

**2.1-5** For each of the following transfer functions, determine and plot the poles and zeros of  $H(s)$ , and use the pole and zero information to predict overall system behavior. Confirm your predictions by graphing the system's frequency response (magnitude and phase).

(c)  $H(s) =$

$$\frac{0.03s^4 + 1.76s^2 + 15.22}{s^4 + 2.32s^3 + 9.79s^2 + 13.11s + 17.08}$$

Using MATLAB,

```
num = [0.03 0 1.76 0 15.22];  
den = [1 2.32 9.79 13.11 17.08];
```

```
H = tf(num,den);  
Hp = pole(H)  
Hz = zero(H)  
|  
subplot(3,1,1);  
pzplot(H); axis([-1 1 -8 8]);
```

Command Window

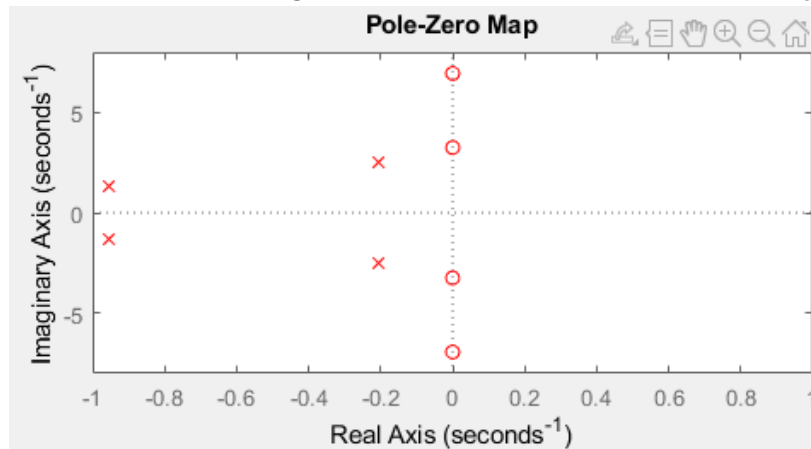
```
>> C2_1_5c
```

```
Hp =
```

```
-0.2063 + 2.4986i  
-0.2063 - 2.4986i  
-0.9537 + 1.3446i  
-0.9537 - 1.3446i
```

```
Hz =
```

```
0.0000 + 6.9372i  
0.0000 - 6.9372i  
0.0000 + 3.2469i  
0.0000 - 3.2469i
```



From looking at the P/Z Map we can see that

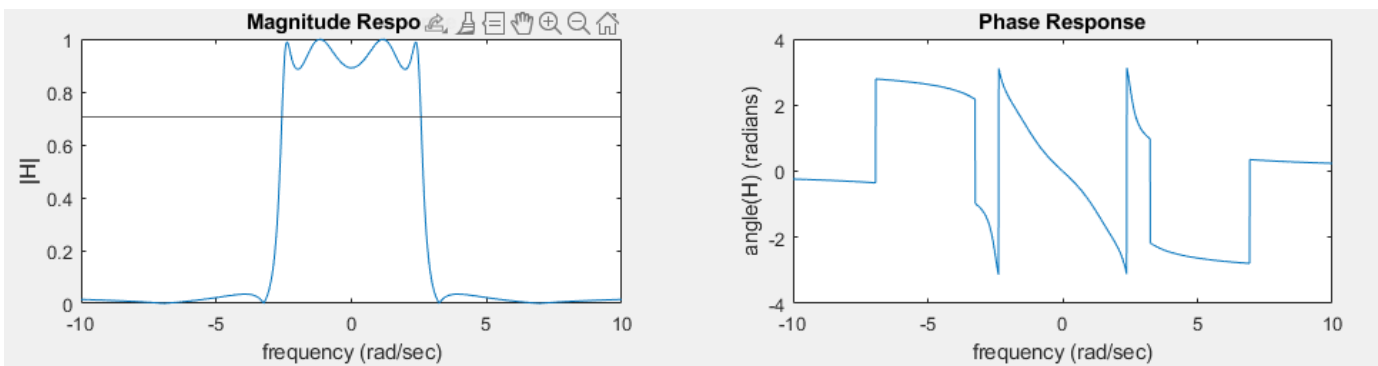
- As  $\omega$  goes to infinity  $|H(s)| = 0$
- There are some poles close to  $\omega = 0$  rad/sec (DC)

From this I would assume

- This transfer function has low pass-ish characteristics. Looking at the graph I would make a guess that it passes frequencies in the 0 – 2 rad/sec band
- Difficult to guess what type of phase response this T.F. just by looking at the P/Z Map

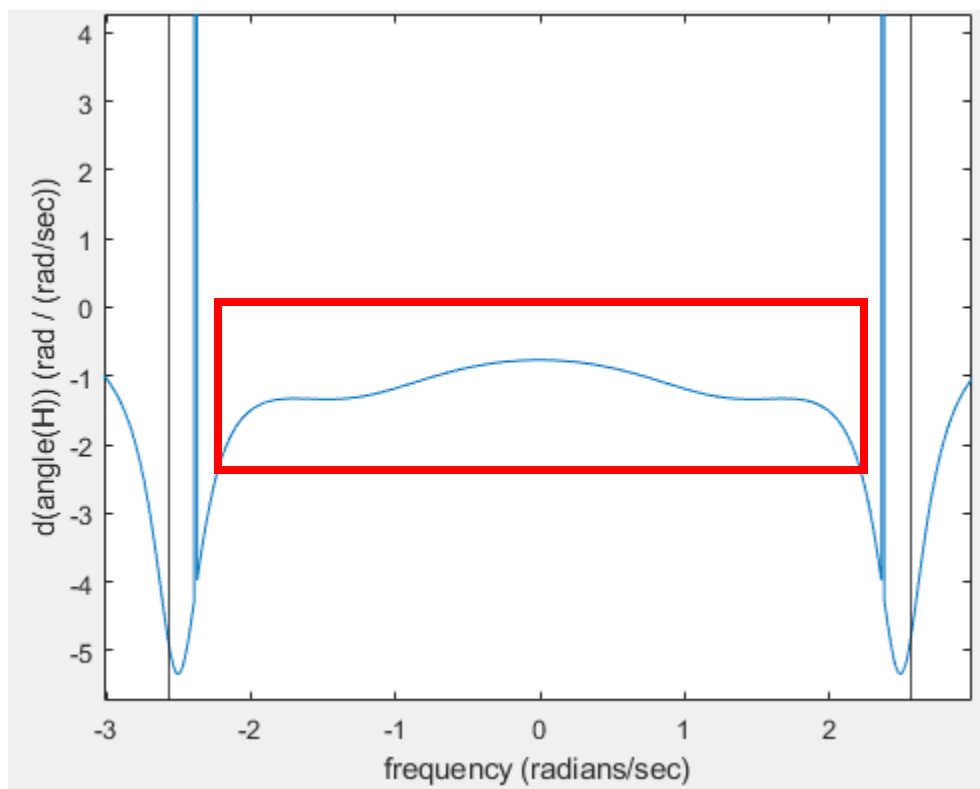
In MATLAB,

```
w = -10:0.1:10;
s = j*w;
H = (0.03.*s.^4 + 1.76.*s.^2 + 15.22) ./ (s.^4 + 2.32.*s.^3 + 9.79.*s.^2 + 13.11.*s + 17.08);
subplot(3,1,2);
plot(w,abs(H)); title("Magnitude Response"); xlabel("frequency (rad/sec)"); ylabel("|H|");
subplot(3,1,3);
plot(w,angle(H)); title("Phase Response"); xlabel("frequency (rad/sec)"); ylabel("angle(H) (degrees)");
```



From these plots we can see that:

- T.F. is a low-pass filter. From its shape it looks epileptic
- Has a  $\frac{1}{2}$  power point at  $\pm 2.57$  rad/sec. So, our guess of the pass band being from 0  $\rightarrow$  2 rad/sec was a somewhat good estimate
- It appears the phase response in the pass band ( $-2.57$  rad/sec  $\rightarrow$  2.57 rad/sec) is somewhat close to linear. Plotting the first derivative of the phase response (seen on next page), we see the slope of phase is sticks close to -1 rad / (rad/sec) . This can help keep distortion less transmission in the pass band



**2.2-1** Consider the LTIC system with transfer function  $H(s) = \frac{3s}{s^2+2s+2}$ . Using Eq. (2.15), plot the delay response of this system. Is the delay response constant for signals within the system's passband?

$$t_g(\omega) = -\frac{d}{d\omega} \angle H(\omega). \quad (2.15)$$

In MATLAB,

```

C2_1_5c.m  C2_2_1.m  +
1  step = 0.001;
2  w = -10:step:10; s = j.*w;
3  H = (3.*s) ./ (s.^2 + 2.*s + 2);
4
5  tg = (-diff(angle(H)))./step;
6
7  subplot(2,2,1); plot(w,abs(H)); xlabel("rad/sec"); title("Magnitude Resp. of H"); axis([-6 6
8  subplot(2,2,2); plot(w,angle(H)); xlabel("rad/sec"); title("Phase Resp. of H"); axis([-6 6 -p
9
10 w(end) = [];
11 subplot(2,2,[3 4]); plot(w,tg); xlabel("rad/sec"); title("Measure of Delay Variation of H");
12 axis([-6 6 -0.5 1.5]); xline(-2.7,'k'); xline(2.7,'k'); xline(-0.73,'k'); xline(0.73,'k');

```

In the bottom plot we can see that this systems delay varies from a max of  $1.2 \frac{\text{rad}}{\text{rad/sec}}$  to  $0.33 \frac{\text{rad}}{\text{rad/sec}}$  over the pass band  $\left(0.73 \frac{\text{rad}}{\text{sec}} \text{ to } 2.7 \frac{\text{rad}}{\text{sec}}\right)$ . Therefore, this system **does not** have a constant delay response in the passband.

