

# **Calculus and Analytical Geometry**

## **Lecture no. 03**

**Amina Komal**

**March 2022**

---

**Topic:** Graph of an equation

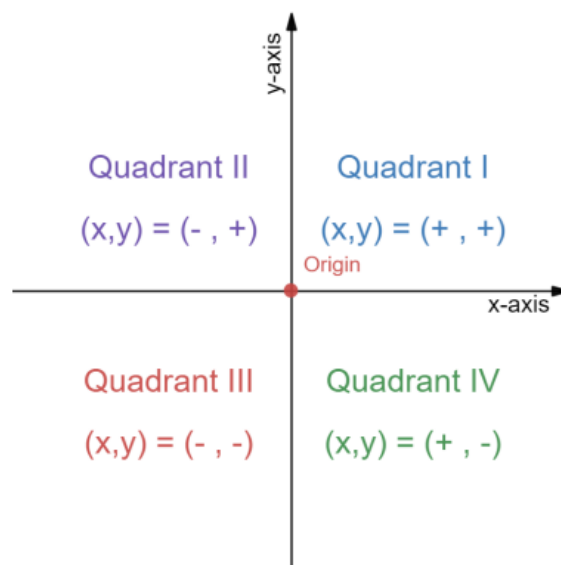
**Outline of the lecture:**

- i. Cartesian plane
- ii. Ordered pair
- iii. Graph of an equation
- iv. Examples involving sketching of graph
- v. Practice questions

## 1) Cartesian Plane:

A cartesian plane is a plane consists of two perpendicular number lines that intersect at the origin.

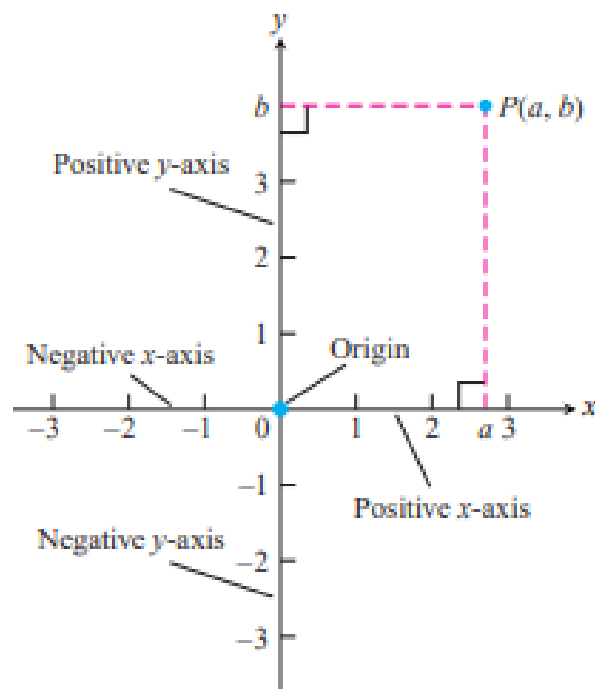
- The horizontal line is known as **x-axis**, whereas the vertical line is known as **y-axis**.
- The horizontal line towards right is positive and towards left its negative.
- The vertical line above the origin is positive and below the origin its negative.
- The plane is divided into 4 equal sections known as **quadrants**.



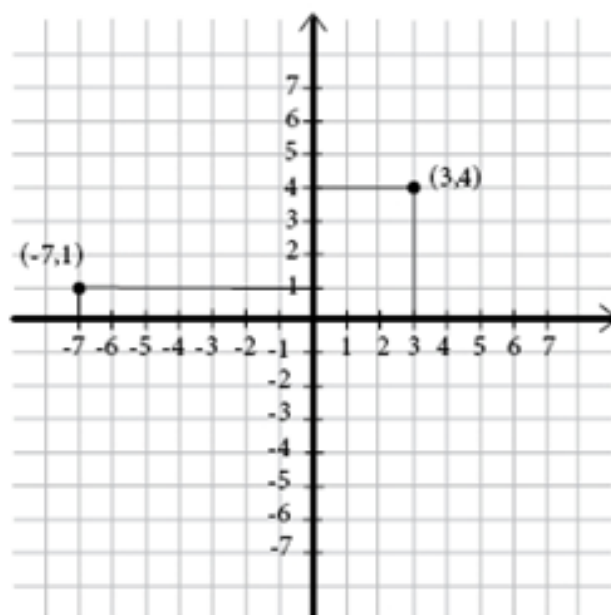
## 2) Ordered pair:

An ordered pair contains the coordinates of one point in the coordinate system. A point is named by its ordered pair of the form  $P=(x, y)$ . The first number corresponds to the x-coordinate and the second to the y-coordinate.

To graph a point, you draw a dot at the coordinates that corresponds to the ordered pair. It's always a good idea to start at the origin. The x-coordinate tells you how many steps you have to take to the right (positive) or left (negative) on the x-axis. And the y-coordinate tells you have many steps to move up (positive) or down (negative) on the y-axis.



**Example:**



The ordered pair  $(3, 4)$  is found in the coordinate system when you move 3 steps to the right on the x-axis and 4 steps upwards on the y-axis.

The ordered pair  $(-7, 1)$  is found in the coordinate system when you move 7 steps to the left on the x-axis and 1 step upwards on the y-axis.

### 3) Graph of an equations:

Graph of an equation is the graph of all the ordered pairs (x,y) that satisfies the equation.

There are infinitely many points that satisfies a particular equation so, we draw reasonable amount of ordered pairs and join them to make the graph of an equation.

### 4) Types of Equations:

- Linear equation.
- Quadratic equation
- Cubic equation
- Reciprocal equation

#### 4.1) Graph of Linear equation:

##### Example:

Draw the graph of the equation  $8x + 4y = 12$ .

##### Solution:

$$8x + 4y = 12$$

$$4y = -8x + 12$$

$$y = \frac{-8x+12}{4}$$

$$y = \frac{-8x}{4} + \frac{12}{4}$$

$$y = -2x + 3$$

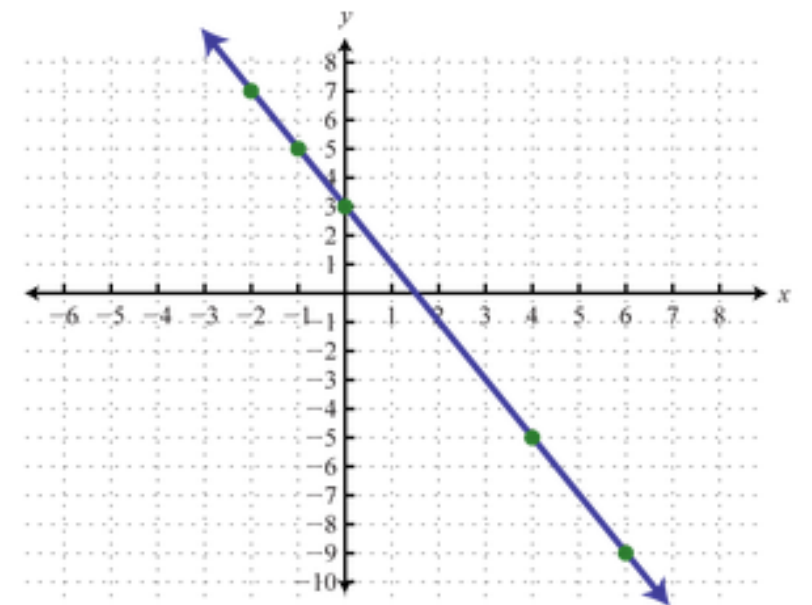
*Subtract 8x on both sides.*

*Divide both sides by 4.*

*Simplify.*

x	y	$y = -2x + 3$	Solutions
-2	7	$y = -2(-2) + 3 = 4 + 3 = 7$	$(-2, 7)$
-1	5	$y = -2(-1) + 3 = 2 + 3 = 5$	$(-1, 5)$
0	3	$y = -2(0) + 3 = 0 + 3 = 3$	$(0, 3)$
4	-5	$y = -2(4) + 3 = -8 + 3 = -5$	$(4, -5)$
6	-9	$y = -2(6) + 3 = -12 + 3 = -9$	$(6, -9)$

Plot the points on graph:

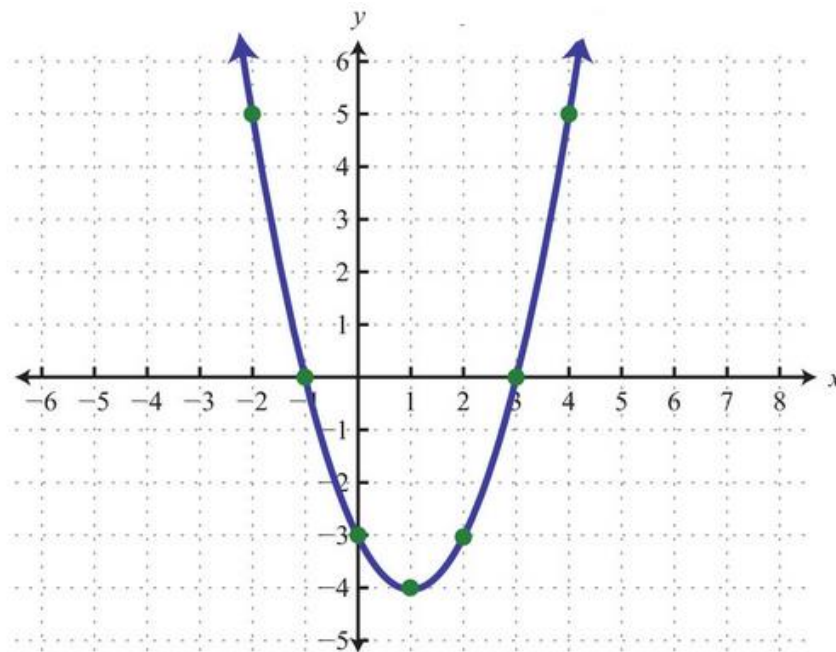


#### 4.2) Graph of Quadratic equation:

**Example:** Draw the graph of  $y = x^2 - 2x - 3$

<b>x</b>	<b>y</b>		<i>Points</i>
-2	5	$y = (-2)^2 - 2(-2) - 3 = 4 + 4 - 3 = 5$	$(-2, 5)$
-1	0	$y = (-1)^2 - 2(-1) - 3 = 1 + 2 - 3 = 0$	$(-1, 0)$
0	-3	$y = (0)^2 - 2(0) - 3 = 0 - 0 - 3 = -3$	$(0, -3)$
1	-4	$y = (1)^2 - 2(1) - 3 = 1 - 2 - 3 = -4$	$(1, -4)$
2	-3	$y = (2)^2 - 2(2) - 3 = 4 - 4 - 3 = -3$	$(2, -3)$
3	0	$y = (3)^2 - 2(3) - 3 = 9 - 6 - 3 = 0$	$(3, 0)$
4	5	$y = (4)^2 - 2(4) - 3 = 16 - 8 - 3 = 5$	$(4, 5)$

Plot the points on graph:



#### 4.3) Graph of Cubic equation:

**Example:** Draw the graph of  $y = \frac{-x^3}{6} + 2x + 5$

**Solution:**

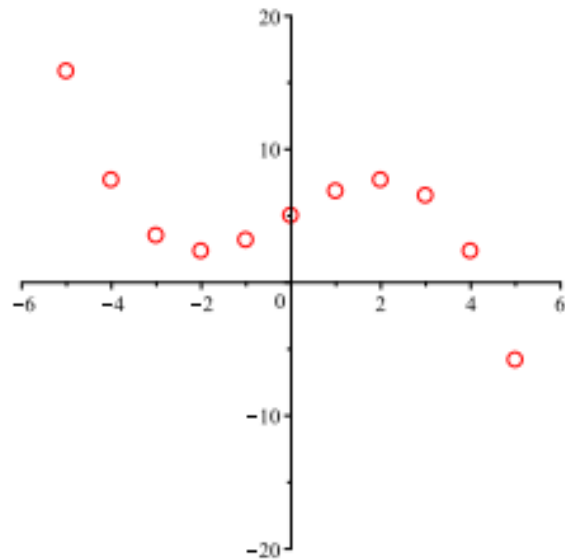
The y-values will be obtained by putting the x-values in the given equation. For instance, if we take  $x = -5$ , then

$$\begin{aligned} y &= \frac{-(-5)^3}{6} + 2(-5) + 5 \\ &= \frac{-(-125)}{6} - 10 + 5 \\ &= \frac{125}{6} - 5 \approx 15.8 \end{aligned}$$

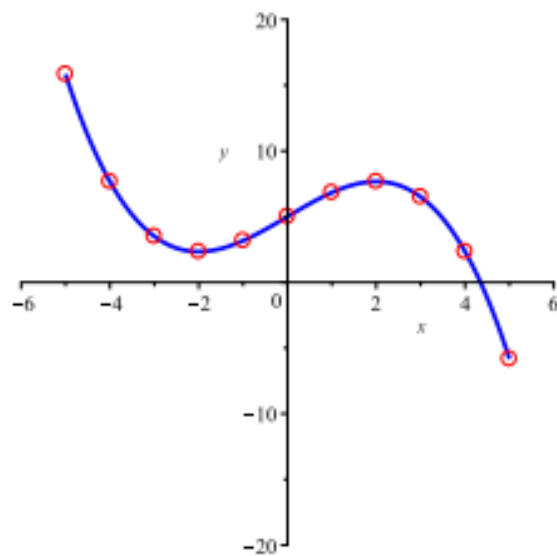
Similarly, find the other values of y and write them in the table.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	15.8	7.6	3.5	2.3	3.1	5	6.8	7.6	6.5	2.3	-5.8

Mark the ordered pairs on graph.



Join the points to make graph.



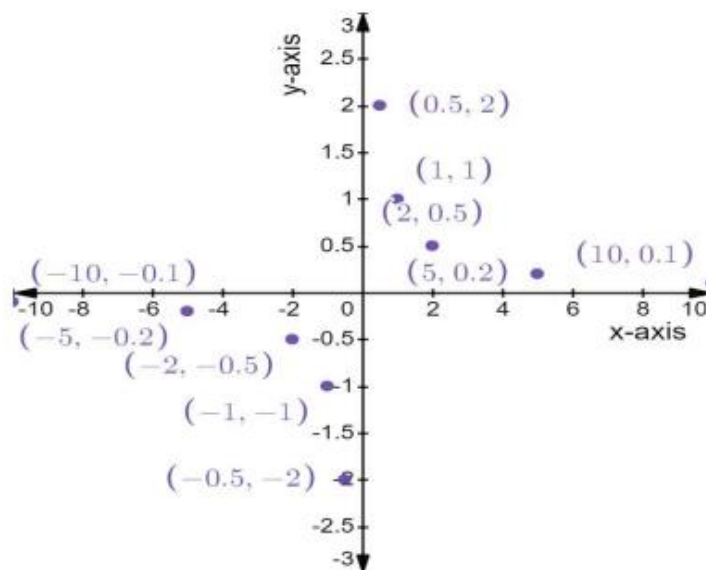
#### 4.4) Graph of Reciprocal equation:

**Example:** Draw the graph of  $y = \frac{1}{x}$

**Solution:**

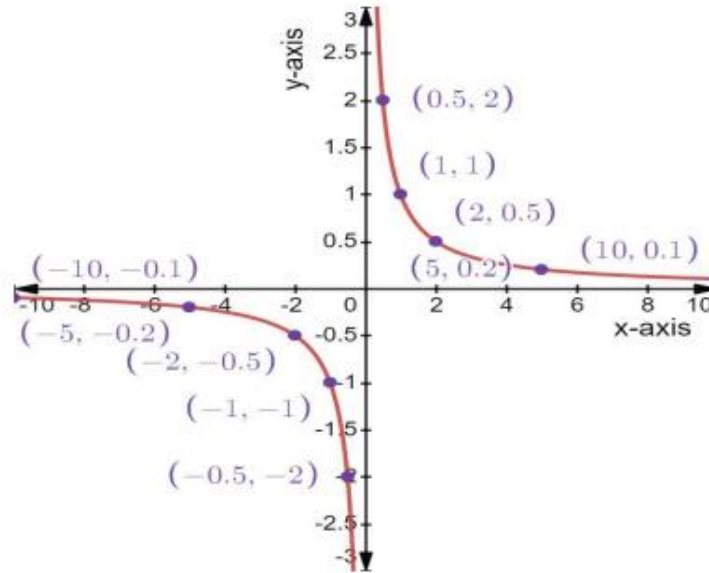
x	y	Ordered pairs
- 10	- 0.1	(- 10,- 0.1)
- 5	- 0.2	(- 5,- 0.2)
- 2	- 0.5	(- 2,- 0.5)
- 1	- 1	(- 1,- 1)
- 0.5	- 2	(- 0.5,- 2)
+ 0.5	+ 2	(+ 0.5, + 2)
+ 1	+ 1	(+ 1, + 1)
+ 2	+ 0.5	(+ 2, + 0.5)
+ 5	+ 0.2	(+ 5, + 0.2)
+ 10	+ 0.1	(+ 10, + 0.1)
+ 20	+ 0.05	(+ 20, + 0.05)

Mark the ordered pairs.





Join the points to make the curves:



**Practice Questions:**

- i.  $y = -6x^2 + 11x - 4$
- ii.  $y = 4x^2 - 25$
- iii.  $y = -x^2 + 10x - 34$
- iv.  $y = -2(x - 4)^2 + 22$
- v.  $\frac{1}{8}x - \frac{1}{6}y = -\frac{3}{2}$
- vi.  $-8x + 3y = 28$
- vii.  $y = x^3$
- viii.  $y = \frac{1}{x^2}$
- ix.  $y = -x - 2$