ay: Date: / /20

Chapter 06

Formula of Z-Score

$$Z = \frac{X - \mu}{\sigma}$$

$\mu \rightarrow$ mean

$\sigma \rightarrow$ Standard deviation

Example 6.1

$X \sim N(5, 6)$

$X = 17, \mu = 5, \sigma = 6$

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{17 - 5}{6}$$

$$Z = \frac{12}{6}$$

$$\boxed{Z = 2}$$

Try It 6.1

$X \sim N(12, 3)$, $x = 1$, $z = ?$

$$Z = \frac{X - \mu}{\sigma}$$

$x = 1, \mu = 12, \sigma = 3$

$$Z = \frac{1 - 12}{3}$$

$$Z = \frac{-11}{3}$$

$$\boxed{Z = -3.667}$$

Try It 6.4

$X \sim N(496, 114)$

(A) $X_1 = 325, \mu = 496, \sigma = 114$

$$Z_1 = \frac{X_1 - \mu}{\sigma}$$

$$Z_1 = \frac{325 - 496}{114} \rightarrow \frac{-171}{114}$$

$$\boxed{Z_1 = -1.5}$$

(B) $X_2 = 366.21, \mu = 496, \sigma = 114$

$$Z_2 = \frac{X_2 - \mu}{\sigma}$$

Day: Date: __/__/__

$$z_2 = \frac{366.21 - 496}{114} \rightarrow \frac{-129.79}{114}$$

$$\boxed{z_2 = -1.14}$$

Student(2) scored closer to the mean then student(1) and since they both had negative z-score student(2) had better score.

Try it 6.6

(a) 68%

$68\% = \mu \pm 1\sigma$

One standard Deviation

$\mu = 52$, $\sigma = 11$

$52 + 11 = 63$, $52 - 11 = 41$

63 and 41

(b) 95%

$95\% = \mu \pm 2\sigma$

$\mu = 52$, $\sigma = 11$

$52 + 11 + 11 = 74$, $52 - 11 - 11 = 30$

74 and 30

(c) 99.7%

$99.7\% = \mu \pm 3\sigma$

$\mu = 52$, $\sigma = 11$

$52 + 11 + 11 + 11 = 85$, $52 - 11 - 11 - 11 = 19$

85 and 19

Day: _____ Date: __/__/20__

Normal Distribution

Formulas

(1) $P(z \leq a) = \phi(a)$

(2) $P(z \geq a) = 1 - P(z \leq a)$ Right

(3) $P(a \leq z \leq b) = \phi(b) - \phi(a)$

Example 6.7

If the area to the left of x is 0.0228, then what to the right

Solution:

$$P(z \geq a) = 1 - P(z \leq a)$$

$$= 1 - 0.0228$$

The area to the right is 0.9772

Try It 6.7

If the area to the left of x is 0.012, then what is the are to the right

Solution:

$$P(z \geq a) = 1 - P(z \leq a)$$

$$= 1 - 0.012$$

The area to the right is 0.988

Day: Date:

Example 6.8

$X \sim N(63, 5)$

$x = 65, \mu = 63, \sigma = 5$

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{65 - 63}{5} \rightarrow \frac{2}{5}$$

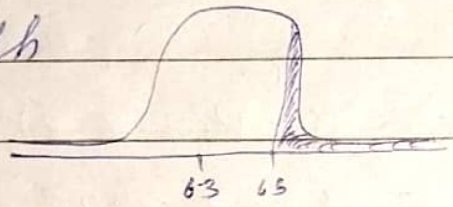
$$\boxed{z = 0.4}$$

$$\therefore P(z \geq 65) = 1 - P(z \leq 65)$$

$$1 - .65542$$

$$= 0.3446$$

Graph



Try It 6.8

$\mu = 68, \sigma = 3$

$$P(x < 65) = P\left(\frac{x - \mu}{\sigma} < \frac{65 - 68}{3}\right)$$

$$= \frac{-3}{3}$$

$$\boxed{z = -1}$$

$$P(z < -1) = 0.15866$$

Try It 6.9

$\mu = 68, \sigma = 3$, between 66 and 70

$$P(a < x < b) = \phi(b) - \phi(a)$$

$$P(66 < x < 70) = P\left(\frac{66 - 68}{3} < z < \frac{70 - 68}{3}\right)$$

$$= P(-2/3 < z < 2/3)$$

$$P(-0.6 < z < 0.6) = \phi(b) - \phi(a)$$

$$P(.27425 < z < .72575) = 0.4515$$

Day: _____ Date: __/__/20__

Example 6.10

Standard deviation 36.9 and 13.9 Smartphone user in the age 13 to 15+ is between 23 and 64.7

Solution:

(a) $P(a < Z < b) = \phi(b) - \phi(a)$

$P(23 < x < 64.7) = \phi(b) - \phi(a)$

$= P\left(\frac{23 - 36.9}{13.9} < Z < \frac{64.7 - 36.9}{13.9}\right)$

$= P\left(\frac{-13.9}{13.9} < Z < \frac{27.8}{13.9}\right)$

$P(-1 < Z < 2) = \phi(2) - \phi(-1)$ (81859)

$P(.15866 < .97725) = 0.8186$

(b) $\mu = 36.9, \sigma = 13.9$

$P(x < 50.8) = P\left(\frac{x - \mu}{\sigma} < \frac{50.8 - 36.9}{13.9}\right)$

$= \frac{13.9}{13.9}$

$Z = 1$

$P(Z < 1) = 0.8413$

(c) 80th Percentile

$\mu = 36.9, \sigma = 13.9$

$P(x < k) = 0.80$

$P\left(\frac{x - \mu}{\sigma} < \frac{k - 36.9}{13.9}\right) = 0.80$

$P\left(Z < \frac{k - 36.9}{13.9}\right) = 0.80$

$\phi\left(\frac{k - 36.9}{13.9}\right) = 0.80$

Day: Date: /

$$\frac{k-36.9}{13.9} = \phi^{-1}(0.80)$$

$$\frac{k-36.9}{13.9} = 0.85$$

$$k-36.9 = 0.85 \times 13.9$$

$$k-36.9 = 11.815 \rightarrow k = 11.815 + 36$$

$$\boxed{k = 48.715}$$

Example 6.9

Between 1.8 and 2.75.

$X \sim N(2, 0.5)$

$\mu = 2, \sigma = 0.5$

$P(a \leq x \leq b) = \phi(b) - \phi(a)$

$P(1.8 < X < 2.75)$

$$= P\left(\frac{1.8-2}{0.5} < Z < \frac{2.75-2}{0.5}\right)$$

$$= P\left(\frac{-0.2}{0.5} < Z < \frac{0.75}{0.5}\right)$$

$$P(-0.4 < Z < 1.5)$$

$\therefore$ Table $-0.4 = .34458, 1.5 = .93319$

$P(-0.4 < Z < 1.5) = 0.5886$

Example 6.12

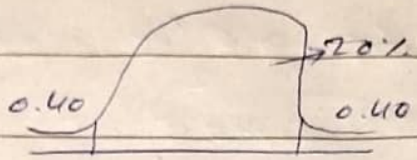
(a)

Day: _____ Date: __/__/20__

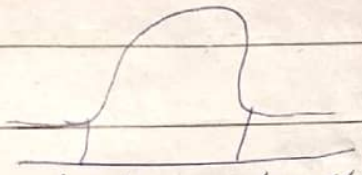
(b) middle 20%

20% = 0.20

$1 - 0.20 = 0.80 \rightarrow \frac{0.80}{2} = 0.40$



Now find k_1 is 40th percentile and k_2 is 60th percentile because 0.40 + 0.20



40th percentile

60th percentile (0.40 + 0.20)

Now

$\mu = 5.85$, $\sigma = 0.24$

40th percentile	60th percentile
$P(X \leq k_1) = 0.40$	$P(X \leq k_2) = 0.60$
$P\left(\frac{x-\mu}{\sigma} \leq \frac{k_1 - 5.85}{0.24}\right) = 0.40$	$P\left(\frac{x-\mu}{\sigma} \leq \frac{k_2 - 5.85}{0.24}\right) = 0.60$
$P\left(Z \leq \frac{k_1 - 5.85}{0.24}\right) = 0.40$	$P\left(Z \leq \frac{k_2 - 5.85}{0.24}\right) = 0.60$
$\phi\left(\frac{k_1 - 5.85}{0.24}\right) = 0.40$	$\phi\left(\frac{k_2 - 5.85}{0.24}\right) = 0.60$
$\frac{k_1 - 5.85}{0.24} = \phi^{-1} 0.40$	$\frac{k_2 - 5.85}{0.24} = \phi^{-1} 0.60$
$\frac{k_1 - 5.85}{0.24} = -0.25$	$\frac{k_2 - 5.85}{0.24} = 0.26$
$k_1 - 5.85 = -0.25 \times 0.24$	$k_2 - 5.85 = 0.26 \times 0.24$
$k_1 - 5.85 = -0.06$	$k_2 - 5.85 = 0.0624$
$k_1 = -0.06 + 5.85$	$k_2 = 0.0624 + 5.85$
$\boxed{k_1 = 5.79}$	$\boxed{k_2 = 5.91}$

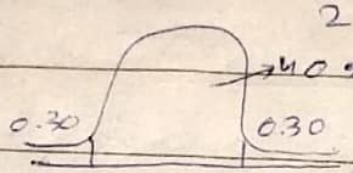
Day:_____ Date: __/__/20

Try It 6.12

(a) middle 40%

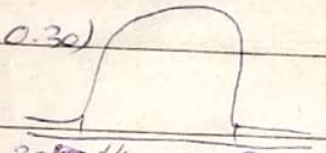
40% = 0.40

$1 - 0.40 = 0.60 \rightarrow \frac{0.60}{2} = 0.30$



Now find k_1 is 40th percentile and k_2 is 60th percentile because

(0.60 + 0.30)



Now

$\mu = 5.85$, $\sigma = 0.24$

30th percentile	70th percentile
$P(X \leq k_1) = 0.30$	$P(X \leq k_2) = 0.70$
$P\left(\frac{X-\mu}{\sigma} \leq \frac{k_1 - 5.85}{0.24}\right) = 0.30$	$P\left(\frac{X-\mu}{\sigma} \leq \frac{k_2 - 5.85}{0.24}\right) = 0.70$
$P\left(Z \leq \frac{k_1 - 5.85}{0.24}\right) = 0.30$	$P\left(Z \leq \frac{k_2 - 5.85}{0.24}\right) = 0.70$
$\Phi\left(\frac{k_1 - 5.85}{0.24}\right) = 0.30$	$\Phi\left(\frac{k_2 - 5.85}{0.24}\right) = 0.70$
$\frac{k_1 - 5.85}{0.24} = \Phi^{-1} 0.30$	$\frac{k_2 - 5.85}{0.24} = \Phi^{-1} 0.70$
$\frac{k_1 - 5.85}{0.24} = -0.52$	$\frac{k_2 - 5.85}{0.24} = 0.53$
$k_1 - 5.85 = -0.52 \times 0.24$	$k_2 - 5.85 = 0.53 \times 0.24$
$k_1 - 5.85 = -0.1248$	$k_2 - 5.85 = 0.1272$
$k_1 = -0.1248 + 5.85$	$k_2 = 0.1272 + 5.85$
$\boxed{k_1 = 5.7252}$	$\boxed{k_2 = 5.9772}$

Date: __/__/20__

(b) 16th percentile

$\mu = 5.85$, $\sigma = 0.24$

$P(x \leq k) = 0.16$

$P\left(\frac{x-\mu}{\sigma} \leq \frac{k-5.85}{0.24}\right) = 0.16$

$P\left(Z \leq \frac{k-5.85}{0.24}\right) = 0.16$

$\Phi\left(\frac{k-5.85}{0.24}\right) = 0.16$

$\frac{k-5.85}{0.24} = \Phi^{-1} 0.16$

$\frac{k-5.85}{0.24} = -0.99$

$k - 5.85 = -0.99 \times 0.24$

$k - 5.85 = -0.2376$

$k = -0.2376 + 5.85$

$k = 5.6124$

Practice Question (8)

What does a z-score measure

A z-score measures the number of standard deviation from the mean. The z-score can be defined as the numerical measurement of the

Day:_____ Date: 1-12-20

relationship between a group of values and the mean.

10

A standardized normal distribution is that when mean is zero and standard deviation is one.

important → So $X \sim N(0, 1)$ is a standardized normal distribution because it has a mean of 0 and a standard deviation of 1. The notation 'X' indicate that the random variable X.

11

In this case the value of X is two standard deviation above the mean. So the z-score would be 2.

12

In this case the value of X is 1.5 standard deviation below the mean, So the z-score would be -1.5

change

Date: __/__/20

13

See Answer to (Q.12)

-2.78

14

See Answer to (Q.12)

-0.133

15

$X \sim N(2, 6)$

$Z = 3, \mu = 2, \sigma = 6$

$x = ?$

$X = Z \times \sigma + \mu$

$X = 3 \times 6 + 2$

$\boxed{X = 20}$

16

$X \sim N(8, 1)$

$Z = -2.25, \mu = 8, \sigma = 1$

$x = ?$

$X = Z \times \sigma + \mu$

$X = -2.25 \times 1 + 8$

$\boxed{X = 5.75}$

17

$X \sim N(9, 5)$

$Z = -0.5, \mu = 9, \sigma = 5$

$x = ?$

Day: Date: 1/20

$x = z \times \sigma + \mu$

$x = -0.5 \times 5 + 7$

$\boxed{x = 6.5}$

18

$X \sim N(2, 3)$

$z = -0.67, \mu = 2, \sigma = 3$

$x = ?$

$x = z \times \sigma + \mu$

$x = -0.67 \times 3 + 2$

$\boxed{x = -0.01}$

19

$X \sim N(4, 2)$

$x = ?$ [redacted] left

$z = 1.5, \mu = 4, \sigma = 2$

$x = (\mu - z \times \sigma)$

$x = 4 - 1.5 \times 2$

$\boxed{x = 1}$

20

$X \sim N(4, 2)$

$x = ?$ right [redacted]

$z = 2, \mu = 4, \sigma = 2$

[redacted] $x = z \times \sigma + \mu$

[redacted] $x = 2 \times 2 + 4$

$\boxed{x = 8}$

Date: __/__/20__

21

$X \sim N(8, 9)$ Left

$X = ?$

$Z = 0.67, \mu = 8, \sigma = 9$

$X = \mu - Z \times \sigma$

$X = 8 - 0.67 \times 9$

$\boxed{X = 1.97}$

22

$X \sim N(-1, 2)$

$Z = ?$

$X = 2, \mu = -1, \sigma = 2$

$Z = \frac{X - \mu}{\sigma}$

$Z = \frac{2 - (-1)}{2}$

$Z = \frac{2 + 1}{2}$

$Z = 3/2$

$\boxed{Z = 1.5}$

23

$X \sim N(12, 6)$

$Z = ?$

$X = 2, \mu = 12, \sigma = 6$

$Z = \frac{X - \mu}{\sigma}$

$Z = \frac{2 - 12}{6}$

Day:_____ Date: __/__/20__

$$Z = -10/6$$

$$\boxed{Z = -1.67}$$

24

$$X \sim N(9, 3)$$

$$Z = ?$$

$$X = 9, \mu = 9, \sigma = 3$$

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{9 - 9}{3}$$

$$Z = 0/3$$

$$\boxed{Z = 0}$$

25

$$\mu = 6, \sigma = 1.5, X = 5.5$$

$$Z = ?$$

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{5.5 - 6}{1.5}$$

$$Z = -0.5/1.5$$

$$\boxed{Z = -0.33}$$

Date: __/__/20__

Try It 6.10

(a) Find 30th percentile

$P(x < k) = 0.30$

$\mu = 36.9, \sigma = 13.9$

$P\left(\frac{x-\mu}{\sigma} \leq \frac{k-36.9}{13.9}\right) = 0.30$

$P\left(Z \leq \frac{k-36.9}{13.9}\right) = 0.30$

$\Phi\left(\frac{k-36.9}{13.9}\right) = 0.30$

$\frac{k-36.9}{13.9} = \Phi^{-1} 0.30$

$\frac{k-36.9}{13.9} = -0.52$

$k - 36.9 = -0.52 \times 13.9$

$k - 36.9 = -7.228$

$k = -7.228 + 36.9$

$\boxed{k = 29.672}$

(b)

$\mu = 36.9, \sigma = 13.9$

$P(x < 27)$

$P(x < 27) = P\left(\frac{x-\mu}{\sigma} < \frac{27-36.9}{13.9}\right)$

$= \frac{-9.9}{13.9}$

$Z = -0.7122$

$P(Z < -0.7122) = 0.24196$