Intersection of a Line &a Plane:

Intersection of line & plane is either a line or a point.

Example:

Find the point where $x = 8/3 + 2t$, $y = -2t$, $z = 1 + t$ intersects the plane $3x + 2y + 6z = 6$.

Solution:

$3x + 2y + 6z = 6$

$3(8/3 + 2t) + 2(-2t) + 6(1 + t) = 6$

$8 + 6t - 4t + 6 + 6t = 6$

$8t + 14 = 6$

$8t = 6 - 14$

$8t = -8$

$t = -1$

The point of intersection is

$x = 8/3 + 2(-1) = 2/3$

$y = -2(-1) \quad = 2$

$z = 1 + (-1) \quad = 0$

Point of intersection is $(2/3, 2, 0)$

Ex. 12.5: 53-56

In Exercises 53–56, find the point in which the line meets the plane.

53. $x = 1 - t, \quad y = 3t, \quad z = 1 + t; \quad 2x - y + 3z = 6$

54. $x = 2, \quad y = 3 + 2t, \quad z = -2 - 2t; \quad 6x + 3y - 4z = -12$

55. $x = 1 + 2t, \quad y = 1 + 5t, \quad z = 3t; \quad x + y + z = 2$

56. $x = -1 + 3t, \quad y = -2, \quad z = 5t; \quad 2x - 3z = 7$

Intersection of two Planes:

Intersection of two planes iseither a plane or a line.

Example: Find the parametric equations for the line in which the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$ intersect.

Solution:

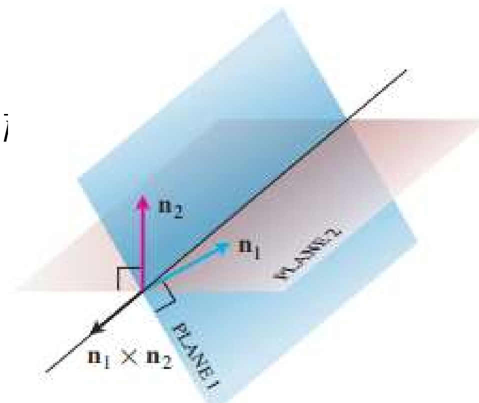
As two planes are not parallel so the intersection is a line.

For the line we need a vector and the point of intersection of these two planes.

The line of intersection of two planes is perpendicular to both the plane normal vectors $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$ and therefore, parallel to $\vec{n}_1 \times \vec{n}_2$.

So $\quad \vec{n}_1 = 3\vec{\imath} - 6\vec{\jmath} - 2\vec{k}$

$$\vec{n}_2 = 2\vec{\imath} + \vec{\jmath} - 2\vec{k}$$



$$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix}$$

$$\vec{v} = \vec{\imath}\begin{vmatrix} -6 & -2 \\ 1 & -2 \end{vmatrix} - \vec{\jmath}\begin{vmatrix} 3 & -2 \\ 2 & -2 \end{vmatrix} + \vec{k}\begin{vmatrix} 3 & -6 \\ 2 & 1 \end{vmatrix}$$

$$\vec{v} = \vec{\imath}(12 + 2) - \vec{\jmath}(-6 + 4) + \vec{k}(3 + 12)$$

$$\vec{v} = 14\vec{\imath} + 2\vec{\jmath} + 15\vec{k}$$

To find the point of intersection we take the equations of these two planes.

$$3x - 6y - 2z = 15$$

$$2x + y - 2z = 5$$

As the number of equations is less than the number of variables so the system of equations has infinite many solutions.From these infinitely many solutions we need only one solution.

Let us substitute z =0, then the equations of planes become

$$3x - 6y = 15 \qquad \rightarrow (\mathbf{1})$$

$$2x + y = 5 \qquad \rightarrow (\mathbf{2})$$

Solving equations (1) and (2) simultaneously

$$3x - 6y = 15$$

$$\pm 12x \pm 6y = \pm 30$$

$$15\,x = 45$$

$$\mathrm{x} = 3$$

put in equation (1)

$3(3) - 6y = 15$

$-6y = 15 - 9$

$y = 1$

So, the point of intersection is

$$P = (3, -1, 0)$$

Equation of line is

$x = x_o + tv_1 = 3 + 14t$

$y = y_o + tv_2 = -1 + 2t$

$z = z_o + tv_3 = 0 + 15t\ ; -\infty < t < \infty$

Ex. 12.5: 57-60

Find parametrizations for the lines in which the planes in Exercises 57–60 intersect.

57. $x + y + z = 1, \quad x + y = 2$

58. $3x - 6y - 2z = 3, \quad 2x + y - 2z = 2$

59. $x - 2y + 4z = 2, \quad x + y - 2z = 5$

60. $5x - 2y = 11, \quad 4y - 5z = -17$