Formula to Calculate Weight for Low-Weight Weight 3 Inputs and Proof

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1 Equation and Proof

Theorem 1.1. Let $Q(x) = x^{a\tau+t}(1 + x^{\beta\tau+1} + x^{\gamma\tau+2})$ be the polynomial representation of a weight 3 RTZ input. The Hamming weight, w_H of a turbo codeword generated by a weight-3 RTZ input is given by

$$7 + 2(\max\{l_1, l_2\} + \max\{l'_1, l'_2\}) \tag{1}$$

Figure 1: weigt-3 RTZ input Hamming weight equation proof

Proof. We know that the impule response of the RSC encoder is given by {1110110110110...} with a cycle of 110. We group the cycles into blocks as shown in Figure 1 (a) and refer to each block as a layer, with the numbering of the layer beginning at 0. If we assume that the weight-3 RTZ begins at the head of the weight-3 RTZ input, then β and γ can occur at either the 1st or second position within a layer.

Let the weight of the parity bit sequence be denoted by w_p. Calculating w_p for the weight-3 RTZ requires 2 steps calculating the weight from a to β (denoted $w_{\Delta\beta}$) and the weight from β to γ denoted (denoted $w_{\Delta\gamma}$) . This is done for 2 cases

case 1. β occupies the first position in its layer and γ, the second position (Figure 1(b))

let l_1 be the layer that β is located in and l_2 be the layer that γ is located in with reference to a. To calculate $w_{\Delta\beta}$ we need to observe that each individual non-overlaping layer adds a weight of 2 to $w_{\Delta\beta}$ whiles the fixed point adds weight 1 to it. This means

$$w_{\Delta\beta} = 2l_1 + 1$$

To calculate $w_{\Delta\gamma}$, observe that each double-overlapping layers and the last layer each contribute a weight of 2 to $w_{\Delta\gamma}$ whiles l_1 adds a weight of 1. Therefore

$$w_{\Delta\gamma} = 2(l_2 - l_1) + 1$$

And

$$\begin{aligned} w_p &= 2l_1 + 1 + 2(l_2 - l_1) + 1 \\ &= 2l_2 + 2 \end{aligned} \tag{2}$$

case 2. β occupies the second position in its layer and γ, the first position Figure 1(c)

let l_1 be the layer that β is located in and l_2 be the layer that γ is located in with reference to a.

To calculate $w_{\Delta\beta}$ we need to observe that each individual non-overlaping layer adds a weight of 2 to $w_{\Delta\beta}$ whiles the fixed point and l_1 adds weight 1 to it. This means

$$w_{\Delta\beta} = 2l_1 + 2$$

To calculate $w_{\Delta\gamma}$, observe that each double-overlapping layer contributes a weight of 2 to $w_{\Delta\gamma}$ whiles l_1 and l_2 each adds a weight of 1. Therefore

$$w_{\Delta\gamma} = 2(l_2 - l_1) + 2$$

And

$$\begin{aligned} w_p &= 2l_1 + 1 + 2(l_2 - l_1) + 1 \\ &= 2l_2 + 2 \end{aligned} \tag{3}$$

Making $\beta > \gamma$ and maintaining the definitions for l_1 and l_2, we get

$$w_p = 2l_1 + 2$$

For both cases listed above. We can see that, w_p depends on wether l_1 or l_2 is furtherst from a, therefore

$$w_p = 2(\max\{l_1, l_2\}) + 2 \tag{4}$$

Assuming that after interleaving $Q(x)$ another weight-3 RTZ input $Q'(x) = x^{a'\tau+t}(1+x^{\beta'\tau+1}+x^{\gamma'\tau+2})$is produced. Let l_1', l_2' be similarly defined like l_1 and l_2. Then w_H of the turbo codeword is given by

$$\begin{aligned} w_H &= 2(\max\{l_1, l_2\}) + 2 + 2(\max\{l_1', l_2'\}) + 2 + 3 \\ &= 7 + 2(\max\{l_1, l_2\} + \max\{l_1', l_2'\}) \end{aligned} \tag{5}$$

$\square$