Lecture #1

INTRODUCTION TO LINEAR ALGEBRA

What do you think about Linear Algebra?

Why we study Linear Algebra in computer science?

provides concepts to many important areas of C.S.

all using technologies build upon Linear Algebra.

including

- graphics (taking a digital photo with your phone / transform the image in photoshop)
- image processing
- Cryptography (transforming data into some unreadable form for privacy / secrets) ↳ encoding and decoding matrix to code & decode a message)
- Web search
- Games (puzzle games)

zoom in, zoom out on phone

or make a phone call

↳ or watch a movie with digital effects.

Linear Algebra → built on two basic elements → MATRIX, → VECTOR

Algebra ⟶ Art of solving equations & systems of equations.

Linear Algebra ⟶ Art of solving systems of Linear equations.

Topics: Systems of linear eqs; Matrices, Gaussian elimination

LINEAR EQUATION

- Equation that has the highest degree of 1
- No variable in a linear equation has an exponent more than 1.
- Graph → straight line always.

NON-LINEAR EQUATIONS

(1) $x + 3y^2 = 4$

(2) $3x + 2y - xy = 5$

(3) $\sin x + y = 0$

(4) $\sqrt{x_1} + x_2 + x_3 = 1$

LINER EQUATIONS

(1) $2x + 1 = 0$ (one variable)

(2) $x + 3y = 7$ (two variables)

(3) $\frac{1}{2}x - y + 3z = -1$ (three variables)

(4) $x_1 - 2x_2 - 3x_3 + x_4 = 2$ (four variables)

(5) $x_1 + x_2 + x_3 + \ldots + x_n = 1$ (n variables)

Note:
- No products or roots of variables.
- Not involve trigonometric, logarithmic, or exponential functions.

Ex. Determine whether the equation is linear in x_1, x_2, and x_3.

(a) $x_1 + 5x_3 - \sqrt{2}x_3 = 1$

Not linear.

(b) $x_1 + 3x_2 + x_1x_3 = 2$

Not linear

(c) $x_1^{-2} + x_2 + 8x_3 = 5$

Not linear

(d) $x_1 = -7x_2 + 3x_3$

Linear.

(e) $x_1^{3/5} - 2x_2 + x_3 = 4$

Not linear.

Linear Equation in General

$$\boxed{a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n = b} \quad \text{— (1)}$$

b → real quantity.

$a_1, a_2, a_3, \dots a_n$ → real constants

$x_1, x_2, x_3, \dots x_n$ → unknowns.

Solution to Linear Equation → sequence of n numbers $s_1, s_2, \dots, s_n$

which satisfies eq (1) when $x_1 = s_1$, $x_2 = s_2$, $x_n = s_n$

For example.

The equation $6x_1 - 3x_2 + 4x_3 = -13$

has the solution $x_1 = 2,\ x_2 = 3,\ x_3 = -4$

because $6(2) - 3(3) + 4(-4) = -13$

$$-13 = -13$$

System of Linear Equation in two Variables

general form $\rightarrow$

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$

(a_1, b_1, c_1
a_2, b_2, c_2
$\downarrow$
real numbers

TECHNIQUES to find solution to a linear system.

METHOD OF ELIMINATION

eliminate some variable by adding a multiple of one equation to another.

METHOD OF SUBSTITUTION

Use value of one variable from one equation in other equation

Example 1.
Find the solution of the linear system by using Method of elimination.

$$5x + y = 3 \quad -①$$

$$2x - y = 4 \quad -②$$

Adding eq ① & ②

$$7x = 7$$

$$\boxed{x = 1}$$

Putting $x = 1$ in eq ①

$$5(1) + y = 3$$

$$\boxed{y = -2}$$

So $(1, -2)$ is solution of the given system.

has solution (CONSISTENT LINEAR SYSTEM)

Example 2. Use method of elimination to solve

$$x - 3y = -7 \quad -①$$

$$2x - 6y = 7 \quad -②$$

Multiplying eq ① by 2

$$2x - 6y = -14$$
$$2x - 6y = 7$$
→ subtracting both.

$$\underline{-\quad + \quad -}$$

$$0 = -21$$

i.e no solution.

(INCONSISTENT LINEAR SYSTEM)

Exercise.

1. In each part, determine whether the given points is a solution of the linear system

$$2x - 4y - z = 1$$
$$x - 3y + z = 1$$
$$3x - 5y - 3z = 1$$

(a) $(3, 1, 1)$

Yes.

$$2(3) - 4(1) - (1) = 1$$
$$1 = 1$$
$$(3) - 3(1) + (1) = 1$$
$$1 = 1$$
$$3(3) - 5(1) - 3(1) = 1$$
$$1 = 1$$

(b) $(3, -1, 1)$

(c) $(13, 5, 2)$

(d) $\left(\frac{13}{2}, \frac{5}{2}, 2\right)$

(e) $(17, 7, 5)$

2. $\begin{cases} x + 2y - 2z = 3 \\ 3x - y + z = 1 \\ -x + 5y - 5z = 5 \end{cases}$

(a) $\left(\frac{5}{7}, \frac{8}{7}, 1\right)$

(b) $\left(\frac{5}{7}, \frac{8}{7}, 0\right)$

(c) $(5, 8, 1)$

(d) $\left(\frac{5}{7}, \frac{10}{7}, \frac{2}{7}\right)$

(e) $\left(\frac{5}{7}, \frac{22}{7}, 2\right)$

Work to do Elementary LA by Howard Anton.
Ex 1.1 : Q 1-9

Q. Solve the linear system using elimination method.

$$2x - 3y + 4z = -12 \quad -①$$

$$x - 2y + z = -5 \quad -②$$

$$3x + y + 2z = 1 \quad -③$$

Step 1 To eliminate x, we will multiply equation ② by 2 and subtract from ①

$$2x - 3y + 4z = -12$$

$$2x - 4y + 2z = -10$$

$$- \quad + \quad - \quad +$$

$$\boxed{y + 2z = -2.} \quad -④$$

Step 2 Now again to eliminate x from eq ② we will multiply it by 3 and subtract from eq ③.

$$3x + y + 2z = 1$$

$$3x - 6y + 3z = -15$$

$$- \quad + \quad - \quad +$$

$$\boxed{7y - z = 16} \quad -⑤$$

Step 3 Multiplying eq ⑤ by 2 and and adding in ④

$$y + 2z = -2$$

$$+ \; 14y - 2z = 32 \longrightarrow 15y = 30 \rightarrow \boxed{y = 2}$$

step 4

Substituting $y=2$ in eq 4 (or 5)

$$y + 2z = -2$$

$$2 + 2z = -2$$

$$2z = -4$$

$$\boxed{z = -2}$$

step 5

Substituting $y = 2$ and $z = -2$ in eq 1 (or 2 or 3)

eq 2 $\Rightarrow$ $x - 2y + z = -5$

$$x - 2(2) + (-2) = -5$$

$$x - 4 - 2 = -5$$

$$\boxed{x = 1}$$

So $(1, 2, -2)$ is solution of the given system.