Try It Σ

$(42, 50, 45, \frac{8}{\sqrt{30}})$

7.1 An unknown distribution has a mean of 45 and a standard deviation of eight. Samples of size $n = 30$ are drawn randomly from the population. Find the probability that the sample mean is between 42 and 50.

$P(42 < \bar{x} < 50) = 0.9797$

Example 7.2

The length of time, in hours, it takes an "over 40" group of people to play one soccer match is normally distributed with a **mean of two hours** and a **standard deviation of 0.5 hours**. A **sample of size $n = 50$** is drawn randomly from the population. Find the probability that the **sample mean** is between 1.8 hours and 2.3 hours.

Solution 7.2

Let X = the time, in hours, it takes to play one soccer match.

The probability question asks you to find a probability for the **sample mean time, in hours,** it takes to play one soccer match.

Let $\bar{X}$ = the **mean** time, in hours, it takes to play one soccer match.

If $\mu_{\bar{X}}$ = 2, $\sigma_{\bar{X}}$ = 0.5, and n = 50, then $\bar{X} \sim N(2, \frac{0.5}{\sqrt{50}})$ by the **central limit theorem for means.**

$\mu_{\bar{X}} = 2$, $\sigma_{\bar{X}} = 0.5$, $n = 50$, and $\bar{X} \sim N\left(2, \frac{0.5}{\sqrt{50}}\right)$

Find $P(1.8 < \bar{x} < 2.3)$. Draw a graph.

$P(1.8 < \bar{x} < 2.3) = 0.9977$

`normalcdf` $\left(1.8, 2.3, 2, \frac{.5}{\sqrt{50}}\right) = 0.9977$

The probability that the mean time is between 1.8 hours and 2.3 hours is 0.9977.

Try It Σ

7.2 The length of time taken on the SAT for a group of students is normally distributed with a mean of 2.5 hours and a standard deviation of 0.25 hours. A sample size of $n = 60$ is drawn randomly from the population. Find the probability that the sample mean is between two hours and three hours.

$P(2, 3, 2.5, \frac{0.25}{\sqrt{60}})$

$P(2 < \bar{x} < 3) = 1$

Using the TI-83, 83+, 84, 84+ Calculator

To find percentiles for means on the calculator, follow these steps. ✓

2nd DIStR
3:invNorm

$$k = \text{invNorm}\left(\text{area to the left of } k\text{, mean, } \frac{\textit{standard deviation}}{\sqrt{\textit{sample size}}}\right)$$

where:

- k = the k^{th} percentile
- *mean* is the mean of the original distribution
- *standard deviation* is the standard deviation of the original distribution
- *sample size* = n

Example 7.3

In a recent study reported Oct. 29, 2012 on the Flurry Blog, the mean age of tablet users is 34 years. Suppose the standard deviation is 15 years. Take a sample of size $n = 100$.

a. What are the mean and standard deviation for the sample mean ages of tablet users?

b. What does the distribution look like?

c. Find the probability that the sample mean age is more than 30 years (the reported mean age of tablet users in this particular study).

d. Find the 95th percentile for the sample mean age (to one decimal place).

Solution 7.3

a. Since the sample mean tends to target the population mean, we have $\mu_{\bar{x}} = \mu = 34$. The sample standard deviation is given by $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{100}} = \frac{15}{10} = 1.5$

b. The central limit theorem states that for large sample sizes(n), the sampling distribution will be approximately normal.

c. The probability that the sample mean age is more than 30 is given by $P(X > 30)$ = `normalcdf`(30,E99,34,1.5) = 0.9962

d. Let k = the 95th percentile.

$$k = \text{invNorm}\left(0.95, 34, \frac{15}{\sqrt{100}}\right) = 36.5$$

Try It Σ

$\left(29, 35, 28, \frac{4.8}{\sqrt{100}}\right)$

$P(29 < \bar{x} < 35) =$ 0.0186

7.3 In an article on Flurry Blog, a gaming marketing gap for men between the ages of 30 and 40 is identified. You are researching a startup game targeted at the 35-year-old demographic. Your idea is to develop a strategy game that can be played by men from their late 20s through their late 30s. Based on the article's data, industry research shows that the average strategy player is 28 years old with a standard deviation of 4.8 years. You take a sample of 100 randomly selected gamers. If your target market is 29- to 35-year-olds, should you continue with your development strategy?

You can conclude there is approximately a 2% chance that your game will be played by men whose mean age is between 29 and 35.

Example 7.4

The mean number of minutes for app engagement by a tablet user is 8.2 minutes. Suppose the standard deviation is one minute. Take a sample of 60.

a. What are the mean and standard deviation for the sample mean number of app engagement by a tablet user?

b. What is the standard error of the mean?

c. Find the 90th percentile for the sample mean time for app engagement for a tablet user. Interpret this value in a complete sentence.

d. Find the probability that the sample mean is between eight minutes and 8.5 minutes.

Solution 7.4

a. $\mu_{\bar{x}} = \mu = 8.2 \; \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{60}} = 0.13$

b. This allows us to calculate the probability of sample means of a particular distance from the mean, in repeated samples of size 60.

c. Let k = the 90th percentile
k = invNorm$\left(0.90, 8.2, \frac{1}{\sqrt{60}}\right) = 8.37$. This values indicates that 90 percent of the average app engagement time for table users is less than 8.37 minutes.

d. $P(8 < \bar{x} < 8.5)$ = normalcdf$\left(8, 8.5, 8.2, \frac{1}{\sqrt{60}}\right) = 0.9293$

Try It Σ

7.4 Cans of a cola beverage claim to contain 16 ounces. The amounts in a sample are measured and the statistics are $n = 34$, $\bar{x} = 16.01$ ounces. If the cans are filled so that $\mu = 16.00$ ounces (as labeled) and $\sigma = 0.143$ ounces, find the probability that a sample of 34 cans will have an average amount greater than 16.01 ounces. Do the results suggest that cans are filled with an amount greater than 16 ounces?

$P(\bar{x} > 16.01) = 0.3417$

7.2 | The Central Limit Theorem for Sums

Suppose X is a random variable with a distribution that may be **known or unknown** (it can be any distribution) and suppose:

a. μ_X = the mean of X

b. σ_X = the standard deviation of X

If you draw random samples of size n, then as n increases, the random variable ΣX consisting of sums tends to be **normally distributed** and $\Sigma X \sim N((n)(\mu_X), (\sqrt{n})(\sigma_X))$.

The **central limit theorem for sums** says that if you keep drawing larger and larger samples and taking their sums, the sums form their own normal distribution (the sampling distribution), which approaches a normal distribution as the sample size increases. **The normal distribution has a mean equal to the original mean multiplied by the sample size and a standard deviation equal to the original standard deviation multiplied by the square root of the sample size.**

The random variable ΣX has the following z-score associated with it:

a. Σx is one sum.

b. $z = \frac{\Sigma x - (n)(\mu_X)}{(\sqrt{n})(\sigma_X)}$

i. $(n)(\mu_X)$ = the mean of ΣX

ii. $(\sqrt{n})(\sigma_X)$ = standard deviation of ΣX

Using the TI-83, 83+, 84, 84+ Calculator

To find probabilities for sums on the calculator, follow these steps.

2nd DISTR
2:normalcdf
normalcdf(lower value of the area, upper value of the area, (n)(mean), $(\sqrt{n})$(standard deviation))