# Calculus and Analytical Geometry

# Lecture no. 09

# Amina Komal

# April 2022

**Topic:** Techniques of Differentiation

**Outline of the lecture:**

i. Derivation of a constant function
ii. Power rule
iii. Constant multiple rule
iv. Sum and difference rules
v. Derivative of trigonometric functions
vi. Examples
vii. Practice questions

**Rules of Differentiation:**

Some basic rules for differentiation are given below.

1. **Derivation of a constant function:**

> Let function is a constant c, then $\frac{d}{dx}(c)=0$

**Example:** Find the derivative of $f(x) = 15$

**Solution:**

$$f'(x) = 0$$

2. **Power Rule:**

> If n is a positive integer, then $\frac{d}{dx}(x^n) = nx^{n-1}$

**Example:** Differentiate $f(x) = x^4$

**Solution:**

$$\frac{d}{dx}(x^4) = 4x^{4-1} = 4x^3$$

3. **Constant multiple rule:**

> If c is a constant and f is a differentiable function, then
> $$\frac{d}{dx}[cf(x)] = c\,\frac{d}{dx}f(x)$$

**Example 1:** Differentiate $5x^3$ with respect to x.

**Solution:**

$$f'(x) = \frac{d}{dx}[5x^3] = 5\frac{d}{dx}x^3$$

$$= 5(3x^{3-1})$$

$$= 5(3x^2)$$

$$= 15x^2$$

**Example 2:** Differentiate $\frac{3}{\sqrt{x}}$ with respect to x.

**Solution:**

$$f'(x) = \frac{d}{dx}\left[\frac{3}{\sqrt{x}}\right] = 3\frac{d}{dx}x^{-\frac{1}{2}}$$

$$= -3\left(\frac{1}{2}x^{\frac{1}{2}-1}\right)$$

$$= -3\left(\frac{1}{2}x^{-\frac{3}{2}}\right) = -\frac{3}{2}x^{-\frac{3}{2}}$$

$$= -\frac{3}{2x^{\frac{3}{2}}}$$

4. **Sum and Difference Rules:**

If f and g are both differentiable functions, then

$$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

**Example 1:** Differentiate $2x^6 + x^{-9}$

**Solution:**

$$f'(x) = \frac{d}{dx}(2x^6 + x^{-9}) = \frac{d}{dx}2x^6 + \frac{d}{dx}x^{-9}$$

$$= \frac{d}{dx}2x^6 + \frac{d}{dx}x^{-9}$$

$$= 2\frac{d}{dx}x^6 + \frac{d}{dx}x^{-9}$$

$$= 2(6x^{6-1}) - 9(x^{-9-1})$$

$$= 12x^5 - 9x^{-10}$$

**Example 2:** Differentiate $\frac{\sqrt{x}-2}{\sqrt{x}}$

**Solution:**

$$f'(x) = \frac{d}{dx}\left(\frac{\sqrt{x}-2x}{\sqrt{x}}\right) = \frac{d}{dx}\left(1 - 2\sqrt{x}\right)$$

$$= \frac{d}{dx}(1) - 2\frac{d}{dx}\sqrt{x}$$

$$= 0 - 2\frac{1}{2}x^{\frac{1}{2}-1} = -x^{-\frac{1}{2}}$$

$$= -\frac{1}{\sqrt{x}}$$

**Some Examples involving these 4 rules:**

I. Differentiate $f(x) = 3x^8 + 2x^2 + 1$

**Solution:**

$$f'(x) = \frac{d}{dx}(3x^8 + 2x^2 + 1)$$

$$= \frac{d}{dx}(3x^8) + \frac{d}{dx}(2x^2) + \frac{d}{dx}(1)$$

$$= 3\frac{d}{dx}(x^8) + 2\frac{d}{dx}(x^2) + \frac{d}{dx}(1)$$

$$= 3(8x^{8-1}) + 2(2x^{2-1}) + 0$$

$$= 24x^7 + 4x$$

II. Differentiate $f(x) = -\frac{1}{3}(x^7 + 2x - 9)$

**Solution:**

$$f'(x) = \frac{d}{dx}\left[-\frac{1}{3}(x^7 + 2x - 9)\right]$$

$$= -\frac{1}{3}\frac{d}{dx}(x^7 + 2x - 9)$$

$$= -\frac{1}{3}\left(\frac{d}{dx}(x^7) + 2\frac{d}{dx}x - \frac{d}{dx}9\right)$$

$$= -\frac{1}{3}(7x^{7-1} + 2(1) - 0)$$

$$= -\frac{7}{3}x^6 - \frac{2}{3}$$

III. Differentiate $f(x) = x^e + \frac{1}{x^{\sqrt{10}}}$

**Solution:**

$$f'(x) = \frac{d}{dx}\left[x^e + \frac{1}{x^{\sqrt{10}}}\right]$$

$$= \left[\frac{d}{dx}x^e + \frac{d}{dx}x^{-\sqrt{10}}\right]$$

$$= ex^{e-1} - \sqrt{10}x^{-\sqrt{10}-1}$$

**IV.** Differentiate $f(x) = \sqrt[3]{\frac{8}{x}}$

**Solution:**

$$f'(x) = \frac{d}{dx}\sqrt[3]{\frac{8}{x}}$$

$$= \frac{d}{dx}\left(\frac{8}{x}\right)^{\frac{1}{3}}$$

$$= \frac{d}{dx}(8x^{-1})^{\frac{1}{3}}$$

$$= \frac{d}{dx}\left(8^{\frac{1}{3}}x^{-\frac{1}{3}}\right)$$

$$= 2x^{\frac{-4}{3}}$$

$$= \frac{2}{x^{\frac{4}{3}}}$$

**V.** Find $\frac{dy}{dx}|_{x=1}$ where $f(x) = x^{24} + 2x^{12} + 3x^8 + 4x^6$

**Solution:**

$$f'(x) = 24x^{24-1} + 2(12)x^{12-1} + 3(8)x^{8-1} + 4(6)x^{6-1}$$

$$= 24x^{23} + 24x^{11} + 24x^7 + 24x^5$$

$substitute\ x = 1$

$$\frac{dy}{dx}|_{x=1} = 24(1) + 24(1) + 24(1) + 24(1)$$

$$= 96$$

**Class Practice Question:**

Differentiate $f(x) = (5x^2 - 3)(7x^3 + x)$

**Solution:**

$$f'(x) = \frac{d}{dx}[(5x^2)(7x^3) + 5x^2(x) - 3(7x^3) - 3x]$$

$$= \frac{d}{dx}35x^5 + \frac{d}{dx}5x^3 - \frac{d}{dx}21x^3 - \frac{d}{dx}3x$$

$$= 175x^4 + 15x^2 - 63x^2 - 3$$

**Derivative of trigonometric functions:**

$$\frac{d}{dx}(sinx) = cosx \qquad \frac{d}{dx}(cscx) = -cscxcotx$$

$$\frac{d}{dx}(cosx) = -sinx \qquad \frac{d}{dx}(secx) = secxtanx$$

$$\frac{d}{dx}(tanx) = sec^2x \qquad \frac{d}{dx}(cotx) = -csc^2x$$

**Examples:**

**a.** Find the derivative of $f(x) = 4cosx + 2sinx$

$$f'(x) = \frac{d}{dx}(4cosx + 2sinx)$$

$$= 4\frac{d}{dx}cosx + 2\frac{d}{dx}sinx$$

$$= -4sinx + 2cosx$$

**b.** Find the derivative of $f(x) = secx - \sqrt{2}tanx$

$$f'(x) = \frac{d}{dx}\left(secx - \sqrt{2}tanx\right)$$

$$= secxtanx - \sqrt{2}sec^2x$$

**Practice Questions:**

Differentiate the following functions:

i.      $f(x) = (3x^2 + 3)^2$

ii.      $f(x) = 7x^{-6} - 5\sqrt{x}$

iii.      $f(x) = ax^4 + bx^3 + cx^{-2}$

iv.      $f(x) = \sqrt{x} + \frac{1}{x}$

v.      $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x$

vi.      $f(x) = (x^3 - 5)(2x + 3)$

vii.      $f(x) = 4secx - cotx$