Maximum and Minimum Sums of Subsequences of At Most Size K

1 Problem Description

You are given an integer array `nums` and a positive integer k. Your task is to find the sum of the maximum and minimum elements of **all subsequences** of `nums` that contain **at most k elements**.

Since the sum may be very large, return the result modulo $10^9 + 7$.

1.1 Definitions

- A **subsequence** is a sequence derived from `nums` by deleting some or no elements **without changing the order** of the remaining elements.

- The **minimum and maximum** elements of each subsequence are considered, and their sum is added to the final total.

2 Examples

2.1 Example 1

Input:
$$\text{nums} = [1, 2, 3], \quad k = 2$$

Subsequences with at most 2 elements:

Subsequence	Min	Max	Sum (Min + Max)
[1]	1	1	2
	2	2	4
	3	3	6
	1	2	3
	1	3	4
	2	3	5

Total Sum:
$$2 + 4 + 6 + 3 + 4 + 5 = 24$$

Output:
$$24$$

2.2 Example 2

Input:
$$\texttt{nums} = [5, 0, 6], \quad k = 1$$

Subsequences with at most 1 element:

Subsequence	Min	Max	Sum (Min + Max)
[5]	5	5	10
	0	0	0
	6	6	12

Total Sum:
$$10 + 0 + 12 = 22$$

Output:
$$22$$

2.3 Example 3

Input:
$$\texttt{nums} = [1, 1, 1], \quad k = 2$$

Since all numbers are 1, every subsequence has:

$$\text{min} = 1, \quad \text{max} = 1$$

Output:
$$12$$

3 Constraints

- $1 \leq \text{nums.length} \leq 10^5 \, 0 \leq \text{nums[i]} \leq 10^9$

- $1 \leq k \leq \min(100, \text{nums.length})$