Exercise 9.3:

Question 8:

The following table gives the number of calories burned per minute $B = f(w,s)$ of a cyclist as a function of the person's weight w& speed s.

a) Is f_w positive or negative?
Is f_s positive or negative?

What do your answers tell us about the effect of weight and speed on calories burned per minute?

b) Estimate $f_w(160,10)$&$f_s\ (160,10)$. Interpret your answers?

w / s	8 mph	9 mph	10 mph	11 mph
120 lbs	4.2	5.8	7.4	8.9
140lbs	5.1	6.7	8.3	9.9
160 lbs	6.1	7.7	9.2	10.8
180 lbs	7.0	8.6	10.2	11.7
200 lbs	7.9	9.5	11.1	12.6

Solution:

a) We see that f_w is positive since B increases as w increases, when$s$ held constant. $f_s$is also positive since B increases as s increases, when w held constant.

b) We have

$$f_w(160,10) = \frac{\partial f(160,10)}{\partial w} = \frac{[f(160+20,10) - f(160,10)]}{180-160}$$
$$= \frac{(10.2-9.2)}{(180-160)} = 0.05$$

It means if we change the person's weight by 1 lbs then the number of calories burned per minute is increased by 0.05.

For,f_s

$$f_s(160,10) = \frac{\partial f(160,10)}{\partial s} = \frac{[f(160,11) - f(160,10)]}{11-10} = \frac{(10.8-9.2)}{11-10} = 1.6$$

It means if we change the speed of the person by 1 mph then the number of calories burned per minute is increased by 1.6 units.

Exercise 9.3:

Question 5:

The demand for coffee 'Q', in pounds sold per week is a function of the price of coffee, 'c', in dollars per pound & the price of tea, 't' in dollars per pound, so, $Q = f(c,t)$.

a) Do you expect f_c to be positive or negative? What about f_t? Explain.
b) Interpret each of the following statement in terms of the demand for coffee: $f(3,2) = 780, f_c(3,2) = -60, f_t(3,2) = 20$.

Solution:

a) For f_c, we expect the demand for coffee to decrease as the price of coffee increases (assuming the price of tea is fixed). Thus, we expect f_c is negative.
c↑ ⇔ Q↓
c↓ ⇔ Q↑

- Inverse relationship.
- $\frac{\partial f}{\partial c}$ is negative.

For f_t:

t↑ ⇔ Q↑

t↓ ⇔ Q↓

- Direct relationship.
- f is increasing function of t.
- f_t is positive.

b)

- $f(3,2) = 780$

It means if the price of coffee is $3 and price of tea is $2 then we can expect 780 pounds of coffee to be sold each week.

- $f_c(3,2) = -60 = \partial t/\partial c(3,2)$

It means if we change or increase the price of coffee by 1 unit then the demand of coffee is decreased by 60 units.

- $f_t(3,2) = 20$

It means if we increase the price of tea by 1 unit then the demand for coffee is increased by 20 units.

Ex. 9.2: 3-5