

Q-1

$$T(N) = 4T\left(\frac{N}{8}\right) + N^{1/3}$$

$$(T(1) = C)$$

$$= 4\left(4T\left(\frac{N}{64}\right) + \left(\frac{N}{8}\right)^{1/3}\right) + N^{1/3}$$

$$= 16T\left(\frac{N}{64}\right) + 4 \times \frac{N^{1/3}}{2} + N^{1/3}$$

$$= 16\left[4T\left(\frac{N}{512}\right) + \left(\frac{N}{64}\right)^{1/3}\right] + 2N^{1/3} + N^{1/3}$$

$$= 64T\left(\frac{N}{512}\right) + 16 \frac{N^{1/3}}{4} + 2N^{1/3} + N^{1/3}$$

$$= 4^3 T\left(\frac{N}{8^3}\right) + 4N^{1/3} + 2N^{1/3} + N^{1/3}$$

$$= 4^k T\left(\frac{N}{8^k}\right) + N^{1/3}(2^2 + 2^1 + \dots + 2^0)$$

here

$$\left(\frac{N}{8^k}\right) = 1 \Rightarrow N = 8^k$$

$$\Rightarrow \log N = \log 8^k$$

$$\Rightarrow \log N = k \times \log 8$$

$$\Rightarrow k = \frac{\log N}{\log 8}$$

$$\Rightarrow k = \log_8 N$$

$$\Rightarrow 4^k T\left(\frac{N}{8^k}\right) + N^{1/3} \sum_{i=0}^{k-1} (2^i)$$

$$\Rightarrow 4^{(\log_8 N)} \times C + N^{1/3} \sum_{i=0}^{\log_8 N - 1} (2^i)$$

$$\Rightarrow N^{\log_8 4} \times C + N^{1/3} \left[\frac{2^{\log_8 N - 1 + 1} - 1}{2 - 1} \right]$$

$$\Rightarrow N^{0.66} \times C + N^{1/3} \left[2^{\log_8 N} - 1 \right]$$

$$\Rightarrow N^{0.66} \times C + N^{1/3} \left[N^{\log_8 2} - 1 \right]$$

$$\Rightarrow N^{0.66} \times C + N^{0.33} \times N^{0.33} - N^{0.33}$$

$$\Rightarrow C N^{0.66} + N^{0.66} - N^{0.33}$$

$$\Rightarrow C N^{0.66} \approx N^{0.33} \Rightarrow O(N^{0.66})$$

Q-2

— $XYZ(A, s, e)$

if (size) // divide till size has more than 1

$$mid = s + (e - s) / 2$$

$$L = XYZ(A, s, mid)$$

$$R = XYZ(A, mid + 1, e)$$

return L + R

else

[return A[e] // single element return

Now we have done that

— Divide 2 parts

— Repeat till size is 1

— When size 1 return that value

$$T(N) = 2T(N/2) + 1 \rightarrow (O(N)) // \text{we can use } \downarrow \text{ here} \\ (C) // O(1)$$

Mid-Term Solution

Summer 22

Q-3

outer loop \Rightarrow initialization $\rightarrow 1$
 loop end $\rightarrow 1$
 loop successful $\rightarrow (2N-1+1)$ times
 $\rightarrow \lg_2(2N)$
 $\rightarrow \lg_2(2) + \lg_2(N)$
 $\rightarrow 1 + \lg_2 N$ times.

Inner loop Dependent:

$i = 1$	2	4	8	...	$N/2$	N	$2N$
$j = 0 \rightarrow 1$	$0 \rightarrow 2$	$0 \rightarrow 4$	$0 \rightarrow 8$		$0 \rightarrow N/2$	$0 \rightarrow N$	$0 \rightarrow 2N$
$\rightarrow 1(S)$	$2(S)$	$4(S)$	$8(S)$		$N/2(S)$	$N(S)$	$2N(S)$
$1(I)$	"	"	"		"	"	"
$1(E)$	"	"	"		"	"	"

$2 * \lg_2 N \Rightarrow (\text{Initialization} \& \text{End})$

$1 + 2 + 4 + 8 + 16 + \dots + N/2 + N + 2N$ (geometric)

$< 2 * 2N$

$< 4N$

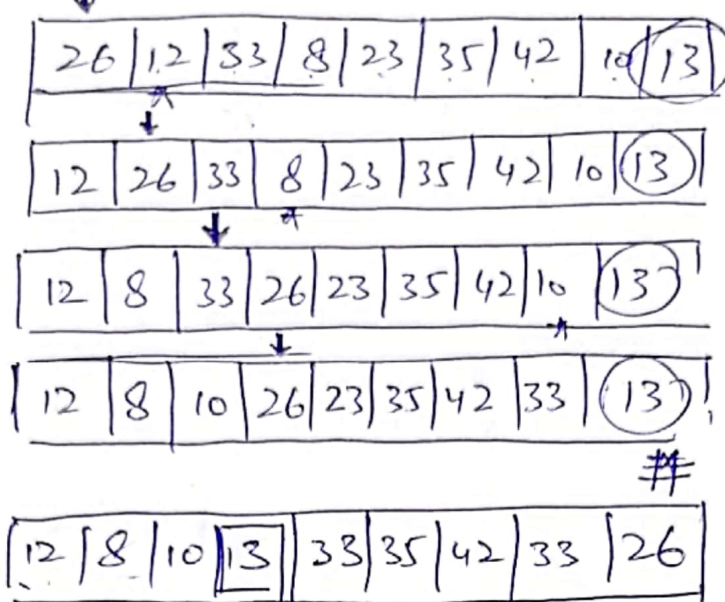
$< O(N)$

$\lg_2 N + N + \lg_2 N$

$O(N)$ answer.

Q-4

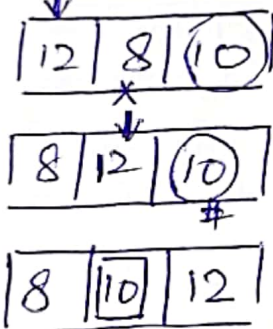
1st call



Quick-Sort(A, 0, 8)

2nd call

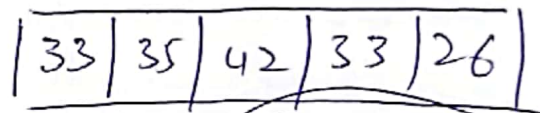
Quick-Sort(A, 0, 2)



Quick-Sort(A, 0, 0) Quick-Sort(A, 2, 2)

* *

Quick-Sort(A, 3, 8) ⇒ ★ (b part)



3rd call