statistical difference between the yield of the two fields.

SPECIFICATIONS IN THE T-TEST: PAIRED AND UNPAIRED T-TEST

- A **paired t-test** is a statistical test that compares **two related or dependent groups** to determine if there is a significant difference between the two groups.
- An **unpaired t-test** is a statistical procedure that compares two **independent or unrelated groups** to determine if there is a significant difference between the two.
- Paired t-tests are considered more powerful than unpaired t-tests because using the same participants or item eliminates variation between the samples that could be caused by anything other than what's being tested.
- The dataset seen before shows an example of an unpaired t-test. We will further see an example of a paired t-test

PERFORMING THE T-TEST : PAIRED T-TEST

- As study was performed to compare the height of boys (having parents shorter than 4 feet tall) at different stages of their childhood.
- The dataset gives the height of 10 boys at age 5 and at age 15 in inches.
- Our null hypothesis says that there is no significant difference between the heights at age 5 and at age 15.

Age 5	Age 15
53	73
56	65
55	71
48	65
39	63
45	70
48	65
43	68
51	63
54	71

Weight in lbs

PERFORMING THE T-TEST: CODE OUTPUT FOR PAIRED T-TEST

```
***********************************************************************
Operation                  Age_5                   Age_15
***********************************************************************
Mean:                 49.2                        67.4
Standard Deviation:   5.6134758493389025          3.6575644598253874
T-Value:              8.590173985951095
***********************************************************************
We reject the null hypothesis
***********************************************************************
```

Fig.6 - Paired T-test Negative output

- From the output we can see that out null hypothesis has been rejected. This means that there is a statistical difference between the heights of boys at age 5 and at age 15. Thus we can conclude that this group of boys (whose parents have dwarfism) have inherited genes marked as "tall".

VARIATIONS IN THE T-TEST: TWO-TAILED TEST

- Two-tailed hypothesis tests are also known as nondirectional and two-sided tests because you can test for variations in both directions. When you perform a two-tailed test, you split the significance level percentage between both tails of the distribution.

- For a two tailed test, the generic null and alternative hypotheses are the following

  1. **Null**: The variation in data equals zero

  2. **Alternative**: The variation in data does not equal zero

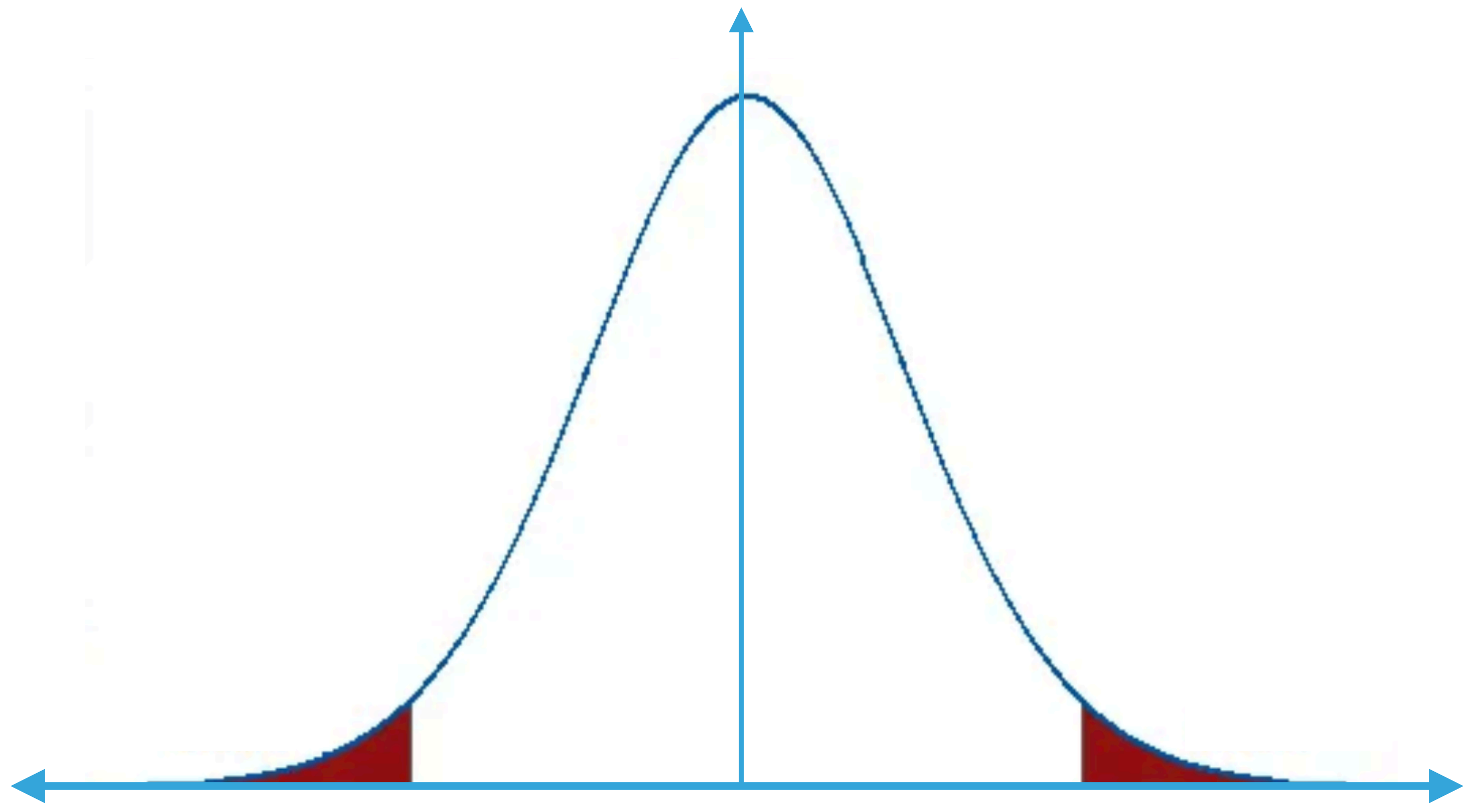


Fig.7 - Generic Two tailed t-test plot

VARIATIONS IN THE T-TEST: ONE-TAILED TEST

- One-tailed hypothesis tests are also known as directional and one-sided tests because you can test for variations in only one direction. When you perform a one-tailed test, the entire significance level percentage goes into the extreme end of one tail of the distribution.

- For a one tailed test, the generic null and alternative hypotheses are the following

  1. **Null**: The variation in data is less than or equal to zero.

  2. **Alternative:** The variation in data is greater than zero.

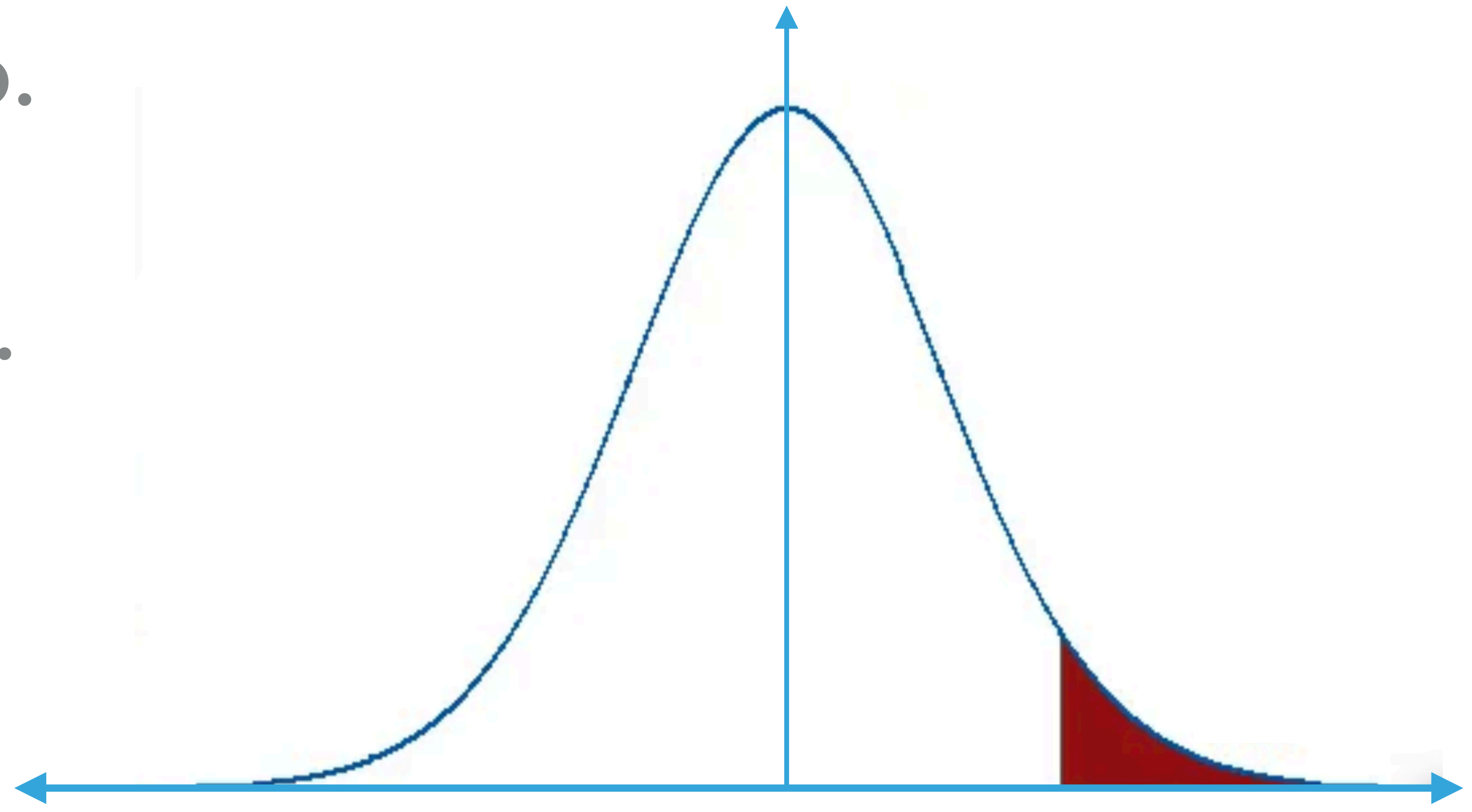


Fig.8 - Generic One tailed t-test plot

PERFORMING THE T-TEST : EXAMPLE OF ONE TAILED T-TEST

- A study was conducted to determine the effectiveness of a weight loss program. The dataset shows the before and after weights of 15 subjects in the program.
- Our Null Hypothesis says that there is no significant difference between the weight of the participants before and after the weight loss program. ie. the mean difference is greater than or equal to zero.
- Alternative hypothesis implies that the mean difference is less than zero. Thus we can conclude that its a left-tail t-test.

Before	After
185	169
192	187
206	193
177	176
225	194
168	171
256	228
239	217
199	204
218	195
193	186
175	178
215	192
200	183
199	180

Weight in lbs

PERFORMING THE T-TEST: CODE OUTPUT FOR ONE TAILED T-TEST

```
************************************************************
Mean Difference:                         -12.933333333333334
Degrees of Freedom [One-tail t-test]:    14
T_value :                                -4.221668723180485
Critical value:                          -1.7613
************************************************************
We reject the null hypothesis
************************************************************
```

Fig.9 - One tail T-test

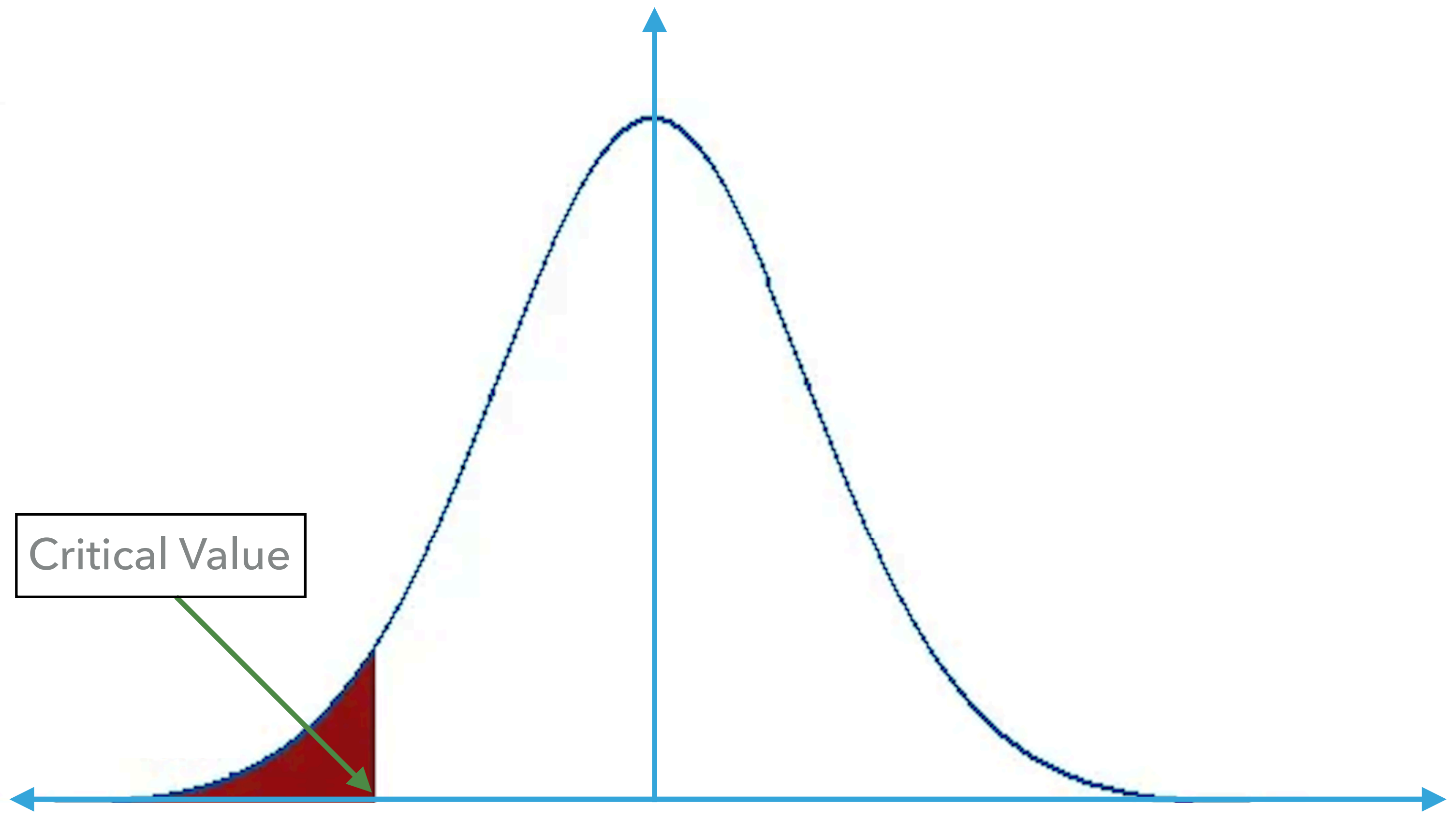


Fig.10 - Left tail T-test

- By performing the t-test on the dataset, we come to the conclusion that we have to reject the null hypothesis. This means that there is a significant difference between the initial and final weight of the participants.
- Keep in mind that it is a left tail t-test and thus if t-value is less than critical value we reject the null hypothesis.

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- Statistics Solutions - LINK
- Technology Networks - LINK

THANK YOU