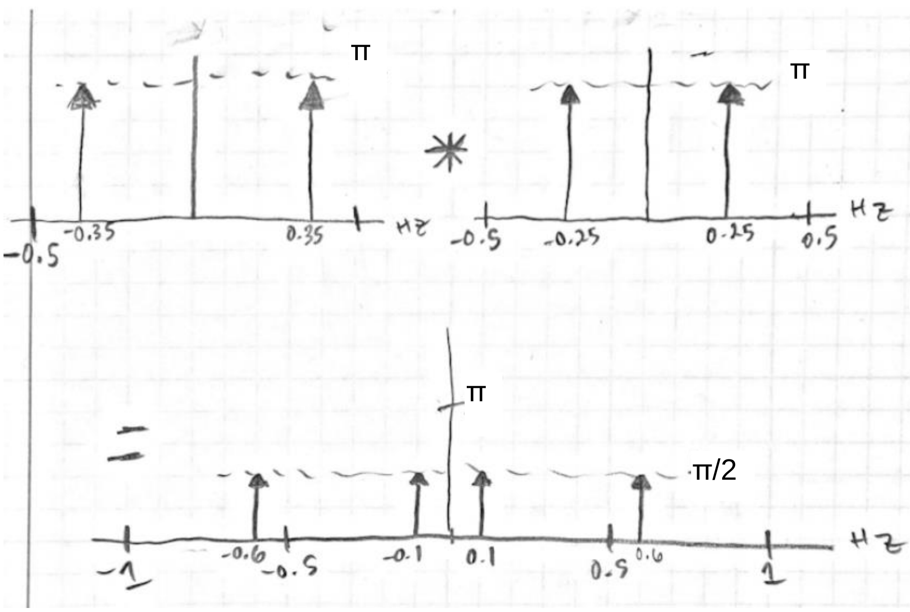
3.2-18 The signal $x(t) = \sin(0.7\pi t)\cos(0.5\pi t)$ is sampled using $F_s = 1$ Hz to yield a discrete-time signal $x[n]$. Next, $x[n]$ is filtered using an ideal high-pass digital filter that eliminates all frequencies below $\frac{3}{10}F_s$, the output of which is called $y[n]$. Finally, $y[n]$ is passed through a perfect reconstruction filter at the rate $F_s = \frac{1}{2}$ Hz. Find a simplified expression for $y(t)$, the output of the reconstructor. Can this system operate in "real time"?

Multiplying in time → Convolution in frequency

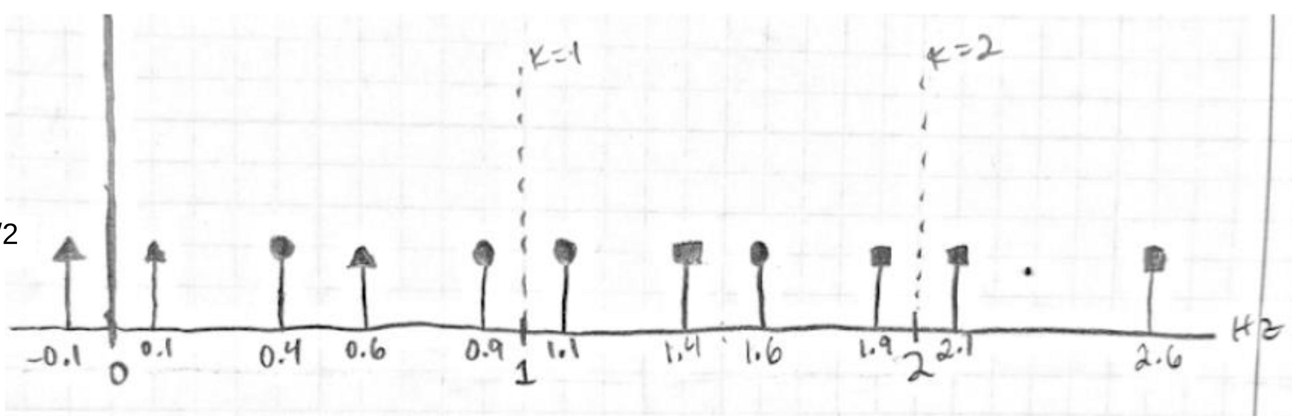
Sinusoids with $F_1 = 0.35Hz$ and $F_2 = 0.25Hz$ will have impulses in the frequency domain at these frequencies

Multiplying these two sinusoids will lead to a $X(\omega)$ of:



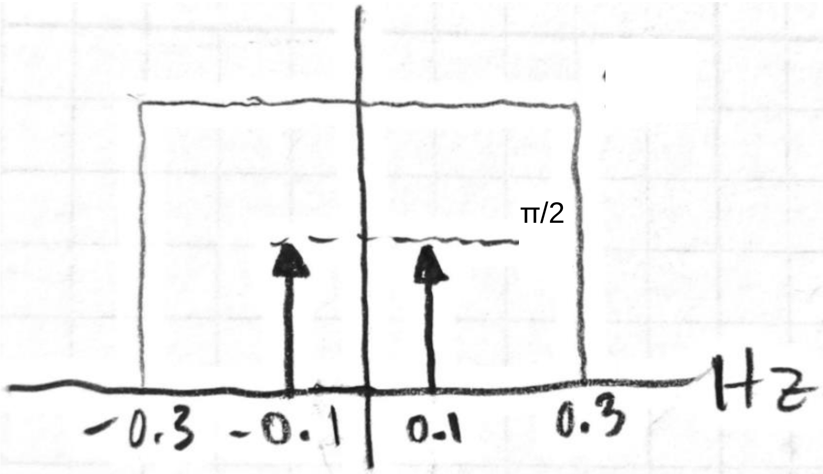
Sampling at $F_s = 1Hz$ will lead to periodic replications of $X(\omega)$ centered around integer multiples of 1Hz

π/2



We can see that the replications do overlap into the original BW of x(t), but they do not add onto the original spectra

If an ideal LPF with $f_0 = 0.3Hz$ is applied:



If this is passed through a perfect reconstruction filter with $F_s = \frac{1}{2}Hz$:

$$H_{reconstruction}(\omega) = T\Pi\left(\frac{\omega T}{2\pi}\right) = 2\Pi\left(\frac{\omega}{\pi}\right) = 2\Pi(2f)$$

Since $F_s = 0.5Hz > 2 * 0.1Hz$ this will reconstruct the signal to be:

$$y(t) = 0.5\cos(0.2\pi t)$$

This could have been achieved by passing our original signal, x(t), through an ideal LPF with pass band gain of 0.5 and a cutoff frequency of 0.3Hz