

Solution of Word Problems Continued....

Question 1

I have 32 bills in my wallet, in the denominations of US \$ 1, 5 and 10, worth \$100 in total. How many do I have of each denomination?

Solution: Let x = no. of 1\$ bills, y = no. of 5\$ bills, z = no. of 10\$ bills.

System looks like:

$$\begin{cases} x + y + z = 32 \longrightarrow (1) \\ x + 5y + 10z = 100 \longrightarrow (2) \end{cases}$$

The augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 32 \\ 1 & 5 & 10 & 100 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 32 \\ 0 & 4 & 9 & 68 \end{array} \right] \quad \sim \sim \sim \sim \sim \sim \sim \quad R_2 - R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 32 \\ 0 & 1 & \frac{9}{4} & 17 \end{array} \right] \quad \sim \sim \sim \sim \sim \sim \sim \quad \frac{1}{4}R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{5}{4} & 15 \\ 0 & 1 & \frac{9}{4} & 17 \end{array} \right] \quad \sim \sim \sim \sim \sim \sim \sim \quad R_1 - R_2$$

$$x - \frac{5}{4}z = 15 \quad \Rightarrow x = \frac{5}{4}z + 15$$

$$y + \frac{9}{4}z = 17 \quad \Rightarrow y = -\frac{9}{4}z + 17$$

Let $z = t$,

x, y, z should be positive integers.

Now we see the conditions on z or t .

- First of all, let $z = t$ should be multiple of 4.
- Further z must be positive and $y = 17 - \frac{9}{4}t$ must be positive as well.
- $t = 4, 8, 12, 16 \dots \dots$

Only $t = 4$ works here, which gives $z = 4$ and

$$x = 15 + \frac{5}{4}(4) = 20$$

$$y = 17 - \frac{9}{4}(4) = 8.$$

So I have 20 one dollar bills, 8 five dollar bills, and 4 ten dollar bills.

Question 2

Ali is getting some flowers for his office. Being of a precise analytical mind, he plans to spend exactly \$24 on a bunch of exactly two dozen flowers. At the flower market they have lilies (\$3 each), roses (\$2 each), and daisies (\$0.50 each). Ali loves lilies, what is he to do?

Solution:

$$\begin{aligned} 3l + 2r + 0.5d &= 24 \\ l + r + d &= 24 \end{aligned}$$

Its Augmented matrix is

$$\left[\begin{array}{ccc|c} 3 & 2 & 0.5 & 24 \\ 1 & 1 & 1 & 24 \end{array} \right]$$

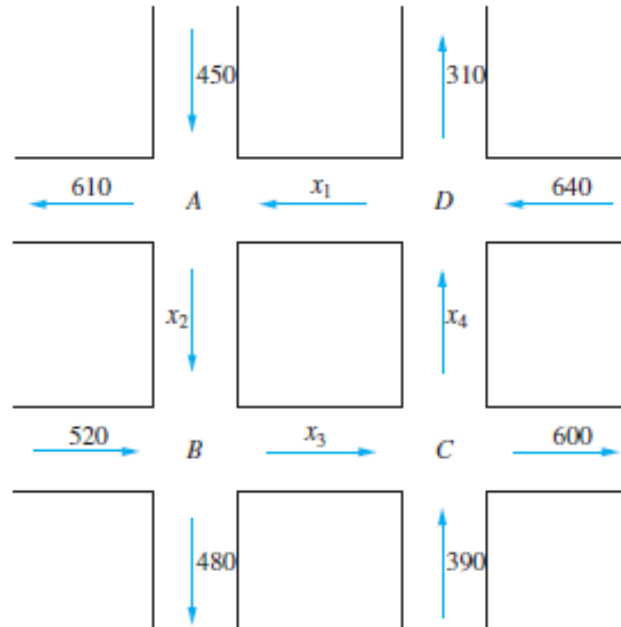
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Complete it yourself

Question 3

In the downtown section of a certain city, two sets of one-way streets intersect as shown in Figure below. The average hourly volume of traffic entering and leaving this section during rush hour is given in the diagram. At each intersection, the number of automobiles entering must be the same as the number leaving.

- (a) Determine the amount of traffic between each of the four intersections.
- (b) The amount of traffic between intersections C and D averages 200 automobiles. Please find the amount of traffic for other intersections.



Solution:

$$\begin{aligned} \text{A: } x_1 + 450 &= x_2 + 610 & \Rightarrow x_1 - x_2 &= 610 - 450 = 160 \\ \text{B: } x_2 + 520 &= x_3 + 480 & \Rightarrow x_2 - x_3 &= 480 - 520 = -40 \\ \text{C: } x_3 + 390 &= x_4 + 600 & \Rightarrow x_3 - x_4 &= 600 - 390 = 210 \\ \text{D: } x_1 + 310 &= x_4 + 640 & \Rightarrow x_1 - x_4 &= 640 - 310 = 330 \end{aligned}$$

$$\begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \left[\begin{array}{cccc} 1 & 0 & 0 & -1:330 \\ 0 & 1 & 0 & -1:170 \\ 0 & 0 & 1 & -1:210 \\ 0 & 0 & 0 & 0:0 \end{array} \right] \end{array}$$

The solution is $x_1 = t+330$, $x_2 = t+170$, $x_3 = t+210$, $x_4 = t$.

(b) Hence, for $x_4 = 200$, amount of traffic for other intersections is $x_1 = 530$, $x_2 = 370$, and $x_3 = 410$.

Work to do:

Question 4

Some parking meters in Milan, Italy, accept coins in the denominations of 20c, 50c, and €2. As an incentive program, the city administrators offer a big reward (a brand new Ferrari Testarossa) to any meter maid who brings back exactly 1,000 coins worth exactly € 1,000 from the daily rounds. What are the odds of this reward being claimed anytime soon?

Question 5

The new animated feature film is now playing 3 times a day in DHA Cinema. One day, there were 20 adults, 30 children and 10 senior citizens and theater made \$600. At the next showing there were 24 adults, 60 children and 20 senior citizens with the theater making it \$800 in ticket sale. At the last showing theater made \$400 with 10 adults, 30 children and 5 senior citizens. How much are the tickets at the movie theater?