

$$X_b[n] = \frac{1}{2\pi} \left[\int_{-\pi/3}^0 \frac{-3\Omega}{\pi} e^{j\Omega n} d\Omega + \int_0^{\pi/3} \frac{3\Omega}{\pi} e^{j\Omega n} d\Omega \right]$$

$$= \frac{3}{2\pi^2} \left[- \int_{-\pi/3}^0 \Omega e^{j\Omega n} d\Omega + \int_0^{\pi/3} \Omega e^{j\Omega n} d\Omega \right]$$

using integration by parts
let;

$$u = \Omega \quad dv = e^{j\Omega n} d\Omega$$

$$du = d\Omega \quad v = \frac{e^{j\Omega n}}{jn}$$

Then,

$$\int \Omega e^{j\Omega n} d\Omega = \frac{-j\Omega e^{j\Omega n}}{n} + \frac{j}{n} \int e^{j\Omega n} d\Omega$$

$$= \frac{-j\Omega e^{j\Omega n}}{n} + \frac{j e^{j\Omega n}}{jn^2} = \frac{e^{j\Omega n}}{n^2} - \frac{j\Omega e^{j\Omega n}}{n}$$

$$X_b[n] = \frac{3}{2\pi^2} \left[\underbrace{\left[\frac{e^{j\Omega n}}{n^2} - \frac{j\Omega e^{j\Omega n}}{n} \right]_0^{\pi/3}}_A + \underbrace{\left[\frac{e^{j\Omega n}}{n^2} - \frac{j\Omega e^{j\Omega n}}{n} \right]_{-\pi/3}^0}_B \right]$$

$$A = \left(\frac{1}{n^2} \right) - \left(\frac{e^{j\frac{\pi}{3}n}}{n^2} + \frac{j\frac{\pi}{3} e^{j\frac{\pi}{3}n}}{n} \right) = \frac{1}{n^2} - \frac{e^{j\frac{\pi}{3}n}}{n^2} + \frac{jn\frac{\pi}{3} e^{j\frac{\pi}{3}n}}{n^2}$$

$$A = \frac{1 - e^{j\frac{\pi}{3}n} + jn\frac{\pi}{3} e^{j\frac{\pi}{3}n}}{n^2}$$

$$B = \left(\frac{e^{j\frac{\pi}{3}n}}{n^2} - \frac{j\frac{\pi}{3} e^{j\frac{\pi}{3}n}}{n} \right) - \left(\frac{1}{n^2} \right) = \frac{e^{j\frac{\pi}{3}n} - jn\frac{\pi}{3} e^{j\frac{\pi}{3}n} - 1}{n^2}$$

$$X_b[n] = \frac{3}{2\pi^2} [-A + B] = \frac{3}{2\pi^2} \left[\frac{e^{-j\frac{\pi}{3}n} + e^{j\frac{\pi}{3}n} - jn\frac{\pi}{3} e^{-j\frac{\pi}{3}n} - jn\frac{\pi}{3} e^{j\frac{\pi}{3}n} - 2}{n^2} \right]$$

remember Euler's that says $\cos x = \frac{e^{jx} + e^{-jx}}{2}$

$$X_b[n] = \frac{3}{2n^2\pi^2} \left[\frac{1}{2} \cos\left(\frac{\pi}{3}n\right) - jn\frac{1}{2} \cos\left(\frac{\pi}{3}n\right) - \frac{4}{2} \right]$$

$$X_b[n] = \frac{3}{4n^2\pi^2} \left[\cos\left(\frac{\pi}{3}n\right) - n \sin\left(\frac{\pi}{3}n\right) - 4 \right]$$