University of Central Punjab
FOIT
Final Term Exam

Course Title: Differential Equations - (All Sections)

Course Instructors: Dr. Zain Ul Abadin Zafar, Rohsha Tahir, Mahnoor Akram, Muhammad Abdullah			
Course Code: CSSS2763	**Marks: 60**	**Time: 2.5 hr.**	**Semester: Spring 2022**

Name: **Registration Number:**

INSTRUCTIONS
1. Write your name and registration number on the Question Paper and Answer Sheet.
2. Write with blue/black permanent ink pen.
3. All your rough work and calculations should also be available on the answer sheet.
4. Make sure your calculator is in radian mode. Exchange of calculators is not allowed.
- **No cheat sheet, notes, handbooks or any kind of sharing allowed.**

Q1. Marks: [4+5+1]

A herd of wolves has 1000 wolves in it, and the population is growing exponentially. At time $t = 4$ months, it has 2000 wolves. Write a formula for the number of wolves at arbitrary time t.

a. Write an exponential model for the given scenario.

b. Use this model to find the number of wolves after 1 year.

c. When will the number of wolves be zero?

Q2. Marks: [4+4+4+2]

a. Write the general solution if the ODE and its solutions are given as:

$$4y'' - 4y' + y = 0 \;\; ; \;\; y_1 = e^{(x/2)} \quad , \;\; y_2 = xe^{(x/2)}$$

b. Find the second solution y_2 if $y'' + 2y' + y = 0$ and $y_1 = xe^{-x}$.

c. Find the general solution of the following homogeneous linear differential equation with constant coefficients.

$$y'' - 16y = 0 \;\; ; \; y(0) = 2 \;\; ; y'(0) = -2$$

d. If the Wronskian of two solutions is zero, can variation of parameters be used to find the particular solution using those two functions? Explain.

Q3. Marks: [12]
Solve the following differential equation using method of undetermined coefficients.

$$y'' + 4y = x(cosx)$$

Q4. Marks: [12]
Use power series to solve the following differential equation.

$$y'' - (1 + x)y = 0$$

Q5. Marks: [3+3+6]
a. Evaluate the following

i. $\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s-3}\right\}$

ii. $\mathcal{L}\{\cos(5t) + (2t - 1)^2\}$

b. Use the Laplace transform to solve the following initial value problem.

$$y' - y = 2\cos(5t) \quad ; \quad y(0) = 0$$

Note:
Formula for Integration by Parts:

$$\int uv\,dx = u\int v\,dx - \int u'\left(\int v dx\right) dx$$

Laplace Transforms of basic functions:

$$\mathcal{L}\{1\} = \frac{1}{s} \quad , \quad \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \quad , \quad \mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{sinkt\} = \frac{k}{s^2+k^2} \quad , \quad \mathcal{L}\{coskt\} = \frac{s}{s^2+k^2}$$

$$\mathcal{L}\{sinhkt\} = \frac{k}{s^2-k^2} \quad , \quad \mathcal{L}\{coshkt\} = \frac{s}{s^2-k^2}$$

Good Luck!