Solution of Word Problems using Gauss-Jordan Method

Example 1

Ali and Sara are shopping for chocolate bars. Ali observes, "==If I add half my money to yours, it will be enough to buy two chocolate bars.==" Sara naively asks, "If I add half my money to yours, how many can we buy?" Ali replies, "One chocolate bar." How much money did Ali have?

Solution: Let a = Ali's money
s = Sara's money
c = Cost of chocolate

$$\begin{cases} \dfrac{1}{2}a + s = 2c & ----\!\to (1) \\[2mm] a + \dfrac{1}{2}s = c & ------\!\to (2) \end{cases}$$

$$\text{Or} \quad \begin{cases} a + 2s = 4c \\ 2a + s = 2c \end{cases}$$

The augmented matrix is

$$\begin{bmatrix} 1 & 2 & 4c \\ 2 & 1 & 2c \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4c \\ 0 & -3 & -6c \end{bmatrix} \qquad \sim\!\sim\!\sim\!\sim\!\sim\!\sim\!\sim\!\sim \qquad R_2 - 2R_1$$

$$\begin{bmatrix} 1 & 2 & 4c \\ 0 & 1 & 2c \end{bmatrix} \qquad \sim\!\sim\!\sim\!\sim\!\sim\!\sim\!\sim\!\sim \qquad -\dfrac{1}{3}R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2c \end{bmatrix} \qquad \sim\!\sim\!\sim\!\sim\!\sim\!\sim\!\sim\!\sim \qquad R_1 - 2R_2$$

Solution is $(a, s) = (0, 2c)$. It means Ali has no money.

Example 2

Three Alto, two Suzuki, and four City can be rented for $106 per day. At the same rates two Alto, four Suzuki, and three City cost $I07 per day, whereas four Alto, three Suzuki, and two City cost $102 per day. Find the rental rates for all three kinds of cars?

$$3a + 2s + 4c = 106$$
$$2a + 4s + 3c = 107$$
$$4a + 3s + 2c = 102$$

Its Augmented matrix is

$$\begin{bmatrix} 3 & 2 & 4 & : 106 \\ 2 & 4 & 3 & : 107 \\ 4 & 3 & 2 & : 102 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 & : -1 \\ 2 & 4 & 3 & : 107 \\ 4 & 3 & 2 & : 102 \end{bmatrix} \quad \sim\sim\sim\sim\sim\sim\sim\sim\sim\sim \quad R_1 - R_2$$

$$\begin{bmatrix} 1 & -2 & 1 & : -1 \\ 0 & 8 & 1 & : 109 \\ 4 & 3 & 2 & : 102 \end{bmatrix} \quad \sim\sim\sim\sim\sim\sim\sim\sim\sim\sim \quad R_2 - 2R_1$$

$$\begin{bmatrix} 1 & -2 & 1 & : -1 \\ 0 & 8 & 1 & : 109 \\ 0 & 11 & -2 & : 106 \end{bmatrix} \quad \sim\sim\sim\sim\sim\sim\sim\sim\sim\sim \quad R_3 - 4R_1$$

$$\begin{bmatrix} 1 & -2 & 1 & : -1 \\ 0 & -3 & 3 & : 3 \\ 0 & 11 & -2 & : 106 \end{bmatrix} \quad \sim\sim\sim\sim\sim\sim\sim\sim\sim\sim \quad R_2 - R_3$$

$$\begin{bmatrix} 1 & -2 & 1 & : -1 \\ 0 & 1 & -1 & : -1 \\ 0 & 11 & -2 & : 106 \end{bmatrix} \quad \sim\sim\sim\sim\sim\sim\sim\sim\sim\sim \quad -\frac{1}{3}R_3$$

$$\begin{bmatrix} 1 & -2 & 1 & : -1 \\ 0 & 1 & -1 & : -1 \\ 0 & 0 & 9 & : 117 \end{bmatrix} \quad \sim\sim\sim\sim\sim\sim\sim\sim\sim\sim \quad R_3 - 11R_2$$

$$\begin{bmatrix} 1 & -2 & 1 & : -1 \\ 0 & 1 & -1 & : -1 \\ 0 & 0 & 1 & : 13 \end{bmatrix} \quad \sim\sim\sim\sim\sim\sim\sim\sim\sim\sim \quad \frac{1}{9}R_3$$

$$\begin{bmatrix} 1 & -2 & 1 & : -1 \\ 0 & 1 & 0 & : 12 \\ 0 & 0 & 1 & : 13 \end{bmatrix} \quad \sim\sim\sim\sim\sim\sim\sim\sim\sim\sim \quad R_2 + R_3$$

$$\begin{bmatrix} 1 & -2 & 0 & : -14 \\ 0 & 1 & 0 & : 12 \\ 0 & 0 & 1 & : 13 \end{bmatrix} \quad \sim\sim\sim\sim\sim\sim\sim\sim\sim\sim \quad R_1 - R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & :10 \\ 0 & 1 & 0 & :12 \\ 0 & 0 & 1 & :13 \end{bmatrix} \quad \sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim \quad R_1 + 2R_3$$

Hence, the rental rates for Alto, Suzuki, and City cars are $10, $12 and $13 per day, respectively.

Example 3

A restaurant owner plans to use x tables seating 4, y tables seating 6 and z tables seating 8, for a total 20 tables. When fully occupied, the tables seat 108 customers. If only half of the x tables, half of the y tables and one-fourth of the z tables are used, each fully occupied, then 46 customers will be seated. Find x, y, and z.

Solution:

$$x + y + z = 20$$

$$4x + 6y + 8z = 108$$

$$4\left(\frac{x}{2}\right) + 6\left(\frac{y}{2}\right) + 8\left(\frac{z}{4}\right) = 46$$

Simplifying the system, we have

$$x + y + z = 20$$

$$2x + 3y + 4z = 54$$

$$2x + 3y + 2z = 46$$

$$.$$
$$.$$
$$.$$

The answer is: $x = 10, y = 6$ and $z = 4$

Work to do:

Q1. Students are buying books for the new semester. Asma buys the linear algebra book and the differential equation book for $178. Aiman, who is buying books for herself and her friend, spends $319 on two linear algebra books, one differential equation book, and one educational psychology book. Sara buys the educational psychology book and the differential equation book for $147 in total. How much does each book cost?

Q2. A soap manufacturer wants to spend 60 Lac rupees on radio, magazine, and TV advertising. If he spends as much on TV advertisement as on magazines and radio together, and the amount spend on magazines and TV combined equals 5 times that spent on radio, what is the amount to be spent on each type of advertising?

Q3. Three merchants find a purse lying in the road. One merchant says "If I keep the purse, I shall have twice as much money as the two of you together". "Give me the purse and I shall have three times as much as the two of you together", said the second merchant. The third merchant said, "I shall be much better off than either of you if I keep the purse, I shall have five times as much as the two of you together." If there are 60 coins (of equal value) in the purse, how much money does each merchant have?