

$$X_a(\Omega) = \begin{cases} \cos(\Omega) & -\frac{\pi}{2} < \Omega \leq \frac{\pi}{2} \\ 0 & \text{o.w.} \end{cases}$$

$$X_b(\Omega) = \begin{cases} \left| \frac{3\Omega}{\pi} \right| & -\frac{\pi}{3} < \Omega \leq \frac{\pi}{3} \\ 0 & \text{o.w.} \end{cases}$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} x(\Omega) e^{j\Omega n} d\Omega$$

$$X_a[n] = \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\Omega) e^{j\Omega n} d\Omega$$

with integration by parts:

$$\int u dv = uv - \int v du$$

let:

$$u = \cos(\Omega) \quad dv = e^{j\Omega n} d\Omega$$

$$du = -\sin(\Omega) d\Omega \quad v = \frac{e^{j\Omega n}}{jn}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\Omega) e^{j\Omega n} d\Omega = \cos(\Omega) \frac{e^{j\Omega n}}{jn} \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{1}{jn} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(\Omega) e^{j\Omega n} d\Omega$$

I.B.P.

let:

$$u = \sin(\Omega) \quad dv = e^{j\Omega n} d\Omega$$

$$du = \cos(\Omega) d\Omega \quad v = \frac{e^{j\Omega n}}{jn}$$

$$\text{let } A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\Omega) e^{j\Omega n} d\Omega$$

$$A = \cos(\Omega) \frac{e^{j\Omega n}}{jn} + \frac{1}{jn} \left[ \sin(\Omega) \frac{e^{j\Omega n}}{jn} - \frac{1}{jn} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\Omega) e^{j\Omega n} d\Omega \right]$$

$$A = \cos(\Omega) \frac{e^{j\Omega n}}{jn} - \frac{\sin(\Omega) e^{j\Omega n}}{n^2} + \frac{1}{n^2} A$$

$$A \left( 1 - \frac{1}{n^2} \right) = -j \cos(\Omega) \frac{e^{j\Omega n}}{n} - \frac{\sin(\Omega) e^{j\Omega n}}{n^2}$$

$$A = \frac{-\sin(\Omega) e^{j\Omega n}}{n^2 - 1} - j \frac{n \cos(\Omega) e^{j\Omega n}}{n^2 - 1} = \frac{-\sin(\Omega) e^{j\Omega n} - n \sin(\Omega) e^{j\Omega n}}{n^2 - 1}$$