Assignment 1

Bohulu Kwame Ackah
b1841008

Graduate School of Informatics and Engineering
The University of Electro-Communications

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Other Models

- Shifted Hazard Model : Obtained by replacing t in the hazard model with $(t - t_0$ to fit observed data, with t_0 being adjustable
- Piecewise model estimation: By using the piecewise-linear model, an approximation of the hazard function might be obtained
  - Accuracy is improved by increasing the number of time segments.
- Power series Estimation : Another estimate can be obtained using the power series models
  - Let $\lambda(t) = K_0 + K_1 t + K_2 t^2 + \ldots + K_n t^m$. The reliability function is then
    $R(t) = \exp\left[-\left(K_0 t + K_1 \frac{t^2}{2} + K_2 \frac{t^3}{3} + \ldots + K_m \frac{t^{2+1}}{n+1}\right)\right]$
- Most cases don't need complex models
  - Real Challenge: Keeping the models of individual compenents as simple as possible to prevent analysis of the complete sytem from becoming too complicated

Modelling the Wearout Region

- With reference to the bathtub hazard function, the wearout region corresponds to is the rapidly rising part towards the end of the useful life
    - Alternately modelled using the normal distribution around the mean wearout life W of a component with suitably chosen paramenters
- Using the normal distribution, the associated density function will be $f(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(t-W)^2}{2\sigma^2}\right]$ where t and σ is the age of the component and standard deviation respectively
- We consider a duration t in the lifetime of the component which begins at time T.
- Assume failures occur due to random events during interval $(T, T+t)$. The components reliability is then

$$R_t(t) = e^{-\lambda t}$$

where λ corresponds to the useful lifetime of the component.

Modelling the Wearout Region-2

- Assuming Failure is actually due to wearout during time interval $(T, T+t)$ then then the associated probability given survival up till T is

$$Q_w(t) = \frac{\int_T^{r+1} f(\xi)d\xi}{\int_r^{\infty} f(\xi)d\xi}$$

- The probability of no wearout failure then becomes

$$R_w(t) = 1 - Q_w(t) = \frac{\int_{r+1}^{\infty} f(\xi)d\xi}{\int_r^{\infty} f(\xi)d\xi}$$

- Considering both chance failures and wearout failures, the probability that they do not occur during the same interval is given by

$$R(t) = R_c(t)R_w(t) = e^{-\lambda t}\frac{R_{\text{we}}\,(T+t)}{R_{\text{we}}\,(T)}$$

which simplifies to $e^{-\lambda t}R_{\text{we}}\,(t)$ at $T=0$

Modelling the Wearout Region-3

- To achieve high values of reliability we need to ensure that
  - The system is subjected to high burn and debugging procedures
  - Strict preventive maintenance and replacement schedules are to be adhered to to prevent components from entering wearout region.

Reliability and Maintenance

- Scheduled maintenance is done at constant intervals of time with the aim of prolonging the life of components etc.
- Forced Maintenance is done only when there is an in-service failure and thus may be thought of as repair
- Components that have a increasing hazard are the only ones that should be considered for scheduled maintenance
- After incorporating maintenance, the density function may be written as

$$f_T^*(t) = \sum_{k=0}^{\infty} f_1(t - kT_M) R^k(T_M)$$

where
$f_T(t) =$ failure density function , $R(t) =$ component reliability function
$T_M =$ fixed time interval between maintenances

$$f_1(t) = \begin{cases} f_T(t) & \text{for } 0 < t \leq T_M \\ 0 & \text{otherwise} \end{cases}$$

Ideal Repair

- 2 conditions are assumed for ideal repair
  - duration of repair after failure is much smaller when compared to time between failures and canbe assumed to be zero
  - Component restored to 'as new' condition post repair
- Time to *kth* failure may be calculated as

$$f_k(t) = \int_0^1 f_{k-1}(\tau) f_1(t-\tau) d\tau; \quad k \geq 2$$

- let $L(t)$ be the density function of some failure occuring with ideal repair. Then

$$L(t) = f_1(t) + \sum_{k=2}^{\infty} \int_0^t f_{k-1}(\tau) f_1(t-\tau) d\tau$$

Ideal Repair-2

- if time between failures are exponentially distributed, then $f_k(t)$ becomes special case of Erlangian distribution
- $f_k(t)$ becomes

$$f_k(t) = \lambda^k \frac{t^{k-1}}{(k-1)!} e^{-\lambda t}$$

- Finally for this special case $L(t)$ becomes

$$L(t) = \sum_{k=1}^{*} f_k(t) = \left(\lambda e^{-\lambda t}\right) \sum_{k=1}^{\infty} \frac{(\lambda t)^{k-1}}{(k-1)!}$$
$$= \lambda e^{-\lambda t} e^{\lambda t} = \lambda$$

Ideal Repair and Preventative Maintenance

- Combining Ideal repair and preventive maintenance if possible, will reduce the frequency of repairs
- Assuming ideal maintenance at periodicc intervals of T_M the frequence of repair f_R will be

$$f_R = \frac{1}{T_M} \int_0^{T_M} L(t)dt$$

- Maintenance is only useful if the MTTF which is the reciprocal of f_R increases.