2.7-6 Find the transfer function $H(s)$ and plot the magnitude response for a bandpass Butterworth filter that satisfies $\alpha_p \leq 3$ dB, $\alpha_s \geq 17$ dB, $\omega_{p_1} \leq 100$ rad/s, $\omega_{p_2} \geq 250$ rad/s, $\omega_{s_1} \geq 40$ rad/s, and $\omega_{s_2} \leq 500$ rad/s.

Copying example 2.12 from book:

Editor - C:\Users\thomas.smallarz\Documents\MATLAB\HW3\C2_7_6.m

cost.m | C2_7_6.m

```
omegap1 = 100; omegap2 = 250; omegas1 = 40; omegas2 = 500; omegap = 1;
omegas = abs([omegap*(omegas1^2-omegap1*omegap2)/(omegas1*(omegap2-omegap1)),...
        omegap*(omegas2^2-omegap1*omegap2)/(omegas2*(omegap2-omegap1))])

omegas = min(omegas); alphap = 3; alphas = 17;
K = ceil(log((10^(alphas/10)-1)/(10^(alphap/10)-1))/(2*log(omegas/omegap)))

omegac = [omegap/(10^(alphap/10)-1).^(1/(2*K)),omegas/(10^(alphas/10)-1).^(1/(2*K))]

omegac = mean(omegac); k = 1:K;
pk = (1j*omegac*exp(1j*pi/(2*K)*(2*k-1))); A = poly(pk)

H_lp = @(s) A(1,3) ./ ( A(1,1).*s.^2 + A(1,2).*s + A(1,3));

w = 0:0.1:650; s = j.*w;
s2 = omegap .* ( (s.^2 + omegap1*omegap2) ./ (s.*(omegap2 - omegap1)));

plot(w,abs(H_lp(s2)),'k'); xlabel("Frequency (rad/sec)"); ylabel("Gain");
title("Magnitude Response of Band-pass filter");
rectangle('Position',[100 0 150 0.7071],'FaceColor','red');
rectangle('Position',[500 0.14125 150 (1-0.14125)],'FaceColor','red');
rectangle('Position',[0 0.14125 40 (1-0.14125)],'FaceColor','red');
```

Command Window

```
>> C2_7_6

omegas =

    3.9000    3.0000


K =

     2


omegac =

    1.0012    1.1332


A =

   1.0000 + 0.0000i   1.5092 - 0.0000i   1.1389 - 0.0000i
```

Therefore, our prototype LP Transfer Function is:

$$H_{lp}(s) = \frac{1.1389}{s^2 + 1.5092s + 1.1389}$$

For lowpass to band pass transformation:

$$s_2 = \omega_p \frac{s^2 + \omega_{p1}\omega_{p2}}{s(\omega_{p2} - \omega_{p1})} = \frac{s^2 + 25{,}000}{150s}$$

Then,

$$H_{bp}(s) = \frac{1.1389}{\left(\frac{s^2 + 25{,}000}{150s}\right)^2 + 1.5092\left(\frac{s^2 + 25{,}000}{150s}\right) + 1.1389}$$

Then, plotted in MATLAB with constraints: