Part I

The Hilbert Matrix

See the separate discussion document, “The Hilbert Matrix.”

Part II

Convolutional codes

The Jacobi method for solving the system of equations is notably less efficient. It tends to require more iterations as the size of the code increases. In testing, the Guass-Seidel iteration required a constant number of iterations (2) to produce the output. In this way, the tolerance given has negligible impact on the number of iterations required to decode the bitstring, provided the Guass-Seidel method is used. For the Jacobi method, greater tolerances allows faster solutions, but the size of the input still increases the number of iterations required.

Since we are dealing with bitstrings, high tolerances may not be acceptable. A tolerance of 1.0, for example, would not correctly decode the bitstring, since the maximum difference between 0 and 1 is 1.

Part III

Urban population dynamics

1 The Leslie matrix

A Leslie matrix describes population dynamics in terms of the aging on a population, which is split into age categories. (We will refer to them as generation of the population from this point onward.) It is notable that the first value in the table is zero. This is because every generation of the population produces offspring except for the first generation, so it is logical that the first generation has a multiplier of zero. As well, the first row of the Leslie matrix models the birthrates for the entire population. In this particular matrix, the birthrate multipliers decrease (from 1.2 to 0.1, and then 0). This can accurately model reality, as older members of the population tend to have less children, and eventually lose their ability to have children altogether.

The diagonal values in the matrix model the success of a generation to age past their current generation and into the next one. For instance, the first value in the first column (0.7) multiplies with the first generation of the current population. This number models the number of first-gen kids who either aged past the [0,10) age range or died early. The rest of the diagonal numbers similarly model the transfer of members of one generation to the next, culminating in the penultimate generation, whose members have an unsavory success rate of only 0.40. The final generation being modeled is not kept track of, although a larger matrix could easily model further generations until their deaths, when those generations reach a sub-population count of zero.

Socially, birthrates are fickle. Culture influences the average number of children desired. In largely homogeneous societies, preferences can be more easily modeled, whereas preferences in heterogenous societies tend to be more varied and unpredictable. However, intergenerational transfers of population are largely determined by access to medical care. A developing country with limited access to healthcare would have lower success multipliers across all generations, with the greatest differences more apparent in the first and last few generations as children and the elderly are particularly vulnerable to disease. (The relationship between age and success at "growing up" can be seen in the diagonal of this example matrix, as the multipliers increase up to a cap of 0.9 and then decrease with increasing age.)

2 Population predictions and changes

The table displays the population distributions, totals, and changes in total population for each year.

Table 1: Population distributions $\times 10^5$

Years	2000	2010	2020	2030	2040	2050
Total Population	14.2	17.349	18.285	22.121	26.505	33.879
$\triangle$Total Population	NA	3.149	0.936	3.836	4.384	7.374
Distribution	2.100	6.350	5.188	8.162	9.656	13.413
	1.900	1.470	4.445	3.631	5.714	6.760
	1.800	1.615	1.250	3.778	3.087	4.857
	2.100	1.620	1.454	1.125	3.400	2.778
	2.000	1.890	1.458	1.308	1.012	3.060
	1.700	1.760	1.663	1.283	1.151	0.891
	1.200	1.360	1.408	1.331	1.026	0.921
	0.900	0.924	1.047	1.084	1.025	0.790
	0.500	0.360	0.370	0.419	0.434	0.410

3 Eigenvalues and convergence

The largest eigenvalue of the example Leslie matrix was 1.2886562 with 8 digits of accuracy (tolerance = 0.00000005). Because of this, we know that the population will continue to grow and will not become stable. As a matter of fact, the population count grows past the abillity of the double-precision floating point numbers to store before we simulate 100,000 years of growth (reaching a total population of 1.4×10^{308} before the number reached the maximum double value allowed and was considered infinity).

Since the eigenvalue represents the growth-rate in the direction of the eigenvector, a positive eigenvalue greater than one models increasing population, an eigenvalue approaching zero models a stable population, and an eigenvalue less than one models a decreasing population

4 Stability and population recovery

The largest eigenvalue of the damaged population is 1.16790274.

Based on my conclusions in a previous section, the population will continue increasing. The table above also clearly shows that even with the "disaster", the population continues to grow after a small setback. The total population will

Table 2: Population Distribution $\times 10^5$ Recovery Scenario

Years	2000	2010	2020	2030	2040	2050
Total Population	14.2	17.349	17.4	21.504	25.239	32.311
$\triangle$Total Population	NA	3.149	0.0505	4.104	3.735	7.072
Distribution	2.100	6.350	4.306	8.162	8.916	12.836
	1.900	1.470	4.445	3.014	5.714	6.241
	1.800	1.615	1.250	3.778	2.562	4.857
	2.100	1.620	1.454	1.125	3.400	2.306
	2.000	1.890	1.458	1.308	1.012	3.060
	1.700	1.760	1.663	1.283	1.151	0.891
	1.200	1.360	1.408	1.331	1.026	0.921
	0.900	0.924	1.047	1.084	1.025	0.790
	0.500	0.360	0.370	0.419	0.434	0.410

follow the same pattern as before. While more drastic reductions in birthrate might cause the population to fall, unless the largest eigenvalue of the matrix is negative, the population as a whole will not trend towards zero.