Thomas Smallarz ECE444 Homework 3

3.1-2(c,d,e)

3.1-2 Figure P3.1-2 shows Fourier spectra of real signals $x_1(t)$ and $x_2(t)$. Determine the Nyquist sampling rates for signals

(a) $x_1(t)$ (b) $x_2(t/2)$
(c) $x_1^2(3t)$ (d) $x_2^3(t)$
(e) $x_1(t)x_2(t)$ (f) $1 - x_1(t)$

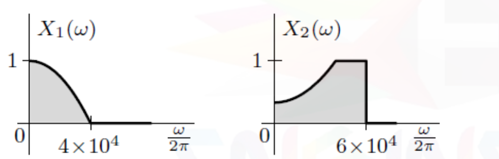

Figure P3.1-2

c) $x_1(3t)x_1(3t)$

Scaling and Reversal:

$$x(at) \Longleftrightarrow \frac{1}{|a|}X\left(\frac{\omega}{a}\right)$$

$$x(t)y(t) \Longleftrightarrow \frac{1}{2\pi}X(\omega) * Y(\omega)$$

$$x(3t) \rightarrow \frac{X\left(\frac{\omega}{3}\right)}{3}$$

This will cause max frequency in Fourier spectra to shrink from 40,000Hz to 13,333Hz

$$x(3t)x(3t) = X(\omega) * X(\omega)$$

Multiplication in time domain leads to convolution in frequency domain. According to width property:

$$\text{Duration } (f_1 * f_2) = \text{Duration } (f_1) + \text{Duration } (f_2)$$

So, Max Frequency in Fourier spectra will be 13,333Hz + 13,333Hz = 26,666Hz

Nyquist’s says that:

$$f_s \geq 2f$$

So,

$$f_s \geq 26{,}666 * 2 = 53{,}333Hz$$

d)

As in problem c, this is equivalent to:

$x_2^3(t) = x_2(t)x_2(t)x_2(t)$

$$x(t)y(t) \Longleftrightarrow \frac{1}{2\pi}X(\omega) * Y(\omega)$$

Multiplication in time-domain leads to convolution in the frequency domain.

According to width property:

$$\text{Duration }(f_1 * f_2) = \text{Duration }(f_1) + \text{Duration }(f_2)$$

So, new Fourier spectra max = (60,000Hz + 60,000Hz) + 60,000Hz = 180,000Hz

Nyquist's says that:

$$f_s \geq 2f$$

So,

$$f_s \geq 180{,}000 * 2 = 360{,}000Hz$$

e)

$$x(t)y(t) \Longleftrightarrow \frac{1}{2\pi}X(\omega) * Y(\omega)$$

Multiplication in time-domain leads to convolution in the frequency domain.

According to width property:

$$\text{Duration }(f_1 * f_2) = \text{Duration }(f_1) + \text{Duration }(f_2)$$

So, new Fourier spectra max = 40,000Hz + 60,000Hz = 100,000Hz

Nyquist's says that:

$$f_s \geq 2f$$

So,

$$f_s \geq 100{,}000 * 2 = 200{,}000Hz$$

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3.1-3(a,c)

3.1-3 Determine the Nyquist sampling rate and the Nyquist sampling interval for

(a) $x_a(t) = \text{sinc}^2(100t)$

(b) $x_b(t) = 0.01\,\text{sinc}^2(100t)$

(c) $x_c(t) = \text{sinc}(100t) + 3\text{sinc}^2(60t)$

(d) $x_d(t) = \text{sinc}(50t)\,\text{sinc}(100t)$

a) We know the Transform Pair for this function is:

	$x(t)$	$X(\omega)$
10.	$\frac{B}{2\pi}\text{sinc}^2\left(\frac{Bt}{2\pi}\right)$	$\Lambda\left(\frac{\omega}{2B}\right)$

Where, $B = 200\pi$ and the Unit Triangle function is described as:

$$\Lambda(t) = \begin{cases} 1 - 2|t| & |t| \leq \frac{1}{2} \\ 0 & |t| > \frac{1}{2} \end{cases}$$

Then we can easily see that $x_a(t)$ will be zero when $\omega = 200\pi\ \frac{rad}{sec}$

Nyquist's says that: $f_s \geq 2f$

So, $f_s \geq 100 * 2 = 200Hz$

And, $T_s \leq 0.005s$

c) Let's say that: $x_{c,1}(t) = sinc(100t)\ and\ x_{c,2}(t) = 3sinc^2(60t)$

We know the Transform Pairs for these two added functions are:

	$x(t)$	$X(\omega)$
8.	$\frac{B}{\pi}\text{sinc}\left(\frac{Bt}{\pi}\right)$	$\Pi\left(\frac{\omega}{2B}\right)$
10.	$\frac{B}{2\pi}\text{sinc}^2\left(\frac{Bt}{2\pi}\right)$	$\Lambda\left(\frac{\omega}{2B}\right)$

Then we can see that: $Max\left(X_{c,1}(\omega)\right) = 100\pi\ \frac{rad}{sec}\ \ and\ Max\left(X_{c,2}(\omega)\right) = 120\pi\ \frac{rad}{sec}$

Nyquist's says that: $f_s \geq 2f$

So, $f_s \geq 60 * 2 = 120Hz$

And, $T_s \leq 0.0083s$