Deterministic Interleaver Design for Turbo Codes

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1. Turbo Codes : Brief Introduction

- AWGN channel capacity approacing code
- Parallel concatenation of 2 convolutional codes via an interleaver
- Good performance depends on interleaver

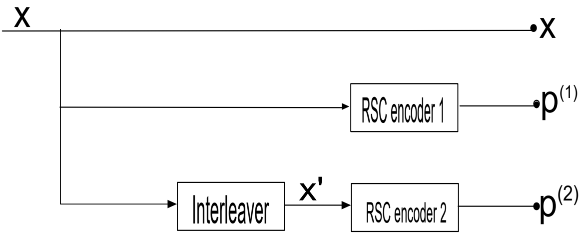
2. Interleavers

- Divided into 2 groups
  - Random Interleavers
    - Advantage: Good performance for large frame sizes
    - Disadvantage: storage of interleaver tables required.
  - Deterministic Interleavers
    - Advantage : Interleaving done via algorithm
    - Disadvantage: For large frame sizes, interleaver better than random not yet found.

3. Purpose of Research

- Purpose of Research
  - Deterministic interleaver that outperforms linear interleaver for large frame sizes
  - multi-shift interleaver is proposed
- Why Linear Interleaver?
  - Better than random interleaver for short frame sizes.
  - Easy to design

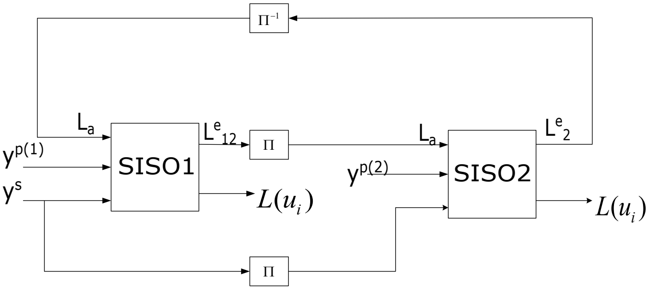
4. Turbo Encoder



- N is interleaver size, M is No. of memory elements
- **x** is information bits with length $N - M$
- $\mathbf{p}^{(1)}$ is upper parity checkbits, $\mathbf{p}^{(2)}$ is lower parity checkbits
  - both have length N

RSC :- Recursive Systematic Convolutional

5. Turbo Decoder

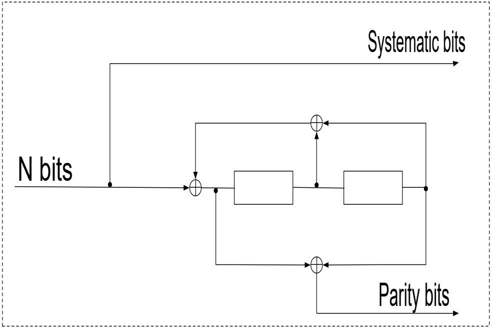


- y^s is systematic bits
- $y^{p(1)}$ is upper parity check bits, $y^{p(2)}$ is lower parity check bits
- $L(u_i)$ is Log-Likelihood Ratio, L_{12}^e, L_{21}^e is extrinsic information

6. RSC Encoders

- cycle length (τ) of RSC encoders
  - length of the cycle with input $[1, 0, 0, 0, 0, ...]$

Example



- $\left[\frac{1+D^2}{1+D+D^2}\right]$ (5/7) RSC Encoder
- output : $[1, |1, 1, 0|, |1, 1, 0|, |1, 1, 0|...]$
- cycle : $[1, 1, 0]$, $\tau = 3$

7. RSC Encoders and $a\tau$-seperated weight 2 errors

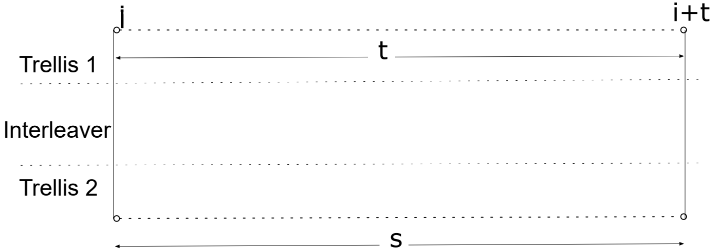
- weight 2 information sequences
  - "1" bit pair seperated by $a\tau - 1$ "0" bits
- effective free distance d_{eff}
  - minimum codeword weight due to weight 2 input

Example

$N = 16$, input$=[1, 0, 0, 1, ..., 0]$

$$
\begin{array}{ll}
1\,0\,0\,0\,0\,... & 1\,1\,1\,0\,1\,1\,0\,1\,1\,0\,1\,1\,0\,1\,1\,0 \\
\underbrace{.\qquad\quad}_{\tau}1\,0\,0\,0\,... & \underbrace{0\,0\,0}_{\tau}\,1\,1\,1\,0\,1\,1\,0\,1\,1\,0\,1\,1\,0 \\
\hline
 & 1\,1\,1\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0
\end{array}
$$

8. t-seperated weight 2 error in Turbo Codes



- $t = a\tau \mapsto s = b\tau$, low-weight turbo codewords.

9. Turbo Codes and Linear Interleavers

- Index mapping function with depth D

$$\Pi_{\mathfrak{L}_N}(i) = Di \mod N, \quad gcd(D, N) = 1 \tag{1}$$

- $t = a\tau \mapsto s = b\tau$
  - Solution: $\underset{1 \leq D < N | gcd(D,N)=1}{\arg\max} \left\{ \min\{a + b \mod N\} \right\}$

10. $a\tau$ weight 2 error : Interleaver Search

Linear Interleaver Search Results

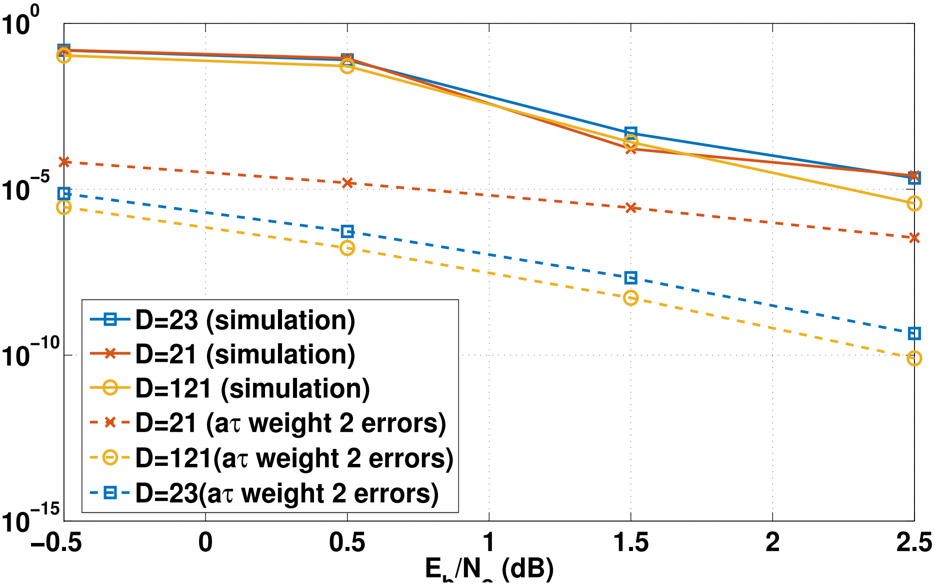
D	13	121	17	23	21
a	19	17	15	11	12
b	9	9	1	3	4
d_{eff}	30	30	15	26	15
N_{free}	1	1	2	1	2

BER Approximation

$$P_b \approx \frac{1}{2} \sum_{w_c} Y_{w_c} erfc\left(\sqrt{w_c \frac{R_c E_b}{N_o}}\right) \tag{2}$$

where

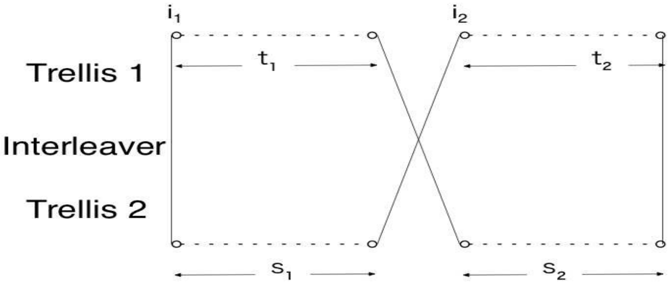
$$Y_{w_c} \triangleq \sum_{w_x + w_p = w_c} \frac{w_x}{N} A_{w_x, w_p}$$



10^0
10^−5
10^−10
10^−15
−0.5
0
0.5
1
1.5
2
2.5
Eb/No (dB)
D=23 (simulation)
D=21 (simulation)
D=121 (simulation)
D=21 (aτ weight 2 errors)
D=121(aτ weight 2 errors)
D=23(aτ weight 2 errors)

12. τ-seperated weight 4 errors

- Dominate BER performance [2]



- $\tau = Dv \mod N$, $v = i_2 - i_1$
  - Weight 4 input :- $(1 + X^v)(1 + X^\tau)$

[2] Oscar Y. Takeshita, Member, IEEE, and Daniel J. Costello , "New Deterministic Interleaver Designs for Turbo Codes",IEEE Trans. Inform. Theory, vol. 46,pp. 1988-2006,Nov. 2000

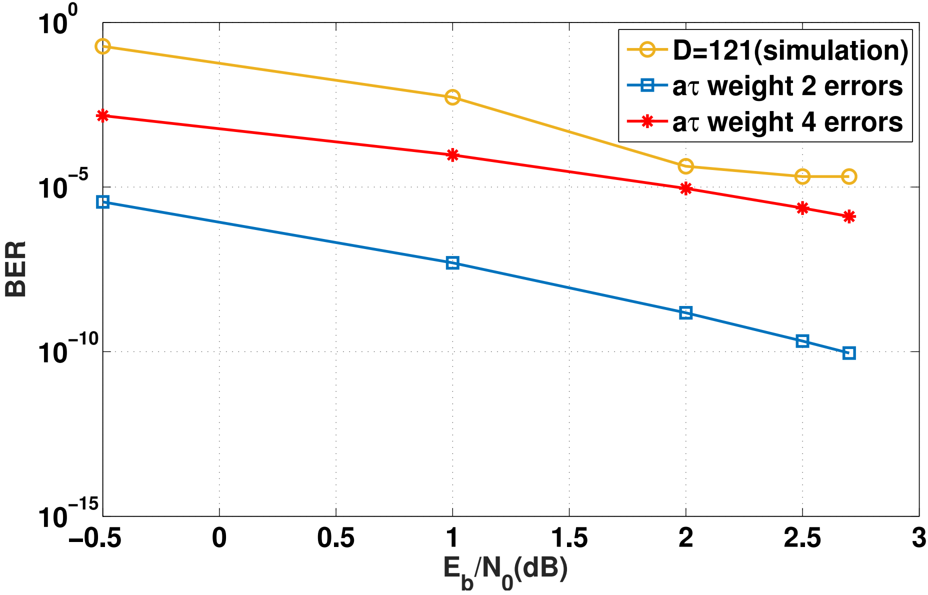
12. τ weight 4 errors

Example

- $N = 32$, $\tau = 3$, $D = 5$, $v = 7$
- input : $(1 + X^3)(1 + X^7)$, output :$(1 + X^3)(1 + X^{15})$

- codeword weight : 20, multiplicity $\approx N$
- same result for different D and N

13. τ weight 4 error : BER Approximation vs Simulation



14. Sequential representation of Linear Interleaver

- Algorithm for linear interleaving
  - 1. $p_0 = 0$
  - 2. $p_i = (p_{i-1} + D) \mod N$
- element positions shifted by constant D

p_i is element position

15. Multi-Shift Interleaver

- For $N = 2^r, \;\; r \in \{1, 2, ...\}$ set $\Delta s = 2^q, \;\; q \in \{2, 3, .., r-1\}$
- cycle set $\mathbb{D} = \{d_0, d_1, ..., d_{V-1}\}, \;\; V = N/\Delta s$
  - $d_0 = D, \quad d_i = d_{i-1} + \Delta s$
- Algorithm for proposed interleaver (multi-shift interleaver)
  - 1. $p_0 = 0$
  - 2. $p_i = p_{i-1} + d_{((i-1) \mod V)} \mod N$, $\quad$ *d_0 is an odd integer*
  - Shift value of D for each position shift

16. Multi-Shift Interleaver : Search for Good Interleaver

- procedure for choosing good interleavers
  - choose d_0 from $(\sqrt{N}, N/2)$
  - calculate hamming weight for $\Delta s \in 2^q$
  - best Δs = largest d_{eff}
  - repeat for d_0 within range
  - best parameter, $(d_0, \Delta s)$ with largest d_{eff}, least value of Δs and multiplicity

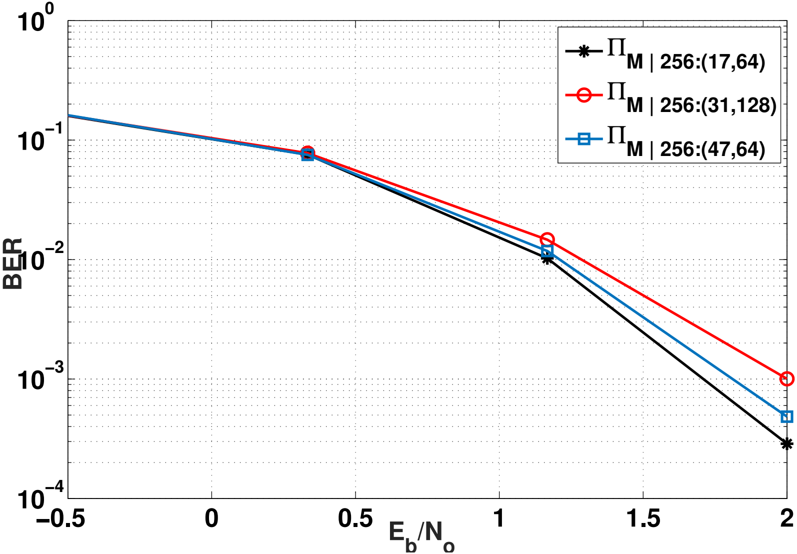
17.MSI Search Results : 5/7 component encoder. $N = 256$

d_0	17	31	47
d_{eff}	38	38	38
Δs	64	128	64
$N_{free,eff}$	207	208	209

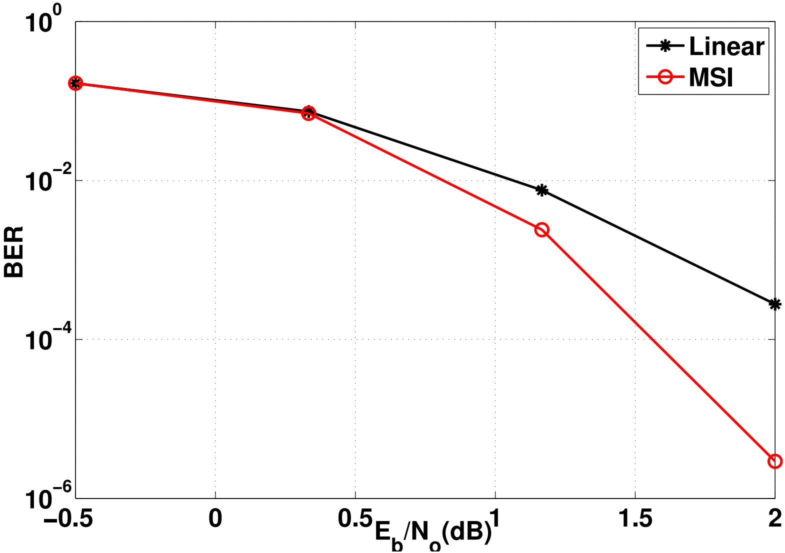
- best parameter $(d_0 = 17, \Delta s = 64)$

$N_{free,eff}$: multiplicity of d_{eff} codewords

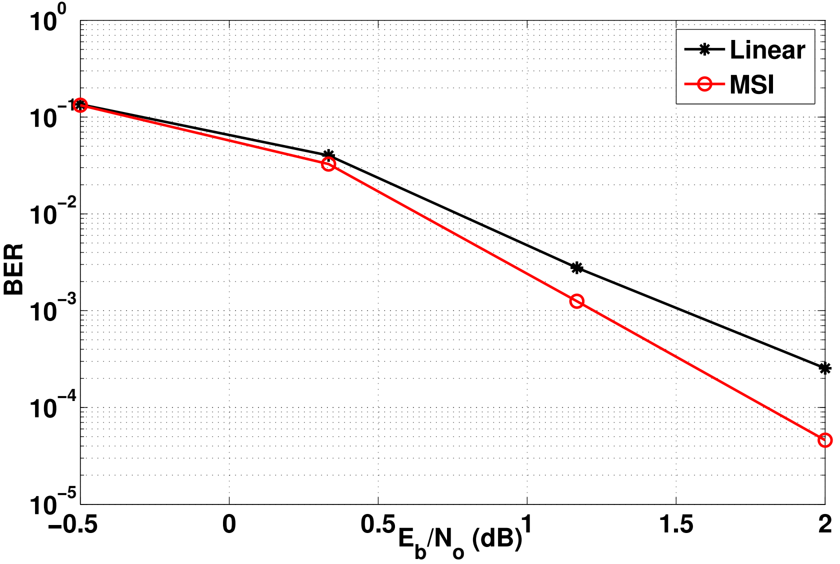
17. Simulation Results for Table



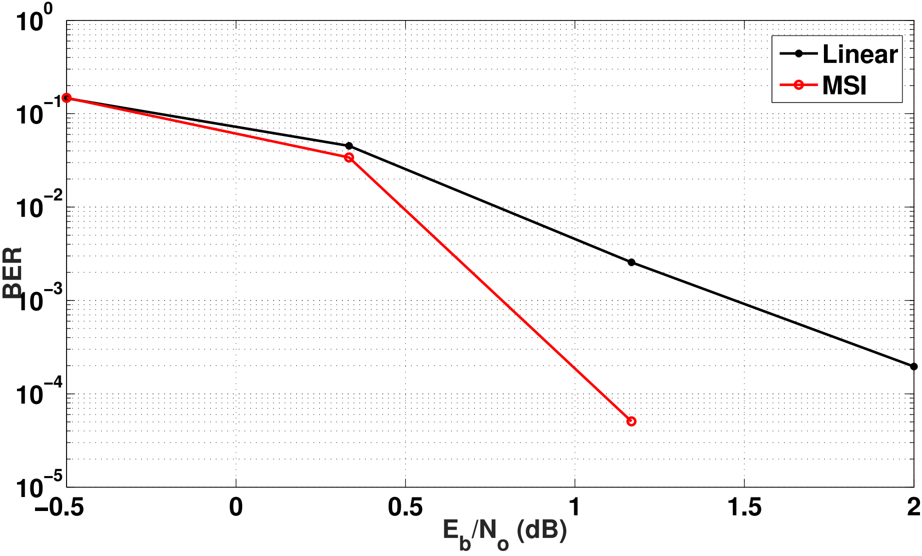
17. Results for 5/7 Component Code. $N = 1024$



17. Results for 7/5 Component Code. $N = 1024$



17. Results for 5/7 Component Code. $N = 16384$



18. Conclusion and Future Works

Conclusion

- The multi-shift interleaver outperforms the linear interleaver for both medium and long frame sizes.

Future Research

- Comparison with other interleavers
- Theoretical BER bound for Interleaver