Intro to Probability

Introduction

1. Developed in 17th century.
2. Reason - increase chances to win games.
3. Nowadays - basis for Data Science.
4. Why we study - without understanding the basics of Probability, it is difficult to continue with Stats and Modelling.
5. Mostly we will use games to understand the concepts.
6. We will solve some tasks using formulas on the paper and code in Python.

Key concepts. Sample Space

The list of all possible outcomes is called the sample space.

Toss a coin: all possible outcomes are heads and tails.
If we roll a die, all possible outcomes are 1, 2, 3, 4, 5, 6.

We use the Greek letter Omega Ω to denote the entire sample space of a random experiment. For the coin, we typically denote the sample space as $\Omega = \{H, T\}$. For the die, we denote the sample space as $\Omega = \{1, 2, 3, 4, 5, 6\}$.

What is the ss: Guess the number, Rock Paper Scissors, Toss the coin twice

Key concepts: Events

Event is basically the outcome which probability we are measuring. In math language we define a subset from sample space.

Event: dice roll is even. What outcome is it? {2,4,6}

Event: while flipping a coin the head fell at least once: {HH, HT, TH}

Another example is the event that a die roll is less than or equal to 2. B = {1, 2}.

Definition of Probability

1. Probability is the likelihood that events will occur - General
2. The relative frequency of occurrence of an experiment's outcome, when repeating the experiment - How often did it rain historically this date having all conditions the same.
3. Degree of belief - It is going to rain today. How much do you believe me?
4. The max price you would pay for a game: 1€ if event occurs, 0€ if not.

Einstein: "I am convinced that God does not play dice".

Union and Intersection

The intersection of sets A and B is denoted by A ∩ B and contains all elements that are both in A and in B.

```
a = set([1, 3, 5])
b = set([5, 6])
a.union(b)
{1, 3, 5, 6}
a.intersection(b)
{5}
```

3 Axioms

1. The probability of an event is a non negative real number.
2. The probability of the entire sample space is 1 or P(Ω) = 1.
3. The union of mutually exclusive events is equal to the sum of these events. In mathematical notation:

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

More concepts

1. Complement - Everything except the event
2. Mutual Exclusivity - events listed do not overlap and complete the sample space
3. Marginal Probability - probability of single event
4. Joint Probability - probability of 2 events happening at a time
5. Conditional Probability - probability of 1 event given that another event has happened
6. Independence - events do not affect each other
7. Venn diagrams - useful tool to understand probabilities

Naive Definition

Probability = Event space / Sample Space.

For example, the probability that a die roll is even is 3/6

The probability that a die roll is less than or equal to 2 is 2/6

```
def even(x):
    return(x % 2 == 0)
sample_space = [1, 2, 3, 4, 5, 6]
die_sides = len(sample_space)
even_roll = len([x for x in sample_space if even(x)])
even_probability = even_roll / die_sides
print(even_probability)
0.5
```

Summary

Summary of probabilities

Event	Probability
A	$P(A) \in [0,1]$
not A	$P(A^{\complement}) = 1 - P(A)$
A or B	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \cup B) = P(A) + P(B) \qquad \text{if A and B are mutually exclusive}$
A and B	$P(A \cap B) = P(A\|B)P(B) = P(B\|A)P(A)$ $P(A \cap B) = P(A)P(B) \qquad \text{if A and B are independent}$
A given B	$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B\|A)P(A)}{P(B)}$