Formula to Calculate Weight for Low-Weight Weight 3 Inputs and Proof

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0.1 Equation and Proof

Proof. Since the impulse response is

$$(1\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ \cdots)$$

Let
$\boldsymbol{\phi}_1 = (0\ 0\ 1),\ \boldsymbol{\phi}'_1 = (0\ 1\ 0),\ \boldsymbol{\phi}''_1 = (1\ 0\ 0),$
$\boldsymbol{\phi}_2 = (0\ 1\ 1),\ \boldsymbol{\phi}'_2 = (1\ 1\ 0),\ \boldsymbol{\phi}''_2 = (1\ 0\ 1),$
$\boldsymbol{\phi}_3 = (1\ 1\ 1).$

Then, the weight-3 RTZ occurs since $\boldsymbol{\phi}_2 + \boldsymbol{\phi}'_2 + \boldsymbol{\phi}''_2 = \mathbf{0}_3$.

Now, we consider the weight of the vector derived by the sumation of the followings vectors.

$$\begin{array}{c}(\mathbf{0}_{3i}\ \boldsymbol{\phi}_1\ \boldsymbol{\phi}'_2\ \cdots) \\ (\mathbf{0}_{3j}\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}''_2\ \cdots) \\ (\mathbf{0}_{3k}\ \boldsymbol{\phi}_3\ \boldsymbol{\phi}_2\ \cdots)\end{array}$$

To simplify calculation, we have included an addition table for all the vectors which is shown in Table 1

	$\boldsymbol{\phi}_1$	$\boldsymbol{\phi}'_1$	$\boldsymbol{\phi}''_1$	$\boldsymbol{\phi}_2$	$\boldsymbol{\phi}'_2$	$\boldsymbol{\phi}''_2$	$\boldsymbol{\phi}_3$
$\boldsymbol{\phi}_1$	$\mathbf{0}_3$	–	–	–	–	–	–
$\boldsymbol{\phi}'_1$	$\boldsymbol{\phi}_2$	$\mathbf{0}_3$	–	–	–	–	–
$\boldsymbol{\phi}''_1$	$\boldsymbol{\phi}''_2$	$\boldsymbol{\phi}'_2$	$\mathbf{0}_3$	–	–	–	–
$\boldsymbol{\phi}_2$	$\boldsymbol{\phi}'_1$	$\boldsymbol{\phi}_1$	$\boldsymbol{\phi}_3$	$\mathbf{0}_3$	–	–	–
$\boldsymbol{\phi}'_2$	$\boldsymbol{\phi}_3$	$\boldsymbol{\phi}''_1$	$\boldsymbol{\phi}'_1$	$\boldsymbol{\phi}''_2$	$\mathbf{0}_3$	–	–
$\boldsymbol{\phi}''_2$	$\boldsymbol{\phi}''_1$	$\boldsymbol{\phi}_3$	$\boldsymbol{\phi}_1$	$\boldsymbol{\phi}'_2$	$\boldsymbol{\phi}_2$	$\mathbf{0}_3$	–
$\boldsymbol{\phi}_3$	$\boldsymbol{\phi}'_2$	$\boldsymbol{\phi}''_2$	$\boldsymbol{\phi}_2$	$\boldsymbol{\phi}''_1$	$\boldsymbol{\phi}_1$	$\boldsymbol{\phi}'_1$	$\mathbf{0}_3$

Table 1: Truth Table

Furthermore, we consider 4 general cases for all possible values of i, j, k These cases are $(=\ \ =)$, $(=\ \ <)$, $(<\ \ =)$ and $(<\ \ <)$

Case 0: $i = j = k$

For this case, the vectors to sum will be

$$\begin{array}{c}(\mathbf{0}_{3i}\ \boldsymbol{\phi}_1\ \boldsymbol{\phi}'_2\ \cdots) \\ (\mathbf{0}_{3j}\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}''_2\ \cdots) \\ (\mathbf{0}_{3k}\ \boldsymbol{\phi}_3\ \boldsymbol{\phi}_2\ \cdots) \\ \hline (\mathbf{0}_{3i}\ \boldsymbol{\phi}''_2\ \mathbf{0}_3\ \cdots)\end{array}$$

and the derived vector will be $(\mathbf{0}_{3i}\ \boldsymbol{\phi}''_2\ \mathbf{0}_3\ \cdots)$ with a weight of $w_p = 2$

Case 1a: $i = j < k$
vector to sum:

$$\begin{array}{r} (\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_1\ \phi_2'\ \cdots\ \phi_2'\ \phi_2'\ \phi_2'\ \cdots) \\ (\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_2\ \phi_2''\ \cdots\ \phi_2''\ \phi_2''\phi_2''\ \cdots) \\ +(\mathbf{0}_3\ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_3\ \phi_2\ \cdots) \\ \hline (\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_1'\ \phi_2\ \cdots\ \phi_2\ \phi_1''\ \mathbf{0}_3\ \cdots) \end{array}$$

derived vector : $(\mathbf{0}_{3j}\ \phi_1'\ (\phi_2)_{k-j-1}\ \phi_1''\ \mathbf{0}_3\ \cdots)$
Parity weight:

$$w_p = 2(k-j) \tag{0-1}$$

Case 1b: $i = k < j$
vectors to sum:

$$\begin{array}{r} (\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_1\ \phi_2'\ \phi_2'\ \phi_2'\ \phi_2'\ \cdots) \\ (\mathbf{0}_3\cdots\cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_2\ \phi_2''\ \cdots) \\ +(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_3\ \phi_2\ \phi_2\ \phi_2\ \phi_2\ \cdots) \\ \hline (\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_2'\ \phi_2''\ \phi_2''\ \phi_2'\ \mathbf{0}_3\ \cdots) \end{array}$$

derived vector : $(\mathbf{0}_{3i}\ \phi_2'\ (\phi_2'')_{j-k-1}\ \phi_2'\ \mathbf{0}_3\ \cdots)$
Parity weight:

$$w_p = 2(j-i) + 2 \tag{0-2}$$

Case 1c: $j = k < i$
vectors to sum:

$$\begin{array}{r} (\mathbf{0}_3\ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_1\ \phi_2'\ \cdots) \\ (\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_2\ \phi_2''\ \cdots\ \phi_2''\ \phi_2''\phi_2''\ \cdots) \\ +(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_3\ \phi_2\ \cdots\ \phi_2\ \phi_2\ \phi_2\ \cdots) \\ \hline (\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_1''\ \phi_2'\ \cdots\ \phi_2'\ \phi_3\ \mathbf{0}_3\ \cdots) \end{array}$$

derived vector : $(\mathbf{0}_{3j}\ \phi_1''\ (\phi_2)_{i-j-1}\ \phi_3\ \mathbf{0}_3\ \cdots)$
Parity weight:

$$w_p = 2(i-j) + 2 \tag{0-3}$$

Case 2a: $i < j = k$
vectors to sum:

$$
\begin{array}{r}
(\mathbf{0}_3 \ \cdots \ \boldsymbol{\phi}_1 \ \boldsymbol{\phi}_2' \ \cdots \ \boldsymbol{\phi}_2' \ \boldsymbol{\phi}_2' \ \boldsymbol{\phi}_2' \ \cdots) \\
(\mathbf{0}_3 \ \cdots \ \cdots \ \cdots \ \cdots \ \mathbf{0}_3 \ \phi_2 \ \phi_2'' \ \cdots) \\
+(\mathbf{0}_3 \ \cdots \ \cdots \ \cdots \ \cdots \ \mathbf{0}_3 \ \phi_3 \ \phi_2 \ \cdots) \\
\hline
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \boldsymbol{\phi}_1 \ \boldsymbol{\phi}_2' \ \cdots \ \boldsymbol{\phi}_2' \ \boldsymbol{\phi}_1' \ \mathbf{0}_3 \ \cdots)
\end{array}
$$

derived vector : $(\mathbf{0}_{3i} \ \boldsymbol{\phi}_1 \ (\boldsymbol{\phi}_2')_{k-i-1} \ \boldsymbol{\phi}_1' \ \mathbf{0}_3 \ \cdots)$
Parity weight:

$$w_p = 2(k - i) \tag{0-4}$$

Case 2b: $j < k = i$
vectors to sum:

$$
\begin{array}{r}
(\mathbf{0}_3 \ \cdots \ \cdots \ \cdots \ \cdots \ \mathbf{0}_3 \ \phi_1 \ \phi_2' \ \cdots) \\
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \phi_2 \ \phi_2'' \ \cdots \ \phi_2'' \ \phi_2'' \ \phi_2'' \ \cdots) \\
+(\mathbf{0}_3 \ \cdots \ \cdots \ \cdots \ \cdots \ \mathbf{0}_3 \ \phi_3 \ \phi_2 \ \cdots) \\
\hline
(\mathbf{0}_3 \ \cdots \mathbf{0}_3 \ \boldsymbol{\phi}_2 \ \boldsymbol{\phi}_2'' \ \cdots \ \boldsymbol{\phi}_2'' \ \boldsymbol{\phi}_2 \ \mathbf{0}_3 \ \cdots)
\end{array}
$$

derived vector : $(\mathbf{0}_{3j} \ \boldsymbol{\phi}_2 \ \boldsymbol{\phi}_2'')_{i-j-1} \ \boldsymbol{\phi}_2 \ \mathbf{0}_3 \ \cdots)$
Parity weight:

$$w_p = 2(i - j) + 2 \tag{0-5}$$

Case 2c: $k < i = j$
vectors to sum:

$$
\begin{array}{r}
(\mathbf{0}_3 \ \cdots \ \cdots \ \cdots \ \cdots \ \mathbf{0}_3 \ \phi_1 \ \phi_2' \ \cdots) \\
(\mathbf{0}_3 \ \cdots \ \cdots \ \cdots \ \cdots \ \mathbf{0}_3 \ \phi_2 \ \phi_2'' \ \cdots) \\
+(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \phi_3 \ \phi_2 \ \cdots \ \phi_2 \ \phi_2 \ \phi_2 \ \cdots) \\
\hline
(\mathbf{0}_3 \ \cdots \mathbf{0}_3 \ \boldsymbol{\phi}_3 \ \boldsymbol{\phi}_2 \ \cdots \ \boldsymbol{\phi}_2 \ \boldsymbol{\phi}_1 \ \mathbf{0}_3 \ \cdots)
\end{array}
$$

derived vector : $(\mathbf{0}_{3k} \ \boldsymbol{\phi}_3 \ (\boldsymbol{\phi}_2)_{j-k-1} \ \boldsymbol{\phi}_1 \ \mathbf{0}_3 \ \cdots)$
Parity weight:

$$w_p = 2(j - k) + 2 \tag{0-6}$$

Case 3a: $i < j < k$
vectors to sum:

$$
\begin{array}{l}
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \phi_1 \ \phi_2' \ \cdots \ \phi_2' \ \phi_2' \ \phi_2' \ \cdots \ \phi_2' \ \phi_2' \ \phi_2' \ \cdots) \\
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \mathbf{0}_3 \ \mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \phi_2 \ \phi_2'' \ \cdots \ \phi_2'' \ \phi_2'' \ \phi_2'' \ \cdots) \\
+(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \mathbf{0}_3 \ \mathbf{0}_3 \ \cdots \ \cdots \ \cdots \ \cdots \ \mathbf{0}_3 \ \phi_3 \ \phi_2 \ \cdots) \\
\hline
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \boldsymbol{\phi}_1 \ \boldsymbol{\phi}_2' \cdots \boldsymbol{\phi}_2' \ \boldsymbol{\phi}_2'' \ \boldsymbol{\phi}_2 \ \cdots \ \boldsymbol{\phi}_2 \ \boldsymbol{\phi}_1'' \ \mathbf{0}_3 \ \cdots)
\end{array}
$$

derived vector : $(\mathbf{0}_{3i} \ \boldsymbol{\phi}_1 \ (\boldsymbol{\phi}_2')_{j-i-1} \ \boldsymbol{\phi}_2'' \ (\boldsymbol{\phi}_2)_{k-j-1} \ \boldsymbol{\phi}_1'' \ \mathbf{0}_3 \ \cdots)$
Parity weight:

$$
\begin{aligned}
w_p &= 2(j-i) + 1 + 2(k-j-1) + 1 \\
&= 2(k-i)
\end{aligned} \tag{0-7}
$$

Case 3b: $i < k < j$
vectors to sum:

$$
\begin{array}{l}
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \phi_1 \ \phi_2' \ \cdots \ \phi_2' \ \phi_2' \ \phi_2' \ \cdots \ \phi_2' \ \phi_2' \ \phi_2' \cdots) \\
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \mathbf{0}_3 \ \mathbf{0}_3 \ \cdots \ \cdots \ \cdots \ \cdots \ \mathbf{0}_3 \ \phi_2 \ \phi_2'' \cdots) \\
+(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \mathbf{0}_3 \ \mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \phi_3 \ \phi_2 \ \cdots \ \phi_2 \ \phi_2 \ \phi_2 \cdots) \\
\hline
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \boldsymbol{\phi}_1 \ \boldsymbol{\phi}_2' \cdots \ \boldsymbol{\phi}_2' \ \boldsymbol{\phi}_1 \ \boldsymbol{\phi}_2'' \ \cdots \ \boldsymbol{\phi}_2'' \ \boldsymbol{\phi}_2' \ \mathbf{0}_3 \cdots)
\end{array}
$$

derived vector : $(\mathbf{0}_{3i} \ \boldsymbol{\phi}_1 \ (\boldsymbol{\phi}_2')_{k-i-1} \ \boldsymbol{\phi}_1 \ (\boldsymbol{\phi}_2'')_{j-i-1} \ \boldsymbol{\phi}_2' \ \mathbf{0}_3 \ \cdots)$
Parity weight:

$$
\begin{aligned}
w_p &= 2(k-i) + 2(j-k) \\
&= 2(j-i)
\end{aligned} \tag{0-8}
$$

Case 3c: $j < k < i$
vectors to sum:

$$
\begin{array}{l}
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \mathbf{0}_3 \ \mathbf{0}_3 \ \cdots \ \cdots \ \cdots \ \cdots \ \mathbf{0}_3 \ \phi_1 \ \phi_2' \cdots) \\
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \phi_2 \ \phi_2'' \ \cdots \ \phi_2'' \ \phi_2'' \ \phi_2'' \ \cdots \ \phi_2'' \ \phi_2'' \ \phi_2'' \cdots) \\
+(\mathbf{0}_3 \ \cdots \mathbf{0}_3 \ \mathbf{0}_3 \ \mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \phi_3 \ \phi_2 \ \cdots \ \phi_2 \ \phi_2 \ \phi_2 \cdots) \\
\hline
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \boldsymbol{\phi}_2 \ \boldsymbol{\phi}_2'' \cdots \ \boldsymbol{\phi}_2'' \ \boldsymbol{\phi}_1' \ \boldsymbol{\phi}_2' \cdots \ \boldsymbol{\phi}_2' \ \boldsymbol{\phi}_3 \ \mathbf{0}_3 \cdots)
\end{array}
$$

derived vector : $(\mathbf{0}_{3k} \ \boldsymbol{\phi}_2 \ (\boldsymbol{\phi}_2'')_{k-j-1} \ \boldsymbol{\phi}_1' \ (\boldsymbol{\phi}_2')_{i-k-1} \ \boldsymbol{\phi}_3 \ \mathbf{0}_3 \ \cdots)$
Parity weight:

$$
\begin{aligned}
w_p &= 2(k-j) + 1 + 2(i-k) + 1 \\
&= 2(i-j) + 2
\end{aligned} \tag{0-9}
$$

Case 3d: $j < i < k$

vectors to sum:

$$\begin{array}{r}
(\mathbf{0}_3\ \cdots \mathbf{0}_3\ \mathbf{0}_3\ \mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_1\ \phi_2'\ \cdots\ \phi_2'\ \phi_2'\ \phi_2' \cdots)\\
(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_2\ \phi_2''\ \cdots\ \phi_2''\ \phi_2''\ \phi_2''\ \cdots\ \phi_2''\ \phi_2''\ \phi_2'' \cdots)\\
+(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \mathbf{0}_3\ \mathbf{0}_3\ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_3\ \phi_2 \cdots)\\
\hline
(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_2'' \cdots\ \boldsymbol{\phi}_2''\ \boldsymbol{\phi}_1''\ \boldsymbol{\phi}_2 \cdots\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_1''\ \mathbf{0}_3 \cdots)
\end{array}$$

derived vector : $(\mathbf{0}_{3j}\ \boldsymbol{\phi}_2\ (\boldsymbol{\phi}_2'')_{i-j-1}\ \boldsymbol{\phi}_1''\ (\boldsymbol{\phi}_2)_{k-i-1}\ \boldsymbol{\phi}_1''\ \mathbf{0}_3\ \cdots)$

Parity weight:

$$\begin{aligned} w_p &= 2(i-j)+1+2(k-i-1)+1 \\ &= 2(k-j) \end{aligned} \tag{0-10}$$

Case 3e: $k < i < j$

vectors to sum:

$$\begin{array}{r}
(\mathbf{0}_3\ \cdots \mathbf{0}_3\ \mathbf{0}_3\ \mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_1\ \phi_2'\ \cdots\ \phi_2'\ \phi_2'\ \phi_2' \cdots)\\
(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \mathbf{0}_3\ \mathbf{0}_3\ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_2\ \phi_2'' \cdots)\\
+(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_3\ \phi_2\ \cdots\ \phi_2\ \phi_2\ \phi_2\ \cdots\ \phi_2\ \phi_2\ \phi_2 \cdots)\\
\hline
(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_3\ \boldsymbol{\phi}_2 \cdots\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_1'\ \boldsymbol{\phi}_2'' \cdots\ \boldsymbol{\phi}_2''\ \boldsymbol{\phi}_2'\ \mathbf{0}_3 \cdots)
\end{array}$$

derived vector : $(\mathbf{0}_{3k}\ \boldsymbol{\phi}_3\ (\boldsymbol{\phi}_2)_{i-k-1}\ \boldsymbol{\phi}_1'\ (\boldsymbol{\phi}_2'')_{j-i-1}\ \boldsymbol{\phi}_2'\ \mathbf{0}_3\ \cdots)$

Parity weight:

$$\begin{aligned} w_p &= 2(i-k)+2+2(j-i) \\ &= 2(j-k)+2 \end{aligned} \tag{0-11}$$

Case 3f: $k < j < i$

$$\begin{array}{r}
(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \mathbf{0}_3\ \mathbf{0}_3\ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_1\ \phi_2' \cdots)\\
(\mathbf{0}_3\ \cdots \mathbf{0}_3\ \mathbf{0}_3\ \mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_2\ \phi_2''\ \cdots\ \phi_2''\ \phi_2''\ \phi_2'' \cdots)\\
+(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_3\ \phi_2\ \cdots\ \phi_2\ \phi_2\ \phi_2\ \cdots\ \phi_2\ \phi_2\ \phi_2 \cdots)\\
\hline
(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_3\ \boldsymbol{\phi}_2 \cdots\ \boldsymbol{\phi}_2\ \mathbf{0}_3\ \boldsymbol{\phi}_2' \cdots\ \boldsymbol{\phi}_2'\ \boldsymbol{\phi}_3\ \mathbf{0}_3 \cdots)
\end{array}$$

derived vector : $(\mathbf{0}_{3k}\ \boldsymbol{\phi}_3\ (\boldsymbol{\phi}_2)_{j-k-1}\ \mathbf{0}_3\ (\boldsymbol{\phi}_2')_{i-j-1}\ \boldsymbol{\phi}_3\ \mathbf{0}_3\ \cdots)$

Parity weight:

$$\begin{aligned} w_p &= 2(j-k)+1+2(i-j)+1 \\ &= 2(i-k)+2 \end{aligned} \tag{0-12}$$

From all the above cases we can conclude that the parity weight for a weight-3 RTZ sequence may be calculated as

$$w_p = \begin{cases} 2l, & i < k \\ 2l+2 & i \geq k \end{cases} \tag{0-13}$$

where $l = \max\{i, j, k\} - \min\{i, j, k\}$ is known as the layer distance.

We consider two weight-3 RTZ inputs $P(x) = 1 + x^2 + x^4$ and $P'(x) = x^2 + x^4 + x^6$, which is a shifted version of $P(x)$. For $P(x),\ k = 0,\ j = 1,\ i = 0,\ l = 1$ and since $i = k$, we use the equation $w_p = 2l + 2 = 2(1) + 2 = 4$ For $P(x),\ k = 2\ j = 1,\ i = 0,\ l = 2$ and since $i < k$, we use the equation $w_p = 2l = 2(2) = 4$

This means that shifted versions of a weight-3 RTZ input have the same weight and without loss of generality, we may assume that all weight-3 begin at index 0 which means that we may ignore the case where $i < k$. We therefore have

$$w_p = 2l + 2 \tag{0-14}$$

Assuming that after interleaving, another weight-3 RTZ input is produced. Let i', j', k', l' and w'_p be similarly defined. Then the Hamming weight $w_H$of the turbo codeword produced can be calculated as

$$w_H = 7 + 2(l + l') \tag{0-15}$$

□