A General Construction of Z-Concatenative Complete Complementary Codes

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Background

- For (quasi-) synchronous CDMA systems, complete complementary codes (CCCs) and zero correlation zone (ZCZ) sequences provide co-channel and multi-path interference free communications.
- Comparing with ZCZ sequences, CCC leads a CDMA system which has lower implementation complexity but is lacking in spectral efficiency in general.
- As a hybrid of CCC and ZCZ sequence, we proposed the Z-concatenative CCC (Z-CCC) and presented two Z-CCCs consisting of rows of the discrete Fourier transform (DFT) and Hadamard matrices.
- In this work, we generalize the previous work and propose a novel construction of Z-CCCs.

Definition

- **zero-correlation zone sequences**
  A sequence set (SS) with M length-L sequence, denoted by (M, L)-SS, $\mathbb{S}$ is called the *zero correlation zone SS* or $(M, L; Z)$-ZCZ if their periodic correlations satisfying
  $$\tilde{R}_{\mathbf{s}_m,\mathbf{s}_{m'}}(\tau) = \tilde{R}_{\mathbf{s}_m,\mathbf{s}_{m'}}(0)\delta(m-m'), \text{ for } |\tau| \leq Z$$
- complete complementary codes
  A sequence family (M, N, L)-SF $\mathcal{C}$, consisting of M (N, L)-SSs, is called to the *complete complementary code* (CCC) or (M, N, L)-CCC if the sum of the correlation between $\mathbf{s}_n^m$ and $\mathbf{s}_n^{m'}$ satisfying
  $$\mathcal{R}_{\mathcal{C}^m,\mathcal{C}^{m'}}(\tau) = \sum_{n=0}^{N-1} R_{\mathbf{s}_n^m,\mathbf{s}_n^{m'}}(\tau)\delta(m-m',\tau)$$
  for all $0 \leq m, m' < M$.
- Z-concatenative complete complementary codes
  If the (M, NL)-SS $\mathbb{S}$ generated by connecting sequences in each SS of an (M, N, L)-CCC $\mathcal{C}$, *i.e.*, $\mathbb{S} = \{\mathbf{s}^m\}_{m=0}^{M-1} = \{(\mathbf{c}_n^m)_{n=0}^{N-1}\}_{m=0}^{M-1}$, is an $(M, LN; Z)$-ZCZ, then $\mathcal{C}$ is called the Z-concatenative CCC or $(M, N, L; Z)$-CCC.
- Kronecker's product
  For two matrix $\mathbf{A}^{(0)}$ and $\mathbf{A}^{(1)}$ of sizes $M_0 \times N_0$ and $M_1 \times N_1$, respectively, $\otimes$ denotes the Kronecker's product which yields a size $M_0M_1 \times N_0N_1$ matrix by the rule
  $$\mathbf{A}^{(1)} \otimes \mathbf{A}^{(0)} = \left[a_n^{(1,m)}\mathbf{A}^{(0)}\right]_{m=0,n=0}^{M_1-1,N_1-1}$$

Construction Method

Let
$$\mathbf{U} = \bigotimes_{k=0}^{K-1} \mathbf{F}_{N_k} := \mathbf{F}_{N_{(K-1)}} \otimes \mathbf{F}_{N_{(K-2)}} \otimes \cdots \otimes \mathbf{F}_{N_0}$$
where $\mathbf{F}_N$ stands for the N-dimensional DFT matrix. Then, the (N, N, N)-SF, $N = \prod_{k=0}^{K-1} N_k$, constructed by entry-wise multiplication $\odot$ as
$$\mathcal{C} = [\mathbf{c}_n^m]_{m=0,n=0}^{N-1,N-1} = [\mathbf{u}_N^m \odot \mathbf{u}_N^n]_{m=0,n=0}^{N-1,N-1}$$
is an $(N, N, N; Z)$-CCC with $Z = (N_{K-1} - 1)\prod_{k=0}^{K-2} N_k$.

Construction efficiency

For the bound achieving CCCs, *i.e.*, $(N, N, L; Z)$-CCCs, we have theoretical bound $Z \leq L - 1$. Therefore, under definition of the merit figure $\eta := (Z+1)/L$, the construction efficiency of the proposed Z-CCC can be evaluated as
$$\eta = \frac{(N_{K-1}-1)\prod_{k=0}^{K-2} N_k + 1}{\prod_{k=0}^{K-1} N_k} \approx \frac{N_{K-1}-1}{N_{K-1}}$$
There is a tradeoff relationship between merit figure and alphabet size. To achieve high merit figure, a large N_{K-1} is expected at expenses of increasing alphabet size.

Example

Let
$$\mathbf{U} = \mathbf{F}_4 \otimes \mathbf{F}_2 = \begin{bmatrix} \mathbf{u}^0 \\ \mathbf{u}^1 \\ \mathbf{u}^2 \\ \mathbf{u}^3 \\ \mathbf{u}^4 \\ \mathbf{u}^5 \\ \mathbf{u}^6 \\ \mathbf{u}^7 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & j & j & -1 & -1 & -j & -j \\ 1 & -1 & j & -j & -1 & 1 & -j & j \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -j & -j & -1 & -1 & j & j \\ 1 & -1 & -j & j & -1 & 1 & j & -j \end{bmatrix}$$
Then, from the table

Table: Parameters of the considering CCs

$\mathbf{j} \cdot \mathbf{i}$	(00)	(10)	(01)	(11)	(02)	(12)	(03)	(13)
(00)	(00)	(10)	(01)	(11)	(02)	(12)	(03)	(13)
(10)	(10)	(00)	(11)	(01)	(12)	(02)	(13)	(03)
(01)	(01)	(11)	(02)	(12)	(03)	(13)	(00)	(10)
(11)	(11)	(01)	(12)	(02)	(13)	(03)	(10)	(00)
(02)	(02)	(12)	(03)	(13)	(00)	(10)	(01)	(11)
(12)	(12)	(02)	(13)	(03)	(10)	(00)	(11)	(01)
(03)	(03)	(13)	(00)	(10)	(01)	(11)	(02)	(12)
(13)	(13)	(03)	(10)	(00)	(11)	(01)	(12)	(02)

the resulted SF is given by
$$\mathcal{C} = \begin{bmatrix} \mathbf{u}^0 & \mathbf{u}^1 & \mathbf{u}^2 & \mathbf{u}^3 & \mathbf{u}^4 & \mathbf{u}^5 & \mathbf{u}^6 & \mathbf{u}^7 \\ \mathbf{u}^1 & \mathbf{u}^0 & \mathbf{u}^3 & \mathbf{u}^2 & \mathbf{u}^5 & \mathbf{u}^4 & \mathbf{u}^7 & \mathbf{u}^6 \\ \mathbf{u}^2 & \mathbf{u}^3 & \mathbf{u}^4 & \mathbf{u}^5 & \mathbf{u}^6 & \mathbf{u}^7 & \mathbf{u}^0 & \mathbf{u}^1 \\ \mathbf{u}^3 & \mathbf{u}^2 & \mathbf{u}^5 & \mathbf{u}^4 & \mathbf{u}^7 & \mathbf{u}^6 & \mathbf{u}^1 & \mathbf{u}^0 \\ \mathbf{u}^4 & \mathbf{u}^5 & \mathbf{u}^6 & \mathbf{u}^7 & \mathbf{u}^0 & \mathbf{u}^1 & \mathbf{u}^2 & \mathbf{u}^3 \\ \mathbf{u}^5 & \mathbf{u}^4 & \mathbf{u}^7 & \mathbf{u}^6 & \mathbf{u}^1 & \mathbf{u}^0 & \mathbf{u}^3 & \mathbf{u}^2 \\ \mathbf{u}^6 & \mathbf{u}^7 & \mathbf{u}^0 & \mathbf{u}^1 & \mathbf{u}^2 & \mathbf{u}^3 & \mathbf{u}^4 & \mathbf{u}^5 \\ \mathbf{u}^7 & \mathbf{u}^6 & \mathbf{u}^1 & \mathbf{u}^0 & \mathbf{u}^3 & \mathbf{u}^2 & \mathbf{u}^5 & \mathbf{u}^4 \end{bmatrix}$$
One can confirm that $\mathcal{C}$ is $(8, 8, 8)$-CCC and if we let
$$\mathbb{S} = \begin{bmatrix} \mathbf{s}^0 \\ \mathbf{s}^1 \\ \mathbf{s}^2 \\ \mathbf{s}^3 \\ \mathbf{s}^4 \\ \mathbf{s}^5 \\ \mathbf{s}^6 \\ \mathbf{s}^7 \end{bmatrix} = \begin{bmatrix} (\mathbf{u}^0\ \mathbf{u}^1\ \mathbf{u}^2\ \mathbf{u}^3\ \mathbf{u}^4\ \mathbf{u}^5\ \mathbf{u}^6\ \mathbf{u}^7) \\ (\mathbf{u}^1\ \mathbf{u}^0\ \mathbf{u}^3\ \mathbf{u}^2\ \mathbf{u}^5\ \mathbf{u}^4\ \mathbf{u}^7\ \mathbf{u}^6) \\ (\mathbf{u}^2\ \mathbf{u}^3\ \mathbf{u}^4\ \mathbf{u}^5\ \mathbf{u}^6\ \mathbf{u}^7\ \mathbf{u}^0\ \mathbf{u}^1) \\ (\mathbf{u}^3\ \mathbf{u}^2\ \mathbf{u}^5\ \mathbf{u}^4\ \mathbf{u}^7\ \mathbf{u}^6\ \mathbf{u}^1\ \mathbf{u}^0) \\ (\mathbf{u}^4\ \mathbf{u}^5\ \mathbf{u}^6\ \mathbf{u}^7\ \mathbf{u}^0\ \mathbf{u}^1\ \mathbf{u}^2\ \mathbf{u}^3) \\ (\mathbf{u}^5\ \mathbf{u}^4\ \mathbf{u}^7\ \mathbf{u}^6\ \mathbf{u}^1\ \mathbf{u}^0\ \mathbf{u}^3\ \mathbf{u}^2) \\ (\mathbf{u}^6\ \mathbf{u}^7\ \mathbf{u}^0\ \mathbf{u}^1\ \mathbf{u}^2\ \mathbf{u}^3\ \mathbf{u}^4\ \mathbf{u}^5) \\ (\mathbf{u}^7\ \mathbf{u}^6\ \mathbf{u}^1\ \mathbf{u}^0\ \mathbf{u}^3\ \mathbf{u}^2\ \mathbf{u}^5\ \mathbf{u}^4) \end{bmatrix}$$
Then, it is an $(8, 64; 6)$-ZCZ which merit factor is $\eta = 7/8$.

References

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