**6.3-8** Repeat Prob. 6.3-7 for an analog integrator that is realized using the trapezoidal approximation (see Ex. 4.10)

$$y[n] - y[n-1] = \frac{T}{2}\left(x[n] + x[n-1]\right).$$

Find and sketch the magnitude and phase responses of this system. Further, find the responses of this system to the following input sinusoids:

**(a)** $x_{\mathrm{a}}[n] = \cos(0.1n)$

**(b)** $x_{\mathrm{b}}[n] = \sin(\pi n/6)$

**(c)** $x_{\mathrm{c}}(t) = \cos(10^6 \pi t)$ sampled at rate $F_{\mathrm{s}} = 2$ MHz

$$y[n] - y[n-1] = \frac{T}{2}\left[x[n] + x[n-1]\right]$$

$$\Big\downarrow \text{DTFT}$$

$$Y(\Omega) - Y(\Omega)e^{-j\Omega} = \frac{T}{2}\left[X(\Omega) + X(\Omega)e^{-j\Omega}\right]$$

$$Y(\Omega)\left(1 - e^{-j\Omega}\right) = \frac{T}{2}X(\Omega)\left(1 + e^{-j\Omega}\right)$$

$$\frac{Y(\Omega)}{X(\Omega)} = H(\Omega) = \frac{T}{2}\left(\frac{1 + e^{-j\Omega}}{1 - e^{j\Omega}}\right)$$

$$H(\Omega) = \frac{T}{2}\left(\frac{e^{-j\Omega}}{1 - e^{j\Omega}} + \frac{1}{1 - e^{j\Omega}}\right)$$

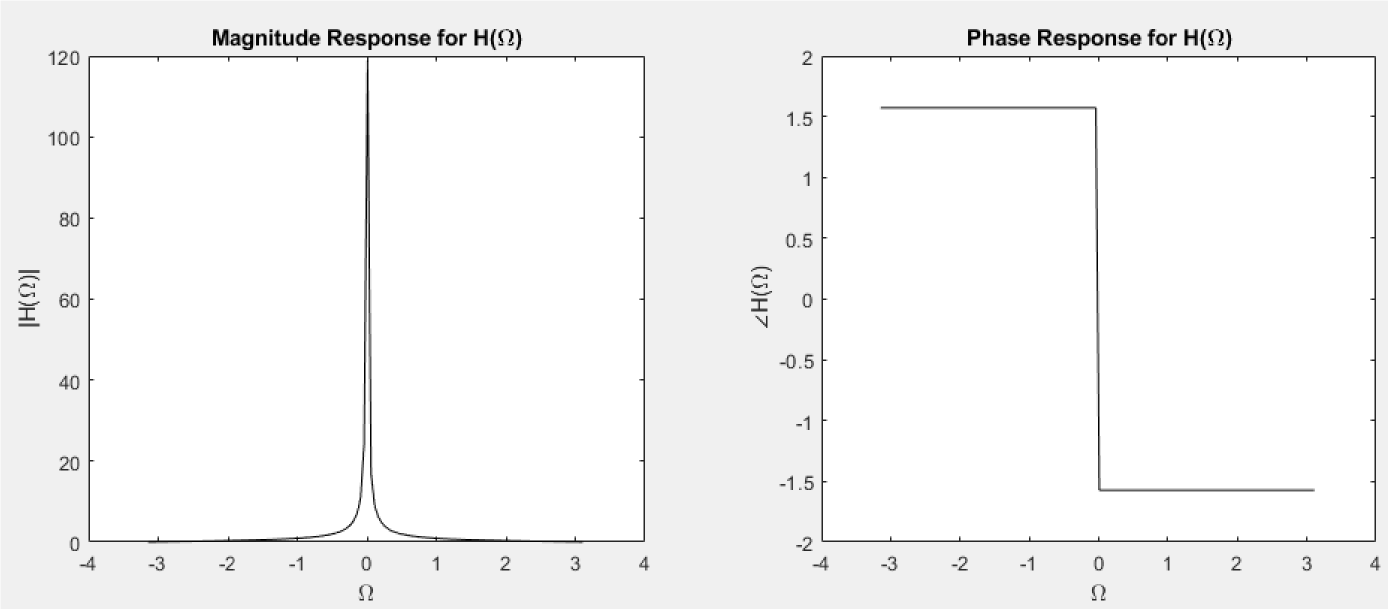
Then in MATLAB, with a period T = 1

C6_3_8.m

```matlab
T = 1;
H = @(W) (T/2).*( ( 1 + exp(-j.*W) ) ./ ( 1 - exp(-j.*W) ) ) ;%.* (W~=0);
W = -pi:0.05:pi;

subplot(321); plot(W,abs(H(W)),'k'); title("Magnitude Response for H(\Omega)");
xlabel("\Omega"); ylabel("|H(\Omega)|");

subplot(322); plot(W,angle(H(W)),'k'); title("Phase Response for H(\Omega)");
xlabel("\Omega"); ylabel("\angleH(\Omega)");
```



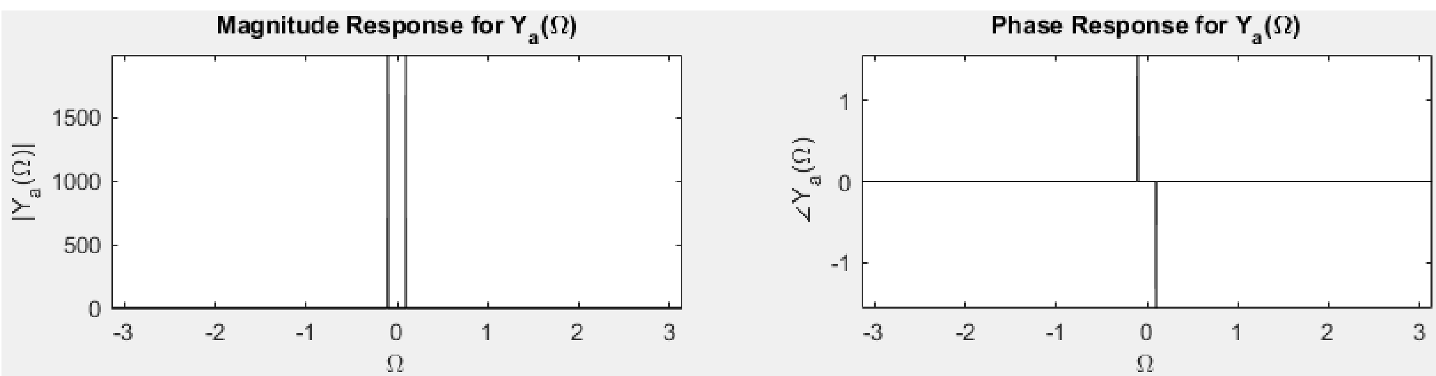

**a)** $x_a[n] = \cos(0.1n)$

$$T = \frac{2\pi}{\omega} = 20\pi$$

$x_a[n] = \cos(0.1n)$

$X_a(\Omega) = \pi\left[\delta(\Omega - 0.1) + \delta(\Omega + 0.1)\right]$

```
Ta = 20*pi;
Xa = @(W) pi.*(delta(W-0.1) + delta(W+0.1));
Ya = @(W) Xa(W) .* H(W,Ta);
Wa = -pi:0.00001:pi;
```



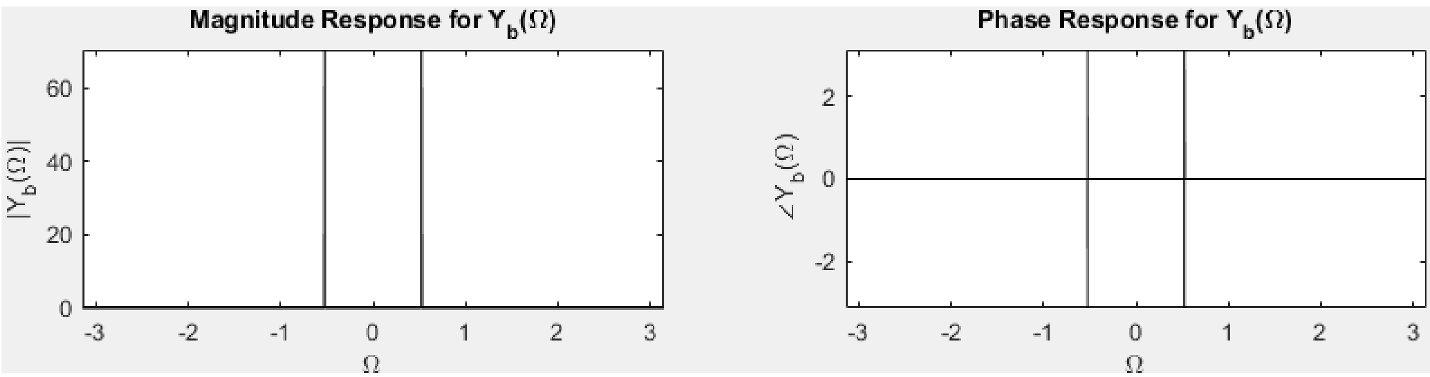
**b)** $x_b[n] = \sin\left(\frac{\pi n}{6}\right)$

$$T = \frac{2\pi}{\omega} = 12$$

$x_b[n] = \sin\left(\frac{\pi n}{6}\right)$

$X_b(\Omega) = \frac{\pi}{j}\left[\delta\left(\Omega - \frac{\pi}{6}\right) - \delta\left(\Omega + \frac{\pi}{6}\right)\right]$

```
Tb = 12;
Xb = @(W) (pi/j).*(delta(W-(pi/6)) - delta(W+(pi/6)));
Yb = @(W) Xb(W) .* H(W,Tb);
Wb = -pi:0.00001:pi;
```




**c)** $x_c(t) = \cos(10^6 \pi t)$ $\quad$ ***sampled at*** $F_s = 2MHz$

$x_c(t) = \cos(10^6 \pi t)$ w/ $F_s = 2MHz$

$T_s = 0.0000005$ s

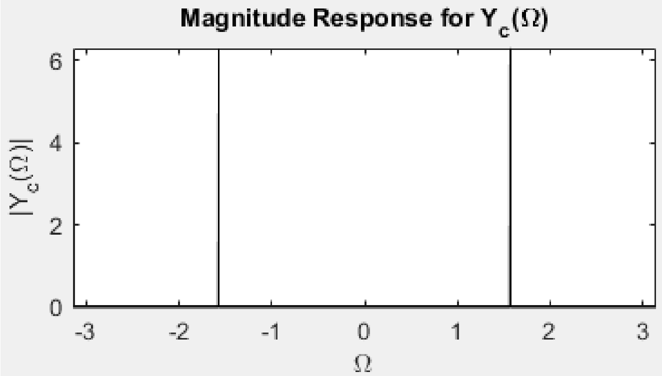
when sampled

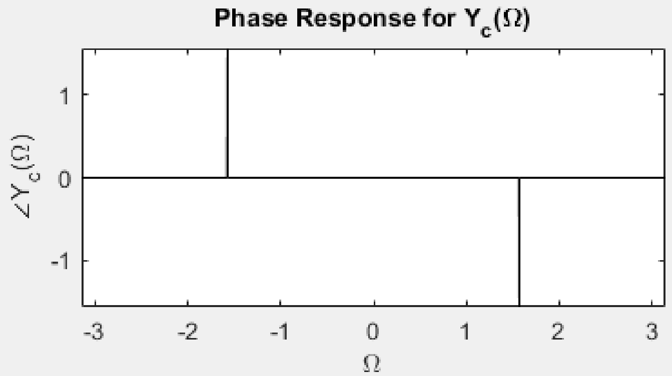
$x_c[n] = \cos\left(\cancel{10^6}\pi \frac{1}{2\cdot\cancel{10^6}} n\right) = \cos\left(\frac{\pi}{2} n\right)$

$X_c(\Omega) = \pi\left[\delta\left(\Omega - \frac{\pi}{2}\right) + \delta\left(\Omega + \frac{\pi}{2}\right)\right]$

$$T = \frac{2\pi}{\omega} = 4$$

```
Tc = 4;
Xc = @(W) pi.*(delta(W-(pi/2)) + delta(W+(pi/2)));
Yc = @(W) Xc(W) .* H(W,Tc);
Wc = -pi:0.00001:pi;
```




MATLAB

```
H = @(W,T) (T/2).*( ( 1 + exp(-j.*W) ) ./ ( 1 - exp(-j.*W) ) ) .* (W~=0);
W = -pi:0.025:pi;
T = 1;
subplot(421); plot(W,abs(H(W,T)),'k'); title("Magnitude Response for H(\Omega)");
xlabel("\Omega"); ylabel("|H(\Omega)|");

subplot(422); plot(W,angle(H(W,T)),'k'); title("Phase Response for H(\Omega)");
xlabel("\Omega"); ylabel("\angleH(\Omega)");

delta = @(w) (w<=0.001 & w>=-0.001);
Ta = 20*pi;
Xa = @(W) pi.*(delta(W-0.1) + delta(W+0.1));
Ya = @(W) Xa(W) .* H(W,Ta);
Wa = -pi:0.00001:pi;

Tb = 12;
Xb = @(W) (pi/j).*(delta(W-(pi/6)) - delta(W+(pi/6)));
Yb = @(W) Xb(W) .* H(W,Tb);
Wb = -pi:0.00001:pi;

Tc = 4;
Xc = @(W) pi.*(delta(W-(pi/2)) + delta(W+(pi/2)));
Yc = @(W) Xc(W) .* H(W,Tc);
Wc = -pi:0.00001:pi;

subplot(423); plot(Wa,abs(Ya(Wa)),'k'); title("Magnitude Response for Y_a(\Omega)");
xlabel("\Omega"); ylabel("|Y_a(\Omega)|"); axis tight;

subplot(424); plot(Wa,angle(Ya(Wa)),'k'); title("Phase Response for Y_a(\Omega)");
xlabel("\Omega"); ylabel("\angleY_a(\Omega)"); axis tight;

subplot(425); plot(Wb,abs(Yb(Wb)),'k'); title("Magnitude Response for Y_b(\Omega)");
xlabel("\Omega"); ylabel("|Y_b(\Omega)|"); axis tight;

subplot(426); plot(Wb,angle(Yb(Wb)),'k'); title("Phase Response for Y_b(\Omega)");
xlabel("\Omega"); ylabel("\angleY_b(\Omega)"); axis tight;

subplot(427); plot(Wc,abs(Yc(Wc)),'k'); title("Magnitude Response for Y_c(\Omega)");
xlabel("\Omega"); ylabel("|Y_c(\Omega)|"); axis tight;

subplot(428); plot(Wc,angle(Yc(Wc)),'k'); title("Phase Response for Y_c(\Omega)");
xlabel("\Omega"); ylabel("\angleY_c(\Omega)"); axis tight;
```