Deterministic Interleaver Design for Turbo Codes

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22-06-2017

1 Abstract

input weight 2m error events with the distance between the bit '1' seperated by a multiple of the componet codes cycle length ($a - \tau$ seperated input weight 2m error events) tend to produce low-weight turbo codewords if present in both component codes. In this research paper, we introduce a method for designing interleavers in such a way that $a-\tau$ seperated input weight 2m error events in the first component encoder are not mapped unto the second component encoder. Using this method, we find good interleavers for specified frame lengths and component codes.

2 Introduction

Turbo Codes are amongst the class of capacity approaching FEC codes that were discovered in 1993 by Claude Bearoux. They are constructed by the parallel concatenation of 2 Recursive Systematic Convolutional (RSC) Encoders via an interleavers. Diagram for

Decoding of Turbo codes is done using the Turbo Decoder. It is made up 2 Soft Input Soft Output (SISO) Decoders. The interleaver plays a very important role in Turbo codes as it reduces the number of low-weight codewords(multiplicity) of the Turbo code [4]. However, the existence of the low-weight codewords causes the turbo codes to have a high error floor in the high SNR region. A lot of research has been done concerning interleavers for turbo codes and they are generally put into 2 groups, Random and Deterministic interleavers.

Turbo codes implemented using Random interleavers have been shown to have good performance, especially for large frame sizes [3]. The disadvantage of using Random interleavers stems from required use of interleaver tables in both the encoder and decoder, which is undesirable in many applications. Deterministic interleaver on the other hand require no such interleaver tables and the logic behind interleaving and deinterleaving can be executed by means of algorithms. Due to this advantage, a lot of Turbo codes employ Deterministic interleavers in their construction. Most noticable amongst these is the Quadratic Permutation Polynomial (PP2) interleaver [3] which is used in LTE applications.

Despite this advantage, Deterministic interleavers that outperform Random interleavers, especially for large frame sizes have yet to be discovered. The aim of this research is to design an interleaver that has a performance that is at least as good as that of random interleavers for large frame sizes.

2.1 Turbo Encoding and Decoding

The Turbo encoder is shown in figure (1).

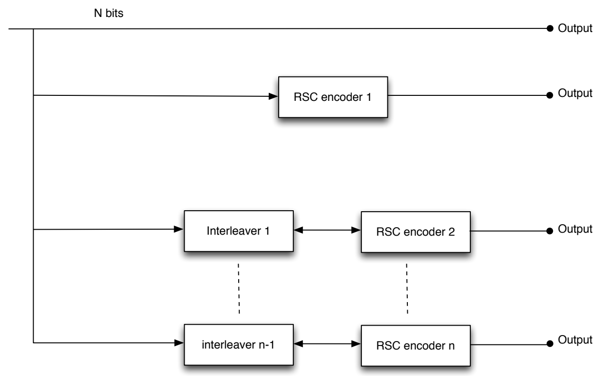

Figure 1: Turbo Encoder

The input information bits of length N is fed into the first RSC encoder which produces the first parity check bits. The input to the second RSC encoder is an interleaved version input information bits and this is used to generate the second parity check bits. Both parity check bits have length N which gives an output rate of 1/3.The RSC encoders used are known as component encoders. Usually identical component encoders are used.

Decoding of Turbo Codes is done using the Turbo Decoder which is made up of 2 Soft Input Soft Output (SISO) decoders. The Turbo decoder is shown in (2). An iterative decoding scheme based on the BCJR algorithm is employed in the decoding of turbo codes.

The turbo decoding process is as follows

1 The input to the first component decoder is $(\boldsymbol{y}^s, \boldsymbol{y}^p)$ and relates to the systematic bits and the parity bits of the first component encoder of the Turbo encoder.

2 The Log-Likelihood Ratio (LLR),$L(u_i)$ is calculated. For the first iteration, it is assumed that the input information bits have equal probability and the a-priori LLR value $L^{(a)}(u_i)$ is set to 0

3 At the output of the first componet decoder, the channel LLR values, $L_c y_i^s$ is subtracted from $L(u_i)$ to yield the extrinsic LLR values of the first component decoder, $L_{12}^{(e)}(u_i)$. $L_{12}^{(e)}(u_i)$ is then interleaved and fed into the second component decoder as the value for $L^{(a)}(u_i)$.

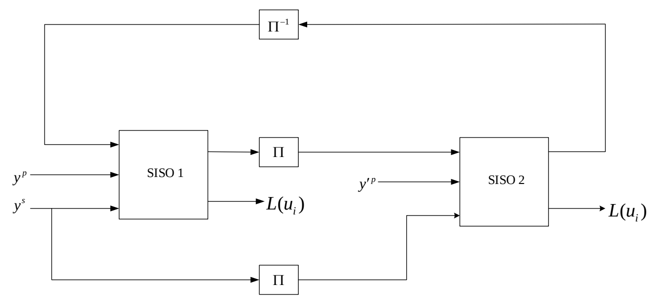

Figure 2: Turbo Decoder

4 The other inputs to the second component decoder are $(\boldsymbol{y'}^s, \boldsymbol{y'}^p)$ which correspond to the interleaved systematic bits and the parity bits of the second component encoder. These are used to calculate the extrinsic LLR values of the second component decoder,$L_{21}^{(e)}(u_i)$.

5 $L_{21}^{(e)}(u_i)$ is deinterleavedand fedback into the first component encoder as the new $L^{(a)}(u_i)$ value.

The iteration is either repeated for a predetermined number of times, or untill a certain condition is met. At the final iteration $L(u_i)$ (from the second component decoder)is deinterleaved and used to estimate the value ofu_i.

3 Methodology

Turbo codes have error floor in the high SNR region. This has been attributed to the prescence of low-weight codewords. The error floor of the Turbo codes can be raised by increasing the interleaver size whiles maintaining the effective free distance. Alternatively increasing the effective free distance of the Turbo while maintaining the multiplicity serves a similar purpose [2]. The effective free distance of a code is the minimum distance associated with an input of weight 2.

In RSC encoders, weight 2 inputs of the form $(1+D^{t\tau})(D^u), 0 \leq u \leq N-\tau, t = \{1, 2, 3, ...\}$ tend to produce low-weight codeword [1]. τ is the cycle length of the

RSC encoder. We shall call such low-weight producing weight 2 inputs $a-\tau$-seperated input weight 2 errors. If the $a-\tau$-seperated input weight 2 errors are input into the Turbo codes component encoders a low-weight codeword will be produced.

To increase effective free distance of the turbo code, we design an interleaver in such a way that the input to the second is of the form

$$(1 + D^{c\tau})(D^u), 0 \leq u \leq N - \tau \tag{1}$$

where c is a large number.

3.1 Linear interleaver design

The mapping function for the linear interleaver is given by

$$\Pi_{\mathbf{L}_n}(i) \equiv bi \mod N,\ 0 \leq i \leq N \tag{2}$$

where b is a positive integer that is co-prime to N. The simplest $a-\tau$-seperated input weight 2 error (t=1) is shown in the figure below.

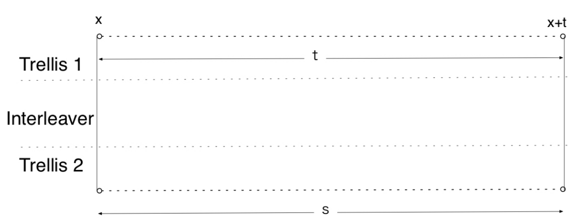

Figure 3: $t = s = \tau$

s is calculated using the equation below.

$$\begin{aligned} s &= \Pi_{\mathbf{L}_n}(x+t) - \Pi_{\mathbf{L}_n}(x) \\ &= b(x+t) - b(x) \mod N \\ &= bt \mod N \end{aligned} \tag{3}$$

The weight of the codeword can be calculated using the equation below.

$$d_{(t_i,s_j)} = w_o\left(3 + \left(\frac{|t_i|}{\tau} + \frac{|s_j|}{\tau}\right)\right) \tag{4}$$

Substituting (3) into (4) and rewriting t as τ gives

$$d_{(t_i,s_j)} = w_o\left(3 + \left(1 + \frac{b\tau \mod N}{\tau}\right)\right) \tag{5}$$

It should be noted that values of b that are considered should satisfy the condition

$$((b\tau \mod N) \mod \tau) \neq 0 \tag{6}$$

The process for choosing the value of b that changes the input to the second component encoder into the form in (1) is outlined below.

1. For a given value of b , $1 \leq b \leq N/2$ which satisfies (6) and all possible inputs of the form $(1 + D^{t\tau})(D^u), 0 \leq u \leq N - \tau, t = 1$, calculate corresponding s using (3)

2. Calculate the Hamming distance for the codeword using equation (4) and select min $d_{(t_i,s_j)}$

3. After min $d_{(t_i,s_j)}$ is selected for all possible values of b, the value of b which corresponds to max(min $d_{(t_i,s_j)_v}$)

The tables below show the values for b selected for $t = \tau$ and $t = 2\tau$ for various frame sizes.

Table 1: $N = 2^m$, $m = \{10, 11, 12, 13, 14\}$, $t = \tau$

b	min(s)	max(s)
511	509	515
1023	1021	1027
2047	2045	2051
4095	4093	4099
8191	8189	8195

Table 2: $N = 2^m$, $m = \{10, 11, 12, 13, 14\}$, $t = 2\tau$

b	min(s)	max(s)
427	510	514
853	1022	1026
1707	2046	2050
3413	4094	4098
6827	8190	8194

4 References

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