1 Average case analysis

In the average case, assume that the pivot is not the desired element until it is the only element, and the pivot is chosen somewhere in the middle. Therefore, assume that the recursion is performed on a list half the size of the previous list.

$$T(n) = 1 + n + T(\frac{n}{2}) \tag{1}$$

Dropping the constant term and following the recursion produces the following series of equations.

$$T(n) = n + T(\frac{n}{2}) \tag{2}$$

$$T(\frac{n}{2}) = \frac{n}{2} + T(\frac{n}{4}) \tag{3}$$

$$T(\frac{n}{4}) = \frac{n}{4} + T(\frac{n}{8}) \tag{4}$$

$$\begin{aligned} T(2) &= 2 + T(1) \\ T(1) &= 1 + ... \end{aligned} \tag{5}$$

Summing the equations and cross-cancelling like values produces a linear result.

$$\begin{aligned} T(n) &= n + \frac{n}{2} + \frac{n}{4} + ... + 2 + 1 + ... \\ &= \sum_{i=0}^{\infty} \frac{n}{2^i} \\ &= n \sum_{i=0}^{\infty} (\frac{1}{2})^i \\ &= n(\frac{1}{1-\frac{1}{2}}) \\ &= 2n \\ &= \mathcal{O}(n) \end{aligned} \tag{6}$$