Formula to Calculate Weight for Low-Weight Weight 3 Inputs and Proof

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0.1 Equation and Proof

Proof. Since the impulse response is

$$(1\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ \cdots)$$

Let
$\boldsymbol{\phi}_1 = (0\ 0\ 1),\ \boldsymbol{\phi}_1' = (0\ 1\ 0),\ \boldsymbol{\phi}_1'' = (1\ 0\ 0),$
$\boldsymbol{\phi}_2 = (0\ 1\ 1),\ \boldsymbol{\phi}_2' = (1\ 1\ 0),\ \boldsymbol{\phi}_2'' = (1\ 0\ 1),$
$\boldsymbol{\phi}_3 = (1\ 1\ 1).$

Then, the weight-3 RTZ occurs since $\boldsymbol{\phi}_2 + \boldsymbol{\phi}_2' + \boldsymbol{\phi}_2'' = \mathbf{0}_3$.

Now, we consider the weight of the vector derived by the sumation of the followings vectors.

$$\begin{array}{c}(\mathbf{0}_{3i}\ \boldsymbol{\phi}_1\ \boldsymbol{\phi}_2'\ \cdots)\\(\mathbf{0}_{3j}\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_2''\ \cdots)\\(\mathbf{0}_{3k}\ \boldsymbol{\phi}_3\ \boldsymbol{\phi}_2\ \cdots)\end{array}$$

To simplify calculation, we have included an addition table for all the vectors which is shown in Table 1

	$\boldsymbol{\phi}_1$	$\boldsymbol{\phi}_1'$	$\boldsymbol{\phi}_1''$	$\boldsymbol{\phi}_2$	$\boldsymbol{\phi}_2'$	$\boldsymbol{\phi}_2''$	$\boldsymbol{\phi}_3$
$\boldsymbol{\phi}_1$	$\mathbf{0}_3$	–	–	–	–	–	–
$\boldsymbol{\phi}_1'$	$\boldsymbol{\phi}_2$	$\mathbf{0}_3$	–	–	–	–	–
$\boldsymbol{\phi}_1''$	$\boldsymbol{\phi}_2''$	$\boldsymbol{\phi}_2'$	$\mathbf{0}_3$	–	–	–	–
$\boldsymbol{\phi}_2$	$\boldsymbol{\phi}_1'$	$\boldsymbol{\phi}_1$	$\boldsymbol{\phi}_3$	$\mathbf{0}_3$	–	–	–
$\boldsymbol{\phi}_2'$	$\boldsymbol{\phi}_3$	$\boldsymbol{\phi}_1''$	$\boldsymbol{\phi}_1'$	$\boldsymbol{\phi}_2''$	$\mathbf{0}_3$	–	–
$\boldsymbol{\phi}_2''$	$\boldsymbol{\phi}_1''$	$\boldsymbol{\phi}_3$	$\boldsymbol{\phi}_1$	$\boldsymbol{\phi}_2'$	$\boldsymbol{\phi}_2$	$\mathbf{0}_3$	–
$\boldsymbol{\phi}_3$	$\boldsymbol{\phi}_2'$	$\boldsymbol{\phi}_2''$	$\boldsymbol{\phi}_2$	$\boldsymbol{\phi}_1''$	$\boldsymbol{\phi}_1$	$\boldsymbol{\phi}_1'$	$\mathbf{0}_3$

Table 1: Truth Table

Furthermore, we consider 4 general cases for all possible values of i, j, k These cases are $(=\quad=)$, $(=\quad<)$, $(<\quad=)$ and $(<\quad<)$

Case 0: $i = j = k$

For this case, the vectors to sum will be

$$\begin{array}{c}(\mathbf{0}_{3i}\ \boldsymbol{\phi}_1\ \boldsymbol{\phi}_2'\ \cdots)\\(\mathbf{0}_{3j}\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_2''\ \cdots)\\(\mathbf{0}_{3k}\ \boldsymbol{\phi}_3\ \boldsymbol{\phi}_2\ \cdots)\\\hline(\mathbf{0}_{3i}\ \boldsymbol{\phi}_2''\ \mathbf{0}_3\ \cdots)\end{array}$$

and the derived vector will be $(\mathbf{0}_{3i}\ \boldsymbol{\phi}_2''\ \mathbf{0}_3\ \cdots)$ with a weight of $w_p = 2$

Case 1a: $i = j < k$
vector to sum:

$$\begin{array}{r} (\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_1\ \boldsymbol{\phi}_2'\ \cdots\ \boldsymbol{\phi}_2'\ \boldsymbol{\phi}_2'\ \boldsymbol{\phi}_2'\ \cdots) \\ (\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_2''\ \cdots\ \boldsymbol{\phi}_2''\ \boldsymbol{\phi}_2''\boldsymbol{\phi}_2''\ \cdots) \\ +(\mathbf{0}_3\ \ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_3\ \boldsymbol{\phi}_2\ \cdots) \\ \hline (\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_1'\ \boldsymbol{\phi}_2\ \cdots\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_1''\ \mathbf{0}_3\ \cdots) \end{array}$$

derived vector : $(\mathbf{0}_{3j}\ \boldsymbol{\phi}_1'\ (\boldsymbol{\phi}_2)_{k-j-1}\ \boldsymbol{\phi}_1''\ \mathbf{0}_3\ \cdots)$
Parity weight:

$$w_p = 2(k-j) \tag{0-1}$$

Case 1b: $i = k < j$
vectors to sum:

$$\begin{array}{r} (\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_1\ \boldsymbol{\phi}_2'\ \boldsymbol{\phi}_2'\ \boldsymbol{\phi}_2'\ \boldsymbol{\phi}_2'\ \cdots) \\ (\mathbf{0}_3 \cdots\cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_2''\ \cdots) \\ +(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_3\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_2\ \cdots) \\ \hline (\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_2'\ \boldsymbol{\phi}_2''\ \boldsymbol{\phi}_2''\ \boldsymbol{\phi}_2'\ \mathbf{0}_3\ \cdots) \end{array}$$

derived vector : $(\mathbf{0}_{3i}\ \boldsymbol{\phi}_2'\ (\boldsymbol{\phi}_2'')_{j-k-1}\ \boldsymbol{\phi}_2'\ \mathbf{0}_3\ \cdots)$
Parity weight:

$$w_p = 2(j-i) + 2 \tag{0-2}$$

Case 1c: $j = k < i$
vectors to sum:

$$\begin{array}{r} (\mathbf{0}_3\ \ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_1\ \boldsymbol{\phi}_2'\ \cdots) \\ (\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_2''\ \cdots\ \boldsymbol{\phi}_2''\ \boldsymbol{\phi}_2''\boldsymbol{\phi}_2''\ \cdots) \\ +(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_3\ \boldsymbol{\phi}_2\ \cdots\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_2\ \cdots) \\ \hline (\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_1''\ \boldsymbol{\phi}_2\ \cdots\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_3\ \mathbf{0}_3\ \cdots) \end{array}$$

derived vector : $(\mathbf{0}_{3j}\ \boldsymbol{\phi}_1''\ (\boldsymbol{\phi}_2)_{i-j-1}\ \boldsymbol{\phi}_3\ \mathbf{0}_3\ \cdots)$
Parity weight:

$$w_p = 2(i-j) + 2 \tag{0-3}$$

Case 2a: $i < j = k$
vectors to sum:

$$\begin{array}{r}
(\mathbf{0}_3\ \cdots\ \boldsymbol{\phi}_1\ \boldsymbol{\phi}_2'\ \cdots\ \boldsymbol{\phi}_2'\ \boldsymbol{\phi}_2'\ \boldsymbol{\phi}_2'\ \cdots) \\
(\mathbf{0}_3\ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_2\ \phi_2''\ \cdots) \\
+(\mathbf{0}_3\ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_3\ \phi_2\ \cdots) \\
\hline
(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\boldsymbol{\phi}_1\ \boldsymbol{\phi}_2'\ \cdots\ \boldsymbol{\phi}_2'\ \boldsymbol{\phi}_1'\ \mathbf{0}_3\ \cdots)
\end{array}$$

derived vector : $(\mathbf{0}_{3i}\ \boldsymbol{\phi}_1\ (\boldsymbol{\phi}_2')_{k-i-1}\ \boldsymbol{\phi}_1'\ \mathbf{0}_3\ \cdots)$
Parity weight:

$$w_p = 2(k-i) \tag{0-4}$$

Case 2b: $j < k = i$
vectors to sum:

$$\begin{array}{r}
(\mathbf{0}_3\ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_1\ \phi_2'\ \cdots) \\
(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_2\ \phi_2''\ \cdots\ \phi_2''\ \phi_2''\ \phi_2''\ \cdots) \\
+(\mathbf{0}_3\ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_3\ \phi_2\ \cdots) \\
\hline
(\mathbf{0}_3\ \cdots \mathbf{0}_3\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_2''\ \cdots\ \boldsymbol{\phi}_2''\ \boldsymbol{\phi}_2\ \mathbf{0}_3\ \cdots)
\end{array}$$

derived vector : $(\mathbf{0}_{3j}\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_2'')_{i-j-1}\ \boldsymbol{\phi}_2\ \mathbf{0}_3\ \cdots)$
Parity weight:

$$w_p = 2(i-j) + 2 \tag{0-5}$$

Case 2c: $k < i = j$
vectors to sum:

$$\begin{array}{r}
(\mathbf{0}_3\ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_1\ \phi_2'\ \cdots) \\
(\mathbf{0}_3\ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_2\ \phi_2''\ \cdots) \\
+(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_3\ \phi_2\ \cdots\ \phi_2\ \phi_2\ \phi_2\ \cdots) \\
\hline
(\mathbf{0}_3\ \cdots \mathbf{0}_3\ \boldsymbol{\phi}_3\ \boldsymbol{\phi}_2\ \cdots\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_1\ \mathbf{0}_3\ \cdots)
\end{array}$$

derived vector : $(\mathbf{0}_{3k}\ \boldsymbol{\phi}_3\ (\boldsymbol{\phi}_2)_{j-k-1}\ \boldsymbol{\phi}_1\ \mathbf{0}_3\ \cdots)$
Parity weight:

$$w_p = 2(j-k) + 2 \tag{0-6}$$

Case 3a: $i < j < k$
vectors to sum:

$$\begin{array}{l}
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \phi_1 \ \phi_2' \ \cdots \ \phi_2' \ \phi_2' \ \phi_2' \ \cdots \ \phi_2' \ \phi_2' \ \phi_2' \ \cdots) \\
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \mathbf{0}_3 \ \mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \phi_2 \ \phi_2'' \ \cdots \ \phi_2'' \ \phi_2'' \ \phi_2'' \ \cdots) \\
+(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \mathbf{0}_3 \ \mathbf{0}_3 \ \cdots \ \cdots \ \cdots \ \cdots \ \mathbf{0}_3 \ \phi_3 \ \phi_2 \ \cdots) \\
\hline
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \boldsymbol{\phi}_1 \ \boldsymbol{\phi}_2' \cdots \boldsymbol{\phi}_2' \ \boldsymbol{\phi}_2'' \ \boldsymbol{\phi}_2 \ \cdots \ \boldsymbol{\phi}_2 \ \boldsymbol{\phi}_1'' \ \mathbf{0}_3 \ \cdots)
\end{array}$$

derived vector : $(\mathbf{0}_{3i} \ \boldsymbol{\phi}_1 \ (\boldsymbol{\phi}_2')_{j-i-1} \ \boldsymbol{\phi}_2'' \ (\boldsymbol{\phi}_2)_{k-j-1} \ \boldsymbol{\phi}_1'' \ \mathbf{0}_3 \ \cdots)$
Parity weight:

$$\begin{aligned} w_p &= 2(j-i) + 1 + 2(k-j-1) + 1 \\ &= 2(k-i) \end{aligned} \tag{0-7}$$

Case 3b: $i < k < j$
vectors to sum:

$$\begin{array}{l}
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \phi_1 \ \phi_2' \ \cdots \ \phi_2' \ \phi_2' \ \phi_2' \ \cdots \ \phi_2' \ \phi_2' \ \phi_2' \cdots) \\
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \mathbf{0}_3 \ \mathbf{0}_3 \ \cdots \ \cdots \ \cdots \ \cdots \ \mathbf{0}_3 \ \phi_2 \ \phi_2'' \cdots) \\
+(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \mathbf{0}_3 \ \mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \phi_3 \ \phi_2 \ \cdots \ \phi_2 \ \phi_2 \ \phi_2 \cdots) \\
\hline
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \boldsymbol{\phi}_1 \ \boldsymbol{\phi}_2' \cdots \ \boldsymbol{\phi}_2' \ \boldsymbol{\phi}_1 \ \boldsymbol{\phi}_2'' \ \cdots \ \boldsymbol{\phi}_2'' \ \boldsymbol{\phi}_2' \ \mathbf{0}_3 \cdots)
\end{array}$$

derived vector : $(\mathbf{0}_{3i} \ \boldsymbol{\phi}_1 \ (\boldsymbol{\phi}_2')_{k-i-1} \ \boldsymbol{\phi}_1 \ (\boldsymbol{\phi}_2'')_{j-i-1} \ \boldsymbol{\phi}_2' \ \mathbf{0}_3 \ \cdots)$
Parity weight:

$$\begin{aligned} w_p &= 2(k-i) + 2(j-k) \\ &= 2(j-i) \end{aligned} \tag{0-8}$$

Case 3c: $j < k < i$
vectors to sum:

$$\begin{array}{l}
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \mathbf{0}_3 \ \mathbf{0}_3 \ \cdots \ \cdots \ \cdots \ \cdots \ \mathbf{0}_3 \ \phi_1 \ \phi_2' \cdots) \\
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \phi_2 \ \phi_2'' \ \cdots \ \phi_2'' \ \phi_2'' \ \phi_2'' \ \cdots \ \phi_2'' \ \phi_2'' \ \phi_2'' \cdots) \\
+(\mathbf{0}_3 \ \cdots \mathbf{0}_3 \ \mathbf{0}_3 \ \mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \phi_3 \ \phi_2 \ \cdots \ \phi_2 \ \phi_2 \ \phi_2 \cdots) \\
\hline
(\mathbf{0}_3 \ \cdots \ \mathbf{0}_3 \ \boldsymbol{\phi}_2 \ \boldsymbol{\phi}_2'' \cdots \ \boldsymbol{\phi}_2'' \ \boldsymbol{\phi}_1' \ \boldsymbol{\phi}_2' \cdots \ \boldsymbol{\phi}_2' \ \boldsymbol{\phi}_3 \ \mathbf{0}_3 \cdots)
\end{array}$$

derived vector : $(\mathbf{0}_{3k} \ \boldsymbol{\phi}_2 \ (\boldsymbol{\phi}_2'')_{k-j-1} \ \boldsymbol{\phi}_1' \ (\boldsymbol{\phi}_2')_{i-k-1} \ \boldsymbol{\phi}_3 \ \mathbf{0}_3 \ \cdots)$
Parity weight:

$$\begin{aligned} w_p &= 2(k-j) + 1 + 2(i-k) + 1 \\ &= 2(i-j) + 2 \end{aligned} \tag{0-9}$$

Case 3d: $j < i < k$
vectors to sum:

$$
\begin{array}{r}
(\mathbf{0}_3\ \cdots \mathbf{0}_3\ \mathbf{0}_3\ \mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_1\ \phi_2'\ \cdots\ \phi_2'\ \phi_2'\ \phi_2' \cdots) \\
(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_2\ \phi_2''\ \cdots\ \phi_2''\ \phi_2''\ \phi_2''\ \cdots\ \phi_2''\ \phi_2''\ \phi_2'' \cdots) \\
+(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \mathbf{0}_3\ \mathbf{0}_3\ \ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_3\ \phi_2 \cdots) \\
\hline
(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_2'' \cdots\ \boldsymbol{\phi}_2''\ \boldsymbol{\phi}_1''\ \boldsymbol{\phi}_2 \cdots\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_1''\ \mathbf{0}_3 \cdots)
\end{array}
$$

derived vector : $(\mathbf{0}_{3j}\ \boldsymbol{\phi}_2\ (\boldsymbol{\phi}_2'')_{i-j-1}\ \boldsymbol{\phi}_1''\ (\boldsymbol{\phi}_2)_{k-i-1}\ \boldsymbol{\phi}_1''\ \mathbf{0}_3\ \ \cdots)$
Parity weight:

$$
\begin{aligned}
w_p &= 2(i-j) + 1 + 2(k-i-1) + 1 \\
&= 2(k-j)
\end{aligned}
\tag{0-10}
$$

Case 3e: $k < i < j$
vectors to sum:

$$
\begin{array}{r}
(\mathbf{0}_3\ \cdots \mathbf{0}_3\ \mathbf{0}_3\ \mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_1\ \phi_2'\ \cdots\ \phi_2'\ \phi_2'\ \phi_2' \cdots) \\
(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \mathbf{0}_3\ \mathbf{0}_3\ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_2\ \phi_2'' \cdots) \\
+(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_3\ \phi_2\ \cdots\ \phi_2\ \phi_2\ \phi_2\ \cdots\ \phi_2\ \phi_2\ \phi_2 \cdots) \\
\hline
(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_3\ \boldsymbol{\phi}_2 \cdots\ \boldsymbol{\phi}_2\ \boldsymbol{\phi}_1'\ \boldsymbol{\phi}_2'' \cdots\ \boldsymbol{\phi}_2''\ \boldsymbol{\phi}_2'\ \mathbf{0}_3 \cdots)
\end{array}
$$

derived vector : $(\mathbf{0}_{3k}\ \boldsymbol{\phi}_3\ (\boldsymbol{\phi}_2)_{i-k-1}\ \boldsymbol{\phi}_1'\ (\boldsymbol{\phi}_2'')_{j-i-1}\ \boldsymbol{\phi}_2'\ \mathbf{0}_3\ \ \cdots)$
Parity weight:

$$
\begin{aligned}
w_p &= 2(i-k) + 2 + 2(j-i) \\
&= 2(j-k) + 2
\end{aligned}
\tag{0-11}
$$

Case 3f: $k < j < i$

$$
\begin{array}{r}
(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \mathbf{0}_3\ \mathbf{0}_3\ \cdots\ \cdots\ \cdots\ \cdots\ \mathbf{0}_3\ \phi_1\ \phi_2' \cdots) \\
(\mathbf{0}_3\ \cdots \mathbf{0}_3\ \mathbf{0}_3\ \mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_2\ \phi_2''\ \cdots\ \phi_2''\ \phi_2''\ \phi_2'' \cdots) \\
+(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \phi_3\ \phi_2\ \cdots\ \phi_2\ \phi_2\ \phi_2\ \cdots\ \phi_2\ \phi_2\ \phi_2 \cdots) \\
\hline
(\mathbf{0}_3\ \cdots\ \mathbf{0}_3\ \boldsymbol{\phi}_3\ \boldsymbol{\phi}_2 \cdots\ \boldsymbol{\phi}_2\ \mathbf{0}_3\ \boldsymbol{\phi}_2' \cdots\ \boldsymbol{\phi}_2'\ \boldsymbol{\phi}_3\ \mathbf{0}_3 \cdots)
\end{array}
$$

derived vector : $(\mathbf{0}_{3k}\ \boldsymbol{\phi}_3\ (\boldsymbol{\phi}_2)_{j-k-1}\ \mathbf{0}_3\ (\boldsymbol{\phi}_2')_{i-j-1}\ \boldsymbol{\phi}_3\ \mathbf{0}_3\ \ \cdots)$
Parity weight:

$$
\begin{aligned}
w_p &= 2(j-k) + 1 + 2(i-j) + 1 \\
&= 2(i-k) + 2
\end{aligned}
\tag{0-12}
$$

From all the above cases we can conclude that the parity weight for a weight-3 RTZ sequence may be calculated as

$$
w_p = \begin{cases} 2l, & i < k \\ 2l+2 & i \geq k \end{cases}
\tag{0-13}
$$

where $l = \max\{i, j, k\} - \min\{i, j, k\}$ is known as the layer distance. Assuming that after interleaving, another weight-3 RTZ input is produced. Let i', j', k', l' and w'_p be similarly defined. Then the Hamming weight $w_h$of the turbo codeword produced can be calculated as

$$w_H = \begin{cases} 3 + 2(l + l'), & i < k,\ i' < k' \\ 7 + 2(l + l') & i \geq k,\ i' \geq k' \\ 5 + 2(l + l') & i \geq k,\ i' < k' \text{ or } i < k\ i' \geq k' \end{cases} \tag{0-14}$$

□