

# Theoretical Performance Models for Graphics Processing Units

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# Background

## General-purpose computing on graphics processing units

- ▶ GPGPU is commonly used in many research and industry fields across various disciplines.
- ▶ A major issue with GPGPU and programmers is the **ability to identify performance bottlenecks accurately**.
- ▶ GPUs are also increasing in power; Taking full advantage of this performance is sufficiently difficult.
- ▶ Thus, tools for analyzing for both algorithms and code targeting GPUs are needed.

## Performance models

- ▶ Performance models (like the MWP-CWP model) require the **availability of machine-like code** while algorithm models (like PRAM, MCM) do not.
- ▶ These analytical models give performance estimates (running time, memory consumption, etc.) which are more precise than those provided by algorithm models.

# Challenges in designing a model of computation

## Theoretical aspects

- ▶ GPU-like architectures introduces many **machine parameters** (like memory sizes, number of cores), and too many could lead to intractable calculations.
- ▶ GPU-like code depends also on **program parameters** (like number of threads per thread-block) which specify how the work is divided among the computing resources.

## Practical aspects

- ▶ Analyzing parametric programs (with unknown machine and program parameters) require **handling non-linear algebraic constraints**.

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# MWP-CWP Background

- ▶ At a high level, MWP-CWP takes in several low-level performance metrics to estimate the execution time of a program.
- ▶ The basic idea behind MWP-CWP is estimating the cost of memory operations which in theory allows for the estimation of performance for parallel CUDA applications.
- ▶ Memory Warp Parallelism (MWP) uses the number of concurrently running threads and memory bandwidth consumption to estimate the cost of these memory operations.
- ▶ On the other hands, Computation Warp Parallelism (CWP) represents the amount of computation done by warps while one warp is waiting for memory values.

# MWP-CWP goals

The MWP-CWP model (Sunpyo Hong & Hyesoon Kim, ISCA 2009)

- ▶ aims at estimating  $T_{\text{exec}}$  as

$$T_{\text{exec}} = T_{\text{comp}} + T_{\text{mem}} - T_{\text{overlap}}$$

- ▶ determining  $T_{\text{overlap}}$  requires to understand whether computations hide memory latency or not
- ▶ thus requires hardware characteristics and instruction counts, thus access to the IR of a program.



# Main observation of MWP-CWP

As we know, memory accesses can be overlapped between warps

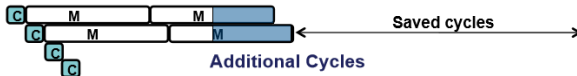
No Parallelism



Infinite Parallelism



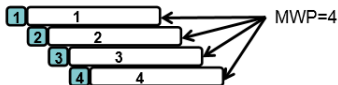
Finite Parallelism  
(Two)



Performance can be predicted by knowing the amount of *memory-level parallelism*.

# Memory Warp Parallelism (MWP)

MWP is the maximum number of warps that can overlap memory accesses.

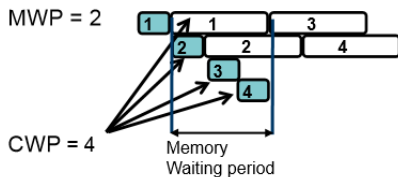


**Four warps are overlapped during memory accesses**

- ▶ Here, we  $MWP = 4$ .
- ▶ MWP is determined by #Active SMs, #Active warps, Bandwidth, Types of memory accesses (Coalesced, Uncoalesced)

# Computation Warp Parallelism (CWP)

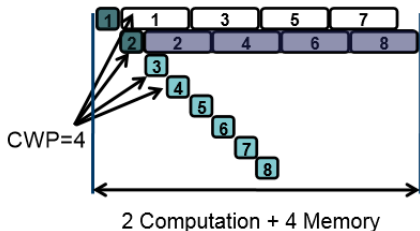
CWP is the number of warps that execute instructions during one memory access period plus one.



Here, we CWP = 4.

# Estimating the number of cycles of a kernel (1/2)

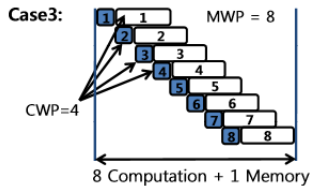
$$MWP \leq CWP$$



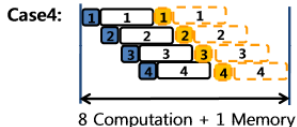
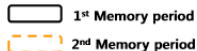
- ▶ Computation cycles are hidden by memory waiting periods
- ▶ Overall performance is dominated by the memory cycles

# Estimating the number of cycles of a kernel (2/2)

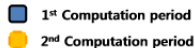
$MWP > CWP$



(a)



(b)



- ▶ Memory accesses are mostly hidden due to high MWP
- ▶ Overall performance is dominated by the computation cycles

See also (Jaewoong Sim & Aniruddha Dasgupta & Hyesoon Kim & Richard Vuduc, PPOPP 12)

# Determining MWP and CWP

Model Parameter	Obtained	Value
Mem_LD	Machine conf.	420
Departure_del_uncoal	Machine conf.	10
#Threads_per_block	Figure 12 Line 1	128
#Blocks	Figure 12 Line 1	80
#Active_blocks_per_SM	Occupancy [22]	5
#Active_SMs	Occupancy [22]	16
#Active_warps_per_SM	$128/32(\text{Table 1}) \times 5$	20
#Comp_insts	Figure 13	27
#Uncoal_Mem_insts	Figure 12 Lines 13, 14	6
#Coal_Mem_insts	Figure 12 Lines 13, 14	0
#Synch_insts	Figure 12 Lines 16, 21	$6 = 2 \times 3$
#Coal_per_mw	see Sec. 3.4.5	1
#Uncoal_per_mw	see Sec. 3.4.5	32
Load_bytes_per_warp	Figure 13 Lines 4, 6	$128B = 4B \times 32$
Departure_delay	Equation (15)	$320 = 32 \times 10$
Mem_L	Equations (10), (12)	$730 = 420 + (32 - 1) \times 10$
MWP_without_BW_full	Equation (16)	$2.28 = 730/320$
BW_per_warp	Equation (7)	$0.175 \text{ GB/S} = \frac{1G \times 128B}{730}$
MWP_peak_BW	Equation (6)	$28.57 = \frac{80GB/s}{0.175GB \times 16}$
MWP	Equation (5)	$2.28 = \text{MIN}(2.28, 28.57, 20)$
Comp_cycles	Equation (19)	$132 \text{ cycles} = 4 \times (27 + 6)$
Mem_cycles	Equation (18)	$4380 = (730 \times 6)$
CWP_full	Equation (8)	$34.18 = (4380 + 132)/132$
CWP	Equation (9)	$20 = \text{MIN}(34.18, 20)$
#Rep	Equation (21)	$1 = 80/(16 \times 5)$
Exec_cycles_app	Equation (23)	$38450 = 4380 \times \frac{20}{2.28} + \frac{132}{6} \times (2.28 - 1)$
Synch_cost	Equation (26)	$12288 = 320 \times (2.28 - 1) \times 6 \times 5$
<b>Final Time</b>	Equation (27)	$50738 = 38450 + 12288$

## MWP-CWP: concluding remarks

- ▶ First analytic model that estimates the execution cycles for GPU
- ▶ Experimentally, quite successful on “average-size” kernels
- ▶ Recalibration for new hardware and in-depth analysis for new applications code
- ▶ Evaluation is fast
- ▶ The model only predicts execution time, no bottlenecks highlighting

# GPUPerf

- ▶ Successor to MWP-CWP (built on top of MWP-CWP with more optimization techniques)
- ▶ Three main components: A frontend data collector, an analytical model, and a performance advisor.
- ▶ MWP-CWP falls short due the assumption that there is "enough intruction-level parallelism"; results in difficulty predicting the effect of prefetching or other optimization techniques revolving around increasing intruction/memory-level parallelism



# Threaded many-core memory (TMM) model

Ma, Agrawal and Chamberlain (2014) introduce the TMM model which retains many important characteristics of GPU-type architectures.

	Description
L	Time for a global memory access
P	Number of processors (cores)
C	Memory access width
Z	Size of fast private memory per core group
Q	Number of cores per core group
X	Hardware limit on number of threads per core

Table: Machine parameters of the TMM model

- ▶ In TMM analysis, the running time of algorithm is estimated by choosing the maximum quantity among the work, span and amount of memory accesses. No Graham-Brent theorem-like is provided.
- ▶ Such running time estimates depend on the machine parameters.

# The MCM model

(S. A, Haque, N, Xie, M., 2013) proposes a many-core machine (MCM) model which aims at

- ▶ **tuning program parameters** to minimize parallelism overheads of algorithms targeting GPU-like architectures as well as
- ▶ **comparing different algorithms** independently of the value of machine parameters of the targeted hardware device.

In the design of this model, we insist on the following features:

- ▶ Two-level DAG programs
- ▶ Parallelism overhead
- ▶ A Graham-Brent theorem

# Characteristics of the abstract many-core machines (1/2)

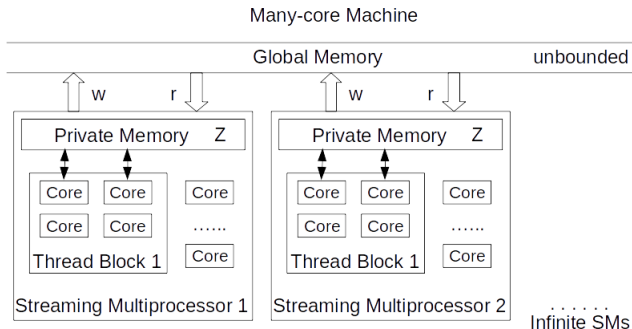


Figure: A many-core machine

- ▶ It has a global memory with high latency, while private memories have low latency.

## Characteristics of the abstract many-core machines (2/2)

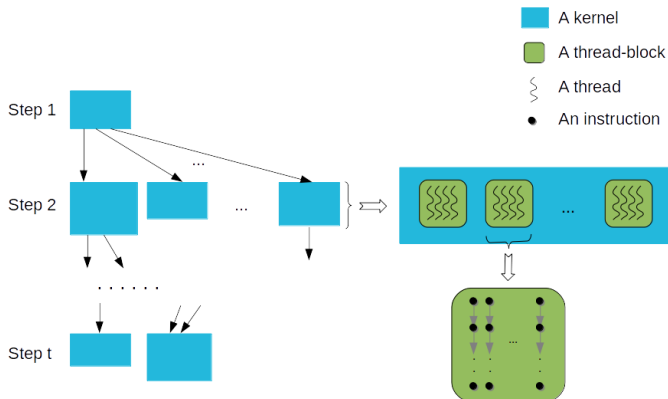


Figure: Overview of a many-core machine program, also called *kernel DAG*

# Machine parameters and complexity measures

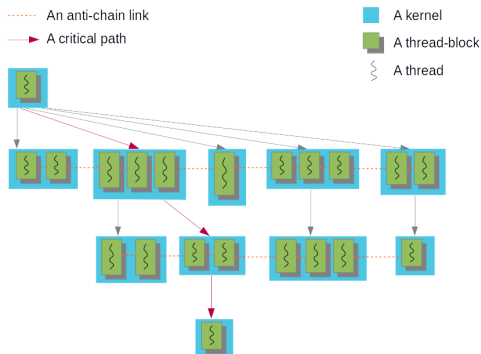
## Machine parameters

- ▶  $Z$ : Private memory size of any SM
- ▶  $U$ : Data transfer time

## Complexity measures

- ▶ **work**  $W(\mathcal{P})$  is the total work of all its kernels;
- ▶ **span**  $S(\mathcal{P})$  is the longest path, counting the weight (span) of each vertex (kernel), in the kernel DAG;
- ▶ **parallelism overhead**  $O(\mathcal{P})$  is the total parallelism overhead (i.e. data transfer time) of all its kernels.

# Characteristic quantities of the thread-block DAG



## A Graham-Brent Theorem

$N(\mathcal{P})$ : number of vertices in the thread-block DAG of  $\mathcal{P}$ ,

$L(\mathcal{P})$ : critical path length (where length of a path is the number of edges in that path) in the thread-block DAG of  $\mathcal{P}$ .

Let  $K$  be the maximum number of thread-blocks along an anti-chain

$$T_{\mathcal{P}} \leq (N(\mathcal{P})/K + L(\mathcal{P}))C(\mathcal{P}) \quad (1)$$

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# Matrix Multiplication

Kernel Input				Elapsed Time (ms)	execution cycles
1024,	4	*	4	101.15	1410580510
1024,	8	*	8	78.11	218486863
1024,	16	*	16	75.79	66568741
1024,	32	*	32	75.15	16295843
512,	4	*	4	71.98	109441566
512,	8	*	8	70.24	19019599
512,	16	*	16	68.76	4381830
512,	32	*	32	69.46	1935861
256,	4	*	4	68.52	9526430
256,	8	*	8	68.69	1253278
256,	16	*	16	68.60	829293
256,	32	*	32	67.39	177923



# 2D Convolution

Kernel Input				Elapsed Time (ms)	execution cycles
4096,	2	*	2	79.38	109913262
4096,	4	*	4	74.28	55541774
4096,	8	*	8	71.30	28262428
4096,	16	*	16	72.26	24299618
4096,	32	*	32	70.99	22364518
2048,	2	*	2	69.59	27478336
2048,	4	*	4	68.63	13885454
2048,	8	*	8	69.57	7065628
2048,	16	*	16	68.79	6074978
2048,	32	*	32	68.71	5591398
1024,	2	*	2	70.21	6869605
1024,	4	*	4	71.78	3471374
1024,	8	*	8	72.08	1766428
1024,	16	*	16	68.26	1518818
1024,	32	*	32	68.61	1397939

# SYRK (Symmetric rank-k update)

Kernel Input				Elapsed Time (ms)	execution cycles
1024,	2	*	2	181.57	883895500801
1024,	4	*	4	126.87	440367566849
1024,	8	*	8	108.14	226630428675
1024,	16	*	16	114.32	206762846988
1024,	32	*	32	157.07	211636194456
512,	2	*	2	88.13	55726686209
512,	4	*	4	75.51	27665691649
512,	8	*	8	72.11	14236023299
512,	16	*	16	73.62	14840789511
512,	32	*	32	77.77	16352705031
256,	2	*	2	69.26	3543429121
256,	4	*	4	69.91	1746976001
256,	8	*	8	68.22	898719361
256,	16	*	16	69.49	936566401
256,	32	*	32	69.53	1031184001

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# Future Work

- ▶ Saturated vs Unsaturated kernels
- ▶ Taking a deeper dive into more performance models focused on program analysis
- ▶ KLARAPTOR

# Concluding Remarks

- ▶ Determining MWP and CWP in the presence of parameters is work in progress
- ▶ Need to explore models that may scale better with newer Nvidia architectures.