Covariant Phase Space (CPS) Formalism
[Phase Space without Time slicing]

References

1. Witten (1986) : Interacting F.T of Open SuperStrings.
2. Zuckerman (1986) : Action principles & Global Symmetries
3. Crnkovic & Witten (1987) : "300 yrs of Gravitation"
4. Crnkovic (1988) : Symplectic Geometry of CPS, Superstring & Superspace.

→ Core CPS Foundations

(5) Lee & Wald (1990) → Local Symmetries & Constraints.
(6) Iyer & Wald (1993-95) → BH Entropy
(7) Wald & Zoupas (2000) → Charges & flux

→ Noether charge technology (used everywhere in GR)

(12) Barnich & Brandt (2001-2002)
(13) Compère (2018-19) Lec notes

→ Asymptotic Symmetries & Covariant Surface Charge methods.

(14) Modern CPS

"CPS with boundaries" done Carefully →

Harlow & Wu [2020, JHEP]

REVIEW

Regge & Teitelboim → "Role of surface Integrals in the Hamiltonian formulation of GR".

Arnold : Mathematical aspects of classical & celestial Mechanics.

Application :

* Entropy of BH = $\int_{\text{Horizon}}$ Noether charges associated with Horizon Killing Vector field.

* Asymptotic Symmetries @ bdy of AdS & Flat Spacetimes.

CPS is classical, covariant way to define phase space and symplectic str. from the Lagrangian.

∃ some confusion in literature when we deal with boundaries & tot. derivative terms in the formalism.

Quantization (Canonical, Peierls/pAQFT/path int) uses that str. to define the quantum op. alg & charges.

In String th., the same ideas appear for the worldsheet QFT, & even more structurally in SFT via Symplectic/BV forms.

Hamiltonian Mechanics : the split of coordinates into position & momenta makes it difficult to preserve covariance.

↓

geometric formulation of Hamiltonian Mechanics

We construct, Phase space (Abstract space) endowed with a closed & non-degenerate 2-form Ω, which is called Symplectic form.

→ Manifold with such form is called Symplectic Manifold.

What we do in Hamiltonian formulation,

$$M \sim \mathbb{R} \times \Sigma_t$$

then you define canonical data on each Σ_t.

$\phi(t, \vec{x})$ restricted to Σ_t.

$$\Pi(t, \vec{x}) = \frac{\partial \mathcal{L}}{\partial(\partial_t \phi)}$$ depends on split.

evolve $\Sigma_{t_0} \rightarrow \Sigma_{t_1}$ using Hamilton's eqn.

$$\mathcal{L} = \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}(\nabla \phi)^2 - \frac{1}{2}m^2\phi^2 , \quad \pi = \partial_t \phi.$$

$$H = \int_{\Sigma_t} d^3x \left[\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2 \right].$$

time slicing.

Equal time poisson bracket

$$\{\phi(t,\vec{x}), \pi(t,\vec{y})\} = \delta^{(3)}(x-y) \quad ! \ \Sigma_t$$

not space-time covariant.

$$H(t) = \int_{\Sigma_t} d^3x \, \mathcal{H}(\phi, \pi, \nabla\phi, \ldots)$$

Q. What if system is not closed on a slice?

eg. gauge symmetry, rad^n leaking from boundary?

Boundaries → generators are not differentiable.

Σ has a bdy.

$$\delta H \sim \Omega(\delta, \delta_\xi) + (\text{boundary term}).$$

if $\neq 0$ then gen is not well-defined.

Rad^n/flux → Hamiltonian is not conserved / not integrable.

Charges change with slices.

Benefits of Hamiltonian Mechanics.

- IV Problem Simple. { if IVP is well-posed then set of initial Data on Cauchy Slice
- D.O.F Counting Simpler. = set of sol^ns of EOMs.
- ⚠ - Breaks Covariance.

IVP needs Cauchy surfaces. If spacetime isn't globally hyperbolic (horizons / null boundaries), data specifications on Σ_t is tricky / fails.!!

So, Gravity its a problem because Hamiltonian is always a boundary term.

? Question is whether you're constructing right Hamiltonian or not!!

Also, OPs at bdy are asymptotically dressed to make them diffeo Invariant. So. One has to deal with bdy. CAREFULLY !!

Explain Hamiltonian Mechanics in a covariant way from the Point of view of CPS formalism.

Phase Space

Defined as the space of solutions, with a symplectic form built from Lagrangian.

So you don't have to choose a time foliation to do Hamiltonian Mechanics.

is Dynamics of a phase space is labelled by $\{q^a, p^a\}$

with any Scalar function $H : P \longrightarrow \mathbb{R}$,

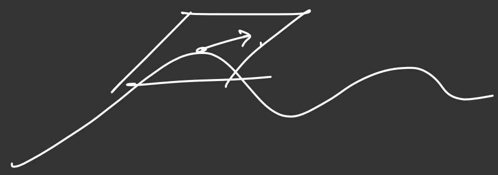
generating dynamical evolutions via Hamilton's eqn.

$$\dot{q}^a = \frac{\partial H}{\partial p^a} \quad ; \quad \dot{p}^a = -\frac{\partial H}{\partial q^a} .$$

Lagrangian mechanics $\rightarrow$ Let's consider a n-dim smooth & Differentiable manifold.

$$\{q_i\} \in M \quad , \quad i = 1 \dots n .$$

$\uparrow$ n-real numbers



Tangent Space : $T_q M$

$\uparrow$ n-dimensional.

Basis : $\dfrac{\partial}{\partial q_i} \equiv \partial_i$

$v = v^i \partial_i \rightarrow$ vector in tangent space

If v is the tangent to trajectory then v is velocity $= v^i = \dot{q}^i$

$(q^i, v^i) \Rightarrow$ collectively product of manifold and the tangent

$=$ Tangent bundles. (set)

$$: \bigcup_q T_q M = TM$$

Lagrangian $L : TM \rightarrow \mathbb{R}$

However, we know that for Hamiltonian we need <u>Canonical Momenta</u> -

map : $(q^i, \dot{q}^i) \to (q^i, p_i)$ invertible map.

$$p_i = \frac{\partial L}{\partial \dot{q}^i} \in \underline{\underline{T_q^* M}} \quad \text{(cotangent bundle)}$$

$$\equiv \underline{\underline{a_{ij}}}\, q_j$$

(if singular Lagrangian.

$\Rightarrow$ non-invertible map (degenerate)

Primary constraint : $\phi_a(q, p) = 0$

Hamiltonian non-unique

$$H_T = H_c + u^a \phi_a$$

Canonical Hamiltonian — Lagrange's multipliers reflecting degeneracy

$$\dot{\phi}^a = \{\phi_a, H_T\} \approx 0$$

Consistency generates more constraints ←

Constraint hold in time

2ndary constraints → fix some u^a.

Singular Lagrangian comes from Gauge theories

(EM, Yang-Mills, GR)

- 1st Class → generates gauge transf^{n}s.
- 2nd Class → must be eliminated (via Dirac brackets)

* Hamiltonian Mechanics ⟶ Dirac-Bergmann Constrained Hamiltonian th.

20th century physicists → How to quantize sing. Lagrangian.

Phase space is replaced by Constraint surface.

& evolution is generated by Hamiltonian + Constraints.

& gauge freedom show up as non-uniqueness of sol^{n}s until you fix a gauge.

Let's $\quad f = f(\{q^i\}) \qquad f \in M$ → 0-form.

function in phase space.

$$df = \frac{\partial f}{\partial q^i} dq^i \Rightarrow \underbrace{df}_{\text{(1-form)}} \in T_q^* M$$

(vector in cotengent bundle)

$v \in T_q M$

$$df(v) \equiv \underbrace{(\,df, v\,)}_{\text{Scalar product}} = v_i \frac{\partial f}{\partial q^i} \in \mathbb{R}$$

df takes a vector & gives a number.
(1-form)

$$\boxed{\bullet : T_q^* M \longrightarrow \mathbb{R}}$$

Basis for 1-form : $\{dq^i\}$, $\boxed{p = p_i \, dq^i}$ (1 form)

2-forms

$u, v \longrightarrow$ vectores. $\in T_q M$

$\Omega(u, v)$
↑ ↑
arguements as two vectores.

$A, B \longrightarrow$ 1-forms.

$A(u) \cdot B(v)$
$A(v) \cdot B(u)$.

$$\Omega(u,v) = \underbrace{A(u)\,B(v) - A(v)B(u)}_{\text{outere producete.}}$$

Antisymmetric Bilineare rules.

$u = u_i \partial_i \qquad v_i \partial_i = v_i$

$A = A_i \, dx^i \quad , \quad B = B_i \, dx^i$

$$\Omega(u,v) = (A_i B_j - A_j B_i)\, u^i v^j$$

$$= \underset{\text{Antisymmetric}}{\Omega_{ij}}\, u^i v^j$$

<u>Wedge product</u>

$$dq^i \wedge dq^j = dq^i \otimes dq^j - dq^j \otimes dq^i$$

outer product.

Claim

$$\Omega(u,v) = \frac{1}{2!}\Omega_{ij}\, dq^i \wedge dq^j (u,v)$$

$$= \frac{1}{2!}\Omega_{ij}\,(u^i v^j - u^j v^i) = \Omega_{ij}\, u^i v^j$$

Basis of 2-form : $dq^i \wedge dq^j$

r form : $\Omega_r = \frac{1}{r!}\Omega_{k_1 \dots k_r}\, dq^{k_1} \wedge dq^{k_2} \wedge \dots dq^{k_r}$

$\rightarrow$ How P acts on velocity vectors ?

$P \in T_q^* M$, velocity $\in T_q M$

$$P\dot{q} = (p_i\, dq^i)\left(\dot{q}^j \frac{\partial}{\partial q^j}\right)$$

$$= p_i\, \dot{q}^i$$

Interior product

$$i_X \Omega(Y) = \Omega(X, Y)$$

$$i_{\frac{\partial}{\partial p_j}} dp_i = \delta^i_j \,, \quad i_{\partial/\partial q_i} dq_i = \delta^j_i$$

mixed vanishes.

Interior Product on Wedge product.

$$i_X(\alpha \wedge \beta) = (i_X \alpha) \wedge \beta + (-1)^{\deg \alpha} \alpha \wedge (i_X \beta)$$

On (M, Ω) for every smooth function f $\exists$ a unique vector field X_f such that $i_X \Omega = df$.

$$\{f, g\} = X_g(f) = df(X_g) = (i_{X_f} \Omega)(X_g) = \Omega(X_g, X_f)$$

$\mathbb{P}$ has a closed, non-degenerate two form Ω called symplectic form & the manifold is symplectic Manifold. (M, Ω)

(Arnold Book)

Given, $\Theta = p_i \, dq^i \rightarrow$ 1-form.

2-form, $\Omega \equiv d\Theta$ "symplectic potential.

Closed: $d\Omega = 0 \implies$ (∵ Any closed form is locally exact.) Poincaré Lemma

$d^2\Theta = 0$.

$$\Omega = dq^i \wedge dp_i \quad \text{(Symplectic form)}$$

We will show that time-evolution gives trajectories in phase space

Given $H: P \longrightarrow \mathbb{R}$, we can define a vector X_H on P

$$X_H(f) \equiv \Omega^{-1}(df, dH)$$

$\{f, g\} \equiv$ Apply transf^n generated by g on f.

where $f : P \longrightarrow \mathbb{R}$. $\longrightarrow$ f is observable.

So, the integral curves of X_H in P gives the time evolution of system generated by viewing H as Hamiltonian.

$\partial_q f \partial_p g - \partial_p f \cdot \partial_q g$

$$= df(X_H)$$

We know $\{f, g\} \equiv \Omega^{-1}(df, dg) = \Omega(X_g, X_f)$

$$= X_g(f)$$

So, $\dfrac{df}{dt} = \{f, H\} + \cancel{\dfrac{\partial f}{\partial t}}^{\,0} \Rightarrow \{f, H\} = \dot{f}$

$$= X_H(f) + \frac{\partial f}{\partial t}$$

$X_H^\mu \partial_\mu f$

$$X_H(f) = \left(X^i_{(q)} \frac{\partial}{\partial q^i} + X^{(p)}_i \frac{\partial}{\partial p_i}\right) f$$

$\to$ direction in p-direction

$\therefore$ Derivative along the Hamiltonian flow $= \{f, H\}$

$X \in \mathbb{P}$: $X_q \in T_q M$

vector in Phase space.

$$\begin{cases} X^i_{(q)} = \dot{q}^i \\ X^{(p)}_i = \dot{p}_i \end{cases}$$

$\Downarrow$

Tangent vect on the flow in Hamiltonian space.

$\|$ Tangent on the trajectory in phase space.

Given $f = q^i$ & $f = p_i$, $\{q^i, p_j\} = \delta^i_j$.

$\dot{q}^i = \{q^i, H\}$ & $\dot{p}_i = \{p_i, H\}$

Direction Derivative along X vector.

Cartan's magic formula | $\mathcal{L}_X \Omega = i_X(d\Omega) + d(i_X \Omega)$

$\uparrow$ interior product.

$$\equiv X \cdot d\Omega + d(X \cdot \Omega)$$

For X_H, Hamiltonian Vector field.

we have, $$i_{X_H}\Omega = dH \quad \text{(Hamiltonian Eq}^n\text{.)}$$

Conservation of Poisson's bracket is equivalent as saying that the Lie derivative $\mathcal{L}_{X_H}\Omega = 0$

$$\Rightarrow \mathcal{L}_{X_H}\Omega = d(dH) + X_H \cdot d\Omega$$
$$= 0$$
$$\Rightarrow \boxed{d\Omega = 0}$$

$\therefore$ Closeness of symplectic form is an artifact of conservation of Poisson's bracket.

Given $\Omega = dq^i \wedge dP_i$,

$$X_H \cdot \Omega \equiv i_X \Omega = i_X(dq^i \wedge dP_i)$$
$$= (i_X dq^i)\, dP_i - dq^i (i_X dq^i)$$
$$= \dot{q}^i dP_i - \dot{P}_i dq^i$$

$d \equiv$ Exterior derivative on H

and,
$$dH = \frac{\partial H}{\partial P_i} dP_i + \frac{\partial H}{\partial q^i} dq^i$$

Comparing, $$\left.\dot{q}^i = \frac{\partial H}{\partial P_i} \quad \& \quad \dot{P}_i = -\frac{\partial H}{\partial q^i}\right\} \text{Hamilton's Eq}^n$$

$\square$.

ANY QUESTIONS ??

$\boxed{\text{So, Phase space can mean "space of solutions" not data on slice. (in a covariant way) Hamiltonian Mechanics.}}$

1980's Witten et al. — Problem comes in case of continuous local symmetries....

Set of IVPs $\xrightarrow[\text{correspondence.}]{(1-1)}$ Set of Solutions

(non-covariant way) (covariant way)

(M, Ω)

Problem

The problem comes in gauge theories.

IVPs $\not\longrightarrow$ Set of solutions.

$A = 0$

in neighborhood $A \neq 0$.

$\Rightarrow$ for a given IVP there could be diff set of solutions.

So, Let us construct general set of solution space.

$\tilde{\mathbb{P}}$ = Pre-symplectic Phase space

$\tilde{\Omega}$ = " form (could be degenerate).

$(\tilde{P}, \tilde{\Omega})$

$\exists$ zero modes for $\tilde{\Omega}$.

$\Rightarrow$ Solution is not unique.

$\leftarrow$ When $\exists$ gauge symmetries that leads to ambiguities in solutions of EOMs at-least if its a continuous gauge transformation.

These zero modes are vector fields because $\tilde{\Omega}$ is a 2-form.

Lie brackets. $[\text{zero mode}, \text{zero mode}] \longrightarrow$ Zero mode.

$\hookrightarrow$ generates Lie Algebra.

exponentiate to get Gauge gr $\tilde{G}$. which act on $\tilde{P}$

Set of transf'ns on $\tilde{\mathbb{P}}$ that generated by zero modes.

$\tilde{G}$ of $\tilde{\Omega}$. $\leftarrow$

$P = \tilde{P}/\tilde{G}$

[written 1980]

Recall!

TREAT BDY TERM SYSTEMATICALLY!!

Field Theory :

Goal : Start from Lagrangian field th., which preserves Covariance at every step.
(no need to pick a frame).

$\partial M = \Gamma \cup \Sigma_+ \cup \Sigma_-$

dynamical   bdy field

$$S = \int_M L(\phi, X) + \int_{\partial M} \ell(\phi, X)$$

with boundary conditions for $\phi^a(x)$ on Γ (spatial bdy)

Idea of Lagrangian Mechanics →

Impose appropriate BCs @ ∂M ⟶ look for configurations ϕ_c about which the action is stationary under arbitrary variations of dynamical fields which obey BCs.

action is stationary upto a bdy term.
⇒ $\delta S = \int_{\Sigma_+} \Psi - \int_{\Sigma_-} \Psi$

BCs @ Γ	BCs @ $\Sigma_\pm$
Specify the theory	Specify state

If we allow variations of state, then we shouldn't put BCs on $\Sigma_\pm$.

Given Act^n & Bdy cond^{n}s construct Hamiltonian functions

Six Step Program

① $\tilde{C}$: Configuration space — set of $\phi^a(x)$ obeying BCs (not EOMs).

② $\tilde{P}$ $\equiv$ " Pre-sym Space" $\equiv$ Set of sol^{n}s of EOMs (does obey BCs).

③ $\tilde{\Omega}$ $\equiv$ " " form" on $\tilde{P}$ (closed, degenerate)

④ $\tilde{G} \subset \mathrm{diff}(\tilde{P})$ generating zero modes of $\tilde{\Omega}$.

⑤ $P = \tilde{P}/\tilde{G}$ $\Omega = \tilde{\Omega}/\tilde{G}$.

⑥ Given diff gen ξ^{μ}, Construct H_{ξ} s.t $dH_{\xi} = i_X \Omega$

Q. How to construct $\tilde{\Omega}$ & H_{ξ} ?

As we are not imposing BCs at $\Sigma_\pm$, so there will be boundary terms at $\Sigma_\pm$.

$$S = \int_M L + \int_{\partial M} \ell$$

(Action is well-posed & stationary upto-boundary term. modulo future/past bdy.

$$\delta L = E_a \delta\phi^a + d\underset{\sim}{\Theta}$$

δL: d-form Lagrangian

Θ: Symplectic Potential.

$$\Rightarrow \delta S = \int_M E_a \delta\phi^a + \int_{\Sigma_\pm} (\Theta + \delta\ell) + \int_{\Gamma} (\Theta + \delta\ell)$$

(using Stoke's thm)

Stationarity of Action tells us,

1) $E_a = 0$ (EOM) (variations in the bulk that vanishes at bdy)

2) $\underbrace{(\Theta + \delta \ell)\Big|_{\Gamma} = dC}$ → Consistency cond^n/ BCs for the choice of boundary term ℓ.

$$\int_{\Gamma} (\Theta + \delta\ell) = \int_{\Gamma} dC \xrightarrow[\text{theorem}]{\text{Stokes}} \int_{\partial\Gamma} C \quad (\text{bdy of corners of } \Sigma_t)$$

From now on we will impose this eq^n.

Let us define,

$$\omega \equiv \text{Pre-Symplectic current.}$$

$$= \delta(\Theta - dC)\Big|_{\tilde{P}}$$

δ : exterior derivative in configuration space: $\mathcal{C}$

Set of config. of ϕ^a that obey BCs but not EOMs

$\tilde{\mathcal{C}} \supset \tilde{P}$

d-1 form in Spacetime but 2-form in config. space.

1-form on $\tilde{\mathcal{C}}$.

Properties

① $\delta\omega = 0$ (closed 2-form)

② $d\omega = \delta(d\Theta)$

$= \delta(\delta L - E_a \delta\phi^a)$

$= -\delta E_a \delta\phi^a = 0$

$[\delta, d] = 0.$

③ $\omega\Big|_{\Gamma} = \delta(-\delta\ell) = 0$

$\therefore$ $\boxed{\tilde{\Omega} \equiv \int_{\Sigma} \omega}$ → Pre symplectic form.

↳ Covariant construction.

Properties : closed, independent of choice of Σ slice.

$$\int_{\Sigma'} \omega - \int_{\Sigma} \omega = \int_{\text{inside}} d\omega + \underbrace{\text{flux at } \infty}_{\downarrow\, 0}$$

(by cond^n ② & ③ of ω)

Hamiltonian → Generators of Diffeomorphisms of the theory.

Diffeos must obey BCs / if not they are not symmetries.

Every diffeos are not allowed.

eg. metric might not be dynamical, then most diffeos aren't going to give you symmetries.

Diffeo generators ξ^μ s.t.

Noether charge of diffeomorphisms!

ξ^μ : vector field on M.

for diffeo by ξ^a : $\delta_{\xi} \phi$. (Diffeomorphisms generated by ξ_a.

M → M'

$$\Rightarrow \delta_{\xi} \phi = \mathcal{L}_{\xi} \phi .$$

Lie dragged.

Hamiltonian flow for H_{ξ}

= Diffeo flow generated by ξ

We know that,

$$\delta H_{\xi} = i_{X_{\xi}} \tilde{\Omega} \equiv X_{\xi} \cdot \tilde{\Omega} \quad \text{: Hamilton's Eq}^n.$$

X_{ξ} is vect field that corresponds to implementing diffeo on dynamical fields.

$$J_{\xi} \equiv X_{\xi} \cdot \Theta - \xi \cdot L \quad \leftarrow \text{Noether current}$$

$$= \theta(\phi, \mathcal{L}_{\xi}\phi) - \xi \cdot L$$

(d-1) form

• : insertion of vect into diff. form.

$$\mathcal{L}_{\xi} V = d(\xi \cdot V) + \xi \cdot dV$$

$(X_{\xi} \cdot \Omega)(Y) = \Omega(Y, X_{\xi})$

Using identity (Cartan's Magic formula)

$$X_{\xi} \cdot \omega = X_{\xi} \cdot \delta(\theta - dC)$$

$$= X_{\xi} \cdot \delta\Theta - \underbrace{X_{\xi} \cdot \delta(dC)}_{X_{\xi} \cdot d(\delta C)}$$

$$= \mathcal{L}_{X_{\xi}}\Theta - \delta(X_{\xi} \cdot \theta) - d(\mathcal{L}_{X_{\xi}} C - \delta(X_{\xi} \cdot C))$$

$$= (d(\xi \cdot \theta) + \xi \cdot d\theta) - \delta(J_{\xi} + \xi \cdot L) - d(\mathcal{L}_{X_{\xi}} C - \delta(X_{\xi} \cdot C))$$

Covariant Lagrangian

$\mathcal{L}_{X_{\xi}} \sigma(\phi, x) = \mathcal{L}_{\xi} \sigma(\phi, x)$

phase space

spacetime

$\mathcal{L}_{X_H}$, acts only on dynamical fields.

$$= d(\xi \cdot \theta) + \xi \cdot (\cancel{\delta L} - E\,\cancel{\delta\phi}^{\,0}) - \delta J_{\xi} - \cancel{\xi \cdot \delta L} - d(\mathcal{L}_{\xi} C - \delta(X_{\xi} \cdot C))$$

$$= d(\xi \cdot \theta) - \delta J_{\xi} - d(\mathcal{L}_{\xi} C - \delta(X_{\xi} \cdot C))$$

$$\Rightarrow X_{\xi} \cdot \int_{\Sigma} \omega = \delta \int_{\Sigma} J_{\xi} + \int_{\partial\Sigma} \left(-\xi \cdot \theta + \xi \cdot dC + \cancel{d(\xi \cdot C)}^{\,0} - \delta(X_{\xi} \cdot C)\right)$$

$$\Rightarrow X_{\xi} \cdot \tilde{\Omega} = \delta \int_{\Sigma} J_{\xi} + \int_{\partial\Sigma} \xi \cdot (dC - \theta) - \delta(X_{\xi} \cdot C)$$

($\because$ $\partial\Sigma$ is closed manifold)

$$\Rightarrow \quad \delta H = \delta \int_\Sigma J_{\xi} + \int_{\partial\Sigma} \xi \cdot \delta \ell - \delta(X_\xi \cdot C)$$

$$\Rightarrow \quad \boxed{H_\xi = \int_\Sigma J_\xi + \int_{\partial\Sigma} \left(\xi \cdot \ell - X_\xi \cdot C \right)}$$

Here we just assumed that various quantities are covariant under ξ !!

QFT : $\xi \longrightarrow$ boost / rotn / transln etc.

Sym. of th.

If the system is generally covariant (GR)

$\Downarrow$

L etc. are covariant under arbitrary ξ^μ.

then $\quad J_\xi = dQ_\xi$.

$\because \quad J_\xi = X_\xi \cdot \Theta - \xi \cdot L$

$$\begin{aligned} \delta J_\xi &= X_\xi \cdot \delta\Theta - \xi \cdot \delta L \\ &= X_\xi \left(\delta L - E_a \delta\phi^a \right) - \xi \cdot \delta L \\ &= 0 \qquad \text{(on-shell)} \end{aligned}$$

for Local Lagrangian : a closed $(d-1)$ form current is locally exact.

$J_\xi = dQ_\xi$. (Locally, on-shell)
(Noether charge)

Gen Cov. th.

$$H_{\xi_\varphi} = \int_{\partial\Sigma} \left(Q_{\xi_\varphi} + \xi_\varphi \cdot \ell - X_{\xi_\varphi} \cdot C \right)$$

$\Rightarrow$ Generators of diffeos in Gravity theories are always going to be boundary terms.

Arbitrary continuous diffeomorphism where L is covariant.

In such theories Hamiltonian is a pure boundary term.

Analogy: Electric charge is total flux at spatial infinity.