<u>Intersection of Lines:</u>

Intersection of two lines:

Given two lines in the two-dimensional plane, the lines are equal, they are parallel but not equal, or they intersect in a single point. In three dimensions, a fourth case is possible. If two lines in space are not parallel, but do not intersect, then the lines are said to be **skew lines.**

Example:

Find the point of intersection of the lines if any.

L_1: $x = 3 + 2t$	L_2: $x = 1 + 4s$	L_3: $x = 3 + 2r$
$y = -1 + 4t$	$y = 1 + 2s$	$y = 2 + r$
$z = 2 - t$	$z = -3 + 4s$	$z = -2 + 2r$
$-\infty < t < \infty$	$-\infty < s < \infty$	$-\infty < r < \infty$

Solution:

First, we check L_1 & L_2

From L_1: $\overrightarrow{v_1} = 2i + 4j - k$

From L_2: $\overrightarrow{v_2} = 4i + 2j + 4k$

$\overrightarrow{v_1}$ & $\overrightarrow{v_2}$ are not parallel, since they are not scalar multiple of each other.

Now, check if they intersect each other.

$x = 3 + 2t = 1 + 4s \quad \Rightarrow 2t - 4s = -2 \ldots (1)$

$y = -1 + 4t = 1 + 2s \quad \Rightarrow 4t - 2s = 2 \ldots. (2)$

$z = 2 - t = -3 + 4s \quad \Rightarrow -t - 4s = -5 \ldots. (3)$

From (1) & (2)

$2t - 4s = -2$

$\pm \underline{8t \mp 4s = \pm 4}$ "multiply (2) by 2"

$-6t = -6$ "changing signs"

$t = 1$ Put "t = 1" in (1)

$2(1) - 4s = -2$

$-4s = -2 - 2$

$-4s = -4$

$s = 1$

Put "$t=1$", "$s=1$" in (3)

$-t - 4s = -5$

$-1 - 4 = -5$

$-5 = -5$

L_1 & L_2 are intersecting.

Point of intersections:

$x = 3 + 2(1) = 5$

$y = -1 + 4(1) = 3$

$z = 2 - 1 = 1$

So, (5, 3, 1) is point of intersection.

Now, we have to check L_1 & L_3.

From L_1: $\overrightarrow{v_1} = 2i + 4j - k$

From L_3: $\overrightarrow{v_3} = 2i + j + 2k$

As $\overrightarrow{v_1}$ & $\overrightarrow{v_3}$ are not parallel, so L_1 & L_3 are not parallel.

Now check if they intersect or not?

$x = 3 + 2t = 3 + 2r \Rightarrow 2t - 2r = 0$ …. (1)

$y = -1 + 4t = 2 + r \Rightarrow 4t - r = 3$ …... (2)

$z = 2 - t = -2 + 2r \Rightarrow -t - 2r = -4$ …. (3)

From (1) & (3)

$2t - 2r = 0$

$\underline{-_{+}t - _{+}2r = -_{+}4}$

$3t = 4$ "changing signs"

$t = 4/3$

Put it in (1)

$2\,(4/3) - 2r = 0$

$8/3 = 2r$

$r = 8/6$

$r = 4/3$

Put "t" & "r" in (2)

$4\,(4/3) - (4/3) = 3$

$16/3 - 4/3 = 3$

$12/3 = 3$

$4 \neq 3$

L_1 & L_3 do not intersect.

Now check L_2& L_3.

$$\overrightarrow{v_2} = 4i + 2j + 4k$$

$\overrightarrow{v_3} = 2i + j + 2k$

$\overrightarrow{v_2} = 2\,(2i + j + 2k)$

$\overrightarrow{v_2} = 2\,\overrightarrow{v_3}$

$\overrightarrow{v_2}$&$\overrightarrow{v_3}$ are parallel; therefore, L_3& L_2 are parallel.

Ex. 12.5; 61 – 62

61. $L1$: $x = 3 + 2t$, $y = -1 + 4t$, $z = 2 - t$; $-\infty < t < \infty$
$L2$: $x = 1 + 4s$, $y = 1 + 2s$, $z = -3 + 4s$; $-\infty < s < \infty$
$L3$: $x = 3 + 2r$, $y = 2 + r$, $z = -2 + 2r$; $-\infty < r < \infty$

62. $L1$: $x = 1 + 2t$, $y = -1 - t$, $z = 3t$; $-\infty < t < \infty$
$L2$: $x = 2 - s$, $y = 3s$, $z = 1 + s$; $-\infty < s < \infty$
$L3$: $x = 5 + 2r$, $y = 1 - r$, $z = 8 + 3r$; $-\infty < r < \infty$