Introduction to Sinusoidal Waveform

THE SINUSOIDAL WAVEFORM

The sinusoidal waveform or sine wave is the fundamental type of alternating current (ac) and alternating voltage. It is also referred to as a sinusoidal wave or, simply, sinusoid. The electrical service provided by the power company is in the form of sinusoidal voltage and current.

Sinusoidal voltages are produced by two types of sources: rotating electrical machines (ac generators) or electronic oscillator circuits, which are used in instruments commonly known as electronic signal generators.

Figure 2 is a graph showing the general shape of a sine wave, which can be either an alternating current or an alternating voltage. Voltage (or current) is displayed on the vertical axis and time (t) is displayed on the horizontal axis. Notice how the voltage (or current) varies with time. Starting at zero, the voltage (or current) increases to a positive maximum (peak), returns to zero, and then increases to a negative maximum (peak) before returning again to zero, thus completing one full cycle.

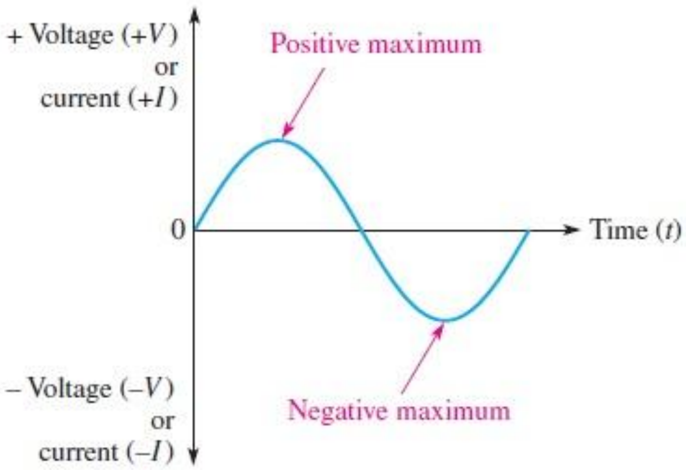
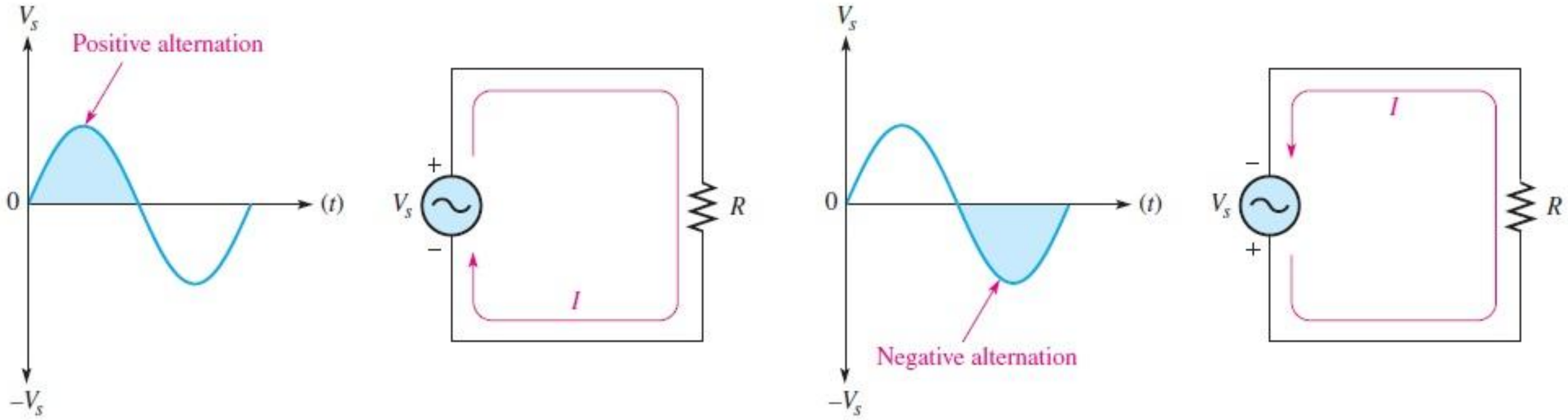


FIGURE 2

Graph of one cycle of a sine wave.

Polarity of a Sine Wave



(a) During a positive alternation of voltage, current is in the direction shown.

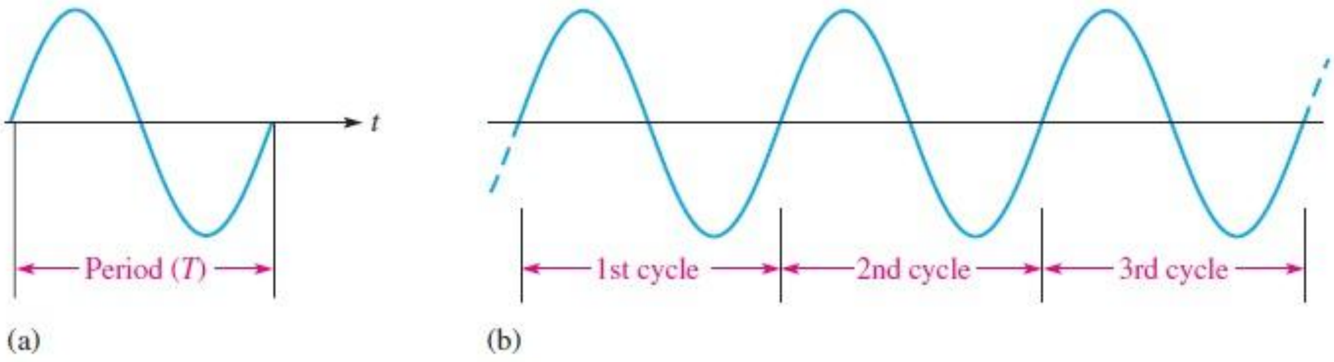
(b) During a negative alternation of voltage, current reverses direction, as shown.

Time Period of a Sine Wave

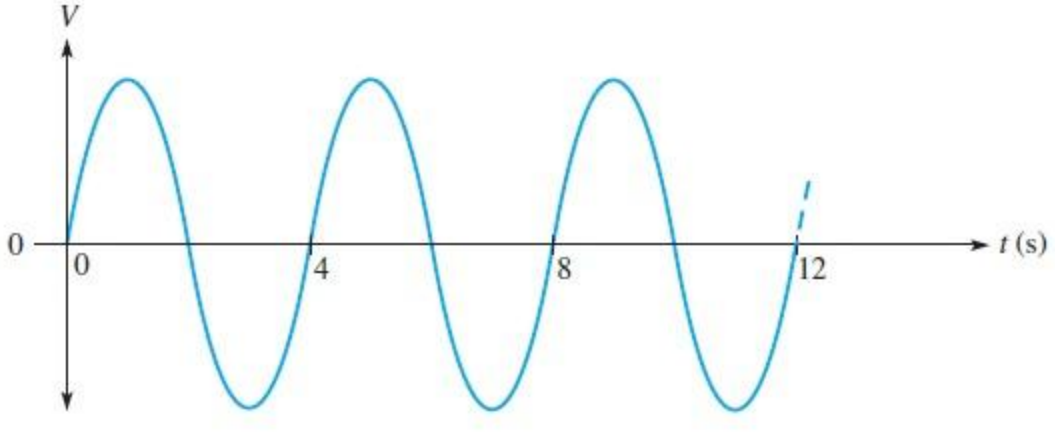
The time required for a sine wave to complete one full cycle is called the period (T).

▶ FIGURE 4

The period of a sine wave is the same for each cycle.

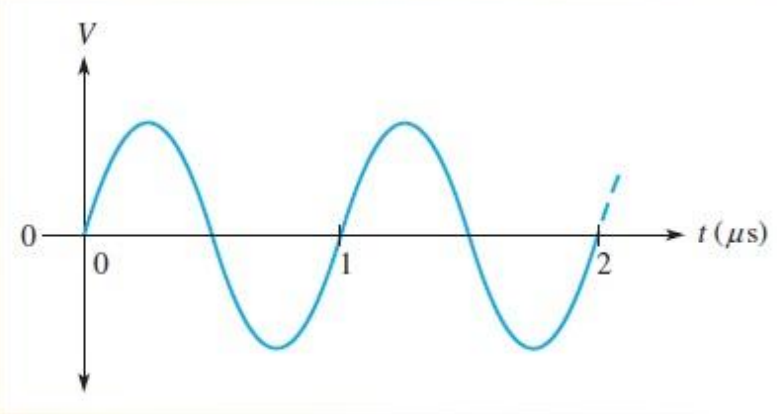


What is the period of the sine wave in Figure 5?



Show three possible ways to measure the period of the sine wave in Figure 6. How many cycles are shown?

▶ FIGURE 6



Solution

Method 1: The period can be measured from one zero crossing to the corresponding zero crossing in the next cycle (the slope must be the same at the corresponding zero crossings).

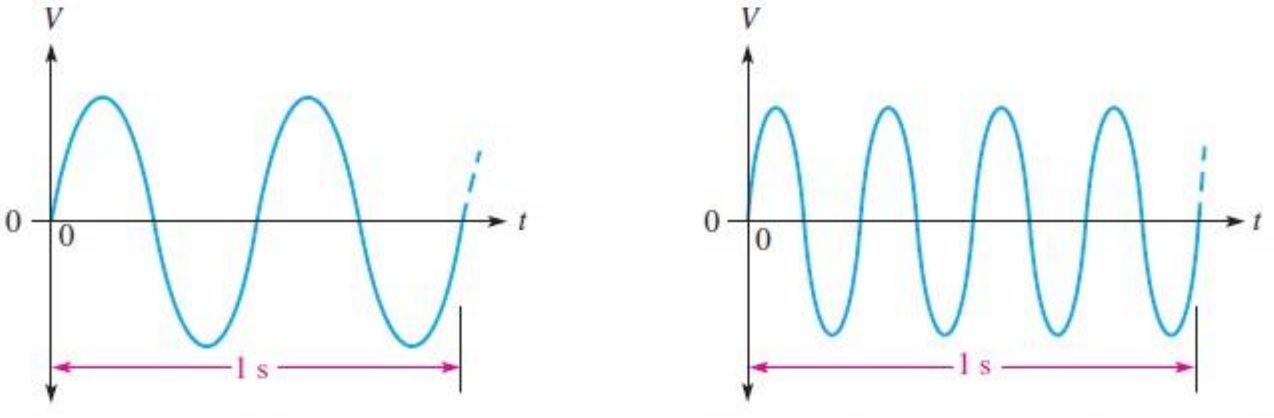
Method 2: The period can be measured from the positive peak in one cycle to the positive peak in the next cycle.

Method 3: The period can be measured from the negative peak in one cycle to the negative peak in the next cycle.

Frequency of a Sine Wave

Frequency (f) is the number of cycles that a sine wave completes in one second.

The more cycles completed in one second, the higher the frequency. Frequency (f) is measured in units of hertz. One hertz (Hz) is equivalent to one cycle per second; 60 Hz is 60 cycles per second,



(a) Lower frequency: fewer cycles per second

(b) Higher frequency: more cycles per second

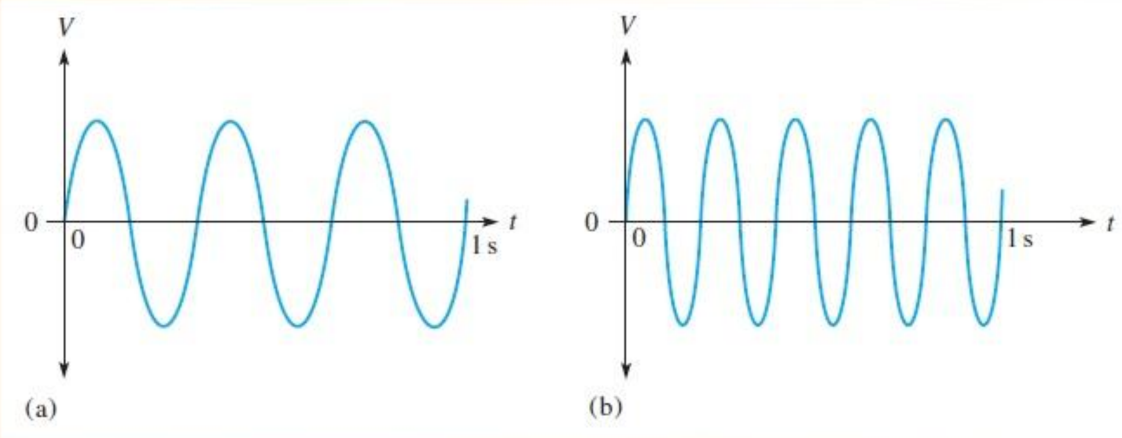
Relationship of Frequency and Period

The formulas for the relationship between frequency (f) and period (T) are as follows:

$$f = \frac{1}{T}$$

$$T = \frac{1}{f}$$

Which sine wave in Figure 9 has a higher frequency? Determine the frequency and the period of both waveforms.



The period of a certain sine wave is 10 ms. What is the frequency?

A certain sine wave goes through four cycles in 20 ms. What is the frequency?

The frequency of a sine wave is 60 Hz. What is the period?

If $T = 15\ \mu s$, what is f?

SINUSOIDAL VOLTAGE AND CURRENT VALUES

Instantaneous Value

Any point in time on a sine wave, the voltage (or current) has an instantaneous value.

This instantaneous value is different at different points along the curve. Instantaneous values are positive during the positive alternation and negative during the negative alternation

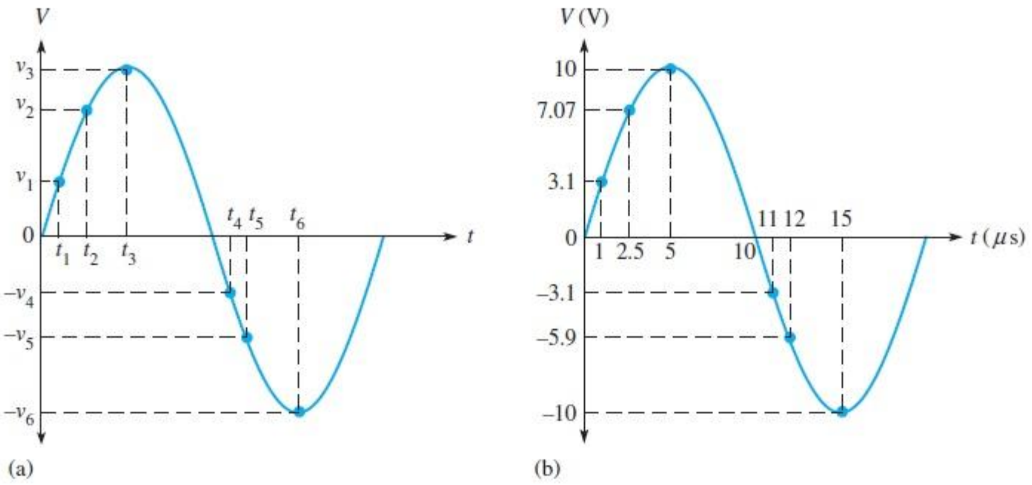


FIGURE 11
Instantaneous values.

Peak Value

The peak value of a sine wave is the value of voltage (or current) at the positive or the negative maximum (peak) with respect to zero.

Since the positive and negative peak values are equal in magnitude, a sine wave is characterized by a single peak value. This is illustrated in Figure 12. For a given sine wave, the peak value is constant and is represented by or Ip. The peak value is also called the amplitude.

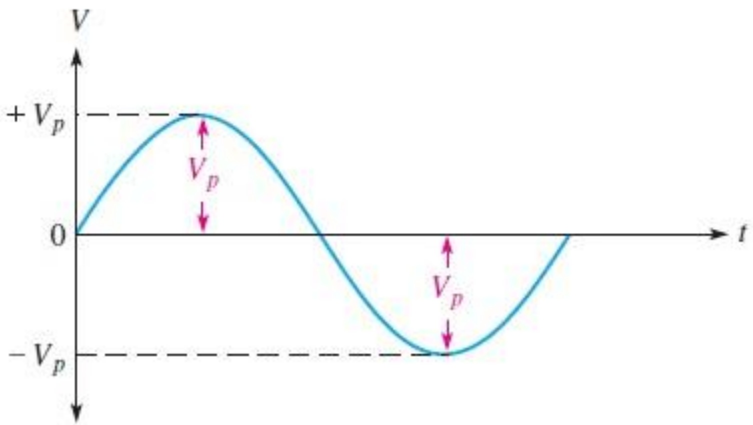


FIGURE 12

Peak values.

Peak-to-Peak Value

The peak-to-peak value of a sine wave, as shown in Figure 13, is the voltage or current from the positive peak to the negative peak. It is always twice the peak value

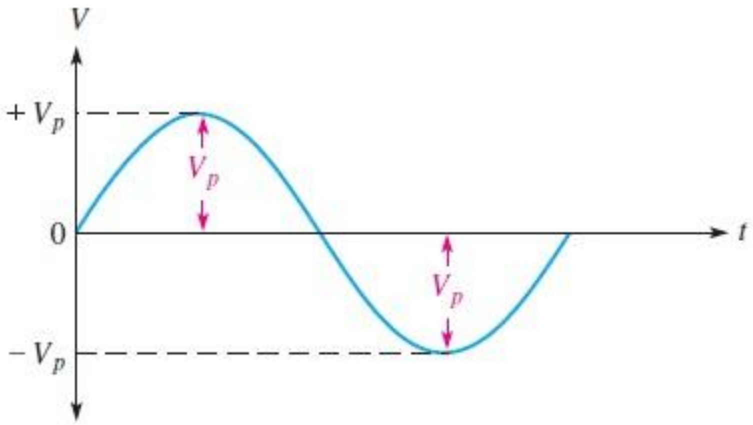


FIGURE 12

Peak values.

Average Value

The average value of a sine wave taken over one complete cycle is always zero because the positive values (above the zero crossing) offset the negative values (below the zero crossing). To be useful for certain purposes such as measuring types of voltages found in power supplies, the average value of a sine wave is defined over a half-cycle rather than over a full cycle.

The average value is the total area under the half-cycle curve divided by the distance in radians of the curve along the horizontal axis

$$V_{avg} = \left(\frac{2}{\pi}\right)V_p$$

$$V_{avg} = 0.637V_p$$

$$I_{avg} = \left(\frac{2}{\pi}\right)I_p$$

$$I_{avg} = 0.637I_p$$

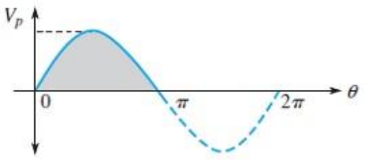
Derivation of Average Value of a Half-Cycle Sine Wave

The equation for a sine wave is

$$v = V_p \sin\theta$$

The average value of the half-cycle is the area under the curve divided by the distance of the curve along the horizontal axis (see Figure 2).

$$V_{avg} = \frac{\text{area}}{\pi}$$



▲ FIGURE 2

To find the area, we use integral calculus.

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} V_p \sin\theta\, d\theta = \frac{V_p}{\pi}(-\cos\theta)\Big|_0^{\pi}$$

$$= \frac{V_p}{\pi}[-\cos\pi - (-\cos 0)] = \frac{V_p}{\pi}[-(-1) - (-1)]$$

$$= \frac{V_p}{\pi}(2) = \frac{2}{\pi}V_p = 0.637V_p$$

RMS Value

The term rms stands for root mean square. Most ac voltmeters display rms voltage. The 220 V at your wall outlet is an rms value. The rms value, also referred to as the effective value, of a sinusoidal voltage is actually a measure of the heating effect of the sine wave.

The rms value of a sinusoidal voltage is equal to the dc voltage that produces the same amount of heat in a resistance as does the sinusoidal voltage.

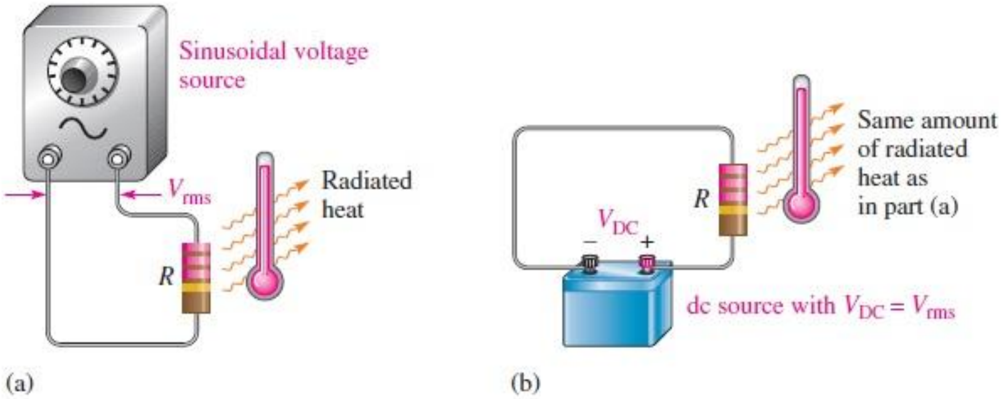


FIGURE 14

When the same amount of heat is produced in both setups, the sinusoidal voltage has an rms value equal to the dc voltage.

The peak value of a sine wave can be converted to the corresponding rms value using the following relationships for either voltage or current:

$$V_{\text{rms}} = 0.707V_p$$

$$I_{\text{rms}} = 0.707I_p$$

Using these formulas, you can also determine the peak value if you know the rms value.

$$V_p = \frac{V_{\text{rms}}}{0.707}$$

$$V_p = 1.414V_{\text{rms}}$$

Similarly,

$$I_p = 1.414I_{\text{rms}}$$

To get the peak-to-peak value, simply double the peak value.

$$V_{pp} = 2.828V_{\text{rms}}$$

and

$$I_{pp} = 2.828I_{\text{rms}}$$

Derivation of RMS Value

The abbreviation "rms" stands for the root mean square process by which this value is derived. In the process, we first square the equation of a sine wave.

$$v^2 = V_p^2 \sin^2\theta$$

Next, we obtain the mean or average value of v^2 by dividing the area under a half-cycle of the curve by π (see Figure 1). The area is found by integration and trigonometric identities.

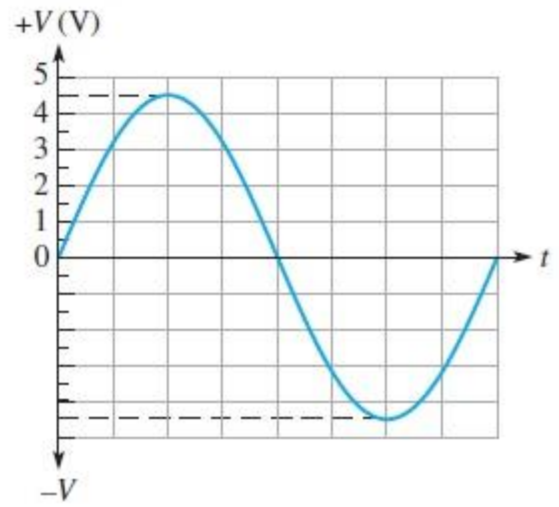
$$\begin{aligned}
V_{\text{avg}}^2 &= \frac{\text{area}}{\pi} = \frac{1}{\pi}\int_0^{\pi} V_p^2 \sin^2\theta \, d\theta \\
&= \frac{V_p^2}{2\pi}\int_0^{\pi}(1 - \cos 2\theta)d\theta = \frac{V_p^2}{2\pi}\int_0^{\pi} 1 \, d\theta - \frac{V_p^2}{2\pi}\int_0^{\pi}(-\cos 2\theta)\, d\theta \\
&= \frac{V_p^2}{2\pi}(\theta - \tfrac{1}{2}\sin 2\theta)_0^{\pi} = \frac{V_p^2}{2\pi}(\pi - 0) = \frac{V_p^2}{2}
\end{aligned}$$

Finally, the square root of V_{avg}^2 is V_{rms}.

$$V_{\text{rms}} = \sqrt{V_{\text{avg}}^2} = \sqrt{V_p^2/2} = \frac{V_p}{\sqrt{2}} = 0.707 V_p$$

Determine V_p, V_{pp}, V_{rms}, and the half-cycle V_{avg} for the sine wave in Figure 15.

FIGURE 15



1. Determine V_{pp} in each case when
   (a) $V_p = 1$ V (b) $V_{rms} = 1.414$ V (c) $V_{avg} = 3$ V
2. Determine V_{rms} in each case when
   (a) $V_p = 2.5$ V (b) $V_{pp} = 10$ V (c) $V_{avg} = 1.5$ V
3. Determine the half-cycle V_{avg} in each case when
   (a) $V_p = 10$ V (b) $V_{rms} = 2.3$ V (c) $V_{pp} = 60$ V

8. For the sine wave in Figure 77, determine the peak, peak-to-peak, rms, and average values.

FIGURE 77