Partial Derivatives

Example 1:

$$R = f(x, y)$$

where R = revenue of airline, x = number of full price tickets, and y = number of discounted tickets.

$$R = f(x, y) = 350x + 200y$$

If we fix the number of discountedtickets at y = 10, we have a function one variable which is $R = f(x, 10) = g(x) = 350x + 2000$.

The rate of change of revenue with respect to 'x' is given by $g'(x) = 350$.

This tells us that if we increase the number of full price tickets by one unit then the revenue of airline is increased by $350 while the number of discounted tickets is fixed at 10.We call $g'(x)$*the partial derivative of R with respect to 'x' at point* $(x, 10)$.If$R = f(x, y)$, we can write;

$$\frac{\partial R}{\partial x} = f_x(x, 10) = \frac{\partial}{\partial x}(350x + 2000) = 350$$

Example 2: Find the rate of change of revenue 'R' as 'y' increases with 'x' fixed at $x = 20$.

Solution:

$$R = f(x, y) = 350x + 200y$$

$$R = f(20, y) = 350\,(20) + 200y$$

$$= 7000 + 200y$$

$$\frac{\partial}{\partial y} f(20, y) = 200$$

Partial Derivatives of $f(x, y)$:

The partial derivative of 'f' with respect to 'x' at (a, b) with 'y' fixed or constant is defined as:

$$\frac{\partial f}{\partial x}|_{(a,b)} = \lim_{h \to 0} \frac{f(a+h, b) - f(a, b)}{h}$$

The partial derivative of 'f' with respect to 'y' at (a, b) with 'x' fixed or constant is defined as:

$$\frac{\partial f}{\partial y}|_{(a,b)} = \lim_{k \to 0} \frac{f(a, b+k) - f(a, b)}{k}$$

Alternative Notations for Partial Derivatives:

If $z = f(x, y)$

$$f_x(x, y) = \frac{\partial z}{\partial x} \& f_y(x, y) = \frac{\partial z}{\partial y}$$

$$f_x\ (a, b) = \frac{\partial z}{\partial x}|_{(a,b)} \& f_y(a, b) = \frac{\partial z}{\partial y}|_{(a,b)}$$

Example 3:

An experiment done on rats to measure the toxicity of formaldehyde yielded the data shown in the given table. The values in the table show the percent 'P' of rats that survived an exposure with concentration 'c' (in parts per million) after time, 't' month, so, $P = f(t, c)$.

Estimate $f_t(18, 6) \& f_c\ (18, 6)$ from the table. Interpret your answers.

c (ppm)		**Time 't' months**												
		0	**2**	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**	**20**	**22**	**24**
	0	100	100	100	100	100	100	100	100	100	100	99	97	95
	2	100	100	100	100	100	100	100	100	99	98	97	95	92
	6	100	100	100	99	99	98	96	96	95	93	90	86	80
	15	100	100	100	99	99	99	99	96	93	82	70	58	36

Solution:

$$f_t(18,6) = \frac{\partial f}{\partial t}|_{(18,6)} \approx \frac{f(t + h, c) - f(t,c)}{h}$$

$$\approx \frac{[f(18 + 2, 6) - f(18,6)]}{2}$$

$$\approx \frac{[f(20,6) - f(18,6)]}{2}$$

$$\approx \frac{(90 - 93)}{(20 - 18)} = -1.5\ \%\ per\ month.$$

This partial derivative tells us that after 18 months of exposure to formaldehyde at a concentration of 6 ppm, P decreases by 1.5% for every additional month of exposure.

$$f_c(18,6) = \frac{\partial f}{\partial t}|_{(18,6)} \approx \frac{[f(t, c + k) - f(t,c)]}{k}$$

$$\approx \frac{[f(18, 6 + 9) - f(18,6)]}{9}$$

$$\approx \frac{[f(18,15) - f(18,6)]}{9}$$

$$\approx \frac{(82 - 93)}{(15 - 6)} = -1.22\ \%\ per\ ppm$$

It means that after 18 months of exposure to formaldehyde at a concentration of 6 ppm, P decreases by 1.22% for every additional ppm of concentration.

Estimate $f_t(14, 2)$ & $f_c(14, 2)$ & interpret it.

Estimate $f_t(4, 0)$ & $f_c(4, 0)$ & interpret it.