情報・ネットワーク工学専攻基礎

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Q2

i. The MAP decision rule is given by

$$\max_{l \in (1,\ldots,k)} P(x_l|y)$$

The ML decision rule is given by

$$\max_{l \in (1,\ldots,k)} P(y|x_l)$$

ii. The MAP decision rule uses the a posteriori probability of the information bit $x_l, l \in (1, ..., k)$ to determine which information bit the received signal y corresponds to. In most cases the probability of x_l is easily calculated making it a suitable decision variable. To determine which x_l the received signal y corresponds to, we calculate the conditional probability $P(x_l|y)$ for all $l \in (1, ..., k)$ and choose the x_l with the maximum probability.

iii. The MAP decision rule is given by

$$\max_{l \in (1,\ldots,k)} P(x_l|y)$$

By Bayes rule

$$P(x_l|y) = \frac{P(x_l)P(y|x_l)}{P(y)}$$

since P(y) is constant, the above formula reduces to

$$P(x_l|y) = P(x_l)P(y|x_l)$$

in the case of equiprobable information bits, $P(x_l) = 1/K$ and $P(x_l|y) \propto P(y|x_l)$.
This is implies that in the case of equiprobable information bits MAP=ML.

Q3

i.

$$\begin{aligned} H(x) &= -\sum_{x \in X} P_X(x) log P_X(x) \\ &= \sum_{x \in X} P_X(x) i(x) \end{aligned}$$

From the information viewpoint, entropy is a measure of information that is acquired by the knowledge of X

ii. For a function to be considered convex, the value at the midpoint of every interval in its domain should not exceed the arithmetic mean of its values at the ends of the interval. The domain for $H(x)$ is $P_X(x)$. The values at the ends of the interval are 0 and 1 respectively. The mean of these values is 0.5. Since this is equal to the midpoint value of the interval, it implies that $H(x)$ is a convex function.

iii. The maximum value for binary entropy is 1. The formula for binary entropy is given as

$$h(p) = -plogp - (1-p)log(1-p)$$

when $p = 1/2$

$$\begin{aligned} h(1/2) &= -1/2log(1/2) - (1-(1/2))log(1-(1/2)) \\ &= 1/2 + 1/2 \\ &= 1 \end{aligned}$$

This shows that when p=1/2, h(p) is maximum.

Q7

i. Using the generator matrix **G**

$$\mathbf{x} = \mathbf{aG}$$

information bit **a**	codeword **x**
000	0000000
001	0011110
010	0101011
011	0110101
100	1001101
101	1010011
110	1100110
111	1111000

ii. Using the MAP decision rule

$$x_{\widehat{m}} = arg \max_{x_m \in \mathbf{x}} P(x_m|y)$$

received signal,y = 1011110, cross-over probability, $\varepsilon = 0.1$

$$\begin{aligned}
P(x_0|y) &= P(0000000|y) = \varepsilon^5(1-\varepsilon)^2 = 8.1 \times 10^{-6} \\
P(x_1|y) &= P(0011110|y) = \varepsilon(1-\varepsilon)^6 = 53144.1 \times 10^{-6} \\
P(x_2|y) &= P(0101011|y) = \varepsilon^5(1-\varepsilon)^2 = 8.1 \times 10^{-6} \\
P(x_3|y) &= P(0110101|y) = \varepsilon^5(1-\varepsilon)^2 = 8.1 \times 10^{-6} \\
P(x_4|y) &= P(1001101|y) = \varepsilon^3(1-\varepsilon)^4 = 656.1 \times 10^{-6} \\
P(x_5|y) &= P(1010011|y) = \varepsilon^3(1-\varepsilon)^4 = 656.1 \times 10^{-6} \\
P(x_6|y) &= P(1100110|y) = \varepsilon^3(1-\varepsilon)^4 = 656.1 \times 10^{-6} \\
P(x_7|y) &= P(1111000|y) = \varepsilon^3(1-\varepsilon)^4 = 656.1 \times 10^{-6}
\end{aligned}$$

therefore by MAP decision rule, $x_{\widehat{m}} = x_1$ which corresponds to the information bit 001