Communication Systems
(ECE4572)
Fall 2013

Homework 5
Assigned Oct. 17, due Oct. 23.

Objective: Review measures of information, Huffman and Lempel Ziv source compression.

1. A source A has alphabet $a_1, a_2, \ldots a_6$, with all letters equally likely.

- How much information, measured in bits, is contained in letter a_6? How much in a_1?
- What is the entropy of this source?
- If the source letters are to be encoded using binary (0/1) codewords of equal length, how many bits per codeword are needed at least?
- Is it possible to devise a binary coding scheme whose average codeword length is less than 3? Why?
- If you answered "yes" to the above question, can you specify such a coding scheme (i.e. list the codewords)?
- If the codewords are transmitted over a communication link that supports 3 Mbits/sec, how many letters are being transmitted on average using each of the two coding schemes (equal-length/unequal-length codewords)?

2. The entropy of a source A with alphabet $\{a_1, a_2, \ldots a_M\}$ is defined as $H_A = \sum_{i=1}^{M} P_i \log_2 \frac{1}{P_i}$, where $P_i = P\{A = a_i\}$. With the understanding that $\sum_{i=1}^{M} P_i = 1$, determine the probabilities $P_1, P_2, \ldots P_M$ for which the entropy H_A is maximal. Support your answer by a mathematical proof.

3. A binary source gives as its output a 0/1 sequence. This sequence is encoded using the Lempel-Ziv algorithm with a dictionary of size 8, and transmitted over a noiseless channel. The sequence observed at the receiver is 0000001101000010000101011011. What was the original sequence?

4. (Bonus.) Mutual information (average) between sources A and B with respective alphabets $\{a_1, a_2, \ldots a_I\}$ and $\{b_1, b_2, \ldots b_J\}$ is defined as

$$\bar{I}_{A;B} = \sum_i \sum_j P_{A,B}(a_i, b_j) \log_2 \frac{P_{A,B}(a_i, b_j)}{P_A(a_i) P_B(b_j)}$$

Prove the following:

- $\bar{I}_{A;B} = \bar{I}_{B;A}$
- $\bar{I}_{A;B} \geq 0$, where the equality holds if and only if A and B are independent.
  (You may find the following fact to be useful: $\ln \xi \leq \xi - 1$ with equality only at $\xi = 1$.)
- $\bar{I}_{A;B} = H_B - H_{B|A} = H_A - H_{A|B}$
- $H_{A,B} \leq H_A + H_B$, with equality if and only if A and B are independent.

Report: Typed reports will be appreciated.