6.1-8 $X_a(\Omega) = \begin{cases} \cos(\Omega) & \frac{-\pi}{2} < \Omega \le \frac{\pi}{2} \\ 0 & o.w. \end{cases}$

$$x_b(\Omega) = \begin{cases} \left|\frac{3\Omega}{\pi}\right| & \frac{-\pi}{3} < \Omega \le \frac{\pi}{3} \\ 0 & o.w. \end{cases}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} x(\Omega)\, e^{j\Omega n}\, d\Omega$$

$$x_a[n] = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\pi/2} \cos(\Omega)\, e^{j\Omega n}\, d\Omega$$

with integration by parts:

$$\int u\,dv = uv - \int v\,du$$

let:

$$u = \cos(\Omega) \qquad dv = e^{j\Omega n}\, d\Omega$$

$$du = -\sin(\Omega)\,d\Omega \qquad v = \frac{e^{j\Omega n}}{jn}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\Omega)\, e^{j\Omega n}\, d\Omega = \cos(\Omega)\frac{e^{j\Omega n}}{jn} + \frac{1}{jn}\underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(\Omega)\, e^{j\Omega n}\, d\Omega}_{I.B.P.}$$

let:

$$u = \sin(\Omega) \qquad dv = e^{j\Omega n}\, d\Omega$$

$$du = \cos(\Omega)\, d\Omega \qquad v = \frac{e^{j\Omega n}}{jn}$$

let $A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\Omega) e^{j\Omega n}\, d\Omega$

$$A = \cos(\Omega)\frac{e^{j\Omega n}}{jn} + \frac{1}{jn}\left[\sin(\Omega)\frac{e^{j\Omega n}}{jn} - \frac{1}{jn}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\Omega)e^{j\Omega n}\, d\Omega\right]$$

$$A = \cos(\Omega)\frac{e^{j\Omega n}}{jn} - \frac{\sin(\Omega)\, e^{j\Omega n}}{n^2} + \frac{1}{n^2} A$$

$$A\left(1 - \frac{1}{n^2}\right) = -j\cos(\Omega)\frac{e^{j\Omega n}}{n} - \frac{\sin(\Omega) e^{j\Omega n}}{n^2}$$

$$A = \frac{-\sin(\Omega)\, e^{j\Omega n}}{n^2 - 1} - j\frac{n\cos(\Omega)\, e^{j\Omega n}}{n^2 - 1} = \frac{-\sin(\Omega)\, e^{j\Omega n} - n\sin(\Omega)\, e^{j\Omega n}}{n^2 - 1}$$

$$x_a[n] = \frac{-1}{2\pi}\left[\left[\frac{e^{j\frac{\pi}{2}n} + \frac{\pi}{2}e^{j\frac{\pi}{2}n}}{n^2-1}\right] - \left[\frac{-e^{-j\frac{\pi}{2}n} + \frac{\pi}{2}e^{-j\frac{\pi}{2}n}}{n^2-1}\right]\right]$$

$$x_a[n] = \frac{-1}{2\pi(n^2-1)}\left[e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n} + j\pi\left(\frac{e^{j\frac{\pi}{2}n} - e^{-j\frac{\pi}{2}n}}{j2}\right)\right]$$

$$= \frac{-1}{2\pi(n^2-1)}\left[\frac{\cos\left(\frac{\pi}{2}n\right)}{2} - \pi\cos\left(\frac{\pi}{2}n\right)\right]$$

$$\boxed{x_a[n] = \frac{-\cos\left(\frac{\pi}{2}n\right)\left(\frac{1}{2}-\pi\right)}{2\pi(n^2-1)}}$$

$$x_b[n] = \frac{1}{2\pi}\left[\int_{-\frac{\pi}{3}}^{0} \frac{-3\Omega}{\pi} e^{j\Omega n}\, d\Omega + \int_{0}^{\frac{\pi}{3}} \frac{3\Omega}{\pi} e^{j\Omega n}\, d\Omega\right]$$

$$= \frac{3}{2\pi^2}\left[-\int_{-\frac{\pi}{3}}^{0} \Omega e^{j\Omega n}\, d\Omega + \int_{0}^{\frac{\pi}{3}} \Omega e^{j\Omega n}\, d\Omega\right]$$

using integration by parts

let:

$u = \Omega \qquad dv = e^{j\Omega n}\, d\Omega$

$du = d\Omega \qquad v = \dfrac{e^{j\Omega n}}{jn}$

Then,

$$\int \Omega e^{j\Omega n}\, d\Omega = \left.\frac{-j\Omega e^{j\Omega n}}{n}\right| + \frac{j}{n}\int e^{j\Omega n}\, d\Omega$$

$$= \left.\frac{-j\Omega e^{j\Omega n}}{n}\right| + \left.\frac{\cancel{j} e^{j\Omega n}}{\cancel{j} n^2}\right| = \left.\frac{e^{j\Omega n}}{n^2} - \frac{j\Omega e^{j\Omega n}}{n}\right|$$

$$x_b[n] = \frac{3}{2\pi^2}\left[-\underbrace{\left[\frac{e^{j\Omega n}}{n^2} - \frac{j\Omega e^{j\Omega n}}{n}\right]_{-\frac{\pi}{3}}^{0}}_{A} + \underbrace{\left[\frac{e^{j\Omega n}}{n^2} - \frac{j\Omega e^{j\Omega n}}{n}\right]_{0}^{\frac{\pi}{3}}}_{B}\right]$$

$$A = \left(\frac{1}{n^2}\right) - \left(\frac{e^{-j\frac{\pi}{3}n}}{n^2} + \frac{j\frac{\pi}{3} e^{-j\frac{\pi}{3}n}}{n}\right) = \frac{1}{n^2} - \frac{e^{-j\frac{\pi}{3}n}}{n^2} + \frac{jn\frac{\pi}{3} e^{-j\frac{\pi}{3}n}}{n^2}$$

$$A = \frac{1 - e^{-j\frac{\pi}{3}n} + jn\frac{\pi}{3} e^{-j\frac{\pi}{3}n}}{n^2}$$

$$B = \left(\frac{e^{j\frac{\pi}{3}n}}{n^2} - \frac{j\frac{\pi}{3} e^{j\frac{\pi}{3}n}}{n}\right) - \left(\frac{1}{n^2}\right) = \frac{e^{j\frac{\pi}{3}n} - jn\frac{\pi}{3} e^{j\frac{\pi}{3}n} - 1}{n^2}$$

$$x_b[n] = \frac{3}{2\pi^2}\left[-A + B\right] = \frac{3}{2\pi^2}\left[\frac{e^{-j\frac{\pi}{3}n} + e^{j\frac{\pi}{3}n} - jn\frac{\pi}{3} e^{-j\frac{\pi}{3}n} - jn\frac{\pi}{3} e^{j\frac{\pi}{3}n} - 2}{n^2}\right]$$

remember Eulers that says $\cos x = \dfrac{e^{jx} + e^{-jx}}{2}$

$$x_b[n] = \frac{3}{2n^2\pi^2}\left[\frac{1}{2}\cos\left(\frac{\pi}{3}n\right) - jn\frac{1}{2}\cos\left(\frac{\pi}{3}n\right) - \frac{4}{2}\right]$$

$$\boxed{x_b[n] = \frac{3}{4n^2\pi^2}\left[\cos\left(\frac{\pi}{3}n\right) - n\sin\left(\frac{\pi}{3}n\right) - 4\right]}$$