

Example 7.3

(a)

$$\mu = 34, \sigma = 15, n = 100$$

$$\mu_{\bar{x}} = \mu = 34$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_{\bar{x}} = \frac{15}{\sqrt{100}} \rightarrow \frac{15}{10}$$

$$\sigma_{\bar{x}} = 1.5$$

(b) The ~~center~~ central limit theorem states that for large sample size (n), ~~the~~ the sampling distribution will be approximately normal.

(c)

$$P(\bar{x} > 30) = 1 - P(\bar{x} < a)$$

$$Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

$$\bar{x} = 30, \mu_{\bar{x}} = 34, \sigma_{\bar{x}} = 1.5, n = 100$$

$$Z = \frac{30 - 34}{1.5}$$

$$Z = \frac{-4}{1.5}$$

$$Z = -2.667$$

$$P(\bar{x} > 30) = 1 - 0.00379$$

$$P(\bar{x} > 30) = 0.9962$$

(d) Find 95 percentile

$$P(\bar{x} < K) = 0.95$$

$$\mu = 34, \sigma = 15, n = 100$$

$$P\left(\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < \frac{k - 34}{\frac{1.5}{\sqrt{100}}}\right) = 0.95$$

$$P\left(Z < \frac{k - 34}{1.5}\right) = 0.95$$

$$\Phi\left(\frac{k - 34}{1.5}\right) = 0.95$$

$$\frac{k - 34}{1.5} = \Phi 0.95$$

$$\frac{k - 34}{1.5} = 1.65$$

$$k - 34 = 1.65 \times 1.5$$

$$k - 34 = 2.475$$

$$k = 2.475 + 34$$

$$\boxed{k = 36.5}$$

Try 9.7.4.

$$\bar{x} = 16.01, n = 34, \mu = 16.00$$

$$\sigma = 0.143$$

$$P(\bar{x} > 16.01) = 1 - P(\bar{x} < 16.01)$$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$Z = \frac{16.01 - 16.00}{\frac{0.143}{\sqrt{34}}}$$

$$Z = \frac{0.01}{0.0245}$$

$$\boxed{Z = 0.4081}$$

$$P(\bar{x} > 16.01) = 1 - 0.65542$$

$$\boxed{P(\bar{x} > 16.01) = 0.34}$$

Example 7.1

$$\mu = 90, \sigma = 15, n = 25$$

Find sample mean is between 85 and 92.

Solution:

$$P(a \leq \bar{x} \leq b) = \Phi(b) - \Phi(a)$$

$$P(85 \leq \bar{x} \leq 92) = P\left(\frac{85-90}{\frac{15}{\sqrt{25}}} \leq Z \leq \frac{92-90}{\frac{15}{\sqrt{25}}}\right)$$

$$= P\left(\frac{-5}{3} \leq Z \leq \frac{2}{3}\right)$$

$$P(-5/3 \leq Z \leq 2/3)$$

$$P(-1.67 \leq Z \leq 0.67)$$

Z Table

$$-1.67 = .4746, 0.67 = .74857$$

$$\textcircled{P(-1.67 \leq Z \leq 0.67)} = .74857 - .4746$$

$$P(-1.67 \leq Z \leq 0.67) = 0.2739$$

Example 7.2

$$\mu = 2, \sigma = 0.5, n = 50$$

between 1.8 and 2.3

Solution:

$$P(a \leq \bar{x} \leq b) = \Phi(b) - \Phi(a)$$

$$P(1.8 \leq \bar{x} \leq 2.3) = P\left(\frac{1.8-2}{\frac{0.5}{\sqrt{50}}} \leq Z \leq \frac{2.3-2}{\frac{0.5}{\sqrt{50}}}\right)$$

$$= P\left(\frac{-0.2}{0.0707} \leq Z \leq \frac{0.3}{0.0707}\right)$$

Try 9A 7-1

$$\mu = 45, \sigma = 8, n = 30$$

between 42 and 50

Solution:

$$P(a < \bar{x} < b) = \Phi(b) - \Phi(a)$$

$$P(42 < \bar{x} < 50) = P\left(\frac{42 - 45}{\frac{8}{\sqrt{30}}} < Z < \frac{50 - 45}{\frac{8}{\sqrt{30}}}\right)$$

$$P\left(\frac{-3}{1.4606} < Z < \frac{5}{1.4606}\right)$$