Modeling Oliver's Four Factors

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1. Loading and Exploring Data

1.1 Loading required libraries

```
library(car)
library(caret)
library(corrplot)
library(dplyr)
library(formattable)
library(gbm)
library(GGally)
library(ggplot2)
library(plotly)
library(randomForest)
library(readxl)
library(rmarkdown)
library(scales)
library(sjstats)
library(tidyr)
library(viridis)
```

1.2 Loading our dataset of interest as a dataframe

```
four_factors <- read_excel("four_factors_all_seasons.xlsx")</pre>
```

1.3 Data exploration

```
head(four_factors)
## # A tibble: 6 x 13
## Team Season WINS MOV NRTG `eFG%` `TOV%` `OREB%` FTF `OPPeFG%` `OPPTOV%`
## <chr> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 Milw... 2018 ... 60 8.87 8.6 0.55 12 20.8 0.197 0.503
                                                                11.5
## 2 Toro... 2018 ... 58 6.09 6 0.543 12.4 21.9 0.198 0.509
                                                                13.1
## 3 Gold... 2018 ... 57 6.46 6.4 0.565 12.6 22.5 0.182 0.508
                                                                11.7
## 4 Denv... 2018 ... 54 3.95 4.1 0.527 11.9 26.6 0.175 0.521
                                                                12.3
## 5 Hous... 2018 ... 53 4.77 4.8 0.542 12
                                           22.8 0.221 0.525
                                                                13.4
## 6 Port... 2018 ... 53 4.2 4.2 0.528 12.1 26.6 0.21
                                                       0.516
                                                                11
## # ... with 2 more variables: `DREB%` <dbl>, OPPFTF <dbl>
```

```
summary(four_factors)
```

```
Team
                      Season
                                        WINS
                                                      MOV
##
   Length:330
                   Length:330
                                   Min. : 8.697 Min. :-13.910000
   Class :character Class :character 1st Qu.:31.295 1st Qu.: -3.260000
##
   Mode :character Mode :character Median :42.000 Median : 0.245000
##
                                   Mean :40.997 Mean : 0.000394
##
                                   3rd Qu.:50.000 3rd Qu.: 3.432500
##
                                   Max. :73.000 Max. : 11.630000
##
##
       NRTG
                         eFG%
                                    TOV%
                                                  OREB%
##
   Min. :-15.200000 Min. :0.4390 Min. :10.70 Min. :18.00
##
   1st Qu.: -3.475000
                    1st Qu.:0.4890
                                   1st Qu.:12.60
                                                 1st Qu.:23.10
   Median: 0.250000 Median:0.5020 Median:13.20
                                                Median :25.10
##
   Mean
        : 0.009697 Mean :0.5038 Mean :13.24 Mean :25.07
##
   3rd Qu.: 3.600000 3rd Qu.:0.5188 3rd Qu.:13.90 3rd Qu.:27.20
##
   Max. : 11.600000 Max. :0.5690 Max. :16.00 Max. :32.60
##
       FTF
            OPPeFG% OPPTOV%
                                                DREB%
##
  Min. :0.1430 Min. :0.4500 Min. :10.50 Min. :68.10
##
   1st Qu.:0.1950 1st Qu.:0.4910 1st Qu.:12.60 1st Qu.:73.30
   Median :0.2100 Median :0.5050 Median :13.20 Median :74.80
  Mean :0.2123 Mean :0.5037 Mean :13.24 Mean :74.92
## 3rd Qu.:0.2280 3rd Qu.:0.5160 3rd Qu.:13.90 3rd Qu.:76.40
## Max. :0.2990 Max. :0.5640 Max. :16.30 Max. :81.20
      OPPFTF
##
## Min. :0.1000
##
  1st Ou.:0.1940
## Median :0.2110
## Mean :0.2121
## 3rd Qu.:0.2270
## Max. :0.3150
```

1.4 Renaming columns containing special characters

```
## [1] "Team" "Season" "WINS" "MOV" "NRTG" "eFG" "TOV" "OREB"
## [9] "FTF" "OPPEFG" "OPPTOV" "DREB" "OPPFTF"
```

1.5 Creating a new dataframe

```
four_factors_scaled = four_factors_new
four_factors_scaled
```

```
## # A tibble: 330 x 13
                 Team Season WINS
                                                                              MOV NRTG eFG
                                                                                                                                     TOV OREB
##
                                                                                                                                                                           FTF OPPeFG OPPTOV DREB
##
                  <chr> <chr> <dbl> <
                                                                                                                                                        20.8 0.197 0.503
##
                                                            60 8.87
                                                                                                  8.6 0.55
                                                                                                                                                                                                                  11.5
         1 Milw... 2018 ...
                                                                                                                                   12
         2 Toro... 2018 ...
                                                                  58 6.09 6 0.543 12.4 21.9 0.198 0.509 13.1 77.1
                                                                  57 6.46 6.4 0.565 12.6 22.5 0.182 0.508 11.7
           3 Gold... 2018 ...
           4 Denv... 2018 ...
                                                              54 3.95 4.1 0.527 11.9 26.6 0.175 0.521
                                                                                                                                                                                                                     12.3 78
           5 Hous... 2018 ...
                                                                 53 4.77 4.8 0.542 12
                                                                                                                                                        22.8 0.221 0.525
                                                                                                                                                                                                                     13.4
           6 Port... 2018 ...
                                                                  53 4.2
                                                                                                   4.2 0.528
                                                                                                                                     12.1 26.6 0.21
                                                                                                                                                                                             0.516
                                                                                                                                                                                                                     11
           7 Phil... 2018 ...
                                                                  51 2.7
                                                                                                   2.6 0.532
                                                                                                                                     12.9 24.5 0.241 0.512
                                                                                                                                                                                                                    11.1 78.6
           8 Utah... 2018 ...
                                                                  50
                                                                              5.26
                                                                                                  5.2 0.538
                                                                                                                                     13.4 22.9 0.217 0.507
                                                                                                                                                                                                                     12.4
           9 Bost... 2018 ...
                                                                 49 4.44
                                                                                                  4.4 0.534
                                                                                                                                     11.5 21.6 0.173 0.514
                                                                                                                                                                                                                    13.4 77
## 10 Okla... 2018 ...
                                                                  49 3.4
                                                                                                    3.3 0.514 11.7 26 0.19
                                                                                                                                                                                             0.523
                                                                                                                                                                                                                   14.4 78.2
## # ... with 320 more rows, and 1 more variable: OPPFTF <dbl>
```

1.6 Scaling the efg, ftf, opperg and oppftf features by a factor of 100

```
four_factors_scaled$eFG <- four_factors_new$eFG * 100
four_factors_scaled$FTF <- four_factors_new$FTF * 100
four_factors_scaled$OPPeFG <- four_factors_new$OPPeFG * 100
four_factors_scaled$OPPFTF <- four_factors_new$OPPFTF * 100
summary(four_factors_scaled)</pre>
```

```
##
      Team
                                        WINS
                                                       MOV
                      Season
## Length:330
                  Length:330
                                  Min. : 8.697 Min. :-13.910000
  Class: character Class: character 1st Qu.:31.295 1st Qu.: -3.260000
  Mode :character Mode :character Median :42.000 Median : 0.245000
##
                                   Mean :40.997 Mean : 0.000394
##
                                   3rd Qu.:50.000 3rd Qu.: 3.432500
##
                                   Max. :73.000 Max. : 11.630000
##
       NRTG
                         eFG
                                   TOV
                                                    OREB
   Min. :-15.200000 Min. :43.90 Min. :10.70 Min. :18.00
##
   1st Qu.: -3.475000 1st Qu.:48.90 1st Qu.:12.60 1st Qu.:23.10
##
   Median: 0.250000 Median:50.20 Median:13.20 Median:25.10
##
   Mean : 0.009697 Mean :50.38 Mean :13.24 Mean :25.07
##
   3rd Qu.: 3.600000 3rd Qu.:51.88 3rd Qu.:13.90 3rd Qu.:27.20
##
   Max. : 11.600000 Max. :56.90 Max. :16.00
##
                                                Max. :32.60
##
       FTF
                    OPPeFG OPPTOV
                                                 DREB
##
   Min. :14.30 Min. :45.00 Min. :10.50 Min. :68.10
                1st Qu.:49.10
##
   1st Ou.:19.50
                              1st Qu.:12.60
                                            1st Qu.:73.30
##
   Median :21.00
                Median:50.50
                              Median :13.20 Median :74.80
##
   Mean :21.23
                Mean :50.37 Mean :13.24 Mean :74.92
##
                3rd Qu.:51.60 3rd Qu.:13.90 3rd Qu.:76.40
   3rd Qu.:22.80
##
   Max. :29.90
                Max. :56.40 Max. :16.30 Max. :81.20
##
      ОРРЕТЕ
##
  Min. :10.00
##
   1st Ou.:19.40
   Median :21.10
  Mean :21.21
  3rd Qu.:22.70
  Max. :31.50
```

```
str(four_factors_scaled)
```

```
## tibble [330 × 13] (S3: tbl_df/tbl/data.frame)

## $ Team : chr [1:330] "Milwaukee Bucks" "Toronto Raptors" "Golden State Warriors" "Denver Nuggets" ...

## $ Season: chr [1:330] "2018 - 2019" "2018 - 2019" "2018 - 2019" "2018 - 2019" "...

## $ WINS : num [1:330] 60 58 57 54 53 53 51 50 49 49 ...

## $ MOV : num [1:330] 8.87 6.09 6.46 3.95 4.77 4.2 2.7 5.26 4.44 3.4 ...

## $ NRTG : num [1:330] 8.6 6 6.4 4.1 4.8 4.2 2.6 5.2 4.4 3.3 ...

## $ EFG : num [1:330] 55 54.3 56.5 52.7 54.2 52.8 53.2 53.8 53.4 51.4 ...

## $ TOV : num [1:330] 12 12.4 12.6 11.9 12 12.1 12.9 13.4 11.5 11.7 ...

## $ OREB : num [1:330] 20.8 21.9 22.5 26.6 22.8 26.6 24.5 22.9 21.6 26 ...

## $ FTF : num [1:330] 19.7 19.8 18.2 17.5 22.1 21 24.1 21.7 17.3 19 ...

## $ OPPEFG: num [1:330] 50.3 50.9 50.8 52.1 52.5 51.6 51.2 50.7 51.4 52.3 ...

## $ DREB : num [1:330] 11.5 13.1 11.7 12.3 13.4 11 11.1 12.4 13.4 14.4 ...

## $ DREB : num [1:330] 80.3 77.1 77.1 78 74.4 77.9 78.6 80.3 77 78.2 ...

## $ OPPFFFF: num [1:330] 16.2 19 20.5 19.4 21 19.5 20.6 18.9 19.8 20.6 ...
```

1.7 Creating octiles for the WINS feature

```
wins_octile <- ntile(four_factors_scaled$WINS, 8)
four_factors_scaled$octile <- as.factor(wins_octile)
str(four_factors_scaled)</pre>
```

```
## tibble [330 × 14] (S3: tbl_df/tbl/data.frame)
## $ Team : chr [1:330] "Milwaukee Bucks" "Toronto Raptors" "Golden State Warriors" "Denver Nuggets" ...
## $ Season: chr [1:330] "2018 - 2019" "2018 - 2019" "2018 - 2019" "2018 - 2019" ...
## $ WINS : num [1:330] 60 58 57 54 53 53 51 50 49 49 ...
## $ MOV : num [1:330] 8.87 6.09 6.46 3.95 4.77 4.2 2.7 5.26 4.44 3.4 ...
## $ NRTG : num [1:330] 8.6 6 6.4 4.1 4.8 4.2 2.6 5.2 4.4 3.3 ...
## $ eFG : num [1:330] 15 5 54.3 56.5 52.7 54.2 52.8 53.2 53.8 53.4 51.4 ...
## $ TOV : num [1:330] 12 12.4 12.6 11.9 12 12.1 12.9 13.4 11.5 11.7 ...
## $ OREB : num [1:330] 20.8 21.9 22.5 26.6 22.8 26.6 24.5 22.9 21.6 26 ...
## $ FTF : num [1:330] 19.7 19.8 18.2 17.5 22.1 21 24.1 21.7 17.3 19 ...
## $ OPPEFG: num [1:330] 50.3 50.9 50.8 52.1 52.5 51.6 51.2 50.7 51.4 52.3 ...
## $ DREB : num [1:330] 11.5 13.1 11.7 12.3 13.4 11 11.1 12.4 13.4 14.4 ...
## $ DREB : num [1:330] 80.3 77.1 77.1 78 74.4 77.9 78.6 80.3 77 78.2 ...
## $ OPPFTF: num [1:330] 16.2 19 20.5 19.4 21 19.5 20.6 18.9 19.8 20.6 ...
## $ octile: Factor w/ 8 levels "1","2","3","4",..: 8 8 8 7 7 7 7 6 6 6 ...
```

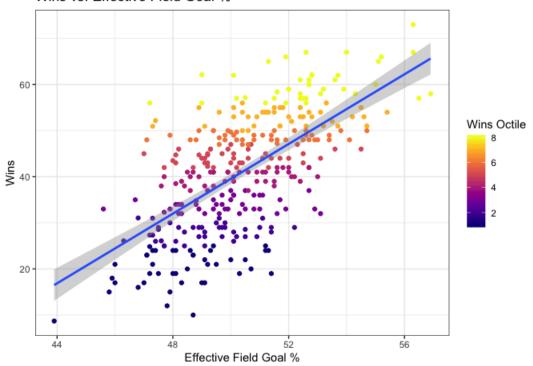
2. Graphical Analysis

2.1 Scatter Plots

Checking graphically to see if there is a relationship between our response variable and predictor variables

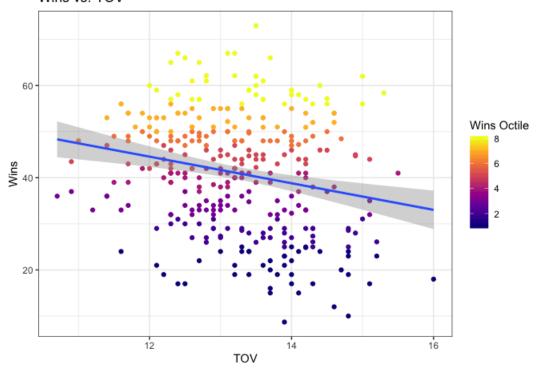
```
ggplot(four_factors_scaled, aes(eFG, WINS)) +
  geom_point(aes(color = wins_octile)) +
  geom_smooth(method ="lm") +
  ggtitle("Wins vs. Effective Field Goal %") +
  xlab("Effective Field Goal %") +
  ylab("Wins") +
  labs(color = "Wins Octile") +
  coord_cartesian() +
  scale_color_viridis(option = "C") +
  theme_bw()
```

Wins vs. Effective Field Goal %



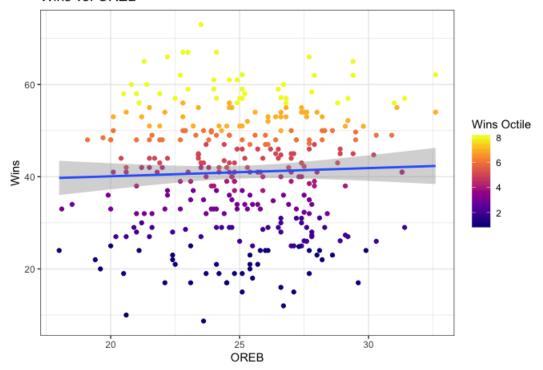
```
ggplot(four_factors_scaled, aes(TOV, WINS)) +
  geom_point(aes(color = wins_octile)) +
  geom_smooth(method ="lm") +
  ggtitle("Wins vs. TOV") +
  xlab("TOV") +
  ylab("Wins") +
  labs(color = "Wins Octile") +
  coord_cartesian() +
  scale_color_viridis(option = "C") +
  theme_bw()
```

Wins vs. TOV



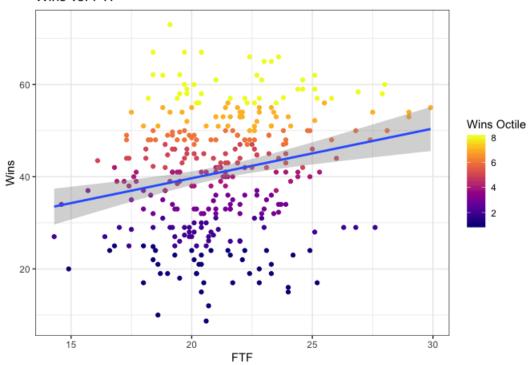
```
ggplot(four_factors_scaled, aes(OREB, WINS)) +
  geom_point(aes(color = wins_octile)) +
  geom_smooth(method ="lm") +
  ggtitle("Wins vs. OREB") +
  xlab("OREB") +
  ylab("Wins") +
  labs(color = "Wins Octile") +
  coord_cartesian() +
  scale_color_viridis(option = "C") +
  theme_bw()
```

Wins vs. OREB



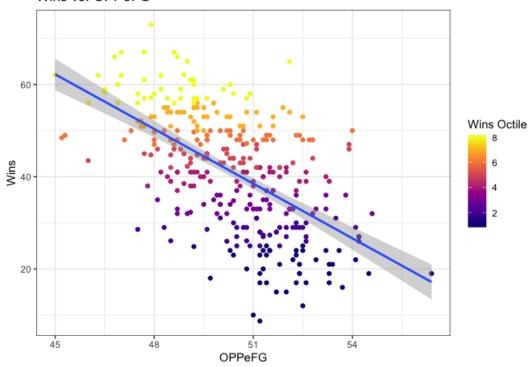
```
ggplot(four_factors_scaled, aes(FTF, WINS)) +
  geom_point(aes(color = wins_octile)) +
  geom_smooth(method ="lm") +
  ggtitle("Wins vs. FTF") +
  xlab("FTF") +
  ylab("Wins") +
  labs(color = "Wins Octile") +
  coord_cartesian() +
  scale_color_viridis(option = "C") +
  theme_bw()
```

Wins vs. FTF



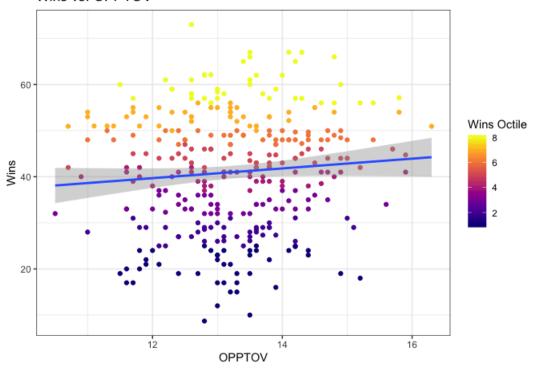
```
ggplot(four_factors_scaled, aes(OPPeFG, WINS)) +
  geom_point(aes(color = wins_octile)) +
  geom_smooth(method ="lm") +
  ggtitle("Wins vs. OPPeFG") +
  xlab("OPPeFG") +
  ylab("Wins") +
  labs(color = "Wins Octile") +
  coord_cartesian() +
  scale_color_viridis(option = "C") +
  theme_bw()
```

Wins vs. OPPeFG



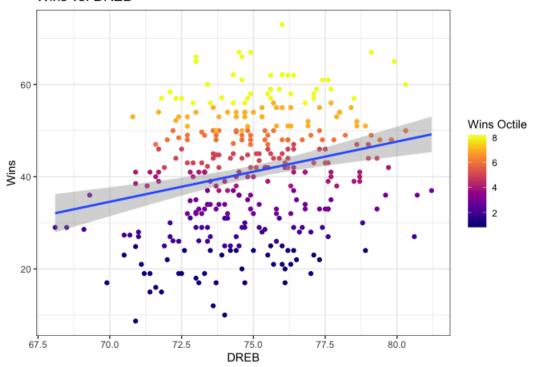
```
ggplot(four_factors_scaled, aes(OPPTOV, WINS)) +
  geom_point(aes(color = wins_octile)) +
  geom_smooth(method ="lm") +
  ggtitle("Wins vs. OPPTOV") +
  xlab("OPPTOV") +
  ylab("Wins") +
  labs(color = "Wins Octile") +
  coord_cartesian() +
  scale_color_viridis(option = "C") +
  theme_bw()
```

Wins vs. OPPTOV



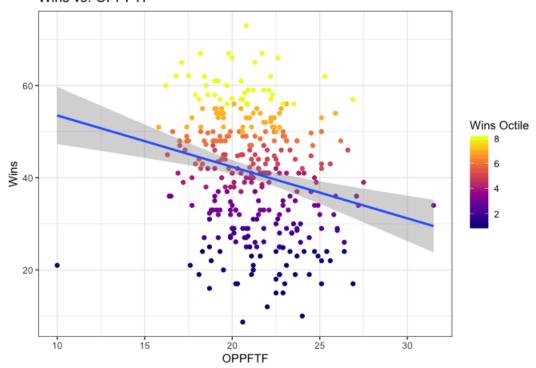
```
ggplot(four_factors_scaled, aes(DREB, WINS)) +
  geom_point(aes(color = wins_octile)) +
  geom_smooth(method ="lm") +
  ggtitle("Wins vs. DREB") +
  xlab("DREB") +
  ylab("Wins") +
  labs(color = "Wins Octile") +
  coord_cartesian() +
  scale_color_viridis(option = "C") +
  theme_bw()
```

Wins vs. DREB



```
ggplot(four_factors_scaled, aes(OPPFTF, WINS)) +
  geom_point(aes(color = wins_octile)) +
  geom_smooth(method ="lm") +
  ggtitle("Wins vs. OPPFTF") +
  xlab("OPPFTF") +
  ylab("Wins") +
  labs(color = "Wins Octile") +
  coord_cartesian() +
  scale_color_viridis(option = "C") +
  theme_bw()
```

Wins vs. OPPFTF

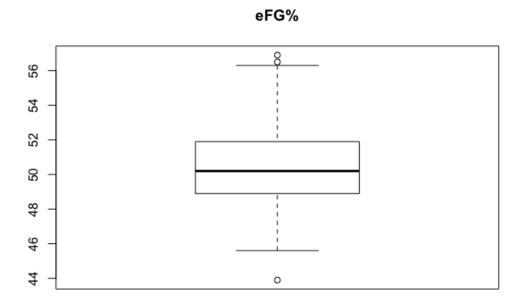


```
new = t(four_factors_scaled)
```

2.2 Box Plots

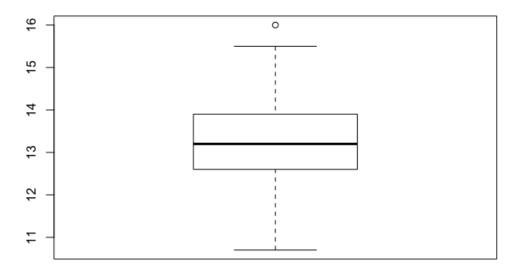
Checking graphically to see if there are outliers in our predictor variables

```
boxplot(four_factors_scaled$eFG, main="eFG%")
```



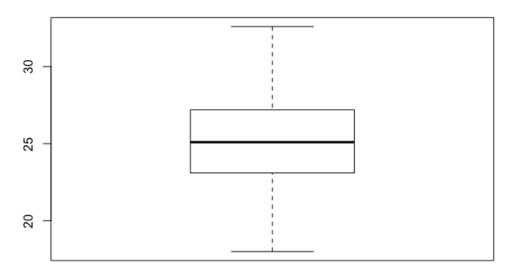
boxplot(four_factors_scaled\$TOV, main="TOV%")





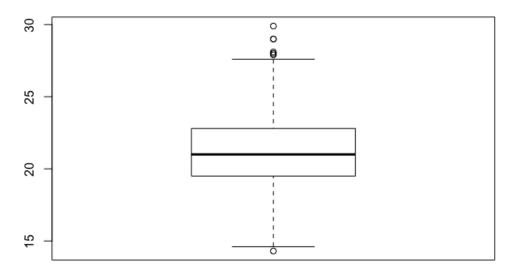
boxplot(four_factors_scaled\$OREB, main="OREB")

OREB



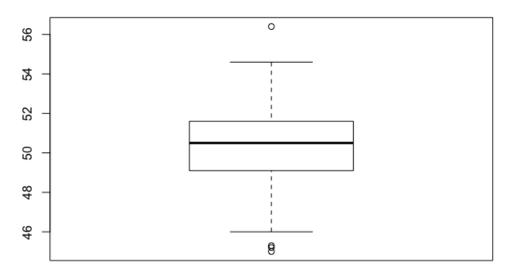
boxplot(four_factors_scaled\$FTF, main="FTF")





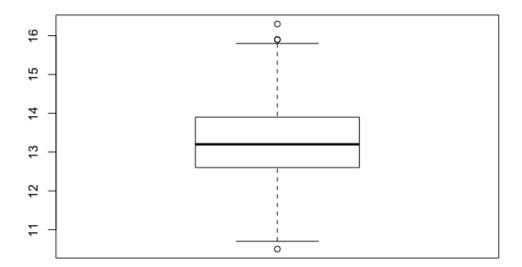
boxplot(four_factors_scaled\$OPPeFG, main="OPPeFG%")

OPPeFG%



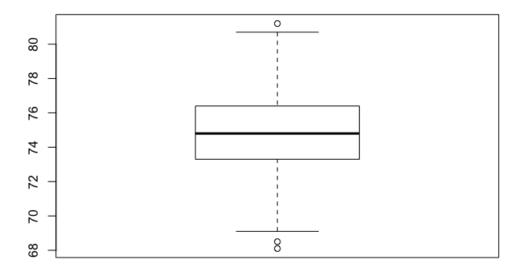
boxplot(four_factors_scaled\$OPPTOV, main="OPPTOV%")

OPPTOV%



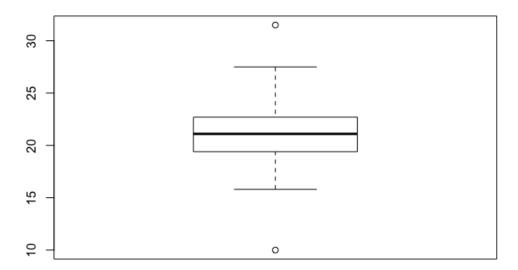
boxplot(four_factors_scaled\$DREB, main="DREB")

DREB



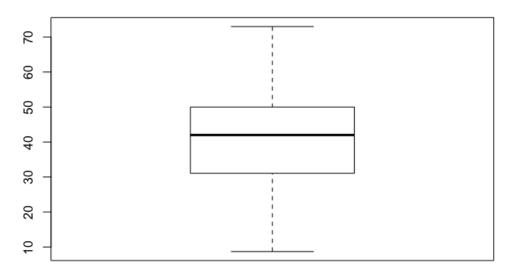
boxplot(four_factors_scaled\$OPPFTF, main="OPPFTF")

OPPFTF



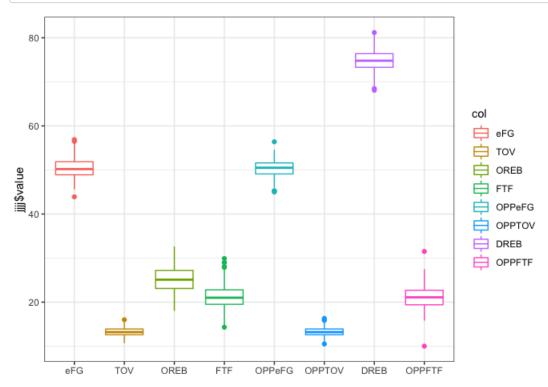
boxplot(four_factors_scaled\$WINS, main="WINS")

WINS



Boxplot for all predictors

```
fact eFG = data.frame("col" = rep("eFG", 330))
fact_TOV = data.frame("col" = rep("TOV", 330))
fact_OREB = data.frame("col" = rep("OREB", 330))
fact_FTF = data.frame("col" = rep("FTF", 330))
fact_OPPeFG = data.frame("col" = rep("OPPeFG", 330))
fact_OPPTOV = data.frame("col" = rep("OPPTOV", 330))
fact_DREB = data.frame("col" = rep("DREB", 330))
fact_OPPFTF = data.frame("col" = rep("OPPFTF", 330))
jjjj <- rbind(fact_eFG, fact_TOV, fact_OREB, fact_FTF, fact_OPPeFG, fact_OPPTOV, fact_DREB, fact_OPPFTF)
jjjj$value[1:330] <- four_factors_scaled$eFG</pre>
jjjj$value[331:660] <- four_factors_scaled$TOV</pre>
jjjj$value[661:990] <- four_factors_scaled$OREB</pre>
jjjj$value[991:1320] <- four_factors_scaled$FTF</pre>
jjjj$value[1321:1650] <- four_factors_scaled$OPPeFG</pre>
jjjj$value[1651:1980] <- four_factors_scaled$OPPTOV</pre>
jjjj$value[1981:2310] <- four_factors_scaled$DREB</pre>
jjjj$value[2311:2640] <- four_factors_scaled$OPPFTF</pre>
#jjjj
h \leftarrow ggplot(data=jjjj, aes(x=jjjjj$col, y=jjjjj$value, color = col)) +
geom boxplot() +
xlab("Factor") +
theme(legend.position="none") +
xlab("") +
theme_bw()
h
```

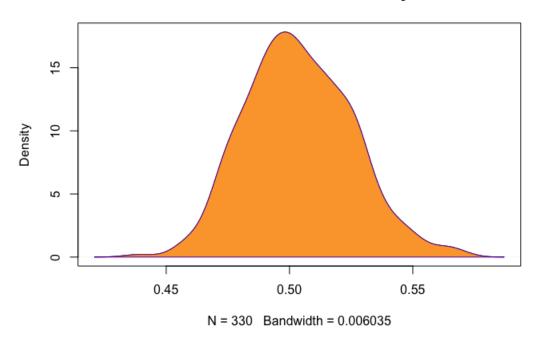


2.3 Density Plots

Checking graphically to see if our feature variables have a normal distribution

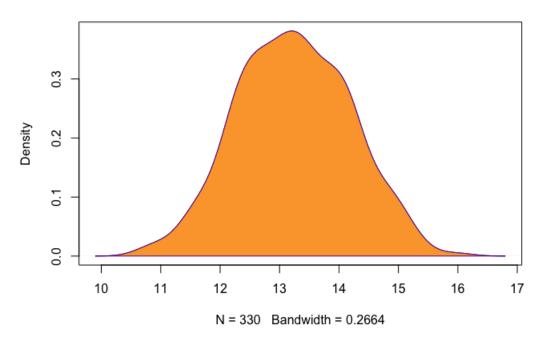
dens_eFG <- density(four_factors_new\$eFG)
plot(dens_eFG, main = "Effective Field Goal % Density")
polygon(dens_eFG, col="#fca538", border="#6721a7")</pre>

Effective Field Goal % Density



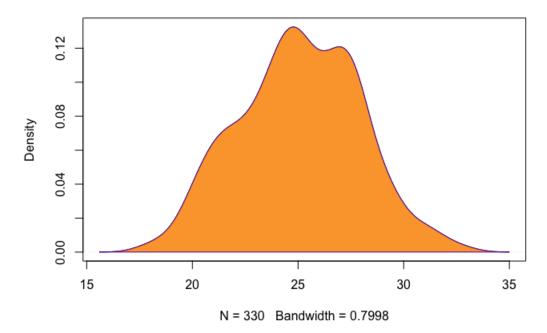
dens_TOV <- density(four_factors_new\$TOV)
plot(dens_TOV, main = "Turnover Density")
polygon(dens_TOV, col="#fca538", border="#6721a7")</pre>

Turnover Density



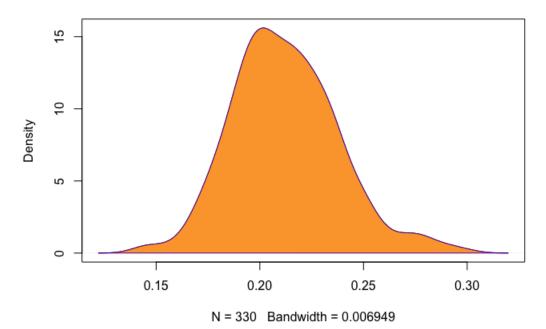
```
dens_OREB <- density(four_factors_new$OREB)
plot(dens_OREB, main = "Offensive Rebound Density")
polygon(dens_OREB, col="#fca538", border="#6721a7")</pre>
```

Offensive Rebound Density



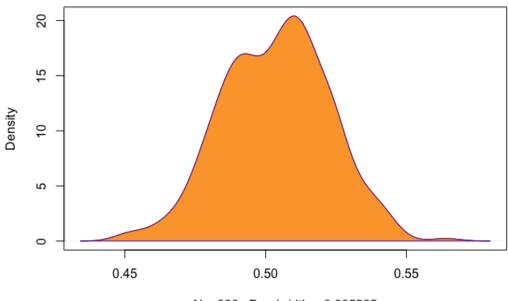
```
dens_FTF <- density(four_factors_new$FTF)
plot(dens_FTF, main = "Free Throw Factor Density")
polygon(dens_FTF, col="#fca538", border="#6721a7")</pre>
```

Free Throw Factor Density



```
dens_OPPeFG <- density(four_factors_new$OPPeFG)
plot(dens_OPPeFG, main = "Opponent Effective Field Goal % Density")
polygon(dens_OPPeFG, col="#fca538", border="#6721a7")</pre>
```

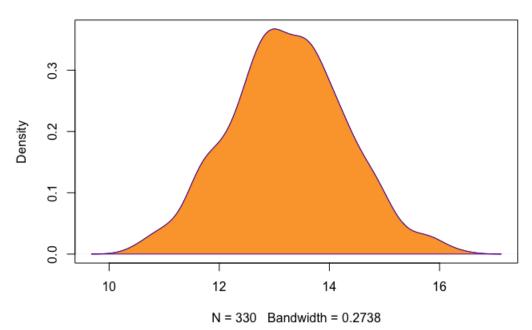
Opponent Effective Field Goal % Density



N = 330 Bandwidth = 0.005265

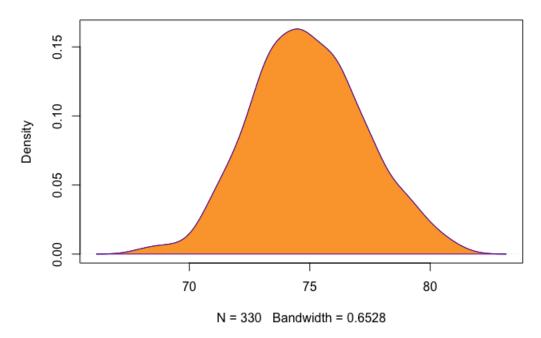
```
dens_OPPTOV <- density(four_factors_new$OPPTOV)
plot(dens_OPPTOV, main = "Opponent Turnover Density")
polygon(dens_OPPTOV, col="#fca538", border="#6721a7")</pre>
```

Opponent Turnover Density



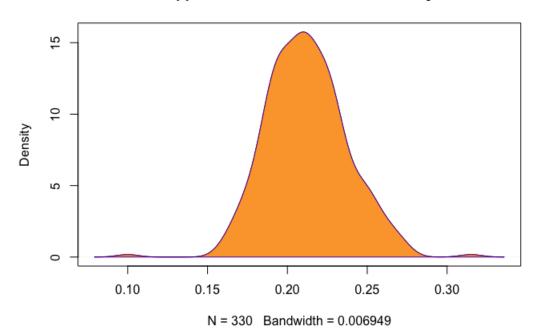
```
dens_DREB <- density(four_factors_new$DREB)
plot(dens_DREB, main = "Defensive Rebound Density")
polygon(dens_DREB, col="#fca538", border="#6721a7")</pre>
```

Defensive Rebound Density

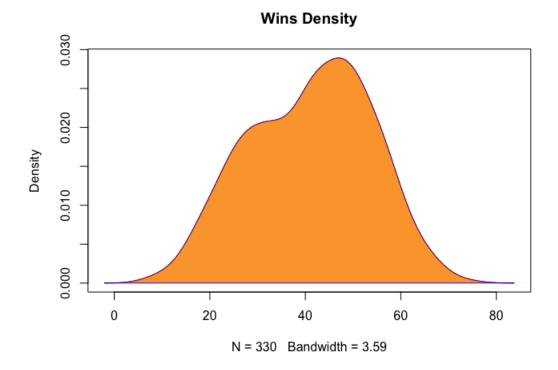


dens_OPPFTF <- density(four_factors_new\$OPPFTF)
plot(dens_OPPFTF, main = "Opponent Free Throw Factor Density")
polygon(dens_OPPFTF, col="#fca538", border="#6721a7")</pre>

Opponent Free Throw Factor Density

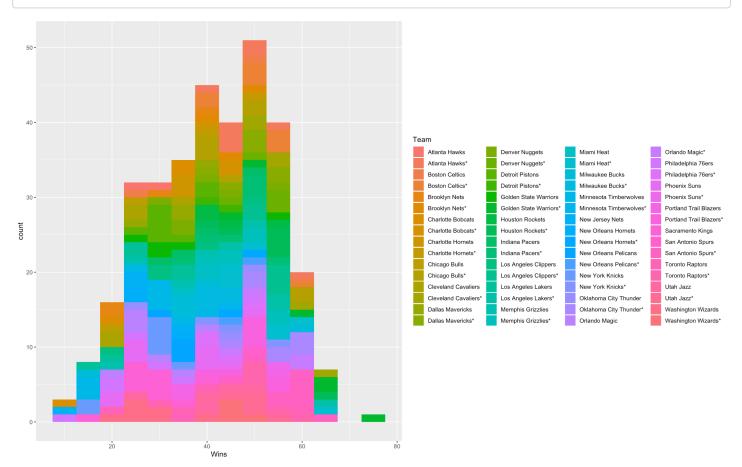


dens_WINS <- density(four_factors_new\$WINS)
plot(dens_WINS, main = "Wins Density")
polygon(dens_WINS, col="#fca538", border="#6721a7")</pre>



2.4 Histogram of wins

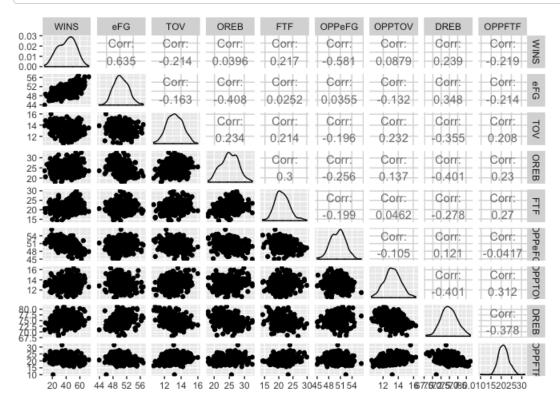
fig2 = ggplot(four_factors_scaled, aes(x=WINS, fill=Team)) + geom_histogram(binwidth=5) + labs(x="Wins")



2.5 Pairplot for all variables

Checking graphically pairwise relationships in our variables

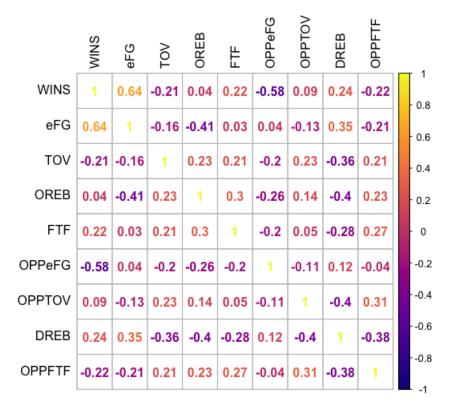
```
ggpairs(four_factors_scaled[c("WINS", "eFG", "TOV", "OREB", "FTF", "OPPeFG", "OPPTOV", "DREB", "OPPFTF")])
```



2.6 Correlation plot for all variables

Checking graphically for multicolinearity between our different feature variables

```
? corrplot
C <- cor(four_factors_scaled[c("WINS", "eFG", "TOV", "OREB", "FTF", "OPPEFG", "OPPTOV", "DREB", "OPPFTF")])
corrplot(C, method = "number", col = plasma(256), tl.col = "black")</pre>
```



There doesn't seem to be any moderate or strong multicolinarity to be aware of in the matrix

3. Modeling and Analysis

3.1 Splitting our dataset into train and test subsets

```
set.seed(8)
rows <- sample(1:nrow(four_factors_scaled), 0.7*nrow(four_factors_scaled))
ff_train = four_factors_scaled[rows,]
ff_test = four_factors_scaled[-rows,]
dim(ff_train)

## [1] 230  14

## [1] 100  14</pre>
```

3.2 Creating a Multiple Linear Regression model regressing the Four Factors on ${\tt WINS}$ using the test dataset

```
lmWINS <- lm(WINS ~ eFG + TOV + OREB + FTF + OPPeFG + OPPTOV + DREB + OPPFTF, data = ff_train)
summary(lmWINS)</pre>
```

```
## Call:
## lm(formula = WINS ~ eFG + TOV + OREB + FTF + OPPeFG + OPPTOV +
     DREB + OPPFTF, data = ff_train)
##
##
## Residuals:
##
      Min
             1Q Median
                             3Q
                                     Max
## -10.6982 -2.1067 -0.1126 2.0078
                                   9.0728
##
## Coefficients:
            Estimate Std. Error t value Pr(>|t|)
## (Intercept) -53.18468 14.54796 -3.656 0.00032 ***
             3.89934 0.11042 35.314 < 2e-16 ***
## eFG
             -3.39168 0.24879 -13.633 < 2e-16 ***
## TOV
             1.11070 0.09013 12.323 < 2e-16 ***
## OREB
             0.70791 0.09304 7.609 7.89e-13 ***
## FTF
## OPPeFG
            -3.76483 0.11439 -32.913 < 2e-16 ***
## OPPTOV
             ## DREB
             ## OPPFTF
             -0.72897 0.10377 -7.025 2.62e-11 ***
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.158 on 221 degrees of freedom
## Multiple R-squared: 0.9435, Adjusted R-squared: 0.9414
## F-statistic: 461.2 on 8 and 221 DF, p-value: < 2.2e-16
```

3.2.1 Interpretation of F-statistic, P-value and R-squared value for the ImWINS model

From the F-statistic and p-value of our **ImWINS** model, we can reject the null hypothesis that our predictor variables have no effect on wins. There is strong evidence to conclude there is a relationship between our predictor and response variables.

Our model explains approximately 94.4% of the variation in WINS using eFG, TOV, OREB, FTF, OPPEFG, OPPTOV, DREB and OPPFTF as predictors.

3.2.2 Interpretation of coefficients for the ImWINS model

- An increase of 1 percentage point of eFG is associated with an average increase of 3.81 wins, holding all else equal
- An increase of 1 percentage point of TOV is associated with an average decrease of 3.64 wins, holding all else equal
- An increase of 1 percentage point of OREB is associated with an average increase of 1.11 WINS, holding all else equal
- · An increase of 1 percentage point of FTF is associated with an average increase of 0.69 wins, holding all else equal
- An increase of 1 percentage point of OPPeFG is associated with an average decrease of 3.86 WINS, holding all else equal
- An increase of 1 percentage point of OPPTOV is associated with an average increase of 2.99 WINS, holding all else equal
- An increase of 1 percentage point of DREB is associated with an average increase of 0.88 WINS, holding all else equal
- · An increase of 1 percentage point of OPPFTF is associated with an average decrease of 0.81 wins, holding all else equal

3.2.3 ANOVA table and confidence interval for the ImWINS model

```
anova(lmWINS)
```

```
## Analysis of Variance Table
##
## Response: WINS
##
       Df Sum Sq Mean Sq F value
            1 16737.8 16737.8 1678.045 < 2.2e-16 ***
## eFG
            1 264.3 264.3 26.502 5.813e-07 ***
## TOV
            1 4722.3 4722.3 473.437 < 2.2e-16 ***
## OREB
                519.4
                       519.4
                              52.076 8.457e-12 ***
             1 12167.9 12167.9 1219.886 < 2.2e-16 ***
             1 1037.7 1037.7 104.037 < 2.2e-16 ***
            1 860.4 860.4 86.260 < 2.2e-16 ***
## OPPFTF 1 492.2 492.2
                              49.349 2.618e-11 ***
## Residuals 221 2204.4 10.0
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

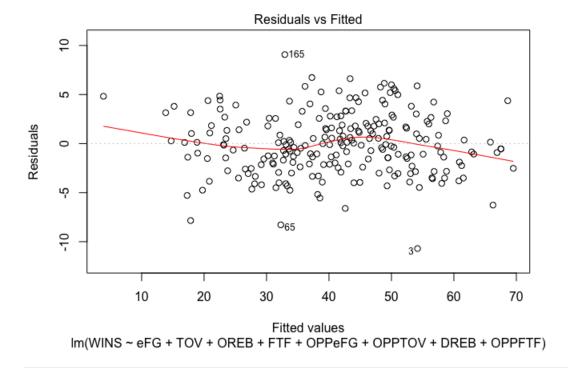
```
confint(lmWINS)
```

From the ANOVA table, we can see that the F-statistic for each predictor variable is significant and adds prediction power to our model. All included variables are relevant to our model.

3.2.4 Checking to see if assumptions of linear regression are reasonably met for the **ImWINS** model

1. The relationship is linear

```
plot(lmWINS, 1)
```



Our residual plot is mostly flat. There are some points that skew the line, such as observation 116, 138 and 188, but overall the relationship is linear.

2. Independence of error terms

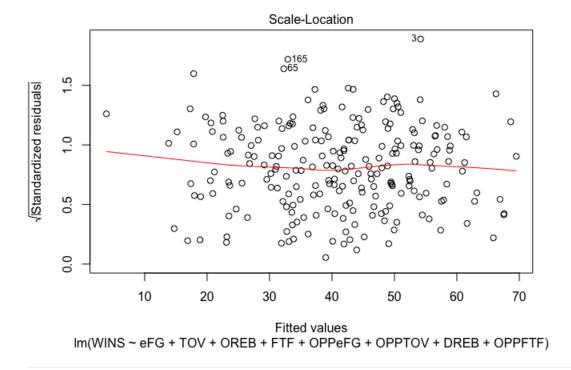
```
durbinWatsonTest(lmWINS)

## lag Autocorrelation D-W Statistic p-value
## 1 -0.01605434 2.02369 0.836
## Alternative hypothesis: rho != 0
```

We fail to reject the null hypothesis that the error terms are not autocorrelated. We have met the independence assumption.

3. Variation of observations' error terms is constant

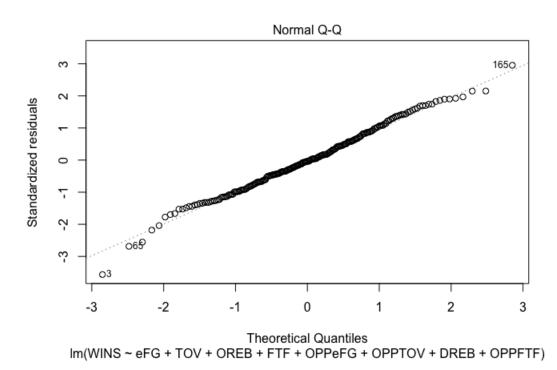
```
plot(lmWINS, 3)
```



The homoskedasticity assumption is met. In the scale-location plot we see points that spread in a normally pattern, there is no evidence of heteroskedasticity.

4. Values are normally distributed

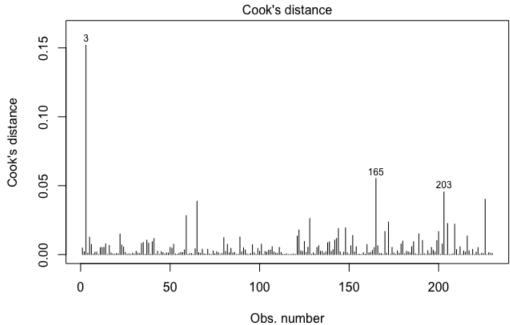
plot(lmWINS, 2)



From the Q-Q plot, we can see that points are very close to the diagonal line and are normally distributed.

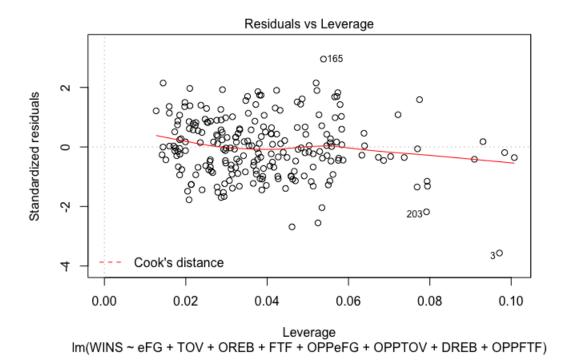
5. Outliers and Influential Points

plot(lmWINS, 4)



Im(WINS ~ eFG + TOV + OREB + FTF + OPPeFG + OPPTOV + DREB + OPPFTF)

plot(lmWINS, 5)



We can look for outliers in our data using Cook's distance. Points 48, 138 and 188 are identified as outliers. These three points also appear in our residual vs. leverage graph as potential points of interest.

6. Multicolinearity

```
vif(lmWINS)

## eFG TOV OREB FTF OPPEFG OPPTOV DREB OPPFTF
## 1.339503 1.229970 1.470495 1.343472 1.120709 1.252527 1.632149 1.341740
```

All of our VIF values are below 4. There is no multicolinearity in our data.

3.3 Creating a Multiple Linear Regression model regressing the Four Factors on MOV using the train dataset

```
lmMOV = lm(MOV ~ eFG + TOV + OREB + FTF + OPPeFG + OPPTOV + DREB + OPPFTF, data = ff_train)
summary(lmMOV)
```

```
##
## Call:
## lm(formula = MOV ~ eFG + TOV + OREB + FTF + OPPeFG + OPPTOV +
     DREB + OPPFTF, data = ff_train)
##
## Residuals:
              1Q Median
                              30
## -2.87201 -0.41263 -0.00396 0.46326 1.89270
##
## Coefficients:
##
      Estimate Std. Error t value Pr(>|t|)
## (Intercept) -38.85131 3.16528 -12.27 <2e-16 ***
## eFG 1.43555 0.02402 59.75 <2e-16 ***
## TOV
             -1.32334 0.05413 -24.45 <2e-16 ***
## OREB
             0.43792 0.01961 22.33 <2e-16 ***
             0.29640 0.02024 14.64 <2e-16 ***
## OPPeFG
            -1.42002 0.02489 -57.06 <2e-16 ***
             1.17049 0.04619 25.34 <2e-16 ***
             0.38560 0.02559 15.07 <2e-16 ***
## DREB
## OPPFTF
            -0.28811 0.02258 -12.76 <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6872 on 221 degrees of freedom
## Multiple R-squared: 0.9806, Adjusted R-squared: 0.9799
## F-statistic: 1398 on 8 and 221 DF, p-value: < 2.2e-16
```

3.3.1 Interpretation of F-statistic, P-value and R-squared value

From the F-statistic and p-value of our **ImMOV** model, we can reject the null hypothesis that our predictor variables have no effect on MoV. There is strong evidence to conclude there is a relationship between our predictor and response variables.

Our model explains approximately 97.9% of the variation in WINS using eFG, TOV, OREB, FTF, OPPEFG, OPPTOV, DREB and OPPFTF as predictors.

3.3.2 Interpretation of coefficients

- An increase of 1 percentage point of erg is associated with an average increase of 1.42 Mov, holding all else equal
- An increase of 1 percentage point of TOV is associated with an average decrease of 1.35 MOV, holding all else equal

- · An increase of 1 percentage point of OREB is associated with an average increase of 0.44 MOV, holding all else equal
- . An increase of 1 percentage point of FTF is associated with an average increase of 0.28 Mov, holding all else equal
- An increase of 1 percentage point of OPPeFG is associated with an average decrease of 1.42 MOV, holding all else equal
- An increase of 1 percentage point of OPPTOV is associated with an average increase of 1.19 MOV, holding all else equal
- An increase of 1 percentage point of DREB is associated with an average increase of 0.42 MOV, holding all else equal
- An increase of 1 percentage point of OPPFTF is associated with an average decrease of 0.28 Mov, holding all else equal

3.3.3 ANOVA table and confidence interval

```
anova(lmMOV)
## Analysis of Variance Table
##
## Response: MOV
##
      Df Sum Sq Mean Sq F value
                                       Pr(>F)
           1 2301.63 2301.63 4874.378 < 2.2e-16 ***
           1 44.78 44.78 94.836 < 2.2e-16 ***
           1 709.96 709.96 1503.552 < 2.2e-16 ***
           1 84.40 84.40 178.739 < 2.2e-16 ***
## OPPeFG 1 1742.09 1742.09 3689.399 < 2.2e-16 ***
## OPPTOV
           1 164.26 164.26 347.865 < 2.2e-16 ***
            1 157.77 157.77 334.123 < 2.2e-16 ***
## DREB
## OPPFTF 1 76.89 76.89 162.835 < 2.2e-16 ***
## Residuals 221 104.35 0.47
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
confint(lmMOV)
```

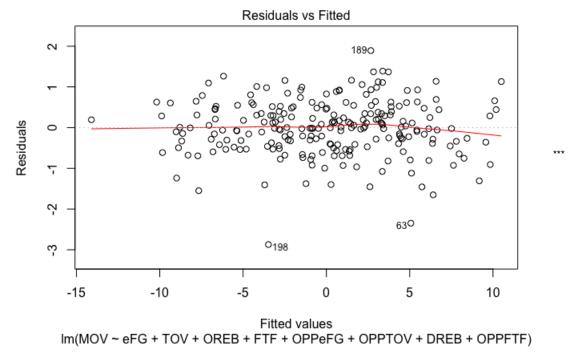
```
2.5 %
## (Intercept) -45.0893055 -32.6133072
             1.3881997
                        1.4828921
## eFG
## TOV
             -1.4300169 -1.2166637
             0.3992758 0.4765695
## OREB
             0.2565094 0.3362945
## FTF
## OPPeFG
             -1.4690670 -1.3709711
## OPPTOV
             1.0794625 1.2615183
## DREB
              0.3351639 0.4360461
## OPPFTF
             -0.3326050 -0.2436138
```

From the ANOVA table, we can see that the F-statistic for each predictor variable is significant and adds prediction power to our model. All included variables are relevant to our model.

3.3.4 Checking to see if assumptions of linear regression for the **ImMOV** model are reasonably met

1. The relationship is linear

```
plot(lmMOV, 1)
```



Our residual plot is mostly flat. There are some points that skew the line, such as observation 63, 189 and 198, but overall the relationship is linear.

2. Independence of error terms

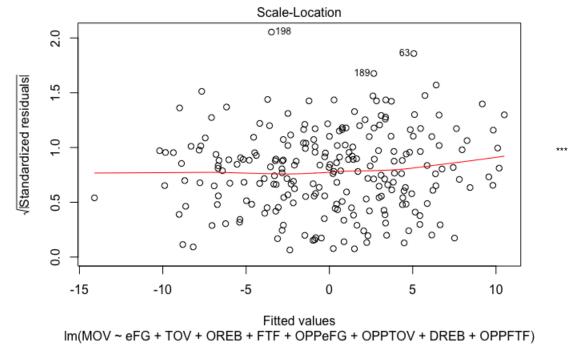
```
durbinWatsonTest(lmMOV)

## lag Autocorrelation D-W Statistic p-value
## 1 -0.01259298 2.014365 0.886
## Alternative hypothesis: rho != 0
```

We fail to reject the null hypothesis that the error terms are not autocorrelated. We have met the independence assumption.

3. Variation of observations' error terms is constant

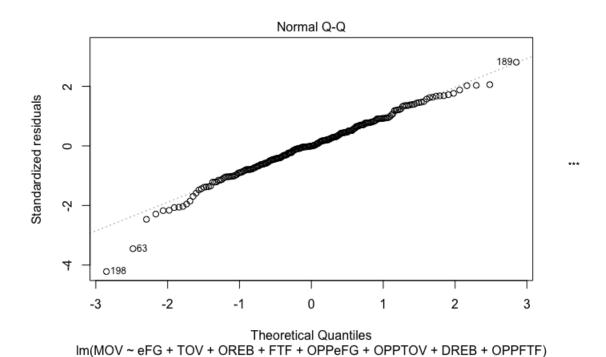
```
plot(lmMOV, 3)
```



The homoskedasticity assumption is met. In the scale-location plot we see see points that are normally spread, there is no evidence of heteroskedasticity.

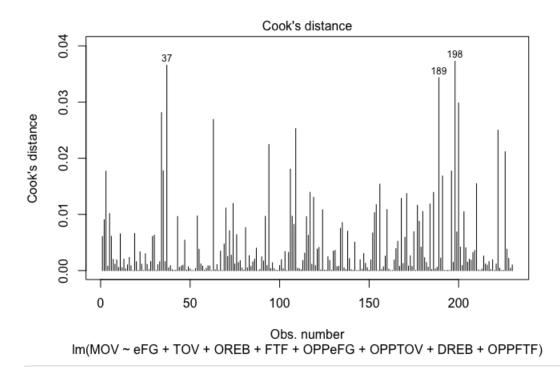
4. Values are normally distributed

plot(lmMOV, 2)

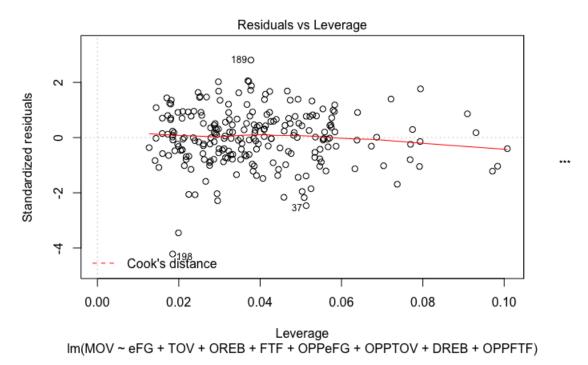


From the Q-Q plot, we can see that points are very close to the diagonal line.

5. Outliers and Influential Points



plot(lmMOV, 5)



We can look for outliers in our data using Cook's distance. Points 48, 138 and 188 are identified as outliers. These three points also appear in our residual vs leverage graph as potential points of interest.

6. Multicolinearity

vif(lmMOV)

```
## eFG TOV OREB FTF OPPEFG OPPTOV DREB OPPFTF
## 1.339503 1.229970 1.470495 1.343472 1.120709 1.252527 1.632149 1.341740
```

All of our VIF values are below 4. There is no multicollinearity in our data.

3.3.5 Linear Regression Models Summary

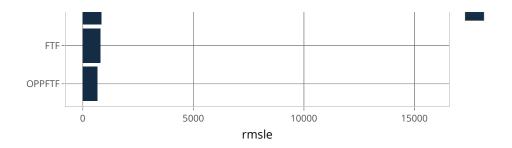
From the ANOVA tables in 3.2.3 and 3.3.3, we can see that all four factors that Dean proposed are significant at threshold of p-value <0.01 for predicted both wins and Mov

3.4 Random Forest Model for WINS

```
set.seed(10)
rfWINS <- randomForest(formula = WINS ~ eFG + TOV + OREB +
FTF + OPPeFG + OPPTOV + DREB +
OPPFTF, type = prob, mtry = 8, ntree = 100, data = ff_train)
rfWINS</pre>
```

Importance of predictors



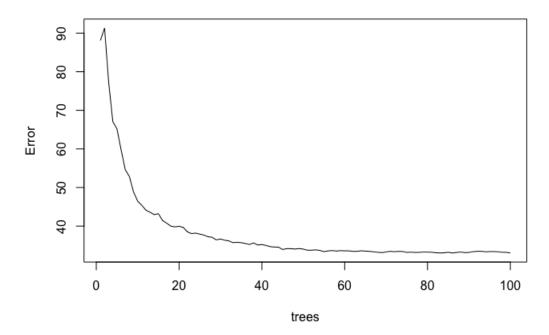


```
predicted_rfWINS = predict(rfWINS, ff_test)
predicted_rfWINS
```

```
2
                         3
                                          5
## 52.99617 43.78906 49.08317 38.65268 37.72000 45.11800 28.95717 29.78150
               10
                        11
                              12
                                         13
                                                14
                                                         15
## 51.47067 53.47717 51.63918 43.44083 42.54587 42.30806 45.12500 28.14433
       17
               18
                        19
                                 20
                                         21
                                                 22
                                                          23
## 47.06783 37.83912 47.22656 44.46980 33.08417 33.71457 29.72049 28.43883
       25
               26
                        27
                                 28
                                        29
                                                 30
                                                         31
## 30.16586 52.91377 49.44632 48.92538 40.72462 41.91182 46.92298 33.05839
       33
              34
                       35
                                 36
                                       37
                                                 38
                                                      39
## 33.57583 26.20969 54.45130 44.35616 45.35495 43.04947 46.37922 43.84762
       41
                42
                        43
                                 44
                                        45
                                                 46
                                                       47
## 40.67612 47.07255 39.06687 31.40869 33.75418 18.75129 43.79155 45.90477
       49
                50
                        51
                                 52
                                         53
                                                  54
                                                          55
## 41.04936 34.39777 29.19262 26.48208 52.36279 55.21718 40.48489 45.21512
        57
                58
                        59
                                 60
                                         61
                                                  62
                                                          63
## 46.49743 46.92894 36.20323 26.10243 24.61030 21.00752 29.63551 58.56314
       65
                66
                        67
                                 68
                                         69
                                                 70
                                                          71
## 47.80291 52.17222 47.25449 45.79075 42.71789 40.44831 28.14403 42.04495
       7.3
              74
                       75
                                 76
                                        77
                                                 78
##
                                                          79
## 40.24382 26.79997 35.26717 57.66292 34.47036 37.69035 39.97372 23.62953
       81
              82
                    83
                                 84
                                      85
                                                 86
                                                        87
## 59.27665 47.52023 47.67037 48.13347 33.84547 37.82014 32.72050 40.61161
       89
                90
                    91
                                 92
                                    93
                                             94
                                                          95
## 30.91467 31.51412 28.12411 53.57304 49.35518 52.68319 48.66120 51.65824
  97
                98
                        99
## 42.09069 42.29389 40.42742 38.33552
```

plot(rfWINS)

rfWINS

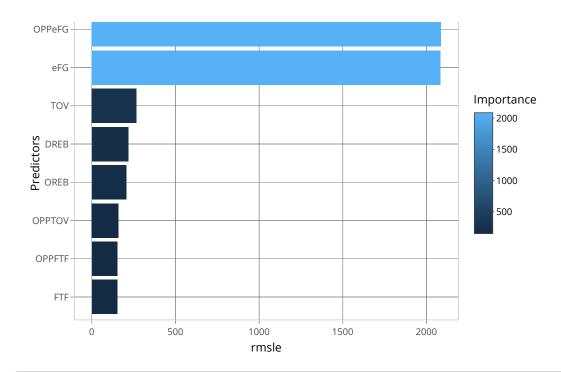


3.5 Random Forest Model for MOV

```
set.seed(12)
rfMOV <- randomForest(
  formula = MOV ~ eFG + TOV + OREB +
  FTF + OPPeFG + OPPTOV + DREB +
  OPPFTF, type = prob, mtry = 5, ntree = 100, data = ff_train)
rfMOV</pre>
```

```
##
## Call:
## randomForest(formula = MOV ~ eFG + TOV + OREB + FTF + OPPeFG +
                                                                        OPPTOV + DREB + OPPFTF, data = ff_train,
type = prob, mtry = 5,
                            ntree = 100)
                  Type of random forest: regression
##
##
                        Number of trees: 100
## No. of variables tried at each split: 5
##
##
             Mean of squared residuals: 4.322187
##
                       % Var explained: 81.54
```

Importance of predictors

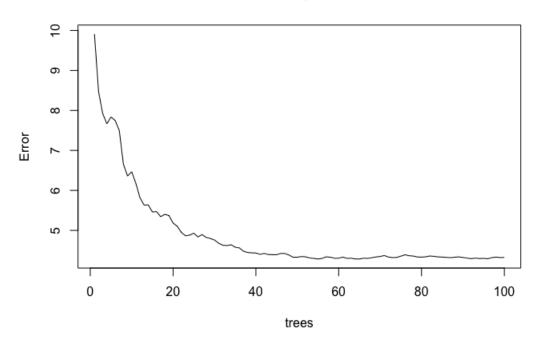


predicted_rfMOV = predict(rfMOV, ff_test)
predicted_rfMOV

```
2
                                      3
    4.70411333
                0.44492000
                            1.75897667 -0.39023667 -0.65458833 1.04610167
             7
##
                         8
                                     9
                                                10
                                                      11
   -5.13784333 -4.64947000
                            3.01612000 5.37060667
                                                      4.40723667
##
            13
                        14
                                     15
                                                 16
                                                              17
    0.47341500
                0.22988500
                            1.07280833 -4.98806500
                                                      3.65733833 -1.29342000
            19
                        20
                                     21
                                                 22
                                                              23
                3.13646333 -2.43330500 -2.40070500 -4.84224167 -4.69837167
    1.10023500
##
            25
                        26
                                     27
                                                 28
                                                              29
               4.81272833 1.17967500 1.60443667 -0.22607167 -0.08688167
##
   -3.50574167
##
                                                 34
                                                              35
            31
                        32
                                     33
    0.83038833 \ -3.54648833 \ -2.97943667 \ -5.30107000 \ \ 4.92724000 \ \ 1.06191667
##
            37
                        38
                                     39
                                                 40
                                                              41
               0.84503667 1.74553667 1.06317167
    1.81877000
                                                      0.26851667
##
            43
                        44
                                     45
                                                 46
                                                              47
   -1.31293167 -3.61436833 -3.26125667 -8.15337167
                                                      0.39171667
                        50
                                     51
    1.53426167 -1.87378167 -4.02905333 -5.35198167
                                                      2.85475333
                        56
                                     57
                                                 58
                                                              59
   -0.58595167
               1.51368000
                            2.33989333
                                        1.82947500 -1.72388833 -4.91215667
##
            61
                        62
                                     63
                                                 64
                                                              65
##
   -5.93165167 -6.72481333 -4.62892333
                                         5.73980333
                                                      2.55964000
##
            67
                        68
                                     69
                                                 70
                                                              71
##
    1.96638333
               2.08326333
                            0.36124667 - 0.63832833 - 4.68654167 - 0.27848833
##
            73
                        74
                                     75
                                                 76
                                                              77
##
    1.12000667 -5.47748000 -0.94587833
                                        6.69601333 -2.18986333 -1.58901667
##
            79
                        80
                                     81
                                                 82
                                                              83
##
   -0.38305167 -5.47842833
                            7.41262333
                                         1.97604167
                                                      2.18967167
            85
                        86
                                     87
                                                 88
                                                              89
   -2.43702333
               0.16829167 -3.36714667
                                        0.41332833 -3.02051667 -4.18188333
##
            91
                        92
                                     93
                                                 94
                                                              95
##
   -4.99639167 \quad 3.64797500 \quad 2.23762167 \quad 2.39703500 \quad 1.67150667 \quad 1.58936167
##
            97
                        98
                                     99
                                                100
   0.67753167 -0.05850833 -0.20965833 -0.78329167
```

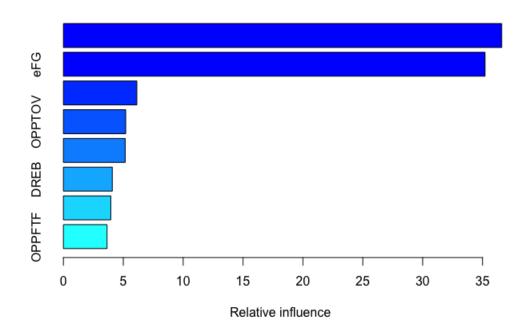
plot(rfMOV)





3.6 Gradient Boosting Machine Model for wins

```
set.seed (14)
gbWINS = gbm(formula = WINS ~ eFG + TOV + OREB +
FTF + OPPeFG + OPPTOV + DREB +
OPPFTF, data = ff_train, distribution =
"gaussian", n.trees = 5000, interaction.depth = 5)
summary(gbWINS)
```

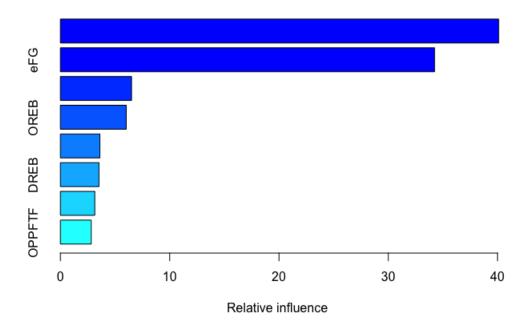


```
predicted_gbWINS = predict(gbWINS, ff_test, n.trees = 5000)
predicted_gbWINS
```

```
## [1] 54.44049 45.89374 54.60644 40.68630 41.00000 43.93951 28.02161 23.42374
## [9] 55.74394 62.25554 52.41642 38.39444 44.04356 47.60842 45.25060 22.51185
## [17] 47.30781 40.48799 43.94208 41.38360 33.91815 37.47008 25.40001 29.22159
## [25] 26.60773 57.48890 47.36590 49.95835 39.05216 45.59009 51.32060 33.49707
## [33] 32.30885 22.26007 54.97178 51.20055 48.93977 47.41121 50.82429 41.43935
## [41] 42.49904 48.98574 36.50340 27.24876 28.30373 11.34907 49.43297 49.14642
## [49] 42.16120 36.78434 30.01739 22.34322 53.87502 57.81247 42.34259 45.60753
## [57] 46.49116 45.47086 40.02824 25.56844 27.17362 17.77033 25.17434 62.65383
## [65] 45.06780 52.47862 46.44692 49.74115 44.18167 40.26226 26.53433 34.56317
## [73] 33.89384 26.78798 30.22813 60.06000 36.13741 36.93802 35.28440 20.91014
## [81] 62.66122 54.20535 45.18643 51.02636 38.68406 37.89987 35.22453 30.79658
## [89] 31.06456 22.50787 25.16126 56.68220 48.24911 48.58269 46.49725 45.94790
## [97] 40.09052 44.84519 38.48797 41.59021
```

3.7 Gradient Boosting Machine Model for MOV

```
set.seed (16)
gbMOV = gbm(formula = MOV ~ eFG + TOV + OREB +
FTF + OPPeFG + OPPTOV + DREB +
OPPFTF, data = ff_train, distribution =
"gaussian", n.trees = 5000, interaction.depth = 5)
summary(gbMOV)
```



```
var
                   rel.inf
## OPPeFG OPPeFG 40.107700
             eFG 34.240521
## TOV
             TOV
                  6.511891
## OREB
            OREB
                  6.021583
   OPPTOV OPPTOV
## DREB
            DREB
                 3.531286
## FTF
             FTF
                  3.147438
## OPPFTF OPPFTF 2.822075
```

```
predicted_gbMOV = predict(gbMOV, ff_test, n.trees = 5000)
predicted_gbMOV
```

```
##
     [1]
          5.41401242
                      2.98954023
                                   4.04905088 -1.66334388 -0.24000078
##
    [6]
          0.11639329
                      -6.03584089 -5.04345189
                                                5.91855454
                                                             8.05355326
##
    [11]
          3.76276710
                       0.42580245
                                   0.96263642
                                                2.46004525
                                                             1.20142592
##
         -6.85072444
                       2.68593527
                                   -1.53143981
                                                1.54774574
                                                            -0.03515514
##
         -2.44130746 -0.78319931
                                  -5.38327137
                                               -4.27373614 -5.27720429
   [21]
                                               -0.81257916
   [26]
          5.02091121
                      1.60751046
                                   3.27731656
                                                             0.71445214
##
   [31]
          0.96450630 -4.18510032
                                  -2.38329799
                                               -7.36643211
                                                             6.04378400
##
   [36]
          2.18513304
                      3.11570476
                                   2.78447763
                                                1.89781985
                                                             0.10434255
##
   [41]
        -0.48908900
                      0.81382123
                                   1.53105953 -5.38207616 -3.96597397
##
   [46] -11.41483236
                      2.35197568
                                   3.47247108
                                                3.05802638 -1.43422693
         -2.52666049 -4.78003448
                                   4.82989906
   [51]
                                                4.76372686
                                                             0.39456892
   [56]
          2.16179877
                      1.36581876
                                  1.60101384 -0.54169606 -5.30367695
   [61]
         -4.02114242 -7.67902734 -5.69591678
                                               6.07631960
                                                            3.84908966
   [66]
          5.33339226
                      1.69472122
                                  2.80727646
                                               -0.80714746 -1.03379846
   [71]
         -4.36500136 -1.92826525 -0.77525729
                                               -5.26684295 -2.17438589
          7.09703563 -1.08193043
   [76]
                                  -0.31680183
                                              -1.56470605 -5.55053625
   [81]
          8.11648806
                      3.64261673
                                   1.57617387
                                                2.40660078 -0.91387736
   [86]
         -0.68600691 -2.50503055
                                  -3.61996388 -3.00463098 -5.68999341
         -7.78111208
                      4.42632578
                                   1.42397457
                                                3.16395713
                                                            1.71156017
   [91]
          2.59497571 -0.04093237 -0.153333345 -1.25801459 -0.44904712
   [96]
```

3.8 Comparison of models

Both wins and mov were predicted using three different models:

- 1. Multiple Linear Regression
- 2. Random Forest
- 3. Gradient Boosting Machine

For the final model, I decided to use multiple linear regression. The **ImWINS** and **ImMOV** models performed better than the **rfWINS**, **rfMOV**, **gbWINS**, and **gbMOV** models. Random forest and gradient boosting may have performed better, had I tweaked and tested different hyperparameter configurations.

Another reason I picked multiple linear regression as the final model type, is that it is a simpler model, which makes it easier to explain. In this case, the other models are black-box models, which can potentially provide more accuracy, but at the expense of explainability compared to a model like multiple linear regression.

Our ImWINS and ImMOV models can be stated mathematically as:

```
WINS = -53.185 + 3.899 * eFG\% - 3.392 * TOV\% + 1.111 * OREB\% + 0.708 * FTF - 3.765 * OPPeFG\% + 2.902 * OPPTOV\% + 0.887 * DREB\% - 0.729 * OPPFTF + <math>\epsilon
MOV = -38.851 + 1.436 * eFG\% - 1.323 * TOV\% + 0.438 * OREB\% + 0.296 * FTF - 1.420 * OPPeFG\% + 1.170 * OPPTOV\% + 0.386 * DREB\% - 0.288 * OPPFTF + <math>\epsilon
```

In the next section, we will evaluate our multiple linear regression model

Comparison of actual values with predicted values and metrics

4.1 Creating a dataframe comparing predicted wins and predicted MOV using our ImWINS and ImMOV models fit the test data, with the observed values of WINS and observed values of MOV

```
predicted_wins = predict(lmWINS, ff_test)
pm = (predict(lmMOV, ff_test))

observed_vs_predicted <- data.frame(predicted_wins)
observed_vs_predicted$observed_wins <- ff_test[["WINS"]]
observed_vs_predicted$predicted_mov <- pm
observed_vs_predicted$observed_mov <- ff_test[["MOV"]]

colnames(observed_vs_predicted) <- c("Predicted Wins", "Observed Wins", "Predicted Margin of Victory", "Observed Margin of Victory")

observed_vs_predicted <- observed_vs_predicted[, c(2, 1, 4, 3)]
observed_vs_predicted</pre>
```

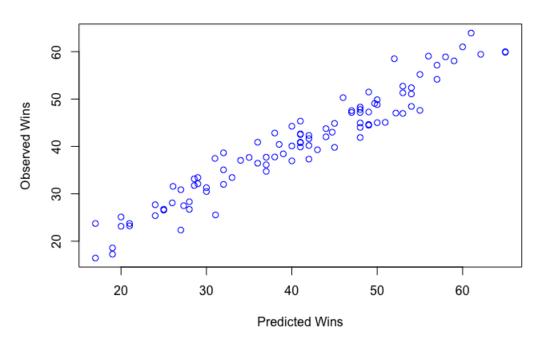
##				Observed Margin of Victory
##		60.00000	61.01426	8.87
##		54.00000	48.44225	3.95
##		49.00000	51.48235	4.44
##		42.00000 39.00000	41.65049	0.71
##		32.00000	38.43315 35.05902	-0.23 -2.90
##		19.00000	17.24568	-9.34
##		19.00000	18.57312	-9.61
##		65.00000	59.85960	8.48
##		59.00000	58.05547	7.78
##		58.00000	58.90024	5.98
##		49.00000	44.46868	2.60
##	13	48.00000	44.94865	1.38
##	14	47.00000	47.55992	2.89
##	15	44.00000	42.03005	-0.30
##	16	27.00000	22.34545	-7.04
##		53.00000	46.97189	2.63
##		43.00000	39.29831	0.49
##		41.00000	45.34191	1.06
##		41.00000	40.81297	-0.52
##		36.00000 31.00000	40.88568 37.46738	0.20
##		28.00000	26.72349	-1.11 -5.70
##		24.00000	27.68881	-5.63
##		20.00000	25.10779	-6.73
##		57.00000	54.16213	6.00
##	27	48.00000	47.17588	2.72
##	28	48.00000	48.30282	1.65
##	29	42.00000	40.20250	-0.30
##	30	41.00000	42.65048	0.20
##	31	40.00000	44.25392	1.79
##		33.00000	33.44204	-4.18
##		30.00000	30.47749	-3.79
##		21.00000	23.73058	-7 . 35
##		55.00000 55.00000	55.19093 47.61074	6.20 3.24
##		50.00000	49.85750	2.90
##		49.00000	47.26998	3.07
##		45.00000	44.86464	2.18
##		41.00000	42.50803	0.43
##		40.00000	40.09710	0.16
##	42	38.00000	42.82806	0.22
##	43	32.00000	38.64799	-1.00
##	44	30.00000	31.29008	-3.55
##		29.00000	32.11757	-3.71
##		17.00000	16.43611	-9.32
##		50.00000	45.01407	1.57
##		48.00000	47.80338	2.63
##		48.00000	44.02758	1.85 -0.79
##		37.00000 29.00000	37.72357 33.43477	-3.66
##		25.00000	26.74820	-4.46
##		57.00000	57.15153	5.09
##		56.00000	59.06653	6.45
##		49.00000	44.64585	1.78
##	56	45.00000	39.80145	0.32
##	57	44.00000	43.75584	0.40
##	58	41.00000	40.88056	-0.22
##		38.00000	37.77610	-1.50
##		28.00000	28.31915	-4.88
##		24.00000	25.38363	-4.68
##		21.00000	23.23027	-9.23
##		20.00000	23.14253	-6 . 99
##		62.12121 52.18182	59.43994 47.04342	7.17 3.30
##		50.93939	45.07695	1.42
" "	50	30.73739	45.07033	1.12

## 67			
	49.69697	49.07492	2.56
## 68	44.72727	43.00650	0.95
## 69	41.00000	40.82477	-0.24
## 70	38.51515	40.42987	0.27
## 71	31.06061	25.54173	-4.79
## 72	28.57576	33.16422	-3.41
## 73	28.57576	31.75337	-3.30
## 74	27.33333	27.48641	-5.68
## 75	26.09091	31.56932	-3.76
## 76	52.00000	58.51056	5.46
## 77	48.00000	41.88999	1.52
## 78	42.00000	42.32496	0.78
## 79	37.00000	36.12199	-1.07
## 80	17.00000	23.72609	-6.63
## 81	61.00000	63.93179	6.52
## 82	53.00000	52.72042	4.66
## 83	53.00000	51.31638	4.09
## 84	50.00000	48.82607	3.30
## 85 ## 86	42.00000	37.32024	-0.37
## 86 ## 87	40.00000 37.00000	36.95088 34.73534	-1.51 -2.46
## 88	32.00000	31.99379	-3.01
## 89	27.00000	30.85953	-3.90
## 90	26.00000	28.09346	-3.60
## 91	25.00000	26.54171	-4.37
## 92	65.00000	59.99583	7.66
## 93	54.00000	51.07413	3.76
## 94	54.00000	52.38515	3.41
## 95	47.00000	47.16779	1.57
## 96	46.00000	50.29970	1.93
## 97	41.00000	39.88191	-0.28
## 98	36.00000	36.45967	-1.11
## 99	35.00000	37.67511	-1.27
## 100	34.00000	37.06367	-1.09
##	Predicted Margin	-	
## 1		7.5019316	
## 2		2.7918936	
## 3		3.8579222	
## 4		0.1291505	
## 5 ## 6		-0.8615088 -2.4180218	
## 7		-2.4100210	
"""		_9 0182489	
## 8		-9.0182489 -8.7940497	
## 8 ## 9		-8.7940497	
## 9		-8.7940497 7.2595134	
		-8.7940497	
## 9 ## 10		-8.7940497 7.2595134 6.3801864	
## 9 ## 10 ## 11		-8.7940497 7.2595134 6.3801864 6.3904133	
## 9 ## 10 ## 11 ## 12		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391	
## 9 ## 10 ## 11 ## 12 ## 13		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198	
## 9 ## 10 ## 11 ## 12 ## 13 ## 14		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804	
## 9 ## 10 ## 11 ## 12 ## 13 ## 14 ## 15 ## 16 ## 17		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235	
## 9 ## 10 ## 11 ## 12 ## 13 ## 14 ## 15 ## 16 ## 17 ## 18		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467	
## 9 ## 10 ## 11 ## 12 ## 13 ## 14 ## 15 ## 16 ## 17 ## 18 ## 19		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129	
## 9 ## 10 ## 11 ## 12 ## 13 ## 14 ## 15 ## 16 ## 17 ## 18 ## 19 ## 20		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054	
## 9 ## 10 ## 11 ## 12 ## 13 ## 14 ## 15 ## 16 ## 17 ## 18 ## 19 ## 20 ## 21		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854	
## 9 ## 10 ## 11 ## 12 ## 13 ## 14 ## 15 ## 16 ## 17 ## 18 ## 19 ## 20 ## 21 ## 22		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486	
## 9 ## 10 ## 11 ## 12 ## 13 ## 14 ## 15 ## 16 ## 17 ## 18 ## 19 ## 20 ## 21 ## 22 ## 23		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226	
## 9 ## 10 ## 11 ## 12 ## 13 ## 14 ## 15 ## 16 ## 17 ## 18 ## 20 ## 21 ## 22 ## 23 ## 24		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247	
## 9 ## 10 ## 11 ## 12 ## 13 ## 14 ## 15 ## 16 ## 17 ## 18 ## 20 ## 21 ## 22 ## 23 ## 24		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247 -6.2136536	
## 9 ## 10 ## 11 ## 12 ## 14 ## 15 ## 16 ## 17 ## 18 ## 20 ## 21 ## 22 ## 23 ## 25 ## 26		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247 -6.2136536 4.9859142	
## 9 ## 10 ## 11 ## 12 ## 13 ## 14 ## 15 ## 16 ## 17 ## 18 ## 20 ## 21 ## 22 ## 23 ## 24		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247 -6.2136536	
## 9 ## 10 ## 11 ## 12 ## 14 ## 15 ## 16 ## 17 ## 20 ## 21 ## 22 ## 23 ## 25 ## 26 ## 27		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247 -6.2136536 4.9859142 2.5351641	
## 9 ## 10 ## 11 ## 12 ## 14 ## 15 ## 16 ## 17 ## 20 ## 21 ## 22 ## 23 ## 25 ## 26 ## 27 ## 28		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247 -6.2136536 4.9859142 2.5351641 2.7661311	
## 9 ## 10 ## 11 ## 12 ## 13 ## 14 ## 15 ## 16 ## 17 ## 20 ## 21 ## 22 ## 23 ## 24 ## 25 ## 28 ## 29		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247 -6.2136536 4.9859142 2.5351641 2.7661311 -0.2998492	
## 9 ## 10 ## 11 ## 12 ## 13 ## 14 ## 15 ## 16 ## 17 ## 20 ## 21 ## 22 ## 24 ## 25 ## 24 ## 25 ## 28 ## 30		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247 -6.2136536 4.9859142 2.5351641 2.7661311 -0.2998492 0.6139207	
## 9 ## 10 ## 11 ## 12 ## 13 ## 14 ## 15 ## 16 ## 17 ## 20 ## 21 ## 22 ## 25 ## 25 ## 25 ## 29 ## 30 ## 31		-8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247 -6.2136536 4.9859142 2.5351641 2.7661311 -0.2998492 0.6139207 1.3555547	

## 34	-6.5288869
## 35	5.3778730
## 36	2.7033167
## 37	3.2481478
## 38	2.3854624
## 39	1.5309146
## 40	0.5272282
## 41	-0.3470279
## 42	0.7780669
## 43	-0.7966378
## 44	-3.6334464
## 45	-3.2572235
## 46	-9.4193199
## 47	1.6106493
## 48	2.7168844
## 49	1.3638212
## 50	-1.3144823
## 51	-2.7455177
## 52	-5.4021469
## 53	6.0919311
## 54	6.7979839
## 55	1.4475761
## 56	-0.3844359
## 57	0.8080249
## 58	-0.2358748
## 59	-1.2575482
## 60	-4.9433063
## 61	-5.8192197
## 62	-6.5466151
## 63	
	-6.9946522
## 64	6.9045917
## 65	2.4790596
## 66	1.5072616
## 67	3.0288613
## 68	0.7386895
## 69	-0.2732132
## 70	-0.2543372
## 71	-5.8070154
## 72	-3.4383673
## 73	-3.5368930 5.1036000
## 74	-5.1236880
## 75	-3.6180666
## 76	6.6350172
## 77	0.5130691
## 78	0.4074916
## 79	-1.8627472
## 80	-6.5704605
## 81	8.6455665
## 82	4.4223505
## 83	3.9850735
## 84	2.9621934
## 85	-1.3352777
## 86	-1.4920864
## 87	-2.4705996
## 88	-3.5617876
## 89	-4.0534494
## 90	-4.9777262
## 91	-5.6076159
## 92	7.2597455
## 93	3.7625920
## 94	4.3145737
## 95	2.2593586
## 96	3.2245813
## 97	-0.5246057
## 98	-1.8442554
## 99	-1.2508650
## 100	-1.2999283
"" 100	2.233,200

```
plot(observed_vs_predicted$"Observed Wins", observed_vs_predicted$"Predicted Wins",
    main="Observed vs. Predicted Wins",
    xlab="Predicted Wins",
    ylab="Observed Wins",
    col="blue")
```

Observed vs. Predicted Wins



4.2 Creating a new dataframe with the team identifier, and their respective observed and predicted WINS and MOV

```
id_observed_vs_predicted <- observed_vs_predicted
id_observed_vs_predicted$Team <- ff_test$Team
id_observed_vs_predicted$Season <- ff_test$Season
id_observed_vs_predicted <- id_observed_vs_predicted[, c(5, 6, 1, 2, 3, 4)]
id_observed_vs_predicted</pre>
```

##		Team		Se	eason	Observed Wins	Predicted Wins
##	1	Milwaukee Bucks	2018	_	2019	60.00000	61.01426
##	2	Denver Nuggets				54.00000	48.44225
##		Boston Celtics				49.00000	51.48235
##		Brooklyn Nets				42.00000	41.65049
##		•				39.00000	
		Charlotte Hornets					38.43315
##		Washington Wizards				32.00000	35.05902
##		Cleveland Cavaliers				19.00000	17.24568
##	8	Phoenix Suns	2018	-	2019	19.00000	18.57312
##	9	Houston Rockets*	2017	-	2018	65.00000	59.85960
##	10	Toronto Raptors*	2017	-	2018	59.00000	58.05547
##	11	Golden State Warriors*	2017	_	2018	58.00000	58.90024
##	12	Portland Trail Blazers*	2017	_	2018	49.00000	44.46868
##	13	Indiana Pacers*	2017	_	2018	48.00000	44.94865
##	14	San Antonio Spurs*	2017	_	2018	47.00000	47.55992
##	15	Milwaukee Bucks*				44.00000	42.03005
##		Chicago Bulls				27.00000	22.34545
##		Boston Celtics*				53.00000	46.97189
##		Memphis Grizzlies*				43.00000	39.29831
		-					
##		Miami Heat				41.00000	45.34191
##		Portland Trail Blazers*				41.00000	40.81297
##		Charlotte Hornets				36.00000	40.88568
##		Minnesota Timberwolves	2016	-	2017	31.00000	37.46738
##	23	Philadelphia 76ers	2016	-	2017	28.00000	26.72349
##	24	Phoenix Suns	2016	-	2017	24.00000	27.68881
##	25	Brooklyn Nets	2016	-	2017	20.00000	25.10779
##	26	Cleveland Cavaliers*	2015	_	2016	57.00000	54.16213
##	27	Charlotte Hornets*	2015	_	2016	48.00000	47.17588
##	28	Miami Heat*	2015	_	2016	48.00000	48.30282
##	29	Dallas Mavericks*	2015	_	2016	42.00000	40.20250
##	30	Houston Rockets*	2015	_	2016	41.00000	42.65048
##	31	Utah Jazz				40.00000	44.25392
##		Milwaukee Bucks				33.00000	33.44204
##		New Orleans Pelicans				30.00000	30.47749
##		Brooklyn Nets				21.00000	23.73058
##		San Antonio Spurs*				55.00000	55.19093
##		Memphis Grizzlies*				55.00000	47.61074
		-					
##		Dallas Mavericks*				50.00000	49.85750
##		Toronto Raptors*				49.00000	47.26998
##		Oklahoma City Thunder				45.00000	44.86464
##		Milwaukee Bucks*				41.00000	42.50803
##		Boston Celtics*				40.00000	40.09710
##		Utah Jazz				38.00000	42.82806
##		Detroit Pistons				32.00000	38.64799
##	44	Denver Nuggets	2014	-	2015	30.00000	31.29008
##	45	Sacramento Kings	2014	-	2015	29.00000	32.11757
##	46	New York Knicks	2014	-	2015	17.00000	16.43611
##	47	Memphis Grizzlies*	2013	-	2014	50.00000	45.01407
##	48	Toronto Raptors*	2013	-	2014	48.00000	47.80338
##	49	Chicago Bulls*	2013	_	2014	48.00000	44.02758
##	50	New York Knicks	2013	_	2014	37.00000	37.72357
##		Detroit Pistons	2013	_	2014	29.00000	33.43477
##		Boston Celtics				25.00000	26.74820
##		Denver Nuggets*				57.00000	57.15153
##		Los Angeles Clippers*				56.00000	59.06653
##						49.00000	
		Brooklyn Nets*					44.64585
##		Chicago Bulls*				45.00000	39.80145
##		Atlanta Hawks*				44.00000	43.75584
##		Boston Celtics*				41.00000	40.88056
##		Milwaukee Bucks*				38.00000	37.77610
##	60	Sacramento Kings	2012	-	2013	28.00000	28.31915
##	61	Cleveland Cavaliers	2012	-	2013	24.00000	25.38363
##	62	Charlotte Bobcats	2012	-	2013	21.00000	23.23027
##	63	Orlando Magic	2012	-	2013	20.00000	23.14253
##	64	San Antonio Spurs*	2011	-	2012	62.12121	59.43994
##	65	Indiana Pacers*	2011	-	2012	52.18182	47.04342
##	66	Los Angeles Lakers*	2011	_	2012	50.93939	45.07695
		=					

##	67	Los Angeles Clippers*	201	1 -	2012	49.69697	49.07492
##	68	Dallas Mavericks*	201	1 -	2012	44.72727	43.00650
##	69	Phoenix Suns	201	1 -	2012	41.00000	40.82477
##	70	Milwaukee Bucks	201	1 –	2012	38.51515	40.42987
##		Detroit Pistons				31.06061	25.54173
##		Golden State Warriors				28.57576	
							33.16422
##		Toronto Raptors				28.57576	31.75337
##		Sacramento Kings				27.33333	27.48641
##	75	New Orleans Hornets	201	1 –	2012	26.09091	31.56932
##	76	Orlando Magic*	201	0 –	2011	52.00000	58.51056
##	77	Portland Trail Blazers*	201	0 –	2011	48.00000	41.88999
##	78	New York Knicks*	201	0 –	2011	42.00000	42.32496
##	79	Indiana Pacers*	201	0 –	2011	37.00000	36.12199
##	80	Minnesota Timberwolves	201	0 –	2011	17.00000	23.72609
##		Cleveland Cavaliers*				61.00000	63.93179
##		Atlanta Hawks*				53.00000	52.72042
##		Denver Nuggets*				53.00000	51.31638
##		Portland Trail Blazers*				50.00000	48.82607
##	85	Houston Rockets	200	9 –	2010	42.00000	37.32024
##	86	Memphis Grizzlies	200	9 –	2010	40.00000	36.95088
##	87	New Orleans Hornets	200	9 –	2010	37.00000	34.73534
##	88	Indiana Pacers	200	9 –	2010	32.00000	31.99379
##	89	Philadelphia 76ers	200	9 –	2010	27.00000	30.85953
##		Golden State Warriors				26.00000	28.09346
##		Sacramento Kings				25.00000	26.54171
##		Los Angeles Lakers*				65.00000	59.99583
##		San Antonio Spurs*				54.00000	51.07413
##	94	Denver Nuggets*	200	8 –	2009	54.00000	52.38515
##	95	Atlanta Hawks*	200	8 –	2009	47.00000	47.16779
##	96	Phoenix Suns	200	8 –	2009	46.00000	50.29970
##	97	Chicago Bulls*	200	8 –	2009	41.00000	39.88191
##	98	Indiana Pacers	200	8 –	2009	36.00000	36.45967
##		Charlotte Bobcats				35.00000	37.67511
	100	Milwaukee Bucks		•	2000		
			200	0	2000	3/1 00000	27 06267
	100					34.00000	37.06367
##		Observed Margin of Victo	ory 1			Margin of Victory	37.06367
##	1	Observed Margin of Victo	ory 1			Margin of Victory 7.5019316	37.06367
## ## ##	1 2	Observed Margin of Victor 8	ory 1 .87 .95			Margin of Victory 7.5019316 2.7918936	37.06367
##	1 2	Observed Margin of Victor 8	ory 1			Margin of Victory 7.5019316	37.06367
## ## ##	1 2 3	Observed Margin of Victo 8 3 4	ory 1 .87 .95			Margin of Victory 7.5019316 2.7918936	37.06367
## ## ## ##	1 2 3 4	Observed Margin of Victor 8 3 4 0	ory 1 .87 .95			Margin of Victory 7.5019316 2.7918936 3.8579222	37.06367
## ## ## ##	1 2 3 4 5	Observed Margin of Victors 8 3 4 0 -0	ory 1 .87 .95 .44			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505	37.06367
## ## ## ## ##	1 2 3 4 5 6	Observed Margin of Victors 8 3 4 0 -0 -2	.87 .95 .44 .71 .23			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088	37.06367
## ## ## ## ## ##	1 2 3 4 5 6 7	Observed Margin of Victors 8 3 4 0 -0 -2 -9	.87 .95 .44 .71 .23 .90			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489	37.06367
## ## ## ## ## ##	1 2 3 4 5 6 7 8	Observed Margin of Victor 8 3 4 0 0 -0 -2 -9 -9	.87 .95 .44 .71 .23 .90			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497	37.06367
## ## ## ## ## ##	1 2 3 4 5 6 7 8	Observed Margin of Victor 8 3 4 0 0 -0 -2 -9 8	.87 .95 .44 .71 .23 .90 .34 .61			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134	37.06367
## ## ## ## ## ## ##	1 2 3 4 5 6 7 8 9	Observed Margin of Victor 8 3 4 0 0 -0 -2 -9 8 7	.87 .95 .44 .71 .23 .90 .34 .61 .48			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864	37.06367
## ## ## ## ## ## ##	1 2 3 4 5 6 7 8 9 10	Observed Margin of Victor 8 8 3 4 0 0 -0 -2 -9 8 7 5	.87 .95 .44 .71 .23 .90 .34 .61 .48			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133	37.06367
## ## ## ## ## ## ##	1 2 3 4 5 6 7 8 9 10 11	Observed Margin of Victor 8 8 3 4 0 0 -0 -2 -9 8 7 5 2	ory 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391	37.06367
## ## ## ## ## ## ## ##	1 2 3 4 5 6 7 8 9 10 11 12 13	Observed Margin of Victor 8 8 3 4 0 0 -0 -2 -9 8 7 5 2 1	ory 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14	Observed Margin of Victor 8 8 3 4 0 0 -0 -2 -9 8 7 5 2 1	ory 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391	37.06367
## ## ## ## ## ## ## ##	1 2 3 4 5 6 7 8 9 10 11 12 13 14	Observed Margin of Victor 8 8 3 4 0 0 -0 -2 -9 8 7 5 2 1	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .98 .60 .38 .89			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	Observed Margin of Victor 8 8 3 4 0 0 -0 -2 -9 8 7 5 2 1 2	Dry 1 .87 .95 .44 .71 .23 .99 .34 .61 .48 .78 .98 .60 .38 .89 .30			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804	37.06367
######################################	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	Observed Margin of Victor 8 8 3 4 0 0 -0 -2 -9 9 8 7 5 2 1 2 -0 0 -7	Dry 1 .87 .95 .44 .71 .23 .99 .34 .61 .48 .78 .98 .60 .38 .89 .30			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17	Observed Margin of Victor 8 8 3 4 0 0 -0 0 -2 9 9 8 7 5 2 1 2 -0 0 -7 2	Dry 1.87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98 .30 .04 .63			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690	37.06367
**************************************	1 2 3 4 4 5 6 6 7 8 9 10 11 12 13 14 15 16 17 18	Observed Margin of Victors 8 3 4 0 0 -0 -2 -9 9 9 8 7 5 2 1 1 2 -0 7 2 0 0	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98 .60 .38 .89 .30 .04 .63 .49			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467	37.06367
**************************************	1 2 3 4 4 5 6 6 7 8 9 10 11 12 13 14 15 16 17 18 19	Observed Margin of Victors 8 3 4 4 0 0 -0 -2 2 -9 9 9 8 7 5 2 1 1 2 2 -0 0 -7 2 0 0 1	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .98 .60 .38 .89 .30 .04 .63 .49 .06			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	Observed Margin of Victors 8 3 4 4 0 0 -0 -2 2 -9 8 7 5 2 1 1 2 2 -0 0 -7 2 0 0 1 1 -0 0	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98 .60 .38 .89 .04 .63 .49 .66 .52			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21	Observed Margin of Victors 8 3 4 4 0 0 -0 -2 -2 -9 8 7 5 2 1 1 2 2 -0 0 -7 2 0 0 1 1 -0 0 0	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98 .60 .38 .89 .04 .63 .49 .06 .52 .20			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22	Observed Margin of Victors 8 3 4 0 -0 -2 -9 -9 8 7 5 2 1 2 -0 -7 2 0 1 -0 0 -1	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98 .60 .38 .89 .30 .04 .63 .49 .06 .52 .20 .11			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23	Observed Margin of Victors 8 3 4 0 -0 -2 -9 -9 8 7 5 2 1 2 -0 -7 2 0 1 -0 0 -1 -5	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98 .60 .38 .89 .30 .04 .63 .49 .06 .52 .20 .11 .70			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23	Observed Margin of Victors 8 3 4 0 -0 -2 -9 -9 8 7 5 2 1 2 -0 -7 2 0 1 -0 0 -1	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98 .60 .38 .89 .30 .04 .63 .49 .06 .52 .20 .11 .70			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24	Observed Margin of Victors 8 3 4 0 -0 -2 -9 -9 8 7 5 2 1 2 -0 -7 2 0 1 -0 0 -1 -5	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98 .60 .38 .89 .30 .04 .63 .49 .06 .52 .20 .11 .70 .63			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	Observed Margin of Victors 8 3 4 0 -0 -2 -9 -9 8 7 5 2 1 2 -0 -7 2 0 1 -0 0 -1 -5 -5 -6	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98 .60 .38 .89 .30 .04 .63 .49 .06 .52 .20 .11 .70 .63			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26	Observed Margin of Victors 8 3 4 0 -0 -2 -9 -9 8 7 5 2 1 2 -0 -7 2 0 1 -0 0 -1 -5 -5 -6 6	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98 .60 .38 .89 .30 .04 .63 .49 .06 .52 .20 .11 .70 .63 .73			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2177854 -1.2279486 -5.5494226 -4.9422247 -6.2136536	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27	Observed Margin of Victors 8 3 4 0 -0 -2 -9 -9 8 7 5 2 1 2 -0 -7 2 0 1 -0 0 1 -5 -5 -6 6 6 2	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .98 .60 .38 .89 .30 .04 .63 .49 .06 .52 .20 .11 .70 .63 .73 .00			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2177854 -1.2279486 -5.5494226 -4.9422247 -6.2136536 4.9859142	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 28 28 29 20 20 20 20 20 20 20 20 20 20 20 20 20	Observed Margin of Victors 8 3 4 0 -0 -2 -9 -9 8 7 5 2 1 2 -0 -7 2 0 1 -0 0 1 -1 -5 -5 -6 6 2 1	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .98 .60 .38 .89 .30 .04 .63 .49 .06 .52 .20 .11 .70 .63 .73 .00 .72 .65			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247 -6.2136536 4.9859142 2.5351641 2.7661311	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 29 20 20 20 20 20 20 20 20 20 20 20 20 20	Observed Margin of Victor 8 3 4 0 -0 -2 -9 -9 -9 8 7 5 2 1 2 -0 -7 2 0 1 -0 0 1 -5 -5 -6 6 2 1 -0	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .98 .60 .38 .89 .30 .04 .63 .49 .06 .52 .20 .11 .70 .63 .73 .00 .72 .65 .30			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247 -6.2136536 4.9859142 2.5351641 2.7661311 -0.2998492	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 29 30 30 30 30 30 30 30 30 30 30 30 30 30	Observed Margin of Victor 8 8 3 4 4 0 0 -0 -0 -2 2 -9 9 9 8 7 7 5 2 2 1 1 2 2 -0 0 7 7 2 2 0 0 1 1 -0 0 0 0 -1 1 -5 5 -5 6 6 6 2 2 1 1 -0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98 .60 .38 .89 .30 .04 .63 .49 .06 .52 .20 .11 .70 .63 .73 .00 .72 .65 .30 .20			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247 -6.2136536 4.9859142 2.5351641 2.7661311 -0.2998492 0.6139207	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 31 31 31 31 31 31 31 31 31 31 31 31	Observed Margin of Victor 8 3 4 0 -0 -2 -9 -9 8 7 5 2 1 2 -0 -7 2 0 1 -0 0 1 -5 -5 -6 6 2 1 -0 0 1	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98 .60 .38 .89 .30 .04 .63 .49 .06 .52 .20 .11 .70 .63 .73 .00 .72 .65 .30 .20 .79			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247 -6.2136536 4.9859142 2.5351641 2.7661311 -0.2998492 0.6139207 1.3555547	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 31 31 31 31 31 31 31 31 31 31 31 31	Observed Margin of Victor 8 3 4 0 -0 -2 -9 -9 8 7 5 2 1 2 -0 -7 2 0 1 -0 0 1 -5 -5 -6 6 2 1 -0 0 0 1 -0 0 1 -1 -5 -5 -6 6 2 1 -0 0 0 1 -1 -5 -5 -6 -6 6 2	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98 .60 .38 .89 .30 .04 .63 .49 .06 .52 .20 .11 .70 .63 .73 .00 .72 .65 .30 .20 .79 .18			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247 -6.2136536 4.9859142 2.5351641 2.7661311 -0.2998492 0.6139207 1.3555547 -2.9487069	37.06367
**************************************	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 31 31 31 31 31 31 31 31 31 31 31 31	Observed Margin of Victor 8 3 4 0 -0 -2 -9 -9 8 7 5 2 1 2 -0 -7 2 0 1 -0 0 1 -5 -5 -6 6 2 1 -0 0 0 1 -0 0 1 -1 -5 -5 -6 6 2 1 -0 0 0 1 -1 -5 -5 -6 -6 6 2	Dry 1 .87 .95 .44 .71 .23 .90 .34 .61 .48 .78 .98 .60 .38 .89 .30 .04 .63 .49 .06 .52 .20 .11 .70 .63 .73 .00 .72 .65 .30 .20 .79			Margin of Victory 7.5019316 2.7918936 3.8579222 0.1291505 -0.8615088 -2.4180218 -9.0182489 -8.7940497 7.2595134 6.3801864 6.3904133 1.3311391 1.4623198 2.6330804 0.3325235 -6.9689690 2.1073394 -0.4522467 1.5203129 -0.2177054 0.2178854 -1.2279486 -5.5494226 -4.9422247 -6.2136536 4.9859142 2.5351641 2.7661311 -0.2998492 0.6139207 1.3555547	37.06367

## 34				
## 36	## 34	-7.35	-6.5288869	
## 36	## 35	6.20	5.3778730	
## 37				
## 33				
## 39				
## 40	## 38	3.07	2.3854624	
## 41	## 39	2.18	1.5309146	
## 41	## 40	0.43	0.5272282	
## 42				
## 43				
## 44	## 42	0.22	0.7780669	
## 45	## 43	-1.00	-0.7966378	
## 45		-3.55	-3.6334464	
## 46				
## 47				
## 48		-9.32	-9.4193199	
## 49	## 47	1.57	1.6106493	
## 49	## 48	2.63	2.7168844	
## 50				
## 51				
## 52				
## 53	## 51	-3.66	-2.7455177	
## 53	## 52	-4.46	-5.4021469	
## 54				
## 55				
## 56				
## 57	## 55	1.78		
## 57	## 56	0.32	-0.3844359	
## 58				
## 59				
## 60				
## 61		-1.50	-1.2575482	
## 62	## 60	-4.88	-4.9433063	
## 62	## 61	-4.68	-5.8192197	
## 63				
## 64				
## 65				
## 66	## 64	7.17	6.9045917	
## 67 ## 68	## 65	3.30	2.4790596	
## 67 ## 68				
## 68				
## 69				
## 70	## 68	0.95	0.7386895	
## 71	## 69	-0.24	-0.2732132	
## 71	## 70	0.27	-0.2543372	
## 72				
## 73				
## 74				
## 75	## 73	-3.30	-3.5368930	
## 75	## 74	-5.68	-5.1236880	
## 76 ## 77	## 75			
## 77 ## 78 0.78 0.4074916 ## 79 -1.07 -1.8627472 ## 80 -6.63 -6.5704605 ## 81 6.52 8.6455665 ## 82 4.66 4.4223505 ## 83 4.09 3.9850735 ## 85 -0.37 -1.3352777 ## 86 -1.51 -1.4920864 ## 87 -2.46 -2.4705996 ## 88 -3.01 -3.5617876 ## 89 -3.90 -4.0534494 ## 90 -3.60 -4.9777262 ## 91 -4.37 -5.6076159 ## 92 7.66 7.2597455 ## 93 3.76 3.7625920 ## 94 94 95 1.57 2.2593586 ## 95 ## 96 1.93 3.2245813 ## 97 -0.28 -0.5246057 ## 98 -1.11 -1.8442554 ## 99				
## 78				
## 79	## 77	1.52	0.5130691	
## 79	## 78	0.78	0.4074916	
## 80				
## 81 6.52 8.6455665 ## 82 4.66 4.4223505 ## 83 4.09 3.9850735 ## 84 3.30 2.9621934 ## 85 -0.37 -1.3352777 ## 86 -1.51 -1.4920864 ## 87 -2.46 -2.4705996 ## 88 -3.01 -3.5617876 ## 89 -3.90 -4.0534494 ## 90 -3.60 -4.9777262 ## 91 -4.37 -5.6076159 ## 92 7.66 7.2597455 ## 93 3.76 3.7625920 ## 94 3.41 4.3145737 ## 95 1.57 2.2593586 ## 96 1.93 3.2245813 ## 97 -0.28 -0.5246057 ## 98 -1.11 -1.8442554 ## 99 -1.27 -1.2508650				
## 82				
## 83	## 81	6.52	8.6455665	
## 84	## 82	4.66	4.4223505	
## 84	## 83	4.09	3.9850735	
## 85				
## 86				
## 87				
## 88	## 86	-1.51	-1.4920864	
## 88	## 87	-2.46	-2.4705996	
## 89				
## 90				
## 91				
## 92	## 90	-3.60	-4.9777262	
## 92 7.66 7.2597455 ## 93 3.76 3.7625920 ## 94 3.41 4.3145737 ## 95 1.57 2.2593586 ## 96 1.93 3.2245813 ## 97 -0.28 -0.5246057 ## 98 -1.11 -1.8442554 ## 99 -1.27 -1.2508650	## 91	-4.37	-5.6076159	
## 93 3.76 3.7625920 ## 94 3.41 4.3145737 ## 95 1.57 2.2593586 ## 96 1.93 3.2245813 ## 97 -0.28 -0.5246057 ## 98 -1.11 -1.8442554 ## 99 -1.27 -1.2508650				
## 94 3.41 4.3145737 ## 95 1.57 2.2593586 ## 96 1.93 3.2245813 ## 97 -0.28 -0.5246057 ## 98 -1.11 -1.8442554 ## 99 -1.27 -1.2508650				
## 95				
## 96		3.41	4.3145737	
## 96	## 95	1.57	2.2593586	
## 97	## 96		3.2245813	
## 98 -1.11 -1.8442554 ## 99 -1.27 -1.2508650				
## 99 -1.27 -1.2508650				
## 100 -1.2999283	## 99	-1.27	-1.2508650	
	## 100	-1.09	-1.2999283	

Team	Season	Observed Wins	Predicted Wins	Observed Margin of Victory	Predicted Margin of Victory
Milwaukee Bucks	2018 - 2019	60	61.01426	8.87	7.5019316
Denver Nuggets	2018 - 2019	54	48.44225	3.95	2.7918936
Boston Celtics	2018 - 2019	49	51.48235	4.44	3.8579222
Brooklyn Nets	2018 - 2019	42	41.65049	0.71	0.1291505
Charlotte Hornets	2018 - 2019	39	38.43315	-0.23	-0.8615088
Washington Wizards	2018 - 2019	32	35.05902	-2.90	-2.4180218
Cleveland Cavaliers	2018 - 2019	19	17.24568	-9.34	-9.0182489
Phoenix Suns	2018 - 2019	19	18.57312	-9.61	-8.7940497
Houston Rockets*	2017 - 2018	65	59.85960	8.48	7.2595134
Toronto Raptors*	2017 - 2018	59	58.05547	7.78	6.3801864
Golden State Warriors*	2017 - 2018	58	58.90024	5.98	6.3904133
Portland Trail Blazers*	2017 - 2018	49	44.46868	2.60	1.3311391
Indiana Pacers*	2017 - 2018	48	44.94865	1.38	1.4623198
San Antonio Spurs*	2017 - 2018	47	47.55992	2.89	2.6330804
Milwaukee Bucks*	2017 - 2018	44	42.03005	-0.30	0.3325235
Chicago Bulls	2017 - 2018	27	22.34545	-7.04	-6.9689690
Boston Celtics*	2016 - 2017	53	46.97189	2.63	2.1073394
Memphis Grizzlies*	2016 - 2017	43	39.29831	0.49	-0.4522467
Miami Heat	2016 - 2017	41	45.34191	1.06	1.5203129
Portland Trail Blazers*	2016 - 2017	41	40.81297	-0.52	-0.2177054
Charlotte Hornets	2016 - 2017	36	40.88568	0.20	0.2178854
Minnesota Timberwolves	2016 - 2017	31	37.46738	-1.11	-1.2279486
Philadelphia 76ers	2016 - 2017	28	26.72349	-5.70	-5.5494226
Phoenix Suns	2016 - 2017	24	27.68881	-5.63	-4.9422247
Brooklyn Nets	2016 - 2017	20	25.10779	-6.73	-6.2136536
Cleveland Cavaliers*	2015 - 2016	57	54.16213	6.00	4.9859142
Charlotte Hornets*	2015 - 2016	48	47.17588	2.72	2.5351641
Miami Heat*	2015 - 2016	48	48.30282	1.65	2.7661311
Dallas Mavericks*	2015 - 2016	42	40.20250	-0.30	-0.2998492
Houston Rockets*	2015 - 2016	41	42.65048	0.20	0.6139207
Utah Jazz	2015 - 2016	40	44.25392	1.79	1.3555547
Milwaukee Bucks	2015 - 2016	33	33.44204	-4.18	-2.9487069
New Orleans Pelicans	2015 - 2016	30	30.47749	-3.79	-3.8951258
Brooklyn Nets	2015 - 2016	21	23.73058	-7.35	-6.5288869
San Antonio Spurs*	2014 - 2015	55	55.19093	6.20	5.3778730
Memphis Grizzlies*	2014 - 2015	55	47.61074	3.24	2.7033167
Dallas Mavericks*	2014 - 2015	50	49.85750	2.90	3.2481478

Toronto Raptors* 2014 - 2015 49 47.26998 3.07 Oklahoma City Thunder 2014 - 2015 45 44.86464 2.18 Milwaukee Bucks* 2014 - 2015 41 42.50803 0.43 Boston Celtics* 2014 - 2015 40 40.09710 0.16 Utah Jazz 2014 - 2015 38 42.82806 0.22 Detroit Pistons 2014 - 2015 32 38.64799 -1.00 Denver Nuggets 2014 - 2015 30 31.29008 -3.55 Sacramento Kings 2014 - 2015 29 32.11757 -3.71 New York Knicks 2014 - 2015 17 16.43611 -9.32 Memphis Grizzlies* 2013 - 2014 48 47.80338 2.63 Chicago Bulls* 2013 - 2014 48 44.02758 1.85						
Oklahoma City Thunder 2014 - 2015 45 44.86464 2.18 Milwaukee Bucks* 2014 - 2015 41 42.50803 0.43 Boston Celtics* 2014 - 2015 40 40.09710 0.16 Utah Jazz 2014 - 2015 38 42.82806 0.22 Detroit Pistons 2014 - 2015 32 38.64799 -1.00 Denver Nuggets 2014 - 2015 30 31.29008 -3.55 Sacramento Kings 2014 - 2015 29 32.11757 -3.71 New York Knicks 2014 - 2015 17 16.43611 -9.32 Memphis Grizzlies* 2013 - 2014 50 45.01407 1.57 Toronto Raptors* 2013 - 2014 48 47.80338 2.63 Chicago Bulls* 2013 - 2014 48 44.02758 1.85	Team	Season	Observed Wins	Predicted Wins	Observed Margin of Victory	Predicted Margin of Victory
Milwaukee Bucks* 2014 - 2015 41 42.50803 0.43 Boston Celtics* 2014 - 2015 40 40.09710 0.16 Utah Jazz 2014 - 2015 38 42.82806 0.22 Detroit Pistons 2014 - 2015 32 38.64799 -1.00 Denver Nuggets 2014 - 2015 30 31.29008 -3.55 Sacramento Kings 2014 - 2015 29 32.11757 -3.71 New York Knicks 2014 - 2015 17 16.43611 -9.32 Memphis Grizzlies* 2013 - 2014 50 45.01407 1.57 Toronto Raptors* 2013 - 2014 48 47.80338 2.63 Chicago Bulls* 2013 - 2014 48 44.02758 1.85	Toronto Raptors*	2014 - 2015	49	47.26998	3.07	2.3854624
Boston Celtics* 2014 - 2015 40 40.09710 0.16 Utah Jazz 2014 - 2015 38 42.82806 0.22 Detroit Pistons 2014 - 2015 32 38.64799 -1.00 Denver Nuggets 2014 - 2015 30 31.29008 -3.55 Sacramento Kings 2014 - 2015 29 32.11757 -3.71 New York Knicks 2014 - 2015 17 16.43611 -9.32 Memphis Grizzlies* 2013 - 2014 50 45.01407 1.57 Toronto Raptors* 2013 - 2014 48 47.80338 2.63 Chicago Bulls* 2013 - 2014 48 44.02758 1.85	Oklahoma City Thunder	2014 - 2015	45	44.86464	2.18	1.5309146
Utah Jazz 2014 - 2015 38 42.82806 0.22 Detroit Pistons 2014 - 2015 32 38.64799 -1.00 Denver Nuggets 2014 - 2015 30 31.29008 -3.55 Sacramento Kings 2014 - 2015 29 32.11757 -3.71 New York Knicks 2014 - 2015 17 16.43611 -9.32 Memphis Grizzlies* 2013 - 2014 50 45.01407 1.57 Toronto Raptors* 2013 - 2014 48 47.80338 2.63 Chicago Bulls* 2013 - 2014 48 44.02758 1.85	Milwaukee Bucks*	2014 - 2015	41	42.50803	0.43	0.5272282
Detroit Pistons 2014 - 2015 32 38.64799 -1.00 Denver Nuggets 2014 - 2015 30 31.29008 -3.55 Sacramento Kings 2014 - 2015 29 32.11757 -3.71 New York Knicks 2014 - 2015 17 16.43611 -9.32 Memphis Grizzlies* 2013 - 2014 50 45.01407 1.57 Toronto Raptors* 2013 - 2014 48 47.80338 2.63 Chicago Bulls* 2013 - 2014 48 44.02758 1.85	Boston Celtics*	2014 - 2015	40	40.09710	0.16	-0.3470279
Denver Nuggets 2014 - 2015 30 31.29008 -3.55 Sacramento Kings 2014 - 2015 29 32.11757 -3.71 New York Knicks 2014 - 2015 17 16.43611 -9.32 Memphis Grizzlies* 2013 - 2014 50 45.01407 1.57 Toronto Raptors* 2013 - 2014 48 47.80338 2.63 Chicago Bulls* 2013 - 2014 48 44.02758 1.85	Utah Jazz	2014 - 2015	38	42.82806	0.22	0.7780669
Sacramento Kings 2014 - 2015 29 32.11757 -3.71 New York Knicks 2014 - 2015 17 16.43611 -9.32 Memphis Grizzlies* 2013 - 2014 50 45.01407 1.57 Toronto Raptors* 2013 - 2014 48 47.80338 2.63 Chicago Bulls* 2013 - 2014 48 44.02758 1.85	Detroit Pistons	2014 - 2015	32	38.64799	-1.00	-0.7966378
New York Knicks 2014 - 2015 17 16.43611 -9.32 Memphis Grizzlies* 2013 - 2014 50 45.01407 1.57 Toronto Raptors* 2013 - 2014 48 47.80338 2.63 Chicago Bulls* 2013 - 2014 48 44.02758 1.85	Denver Nuggets	2014 - 2015	30	31.29008	-3.55	-3.6334464
Memphis Grizzlies* 2013 - 2014 50 45.01407 1.57 Toronto Raptors* 2013 - 2014 48 47.80338 2.63 Chicago Bulls* 2013 - 2014 48 44.02758 1.85	Sacramento Kings	2014 - 2015	29	32.11757	-3.71	-3.2572235
Toronto Raptors* 2013 - 2014 48 47.80338 2.63 Chicago Bulls* 2013 - 2014 48 44.02758 1.85	New York Knicks	2014 - 2015	17	16.43611	-9.32	-9.4193199
Chicago Bulls* 2013 - 2014 48 44.02758 1.85	Memphis Grizzlies*	2013 - 2014	50	45.01407	1.57	1.6106493
	Toronto Raptors*	2013 - 2014	48	47.80338	2.63	2.7168844
N. V. I.V. I. 2010 2014 27 27 70077	Chicago Bulls*	2013 - 2014	48	44.02758	1.85	1.3638212
New York Knicks 2013 - 2014 37 37.72357 -0.79	New York Knicks	2013 - 2014	37	37.72357	-0.79	-1.3144823

4.3 R^2

R2(predicted_wins, ff_test\$WINS)

[1] 0.9287359

R2(pm, ff_test\$MOV)

[1] 0.9707142

 R^2 is the coefficient of determination. It is a measure of the goodness of fit of a model. Both the **ImWINS** and **ImMOV** models perform almost as well on the test data as they did on the training data they were originally fit on. This is a good indicator that our model generalize well, as it predicted well on data it was not trained on.

R^2 values in training and test data sets

Model	Training	Test
ImWINS	0.9435	0.9300
ImMOV	0.9785	0.9707

4.4 Mean Absolute Error

MAE(predicted_wins, ff_test\$WINS)

[1] 2.545743

MAE(pm, ff_test\$MOV)

```
## [1] 0.5472033
```

The MAE values for the **ImWINS** model fit on the test data is 3.392 and for the **ImMOV** model fit on the test data is 0.673. MAE represents the average absolute difference between actual and predicted outcomes.

4.5 Root Mean Squared Error

```
RMSE(predicted_wins, ff_test$WINS)

## [1] 3.260744

RMSE(pm, ff_test$MOV)

## [1] 0.7257216
```

The RMSE values for the **ImWINS** model fit on the test data is 3.392 and for the **ImMOV** model fit on the test data is 0.673. RMSE represents the root of the average squared difference between actual and predicted outcomes.

5. Weights of the four factors

We can calculate the weights of the four factors in our **ImWINS** and **ImMOV** models by calculating the average of each of the coefficents for the four factors.

5.1 Weights for the ImWINS model

```
sum_all_lmwins = (3.89934 + 3.39168 + 1.11070 + 0.70791 + 3.76483 + 2.90168 + 0.88666 + 0.72897)
sum_all_lmwins

## [1] 17.39177

sum_eFG_lmwins = (3.89934 + 3.76483)
weight_eFG_lmwins = 100 * (sum_eFG_lmwins / sum_all_lmwins)
weight_eFG_lmwins
## [1] 44.0678
```

```
sum_TOV_lmWINS = (3.39168 + 2.90168)
weight_TOV_lmWINS = 100 * (sum_TOV_lmWINS / sum_all_lmWINS)
weight_TOV_lmWINS
```

```
## [1] 36.18585

sum_REB_lmWINS = (1.11070 + 0.88666)
weight_REB_lmWINS = 100 * (sum_REB_lmWINS / sum_all_lmWINS)
weight_REB_lmWINS
```

```
## [1] 11.48451
```

```
sum_FTF_lmWINS = (0.70791 + 0.72897)
weight_FTF_lmWINS = 100 * (sum_FTF_lmWINS / sum_all_lmWINS)
weight_FTF_lmWINS
```

```
## [1] 8.261839
```

Weights for the ImWINS model: 44.07% shooting, 36.19% turnovers, 11.48% rebounding, 8.26% foul rate

5.2 Weights for the ImMOV model

```
 \begin{aligned} & \text{sum\_all\_lmMOV} = (1.43555 + 1.32334 + 0.43792 + 0.29640 + 1.42002 + 1.17049 + 0.38560 + 0.28811) \end{aligned} 
sum_all_lmMOV
## [1] 6.75743
sum eFG lmMOV = (1.43555 + 1.42002)
weight eFG lmMOV = 100 * (sum eFG lmMOV / sum all lmMOV)
weight_eFG_lmMOV
## [1] 42.25823
sum_{TOV_{lmMOV}} = (1.32334 + 1.17049)
weight_TOV_lmMOV = 100 * (sum_TOV_lmMOV / sum_all_lmMOV)
weight_TOV_lmMOV
## [1] 36.90501
sum_REB_lmMOV = (0.43792 + 0.38560)
weight_REB_lmMOV = 100 * (sum_REB_lmMOV / sum_all_lmMOV)
weight_REB_lmMOV
## [1] 12.18688
sum_FTF_lmMOV = (0.29640 + 0.28811)
weight_FTF_lmMOV = 100 * (sum_FTF_lmMOV / sum_all_lmMOV)
weight_FTF_lmMOV
## [1] 8.649886
```

Weights for the ImMOV model: 42.26% shooting, 36.91% turnovers, 12.19% rebounding, 8.65% foul rate

5.3 Comparing Model Weights to Oliver's

Factor	Oliver	Küpfer	ImWINS
Shooting	40.00%	45.45%	44.07%
Turnovers	25.00%	27.27%	36.19%
Rebounding	20.00%	13.64%	11.48%
Free Throw Factor	15.00%	13.64%	8.26%

The values we achieved are reasonably close to those Oliver proposed. Shooting is the most important factor for winning games in the NBA. Turnovers are very close in importance as the second most important factor. This makes sense intuitively from a very high level as if you don't have possession of the ball, you can't score the ball and therefore win a game. In the **ImWINS** and **ImMOV** models, we also place rebounding and foul rate as the 3rd and 4th most important factors respectively. However, the weights that we assign those factors in our model is ~ 55% - 63% of the weight that Oliver assigned.

6. Four Factors historical changes

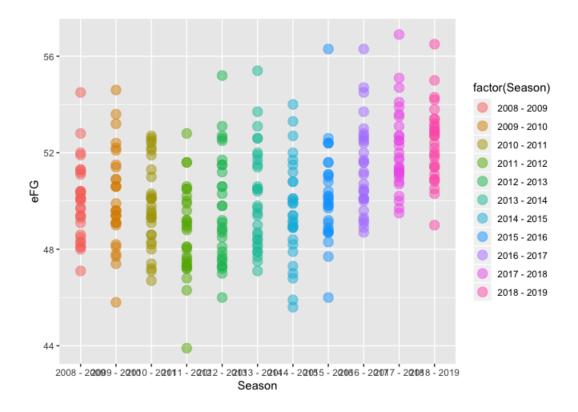
6.1 Preparing data sets for visualizations

6.2 Scatter plots of the four factors for all NBA teams from the 2008 to 2018 seasons

6.2.1 eFG

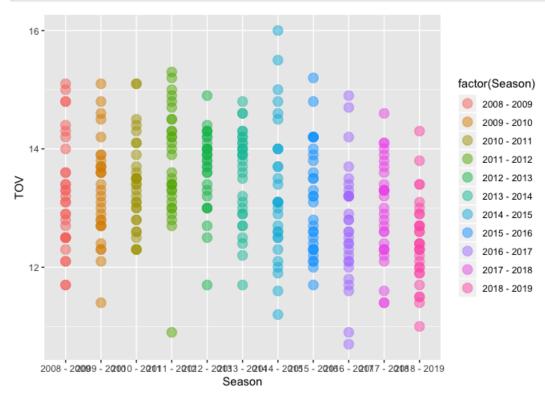
```
eFG_all_hist <- ggplot(data = four_factors_historical, aes(x = Season, y = eFG))

eFG_all_hist + geom_point(alpha=1/2, size=4,
    aes(color=factor(Season)))</pre>
```



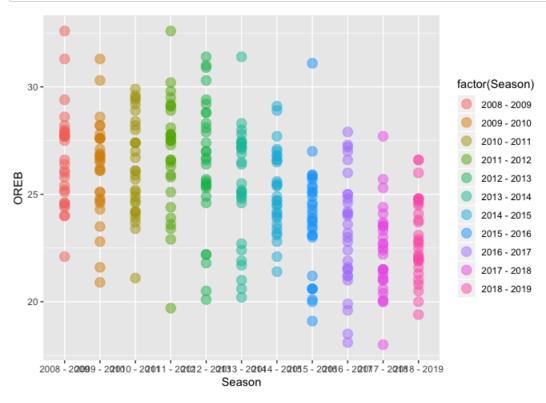
6.2.2 TOV

```
TOV_all_hist <- ggplot(data = four_factors_historical, aes(x = Season, y = TOV))
TOV_all_hist + geom_point(alpha=1/2, size=4,
aes(color=factor(Season)))</pre>
```



```
OREB_all_hist <- ggplot(data = four_factors_historical, aes(x = Season, y = OREB))

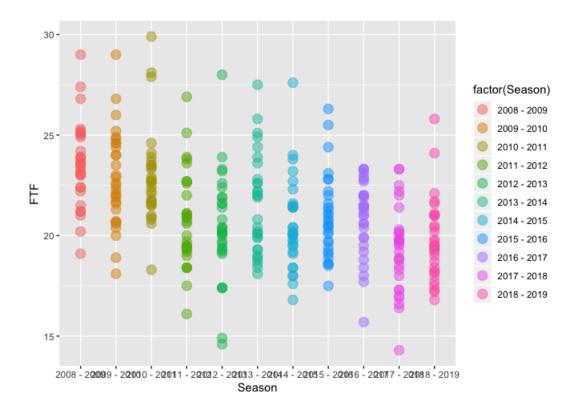
OREB_all_hist + geom_point(alpha=1/2, size=4,
    aes(color=factor(Season)))</pre>
```



6.2.4 FTF

```
FTF_all_hist <- ggplot(data = four_factors_historical, aes(x = Season, y = FTF))

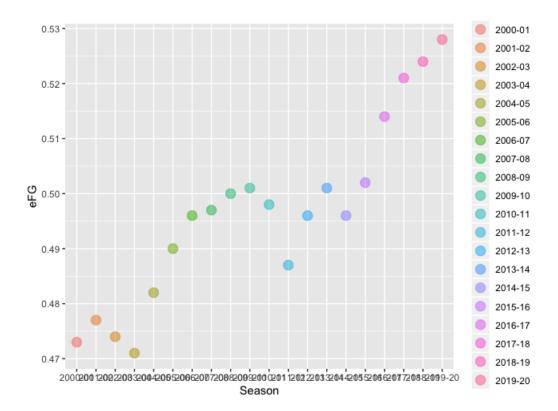
FTF_all_hist + geom_point(alpha=1/2, size=4,
    aes(color=factor(Season)))</pre>
```



6.3 Scatter plots of league averages of the four factors from the 2001 to 2019 seasons

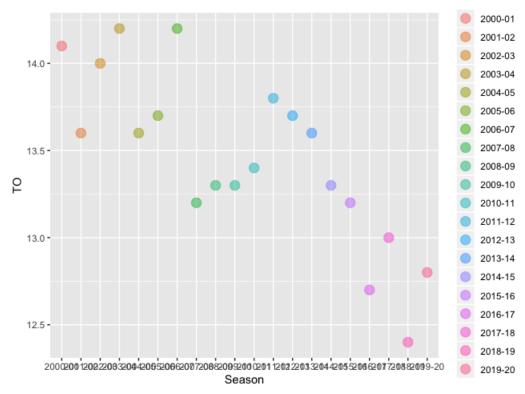
6.3.1 eFG

```
eFG_ave_hist <- ggplot(data = four_factors_hist_new, aes(x = Season, y = eFG))
eFG_ave_hist + geom_point(alpha=1/2, size=4,
aes(color=factor(Season)))</pre>
```



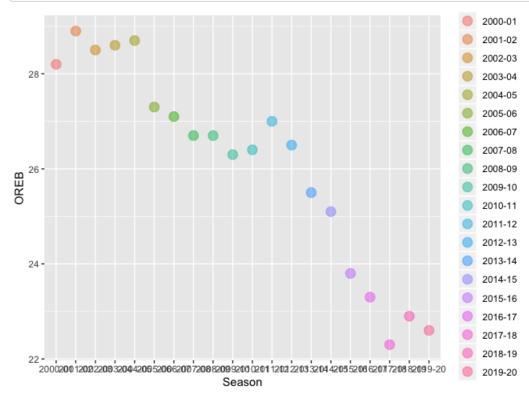
6.3.2 TOV

```
TOV_ave_hist <- ggplot(data = four_factors_hist_new, aes(x = Season, y = TO))
TOV_ave_hist + geom_point(alpha=1/2, size=4,
aes(color=factor(Season)))</pre>
```



```
OREB_ave_hist <- ggplot(data = four_factors_hist_new, aes(x = Season, y = OREB))

OREB_ave_hist + geom_point(alpha=1/2, size=4,
    aes(color=factor(Season)))</pre>
```



6.3.4 FTF

```
FTF_ave_hist <- ggplot(data = four_factors_hist_new, aes(x = Season, y = FTF))

FTF_ave_hist + geom_point(alpha=1/2, size=4,
    aes(color=factor(Season)))</pre>
```

