Reed-Solomon Decoding

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1 Introduction

To begin we assume that a (n,k) t-error correcting Reed-Solomon (RS) code is transmitted over an AWGN channel and received by the receiver. This received sequence can be written as a polynomial as shown below

$$r(x) = r_0 + r_1 X^1 + r_2 X^2 + \cdots + r_{n-1} X^{n-1} \tag{1}$$

The received polynomial can be used to calculate the syndrome by substituting α^i $2t$ times into (1), where $i = 1, 2, ...2t$.

$$\begin{aligned}
&S_1 = r_0 + r_1\alpha^1 + r_2\alpha^2 + \cdots + r_{n-1}\alpha^{n-1} \\
&S_2 = r_0 + r_1(\alpha^1)^2 + r_2(\alpha^2)^2 + \cdots + r_{n-1}(\alpha^{n-1})^2 \\
&. \\
&. \\
&. \\
&S_{2t} = r_0 + r_1(\alpha^1)^{2t} + r_2(\alpha^2)^{2t} + \cdots + r_{n-1}(\alpha^{n-1})^{2t}
\end{aligned} \tag{2}$$

rewriting (2) in matrix form yields

$$\begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_{2t-1} \\ S_{2t} \end{bmatrix} = \begin{bmatrix} r_0 & r_1 & \ldots & r_{n-2} & r_{n-1} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 & 1 \\ \alpha & (\alpha)^2 & (\alpha)^3 & \ldots & (\alpha)^{2t-1} & (\alpha)^{2t} \\ & & & \vdots & & \\ \alpha^{n-2} & (\alpha_{n-2})^2 & (\alpha_{n-2})^3 & \ldots & (\alpha_{n-2})^{2t-1} & (\alpha_{n-2})^{2t} \\ \alpha_{n-1} & (\alpha_{n-1})^2 & (\alpha_{n-1})^3 & \ldots & (\alpha_{n-1})^{2t-1} & (\alpha_{n-1})^{2t} \end{bmatrix} \tag{3}$$

which is

$$\mathbf{S} = \mathbf{r}\mathbf{H}^T$$

where $\mathbf{S}, \mathbf{r}$ are the syndrome vector and received sequence vector respectively and $\mathbf{H}^T$ is the transpose of the parity check vector.

Assuming the received sequence is in error, we may write $\mathbf{r}$ as

$$\mathbf{r} = \mathbf{c} + \mathbf{e} \tag{4}$$

where $\mathbf{c} = \begin{bmatrix} c_0 & c_1 & \ldots & c_{n-2} & c_{n-1} \end{bmatrix}$ is the codeword vector and
where $\mathbf{e} = \begin{bmatrix} e_0 & e_1 & \ldots & e_{n-2} & e_{n-1} \end{bmatrix}$ is the error vector
and $\mathbf{S}$ becomes

$$\begin{aligned}
\mathbf{S} =& \mathbf{c}\mathbf{H}^T + \mathbf{e}\mathbf{H}^T \\
&= \mathbf{0} + \mathbf{e}\mathbf{H}^T = \mathbf{e}\mathbf{H}^T
\end{aligned} \tag{5}$$