1 Initialization of background fields

To initialize the background fields we will start by picking some reference values from the standard solar model, solar S (?). These are chosen to be at the bottom of the convection zone, at a radius $r_r = 0.7R_{\odot}$. These are downloaded from https://users-phys.au.dk/ jcd/solar$_{m}odels/cptrho.l5bi.d.15chttps : //users - phys.au.dk/\ jcd/sc$ by

$$m(r_r) = \int_0^{r_r} dm = 4\pi \int_0^{r_r} \rho(r')r'^2 dr', \tag{1}$$

using cumulative trapezoidal integration. We extract the temperature $T(r_r)$, density $\rho(r_r)$ and pressure $p(r_r)$, and calculate the entropy $s(r_r) = c_p$, where the spesific heat at constant pressure

$$c_p = r_*/(1 - 1/\gamma),$$

$r_* = p/(\rho T)$ is the ideal gas constant normalized by the average mass per particle, and γ is the adiabatic parameter. We can then use these reference values to integrate outward and inward, assuming the background is hydrostatic and composed of an ideal gas. The first-order entropy gradient

$$\frac{ds}{dr} = -\frac{c_p}{H}\Delta\nabla,$$

where $\Delta\nabla$ is the superadiabaticity parameter and

$$H = -\frac{dz}{d\ln p} = -p\frac{dz}{dp}$$

is the pressure scale height (?). This gives us that the governing equations for the background fields are

$$\frac{\partial m}{\partial r} = 4\pi r^2 \rho(r), \tag{2}$$

$$\frac{\partial p}{\partial r} = -\frac{Gm(r)}{r^2}\rho(r), \tag{3}$$

$$\frac{\partial T}{\partial r} = \nabla_* \frac{T}{p}\frac{\partial p}{\partial r}, \tag{4}$$

$$\frac{ds}{dr} = \frac{c_p}{p}\frac{dp}{dr}\Delta\nabla, \tag{5}$$

where G is the universal gravitational constant. These equations, in order, are the mass of a shell with thickness dr, the hydrostatic equilibrium condition, the ????????? and the first-order entropy gradient.

For convective instability we require that the superadiabaticity parameter

$$\Delta\nabla = \left(\frac{\partial \ln T}{\partial \ln p}\right)_* - \left(\frac{\partial \ln T}{\partial \ln p}\right)_{ad} = \nabla_* - \nabla_{ad} > 0,$$

where ∇_* is the adiabatic temperature gradient of the star and $\nabla_{ad} = 0.4$ is the adiabatic temperature gradient for an ideal gas. We therefore set

$$\nabla_* - \nabla_{ad} = k > 0,$$

where k is constant, above $r = 0.7R_{\odot}$ and

$$\nabla_* = \nabla_{ad},$$

for $r < 0.7R_{\odot}$. Using this we can integrate , , and up to a point r_e and down to a point r_b, where we also calculate the gravitational acceleration by newtons law of gravity

$$g(r) = -\frac{Gm(r)}{r^2} \tag{6}$$

for updating the momentum in each timestep.

For stability we will use a variable steplength when integrating the background fields. For all the variables V we have that

$$dV = f dr,$$
$$V_{i+1} = V_i + dV,$$

where f is some function updating the variable. We can then require that

$$\frac{|dV|}{V} < p,$$

where $p < 1$ is constant. Then

$$dr = \frac{pV}{f}.$$

To update the density we will use assume ideal gas and get the equation of state

$$pV = Nk_BT,$$

where V is the volume of the gas, N is the total number of particles and k_B is the boltzmanns constant. The total number of particles can be written as

$$N = \frac{m}{\mu m_u},$$

where μ is the average atomic weight and m_u is the atomic mass unit. Using this and $V = m/\rho$ we get the **equation of state**

$$\rho = \frac{p}{T}\frac{m_u\mu}{k_B}. \tag{7}$$

We set the average atomic weight to 0.6 to.....

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