2.7-14 Repeat Prob. 2.7-1 for an inverse Chebyshev filter.

2.7-1 Determine the transfer function $H(s)$ and plot the magnitude response $H(j\omega)$ for a third-order lowpass Butterworth filter if the 3-dB cutoff frequency is $\omega_c = 100$.

We know:

$$\omega_c = 100\ \frac{rad}{sec}$$

$$\alpha_s = 20\ dB$$

Using eq 2.53 in book:

$$\epsilon^2 = \frac{1}{10^{\alpha_s/10} - 1}. \qquad (2.53)$$

$$\epsilon = 0.1005$$

Then, using eq 2.42 and eq 2.51 in book:

$$C_K(x) = \cos\left[K \cos^{-1}(x)\right] \quad \text{or} \quad C_K(x) = \cosh\left[K \cosh^{-1}(x)\right]. \qquad (2.42)$$

$$|H(j\omega)| = \sqrt{1 - |H_c(-j\omega_p\omega_s/\omega)|^2} = \sqrt{\frac{\epsilon^2 {C_K}^2(\omega_s/\omega)}{1 + \epsilon^2 {C_K}^2(\omega_s/\omega)}}, \qquad (2.51)$$

We know that $|H(j\omega_c)| = \frac{1}{\sqrt{2}}\ so$

$$\frac{1}{\sqrt{2}} = \sqrt{\frac{\epsilon^2 C_k^2\left(\frac{\omega_s}{\omega_c}\right)}{1+\varepsilon^2 C_k^2\left(\frac{\omega_s}{\omega_c}\right)}}$$

$$\frac{1}{2} = \frac{\varepsilon^2 C_k^2\left(\frac{\omega_s}{\omega_c}\right)}{1+\varepsilon^2 C_k^2\left(\frac{\omega_s}{\omega_c}\right)}$$

$$\frac{1}{2} = \frac{1}{2}\varepsilon^2 C_k^2\left(\frac{\omega_s}{\omega_c}\right)$$

$$1 = \varepsilon^2 C_k^2\left(\frac{\omega_s}{\omega_c}\right)$$

$$\frac{1}{\varepsilon^2} = C_k^2\left(\frac{\omega_s}{\omega_c}\right)$$

$$\frac{1}{\varepsilon} = C_k\left(\frac{\omega_s}{\omega_c}\right) = \cos\left(3\cos^{-1}\left(\frac{\omega_s}{\omega_c}\right)\right)$$

$$\cos^{-1}\left(\frac{1}{\varepsilon}\right) = 3\cos^{-1}\left(\frac{\omega_s}{\omega_c}\right)$$

$$\cos\left(\frac{\cos^{-1}\left(\frac{1}{\varepsilon}\right)}{3}\right) = \frac{\omega_s}{\omega_c}$$

$$\omega_s = \omega_c \cos\left(\frac{\cos^{-1}\left(\frac{1}{\varepsilon}\right)}{3}\right) = 100\cos\left(\frac{\cos^{-1}\left(\frac{1}{\varepsilon}\right)}{3}\right)$$

Then our stopband frequency is

```
>> e = sqrt(inv(99))

e =

    0.1005

>> ws = wc*cos( acos(1/e) / 3)

ws =

  153.8459
```

Now, using eq 2.47, 2.55, and 2.56 we can figure out the poles/zeros. Eq 2.55 can be set equal to 2.47 to remove the passband frequency variable.

$$p_k = \frac{\omega_p \omega_s}{p'_k}, \tag{2.55}$$

$$p_k = -\omega_p \sinh\left[\frac{1}{K}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right]\sin\left[\frac{\pi(2k-1)}{2K}\right] + j\omega_p \cosh\left[\frac{1}{K}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right]\cos\left[\frac{\pi(2k-1)}{2K}\right] \quad k = 1, 2, \ldots, K. \tag{2.47}$$

$$z_k = j\omega_s \sec\left(\frac{\pi(2k-1)}{2K}\right) \quad k = 1, 2, \ldots, K. \tag{2.56}$$

$$P_{old} = \frac{\omega_p \omega_s}{P_{new}}$$

$$\frac{\omega_p \omega_s}{P_{new}} = -\omega_p \sinh(\ )\sin(\ ) + j\,\omega_p \cosh(\ )\cos(\ )$$

$$\frac{1}{P_{new}} = \frac{-1}{\omega_s}\sinh(\ )\sin(\ ) + j\frac{1}{\omega_s}\cosh(\ )\cos(\ )$$

$$P_{new} = \frac{1}{\frac{-1}{\omega_s}\sinh(\ )\sin(\ ) + j\frac{1}{\omega_s}\cosh(\ )\cos(\ )}$$

In MATLAB,

```matlab
K = 3; % Order of filter
wc = 100; % 3dB cutoff frequency (rad/sec)
sb_ripple = 20; % 20dB stop band ripple
E = sqrt(abs(inv(10^(sb_ripple/10) - 1))); % stop band ripple parameter
k = 1:K;

ws = wc*cos( acos(1/E) / K);

hyp_inside = asinh(1/E) / K;
reg_inside = (pi.*(2.*k - 1) ./ (2*K));

pk = 1./( (-1/ws).*sinh(hyp_inside).*sin(reg_inside) + (j/ws).*cosh(hyp_inside).*cos(reg_inside));

zk = j.*ws.*sec( (pi.*(2.*k-1)) ./ (2*K));
Gain = 1;
for m = 1:K
    Gain = Gain * (pk(1,m) / zk(1,m));
end

H = @(s) Gain .* ( (s - zk(1,1)).*(s - zk(1,2)).*(s - zk(1,3))) ...
    ./ ( (s - pk(1,1)) .* (s - pk(1,2)) .* (s - pk(1,3)));

w = 0:0.1:200;
s = j.*w;
subplot(2,1,1);
plot(w,abs(H(s)),'k'); yline(1/sqrt(2),'r'); xline(100,'r');
xlabel("frequency (rad/s)"); title("magnitude response of LP Inv. Chebyshev");

subplot(2,1,2);
plot(w,angle(H(s)),'k');
xlabel("frequency (rad/s)"); title("phase response of LP Inv. Chebyshev");
```