

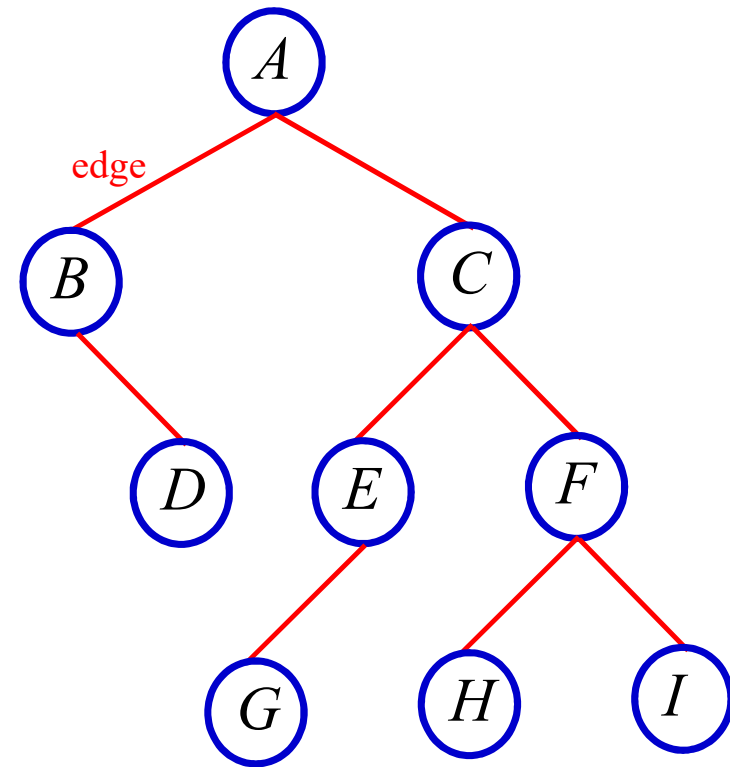
CSE 105: Data Structures and Algorithms-I (Part 2)

Instructor
Dr Md Monirul Islam

Binary Tree: Elements

Edge:

Each node is connected to each of its children by an **edge**

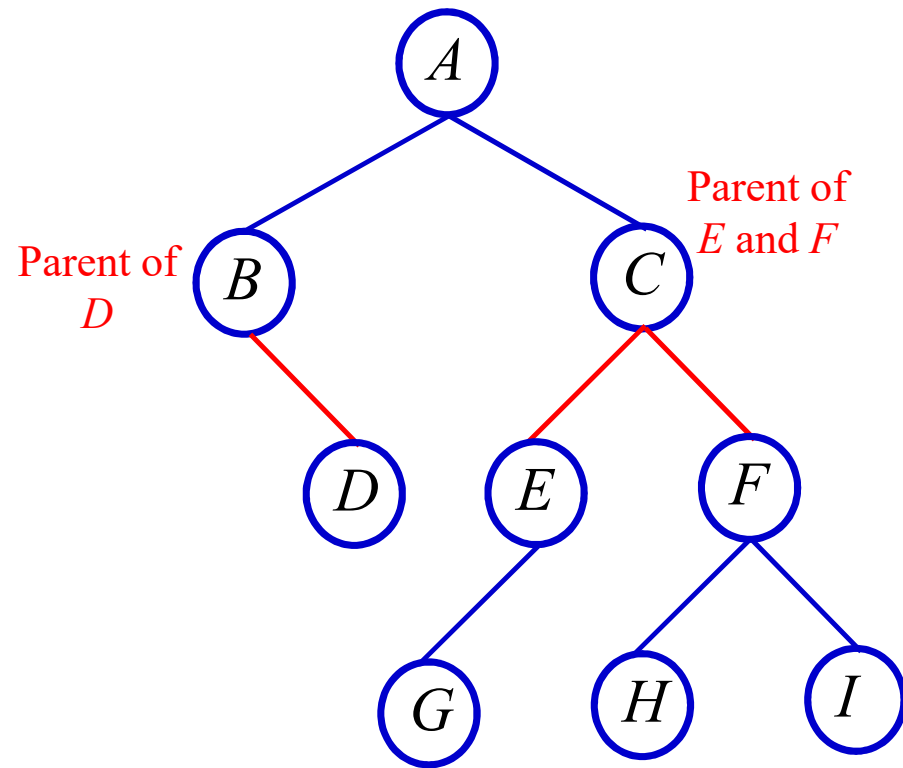


Binary Tree: Elements

Parent:

Root has **no** parent.

Others have parent



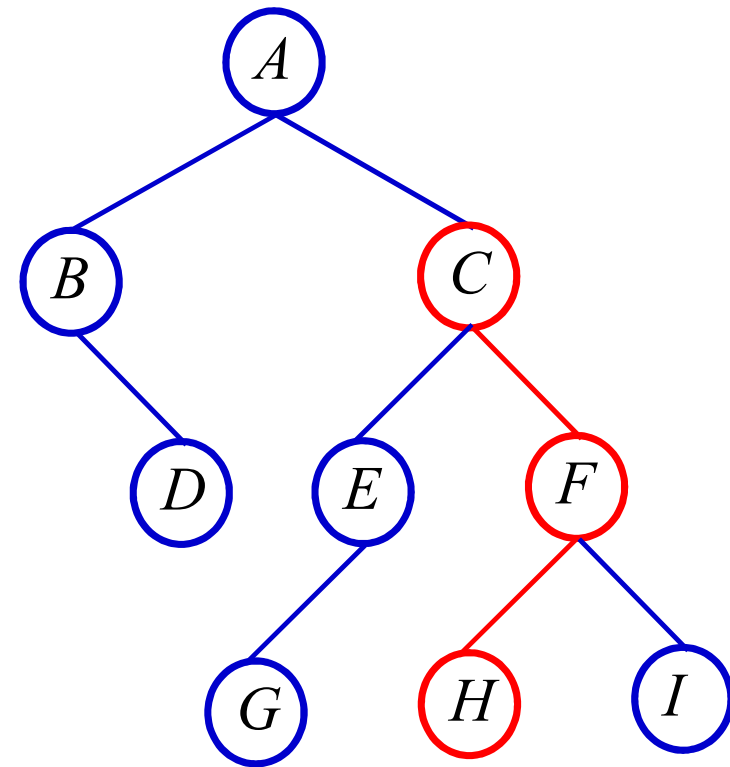
Binary Tree: Elements

Path:

If $n_1, n_2, n_3, \dots, n_k$ is a sequence of nodes where n_i is the parent of n_{i+1} for $1 \leq i < k$, then this sequence is a path

Each path has a length (No. of edges)

Length of path $C-F-H$ is 2



Binary Tree: Elements

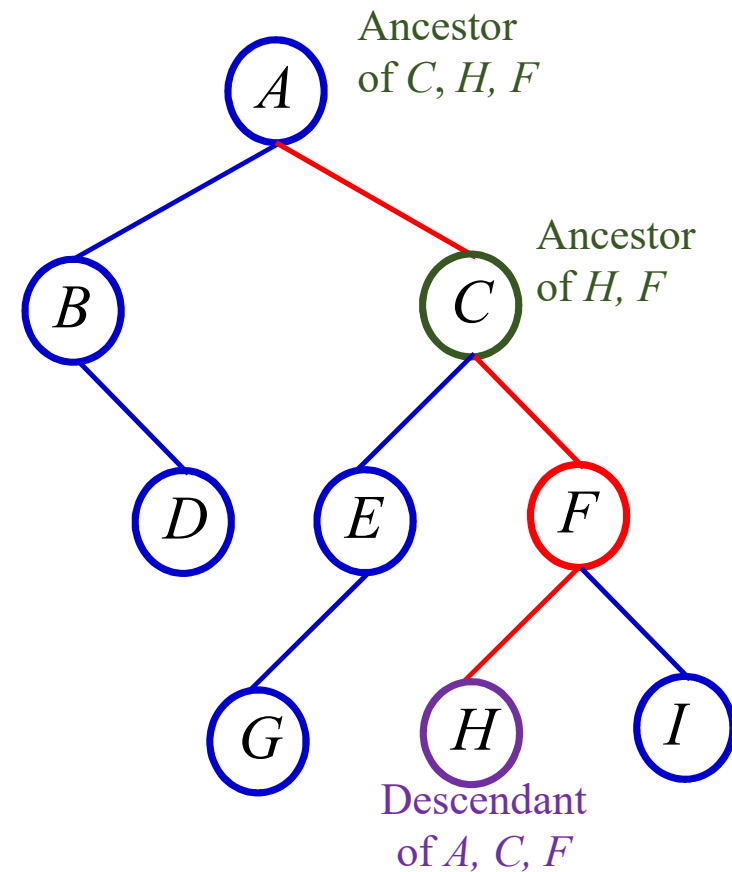
Ancestor/Descendant:

If there is a path from n_i to **parent** of n_j

Then,

n_i is a ancestor of n_j

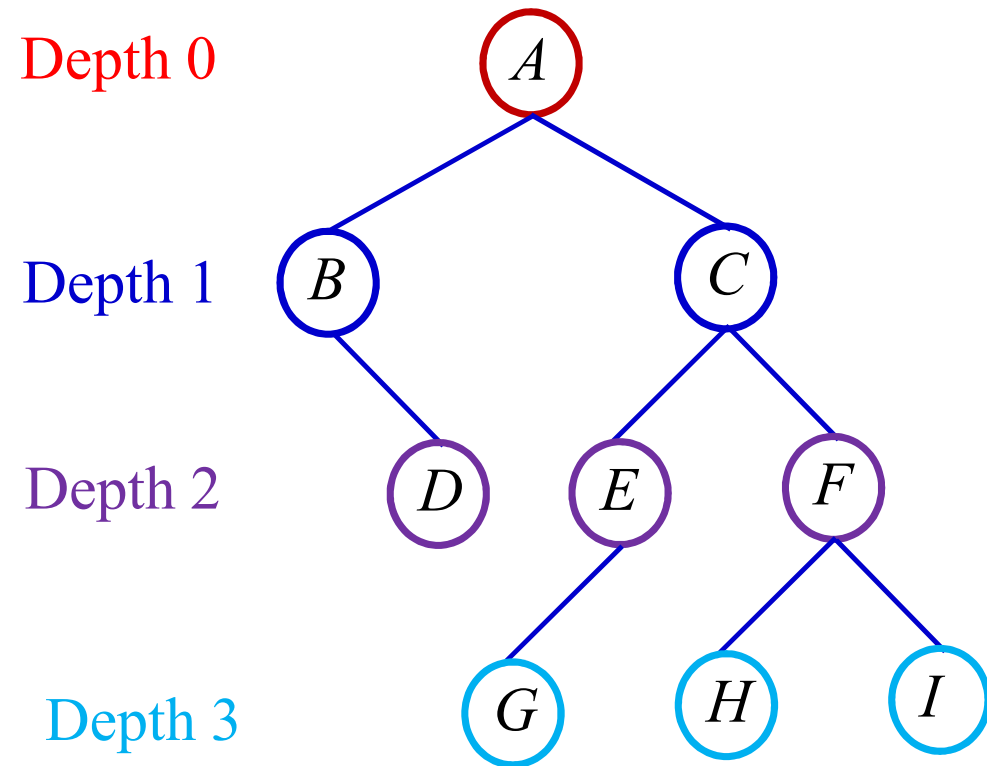
n_j is a descendant of n_i



Binary Tree: Elements

Depth of a node (u):

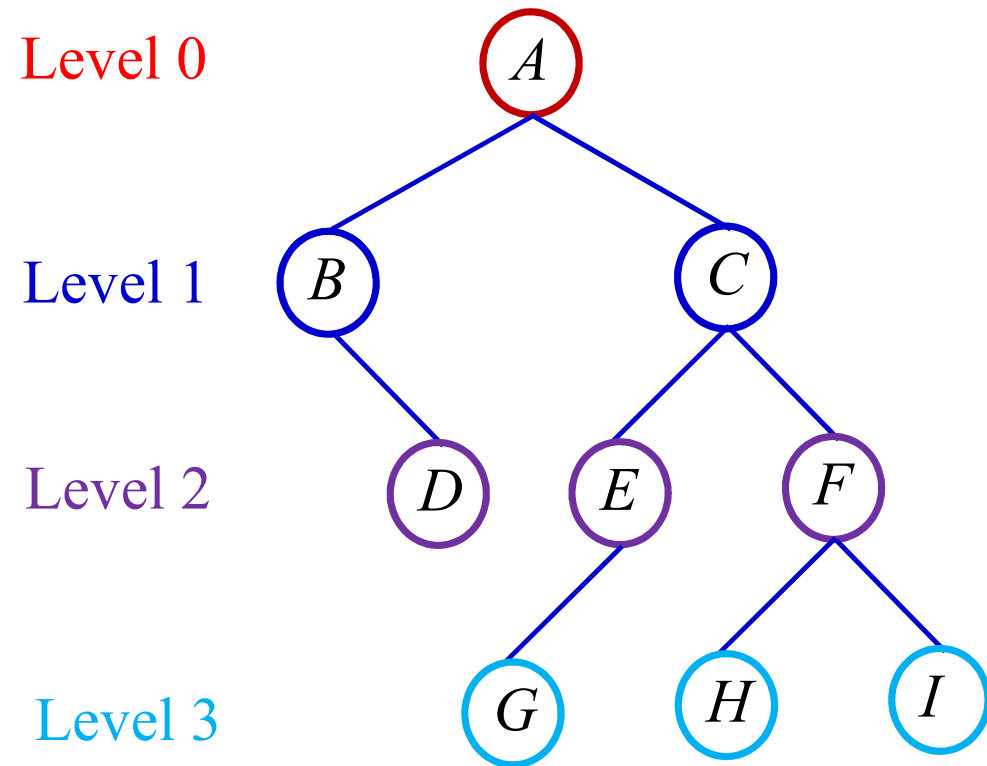
Length of path from root to node u



Binary Tree: Elements

Level of a node:

All nodes of depth n are at level n

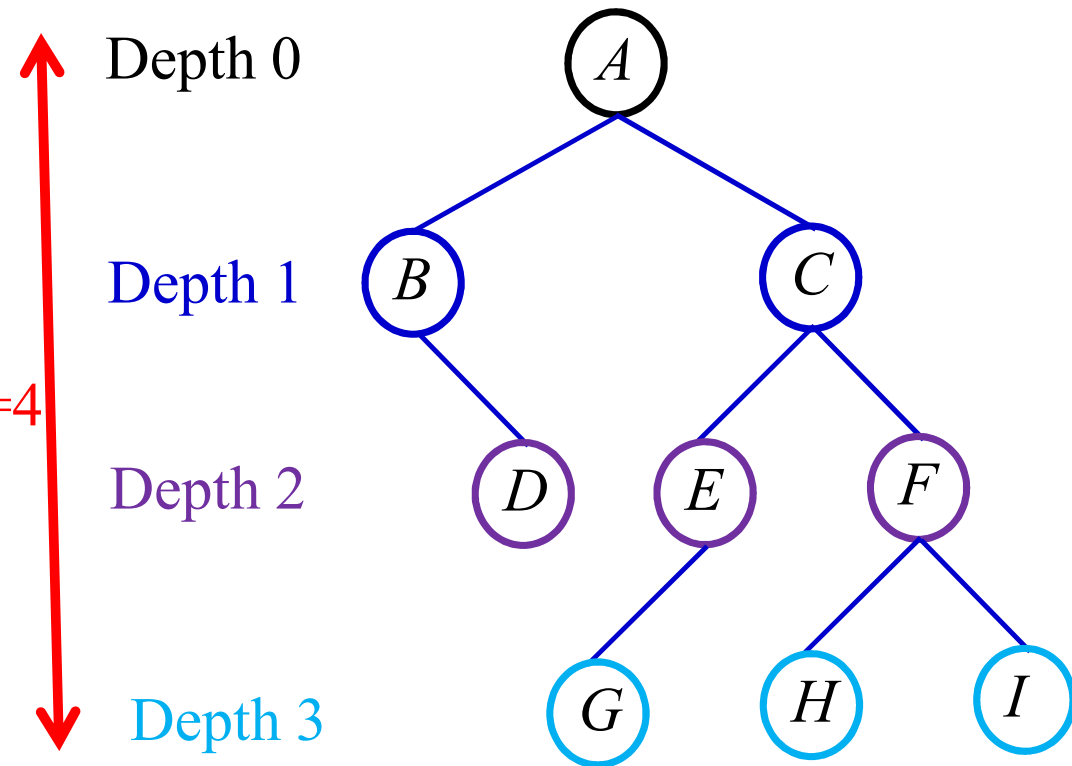


Binary Tree: Elements

Height of a tree:

One more than the depth of the deepest node

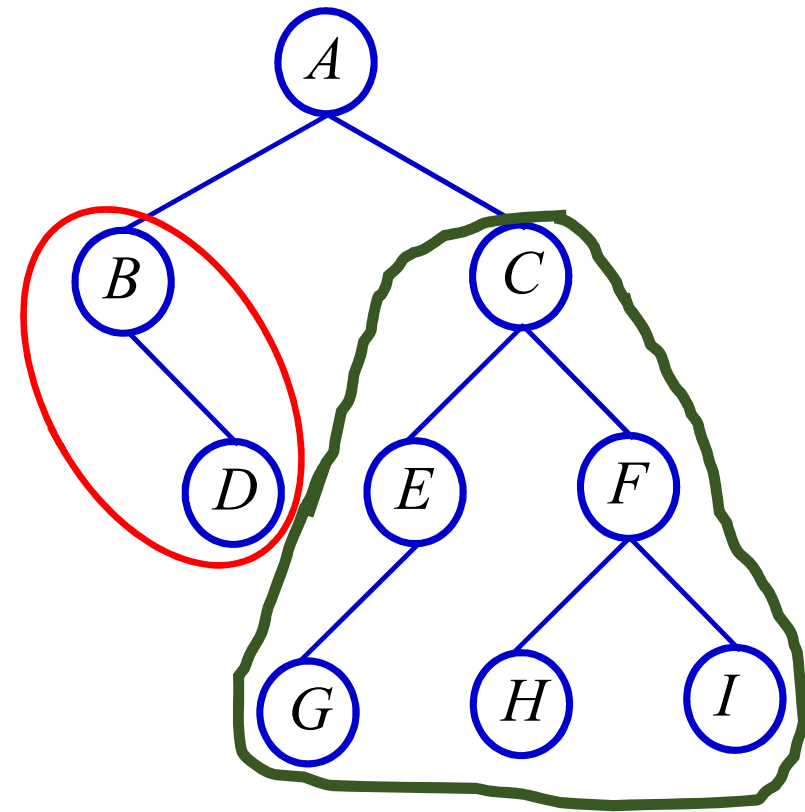
$$\text{Height} = 1 + 3 = 4$$



Binary Tree: Elements

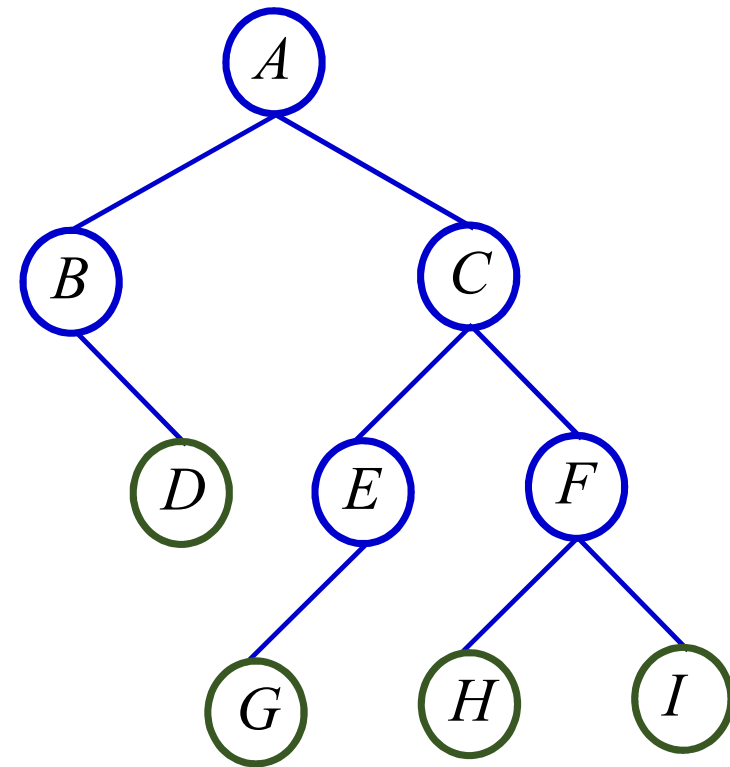
Subtree:

Subset of a tree; **Left** and **right** subtrees



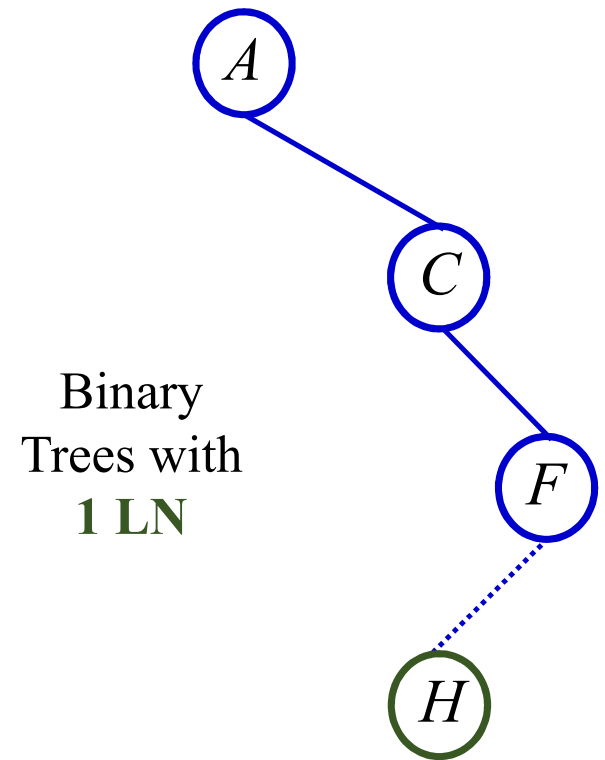
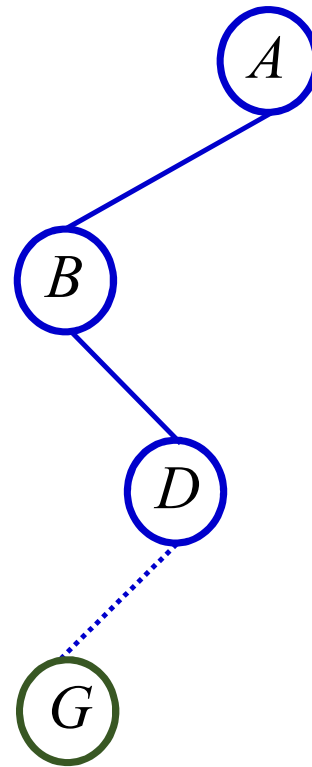
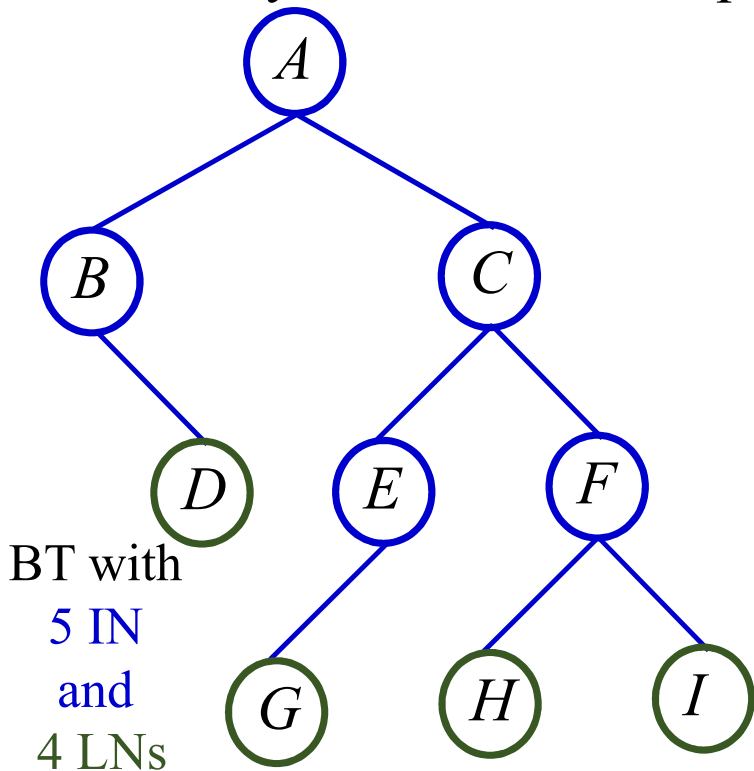
Information storage in Binary Trees

- Some implementations store data
 - at **all nodes**
 - **only at the leaf nodes**, using the internal nodes to provide structure to the tree.
- Space requirements may vary for
 - **internal nodes**
 - **leaf nodes**.



Information storage in Binary Trees

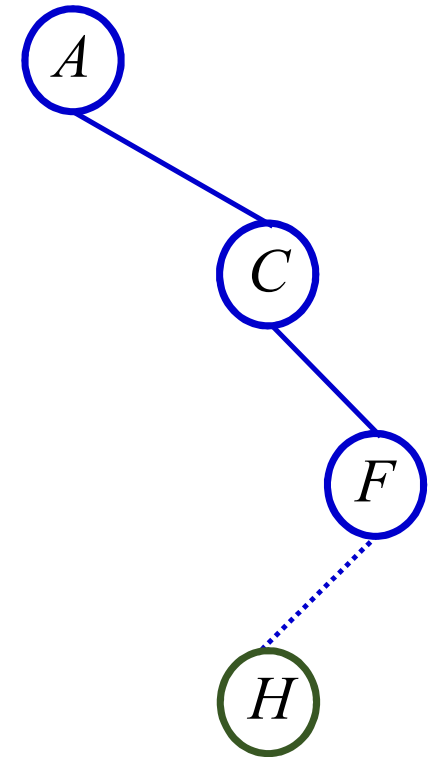
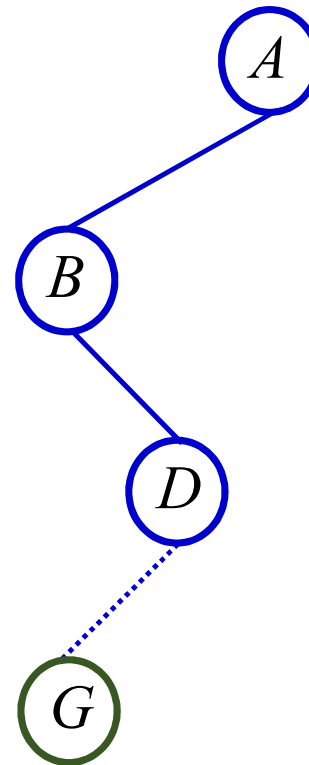
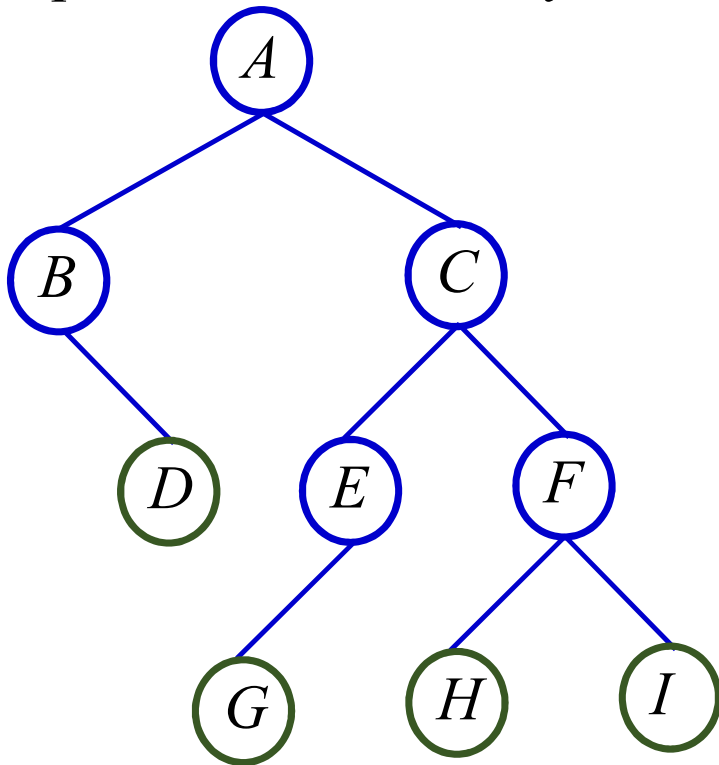
The number of **internal nodes** and **leaf nodes** vary based on the shape of the tree.



Binary
Trees with
1 LN

Information storage in Binary Trees

For **space analysis** and other requirements special class of binary trees are used



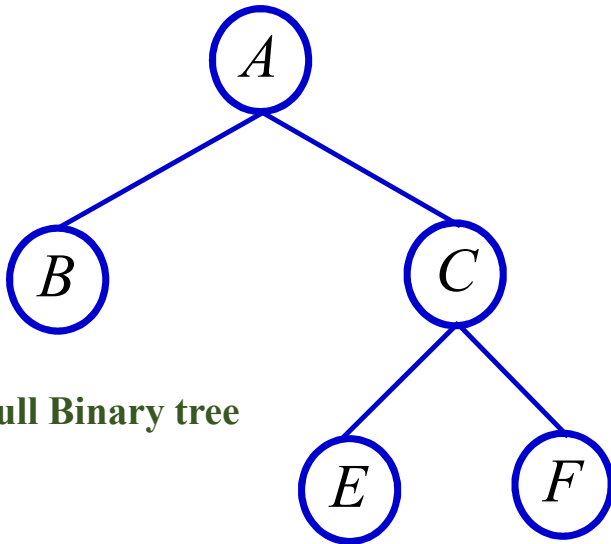
Full and Complete Binary Trees

Full Binary tree

Each node is either

a leaf **OR**

an internal node with exactly two non-empty children.

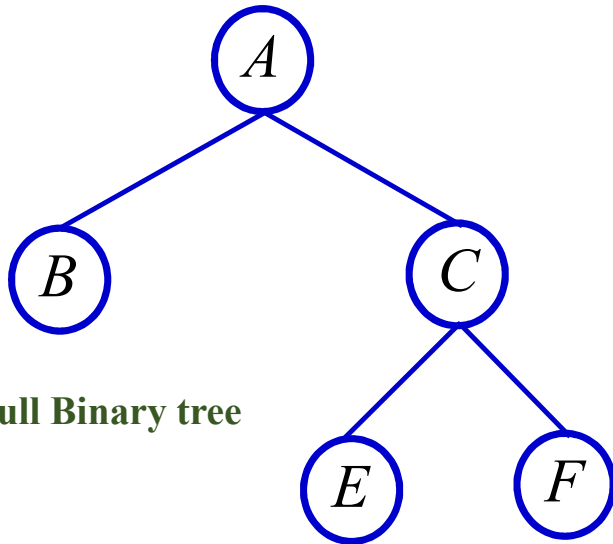


Full Binary tree

Full and Complete Binary Trees

Full Binary tree

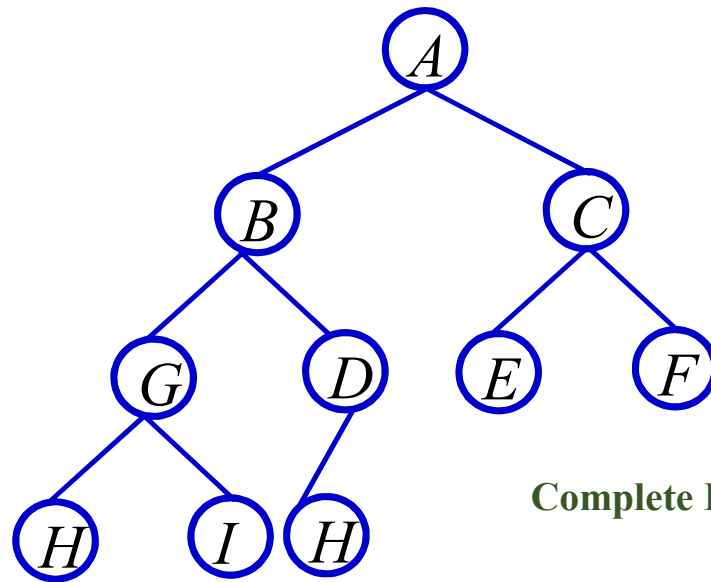
Each node is either a leaf or internal node with exactly two non-empty children.



Full Binary tree

Complete Binary Tree

If the tree has d levels, then all levels except possibly level $d-1$ are completely full. The bottom level has all nodes starting from the left side.

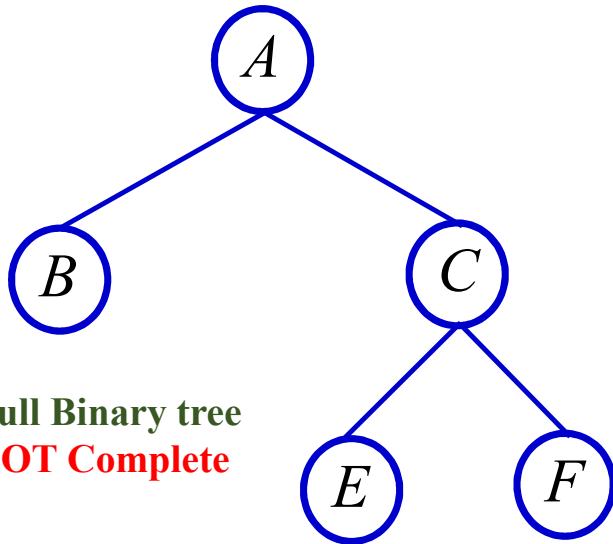


Complete Binary tree

Full and Complete Binary Trees

Full Binary tree

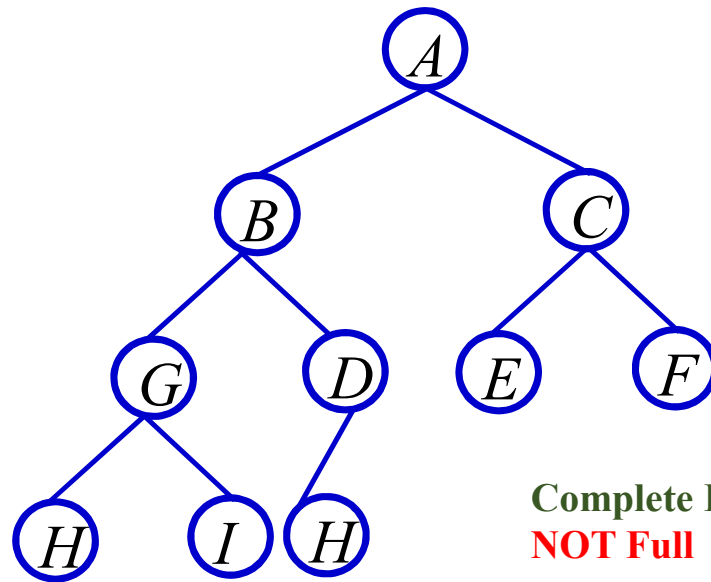
Each node is either a leaf or internal node with exactly two non-empty children.



Full Binary tree
NOT Complete

Complete Binary Tree

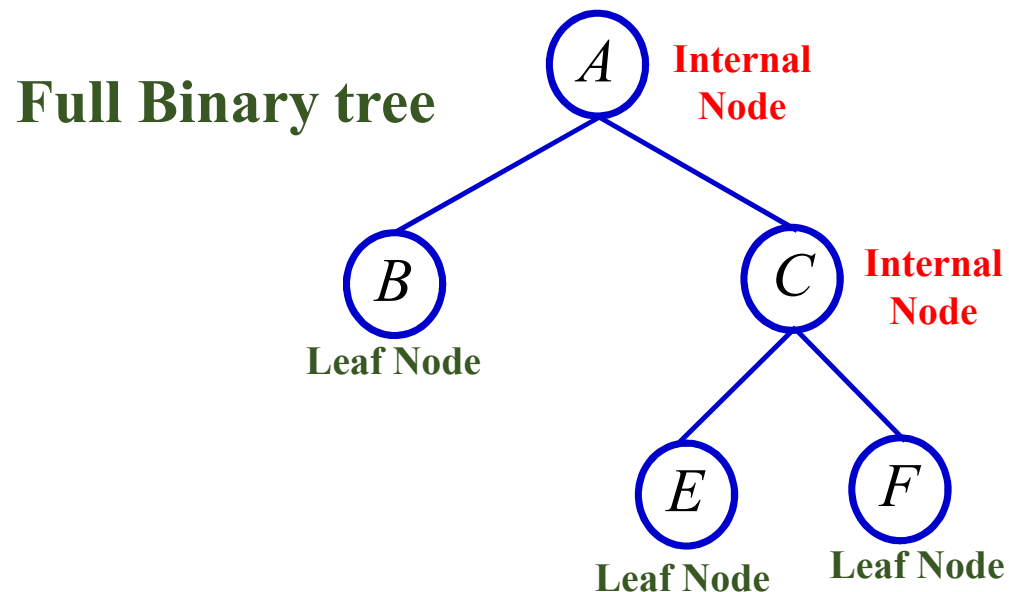
If the tree has d levels, then all levels except possibly level $d-1$ are completely full. The bottom level has all nodes starting from the left side.



Complete Binary tree
NOT Full

Full Binary Tree Theorem

Theorem: The number of leaves in a non-empty full binary tree is one more than the number of internal nodes.



$$LN = 1 + IN$$

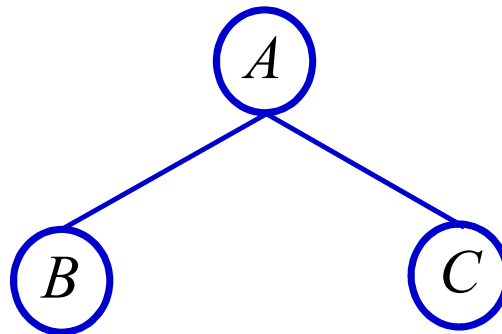
Full Binary Tree Theorem

Theorem: The number of leaves in a non-empty full binary tree is one more than the number of internal nodes.

Proof (by Mathematical Induction):

Base case: A full binary tree with 1 internal node must have two leaf nodes.

Induction Hypothesis: Assume any full binary tree T containing $n-1$ internal nodes has n leaves.



Full Binary Tree Theorem

Induction Step:

We want to prove the following here:

If $n-1$ internal nodes mean n leaves

Then n internal nodes would mean $n+1$ leaves

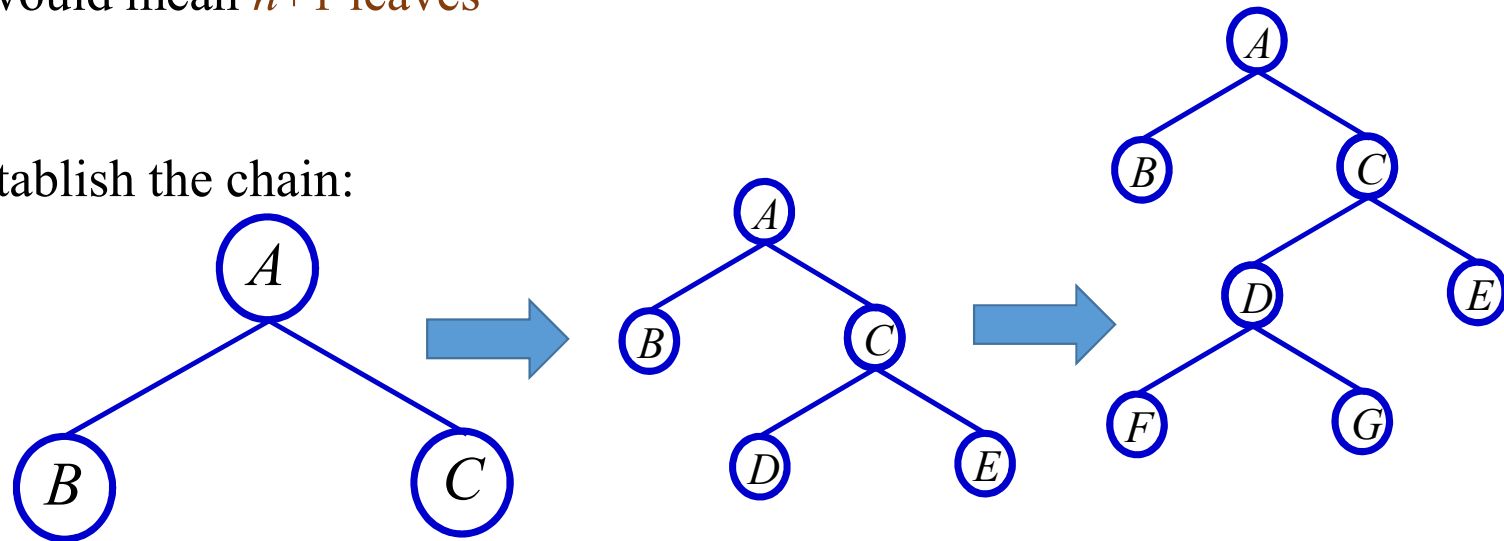
[Recall that this will establish the chain:

1 IN mean 2 L

So, 2 IN mean 3 L

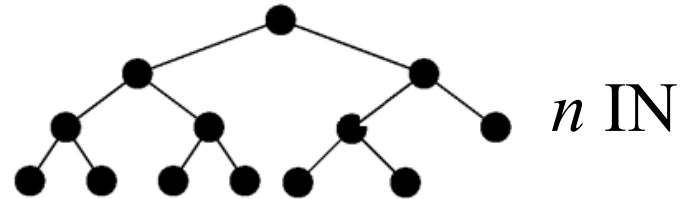
So, 3 IN mean 4

And so on...]



Full Binary Tree Theorem

Induction Step: Given
tree **T** with n internal
nodes

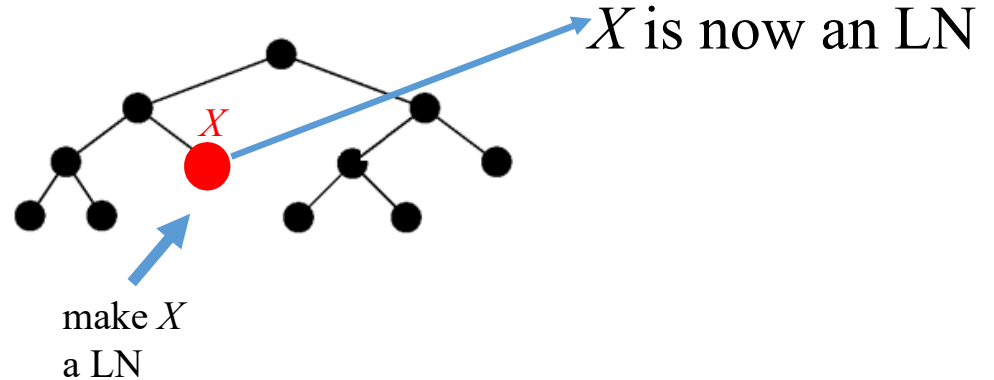
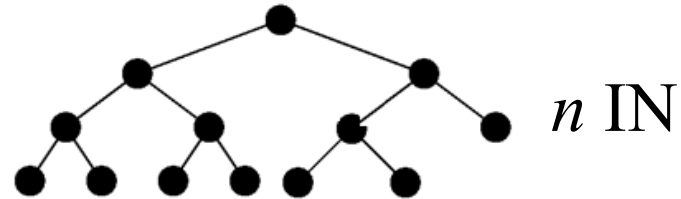


Full Binary Tree Theorem

Induction Step: Given tree T with n internal nodes,

Pick internal node X with two leaf children.

Remove X 's children, call resulting tree T' .

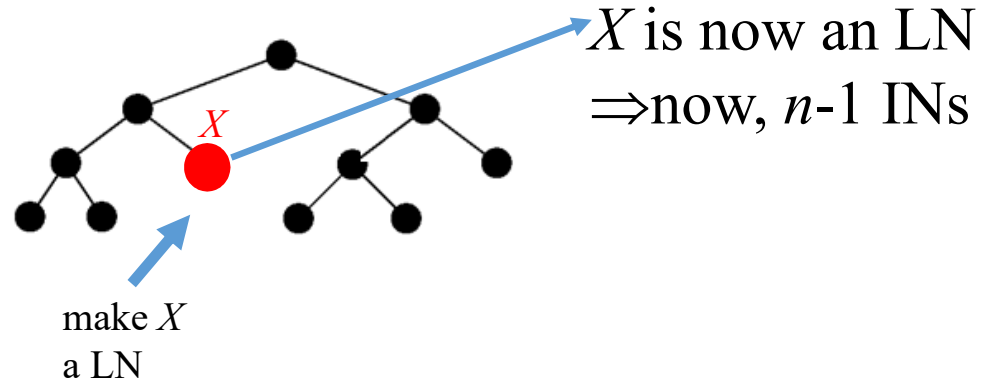
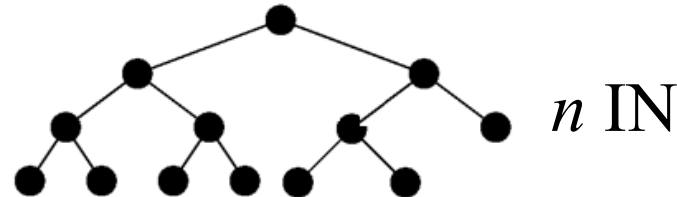


Full Binary Tree Theorem

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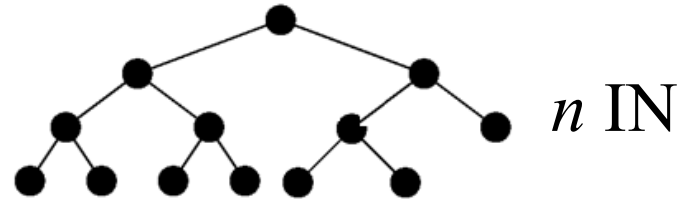
Clearly, T' is a full binary tree with $n-1$ internal nodes



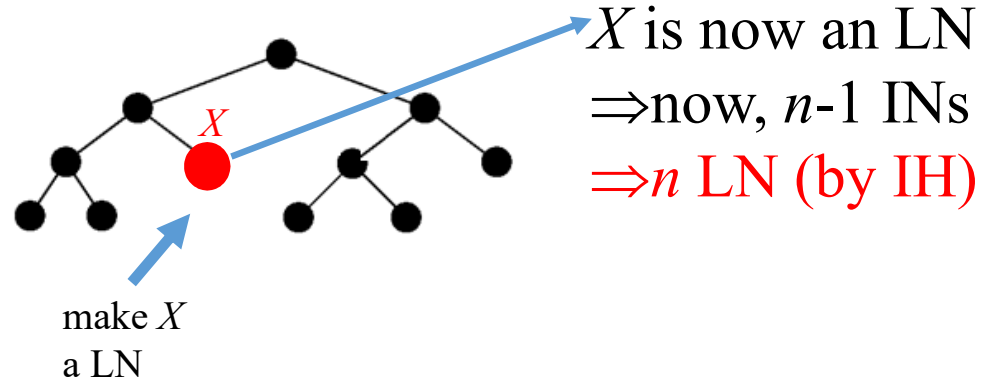
Full Binary Tree Theorem

Induction Step:

Clearly, T' is a full binary tree with $n-1$ internal nodes



By induction hypothesis, it has n leaves.



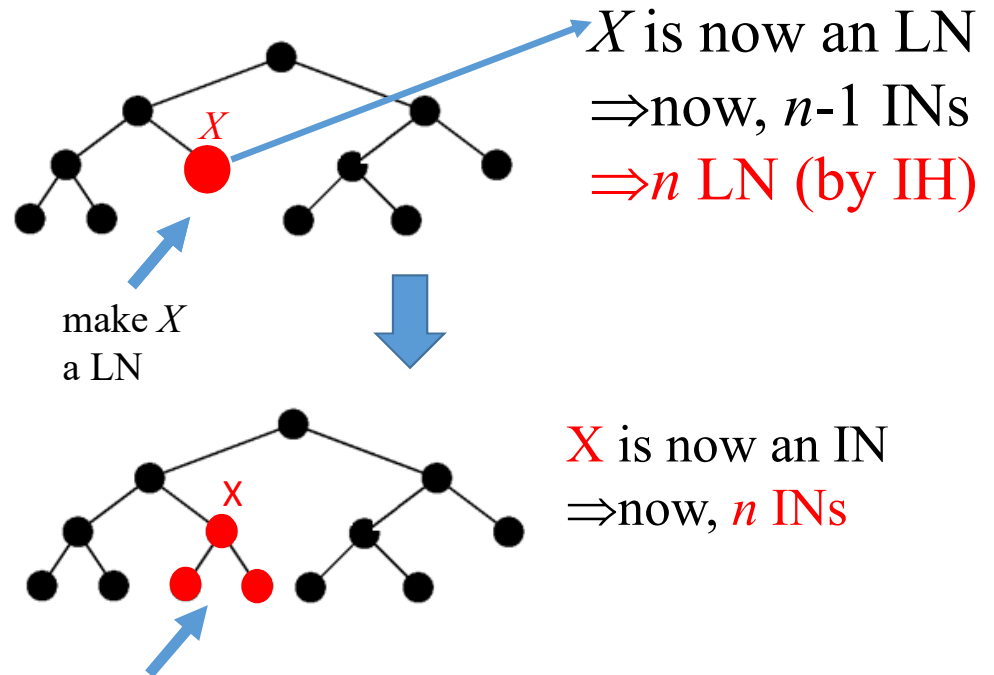
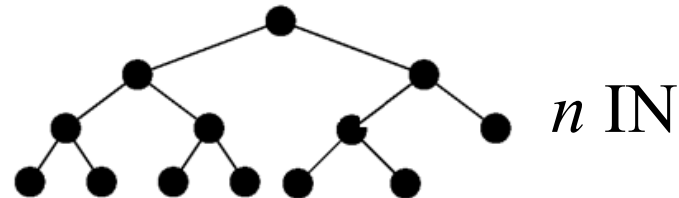
Full Binary Tree Theorem

Induction Step:

Now:

Restore X 's two children.

The number of internal nodes has now gone up by 1 to reach n .



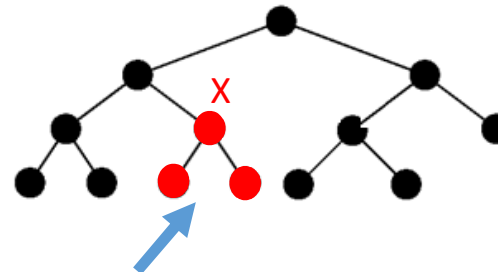
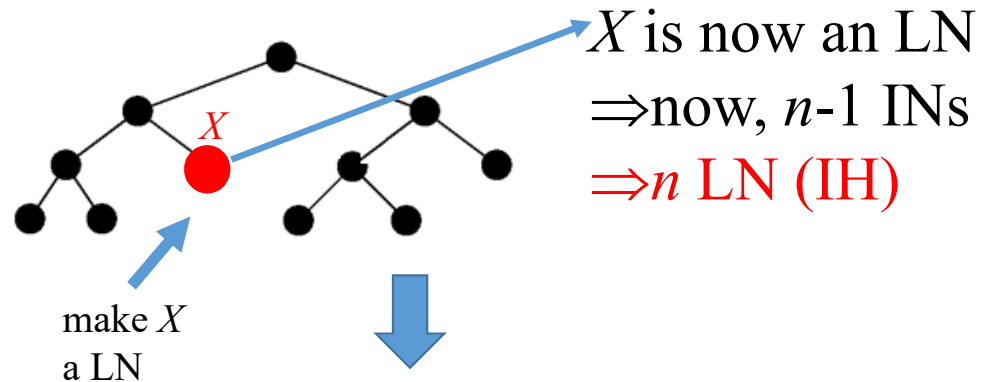
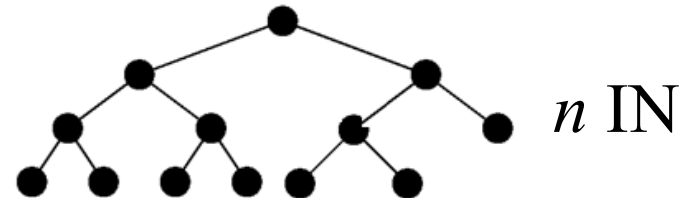
Full Binary Tree Theorem

Induction Step:

Now:

Restore X 's two children. The number of internal nodes has now gone up by 1 to reach n .

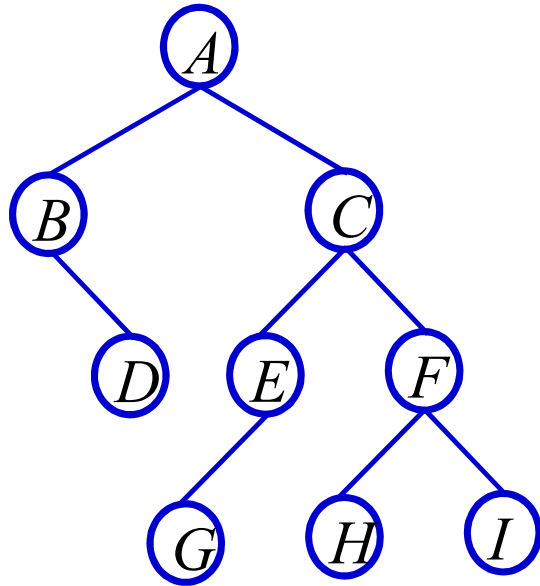
The number of leaves has also gone up by 1.



X is now an IN
 \Rightarrow now, n INs
 X : 1 LN became IN
2 LNs added
LN = $n - 1 + 2 = n + 1$

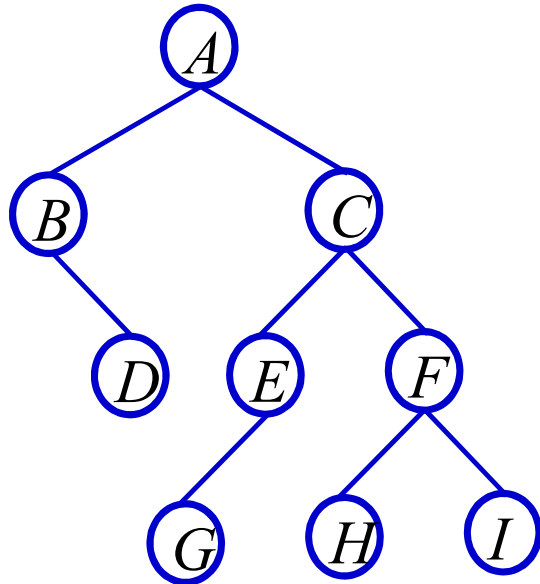
Full Binary Tree Corollary

Theorem: The number of empty subtrees in a non-empty binary tree is one more than the number of nodes in the tree.



Full Binary Tree Corollary

Theorem: The number of empty subtrees in a non-empty **binary tree** is one more than the number of nodes in the tree.



To understand the theorem assume the following binary tree data structure

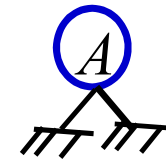
```
// binary tree node in C  
  
struct BTnode {  
    int data;  
    struct BTnode *left, *right;  
}
```

Full Binary Tree Corollary

Theorem: The number of empty subtrees in a non-empty binary tree is one more than the number of nodes in the tree.



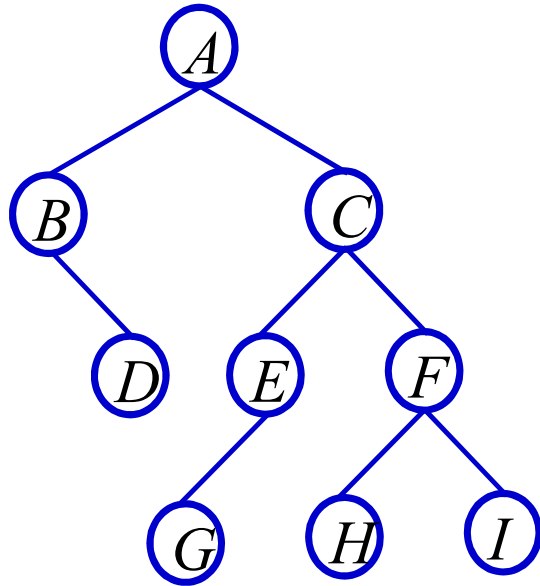
Single node
binary tree



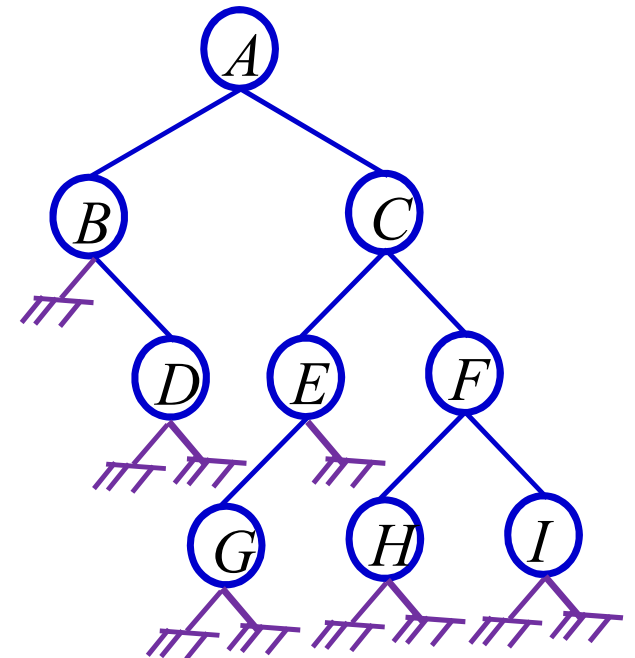
Null pointers
or empty
subtrees

Full Binary Tree Corollary

Theorem: The number of empty subtrees in a non-empty binary tree is one more than the number of nodes in the tree.

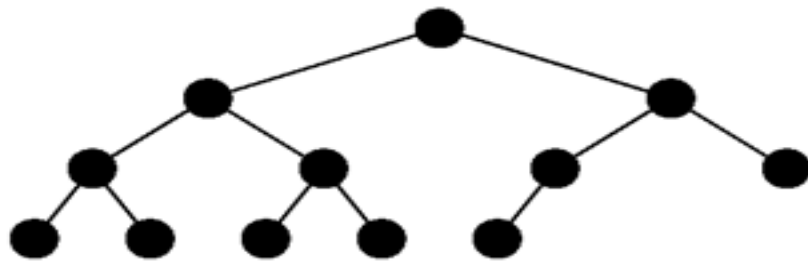


9 Nodes
10 empty
subtrees

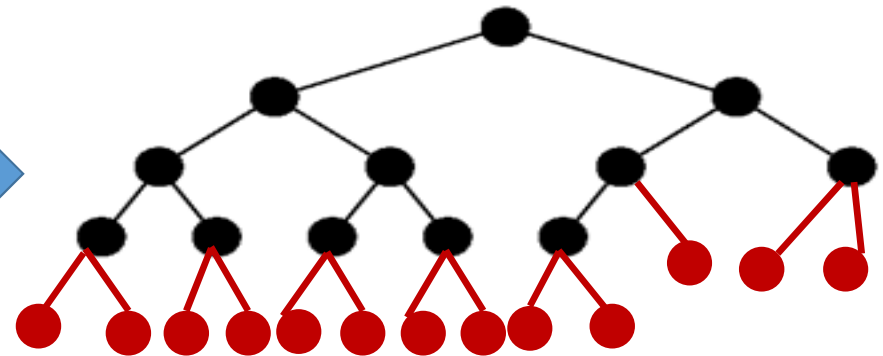
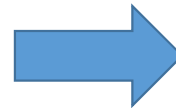


Full Binary Tree Corollary- Proof

- Take an arbitrary binary tree T and replace every empty subtree with a leaf node. Call the new tree T_0 .



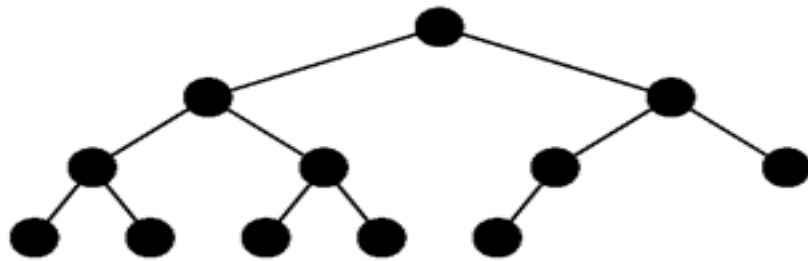
BT: T



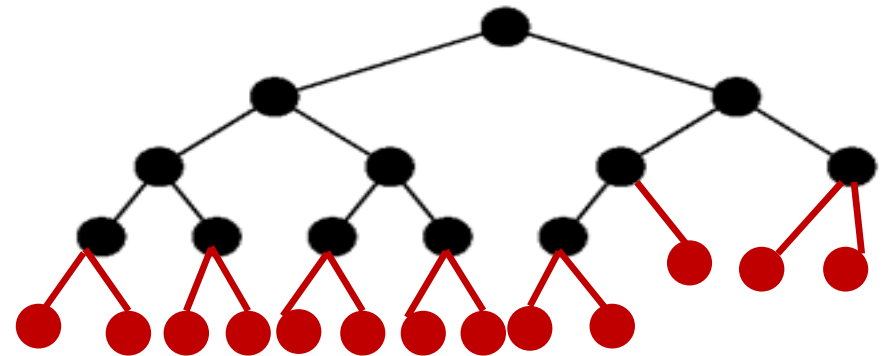
BT: T_0

Full Binary Tree Corollary- Proof

All nodes originally in T will be internal nodes in T0



T: Some blacks are internal nodes
Others are leaf nodes

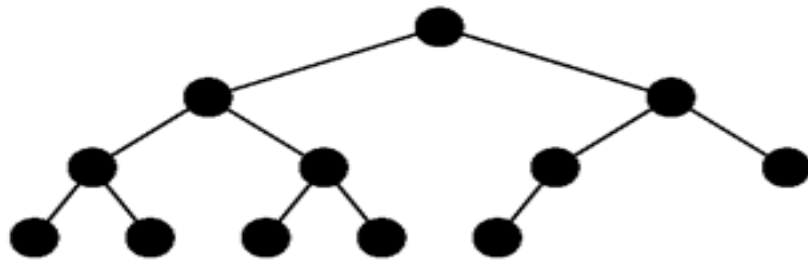


T0: All blacks are internal nodes

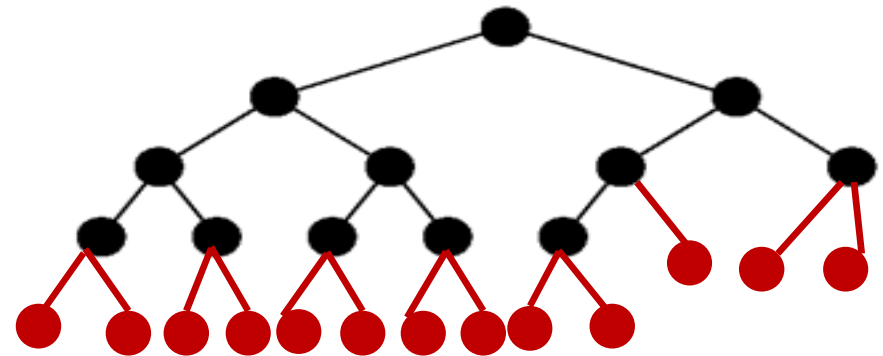
Full Binary Tree Corollary- Proof

All **IN** in T have two children in T0

All LF in T have two children in T0



Some blacks are internal nodes
Others are leaf nodes

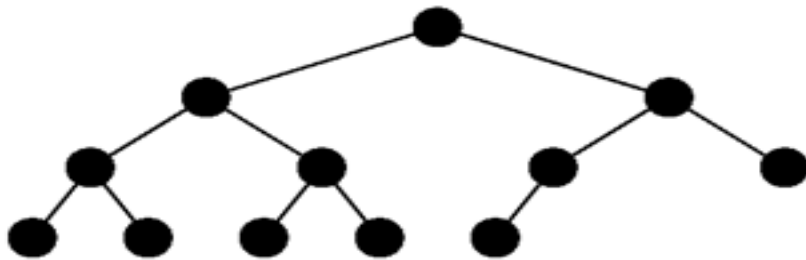


Full Binary tree

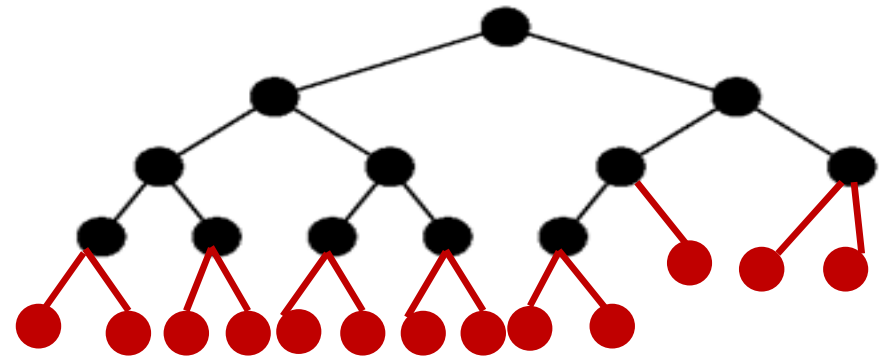
Full Binary Tree Corollary- Proof

The Full Binary Tree Theorem tells:

the number of leaves in a full binary tree is one more than the number of internal nodes.



Some blacks are internal nodes
Others are leaf nodes

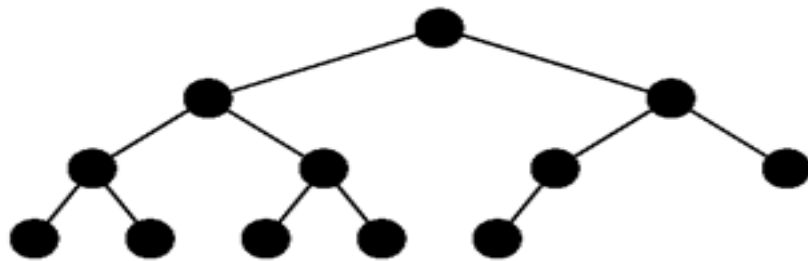


Full Binary Tree Theorem:
 $LN = 1 + IN$

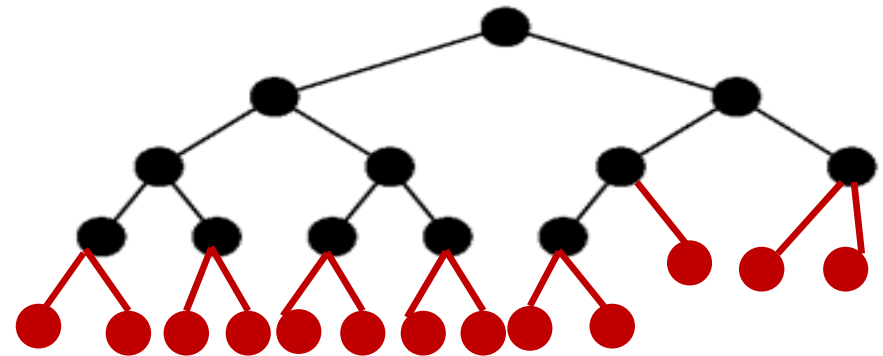
Full Binary Tree Corollary- Proof

leaves in T_0 are newly added.

Their number is one more than the nodes of T



T

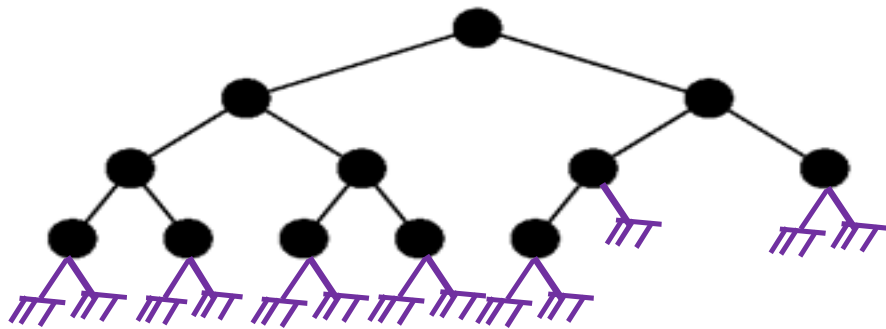


$$T_0: LN = 1 + IN$$

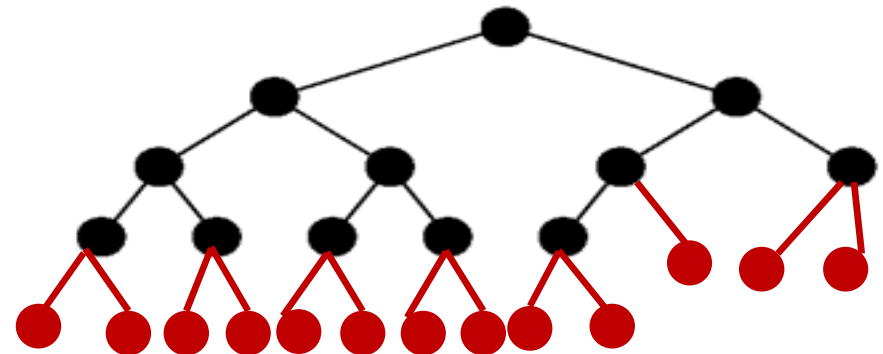
Full Binary Tree Corollary- Proof

leaves in T_0 are empty subtree in T .

Therefore, the number of empty subtrees in T is one more than the number of nodes in T .



T

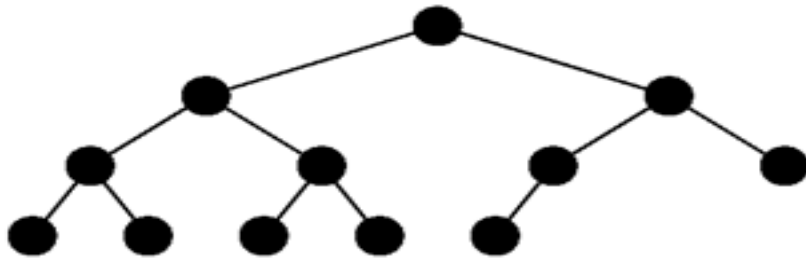


$$T_0: LN = 1 + IN$$

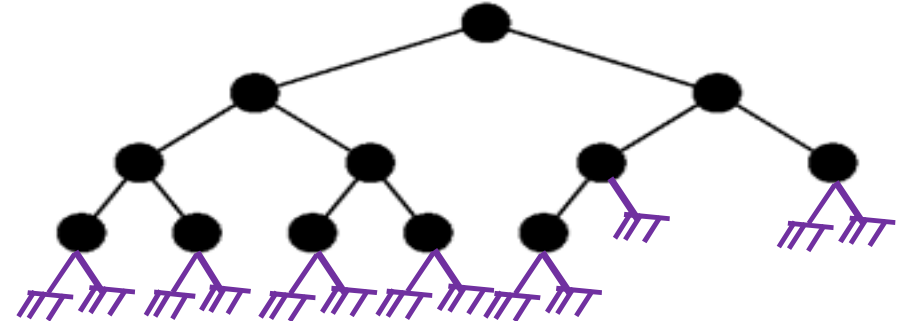
Full Binary Tree Corollary- Proof (2)

Let T have n nodes

Every node has two children: empty or non-empty



T : n nodes



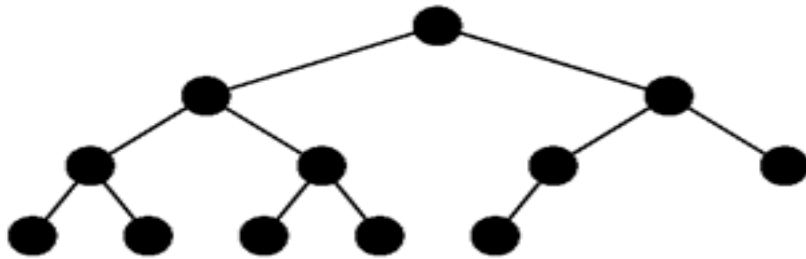
Another view of T

Full Binary Tree Corollary- Proof (2)

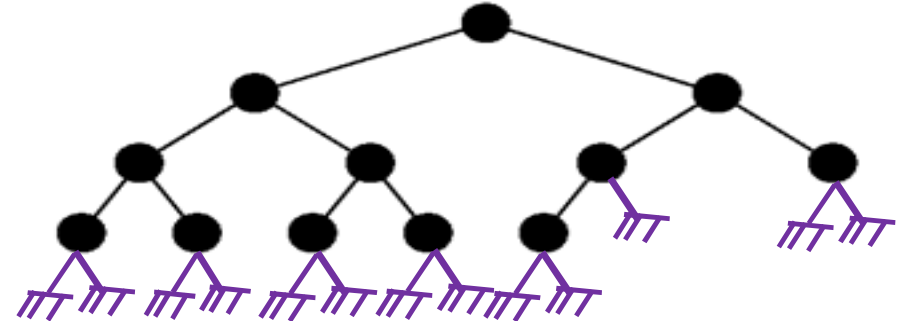
Let T have n nodes

Every node has two children: empty or non-empty

Total children : $2n$



T : n nodes

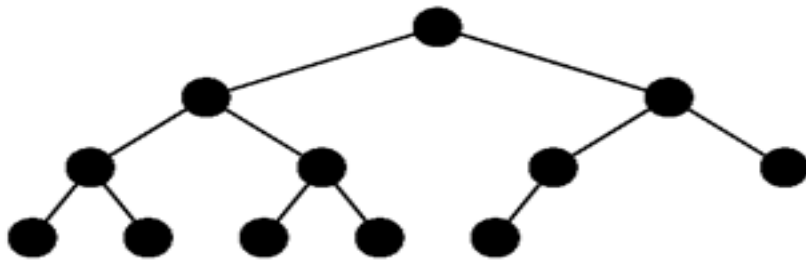


Another view of T : $2n$ Children

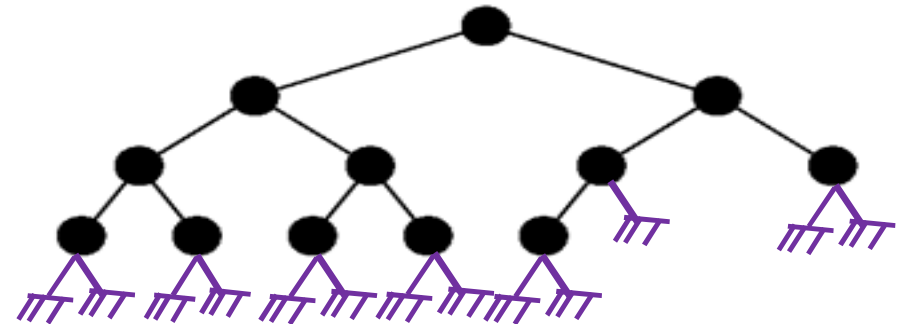
Full Binary Tree Corollary- Proof (2)

Except root, every node has parent.

$n - 1$ nodes have parents: they are nonempty children



T: n nodes



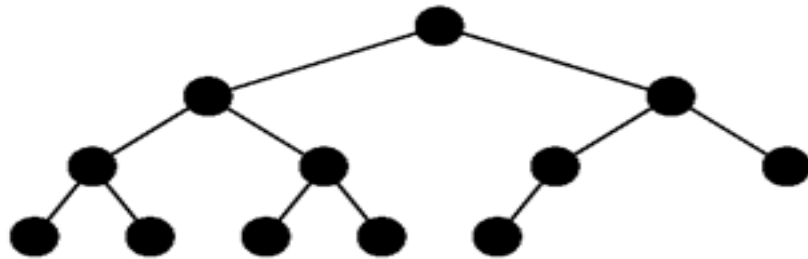
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Full Binary Tree Corollary- Proof (2)

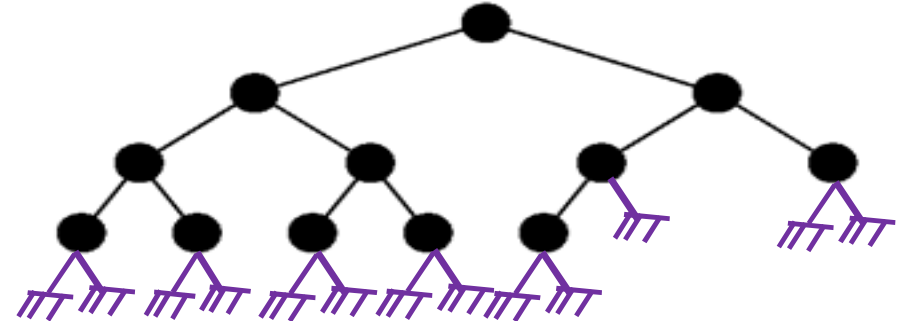
Except root, every node has parent.

$n - 1$ nodes have parents: they are nonempty children

Number of empty children (empty subtrees) = $2n - (n-1) = n + 1$



T: n nodes



Another view of T: $2n$ Children
 $n+1$ empty children

Tree Implementation

We have already seen the following
binary tree data structure

```
struct BTreeNode {  
    int data;  
    struct BTreeNode *left, *right;  
}
```

Tree Implementation

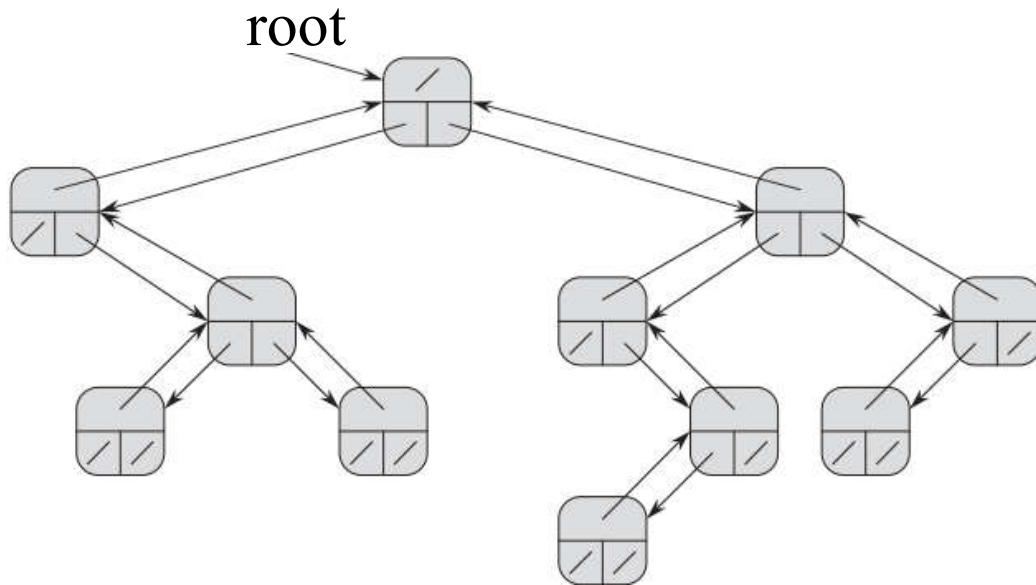
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Some applications may require to store **parent** information in the node

```
struct BTreeNode {  
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}
```


Tree Implementation



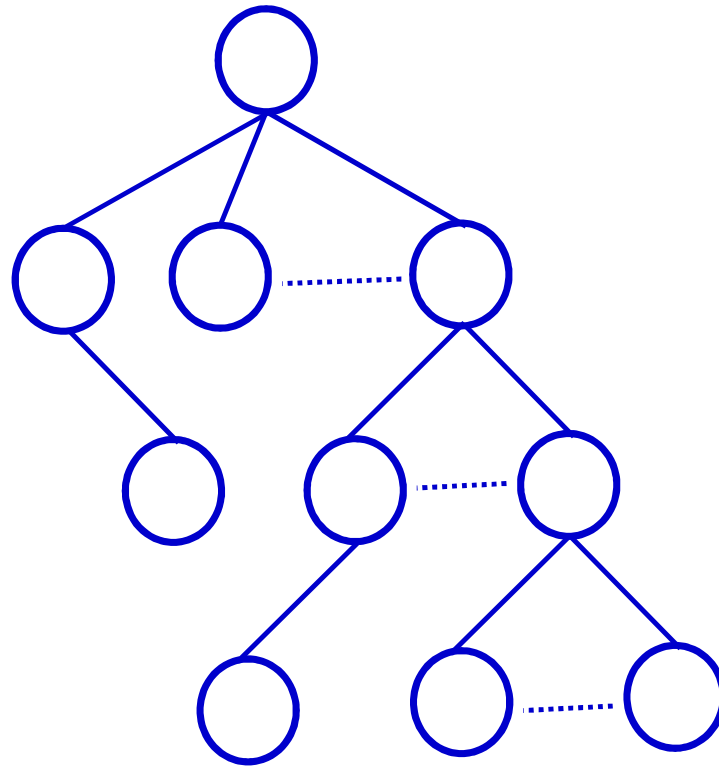
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Tree Implementation

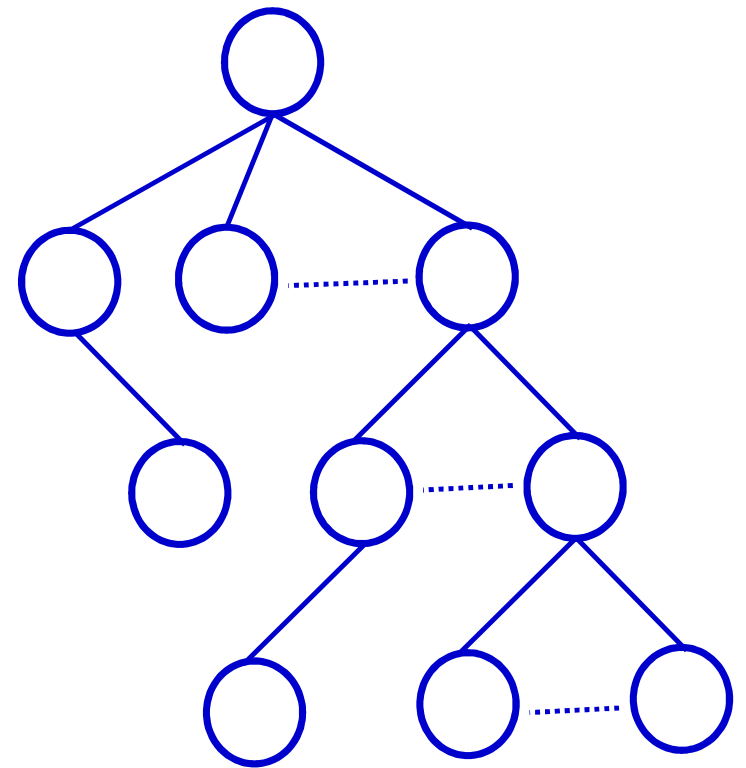
If a tree has unbounded number children



Tree Implementation

If a tree has unbounded number children

```
struct BTreeNode {  
    int data;  
  
    struct BTreeNode *left, *rightsibling, *parent;  
}
```



Tree Implementation

```
struct BTreeNode {  
    int data;  
    struct BTreeNode *left, *rightsibling, *parent;  
}
```

