2.3-3 Consider the simple RC circuit shown in Fig. P2.3-3. Let $R = 1\ \text{k}\Omega$ and $C = 1$ nF.

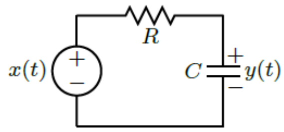


Figure P2.3-3

(a) Find the system transfer function.

(b) Plot the magnitude and phase responses.

(c) Show that a lowpass signal $x(t)$ with bandwidth $W \ll 10^6$ will be transmitted practically without distortion. Determine the output.

(d) Determine the approximate output if a bandpass signal $x(t) = g(t)\cos(\omega_c t)$ is passed through this filter. Assume that $\omega_c = 3 \times 10^6$ and that the envelope $g(t)$ has a very narrow band, on the order of 50 Hz.

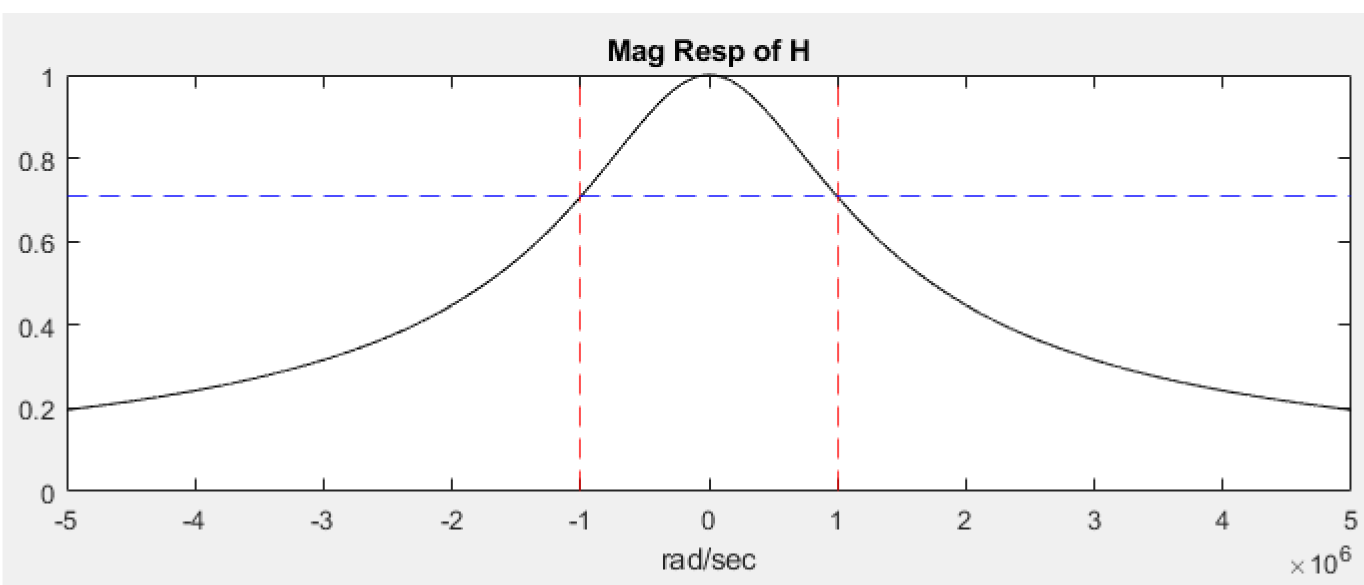
a) Use voltage division

$R \rightarrow R \qquad C \rightarrow \frac{1}{Cs}$

Then, $Y(s) = X(s)\left(\frac{1/Cs}{R + 1/Cs}\right)$

Or, $H(s) = (1 + CRs)^{-1} = (1 + 0.000001s)^{-1}$

b) Using MATLAB, where cutoff frequency is shown with dashed lines



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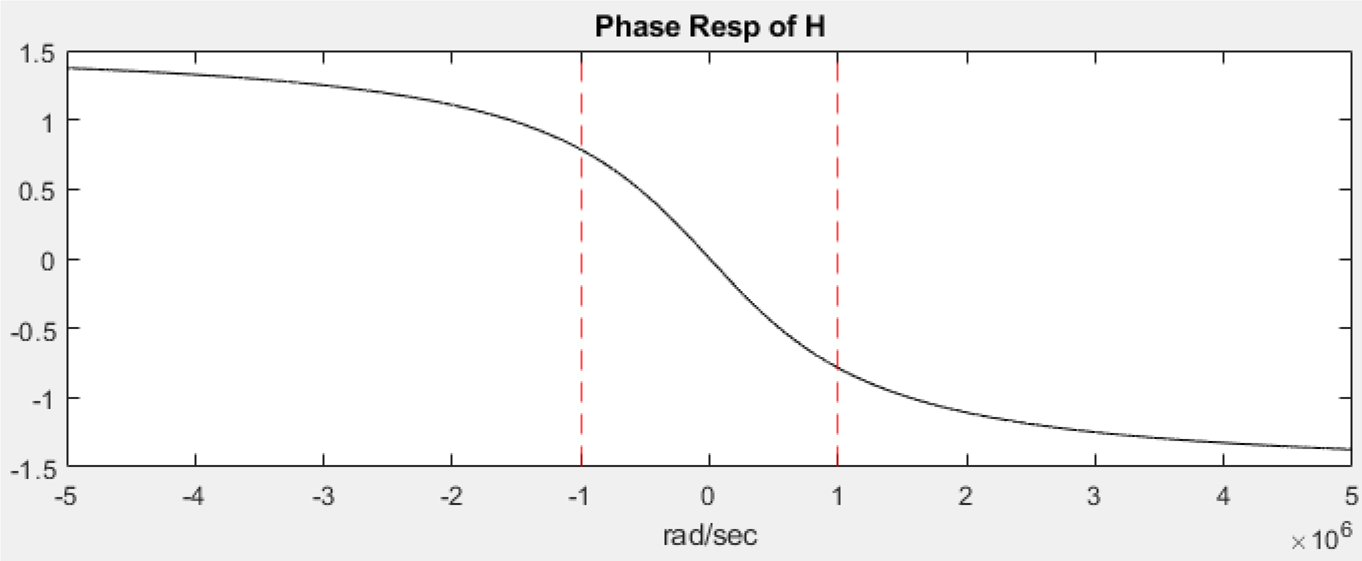
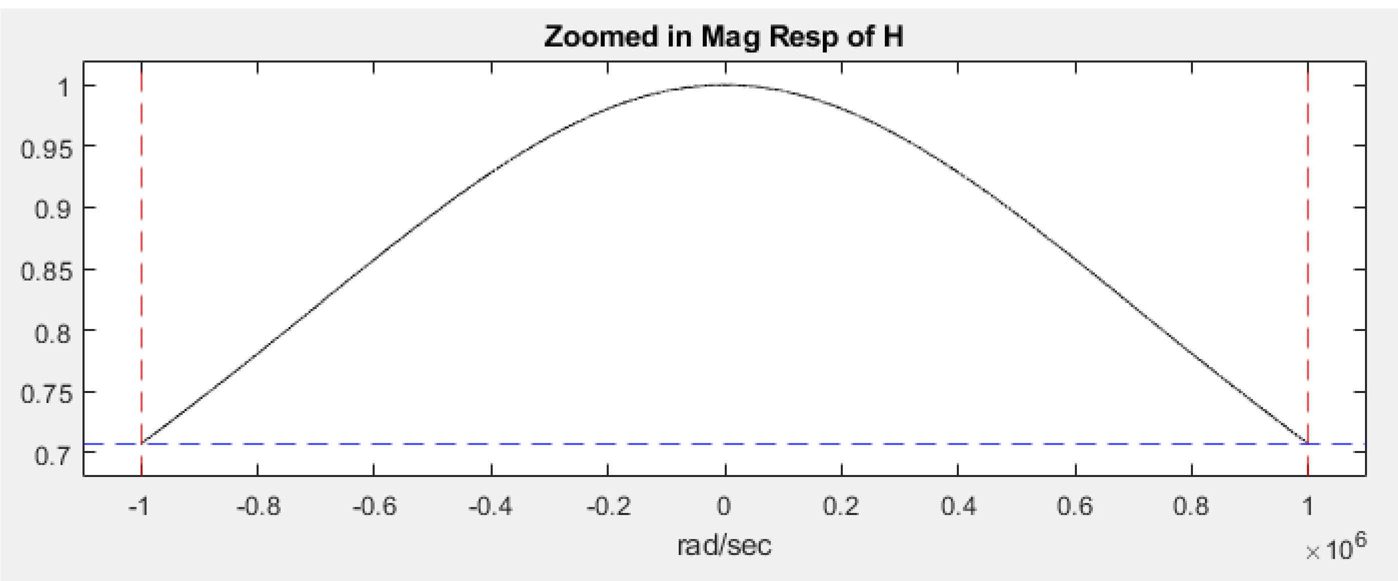


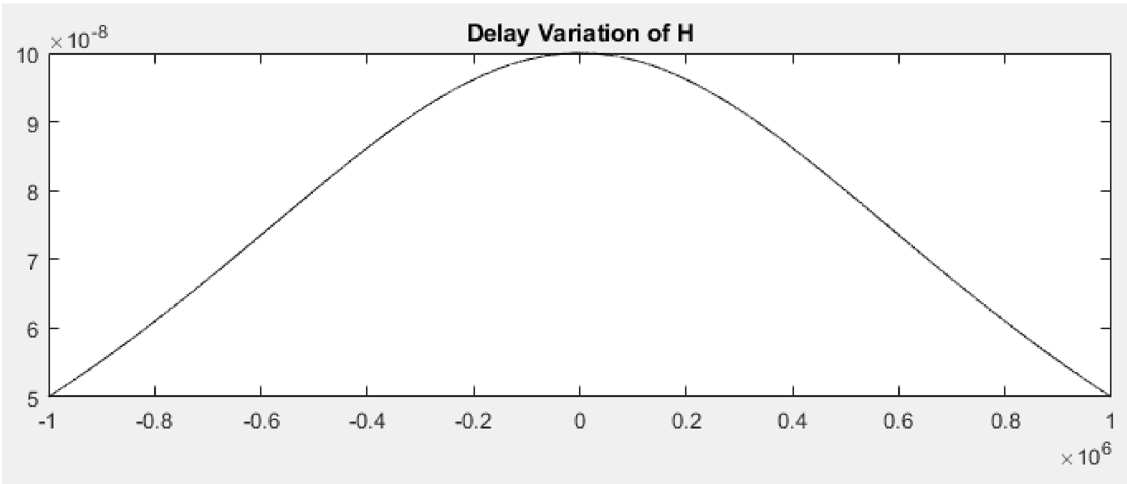
c) To transmit a signal without distortion two things are necessary

1. constant scale factor (magnitude response = constant)
2. constant delay variation (slope of phase response = constant)

Below is the Magnitude response. The scale factor changes from a minimum of 0.707 to a maximum of 1. This isn't ideal but it is somewhat constant-ish over the pass region.



Below is the delay variation (slope of the phase delay). Over the pass band it varies from $10 \times 10^{-8} \frac{rad}{rad/sec}$ $5 \times 10^{-8} \frac{rad}{rad/sec}$



If signal $x(t)$ is inputted to this system, you could expect an output of $y(t) = Ax(t - t_d)$

Where:

$A = 0.7071 \to 1$

$t_d = -0.7854\, rad \to 0.7854\, rad$

d) From page 98 in book, output should have form

$$y_{\mathrm{bp}}(t) = |a| x(t - t_{\mathrm{g}}) \cos\left[\omega_{\mathrm{c}}(t - t_{\mathrm{g}}) + \phi_0\right]$$

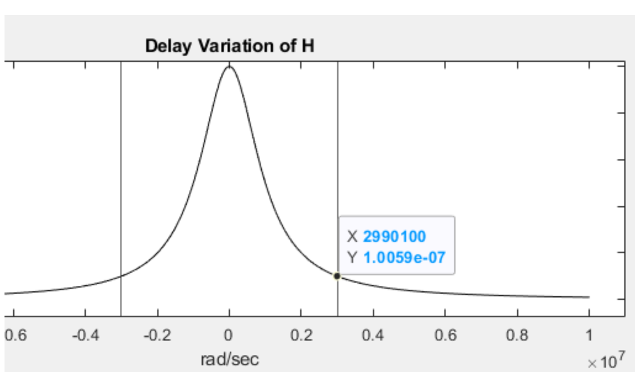
$$|a| = |H(j\omega_c)| = 0.3162$$

```
>> abs(H(j*3*10^6))

ans =

    0.3162
```

$$t_g = d\big(\angle H(j\omega_c)\big) = 1 * 10^{-7}$$



Using point slope we can find ϕ_0

$$y - y_1 = m(x - x_1)$$

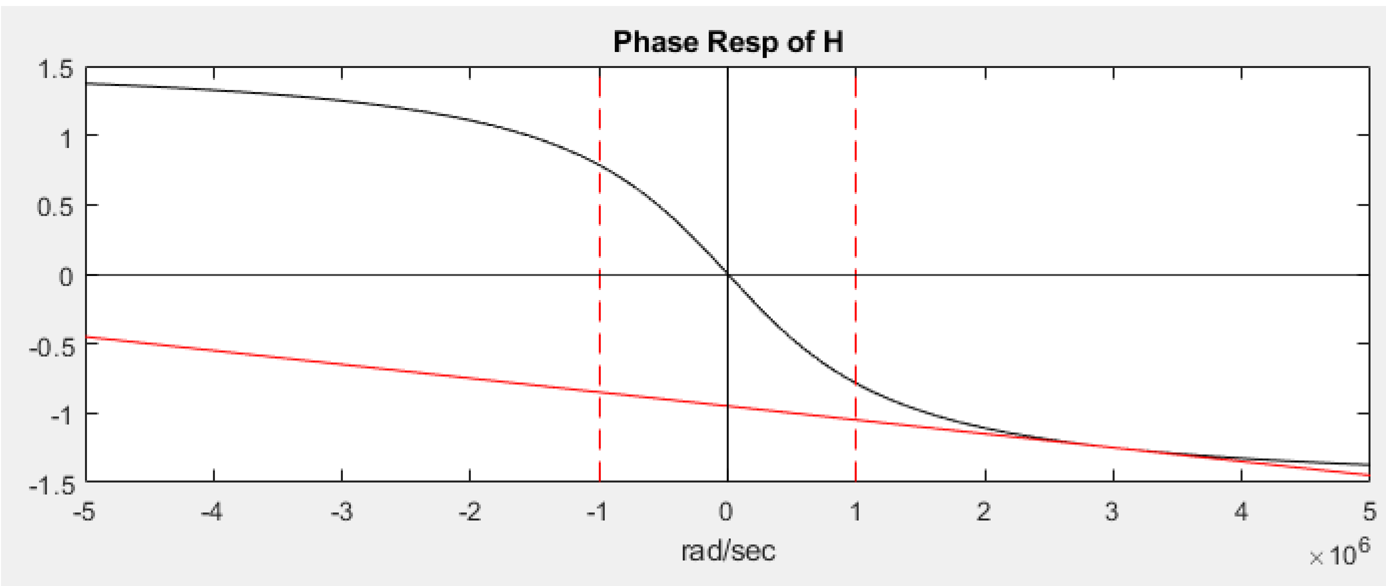
$$y - (-1.249) = 0.0000001(x - 3{,}000{,}000)$$

```
y = @(x) -0.0000001.*(x-3000000) - 1.249;
```

```
>> y(0)

ans =

   -0.9490
```



Therefore, $y_{bp}(t) = 0.3162x(t - 1\times 10^{-7})\cos[3\times 10^{6}(t - 1\times 10^{-7}) - 0.949]$

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C2_3_3.m

```matlab

% Essentials of Digital Signal Proscessing
% Problem 2.3-3
step = 100;
w = -1e7/2:step:1e7/2; s = j.*w;
H = @(s) (s.*1e-6 + 1).^(-1);

subplot(3,2,1); plot(w,abs(H(s)),'k'); title("Mag Resp of H"); xlabel("rad/sec");
xline(1e6,'--r'); xline(-1e6,'--r'); yline(1/sqrt(2),'--b');
subplot(3,2,2); plot(w,angle(H(s)),'k'); title("Phase Resp of H"); xlabel("rad/sec");
xline(1e6,'--r'); xline(-1e6,'--r'); hold on; plot(w,y(w),'r'); xline(0,'k'); yline(0,'k');

w2 = -1e6:step:1e6; s2 = j.*w2;

subplot(3,2,3); plot(w2,abs(H(s2)),'k'); title("Zoomed in Mag Resp of H"); xlabel("rad/sec");
xline(1e6,'--r'); xline(-1e6,'--r'); yline(1/sqrt(2),'--b'); axis([(-1e6-100000) (1e6+100000) 0.680 1.02]);


subplot(3,2,4); plot(w2,angle(H(s2)),'k'); title("Zoomed in Phase Resp of H"); xlabel("rad/sec");
xline(1e6,'--r'); xline(-1e6,'--r'); axis([(-1e6-100000) (1e6+100000) -1 1]);


tg = -diff(angle(H(s2)))./step;
w2_new = w2; w2_new(end) = [];
subplot(3,2,5); plot(w2_new,tg,'k'); title("Delay Variation of H"); xline(1e6); xline(-1e6); xlabel("rad/sec");

w3 = -1e7:step:1e7; s3 = j.*w3;
tg = -diff(angle(H(s3)))./step;
w3_new = w3; w3_new(end)=[];
subplot(3,2,6); plot(w3_new,tg,'k'); title("Delay Variation of H"); xline(3e6); xline(-3e6); xlabel("rad/sec");

y = @(x) -0.0000001.*(x-3000000) - 1.249;

```