Assignment 2

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

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Problem 11.16.4.9

9. If 4-digit numbers greater than 5,000 are randomly formed from the digits 0,1,3,5, and 7, what is the probability of forming a number divisible by 5 when

1) the digits are repeated?
2) the repetition of digits is not allowed?

Solution:

Let X be a random variable denoting four digit number formed from the digits 0,1,3,5,7.

1) digits are repeated
   In this case, X can take values 500 values between 1000 to 7777.
$$\Pr(X) = \frac{1}{500} \tag{1}$$
   Probability required:
$$\Pr((X \ mod\ 5 = 0)|(X > 5000)) \tag{2}$$
   There are 4 digits in the number, number of numbers greater than 5000 is given by:
$$2 \times 5 \times 5 \times 5 - 1 = 249' \tag{3}$$
$$\Pr(X > 5000) = \frac{249}{500} \tag{4}$$
   Of these, numbers which are divisible by 5 are given by:
$$2 \times 5 \times 5 \times 2 - 1 = 99 \tag{5}$$
$$\Pr((X \ mod\ 5)(X > 5000)) = \frac{99}{500} \tag{6}$$
$$Now, \Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \tag{7}$$
$$\Pr(X \ mod\ 5 = 0|X > 5000) = \tag{8}$$
$$\frac{\Pr((X \ mod\ 5 = 0)(X > 5000))}{\Pr(X > 5000)} \tag{9}$$
   Hence required probability is:
$$\frac{99}{249} = \frac{33}{83} \tag{10}$$
2) digits are not repeated
   In this case, X takes 96 different values between 1035 and 7531. Number of numbers greater than 5000 is given by:
$$4 \times 4 \times 3 \times 2 = 96 \tag{11}$$
$$\Pr(X) = \frac{1}{96} \tag{12}$$
   Number of numbers greater than 5000 is given by:
$$2 \times 4 \times 3 \times 2 = 48 \tag{13}$$
$$\Pr(X > 5000) = \frac{48}{96} \tag{14}$$
   To find $\Pr((X \ mod\ 5)(X > 5000))$:
   a) Case 1
      When leading digit is 5, number of possible numbers is:
$$1 \times 3 \times 2 \times 1 = 6 \tag{15}$$
   b) Case 2
      When leading digit is 7, number of possible numbers is:
$$1 \times 3 \times 2 \times 2 = 12 \tag{16}$$
      Thus, total number of numbers are 12+6 = 18.
$$\Pr((X \ mod\ 5)(X > 5000)) = \tag{17}$$
$$\frac{18}{96} \tag{18}$$

Thus $\Pr((X \; mod \; 5)|(X > 5000)) =$

$$\frac{\Pr((X \; mod \; 5 = 0)(X > 5000))}{\Pr(X > 5000)} \tag{19}$$

$$= \frac{18}{48} = \frac{3}{8} \tag{20}$$