Q-1

$T(N) = 4T(N/8) + N^{1/3}$ $\quad (T(1) = C)$

$= 4(4T(N/64) + (N/8)^{1/3}) + N^{1/3}$

$= 16T(N/64) + 4 * \frac{N^{1/3}}{2} + N^{1/3}$

$= 16[4T(N/512) + (N/64)^{1/3}] + 2N^{1/3} + N^{1/3}$

$= 64T(N/512) + 16\frac{N^{1/3}}{4} + 2N^{1/3} + N^{1/3}$

$= 4^3 T(N/8^3) + 4N^{1/3} + 2N^{1/3} + N^{1/3}$

$= 4^k T(N/8^k) + N^{1/3}(2^2 + 2^1 + 2 - \cdots + 2^0)$

Here

$(N/8^k) = 1 \Rightarrow N = 8^k$

$\Rightarrow \log N = \log 8^k$

$\Rightarrow \log N = k * \log 8$

$\Rightarrow k = \log_2 N / \log_2 8$

$\Rightarrow k = \log_8 N$

$\Rightarrow = 4^k T(N/8^k) + N^{1/3} \sum_{i=0}^{k-1} (2^i)$

$\Rightarrow 4^{(\log_8 N)} * C + N^{1/3} \sum_{i=0}^{\log_8 N - 1} (2^i)$

$\Rightarrow N^{\log_8 4} * C + N^{1/3}\left[\frac{2^{\log_8 N - 1 + 1} - 1}{2-1}\right]$

$\Rightarrow N^{0.66} * C + N^{1/3}[2^{\log_8 N} - 1]$

$\rightarrow N^{0.66} * C + N^{1/3}[N^{\log_8 2} - 1]$

$\Rightarrow N^{0.66} * C + N^{0.33} * N^{0.33} - N^{0.33}$

$\Rightarrow CN^{0.66} + N^{0.66} - N^{0.33}$

$\Rightarrow CN^{0.66} - N^{0.33} \Rightarrow O(N^{0.66})$

Q-2

— XYZ(A, S, e)

if (size) // divide till array has more than 1

mid = S + (e-S)/2

L = XYZ(A, S, mid)

R = XYZ(A, mid+1, e)

return L+R

else

return A[e] // single element return

Now we have done that

— Divide 2 parts

— Repeat till size is 1

— When size 1 return that value

$T(N) = 2T(N/2) + 1 \rightarrow (Cost)$ // We can use 1 here

$(C) // O(1)$

Mid-Term Solution

Summer 22

Q-3

outer loop ⇒ initialization → 1
loop end → 1
loop successful → (2N-1+1) times
→ $\log_2(2N)$
→ $\log_2(2) + \log_2(N)$
→ $1 + \log_2 N$ times.

Inner loop Dependent;

i =	1	2	4	8	.....	N/2	N	2N
j =	0→1	0→2	0→4	0→8		0→N/2	0→N	0→2N
→	1(S)	2(S)	4(S)	8(S)		N/2 (S)	N(S)	2N(S)
	1(I)	"	"	"		"	"	"
	1(E)	"	"	"		"	"	"

$2 * \log_2 N$ ⇒ (Initialization & End)

$1 + 2 + 4 + 8 + 16 + \dots + N/2 + N + 2N$ (geometric)

$< 2 * 2N$
$< 4N$
$< O(N)$

$\log_2 N + N + \log_2 N$

$O(N)$ answer.

Q-4