3) Put $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i}$

eq

1. $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho u_x)}{\partial x} = -\rho\frac{\partial u_x}{\partial x} - u_x\frac{\partial \rho}{\partial x}$$

2. First term

$$\frac{\partial \rho \vec{u}}{\partial t} = \frac{\partial \rho u_x}{\partial t}\hat{i} + \frac{\partial \rho u_y}{\partial t}\hat{j} + \frac{\partial \rho u_z}{\partial t}\hat{k}$$

second term

$$\nabla \cdot (\rho u \otimes u) = \nabla \cdot \rho \begin{bmatrix} u_x^2 & u_x u_y & u_x u_z \\ u_y u_x & u_y^2 & u_y u_z \\ u_z u_x & u_z u_y & u_z^2 \end{bmatrix}$$

$$= \frac{\partial \rho u_x^2}{\partial x}\hat{i} + \frac{\partial(\rho u_y u_x)}{\partial x}\hat{j} + \frac{\partial(\rho u_z u_x)}{\partial x}\hat{k}$$

$$= \left(\rho u_x\frac{\partial u_x}{\partial x} + u_x\underbrace{\frac{\partial \rho u_x}{\partial x}}_{\text{split more?}}\right)\hat{i}$$

$$+ \left(\rho u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial \rho u_x}{\partial x}\right)\hat{j}$$

$$+ \left(\rho u_x\frac{\partial u_z}{\partial x} + u_z\frac{\partial \rho u_x}{\partial x}\right)\hat{k}$$

Third term

$$-\nabla P_g = -\frac{\partial P_g}{\partial x}\,\hat{i}$$

Fourth term

$$\vec{J}\times\vec{B} = (\vec{\nabla}\times\vec{B})\times\vec{B}$$

$$= \left(\varepsilon_{ijk}\,\hat{e}_i\,\frac{\partial}{\partial x_j}B_k\right)\times\vec{B} = \left(\varepsilon_{i1k}\,\hat{e}_i\,\frac{\partial}{\partial x}B_k\right)\times\vec{B}$$

$$= \left(\varepsilon_{213}\,\hat{j}\,\frac{\partial}{\partial x}B_z + \varepsilon_{312}\,\hat{k}\,\frac{\partial}{\partial x}B_y\right)\times\vec{B}$$

$$= \left(-\frac{\partial B_z}{\partial x}\,\hat{j} + \frac{\partial B_y}{\partial x}\,k\right)\times\vec{B}$$

$$= \varepsilon_{i2k}\,\hat{e}_i\left(-\frac{\partial B_z}{\partial x}\right)B_k + \varepsilon_{i3k}\,\hat{e}_i\,\frac{\partial B_y}{\partial x}\,B_k$$

$$= \varepsilon_{123}\,\hat{i}\left(-\frac{\partial B_z}{\partial x}\right)B_z + \varepsilon_{321}\,\hat{k}\,\frac{\partial B_y}{\partial x}\,B_x$$

$$+ \varepsilon_{132}\,\hat{i}\,\frac{\partial B_y}{\partial x}\,B_y + \varepsilon_{231}\,\hat{j}\,\frac{\partial B_y}{\partial x}\,B_x$$

$$= -\left(B_z\frac{\partial B_z}{\partial x} + B_y\frac{\partial B_y}{\partial x}\right)\hat{i} + B_x\frac{\partial B_y}{\partial x}\,\hat{j} - B_x\frac{\partial B_y}{\partial x}\,\hat{k}$$

Giving eqs in $\hat{i}, \hat{j}, \hat{k}$:

$\hat{i}$:
$$\frac{\partial \rho u_x}{\partial t} = -\left(\rho u_x \frac{\partial u_x}{\partial x} + u_x \frac{\partial \rho u_x}{\partial x}\right) - \frac{\partial P_g}{\partial x} - \left(B_z \frac{\partial B_z}{\partial x} + B_y \frac{\partial B_y}{\partial x}\right)$$

$\hat{j}$:
$$\frac{\partial \rho u_y}{\partial t} = -\left(\rho u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial \rho u_x}{\partial x}\right) + B_x \frac{\partial B_y}{\partial x}$$

$\hat{k}$:
$$\frac{\partial \rho u_z}{\partial t} = -\left(\rho u_x \frac{\partial u_z}{\partial x} + u_z \frac{\partial \rho u_x}{\partial x}\right) - B_x \frac{\partial B_y}{\partial x}$$

3.
$$\frac{\partial e}{\partial t} = -\frac{\partial e u_x}{\partial x} - P_g \frac{\partial u_x}{\partial x}$$
$$= -u_x \frac{\partial e}{\partial x} - e\frac{\partial u_x}{\partial x} - P_g \frac{\partial u_x}{\partial x}$$

4. Here I don't put negative sign, after perfectly conducting limit from wikipedia on induction eq.

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B)$$
$$= \nabla \times \left((u_y B_z - u_z B_y)\hat{i} - (u_x B_z - u_z B_x)\hat{j} + (u_x B_y - u_y B_x)\hat{k}\right)$$

$$= \varepsilon_{213}\, \hat{j}\, \frac{\partial}{\partial x}\left(u_x B_y - u_y B_x\right) + \varepsilon_{312}\, \hat{k}\, \frac{\partial}{\partial x}\left(u_x B_z - u_z B_x\right)$$

$$\frac{\partial B}{\partial t} = \left(-\frac{\partial u_x B_y}{\partial x} + \frac{\partial u_y B_x}{\partial x}\right)\hat{j} + \left(\frac{\partial u_x B_z}{\partial x} - \frac{\partial u_z B_x}{\partial x}\right)\hat{k}$$

This gives 3 eqs:

$\hat{i}$: $$\frac{\partial B_x}{\partial t} = 0$$

$\hat{j}$: $$\frac{\partial B_y}{\partial t} = -B_y \frac{\partial u_x}{\partial x} - u_x \frac{\partial B_y}{\partial x} + B_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial B_x}{\partial x}$$

$\hat{k}$: $$\frac{\partial B_z}{\partial t} = B_z \frac{\partial u_x}{\partial x} + u_x \frac{\partial B_z}{\partial x} - u_z \frac{\partial B_x}{\partial x} - B_x \frac{\partial u_z}{\partial x}$$