Calculus and Analytical Geometry

Lecture no. 07

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Topic: Secant and tangent lines

Outline of the lecture:

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   - Equation
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iii. Tangent lines
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   - Equation
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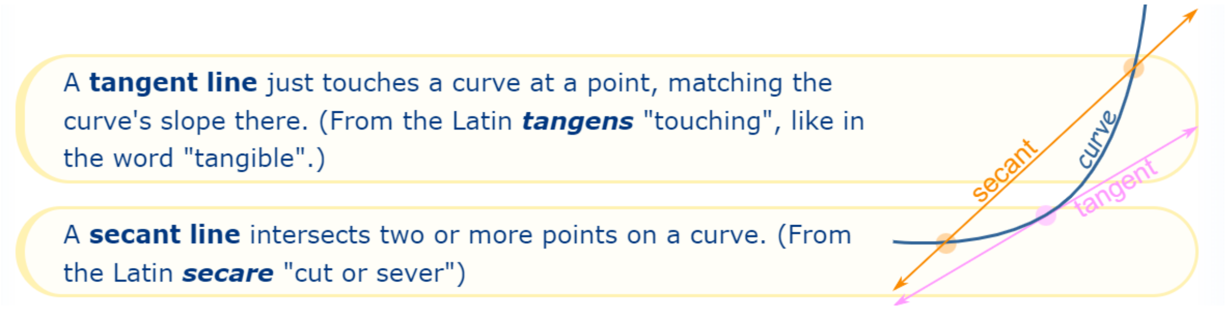
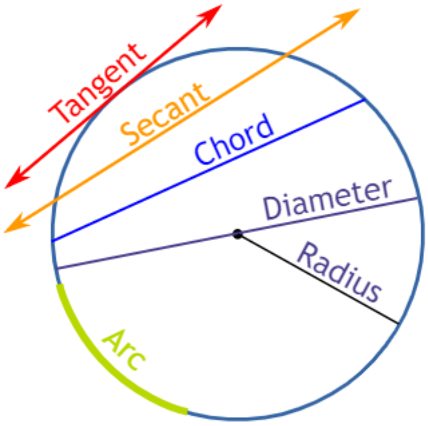
1. Tangent and secant lines:



Example: Consider a circle:



2. Definition of Secant lines:

The line that passes through the two points on a graph of a function is called a secant line.

Equation:

The equation of the secant line passing the points $P(x_o, f(x_o))$ and $Q(x_1, f(x_1))$ on the graph $y = f(x)$ is

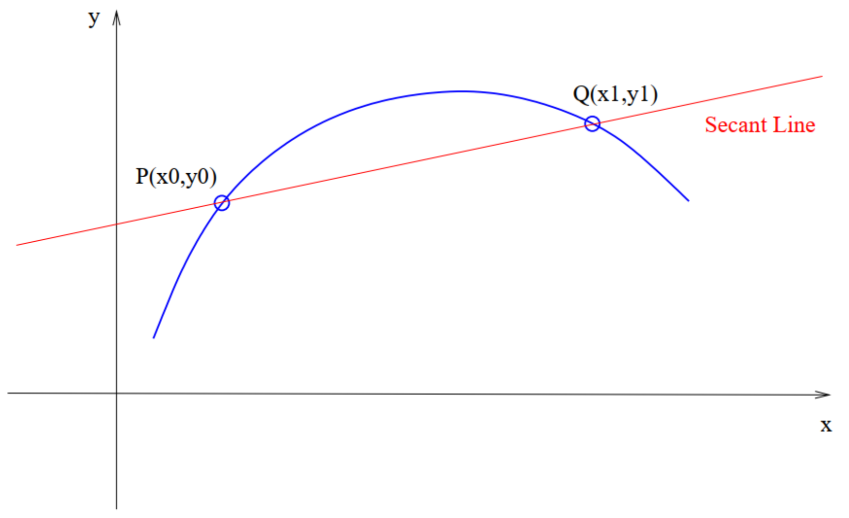
$$y - f(x_o) = m_{sec}(x - x_o)$$

Where,

$$m_{sec} = \frac{f(x_1) - f(x_o)}{(x_1 - x_0)}$$

Is the slope of the line.

Graphical Representation:



Example 2.1:

Find the equation of secant line passing through the points $P(1,1)$ and $Q(3,9)$ on the parabola $f(x) = x^2$, show the line on the graph.

Solution:

Step 1: (slope) Here, $x_0 = 1$, $x_1 = 3$, $f(x_o) = 1, f(x_1) = 9$

$$m_{sec} = \frac{f(x_1) - f(x_o)}{(x_1 - x_0)}$$

$$m_{sec} = \frac{9 - 1}{(3 - 1)}$$

$$m_{sec} = \frac{8}{2}$$

$$m_{sec} = 4$$

Step 2: (Equation of secant line)

The equation of secant line is

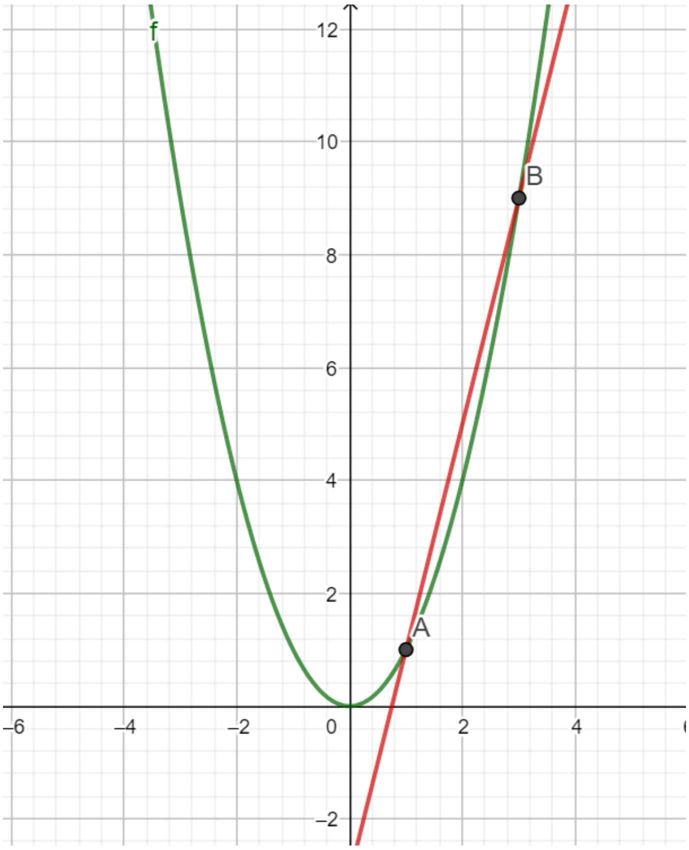
$$y - f(x_o) = m_{sec}(x - x_o)$$

$$y - 1 = 4(x - 1)$$

$$y - 1 = 4x - 4$$

$$y = 4x - 3$$

Step 3: (Graph of secant line)



Example 2.2: Find the equation of secant line on function $f(x) = 4x^2 - 7$ where x=-2 and x=1

Solution:

Step 1: Find $f(x_o)$ and $f(x_1)$

At $x_0 = -2$

$f(x_o) = 4x^2 - 7 = 4(-2)^2 - 7 = 4(4) - 7 = 16 - 7 = 9$

At $x_1 = 1$

$f(x_1) = 4x^2 - 7 = 4(1)^2 - 7 = 4(1) - 7 = 4 - 7 = -3$

Step 2: (slope) Here, $x_0 = -2, x_1 = 1, f(x_o) = 9, f(x_1) = -3$

$$m_{sec} = \frac{f(x_1) - f(x_o)}{(x_1 - x_0)}$$

$$m_{sec} = \frac{-3 - 9}{1 - (-2)}$$

$$m_{sec} = \frac{-12}{3} = -4$$

Step 3: (Equation of secant line)

The equation of secant line is

$$y - f(x_o) = m_{sec}(x - x_o)$$

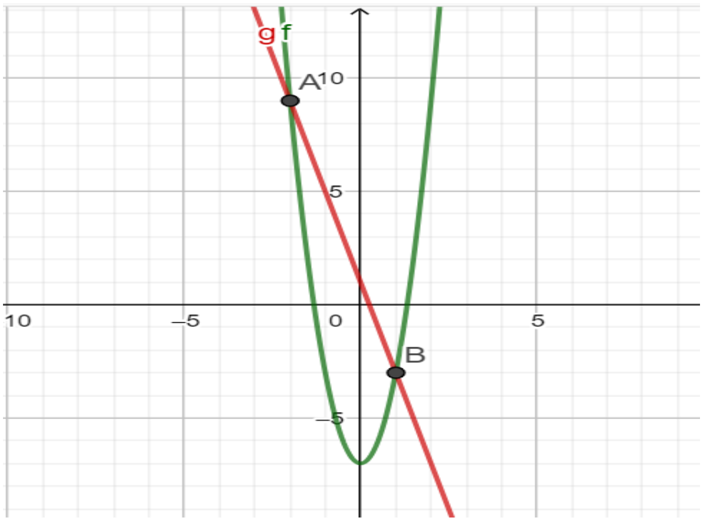
$$y - 9 = -4(x - (-2))$$

$$y - 9 = -4(x + 2)$$

$$y - 9 = -4x - 8$$

$$y = -4x + 1$$

Step 4: (Graph of secant line)



3. Definition of tangent lines:

The line that touches the graph at one point is known as tangent line.

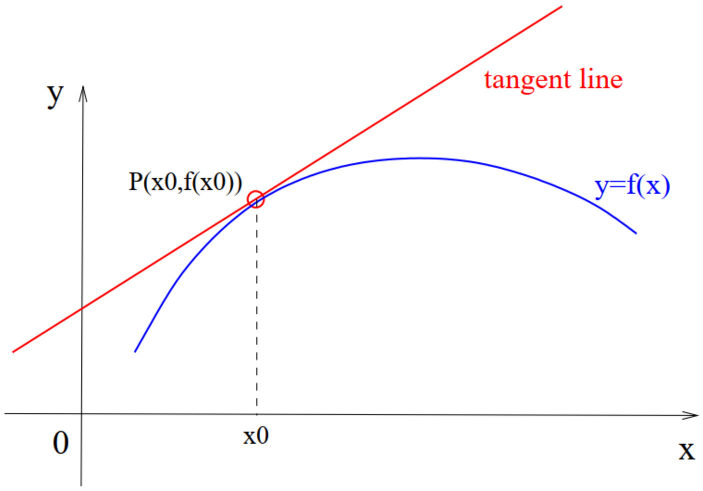
Equation of tangent line:

$$y - f(x_o) = m_{tan}(x - x_0)$$

Where,

$$m_{tan} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Graphical representation:



Example 3.1: Find the equation of tangent line to the curve $f(x) = \frac{1}{x}$ at $x_0 = 2$.

Solution:

Step 1: Finding the value of $f(x_0)$

$$f(x_0) = \frac{1}{x_0} = \frac{1}{2}$$

Step 2: (slope)

$$m_{tan} = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$m_{tan} = \lim_{h\to 0} \frac{1}{h}[f(x_0+h) - f(x_0)]$$

$$m_{tan} = \lim_{h\to 0} \frac{1}{h}[f(2+h) - f(2)]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{2+h} - \frac{1}{2}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{2-(2+h)}{2(2+h)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{2-2-h}{2(2+h)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{-(h)}{2(2+h)}\right]$$

$$= \lim_{h\to 0} \left[\frac{-1}{2(2+h)}\right]$$

$$= -\frac{1}{4}$$

<u>Step 3:</u> (Equation of tangent line)

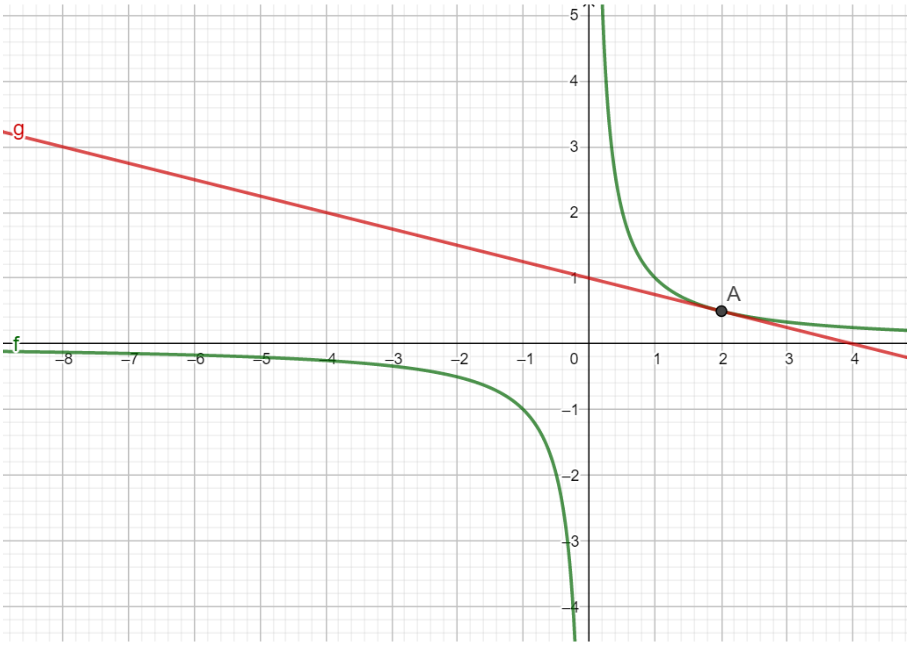
$$y - f(x_o) = m_{tan}(x - x_0)$$

$$y - \frac{1}{2} = -\frac{1}{4}(x - 2)$$

$$y - \frac{1}{2} = -\frac{1}{4}x + \frac{1}{2}$$

$$y = -\frac{1}{4}x + 1$$

<u>Step 4:</u> Graph of tangent line



Example 3.2: Find the equation of tangent line to the curve $f(x) = \sqrt{x}$ at $x_0 = 1$.

Solution:

<u>Step 1:</u> Finding the value of $f(x_0)$

$$f(x_0) = \sqrt{1} = 1$$

<u>Step 2:</u> Slope

$$m_{tan} = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$m_{tan} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{1+h} - \sqrt{1}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{1+h} - \sqrt{1}}{h} \times \frac{\sqrt{1+h} + \sqrt{1}}{\sqrt{1+h} + \sqrt{1}}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{1+h} + \sqrt{1}}$$

$$= \frac{1}{2}$$

Step 3: Equation of the tangent line

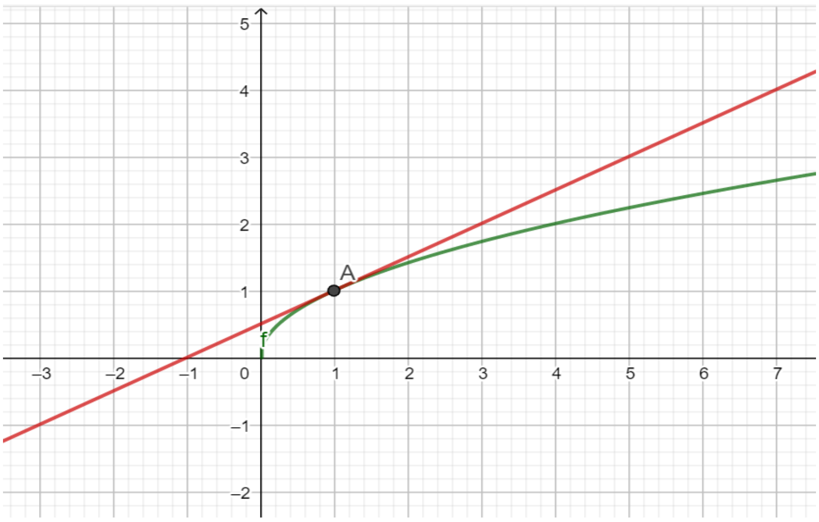
$$y - f(x_o) = m_{tan}(x - x_0)$$

$$y - 1 = \frac{1}{2}(x - 1)$$

$$y - 1 = \frac{1}{2}x - \frac{1}{2}$$

$$y = \frac{1}{2}x + 0.5$$

Step 4: graphical representation



Practice Questions:

1. Find the equation of secant line on the curve of $f(x) = \sqrt{x}$ at $x_0 = 1, x_1 = 4$ and show the line on graph of f.
2. Find the equation of secant line on the curve of $f(x) = x^2 + x$ at $x_0 = 1, x_1 = 2$ and show the line on graph of f.
3. Find the equation of secant line on the curve of $f(x) = |x|$ at $x_0 = -2, x_1 = 1$ and show the line on graph of f.
4. Find the equation of tangent line on the curve of $f(x) = \frac{1}{x^2}$ at $x_0 = -1$ and show the line on graph of f.
5. Find the equation of tangent line on the curve of $f(x) = 3x + 1$ at $x_0 = 3$ and show the line on graph of f.
6. Find the equation of tangent line on the curve of $f(x) = x^2$ at $x_0 = 2$ and show the line on graph of f.