Gaussian Elimination and the RREF

The row reduced echelon form (rref)

- The first non-zero term in each row (the "leading 1") is a 1 ; there are zeros above and below it.
- The leading 1 in any row is to the right of the leading 1 in the row above it.
- Any rows containing all zeros (to the left of the vertical bar) appear at the bottom.

Example 1 Find all 2×2 coefficient matrices in reduced row-echelon form which have two leading 1s.

Solution. Such a matrix must have a leading 1 in each row and each column. Since the leading 1 in the second row must be to the left of the leading 1 in the first row, there is only one possibility:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Example 2 Find all 2×3 coefficient matrices in reduced row-echelon form which have two leading 1s.

Solution. In order for a 2×3 matrix to have two leading 1s, it must have a leading 1 in each row. Since the leading 1 in the second row must be to the left of the leading 1 in the first row, there are three possibilities:

$$\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \end{bmatrix}, \begin{bmatrix} 1 & a & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example 3 Find all 3×2 coefficient matrices in reduced row-echelon form which have two leading 1s.

Solution. The only such matrix is

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

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Solution of Word Problems Continued....

Question 1: I have 32 bills in my wallet, in the denominations of US $1, $5 and $10, worth $100 in total. How many do I have of each denomination?

Solution: Let x = no. of 1$ bills, y = no. of 5$ bills, z = no. of 10$ bills. System looks like:

$$\begin{cases} x + y + z = 32 & (1) \\ x + 5y + 10z = 100 & (2) \end{cases}$$

The augmented matrix is

$$\begin{bmatrix} 1 & 1 & 1 : 32 \\ 1 & 5 & 10 : 100 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 1 & 1 : 32 \\ 0 & 4 & 9 : 68 \end{bmatrix} \xrightarrow{\frac{1}{4}R_1}$$

$$\begin{bmatrix} 1 & 1 & 1 : 32 \\ 0 & 1 & \frac{9}{4} : 17 \end{bmatrix} \xrightarrow{R_1 - R_2} \begin{bmatrix} 1 & 0 & -\frac{5}{4} : 15 \\ 0 & 1 & \frac{9}{4} : 17 \end{bmatrix}$$

$$x - \frac{5}{4}z = 15 \qquad \Rightarrow 4x - 5z = 60$$

$$y + \frac{9}{4}z = 17 \qquad \Rightarrow 4y + 9z = 68$$

Now we'll solve this equation for y and z by trying different positive integer values for z and checking if y is also a positive integer.

If $z = 1$, then $y = (68 - 9)/4 = 14.75$, which is not an integer.
If $z = 2$, then $y = (68 - 18)/4 = 12.5$, which is not an integer.
If $z = 3$, then $y = (68 - 27)/4 = 10.25$, which is not an integer.
If $z = 4$, then $y = (68 - 36)/4 = 8$, which is an integer.
So, $z = 4$, and $y = 8$. Now we can find x :

$$4x - 5z = 60 \Rightarrow 4x = 60 + 5(4) \Rightarrow 4x = 80 \Rightarrow x = 20$$

Thus, we have 20 $\$1$ bills, 8 $\$5$ bills, and 4 $\$10$ bills in your wallet.

(20 one dollar bills, 8 five dollar bills, and 4 ten dollar bills.)

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Solution of Word Problems Continued….

Question 2

Ali is getting some flowers for his office. Being of a precise analytical mind, he plans to spend exactly $24 on a bunch of exactly two dozen flowers. At the flower market they have lilies ($3 each), roses ($2 each), and daisies ($0.50 each). Ali loves lilies, what is he to do?

Solution: Since Ali wants to spend exactly $24 on exactly two dozen flowers (24 flowers in total), he needs to find a combination of lilies, roses, and daisies that will satisfy these conditions. Let's denote the number of lilies as L, the number of roses as R, and the number of daisies as D. We have two equations:

$$3L + 2R + 0.5D = 24$$

$$L + R + D = 24$$

Its corresponding Augmented matrix is

$$\begin{bmatrix} 3 & 2 & 0.5 & : 24 \\ 1 & 1 & 1 & : 24 \end{bmatrix}$$

And the rref is given by:

$$\begin{bmatrix} 1 & 0 & -\frac{3}{2} & : -24 \\ 0 & 1 & \frac{5}{2} & : 48 \end{bmatrix}$$

Ali loves lilies, so we want to maximize the number of lilies (L) in the solution.

To maximize the number of lilies, we need to minimize the number of roses. Since both L and R must be integers, we can try different integer values for R and check if we get an integer value for L..

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Solution of Word Problems Continued....

Question 2

In the downtown section of a certain city, two sets of one-way streets intersect as shown in Figure below. The average hourly volume of traffic entering and leaving this section during rush hour is given in the diagram. At each intersection, the number of automobiles entering must be the same as the number leaving.

(a) Determine the amount of traffic between each of the four intersections.

(b) The amount of traffic between intersections C and D averages 200 automobiles. Please find the amount of traffic for other intersections.

Solution:

A: $x_1 + 450 = x_2 + 610 \quad \Rightarrow x_1 - x_2 = 610 - 450 = 160$

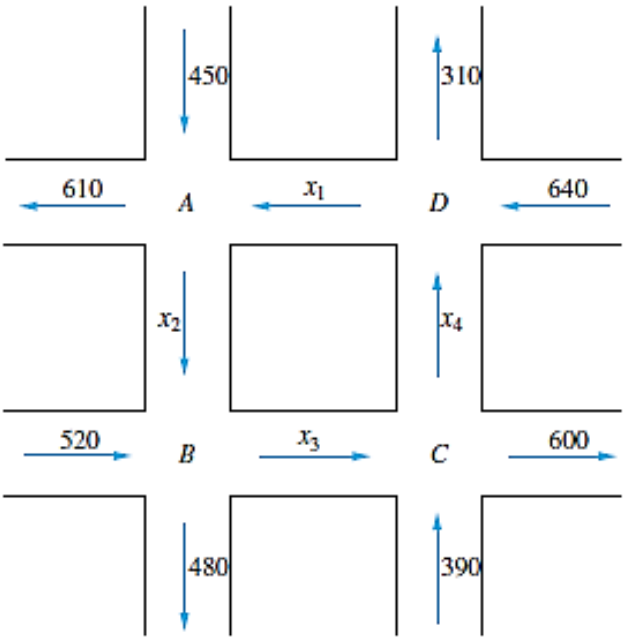
B: $x_2 + 520 = x_3 + 480 \quad \Rightarrow x_2 - x_3 = 480 - 520 = -40$

C: $x_3 + 390 = x_4 + 600 \quad \Rightarrow x_3 - x_4 = 600 - 390 = 210$

D: $x_1 + 310 = x_4 + 640 \quad \Rightarrow x_1 - x_4 = 640 - 310 = 330$

Corresponding Augmented matrix:

$$\begin{bmatrix} 1 & 0 & 0 & -1 : 330 \\ 0 & 1 & 0 & -1 : 170 \\ 0 & 0 & 1 & -1 : 210 \\ 0 & 0 & 0 & 0 : 0 \end{bmatrix}$$



The solution is:

$x_1 =$ t+330, $x_2 =$ t+170,

$x_3 =$ t+210, $x_4 =$ t.

(b) Hence, for $x_4 = 200$,

amount of traffic for other

intersections is $x_1 = 530$, $x_2 = 370$, and $x_3 = 410$.

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Solution of Word Problems Continued....

Work to do:

Question 4

Some parking meters in Milan, Italy, accept coins in the denominations of 20c, 50c, and €2. As an incentive program, the city administrators offer a big reward (a brand new Ferrari Testarossa) to any meter maid who brings back exactly 1,000 coins worth exactly € 1,000 from the daily rounds. What are the odds of this reward being claimed anytime soon?

Question 5

The new animated feature film is now playing 3 times a day in DHA Cinema. One day, there were 20 adults, 30 children and 10 senior citizens and theater made $600. At the next showing there were 24 adults, 60 children and 20 senior citizens with the theater making it $800 in ticket sale. At the last showing theater made $400 with 10 adults, 30 children and 5 senior citizens. How much are the tickets at the movie theater?