NEWTON'S LAW OF COOLING/WARMING

According to Newton's empirical law of cooling/warming, the rate at which the temperature of a body changes is proportional to the difference between the temperature of the body and the temperature
of the surrounding medium, the so-called ambient temperature. Newton's law of cooling/warming translates into the mathematical statement:

$$\frac{dT}{dt} \propto (T - T_m)$$

$$\boldsymbol{\frac{dT}{dt} = k(T - T_m)}$$

where

$T =$ Temperature of body, $t =$ time

$T_m =$ Temperature of environment

Solving such DE gives that its solution takes the form (Do yourself)

$$\boldsymbol{T = T_m + ce^{kt}} \qquad (2)$$

EXAMPLE 4. When a cake is removed from an oven, its temperature is measured at 300° F. Three minutes later its temperature is 200° F. How long will it take for the cake to cool off to a room temperature of 70° F?

SOLUTION. In Eq. (2) we make the identification $\boldsymbol{T_m = 70}$. We must then solve the initial-value problem

$$\boldsymbol{T = T_m + ce^{kt}, \quad T(0) = 300} \qquad (3)$$

and determine the value of k so that $T(3) = 200$. Equation (3) is both linear and separable. If we separate variables, Eq. (3) yields

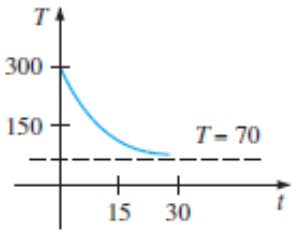
$$\boldsymbol{T = 70 + 230e^{kt}}.$$

Finally, the measurement $T(3) = 200$ leads to $\boldsymbol{e^{3k} = \frac{13}{23}}$, or $k = \frac{1}{3}\ln\frac{13}{23} = -0.19018$. Thus

$$\boldsymbol{T = 70 + 230e^{-0.19018t}} \qquad (4)$$

We note that Eq. (4) furnishes no finite solution to $T(t) = 70$, since $\lim_{t\to\infty} T(t) = 70$.

Yet we intuitively expect the cake to reach room temperature after a reasonably long period of time. How long is "long"? Parts (a) and (b) of Figure 3.1.3 clearly show that the cake will be approximately at room temperature in about one-half hour.



(a)

$T(t)$	t (min)
75°	20.1
74°	21.3
73°	22.8
72°	24.9
71°	28.6
70.5°	32.3

(b)

FIGURE 3.1.3 Temperature of cooling cake approaches room temperature

EXERCISE 3.1

13. A thermometer is removed from a room where the temperature is 70° F and is taken outside, where the air temperature is 10° F. After one-half minute the thermometer reads 50° F. What is the reading of the thermometer at $t = 1$ min? How long will it take for the thermometer to reach 15° F?

14. A thermometer is taken from an inside room to the outside, where the air temperature is 5° F. After 1 minute the thermometer reads 55° F, and after 5 minutes it reads 30° F. What is the initial temperature of the inside room?

15. A small metal bar, whose initial temperature was 20° C, is dropped into a large container of boiling water. How long will it take the bar to reach 90° C if it is known that its temperature increases 2° in 1 second? How long will it take the bar to reach 98° C?