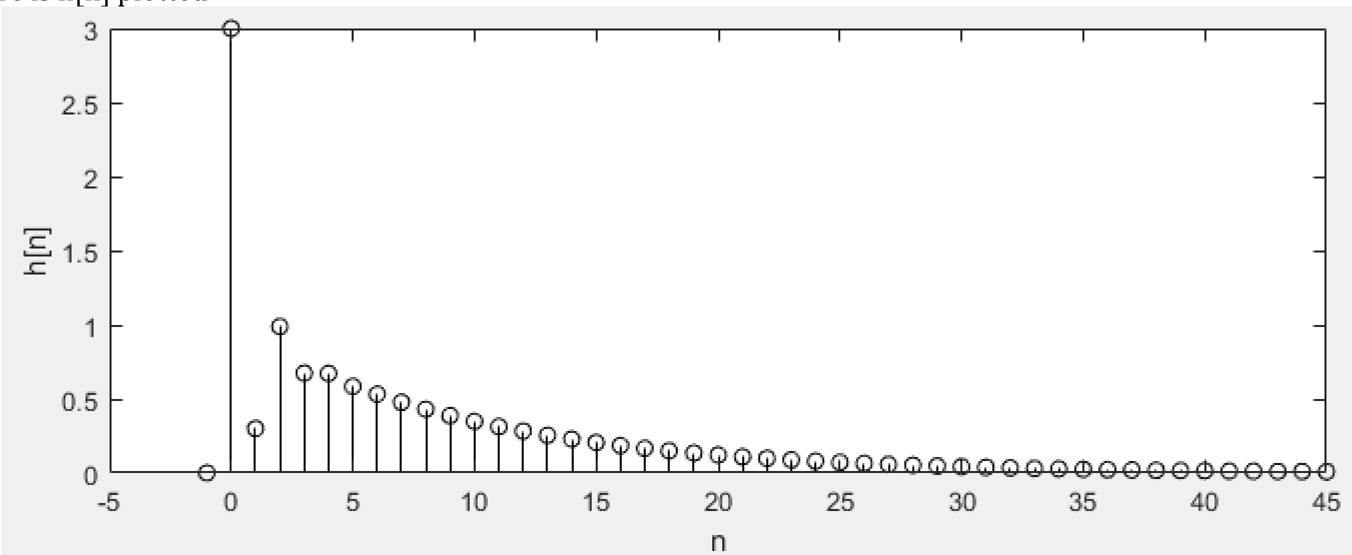
5.8-2 Consider a system with impulse response $h[n] = (0.9)^n u[n] + 2(-0.3)^n u[n]$. Determine this system's time constant, rise time, and pulse dispersion. Identify, if possible, an input $x[n]$ that will cause this system to resonate.

Time Constant)

Here is h[n] plotted



The n=0 value seems out of place… so I'm going to ignore it. I'm going to choose my W_h at where the amplitude is 1% of the peak height of h[n] (still disregarding 3 at n=0). This will be where h[n] = ~0.01, or at n = 44

Then, the width is: $W_h = 0 \rightarrow 44 = 45$

Rise Time)

According to eq. 5.39 the rise time is just equal to the time constant

$$W_r = W_h$$

$$W_r = 45$$

Pulse Dispersion)

According to the book, the pulse dispersion width is equal to the time constant as well

$$W_y = W_x + W_h. \tag{5.40}$$

This result shows that an input pulse spreads out (disperses) as it passes through a system. Since W_h is also the system's time constant or rise time, the amount of spread in the pulse is equal to the time constant (or rise time) of the system.

$$pulse\ dispersion = 45$$

An input x[n] that resonates with this system)

From pg. 316 in the book:

If $h[n] = c\gamma^n u[n] \quad and \quad x[n] = (\gamma - \epsilon)^n u[n]$

Then as $\epsilon \rightarrow 0 \quad y[n] = c(n+1)\gamma^n u[n]$

If we have an input $x[n]$ such that $x[n] = x_1[n] + x_2[n]$. Where,

$$x_1[n] = 0.9^n u[n] \quad and \quad x_2[n] = (-0.3)^n u[n]$$

Then, our $y[n] = y_1[n] + y_2[n] = h[n] * x_1[n] + [n] * x_2[n]$

$$y_1[n] = (n+1)(0.9)^n u[n] + 2\frac{(-0.3)^{n+1} - (0.9)^{n+1}}{-1.2} u[n]$$

$$y_2[n] = 2(n+1)(-0.3)^n u[n] + \frac{(-0.3)^{n+1} - (0.9)^{n+1}}{-1.2} u[n]$$

$$y[n] = (n+1)(0.9)^n u[n] + 2(n+1)(-0.3)^n u[n] + 3\frac{(-0.3)^{n+1} - (0.9)^{n+1}}{-1.2} u[n]$$

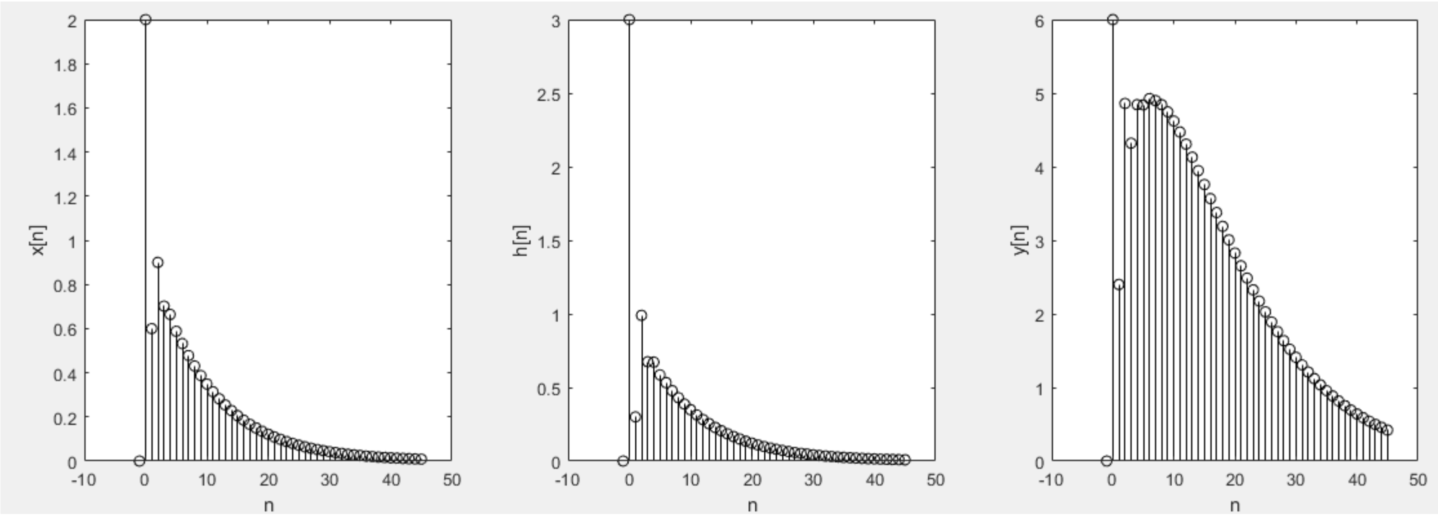
Or in MATLAB,

```
u = @(n) (n>=0);
h = @(n) (0.9).^n.*u(n) + 2.*(-0.3).^n.*u(n);
x = @(n) (0.9).^n.*u(n) + (-0.3).^n.*u(n);
y = @(n) (n+1).*(0.9).^n.*u(n) + 2.*(n+1).*(-0.3).^n.*u(n) ...
    + 3.* ( (-0.3).^(n+1) - (0.9).^(n+1) ) ./ (-1.2) .* u(n);
n = -1:45;

subplot(131);
stem(n,x(n),'k'); xlabel("n"); ylabel("x[n]");

subplot(132);
stem(n,h(n),'k'); xlabel("n"); ylabel("h[n]");

subplot(133);
stem(n,y(n),'k'); xlabel("n"); ylabel("y[n]");
```



It goes to zero, but in our y[n] we can see that there the (n+1) term will go to infinity as n goes to infinity. This output will decay before that can go to infinity since its roots are within the unit circle, but it still resonates with the system.