Example 2: (Revisiting the flight of the glider)
Suppose that we don't know the path of a glider as in the previous example, but only know its acceleration vector

$$\vec{a}(t) = -3cost\vec{\imath} - 3sint\vec{\jmath} + 2\vec{k}$$

We also know that initially (at time t=0). The glider departed from the point (3,0,0) with velocity $\vec{v}(0) = 3\vec{\jmath}$. Find the glider's position as a function of t.

Solution:
Our goal is to find $\vec{r}(t)$.
As we know that

$$\vec{v}(t) = \int \vec{a}(t)\, dt$$

$$\vec{v}(t) = \int (-3cost\vec{\imath} - 3sint\vec{\jmath} + 2\vec{k}\,)dt$$

$$\vec{v}(t) = -3sint\vec{\imath} + 3cost\vec{\jmath} + 2t\vec{k} + c$$

To find c we need to use the initial condition $\vec{v}(0) = 3\vec{\jmath}$

$$\vec{v}(0) = -3\sin{(0)}\vec{\imath} + 3\cos{(0)}\vec{\jmath} + 2(0)\hat{k} + c$$

$$3\vec{\jmath} = 3\vec{\jmath} + c$$

$$c = 0$$

So,

$$\mathbf{\vec{v}(t) = -3sint\vec{i} + 3cost\vec{j} + 2t\vec{k}}$$

$$\vec{r}(t) = \int \vec{v}(t)\, dt$$

$$\vec{r}(t) = \int -3\mathrm{sint}\vec{\mathrm{i}} + 3\mathrm{cost}\vec{\mathrm{j}} + 2\mathrm{t}\vec{\mathrm{k}}\,\mathrm{dt}$$

$$\vec{r}(t) = 3\mathrm{cost}\vec{\mathrm{i}} + 3\mathrm{sint}\vec{\mathrm{j}} + t^2\vec{\mathrm{k}} + d$$

Given that

$$\vec{r}(0) = 3\vec{\mathrm{i}}$$

We have

$$\vec{r}(0) = 3\cos(0)\,\vec{\mathrm{i}} + 3\sin{(0)}\,\vec{\mathrm{j}} + (0)\vec{\mathrm{k}} + d$$

$$3\vec{\mathrm{i}} = 3\vec{\mathrm{i}} + d$$

$$d = 0$$

$$\boldsymbol{\vec{r}(t) = 3\mathbf{cost}\vec{\mathbf{i}} + 3\mathbf{sint}\vec{\mathbf{j}} + t^2\vec{\mathbf{k}} + d}$$

Some important Formulas of Integration

- $\int \boldsymbol{c\ dt = ct}$
- $\int \boldsymbol{t\ dt = \frac{t^2}{2}}$
- $\int \boldsymbol{sinat\ dt = -\frac{cosat}{a}}$
- $\int \boldsymbol{cosat\ dt = \frac{sinat}{a}}$
- $\int \boldsymbol{e^{at}\ dt = \frac{e^{at}}{a}}$

Question: A glider is moving in the air and we don't know the path, but only know its acceleration vector

$$\vec{a}(t) = t\vec{\imath} + e^{t}\vec{\jmath} + e^{-t}\vec{k}$$

We also know that initially (at time t=0). The glider departed from the point (0,1,1) with velocity $\vec{v}(0) = \vec{k}$. Find the glider's position as a function of t.

Solution:

Our goal is to find $\vec{r}(t)$?

As we know

$$\vec{v}(t) = \int \vec{a}(t)\, dt$$

$$\vec{v}(t) = \int t\vec{\imath} + e^{t}\vec{\jmath} + e^{-t}\vec{k}\, dt$$

$$\vec{v}(t) = \frac{t^2}{2}\vec{\imath} + e^{t}\vec{\jmath} - e^{-t}\vec{k} + c$$

To find c we need to use the initial condition $\vec{v}(0) = \vec{k}$

$$\vec{v}(0) = 0\vec{\imath} + e^{0}\vec{\jmath} - e^{0}\vec{k} + c$$

$$\vec{k} = \vec{\jmath} - \vec{k} + c$$

$$\mathbf{c = -\vec{j} + 2\vec{k}}$$

So

$$\vec{\mathrm{v}}(\mathrm{t}) = \frac{t^2}{2}\vec{\imath} + e^{t}\vec{\jmath} - e^{-t}\vec{k} - \vec{\jmath} + 2\vec{\mathrm{k}}$$

$$\boldsymbol{\vec{v}(t) = \frac{t^2}{2}\vec{\imath} + (e^{t} - 1)\vec{\jmath} - (e^{-t} - 2)\vec{k}}$$

$$\vec{r}(t) = \int \vec{v}(t)\, dt$$

$$\vec{r}(t) = \int \frac{t^2}{2}\vec{\imath} + (e^t - 1)\vec{\jmath} - (e^{-t} - 2)\vec{k}\ \mathrm{dt}$$

$$\vec{r}(t) = \frac{\mathrm{t}^3}{6}\vec{\mathrm{i}} + (\mathrm{e}^{\mathrm{t}} - \mathrm{t})\vec{\mathrm{j}} - (-\mathrm{e}^{-\mathrm{t}} - 2\mathrm{t})\vec{\mathrm{k}} + d$$

$$\vec{r}(t) = \frac{\mathrm{t}^3}{6}\vec{\mathrm{i}} + (\mathrm{e}^{\mathrm{t}} - \mathrm{t})\vec{\mathrm{j}} + (\mathrm{e}^{-\mathrm{t}} + 2\mathrm{t})\vec{\mathrm{k}} + d$$

Given

$$\vec{r}(0) = \vec{\mathrm{j}} + \vec{\mathrm{k}}$$

Using this condition

$$\vec{r}(0) = \frac{0}{6}\vec{\imath} + (\mathrm{e}^0 - 0)\vec{\jmath} + (\mathrm{e}^0 + 0)\vec{\mathrm{k}} + d$$

$$\vec{\mathrm{j}} + \vec{\mathrm{k}} = \vec{\mathrm{j}} + \vec{\mathrm{k}} + d$$

$$\mathbf{d = 0}$$

$$\boldsymbol{\vec{r}(t) = \frac{t^3}{6}\vec{i} + (e^t - t)\vec{j} + (e^{-t} + 2t)\vec{k}}$$

<u>Practice Problems</u>

Q1: Find the position vector of the particle that has the given acceleration vector.

$$\vec{a}(t) = sint\vec{\imath} + 2cost\vec{\jmath} + cos2t\vec{k}$$

with $\vec{v}(0) = \vec{\imath}$ *and* $\vec{r}(0) = \vec{\imath} + \vec{\jmath} + \vec{k}$

Q2: Find the position vector of the particle that has the given acceleration vector.

$$\vec{a}(t) = \vec{\imath} + 2\vec{\jmath}$$

with $\vec{v}(0) = \vec{\imath}$ *and* $\vec{r}(0) = \vec{\imath} + \vec{\jmath} + \vec{k}$