Parameterization of a Line segment

Example: Parameterize the line segment joining the points P(-3,2,-3) and Q(1,-1,4).

Solution:

First of all, we will find the parametric equation of the line through the points P(-3,2,-3) and Q(1,-1,4) and then restrict the domain of parameter t to obtain the parametric equation of the line segment from P to Q.

Step-1 (equation of line)

$$\vec{v} = \overrightarrow{PQ} = (1+3)i + (-1-2)j + (4+3)k$$

$$\vec{v} = 4i - 3j + 7k$$

P(-3,2,-3)

Equation of line

$$\begin{aligned} x &= -3 + 4t \\ y &= 2 - 3t \\ z &= -3 + 7t \end{aligned}$$

Step-2 (line segment)

In order to find the value of t for which an arbitrary point (x,y,z) of the line is at P(-3,2,-3) we solve the equation

$$\left.\begin{array}{l} -3 = -3 + 4t \\ 2 = 2 - 3t \\ -3 = -3 + 7t \end{array}\right\} \Rightarrow t = 0$$

Similarly, when (x,y,z) is at Q(1,-1,4) we solve

$$\left.\begin{array}{l} 1 = -3 + 4t \\ -1 = 2 - 3t \\ 4 = -3 + 7t \end{array}\right\} \Rightarrow t = 1$$

So,the parametric equation of the line segment is

$$x = -3 + 4t$$

$$y = 2 - 3t$$

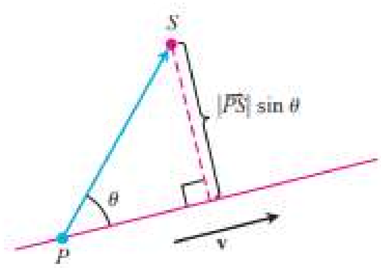
$z = -3 + 7t; \quad 0 \leq t \leq 1$

Question 19: Find the parametric equations of the line segment joining the points P(-2,0,2) and Q(0,2,0).

Ex. 12.5: 13-20

The Distance from aPoint to aLine in Space

The distance from a point S to a line L that passes through a point P and is parallel to a vector $\vec{v}$**is the absolute value of the scalar component of $\overrightarrow{PS}$ in the direction of the vector normal to the line**.



$d = |\overrightarrow{PS}| \sin\theta$**(1)**

As we know that

$$\overrightarrow{PS} \times \vec{v} = |\overrightarrow{PS}||\vec{v}| \sin\theta \, \hat{n}$$

$$|\overrightarrow{PS} \times \vec{v}| = |\overrightarrow{PS}||\vec{v}| \sin\theta \cdot 1$$

$$\frac{|\overrightarrow{PS} \times \vec{v}|}{|\vec{v}|} = |\overrightarrow{PS}| \sin\theta.$$

So, equation **(1)** becomes:

$$d = \frac{|\overrightarrow{PS} \times \vec{v}|}{|\vec{v}|}$$

Example: Find the distance from the point S(1,1,5) to the line:

$$L:\begin{cases} x = 1 + t \\ y = 3 - t \\ z = 2t \end{cases}$$

Solution:

The vector parallel to the line L is

$$\vec{v} = i - j + 2k$$

The Line passes through the point P(1,3,0)

$$\overrightarrow{PS} = (1-1)i + (1-3)j + (5-0)k$$

$$\overrightarrow{PS} = 0i - 2j + 5k.$$

$$\overrightarrow{PS} \times \vec{v} = \begin{vmatrix} i & j & k \\ 0 & -2 & 5 \\ 1 & -1 & 2 \end{vmatrix}$$

$$\overrightarrow{PS} \times \vec{v} = i\begin{vmatrix} -2 & 5 \\ -1 & 2 \end{vmatrix} - j\begin{vmatrix} 0 & 5 \\ 1 & 2 \end{vmatrix} + k\begin{vmatrix} 0 & -2 \\ 1 & -1 \end{vmatrix}$$

$$\overrightarrow{PS} \times \vec{v} = i(-4+5) - j(0-5) + k(0+2)$$

$$\overrightarrow{PS} \times \vec{v} = i + 5j + 2k$$

$$\left|\overrightarrow{PS} \times \vec{v}\right| = \sqrt{(1)^2 + (5)^2 + (2)^2}$$

$$\left|\overrightarrow{PS} \times \vec{v}\right| = \sqrt{30}$$

$$|\vec{v}| = \sqrt{(1)^2 + (-1)^2 + (2)^2} = \sqrt{6}$$

$$d = \frac{\left|\overrightarrow{PS} \times \vec{v}\right|}{|\vec{v}|}$$

$$d = \frac{\sqrt{30}}{\sqrt{6}}$$

$$d = \sqrt{5}$$

Ex. 12.5: 33-38

In Exercises 33–38, find the distance from the point to the line.

33. $(0, 0, 12)$; $x = 4t$, $y = -2t$, $z = 2t$

34. $(0, 0, 0)$; $x = 5 + 3t$, $y = 5 + 4t$, $z = -3 - 5t$

35. $(2, 1, 3)$; $x = 2 + 2t$, $y = 1 + 6t$, $z = 3$

36. $(2, 1, -1)$; $x = 2t$, $y = 1 + 2t$, $z = 2t$

37. $(3, -1, 4)$; $x = 4 - t$, $y = 3 + 2t$, $z = -5 + 3t$

38. $(-1, 4, 3)$; $x = 10 + 4t$, $y = -3$, $z = 4t$

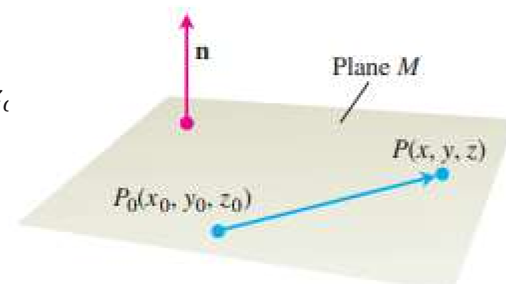
Equation of a Plane in Space:

A plane in space is determined by knowing a point on the plane and its “tilt” or orientation. This “tilt” is defined by specifying a vector that is perpendicular or normal to theplane.

Suppose that a plane *M* passes through a point $P_o\,(x_{o,}y_{o,}z_o)$and is normal to thenon-zero vector $\vec{n} = A\vec{\imath} + B\vec{\jmath} + C\vec{k}$. Then *M* is the set of all points $P(x, y, z)$for which $\overrightarrow{P_O P}$is orthogonal to $\vec{n}$. Thus, the dot product $\vec{n}.\overrightarrow{P_O P} = 0$.

This equation is equivalent to

$$(A\vec{\imath} + B\vec{\jmath} + C\vec{k}).\,[(x - x_o)\vec{\imath} + (y - y_o)\vec{\jmath} + (z - z_($$



$A(x - x_o) + B(y - y_o) + C(z - z_o) = 0$

Remark:

Another form of the equation of plane.

$Ax - Ax_o + By - By_o + Cz - Cz_o = 0$

$Ax + By + Cz - (Ax_o + By_o + Cz_o) = 0$

$Ax + By + Cz = Ax_o + By_o + Cz_o$

$Ax + By + Cz = D$ where $D = \; Ax_o + By_o + Cz_o$

Example 1: Find an equation for the plane through $P(-3,0,7)$ perpendicular to $\vec{n} = 5\vec{\imath} + 2\vec{\jmath} - \vec{k}$.

Solution: The equation of plane is

$$A(x - x_o) + B(y - y_o) + C(z - z_o) = 0$$

$$5(x - (-3)) + 2(y - 0) + (-1)(z - 7) = 0$$

$$5(x + 3) + 2y - z + 7 = 0$$

$5x + 15 + 2y - z + 7 = 0$

$5x + 2y - z + 22 = 0$

$5x + 2y - z = -22$

Example 2: Find an equation for the plane passing through three points A(0,0,1), B(2,0,0) and C(0,3,0).

Solution: A vector normal to the plane is:

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$$

$$= \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & 0 & -1 \\ 0 & 3 & -1 \end{vmatrix} = \vec{\imath} \begin{vmatrix} 0 & -1 \\ 3 & -1 \end{vmatrix} - \vec{\jmath} \begin{vmatrix} 2 & -1 \\ 0 & -1 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix}$$

$$= \vec{\imath}[(0 - (-3)] - \vec{\jmath}(-2 - 0) + \vec{k}(6 - 0)$$

$\vec{n} = 3\vec{\imath} + 2\vec{\jmath} + 6\vec{k}$

Now the equation of plane is

$$A(x - x_o) + B(y - y_o) + C(z - z_o) = 0$$

$$3(x - 0) + 2(y - 0) + 6(z - 1) = 0$$

$3x + 2y + 6z = 6$

Ex. 12.5: 21-26

Find equations for the planes in Exercises 21–26.

21. The plane through $P_0(0, 2, -1)$ normal to $\mathbf{n} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$

22. The plane through $(1, -1, 3)$ parallel to the plane

$$3x + y + z = 7$$

23. The plane through $(1, 1, -1)$, $(2, 0, 2)$, and $(0, -2, 1)$

24. The plane through $(2, 4, 5)$, $(1, 5, 7)$, and $(-1, 6, 8)$

25. The plane through $P_0(2, 4, 5)$ perpendicular to the line

$$x = 5 + t, \quad y = 1 + 3t, \quad z = 4t$$

26. The plane through $A(1, -2, 1)$ perpendicular to the vector from the origin to A