**Communication Systems**
**(ECE4572)**
**Fall 2013**

**Homework 8**
Assigned Nov. 15, due Nov. 21.

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**Objectives/Preparation:** Study the principle of matched filtering. Understand the relationship between bit SNR and bit error rate.

**Problem 1.** The received signal in a PSK or QAM system is given by

$$v(t) = \sum_n d(n)g(t - nT) + w(t) \tag{1}$$

where $d(n)$ are the data symbols, $g(t)$ is the basic pulse (rectangular in time, with amplitude $A$ and duration $T$), and $w_c(t)$ is additive white Gaussian noise (AWGN). The noise is zero-mean, with independent real and imaginary parts, each of p.s.d. $N_0$. The receiver employs a filter matched to the pulse $g(t)$, and samples the filter output at times $nT$. Show that the resulting samples are given by

$$y(nT) = d(n)E_g + z(nT) \tag{2}$$

or shortly,

$$y[n] = d(n)E_g + z[n] \tag{3}$$

where $E_g$ is the energy of the pulse $g(t)$ and $z[n] = z(nT)$ is the sequence of noise samples. Determine the statistical properties of the noise sequence: (i) determine the power $\sigma_z^2 = E\{|z^2[n]|\}$; (ii) specify the p.d.f. of a single sample $z[n]$, and (iii) determine the auto-correlation $R_z[m] = E\{z[n+m]z^*[n]\}$. Make a conclusion as to whether the noise samples $z[n]$ are independent. Hint: For part (iii), begin by expressing the p.s.d. $S_z(f)$ of the noise $z(t)$ at the output of the matched filter in terms of the p.s.d. $S_w(f)$ of the input noise $w(t)$; find $R_z(\tau)$ as the inverse Fourier transform of $S_z(f)$, and finally find $R_z[m]$ as $R_z(mT)$.

**Problem 2.** Consider a BPSK communication system operating over an AWGN channel. Determine the bit error rate (BER), i.e. the probability of bit error $P_e$ as a function of the bit SNR $E_b/N_0$. Use Matlab to plot $P_e$ vs. $E_b/N_0$. Specifically, show $P_e$ on the logarithmic scale vs. $E_b/N_0$ in dB.
(i) If $E_b/N_0$ is 4 dB, what is the resulting $P_e$?
(ii)If $E_b/N_0$ is 8 dB, what is the resulting $P_e$?
(iii)If $E_b/N_0$ is 10 dB, what is the resulting $P_e$?
(iv) If it is required that the BER be kept below $10^{-5}$ what does the bit SNR have to satisfy?
(v) If $N_0 = 4 \cdot 10^{-21}$ W/Hz, and the link attenuation is 144 dB, what energy per bit $E_{b,tx}$ should be used at the transmitter so as to satisfy the requirement (iv)?
(vi) What is the corresponding transmit power if the system uses rectangular pulses and the bit rate is 1 Mbps?
(vii) If the transmit power is limited to 1 W, what is the maximum bit rate that can be supported while keeping the BER below $10^{-5}$?
**Report:** Your typed report should contain a cover page with your name, and a few paragraphs of text (including any figures and equations) describing your solution to each problem.