“ Coset Interleaver Idea Progress March 30th”

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0.1 Introduction

Using the 5/7 RSC interleaver as a reference, we present the idea of coset interleaving for breaking up weight-2 and weight-3 return-to-zero (RTZ) input sequences which produce codewords with weights less than w_H. The notations and the assumptions that will be made throught out this paper are listed below.

0.1.1 Notations

1. N :- Interleaver Length, Interleaver size, $N = n\tau$.
2. $\mathcal{C}^i$:- Coset ,i, $i = \{0, 1, ..., \tau - 1\}$
3. $L = N/\tau$:- Coset length, Coset size.
4. τ :- Cycle length of RSC encoder ($\tau = 3$ for 5/7RSC encoder)
5. Seperation:- Assume that there is a set $\mathcal{A} = \{a, b, c, ...\}$, $b > a, c > b$, etc. For any 2 elements in set, the seperation is defined as the difference between the elements. For example, the seperation between b and a is $b - a$
6. Step:- Assume that there is a set $\mathcal{A} = \{a, b, c, ...\}$, $b > a, c > b$, etc . For any 2 elements in set, the step is defined as the difference between the index of two elements. For example, from a to b a step of 1 is required and from a to c a step of 2 is required.
7. D_1 :-Interleaving step
8. s :- post-interleaving seperation
9. t :- pre-interleaving seperation
10. layer distance (l) :- Assume we have a set $\mathcal{A} = (0, 1, ..., N - 1)$. We proceed to form a $N/\tau \times \tau$ matrix $\mathbf{A}$ from it. We assign indices to each row and refer to each row of $\mathbf{A}$ as a layer and the the layer distance l as the difference betweeen two rows indices of $\mathbf{A}$

0.2 Weight-2 RTZ inputs

In this section, we present the idea for coset interleaving focusing solely on weight-2 RTZ inputs. According to [SunTakeshita] weight-2 RTZ inputs have two "1" bits which are seperated by $n\tau - 1$, $n = 1, 2, ...$ "0" bits. Consequently, the smaller the value of n, the lower the weight of the codeword which will be produced. Based on this, we decided to break down the interleaver into 3 groups depending on the value of $L \mod \tau$, which for the case of the 5/7 RSC is either $0, 1,$ or 2. For each interleaver group, we explore all possible permutation matrices and select the ones that meet a certain design criteria.

0.2.1 Interleavers with coset length L where $L \mod \tau = 0$

From the discussion made in the previous section, we can deduce that the weight-2 RTZ which produces the lowest codeword weight will be of the form $x^t(1 + x^3)$, $t = 0, 1, 2, ...$ The best outcome would be to transform $x^t(1 + x^3)$ into a non-RTZ input. However, as the value of N increases, this becomes possible for only a specified distance within the interleaver. The next best thing we can do is to make sure that anytime we have a RTZ input of the form $x^t(1+x^3)$, it is transformed into another RTZ input which produces a codeword with a heavier weight than $x^t(1+x^3)$. For interleavers where $L \mod \tau = 0$, $N = n\tau^2 = 9n$, $n = 1, 2, ...$ We focus on the case where $n = 1$ and look for unique $N \times N$ permutation matrices such that $x^t(1+x^3)$ is transformed to a non-RTZ input. A permutation matrix is unique if a shift of the elements in the matrix does not produce another another permutation matrix.

A	0	3	6	1	4	7	2	5	8
B	0	3	6	1	4	2	5	8	7
C	0	3	6	1	2	4	5	7	8
D	0	3	6	1	2	5	8	4	7
E	0	3	6	2	1	4	7	5	8
F	0	3	6	2	1	5	4	8	7
G	0	3	6	2	5	1	4	7	8
H	0	3	6	2	5	8	1	4	7
I	0	3	1	4	7	6	2	5	8
J	0	3	1	4	2	6	5	7	8
K	0	3	1	2	4	6	7	5	8
L	0	3	1	2	5	6	4	7	8
M	0	3	2	1	4	6	5	8	7
N	0	3	2	1	5	6	8	4	7
O	0	3	2	5	1	6	4	8	7
P	0	3	2	5	8	6	1	4	7
Q	0	1	3	4	6	7	2	5	8
R	0	1	3	4	6	2	5	8	7
S	0	1	3	4	2	7	5	6	8
T	0	1	3	2	6	4	7	5	8
U	0	1	3	2	6	5	4	8	7
V	0	1	3	2	5	4	7	6	8
W	0	1	4	7	2	3	5	6	8
X	0	2	3	5	6	8	1	4	7

Table 1: All unique permutation matrices for the case when $L \mod \tau = 0$

Table 1 shows all unique permutation matrices that meet the above criteria. Each row is a seperate permutation matrix and the column index corresponds to the rows in each matrix and the column values correspond to the column where a 1 appears.

For example, the first row in Table 1 corresponds to the permutation matrix below.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{0-1}$$

At the 0th row and the 0th column, 1st row and the 3rd column,3rd row and the 6th column,.. and so on, a 1 is present.

For interleaver lengths N where $n > 1$, what we can to is to repeat the decided upon interleaving pattern for all the n blocks of the interleaver. Regardless of which interleaving pattern we choose from Table 1, we are certain that RTZ inputs only occur when any 2 elements have a step of exactly 9 between them.

0.2.2 Interleavers with coset length L where $L \mod \tau = 1$ or $L \mod \tau = 2$

Interlerleavers that meet this criteria, can be further grouped into 2 categories. The first category which is easiest to deal with is the case where N is divisible by 3 and 6. The second category is where N is divisible by 3 only.

For the case where N is divisible by 3 and 6, we choose the largest divisor(6) and set $N = 6n,\ n = 1, 2, \ldots$. Setting $N = 1$ we then follow the procedure in the previous section, and find all permutation matrices which ensure that the weight-2 RTZ input $x^t(1 + x^3)$ is transformed into a non-RTZ input. The permutation matrices which meet this criteria are shown in Table 2. For

A	0	3	1	4	2	5
B	0	3	1	2	4	5
C	0	3	2	1	5	4
D	0	3	2	5	1	4
E	0	1	3	4	2	5
F	0	1	3	2	5	4
G	0	1	4	2	3	5
H	0	2	3	5	1	4

Table 2: All unique permutation matrices for the case when $L \mod \tau = 1$ or $L \mod \tau = 2$ and $N = 6n$

interleaver lengths N where $n > 1$, what we can to is to repeat the decided upon interleaving pattern for all the n blocks of the interleaver. Again, regardless of which interleaving pattern we choose from Table 2, we are certain that RTZ inputs only occur when any 2 are exactly 6 steps away from each other.

For the case where N is divisible by 3 only, we have $N = 3n,\ n = 1, 2, \ldots$. Again, we set $n = 1$ and follow the procedure in the previous section, and find all permutation matrices which ensure that the weight-2 RTZ input $x^t(1 + x^3)$ is transformed into a non-RTZ input. The permutation matrices which meet this criteria are shown in Table 3.

A	0	1	2
B	0	2	1

Table 3: All unique permutation matrices for the case when $L \mod \tau = 1$ or $L \mod \tau = 2$ and $N = 3n$

Again ,for interleaver lengths N where $n > 1$, we repeat the decided upon interleaving pattern for all the n blocks of the interleaver. This time, that RTZ inputs only occur when any 2 are exactly 3 steps away from each other, which means $x^t(1 + x^3)$ is always mapped unto itself and this is a situation we want to avoid as much as possible. In light of this, we present an alternate procedure for coset interleaving for when $L \mod \tau = 1$ or $L \mod \tau = 2$

0.2.3 Alternate Method for Interleavers with coset length L where $L \mod \tau = 1$ or $L \mod \tau = 2$

For interleavers which meet this criteria, we take the elements in each cosest and form a $3 \times \lceil \frac{L}{3} \rceil$ matrix as shown in Figure 0-1

Figure 0-1: Turbo Encoder

Figure 0-1 (a) represents the case where $L \mod \tau = 2$ and Figure 0-1 (b) represents the case where $L \mod \tau = 1$. The task is to generate a permutation matrix which guarantees that the should a weight-2 RTZ $x^t(1 + x^3)$ occur it will not be mapped to the same RTZ sequence at all points during interleaving.

The $L \times N$ permutation matrices which meet the criteria are shown in Table 4 for different values of L and N.

$L = 4, N = 12$	0	3	6	10	
$L = 5, N = 15$	0	3	6	10	12

Table 4: All unique permutation matrices for different values of N and L when $L \mod \tau = 1$ or $L \mod \tau = 2$

The permutation matrices obtained using this method ensures that weight-2 RTZ $x^t(1 + x^3)$ is never mapped to itself at any point in the interleaving process.

0.3 Weight-3 RTZ inputs

In most interleaver design cases, focusing on weight-2 RTZ inputs would be enough. For the 5/7 RSC interleaver however, this is not the case since the minimum weight codeword is caused by the weight -3 RTZ input of the form $x^t(1 + x + x^2),\ t = 1, 2, ...$ In general, any time a weight 3 input has $x \mod 3 = 1$ zeros and/or $x \mod 3 = 1$ between its first 2 "1" bits and and last 2 "1" bits it is a weight 3 RTZ input sequence. This definition is specific to the 5/7 RSC encoder.

Similar to the approach taken for designing interleavers weight-2 RTZ sequences, we group the interleavers into 3 groups, depending on the value of $L \mod \tau$ and find all possible permutation matrices which prevent $x^t(1 + x + x^2)$ from being mapped to itself.

0.3.1 Interleavers with coset length L where $L \mod \tau = 0$

Table 5 list all permutation matrices when which transforms $x^t(1+x+x^2)$ into a non RTZ input.

However, there are other weight 3 RTZ inputs which exist within this span, depending on the permutation matrix that is chosen. For example, $\mathbf{A}$ has $1+x^4+x^8,\ 1+x^5+x^7,\ x+x^3+x^8,\ x+$

A	0	3	6	1	4	7	2	5	8
B	0	3	6	1	4	2	7	5	8
C	0	3	6	1	4	2	8	7	8
D	0	3	6	1	4	2	5	8	7
E	0	3	6	2	5	1	4	7	8
F	0	3	6	2	5	1	4	8	7
G	0	3	6	2	5	1	8	4	7
H	0	3	1	6	4	7	2	5	8
I	0	3	1	4	7	2	5	6	8
J	0	3	2	6	5	8	1	4	7
K	0	3	2	5	8	1	4	6	7
L	0	1	3	4	7	2	5	6	8

Table 5: All unique permutation matrices for the case when $L \mod \tau = 0$

$x^5+x^6,\ x^2+x^4+x^6,\ x^2+x^3+x^7$, whiles **B** has $1+x^4+x^8,\ x+x^3+x^8,\ x+x^3+x^5,\ x+x^6+x^8,\ x^2+x^3+x^7,\ x^2+x^6+x^7$

Regardless of which permutation matrix is picked, we are certain that the weight 3 RTZ $1+x+x^2$ will not be mapped to itself within the chosen span.

0.3.2 Interleavers with coset length L where $L \mod \tau = 1$ or $L \mod \tau = 2$

Similar to weight 2 RTZ, we further divide the abpve case into 2 categories. The first category is when N is divisible by 3 and 6. The second category is where N is divisible by 3 only. Table 6 show all permutation matrices which prevent $1+x+x^2$ from being interleaved to itself when $L \mod \tau = 1$ or $L \mod \tau = 2$ and N is divisible by 3 and 6.

A	0	3	1	4	2	5
B	0	3	2	5	1	4

Table 6: All unique permutation matrices for the case when $L \mod \tau = 1$

The remaining weight 3 RTZ which need to be dealt with is $1+x^2+x^4$.

For the second category (N divisible by 3 only), there are no permutation matrices which meet this criteria.

0.3.3 Alternate Method for Interleavers with coset length L where $L \mod \tau = 1$ or $L \mod \tau = 2$

For the case where $L \mod \tau = 1$, there is a point where $1+x+x^2$ is mapped to itself, depending on N.

For the case where $L \mod \tau = 2$, $1+x+x^2$ is never mapped unto itself, but the lowest weight 3 RTZ that needs to be dealt with is $x+x^3+x^5$

0.4 Maximizing Seperation for weight-2 RTZ inputs

0.4.1 case $L \mod 3 = 0$

From Table 1 we select the permutation matrix **A**. For interleaver length $N > 9$, we repeat this pattern. With this permutation matrix, we are sure that the weight-2 RTZ $1+x^9$ will be mapped to itself. We wish to prevent this by transforming $1+x^9$ into a weight -2 RTZ input which has a large seperation the "1" bits. Let t and s represent the seperation before and after interleaving.It is worth noting that t, s are multiples of $\tau = 3$

The mamimum seperation depends on the value of N. Therefore to achieve the largest separation after interleaving s must meet the following condition.

$$s = \max\left\{a \leq \lfloor \tfrac{N}{2} \rfloor\right.$$

0.4.2 case $L \mod 3 = 1$

0.4.3 case $L \mod 3 = 2$

0.5 Maximizing seperation for weight-3 RTZ inputs

0.5.1 case $L \mod 3 = 0$

0.5.2 case $L \mod 3 = 1$

0.5.3 case $L \mod 3 = 2$

0.6 Maximizing Seperation for weight-2 and weight-3 RTZ inputs

0.6.1 Choice of Permutation polynomial

case $L \mod 3 = 0$

case $L \mod 3 = 1$

case $L \mod 3 = 2$

0.6.2 Range of Maximum Separation

0.7 Interleaver implementation

0.8 Simulation Results