

Functions of Several Variables

$z = f(x, y) \rightarrow$ function of 2 variables (2 independent variables)

$w = f(x, y, z) \rightarrow$ function of 3 variables

Example

Consider an airline's ticket pricing. To avoid flying planes with many empty seats, it sells some tickets at full price and some at a discount. For a particular route, the airline's revenue R , earned in a given time period is determined by the number of full priced tickets, x , and number of discounted tickets, y , sold.

We say that R is a function of x, y and we write:

$R = f(x, y)$ R is the dependent variable whereas, x and y are the independent variables.

- The collection of all possible inputs (x, y) is called domain of f .
- We say a function is an increasing (decreasing) function of one of its variables if the function increases (decreases) as that variable increases while the other independent variables are held constant.

A function of two variables can be represented:

1. Numerically by table.
2. Algebraically by formula.
3. Pictorially by a contour diagram.

Function given Numerically:

$$R = f(x, y)$$

y/x	100	200	300	400
200	75,000	110,000	145,000	180,000
400	115,000	150,000	185,000	220,000
600	155,000	190,000	225,000	260,000
800	195,000	230,000	265,000	300,000
1000	235,000	270,000	305,000	340,000

a) Find the value of $f(300, 600) = 225,000$ & interpret it.

$$f(300, 600) = 225,000$$

The revenue from 300 full price tickets & 600 discounted tickets is \$ 225,000.

b) Is f an increasing or decreasing function of x ?

Increasing

c) Is f an increasing or decreasing function of y ?

Increasing

Function given Algebraically:

$$R = 350x + 200y$$

Example 2

A car rental company charges \$ 40 a day & 15 cents a mile for its cars.

a) Write a formula for the cost C , of renting a car as a function of the number of days d , and the number of miles driven m .

Solution: $C = f(d, m) = 40d + 0.15m$

b) If $C = f(d, m)$. Find $f(5, 300)$ and interpret it.

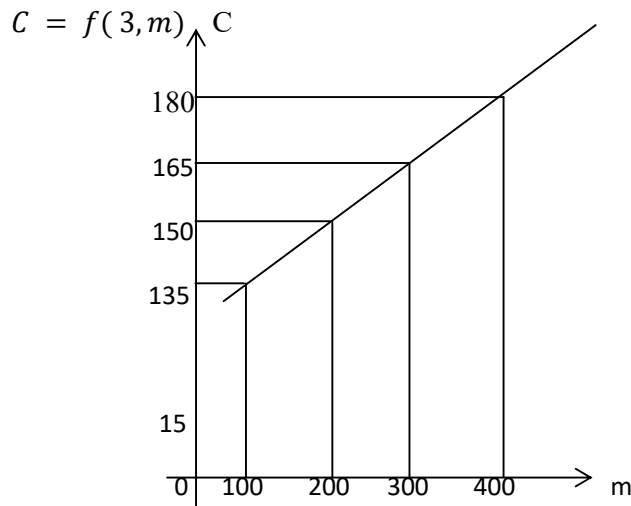
Solution: $f(5, 300) = 40(5) + 0.15(300) = 245$

It means if the car is rented for 5 days & is driven 300 miles then it costs \$ 245.

c) Explain the significance of $f(3, m)$ in terms of rental cost. Graph this function with C as a function of m .

Solution: $f(3, m)$ is the value of C for various values of m with d fixed at 3. In other words, it's the cost of renting a car for three days & driven for variable number of miles.

Graph:



m/d	3
100	135
200	150
300	165
400	180

d) Explain the significance of $f(d, 100)$ in terms of rental car costs. Graph this function with C as a function of d .

Do it yourself.

Exercise:9.1 Q3 – Q5 from Applied Calculus

Question 9 (Ex. 9.1):

The total sales of a product 'S' can be expressed as a function of the price 'P' charged for the product and amount 'a' spent on advertisement, so $S = f(p, a)$.

- a) Do you expect f to be an increasing or decreasing function of P?
- b) Do you expect f to be an increasing or decreasing function of 'a', why?

Solution: Since, $S = f(p, a)$

a) $S = f(p, \$20)$

As S is a function of P, i.e., 'a' = advertisement amount is fixed, we supposed it \$20. Now if the price of product increases, its sale decreases so 'f' is decreasing function of 'P'.

$$P \uparrow \Leftrightarrow S \downarrow$$

$$P \downarrow \Leftrightarrow S \uparrow$$

- 'P' and 'S' are inversely related.
- 'f' is decreasing function of 'P', while 'a' is fixed.

b) $S = f(\$20, a)$

If we increase 'a' = the amount spent on advertisement we can expect 'S' to be increasing.

$$a \uparrow \Leftrightarrow S \uparrow$$

$$a \downarrow \Leftrightarrow S \downarrow$$

- 'a' and 'S' are directly related.
- 'f' is an increasing function of 'a'.

Practice Problems

Ex. 9.1: 3-5, 10, 11 from Applied Calculus