“ Coset Interleaver Idea Progress”

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March 19, 2020

0.1 March 16th

An idea for the coset interleaving method is presented for interlearver length $N = n\tau,\ n = \{9, 10, ...\}$. The notations to be used are listed below.

0.1.1 Notations

1. N :- Interleaver Length, Interleaver size.
2. $\mathcal{C}^i$:- Coset $,i,\ i = \{0, 1, ..., \tau - 1\}$
3. $L = N/\tau$:- Coset length, Coset size.
4. τ :- Cycle length of RSC encoder ($\tau = 3$ for 5/7RSC encoder)
5. $p(x) = x^{p_0} + x^{p_1} + x^{p_2}$: - Polynomial representation of a weight 3-input message, where p_0, p_1, p_2 are integer values and $p_0 < p_1 < p_2$
6. $\mathcal{P}$: - the set representing the indices of the "1" bits in a weight 3-error event. $\mathcal{P} = \{p_0, p_1, p_2\}$
7. $\bar{p}(x)$:- is the normalized version of $p(x)$. $\bar{p}(x) = \frac{p(x)}{x^{\min\{\mathcal{P}\}}}$
8. Seperation:- Assume that there is a set $\mathcal{A} = \{a, b, c, ...\},\ b > a, c > b$, etc. For any 2 elements in set, the seperation is defined as the difference between the elements. For example, the seperation between b and a is $b - a$
9. Step:- Assume that there is a set $\mathcal{A} = \{a, b, c, ...\},\ b > a, c > b$, etc . For any 2 elements in set, the step is defined as the difference between the index of two elements. For example, from a to b as step of 1 is required and from a to c a step of 2 is required.
10. Strictly divisible:- Lets assume we have an integer a, and set of n integers $\mathcal{B} = \{b, c, d, ...\}$. a is strictly dividible by b if b is the only element in set $\mathcal{B}$ which divides a without a remainder.
11. D_1 :-Interleaving step
12. s :- post-interleaving seperation
13. t :- pre-interleaving seperation

0.1.2 Weight-3 input errors

Assume we have a weight 3 input message, written in polynomial form as $p(x) = x^{p_0} + x^{p_1} + x^{p_2}$ as defined above. This weight 3 input message has a potential of producing low-weight parity bits if $\mathcal{P} \mod 3 = (0, 1, 2)$. We refer to to such weight 3 inputs as weight-3 input errors. The most common of such inputs is any input such that $\bar{p}(x) = 1 + x + x^2$, which corresponds to 3 consecutive "1" bits in the input bit sequence. We wish to design an interleaver to break such inputs.

0.1.3 Interleaving Idea and Procedure

As stated above, we wish to break errors of the type $\bar{p}(x) = 1 + x + x^2$. We will break the cases into 3.

1. $N = 3n,\ n = \{0, 1, ...\}$ divisible by $3, 6$
2. $N = 3n,\ n = \{0, 1, ...\}$ and is divisible by $3, 6, 9$
3. $N = 3n,\ n = \{0, 1, ...\}$ strictly divisible 3

For each case we wish to come up with a coset interleaving pattern such that even when the interleaving step $D1$ is set to 1, there is no time when $\bar{p}(x) = 1 + x + x^2$ is interleaved to $\bar{p}(x)' = 1 + x + x^2$

case 1: $N = 3n,\ n = \{0, 1, ...\}$ **and is divisible by** $3, 6$**:-** Let $\mathcal{A}$ and $\mathcal{A}'$ represent the set before interleaving and the set after interleaving. Also, Let $\mathcal{C}^i = (c_0^i, c_1^i, ..., c_{L-1}^i)$ and $\mathcal{C}'^j = (c_0'^j, c_1'^i, ..., c_{L-1}'^i)$ prime represent the coset before interleaving and the coset after interleaving respectively. $\mathcal{C}^i \rightarrow \mathcal{C}'^j$ represents the mapping of an element in $\mathcal{C}^i$ to an element in $\mathcal{C}'^j$.

by adapting the interleaving pattern to be described,we can get rid rid of errors of the form $\bar{p}(x) = 1 + x + x^2$. However, the next dominant weight-3 input error which remains is

$$p(x) = (1 + x^2 + x^4)x^t,\ t = 0, 1, ..., N - 4 \tag{0-1}$$

The interleaving procedure is as follows

1. seperate the elements in $\mathcal{A}$ and seperate them into even-numbers and odd numbers.
2. take the even-numbered elements in $\mathcal{A}$, $(a_0, a_2, a_4, ...)$. in coset notation, it can be written as $\{c_0^0, c_0^2, c_1^1, c_2^0, c_2^2, c_3^1, ...\}$
3. Map the first even-numbered element c_0^0 to $c_0'^{(0)}$, the second element c_0^2 to and $c_{D_1}'^{(1)}$, the third element c_1^1 to $c_{2D_1}'^{(2)}$, the fourth element c_2^0 to $c_{3D_1}'^{(0)}$ and continue this process untill all the even-numbered elements in $\mathcal{A}$ are mapped to elements in $\mathcal{A}'$
4. For the odd-numbered elements repeat steps 3 with the first odd-numbered element c_0^1 being mapped to $c_{ND_1/2}'^{(0)}$untill all odd-numbered elements in $\mathcal{A}$ have been mapped to elements in $\mathcal{A}'$

It is worth noting that the product $a_i D_1$ is calculated modulo L.

We need to select $D1$ such that for all values of t, $p(x) = (1 + x^2 + x^4)x^t$ is mapped to a weight 3 input which produces a parity bit sequence with a large weight.

0.1.4 Maximizing the Seperation for Weight 3 error events for case 1

Let $\bar{p}(x) = 1 + x^{p_1} + x^{p_2}$ be a normalized weight-3 input error event polynomial. Using the interleaving procedure described above, we wish to transform it into a normalized weight 3 output error event $\bar{p}(x)' = 1 + x^{q_1} + x^{q_2}$, where $q_1 >> p_1$ and $q_2 >> p_2$ The corresponding Hamming weight for the turbo codeword can be calculated using the formula below.

$$
\begin{aligned}
w_H &= 2\left(\frac{p_2 - (p_2 \mod 3)}{3}\right) + 2 + 2\left(\frac{q_2 - (q_2 \mod 3)}{3}\right) + 2 + 3 \\
&= 2\left(\frac{\Big(p_2 - (p_2 \mod 3)\Big) + \Big(q_2 - (q_2 \mod 3)\Big)}{3}\right) + 7
\end{aligned} \tag{0-2}
$$

to maximize the values of q_1 and q_2, using the interleaving method described above, we choose

$$D_1 = \max\left\{a \leq \lfloor \tfrac{L}{3} \rfloor \mid \gcd(a, L) = 1\right.$$

0.1.5 Codeword Weight for Weight-3 input events

The equation for calculating the hamming weight for weight-3 error events is

$$w_H = 2\left(\frac{\Big(p_2 - (p_2 \mod 3)\Big) + \Big(q_2 - (q_2 \mod 3)\Big)}{3}\right) + 7 \tag{0-3}$$

From the interleaving patern explained above,we already know that $p_2 = 4$, what we are left with is finding the value of q_2. By writing q_2 in terms of D_2, we should be able to easily calculate the corresponding hamming weights. let $p(x)' = x^{r_0} + x^{r_1} + x^{r_2}$ and $\bar{p}(x)' = 1 + x^{q_1} + x^{q_2}$ be the polynomial representation of the interleaved weight 3 error sequence and its normalized polynomial representation respectively. Depending on wether the minimum power of $p(x)'$ is odd or even, the value of q_2 for $\bar{p}(x)'$ is affected. Therefore we need to find the value of q_2 for both even and odd cases to be able to accurately predict the minimum weight of the interleaved sequence.

1. Case where the minimum power of $p(x)'$ is even For this case, q_2 is very easy to calculate if we assume that the minimum power of $p(x)'$ is 0. This means $q_2 = (\tau D_1 p_2/2) + (p_2/2 \mod \tau) \mod N$ and

2. Case where the minimum power of $p(x)'$ is odd This case is a little more comlpicated. First, we need to find the values for r_0, r_1, r_2, whic will be equal to $\tau D_1(N/2)(N/2 \mod \tau) \mod N$, $, \tau D_1(N/2+1)(N/2+1 \mod \tau) \mod N$, $, \tau D_1(N/2+2)(N/2+2 \mod \tau) \mod N$ respectively.

Next, find $z = \min\{r_1, r_2, r3\}$. Then $q2 = \max\{r_1 - z, r2 - z, r3 - z\}$

Using this procedure for Interleaver of length $N = 258$ and $D = 29$ for the even case, $q_2 = 176$ and for the odd case $r_1 =, 129, r_2 = 217, r_3 = 47$ and $q_2 = 170$

The corresponding Hamming distances for both cases are 125 for the even case and 121 for the even case. This means we can have a minimum hamming weight of up to 121 for weight-3 error events using the above interleaving procedure.