Calculus and Analytical Geometry

Lecture no. 6

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Topic: Continuity of a function

1. Continuity
2. Discontinuity
3. Graphical concept of continuity and discontinuity
4. Examples
5. Practice questions

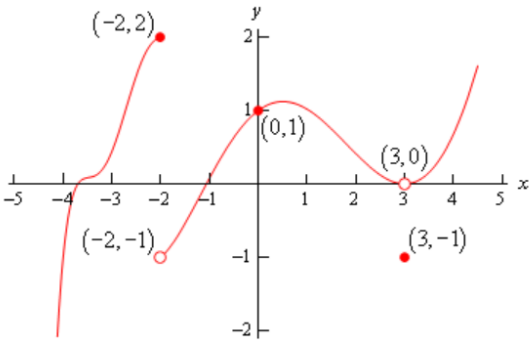
1. **Continuity:**
   A function *f* is said to be *continuous* at a point *x* = *a* if the value of the function is equal to the limit of the function at that point. That is,

$$f(a) = \lim_{x \to a} f(x)$$

2. **Discontinuity:**
   A function *f* is said to be *discontinuous* at a point *x* = *a* if
   a) $f(a)$ does not exist, or
   b) $\lim_{x \to a} f(x)$ does not exist, or
   c) $f(a) \neq \lim_{x \to a} f(x)$

3. **Graphical representation of continuity and discontinuity**

Example 3.1: Determine if the function is continuous or discontinuous at x=-2,0,3



Solution:

a) $\lim_{x \to -2^-} f(x) = 2$

b) $\lim_{x \to -2^+} f(x) = -1$

c) Limit does not exist

d) $f(2) = 2$

e) Function is not continues at -2

f) $\lim_{x \to 0^-} f(x) = 1$

g) $\lim_{x \to 0^+} f(x) = 1$

h) Limit exist

i) $f(0) = 1$

j) Function is continuous at 0

k) $\lim_{x \to 3^-} f(x) = 0$

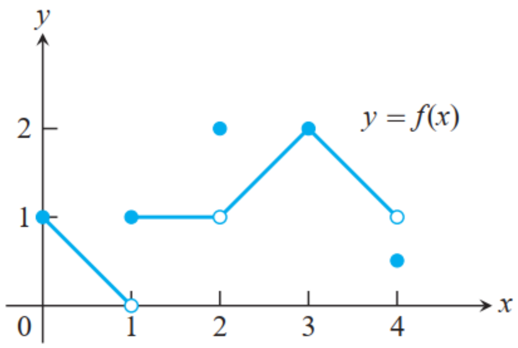
l) $\lim_{x \to 3^+} f(x) = 0$

m) limit exists

n) $f(3) = -1$

o) function is not continuous at 3

Example 3.2: Determine if the given function is continuous or not at the points [0,4]



Solution:

a) $\lim_{x \to 0^-} f(x) = does\ not\ exist$

b) $\lim_{x \to 0^+} f(x) = 1$

c) Limit does not exist

d) $f(0) = 1$

e) Function is not continues at x=0

f) $\lim_{x \to 1^-} f(x) = 0$

g) $\lim_{x \to 1^+} f(x) = 1$

h) Limit does not exist.

i) $f(1) = 1$

j) Function is discontinuous at x=1

k) $\lim_{x \to 2^-} f(x) = 1$

l) $\lim_{x \to 2^+} f(x) = 1$

m) limit exists

n) $f(2) = 2$

o) function is not continuous at x=2

p) $\lim_{x \to 3^-} f(x) = 2$

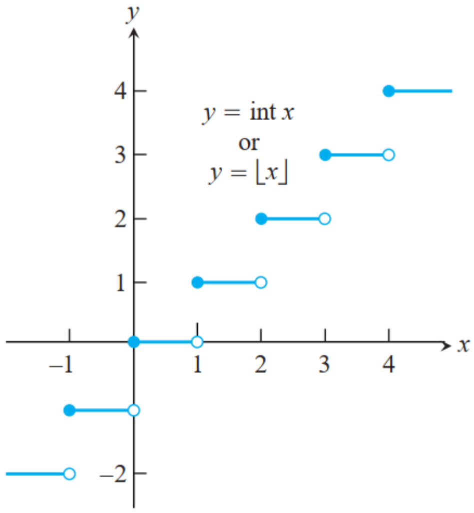
q) $\lim_{x \to 3^+} f(x) = 2$

r) limit exists

s) $f(3) = 2$

t) function is continuous at x=3

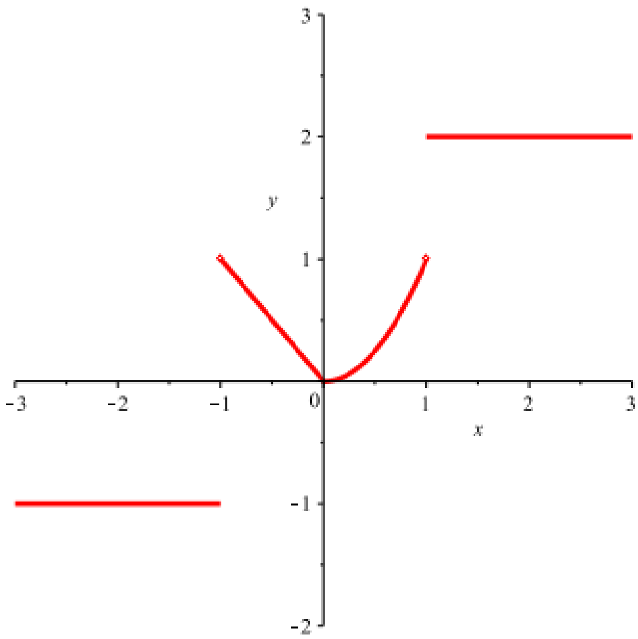
Example 3.3: The greatest integer function



The function is continuous at every non-integer value. It is right continuous but not left continuous for every integer value.

Example 3.4: Consider the following function with its graph:

$$f(x) = \begin{cases} -1 & x < -1 \\ -x & -1 \le x < 0 \\ x^2 & 0 \le x < 1 \\ 2 & 1 \le x \end{cases}$$



Solution:

a) $\lim_{x \to 2^-} f(x) = 2$

b) $\lim_{x \to 2^+} f(x) = 2$

c) $f(2) = 2$

d) f is continuous at $x = 2$.

e) $\lim_{x \to 1^-} f(x) = 1$

f) $\lim_{x \to 1^+} f(x) = 2$

g) $f(1) = 2$

h) f is discontinuous at $x = 1$.

i) $\lim_{x \to 0^+} f(x) = 0$

j) $\lim_{x \to 0^-} f(x) = 0$

k) $f(0) = 0$

l) f is continuous at $x = 0$.

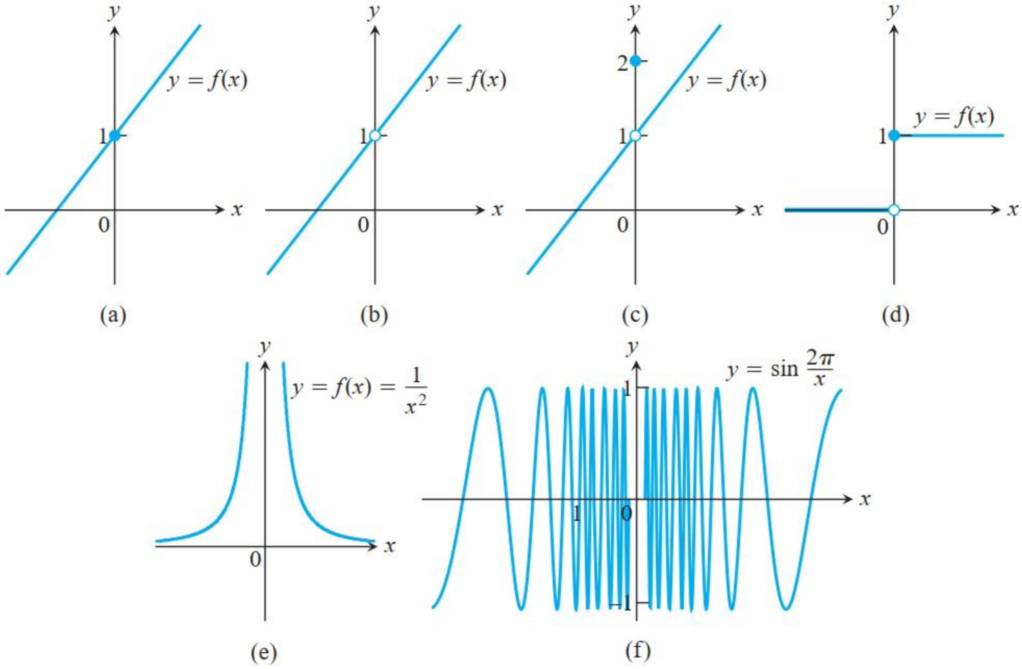
m) $\lim_{x \to -1^+} f(x) = 1$

n) $\lim_{x \to -1^-} f(x) = -1$

o) $f(-1) = 1$

p) f is discontinuous at $x = -1$.

Example 3.5: Determine whether the following functions are continuous on x=0.



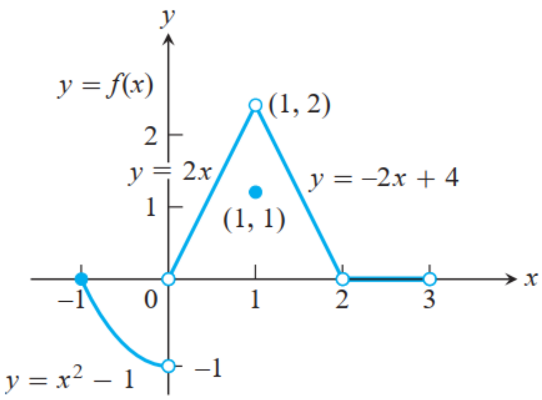
The function (a) is continuous at x=0 and the functions b to f are not continuous at x=0. Why?

Answer: The value of function at x=0 is not defined.

Practice questions:

I. Consider the following function and determine whether the function is continuous at [-1,3] or not:

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$$



II. Consider the following function and find the function is continuous from [-2,5] or not:

$$f(x) = \begin{cases} -1 & x < -2 \\ x + 2 & -2 \le x < -1 \\ 0 & -1 \le x < 0 \\ \sqrt{x} & 0 \le x \le 4 \\ 3 & 4 < x \end{cases}$$