Vector Valued Function

Here we study functions whose values are vectors because such functions are needed to describe curves and surfaces in space. We will also use vector valued functions to describe the motion of objects through space.

- A vector valued function is a function whose domain is the set of real numbers and whose range is the set of vectors.

Vector Valued function is denoted by $\vec{r}(t)$

$$\vec{r}(t) = f(t)\vec{\imath} + g(t)\vec{\jmath} + h(t)\vec{k}$$

Here t is the parameter for some interval I on the common domain.

We can write $\vec{r}(t)$ as

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$

- The terminal point of a vector valued function creates a curve.
- A vector valued function is a parametrically defined function, which generates vectors whose terminal points trace a curve in 3D.

For example, $\vec{r}(0) = \langle -3,5,2 \rangle$is a vector with

Terminal Point: (-3,5,2) and Initial Point: (0,0,0)

Domain of a Vector Valued Function

Example 1:

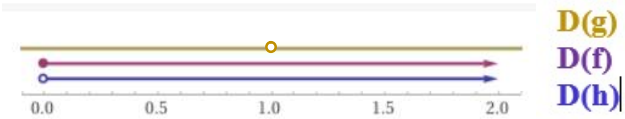
$$\vec{r}(t) = \langle \sqrt{t}\,, \frac{1}{t-1}, \ln t \rangle$$

Solution: Domain of $f(t)$= $[0, \infty)$

Domain of $g(t)$= $R\backslash\{1\}$

Domain of $h(t)$= $(0, \infty)$

Domain of $\vec{r}(t)$= $(0,1) \cup (1, \infty)$



Can you figure out two specific vectors?

Find the vectors for $t = 2,4$

- $t = 2, \quad \vec{r}(2) = \langle \sqrt{2}, 1, ln2 \rangle$
- $t = 4, \quad \vec{r}(4) = \langle 2, \frac{1}{3}, ln4 \rangle$

Example 2:

$$\vec{r}(t) = \langle t, t^2, t^3 \rangle$$

Solution:

Domain of $f(t)$= R

Domain of $g(t)$= R

Domain of $h(t)$= R

Domain of $\vec{r}(t)$= R

Example 3:

$$\vec{r}(t) = \langle t^3, \ln(3-t), \sqrt{t} \rangle$$

Solution:

Domain of $f(t)$= R

Domain of $g(t)$= $(-\infty, 3)$

Domain of $h(t)$= $[0, \infty)$

Domain of $\vec{r}(t)$= $[0,3)$

Example 4:

$$\vec{r}(t) = \langle \sqrt{4-t^2}, e^{-3t}, \ln(t+1) \rangle$$

Solution:

Domain of $f(t)$: $\quad 4 - t^2 \geq 0$

$$-t^2 \geq -4$$

$$t^2 \leq 4$$

$$\pm t \leq 2$$

Domain of $f(t) = [-2,2]$

Domain of $g(t)$= R

Domain of $h(t)$= $(-1, \infty)$

Domain of $\vec{r}(t)$= $(-1,2]$

Example 5:

$$\vec{r}(t) = \frac{t-2}{t+2}\hat{\imath} + sint\hat{\jmath} + \ln(9-t^2)\hat{k}$$

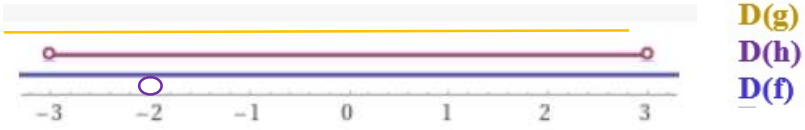
Solution:

Domain of $f(t)$= $R\backslash\{-2\}$

Domain of $g(t)$= R

Domain of $h(t) = \ln(9-t^2)$

$9 - t^2 > 0$



$t^2 < 9$

$\pm t < 3$

Domain of $h(t) = (-3,3)$

Domain of $\vec{r}(t)$= $(-3,-2) \cup (-2,3)$