Derivative of a Vector Valued Function

Example 1: Find the derivative of

$$\vec{r}(t) = \langle\, t, t^2, t^3 \rangle$$

Solution:

$$\vec{r}'(t) = \langle\, 1, 2t, 3t^2 \rangle$$

Example 2: Find the derivative of

$$\vec{r}(t) = (t+1)\vec{\imath} + (t^2 - 1)\vec{\jmath}$$

Solution:

$$\vec{r}'(t) = \vec{\imath} + 2t\vec{\jmath}$$

Example 3: Find the derivative of

$$\vec{r}(t) = e^t\vec{\imath} + \frac{2}{9}e^{2t}\vec{\jmath}$$

Solution:

$$\vec{r}'(t) = e^t\vec{\imath} + \frac{4}{9}e^{2t}\vec{\jmath}$$

Example 4: Find the derivative of

$$\vec{r}(t) = cos2t\vec{\imath} + 3sin2te^{2t}\vec{\jmath}$$

Solution:

$$\vec{r}'(t) = -2sin2t\vec{\imath} + 6e^{2t}[sin2t + cos2t]\vec{\jmath}$$

Formulas of Derivative

1. $\frac{d}{dt}(c) = 0$
2. $\frac{d}{dt}(t) = 1$
3. $\frac{d}{dt}(sinat) = acosat$
4. $\frac{d}{dt}(cosat) = -a\ sinat$
5. $\frac{d}{dt}(lnt) = \frac{1}{t}$
6. $\frac{d}{dt}(e^{at}) = ae^{at}$

Velocity, Speed, Acceleration and Direction

If $\vec{r}(t)$ is a position vector of a particle moving along a smooth curve in space, then

Velocity $\vec{v}(t) = \frac{d\vec{r}}{dt}$

Speed $s(t) = |\vec{v}(t)|$

Acceleration $\vec{a}(t) = \frac{d\vec{v}}{dt}$

Direction The unit vector $\hat{v} = \frac{\vec{v}}{|\vec{v}|}$ is the direction of motion at time t.

Example 1: A person on a hang glider is spiralling upward due to rapidly rising air on a path having position vector

$$\vec{r}(t) = 3cost\vec{i} + 3sint\vec{j} + t^2\vec{k}$$

Find

- The velocity and acceleration vectors.
- The glider's speed at any time t.

Solution:

$$\vec{r}(t) = 3cost\vec{i} + 3sint\vec{j} + t^2\vec{k}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3cost\vec{i} + 3sint\vec{j} + t^2\vec{k})$$

$$\vec{v}(t) = -3sint\vec{i} + 3cost\vec{j} + 2t\vec{k})$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}(-3sint\vec{i} + 3cost\vec{j} + 2t\vec{k})$$

$$\vec{a}(t) = -3cost\vec{i} - 3sint\vec{j} + 2\vec{k}$$

Speed is the magnitude of $\vec{v}(t)$:

$$|\vec{v}(t)| = \sqrt{(-3sint)^2 + (3cost)^2 + (2t)^2}$$

$$|\vec{v}(t)| = \sqrt{9sin^2t + 9cos^2t + 4t^2}$$

$$|\vec{v}(t)| = \sqrt{9 + 4t^2}$$

Practice Problems

Q1: The position vector of an object in a plane is given by $\vec{r}(t) = t^3\vec{i} + t^2\vec{k}$. Find its velocity, speed and acceleration when t = 1.

Q2: Find the velocity, acceleration and speed of a particle with position vector $\vec{r}(t) = t^2\vec{i} + e^t\vec{j} + te^t\vec{k}$.

Ex. 13.1: 1-4,9-14