

Second Order Partial Derivatives

$$z = f(x, y)$$

First order derivatives:

$$f_x = \frac{\partial f}{\partial x} \quad f_y = \frac{\partial f}{\partial y}$$



$$f_{xx} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial x} \right) \quad f_{yy} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial y} \right)$$

$$f_{xy} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right) \quad f_{yx} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right)$$

So, f_{xx}, f_{yx}, f_{xy} and f_{yy} are 2nd order partial derivatives of

$$z = f(x, y)$$

f_{yx}, f_{xy} are called the mixed partial derivatives.

Example: Compute the 2nd order partial derivatives of $f(x, y) = xy^2 + 3x^2e^y$.

Solution:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} (xy^2 + 3x^2e^y)$$

$$= y^2 \frac{\partial}{\partial x} (x) + 3e^y \frac{\partial}{\partial x} (x^2)$$

$$= y^2(1) + 3e^y(2x)$$

$$f_x = y^2 + 6e^yx$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} (xy^2 + 3x^2e^y)$$

$$= x \frac{\partial}{\partial y} (y^2) + 3x^2 \frac{\partial}{\partial y} (e^y)$$

$$= x(2y) + 3e^yx^2$$

$$f_y = 2xy + 3e^yx^2$$

$$\frac{\partial}{\partial x} \left(\frac{\partial f}{\partial x} \right) = \frac{\partial}{\partial x} (y^2 + 6e^yx)$$

$$f_{xx} = 0 + 6e^y \frac{\partial}{\partial x} (x)$$

$$\frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial x} (2xy + 3e^yx^2)$$

$$f_{yx} = 2y \frac{\partial}{\partial x} (x) + 3e^y \frac{\partial}{\partial x} (x^2)$$

$f_{xx} = 6e^y$	$f_{yx} = 2y + 6xe^y$
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$\frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right) = \frac{\partial}{\partial y} (y^2 + 6e^y x)$ $= \frac{\partial}{\partial y} (y^2) + \frac{\partial}{\partial y} (6xe^y)$	$\frac{\partial}{\partial y} \left(\frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial y} (2xy + 3e^y x^2)$ $= 2x \frac{\partial}{\partial y} (y) + 3x^2 \frac{\partial}{\partial y} (e^y)$
$f_{xy} = 2y + 6xe^y$	$f_{yy} = 2x + 3x^2 e^y$

Calculate the 2nd order partial derivatives of f .

- i. $f(x, y) = x^2 y$
- ii. $f(x, y) = x^2 + 2xy + y^2$
- iii. $f(x, y) = xe^y$
- iv. $f(x, y) = \frac{2x}{y}$
- v. $f(x, y) = 5 + x^2 y^2$
- vi. $f(x, y) = e^{yx}$
- vii. $f(x, y) = 100e^{xy}$
- viii. $f(x, y) = 5xe^{-2y}$
- ix. $f(x, y) = \sin(xy)$
- x. $f(x, y) = x^2 y + \cos y + y \sin x$
- xi. $f(x, y) = \ln(x + y)$
- xii. $f(x, y) = xe^y + y + 1$