



University of Central Punjab  
FOIT&CS  
Final Term Exam

Course Title: Differential Equations - (All Sections)

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Course Code: CSSS2763	Marks: 60	Time: 2.5 hr.	Semester: Fall 2022

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INSTRUCTIONS

1. Write your name and registration number on the Question Paper and Answer Sheet.
  2. Write with blue/black permanent ink pen.
  3. All your rough work and calculations should also be available on the answer sheet.
  4. Make sure your calculator is in radian mode. Exchange of calculators is not allowed.
- No cheat sheet, notes, handbooks or any kind of sharing allowed.

**ATTACH THE QUESTION PAPER WITH ANSWER SHEET.**

**Q1. [Marks:  $8 \times 3 = 24$ ] SHORT QUESTIONS**

- ✓ a. Write the general solution if the ODE and its solutions are given as:  
 $4y'' - 4y' + y = 0$  ;  $y_1 = e^{(x/2)}$  ,  $y_2 = xe^{(x/2)}$
- ✓ b. At which interval of  $x$ , the following functions are linearly independent?  
 $f_1(x) = 0$  ;  $f_2(x) = x$  ;  $f_3(x) = e^x$
- c. If the auxiliary equation of an ODE with constant coefficients has roots  $2, 1, 1, \pm i, 2, -2$ , write its complementary solution.
- ✓ d. Find the second solution  $y_2$  if  $x^2y'' - 3xy' + 4y = 0$ ,  $y_1 = x^2$ ,  $x \in (0, \infty)$ .
- ✓ e. If the Wronskian of two solutions is zero, can variation of parameters be used to find the particular solution using those two functions? Explain.
- f. Write the structure of the particular solution using method of undetermined coefficients for the following differential equation without solving it.  
 $y'' + y = x(\sin(x))$
- ✓ g. Which method can be used to find the solution of  $y'' + (x^2)y = 2$ . Explain the reason without solving it.
- ✓ h. Find the inverse Laplace transform.

$$\mathcal{L}^{-1} \left\{ \frac{1}{s^2 + 5} + \frac{s}{s^2 + 5} + \frac{7}{s^2 - 5} \right\}$$

## LONG QUESTIONS

**Q2. [Marks: 4 + 4 = 8]**

The bear population in a certain country increases at a rate of 2% per year. There are 1573 bears this year.

- a. Write the differential model and its solution for the given scenario.
- b. How many bears will there be in 10 years?

**Q3. [Marks: 8]**

Solve the given differential equation using the **most appropriate** method among 'Variation of Parameters' and 'Method of Undetermined Coefficients'.

$$y'' + y = \sec(x)$$

Please note that  $\sec(x) = \frac{1}{\cos(x)}$  and  $\sin^2 x + \cos^2 x = 1$ .

**Q4. [Marks: 10]**

Use power series to solve the following differential equation.

$$y'' - (1 + x)y = 0$$

**Q5. [Marks: 10]**

Use the Laplace transform to solve the given initial value problem.

$$y' + 3y = 13[\sin(2t)]; \quad y(0) = 6$$

*Laplace Transforms of basic functions:*

$$\begin{aligned} \mathcal{L}\{1\} &= \frac{1}{s}, \quad \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}, \quad \mathcal{L}\{e^{at}\} = \frac{1}{s-a} \\ \mathcal{L}\{\sin kt\} &= \frac{k}{s^2 + k^2}, \quad \mathcal{L}\{\cos kt\} = \frac{s}{s^2 + k^2} \\ \mathcal{L}\{\sinh kt\} &= \frac{k}{s^2 - k^2}, \quad \mathcal{L}\{\cosh kt\} = \frac{s}{s^2 - k^2} \end{aligned}$$

*Laplace Transform of derivative:*

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0) = s\mathcal{L}\{f(t)\} - f(0)$$

*Formula for Reduction of Order:*

$$y_2 = y_1 \int \frac{e^{-\int p(x)dx}}{(y_1)^2} dx$$

Happy Exam Season! ☺