

Chapter 8

Example 8.1

$$\bar{x} = 7 \quad EBM = 2.5$$

Find Confidence Interval (C.I.) - ?

$$C.I. = \bar{x} \pm EBM$$

$$C.I. = \bar{x} - EBM$$

$$C.I. = 7 - 2.5$$

$$C.I. = 4.5$$

$$C.I. = \bar{x} + EBM$$

$$C.I. = 7 + 2.5$$

$$C.I. = 9.5$$

Try 9.1 8.1

$$\bar{x} = 15, \quad EBM = 3.2$$

Find C.I. = ?

$$C.I. = \bar{x} \pm EBM$$

$$C.I. = \bar{x} + EBM$$

$$C.I. = 15 + 3.2$$

$$C.I. = 18.2$$

$$C.I. = \bar{x} - EBM$$

$$C.I. = 15 - 3.2$$

$$C.I. = 11.8$$

Example 8.2

$$\sigma = 3, \quad n = 36, \quad \bar{x} = 68$$

Find C.I. 90%.

$$C.I. = \bar{x} \pm EBM$$

$$EBM = (Z_{1-\alpha/2})(\sigma/\sqrt{n})$$

$$C.I. = 90\%$$

$$1 - \alpha = 90\%$$

$$\alpha = 10\%$$

$$\alpha = 0.10\%$$

$$\alpha/2 = \frac{0.10}{2}$$

$$\alpha/2 = 0.05$$

$$1 - \alpha/2 = 1 - 0.05$$

$$1 - \alpha/2 = 0.95 \rightarrow \text{Find Table}$$

$$1 - \alpha/2 = 1.65$$

$$EBM = (1.65) \left(\frac{3}{\sqrt{36}} \right)$$

$$EBM = (1.65)(0.5)$$

$$EBM = 0.825$$

$$C.I = \bar{X} + EBM \quad | \quad C.I = \bar{X} - EBM$$

$$C.I = 68 + 0.825 \quad | \quad C.I = 68 - 0.825$$

$$C.I = 68.825 \quad | \quad C.I = 67.175$$

Try It 8.2

$$\sigma = 6, n = 28, \bar{X} = 36$$

Find C.I 90%

$$C.I = \bar{X} \pm EBM$$

$$EBM = (Z_{1-\alpha/2}) \left(\frac{\sigma}{\sqrt{n}} \right)$$

$$C.I = 90\%$$

$$1 - \alpha = 90\%$$

$$1 - \alpha = 10\%$$

$$1 - \alpha = 0.10\%$$

$$1 - \alpha/2 = 0.10$$

$$1 - \alpha/2 = 0.05$$

$$1 - \alpha/2 = 1 - 0.05$$

$$1 - \alpha/2 = 0.95$$

$$1 - \alpha/2 = 1.65$$

$$EBM = (1.65)(6/\sqrt{28})$$

$$EBM = (1.65)(1.1339)$$

$$EBM = 1.8709$$

$$C.I = \bar{X} + EBM$$

$$C.I = \bar{X} - EBM$$

$$C.I = 36 + 1.8709$$

$$C.I = 36 - 1.8709$$

$$C.I = 37.8709$$

$$C.I = 34.1291$$

Example 8.4

$$\sigma = 3, n = 36, \bar{x} = 68$$

Find C.I 95%

$$C.I = \bar{X} \pm EBM$$

$$EBM = (Z_{1-\alpha/2})(\sigma/\sqrt{n})$$

$$C.I = 95\%$$

$$C.I = 0.95$$

$$1 - \alpha = 1 - 0.95$$

$$1 - \alpha = 0.05$$

$$1 - \alpha/2 = 0.025$$

$$1 - \alpha/2 = 0.025$$

The area is right and the area is left

$$1 - \alpha/2 = 1 - 0.025$$

$$1 - \alpha/2 = 0.975$$

$$1 - \alpha/2 = 1.96$$

$$EBM = (1.96)(\sqrt{36})$$

$$EBM = (1.96)(0.5)$$

$$EBM = 0.98$$

$$C.I = \bar{X} + EBM \quad | \quad C.I = \bar{X} - EBM$$

$$C.I = 68 + 0.98 \quad | \quad C.I = 68 - 0.98$$

$$C.I = 68.98 \quad | \quad C.I = 67.02$$

Try ~~9.4~~ 8.4

$$\sigma = 6, \bar{X} = 36, n = 20$$

Find C.I 95%

$$C.I = 95\% \quad | \quad C.I = \bar{X} - EBM$$

$$EBM = (2.1 - \alpha/2)(\sigma/\sqrt{n})$$

$$C.I = 0.95$$

$$1 - \alpha = 1 - 0.95$$

$$1 - \alpha = 0.05$$

$$1 - \alpha/2 = \frac{0.05}{2}$$

The area is right $1 - \alpha/2 = 0.025$

$$1 - \alpha/2 = 1 - 0.025$$

$$1 - \alpha/2 = 0.975$$

The area is left $1 - \alpha/2 = 1.96$

$$EBM = (1.96)(\frac{6}{\sqrt{20}})$$

$$EBM = (1.96)(1.3416)$$

$$EBM = 2.6295$$

$$C.I = \bar{X} + EBM \quad | \quad C.I = \bar{X} - EBM$$

$$C.I = 36 + 2.6295 \quad | \quad C.I = 36 - 2.6295$$

$$C.I = 38.6295 \quad | \quad C.I = 33.3705$$

Example 8.5

$$C.I = 90\%, \sigma = 3, n = 100, \text{ ~~Find EBM~~ }$$

$$n = 25 \text{ Find EBM}$$

$$C.I = 90\%$$

$$1 - \alpha = 90$$

$$\alpha = 10 \Rightarrow \alpha = 0.10$$

$$\alpha/2 = \frac{0.10}{2}$$

$$\alpha/2 = 0.05$$

$$1 - \alpha/2 = 1 - 0.05$$

$$1 - \alpha/2 = 0.95$$

$$1 - \alpha/2 = 1.65$$

$$EBM = (Z_{1-\alpha/2}) \sigma / \sqrt{n}$$

$$EBM = (Z_{1-\alpha/2}) (\sigma / \sqrt{n}) \quad EBM = (Z_{1-\alpha/2}) (\sigma / \sqrt{n})$$

$$EBM = (1.65) (3 / \sqrt{100}) \quad EBM = (1.65) (3 / \sqrt{25})$$

$$EBM = (1.65) (0.3) \quad EBM = (1.65) (0.6)$$

$$EBM = 0.495 \quad EBM = 0.99$$

Try 8.5

$$\bar{x} = 36, \sigma = 6, n = 50$$

Find C.I 90%.

Solution: yeh question Exp 8.2 ki tarah karvi hai bus value change hain

Example 8.6

$$C.I = (67.18, 68.82), \bar{x} = 68$$

Use sample mean

$$EBM = 68.82 - 68$$

$$EBM = 0.82$$

Don't Use sample mean

$$EBM = 68.82 - 67.18$$

$$EBM = \frac{1.64^2}{2}$$

$$EBM = 0.82$$

Use EBM

$$\bar{x} = 68.82 - 0.82$$

$$\bar{x} = 68$$

Don't Use EBM

$$\bar{x} = \frac{67.18 + 68.82}{2}$$

$$\bar{x} = \frac{136}{2}$$

$$\bar{x} = 68$$

Try 9+ 8.6

$$C.T = (42.12, 47.88)$$

Find EBM and \bar{x}

$$EBM = 47.88 - 42.12$$

$$EBM = \frac{5.76^2}{2}$$

$$EBM = 2.88$$

$$\bar{x} = \frac{42.12 + 47.88}{2}$$

$$\bar{x} = \frac{90}{2}$$

$$\bar{x} = 45$$

Example 8-3

$$\text{Total amount of phones} = 1.11 + 1.48 + 1.43 + \dots + 1.38$$

$$= 30.712$$

$$\text{Total number of phones} = 30$$

$$\sigma = 0.337, \text{ Find } 98\% \text{ C.I.}$$

Solution:

$$\bar{x} = \frac{30.712}{30}$$

$$\bar{x} = 1.024$$

$$n = 30$$

$$EBM = (Z_{1-\alpha/2}) \left(\frac{\sigma}{\sqrt{n}} \right)$$

$$C.I = 98\%$$

$$C.I = 0.98$$

$$1 - \alpha = 1 - 0.98$$

$$1 - \alpha = 0.02$$

$$1 - \alpha/2 = \frac{0.02}{2}$$

$$1 - \alpha/2 = 0.01 \rightarrow \text{The area is right}$$

$$1 - \alpha/2 = 1 - 0.01$$

$$1 - \alpha/2 = 0.99 \rightarrow \text{The area is left}$$

$$0.99 \text{ is equal to } (2.326)$$

$$EBM = (2.326) \frac{0.337}{\sqrt{30}}$$

$$EBM = (2.326)(0.0615)$$

$$EBM = 0.1430$$

$$\bar{x} \pm EBM$$

$$\bar{x} + EBM \rightarrow 1.024 + 0.1430 = 1.167$$

$$\bar{x} - EBM \rightarrow 1.024 - 0.1430 = 0.881$$

Try It 8.3

Exo

Total amount of phones $1.48 + 0.8 + 1.15 + \dots + 0.869 = 18.801$

Total number of phones = 20

$\sigma = 0.337$, Find
CI 93%

Solution:

$$\bar{x} = \frac{18.801}{20}$$

$$\bar{x} = 0.9401$$

$$n = 20$$

$$EBM = (Z_{1-\alpha/2})\left(\frac{\sigma}{\sqrt{n}}\right)$$

~~CI = 93%~~

$$CI = 0.93$$

$$1 - \alpha = 1 - 0.93$$

$$1 - \alpha = 0.07$$

$$1 - \alpha/2 = \frac{0.07}{2}$$

$$1 - \alpha/2 = 0.035$$

$$1 - \alpha/2 = 1 - 0.035$$

$$1 - \alpha/2 = 0.965$$

$$Z_{1-\alpha/2} = 1.82$$

$$EBM = (1.82)\left(\frac{0.337}{\sqrt{20}}\right)$$

$$EBM = (1.82)(0.0754)$$

$$EBM = 0.1372$$

$$\bar{x} \pm EBM$$

$$\bar{x} + EBM \rightarrow 0.9401 + 0.1372 = 1.0773$$

$$\bar{x} - EBM \rightarrow 0.9401 - 0.1372 = 0.8029$$

Example 8.7

$$\sigma = 15, EBM = 2, CI = 95\%$$

$$n = ?$$

Solution

$$n = (Z_{1-\alpha/2})^2 \left(\frac{\sigma}{EBM} \right)^2$$

$$n = \frac{Z_{1-\alpha/2}^2 \cdot \sigma^2}{EBM^2}$$

$$CI = 95\%$$

$$CI = 0.95$$

$$1 - \alpha = 1 - 0.95$$

$$1 - \alpha = 0.05$$

$$1 - \alpha/2 = \frac{0.05}{2}$$

$$1 - \alpha/2 = 0.025$$

$$1 - \alpha/2 = 1 - 0.025$$

$$1 - \alpha/2 = 0.975$$

$$Z_{1-\alpha/2} = 1.96$$

$$n = \frac{(1.96)^2 (15)^2}{(2)^2}$$

$$n = \frac{(3.8416)(225)}{4}$$

$$n = \frac{864.36}{4}$$

$$n = 216.09$$

$$n = 217$$

~~Example 8.7~~ Try Ex 8.7

$$\sigma = 3, EBM = 1, CI = 95\%$$

$$n = ?$$

Solution:

$$n = \frac{Z_{1-\alpha/2}^2 \cdot \sigma^2}{EBM^2}$$

95% is equal to 1.96

$$Z_{1-\alpha/2} = 1.96$$

$$n = \frac{(1.96)^2 (3.8)}{(1)^2}$$

$$n = \frac{(3.8416)(4)}{1}$$

$$n = 34.5744$$

Example 8.8

$$\bar{X} = \frac{\sum X}{n} = \frac{8.6 + 9.4 + 7.9 + 6.8 + 8.3 + 7.3 + 9.2 + 9.6 + 8.7 + 11.4 + \dots + 6.9}{15}$$

$$\bar{X} = \frac{123.4}{15}$$

$$\bar{X} = 8.2267$$

$$n = 15 \quad S = \sqrt{\frac{\sum (X - \bar{X})^2}{n-1}}$$

$$S = \sqrt{\frac{(X - \bar{X})^2 + (X - \bar{X})^2 + \dots + (X - \bar{X})^2}{n-1}}$$

$$S = \sqrt{\frac{(8.6 - 8.2267)^2 + (9.4 - 8.2267)^2 + \dots + (6.9 - 8.2267)^2}{15-1}}$$

$$S = 1.6722 \quad EBM = (t_{1-\alpha/2}) \left(\frac{S}{\sqrt{n}} \right) \quad t_{1-\alpha/2} = 2.14$$

$$CI = 95\%$$

$$CI = 0.95$$

$$1 - \alpha = 1 - 0.95$$

$$1 - \alpha = 0.05$$

$$1 - \alpha/2 = \frac{0.05}{2}$$

$$1 - \alpha/2 = 0.025$$

$$1 - \alpha/2 = 1 - 0.025$$

$$1 - \alpha/2 = 0.975$$

$$t_{1-\alpha/2} = 2.14$$

$$EBM = (2.14) \frac{1.6722}{\sqrt{15}}$$

$$EBM = (2.14) \left(\frac{1.6722}{3.8729} \right)$$

$$EBM = (2.14)(0.4318)$$

$$EBM = 0.9241$$

$$\bar{X} \pm EBM$$

$$\bar{X} + EBM \rightarrow 8.2267 + 0.9241 = 9.1508$$

$$\bar{X} - EBM \rightarrow 8.2267 - 0.9241 = 7.3026$$

Ex 8.8

8.2, 9.1, 7.7, 8.6, 6.9, 11.2, 10.1, 9.9, 8.9, 9.2, 7.5, 10.5
 $n = 12$



$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{8.2 + 9.1 + 7.7 + 8.6 + 6.9 + 11.2 + 10.1 + 9.9 + 8.9 + 9.2 + 7.5 + 10.5}{12}$$

$$\bar{x} = \frac{107.8}{12}$$

$$\bar{x} = 8.98$$

$$S = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

$$S = \sqrt{\frac{(x - \bar{x})^2 + (x - \bar{x})^2 + \dots + (x - \bar{x})^2}{n - 1}}$$

$$S = \sqrt{\frac{(8.2 - 8.98)^2 + (9.1 - 8.98)^2 + (7.7 - 8.98)^2 + \dots + (10.5 - 8.98)^2}{12 - 1}}$$

Example 8.9

$$n = 20$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{129 + 145 + 147 + 160 + \dots + 199}{20}$$

$$\bar{x} = \frac{2549}{20}$$

$$\bar{x} = 127.45$$

$$S = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

$$S = \sqrt{\frac{(129 - 127.45)^2 + (145 - 127.45)^2 + \dots + (199 - 127.45)^2}{20 - 1}}$$

$$S = \sqrt{\frac{12808.95}{19}}$$

$$S = \sqrt{674.1553}$$

$$S = 25.965$$

$$EBM = (t_{1-\alpha/2, n-1}) \left(\frac{S}{\sqrt{n}} \right)$$

$$CI = 90\%$$

$$CI = 0.90$$

$$1 - \alpha = 1 - 0.90$$

$$1 - \alpha = 0.10$$

$$2 - \alpha/2 = \frac{0.10}{2}$$

$$1 - \alpha/2 = 0.05$$

$$1 - \alpha/2 = 1 - 0.05$$

$$1 - \alpha/2 = 0.95$$

$$EBM = (0.95(20-1)) \left(\frac{25.965}{\sqrt{20}} \right)$$

$$EBM = (0.95(19)) \left(\frac{25.965}{4.4721} \right)$$

$$EBM = (1.724)(5.8059)$$

$$EBM = 10.0384$$

$$\bar{x} \pm EBM$$

$$\bar{x} + EBM$$

$$127.45 + 10.0384 = 137.4884$$

$$\bar{x} - EBM$$

$$127.45 - 10.0384 = 117.4116$$

Try # 8-9

$$n = 15, CI = 98\%$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{0+3+1+20+9+\dots+5}{15}$$

$$\bar{x} = \frac{92}{15}$$

$$\bar{x} = 6.1333 \rightarrow EBM = (t_{1-\alpha/2}(n-1))\left(\frac{S}{\sqrt{n}}\right)$$

$$S = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

$$S = \sqrt{\frac{(0-6.1333)^2 + (3-6.1333)^2 + \dots + (5-6.1333)^2}{15-1}}$$

$$S = \sqrt{\frac{(-6.1333)^2 + (-3.1333)^2 + (-5.1333)^2 + (13.8667)^2 + (2.8667)^2 + (-1.1333)^2 + (3.8667)^2 + (-5.1333)^2 + (3.8667)^2 + (-2.1333)^2 + (7.8667)^2 + (-4.1333)^2 + (-2.1333)^2 + (-2.1333)^2 + (-1.1333)^2}{15-1}}$$

$$S = \sqrt{\frac{37.6174 + 9.8176 + 26.3508 + 192.2854 + 8.21797 + 1.2844 + 14.9514 + 26.3508 + 14.9514 + 4.55097 + 61.88497 + 17.0842 + 4.55097 + 4.55097 + 1.2844}{14}}$$

$$S = \sqrt{\frac{425.7337}{14}}$$

$$S = \sqrt{30.4096}$$

$$S = 5.5145$$

$$CI = 0.98$$

$$1-\alpha = 1-0.98$$

$$1-\alpha = 0.02$$

$$1-\frac{\alpha}{2} = \frac{0.02}{2}$$

$$1 - \alpha/2 = 0.01$$

$$1 - \alpha/2 = 1 - 0.01$$

$$t_{1-\alpha/2} = 0.99$$

$$EBM = (0.99)(15-1) \left(\frac{5.5145}{\sqrt{15}} \right)$$

$$EBM = (0.99)(14) \left(\frac{5.5145}{3.8729} \right)$$

$$EBM = (2.624)(1.4239)$$

$$EBM = 3.7363$$

$$\bar{X} \pm EBM$$

$$\bar{X} + EBM$$

$$6.1333 + 3.7363 = 9.8696$$

$$\bar{X} - EBM$$

$$6.1333 - 3.7363 = 2.397$$

Practice

Question. 17

$$\sigma = 0.2, n = 20, CI = 90\%, EBM = ?$$

$$EBM = (2_{1-\alpha/2}) \left(\frac{\sigma}{\sqrt{n}} \right)$$

$$CI = 90\%$$

$$CI = 0.90$$

$$1 - \alpha = 1 - 0.90$$

$$1 - \alpha = 0.10$$

$$1 - \alpha/2 = \frac{0.10}{2}$$

$$1 - \alpha/2 = 0.05$$

$$1 - \alpha/2 = 1 - 0.05$$

$$1 - \alpha/2 = 0.95$$

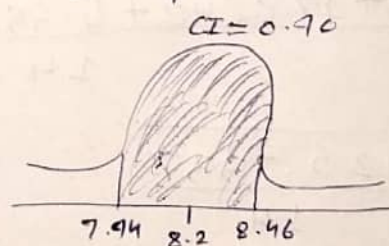
$$2_{1-\alpha/2} = 1.65$$

$$EBM = (1.65) \left(\frac{0.2}{\sqrt{20}} \right)$$

$$EBM = (1.65) \left(\frac{0.2}{4.4721} \right)$$

$$EBM = (1.65)(0.0447)$$

Graph



$$EBM = 0.07$$

Question. 18

$\sigma = 0.2$, $n = 20$ CI = 95%, EBM = ?

$$EBM = (Z_{1-\alpha/2}) \left(\frac{\sigma}{\sqrt{n}} \right)$$

$$CI = 0.95$$

$$1 - \alpha = 1 - 0.95$$

$$1 - \alpha = 0.05$$

$$1 - \alpha/2 = \frac{0.05}{2}$$

$$1 - \alpha/2 = 0.025$$

$$1 - \alpha/2 = 1 - 0.025$$

$$1 - \alpha/2 = 0.975$$

$$Z_{1-\alpha/2} = 1.96$$

$$EBM = (1.96) \left(\frac{0.2}{\sqrt{20}} \right)$$

$$EBM = (1.96) \left(\frac{0.2}{4.4721} \right)$$

$$EBM = (1.96) (0.0447)$$

$$EBM = 0.08$$

Question. 19

The interval is greater because the level of confidence increased. If the only changed in the analysis is a change in confidence level then all we are doing is changing how much area is being calculated for the normal distribution. Therefore a large result area and larger intervals.

Question. 20

The confidence interval denotes the range of possibilities for the population parameter. We are 95% convinced that the population parameter is between x and x is the accurate reading of a 95% confidence intervals.

Question. 21

The confidence level would increase.

Question. 22

If 40 heads of lettuce were sampled instead of 20, it means the sample size has increased. Also the confidence interval is the same.

Question. 44

A random variable is defined as the value of the given variable which represented the outcome of a statistical experiment. It is usually represented by X .

Question. 45

\bar{X} is the mean number of hours spent watching television per month from a sample of 108 Americans.

Question. 46

The distribution required for this problem is 't' distribution with parameter $n-1$.

Question. 47

$$\bar{X} = 151, S = 32, n = 108, CI = 99\%$$

$$CI = 99\%, EBM = (t_{1-\alpha/2}) \frac{S}{\sqrt{n}}$$

$$CI = 0.99$$

$$1 - \alpha = 1 - 0.99$$

$$1 - \alpha = 0.01$$

$$1 - \alpha/2 = \frac{0.01}{2}$$

$$1 - \alpha/2 = 5 \times 10^{-3}$$

$$1 - \alpha/2 = 0.005$$

$$1 - \alpha/2 = 1 - 0.005$$

$$1 - \alpha/2 = 0.995$$

$$t_{1-\alpha/2} = 2.575$$

$$EBM = (2.575) \frac{32}{\sqrt{108}}$$

$$EBM = (2.575) \left(\frac{32}{10.3923} \right)$$

$$EBM = (2.575) (3.0792)$$

$$EBM = 7.9289$$

$$\bar{X} \pm EBM$$

$$\bar{X} + EBM$$

$$151 + 7.9289 = 158.9289$$

$$\bar{X} - EBM$$

$$151 - 7.9289 = 143.0711$$