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[Q No 01]

Array = [50, 20, 70, 30, 10, 80, 60]

Selection Sort

iterations	Swaps	comparisons	Result
1	1	6	10, 20, 70, 30, 50, 80, 60
2	0	5	10, 20, 70, 30, 50, 80, 60
3	1	4	10, 20, 30, 70, 50, 80, 60
4	1	3	10, 20, 30, 50, 70, 80, 60
5	1	2	10, 20, 30, 50, 60, 80, 70
6	1	1	10, 20, 30, 50, 60, 70, 80

f(n) for selection sort

$$(n-1) + (n-2) + (n-3) + \cdots + 3 + 2 + 1$$

$$f(n) = \frac{(n-1)((n-1)+1)}{2} = \frac{n^2-1}{2} = \frac{n^2}{2} = \frac{1}{2}$$

Upper Bound:

$$f(n) \leq c.g(n) \Rightarrow g(n) = n^2 + c = 1$$

$$\frac{n^2}{2} = \frac{1}{2} \leq 1.n^2 \Rightarrow \boxed{O(n^2)}$$

Insertion Sort

Array = [50, 20, 70, 30, 10, 80, 60]

iterations	swaps	Comparisons	Result
1	1	1	20, 50, 70, 30, 10, 80, 60
2	0	1	20, 50, 70, 30, 10, 80, 60
3	2	3	20, 30, 50, 70, 10, 80, 60
4	4	4	10, 20, 30, 50, 70, 80, 60
5	0	1	10, 20, 30, 50, 70, 80, 60
6	2	3	10, 20, 30, 50, 60, 70, 80

$f(n) = k \cdot \frac{n^2}{2} - \frac{k}{2}$

Upper Bound

$f(n) \leq c \cdot g(n) \rightarrow g(n) = n^2 \quad + \quad c = 1$

$f(n) \leq 1 \cdot n^2$

$\boxed{O(n^2)}$

[Q. No. 02]

$a \rightarrow T(n) = 2T(n/3) + 1$

$T(n) = 2T(n/3) + 1$

$T(n/3) = 2T(n/3^2) + 1$

$T(n/3^2) = 2T(n/3^3) + 1$

$T(n/3^3) = 2T(n/3^4) + 1$

$T(n/3^4) = 2T(n/3^5) + 1$

Back Substituting

$T(n) = 2^2 T(n/3^2) + 3$

$T(n) = 2^2[2T(n/3^3) + 1] + 3$

$= 2^3 T(n/3^3) + 7$

$T(n) = 2^3[2T(n/3^4) + 1] + 7$

$= 2^4 T(n/3^4) + 15$

$T(n) = 2^4[2T(n/3^5) + 1] + 15$

$= 2^5 T(n/3^5) + 31$

$$T(n) = 2^k T(n/3^k) + (2^k - 1)$$

$$= 2^{\log_3(n)}(1) + 2^{\log_3(n)} - 1$$

$$= n^{\log_3(2)} + n^{\log_3(2)} - 1$$

$$= 2n^{\log_3(2)} - 1$$

$$\boxed{O(n^{\log_3(2)})}$$

$n/3^k = 1$

$n = 3^k$

$\log_3(n) = \log_3(3^k)$

$\log_3(n) = k$

$2 = 3^{\log_3(2)}$

$= (3^{\log_3(2)})^{\log_3(n)}$

$= (3^{\log_3(n)})^{\log_3(2)}$

$= n^{\log_3(2)}$

$b \rightarrow T(n) = 9T(n/3) + n^2$

$$T(n/3) = 9T(n/3^2) + (n/3)^2$$

$$= 9T(n/3^2) + (n^2/3^2)$$

$$T(n/3^2) = 9T(n/3^3) + n^2/3^4$$

$$T(n/3^3) = 9T(n/3^4) + n^2/3^6$$

Back Substituting

$$T(n) = 9[9T(n/3^2) + n^2/3^2] + n^2$$

$$= 9^2 T(n/3^2) + 2n^2$$

$$T(n) = 9^2[9T(n/3^3) + n^2/3^4] + 2^2$$

$$= 9^3 T(n/3^3) + 3n^2$$

$$T(n) = 9^3[9T(n/3^4) + n^2/3^6] + 3n^2$$

$$= 9^4 T(n/3^4) + 4n^2$$

$$T(n) = 9^k T(n/3^k) + kn^2$$

$$T(n) = 9^{\log_3(n)}(1) + \log_3(n) \cdot n^2$$
$$= 3^{2\log_3(n)} + n^2 \log_3(n)$$
$$= 3^{\log_3(n^2)} + n^2 \log_3(n)$$
$$= n^2 + n^2 \log_3(n)$$

$$n/3^k = 1$$
$$n = 3^k$$
$$\log_3(n) = \log_3(3^k)$$
$$\log_3(n) = k$$

$$\boxed{O(n^2 \log_3(n))}$$

c → $T(n) = 49T(n/25) + n^3/(2\log(n))$

$$T(n/25) = 49T(n/25^2) + \left[(n/25)^3 / 2\log(n/25)\right]$$

$$T(n/25^2) = 49T(n/25^3) + \left[(n/25^2)^3 / 2\log(n/25^2)\right]$$

$$T(n/25^3) = 49T(n/25^4) + \left[(n/25^3)^3 / 2\log(n/25^3)\right]$$

Back Substituting

$$T(n) = 49\left[49T\left(\frac{n}{25^2}\right) + \left(\frac{n}{25}\right)^3 \Big/ 2\log\left(\frac{n}{25}\right)\right] + \frac{n^3}{2\log(n)}$$

$$= 49^2 T\left(\frac{n}{25^2}\right) + 49\left(\frac{n}{25}\right)^3 \Big/ 2\log\left(\frac{n}{25}\right) + n^3 \Big/ 2\log(n)$$

$$T(n) = 49^3\left[T\left(\frac{n}{25^3}\right) + \frac{49^2\left(\frac{n}{25^2}\right)^3}{2\log\left(\frac{n}{25^2}\right)} + \frac{49\left(\frac{n}{25}\right)^3}{2\log\left(\frac{n}{25}\right)} + \frac{n^3}{2\log(n)}\right.$$

$$T(n) = 49^4 T\left(\frac{n}{25^4}\right) + \frac{49^3\left(\frac{n}{25^3}\right)^3}{2\log\left(\frac{n}{25^3}\right)} + \frac{49^2\left(\frac{n}{25^2}\right)^3}{2\log\left(\frac{n}{25^2}\right)} + \frac{49\left(\frac{n}{25}\right)^3}{2\log\left(\frac{n}{25}\right)}$$

$$+ \frac{n^3}{2\log(n)}$$

$$= 49^k T\left(\frac{n}{25^k}\right) + \frac{49^{k-1}\left(\frac{n}{25^{k-1}}\right)^3}{2\log\left(\frac{n}{25^{k-1}}\right)}$$

$$= 49^{\log_{25}(n)}(1) \cdots \frac{49^2\left(\frac{n}{25^2}\right)^3}{2\log\left(\frac{n}{25^2}\right)} + \frac{49\left(\frac{n}{25}\right)^3}{2\log\left(\frac{n}{25}\right)}$$

$\cancel{T(n)} =$

$$\frac{n}{25^k} = 1$$

$$n = 25^k$$

$$\log_{25}(n) = \log_{25}(25^k)$$

$$\log_{25}(n) = k$$

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$$T(n) = 49^{k}\, T\left(\frac{n}{25^{k}}\right) + \frac{49^{(k-1)} \cdot n^3}{25^{3(k-1)}\, 2\log\left(\frac{n}{25^{k-1}}\right)} + \dots$$

$$+ \frac{49^2 n^3}{25^6\, 2\log\left(\frac{n}{25^2}\right)} + \frac{49 n^3}{25^3\, 2\log\left(\frac{n}{25}\right)} + \frac{n^3}{2\log(n)}$$

$$= 49^{\log_{25}(n)}(1) + 49^{\log_{25}(n)} \frac{n^3}{\log n}$$

$$= n^{\log_{25}(49)} + \frac{n^3}{2\log n}\left[\frac{49^{(k-1)}}{25^{3(k-1)}\left(1 - \frac{\log(25)}{\log(n)}\right)^{(k-1)}} + \dots\right.$$

$$\left. + \frac{49^2}{25^6\left(1 - \frac{\log(25)}{\log(n)}\right)} + \frac{49}{25^3\left(1 - \frac{\log(25)}{\log(n)}\right)} + 1\right.$$

So

$$\boxed{\Theta\left(\left(n^2\right)^{\log_{25}(7)}\right)}$$