Møte October 21, 2023

1 Møte

1.1 Conservation of entropy

Advective term:

$$(\partial_t s_1)_{adv} = -\left[v_y \partial_y s_1 + v_z \partial_z s_1 + v_z \partial_z s_0\right] \tag{1}$$

Viscous term:

$$(\partial_t s_1)_{vis} = \frac{\mu}{\rho_0 T_0} (\partial_z v_y + \partial_y v_z)^2 \tag{2}$$

Thermal term:

$$(\partial_t s_1)_{thermal} = \frac{1.866 \cdot 10^6}{\rho_0 T_0} \left[\partial_z (T_0)(\partial_y s_1 + \partial_z s_1) + T_0(\partial_y^2 s_1 + \partial_z^2 s_1) \right]$$
(3)

Magnetic field term:

$$(\partial_t s_1)_B = \frac{\eta}{\rho_0 T_0} \frac{1}{4\pi} \left[\partial_y B_z - \partial_z B_y \right]^2 \tag{4}$$

1.2 Conservation of momentum

Advective term:

$$\hat{\mathbf{j}}: (\partial_t v_y)_{adv} = -\left[v_y \partial_y v_y + v_z \partial_z v_y\right] \tag{5}$$

$$\hat{\mathbf{k}} : (\partial_t v_z)_{adv} = -\left[v_y \partial_y v_z + v_z \partial_z v_z \right] \tag{6}$$

Pressure term:

$$\hat{\mathbf{j}}: (\partial_t v_y)_p = -\frac{1}{\rho_0} \partial_y p_1 \tag{7}$$

$$\hat{\mathbf{k}}: (\partial_t v_z)_p = -\frac{1}{\rho_0} \partial_z p_1 \tag{8}$$

Gravitational term:

$$\hat{\mathbf{j}}: (\partial_t v_u)_q = 0 \tag{9}$$

$$\hat{\mathbf{k}}: (\partial_t v_z)_g = -\frac{\rho_1}{\rho_0} g \tag{10}$$

Viscous term:

$$\hat{\mathbf{j}}: (\partial_t v_y)_{vis} = \frac{\mu}{\rho_0} \left[\partial_z^2 v_y + \partial_y \partial_z v_z \right]$$
(11)

$$\hat{\mathbf{k}}: (\partial_t v_z)_{vis} = \frac{\mu}{\rho_0} \left[\partial_y^2 v_z + \partial_y \partial_z v_y \right] \tag{12}$$

Magnetic field term:

$$\hat{\mathbf{j}}: (\partial_t v_y)_B = \frac{1}{\rho_0} \frac{B_z}{4\pi} \left[\partial_z B_y - \partial_y B_z \right] \tag{13}$$

$$\hat{\mathbf{k}}: (\partial_t v_z)_B = \frac{1}{\rho_0} \frac{B_y}{4\pi} \left[\partial_y B_z - \partial_z B_y \right]$$
 (14)

Møte October 21, 2023

1.3 Elliptic equation

Advective term:

$$(\nabla^{2} p_{1})_{adv} = -\rho_{0} [v_{y} \partial_{y}^{2} v_{y} + v_{z} \partial_{z}^{2} v_{z} + (\partial_{y} v_{y})^{2} + (\partial_{z} v_{z})^{2}$$

$$+ 2\partial_{y} (v_{z}) \partial_{z} (v_{y}) + v_{y} \partial_{y} \partial_{z} v_{z} + v_{z} \partial_{y} \partial_{z} v_{y}]$$

$$- \partial_{z} (\rho_{0}) [v_{y} \partial_{y} v_{z} + v_{z} \partial_{z} v_{z}]$$

Viscous term:

$$(\nabla^2 p_1)_{vis} = 2\mu(\partial_y \partial_z^2 v_y + \partial_y^2 \partial_z v_z) \tag{15}$$

Magnetic field term:

$$(\nabla^2 p_1)_B = -\frac{1}{4\pi} \left[(\partial_y B_z)^2 + (\partial_z B_y)^2 - 2\partial_y (B_z)\partial_z (B_y) + B_z (\partial_y^2 B_z - \partial_y \partial_z B_y) + B_y (\partial_z^2 B_y - \partial_y \partial_z B_z) \right]$$
(16)