Linear Equations Continued…

Example 5

Solve $\dfrac{dy}{dx} + y = x, \quad y(0) = 4.$

SOLUTION The equation is in standard form, and $P(x) = 1$ and $f(x) = x$ are continuous on $(-\infty, \infty)$. The integrating factor is $e^{\int dx} = e^x$, so integrating

$$\frac{d}{dx}[e^x y] = xe^x$$

gives $e^x y = xe^x - e^x + c$. Solving this last equation for y yields the general solution $y = x - 1 + ce^{-x}$. But from the initial condition we know that $y = 4$ when $x = 0$. Substituting these values into the general solution implies that $c = 5$. Hence the solution of the problem is

$$y = x - 1 + 5e^{-x}, \quad -\infty < x < \infty.$$

Practice Questions:

[Exercise 2.3 of Book: Differential Equations by D.G. Zill]

5. $y' + 3x^2 y = x^2$
6. $y' + 2xy = x^3$
7. $x^2 y' + xy = 1$
8. $y' = 2y + x^2 + 5$
9. $x\dfrac{dy}{dx} - y = x^2 \sin x$
10. $x\dfrac{dy}{dx} + 2y = 3$
11. $x\dfrac{dy}{dx} + 4y = x^3 - x$
12. $(1 + x)\dfrac{dy}{dx} - xy = x + x^2$
13. $x^2 y' + x(x + 2)y = e^x$

14. $xy' + (1 + x)y = e^{-x} \sin 2x$

15. $y\,dx - 4(x + y^6)\,dy = 0$

16. $y\,dx = (ye^y - 2x)\,dy$

17. $\cos x \dfrac{dy}{dx} + (\sin x)y = 1$

18. $\cos^2 x \sin x \dfrac{dy}{dx} + (\cos^3 x)y = 1$

19. $(x + 1)\dfrac{dy}{dx} + (x + 2)y = 2xe^{-x}$

20. $(x + 2)^2 \dfrac{dy}{dx} = 5 - 8y - 4xy$

21. $\dfrac{dr}{d\theta} + r \sec \theta = \cos \theta$

22. $\dfrac{dP}{dt} + 2tP = P + 4t - 2$

23. $x\dfrac{dy}{dx} + (3x + 1)y = e^{-3x}$

24. $(x^2 - 1)\dfrac{dy}{dx} + 2y = (x + 1)^2$

In Problems 25–30 solve the given initial-value problem. Give the largest interval I over which the solution is defined.

25. $xy' + y = e^x, \quad y(1) = 2$

26. $y\dfrac{dx}{dy} - x = 2y^2, \quad y(1) = 5$

27. $L\dfrac{di}{dt} + Ri = E, \quad i(0) = i_0,$
L, R, E, and i_0 constants

28. $\dfrac{dT}{dt} = k(T - T_m); \quad T(0) = T_0,$
k, T_m, and T_0 constants

29. $(x + 1)\dfrac{dy}{dx} + y = \ln x, \quad y(1) = 10$

30. $y' + (\tan x)y = \cos^2 x, \quad y(0) = -1$