

# Review of Deterministic Interleavers

Kwame Ackah Bohulu

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## 1 Introduction

Turbo codes are created by the parallel concatenation of two Recursive Systematic Convolutional (RSC) codes via an interleaver. The performance of turbo codes is dependent on its effective free distance. The effective free distance is the minimum distance associated with input weight 2 error events [2]. Input weight 2 error events that are small multiples of the cycle length of the component code tend to produce low-weight codewords. It is desirable to prevent the occurrence of such error events in both component codes to prevent low-weight Turbo codewords. The interleaver reorders input information bits before feeding them into the second component encoder. By designing an interleaver in such a way that it prevents input weight 2 error events present in the first component encoder from occurring in the second component encoder we will be able to increase the effective free distance of the turbo code and improve its performance.

## 2 Case 1: Prevention of duplication of weight 2 error events of length $d$

An input weight error event is defined as an error event with two information bits in error[2]. Each input weight error event in a turbo code corresponds to one input weight error event in each of the two component codes. Consider a typical input weight 2 error event shown in the diagram below. In the first component code, it begins at position  $x$  and ends at position  $x + d$  where

$$x \in \mathbb{Z}$$

and

$$d \in \tau \cdot \mathbb{Z} \triangleq \mathbb{C}$$

where  $\tau$  is the cycle length of the component encoder. We represent the start and end of the error event with the integer pair  $(x, x + d)$ . These points are then rearranged via the interleaver to positions  $\alpha x$  and  $\alpha x + d$  and the distance between these positions is represented by

$$(\alpha(x + d), \alpha(d)) \triangleq \alpha(x + d) - \alpha(d)$$

. Since  $d \in \mathbb{C}$ , we may rewrite  $d$  as  $a\tau$ , where  $a$  is a small integer number. We wish to design an interleaver such that

$$(\alpha(x+d), \alpha(d)) \notin \mathbb{C} \triangleq \mathbb{E}(\alpha(x+d), \alpha(d)) \quad \forall x \in \mathbb{Z}, d \in \mathbb{C}$$

## 2.1 Linear Interleaver design

The index mapping equation for the proposed linear interleaver is given by (1)

$$\alpha(x)_{\mathcal{L}_N} = x(\tau^2 - z\tau + 1) \mod N \quad (1)$$

where  $N = 2^n$ ,  $n \in \mathbb{R}$  is the interleaver size,  $\tau$  is the cycle length of the component encoder,  $z$  is the largest odd integer within the range 1 to  $\sqrt{2N}$ .

$N = 2^n, (\alpha(x+d), \alpha(x))$								
n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	$\tau$
5	13	25	55	113	235	481	981	1
6	6	6	30	70	174	390	854	2
1	1	13	59	5	79	245	649	3
4	4	20	20	52	212	52	372	4
5	5	1	47	89	67	329	29	5
2	10	26	18	122	162	58	650	6
1	9	5	3	29	247	269	193	7
0	8	8	8	72	72	456	712	8

Table 1: Values for  $(\alpha(x+d), \alpha(x))$ ,  $a=1$

$N = 2^n, (\alpha(x+d), \alpha(x))$								
n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	$\tau$
2	10	18	46	98	214	450	938	1
4	12	12	60	12	92	268	684	2
2	2	26	54	10	158	490	274	3
0	8	8	40	104	168	104	744	4
2	10	2	30	50	134	146	58	5
4	4	20	36	116	68	116	276	6
2	2	10	6	58	238	26	386	7
0	0	16	16	16	144	400	400	8

Table 2: Values for  $(\alpha(x+d), \alpha(x))$ ,  $a=2$

$N = 2^n, (\alpha(x+d), \alpha(x))$								
n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	$\tau$
7	7	11	37	83	193	419	895	1
2	2	18	26	82	10	146	514	2
3	3	7	49	15	237	223	923	3
4	12	28	60	28	124	156	92	4
7	15	3	13	11	201	475	87	5
6	14	14	54	110	230	174	926	6
3	11	15	9	87	229	295	579	7
0	8	24	24	88	216	344	88	8

Table 3: Values for  $(\alpha(x+d), \alpha(x))$ ,  $a=3$

### 3 References

- [1] Oscar Y. Takeshita, Member, IEEE, and Daniel J. Costello, "New Deterministic Interleaver Designs for Turbo Codes", IEEE Trans. Inform. Theory, vol. 46, pp. 1988-2006, Nov. 2000
  
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- [3] Jing Sun, Oscar Y. Takeshita " Interleavers for Turbo Codes Using Permutation Polynomials over Integer Rings" , IEEE Trans. Inform. Theory, vol. 51, pp. 101 - 119 Jan. 2005