Communication Systems
(ECE4572)
Fall 2013

Homework 3
Assigned Sept. 19, due Sept. 25.

Objective: To observe the operation of a phase locked loop (PLL).

Description:
The input signal to a PLL is given by

$$r(t) = A\cos(2\pi f_c t + \varphi(t))$$

where $A = 1$, f_c= 10 kHz, and $\varphi(t)$ is the phase that has to be detected. We will study a special case in which

$$\varphi(t) = \varphi_m \cos(2\pi f_m t) \tag{1}$$

Preparation:
Define the operation of a PLL in discrete time ($t \to n\Delta t$):

$$\tilde{\varphi}_e(n\Delta t) = -2\sin(2\pi f_c n\Delta t + \hat{\varphi}(n\Delta t)) \cdot r(n\Delta t) \tag{2}$$

$$\varphi_e(n\Delta t) = \tilde{\varphi}_e(n\Delta t)|_{LP} = \frac{1}{M}\sum_{m=n-M+1}^{n} \tilde{\varphi}_e(m\Delta t) \tag{3}$$

$$\hat{\varphi}(n\Delta t + \Delta t) = \hat{\varphi}(n\Delta t) + K_1\varphi_e(n\Delta t) + K_2\sum_{0}^{n}\varphi_e(i\Delta t) \tag{4}$$

The time index n goes from 0 to some N, which captures a total interval of time $T = N\Delta t$. The time step is $\Delta t = 1/f_s$, where f_s is the sampling frequency used to represent the signals, e.g. $f_s = 4f_c$. Expression (3) represents low-pass filtering, implemented simply as a running sum over M samples, where $M\Delta t$ is sufficient to span several carrier periods, e.g., $M = 2f_s/f_c$. All the variables are equal to 0 for negative time indices.

Exercise:

- Set $\varphi_m = 3$ and $f_m = f_c/1000$. Generate the phase $\varphi(n\Delta t)$ and the signal $r(n\Delta t)$ over the time interval $T = 1000/f_c$.
- Implement the operations (2)-(4) in Matlab. Set the PLL constants to K_1=0.1 and $K_2 = K_1/10$.
- Run the PLL and observe its output. How long does it take the PLL to "lock on," i.e. to provide a reasonable estimate of the phase? What happens if you reduce the PLL constants ten times? What if you increase them?
- Repeat with $f_m = f_c/100$.

Report:
Your typed report should include figures showing $\varphi(n\Delta t)$ and $\hat{\varphi}(n\Delta t)$ on the same plot. Make sure that time is labeled in seconds. With the figures, provide a comment on the PLL performance. Include your Matlab code as an appendix.