MAD101–ASSIGNMENT 01 [deadline: 25/09/2023]

Exercise 01. Construct a truth table for each of these compound propositions:

a/ $\Big((p \vee q) \wedge (\neg p \vee \neg q)\Big) \longleftrightarrow (p \leftrightarrow q)$

b/ $\Big((p \vee q) \oplus (\neg p \vee \neg q)\Big) \longleftrightarrow (p \leftrightarrow q)$

c/ $(p \to r) \wedge (q \to s) \wedge (p \to q) \wedge (r \to s)$

Exercise 02. Suppose P and Q are the statements: P: Jack passed math. Q: Jill passed math.

(a) Translate "Jack and Jill both passed math" into symbols.

(b) Translate "If Jack passed math, then Jill did not" into symbols.

(c) Translate $P \vee Q$ into English.

(d) Translate $\neg(P \wedge Q) \to Q$ into English.

(e) Suppose you know that if Jack passed math, then so did Jill. What can you conclude if you know that Jill did not pass math?

Exercise 03. Assume that $P(x, y)$ is a propositional function with variables x, y. Explain why

$$(\forall z P(z, z)) \longrightarrow \forall x \exists y P(x, y)$$

is valid.

Exercise 04. Find the negation of

a/ $\forall x \exists y \big(\forall z \neg T(x, y, z) \ \wedge \ \neg Q(x, y)\big)$

b/ $\exists x \exists y \big(\forall z T(x, y, z) \ \vee \ Q(x, y)\big)$

Exercise 05. Consider the set

$$X = \Big\{-7, \ -6, \ -5, \ -4, \ -3, \ -2, \ -1, \ 0, \ 1, \ 2, \ 3, \ 4, \ \ldots\Big\}.$$

a/ Prove that the following functions are one-to-one from X to $\mathbb{Z}$:

$$g(n) = n^3 - 1; \qquad h(n) = n^2 + 14n + 5$$

b/ Let $f : X \longrightarrow \mathbb{Z}$ be a function defined as follows

$$f(n) = \begin{cases} -8(n^3 + 1) & \text{if } n \leq 1, \\ 6n - 1 & \text{if } n > 1. \end{cases}$$

Is the function f one-to-one? Give reasons for your answers.

Exercise 06. Determine whether $f : \mathbb{Z} \to \mathbb{Z}$ is onto if:

a/. $f(n) = \begin{cases} -2n & \text{if } n \leq 0, \\ 2n - 1 & \text{if } n > 0. \end{cases}$ b/. $f(n) = \left\lfloor \dfrac{n+1}{3} \right\rfloor$.

Exercise 07. Let $S = \Big\{-9, \ -5, \ 0, \ 2, \ 6, \ 7, \ 9\Big\}$. Find $g(S)$ if $g(x) = \left\lfloor 15 + \left\lceil \dfrac{x}{4 + \sin \pi x} \right\rceil \right\rfloor$.

Exercise 08. Assume $f : \mathbb{R} \longrightarrow \mathbb{R}$, is given by

$$f(x) = 7x^3 + 19$$

Prove that the function f is bijection and find the inverse function f^{-1}.