Input-Structure Distance spectrum based on RTZ and Parity-check sequences of weight 2

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0.1 Input-Structure Distance spectrum based on RTZ and Parity-check sequences of weight 2

5/7 RSC code, $f(x) = 1 + x^2,\ g(x) = 1 + x + x^2$

$f(x)$ is Case3 whiles $g(x)$ is Case1 if $a(x) = \frac{h(x)}{f(x)},\ a(x) = \sum_{i=1}^{I} x^{a-2i}$

if $a(x) = \frac{b(x)}{g(x)},\ a(x) = \sum_{i=1}^{I} x^{b-3i+1} + x^{b-3i}$

Table 1: $a(x),\ b(x)$ for $h(x) = 1 + x^a$ generated via $f(x)$. $d_{\max} = 8$

$a(x)$	$b(x)$	$h(x)$
1	$1 + x + x^2$	$1 + x^2$
$1 + x^2$	$1 + x + x^3 + x^4$	$1 + x^4$
$1 + x^2 + x^4$	$1 + x + x^3 + x^5 + x^6$	$1 + x^6$
$1 + x^2 + x^4 + x^6$	$1 + x + x^3 + x^5 + x^7 + x^8$	$1 + x^8$

Table 2: $a(x),\ h(x)$ for $b(x) = 1 + x^b$ generated via $g(x)$. $d_{\max} = 8$

$a(x)$	$b(x)$	$h(x)$
$1 + x$	$1 + x^3$	$1 + x + x^2 + x^3$
$1 + x + x^3 + x^4$	$1 + x^6$	$1 + x + x^2 + x^4 + x^5 + x^6$

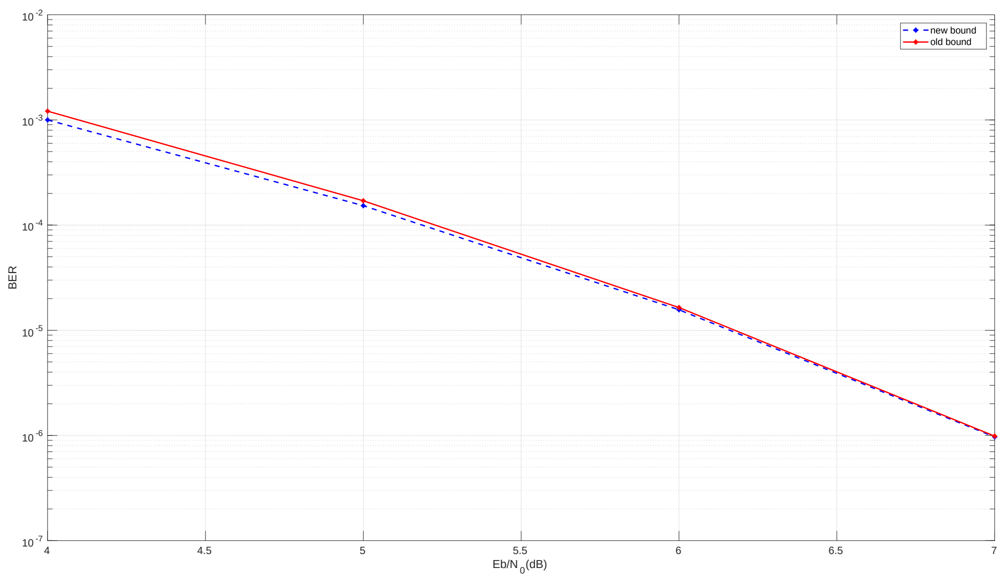

Figure 0-1: Old Bound vs New Bound for 5/7 RSC Code

37/21 RSC code, $f(x) = 1 + x + x^2 + x^3 + x^4,\ g(x) = 1 + x^4$

$f(x)$ is Case2 whiles $g(x)$ is Case3

if $a(x) = \frac{h(x)}{f(x)},\ a(x) = \sum_{i=1}^{I} x^{b-5i+1} + x^{b-5i}$

if $a(x) = \frac{b(x)}{g(x)},\ a(x) = \sum_{i=1}^{I} x^{b-4i}$

Table 3: $a(x),\ b(x)$ for $h(x) = 1 + x^a$ generated via $f(x)$. $d_{\max} = 9$

$a(x)$	$b(x)$	$h(x)$
$1 + x$	$1 + x + x^4 + x^5$	$1 + x^5$
$1 + x + x^5 + x^6$	$1 + x + x^4 + x^6 + x^9 + x^{10}$	$1 + x^{10}$

Table 4: $a(x),\ h(x)$ for $b(x) = 1 + x^b$ generated via $g(x)$. $d_{\max} = 9$

$a(x)$	$b(x)$	$h(x)$
1	$1 + x^4$	$1 + x + x^2 + x^3 + x^4$

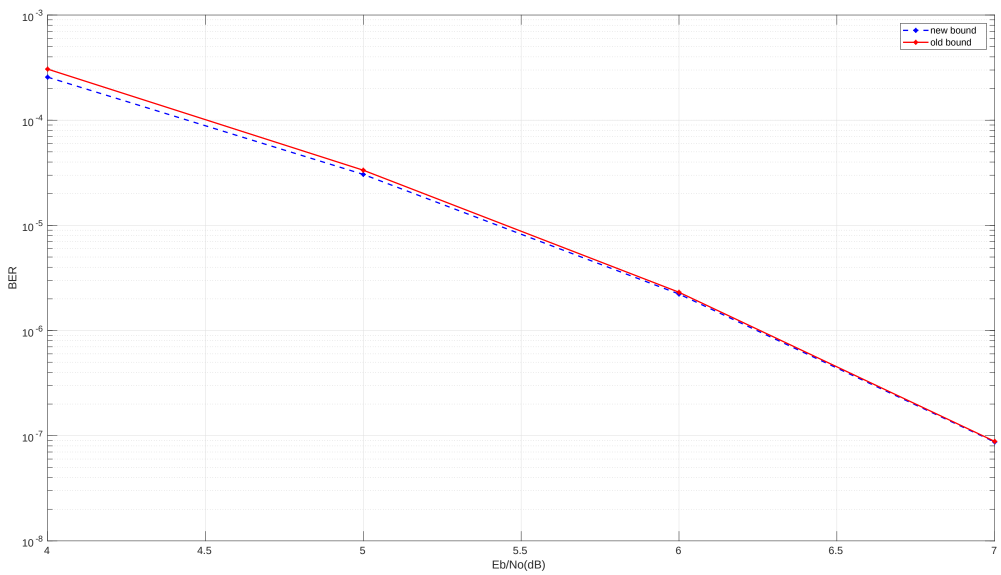

Figure 0-2: Old Bound vs New Bound for 37/21 RSC Code

23/35 RSC code, $f(x) = 1 + x + x^4,\ g(x) = 1 + x^2 + x^3 + x^4$

$f(x)$ is Case1 whiles $g(x)$ is Case4

if $a(x) = \frac{b(x)}{g(x)},\ a(x) = \sum_{i=1}^{I} x^{b-7i+3} + x^{b-7i+2} + x^{b-7i}$

Table 5: $a(x),\ b(x)$ for $h(x) = 1 + x^a$ generated via $f(x)$. $d_{\max} = 10$

$a(x)$	$b(x)$	$h(x)$
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Table 6: $a(x),\ h(x)$ for $b(x) = 1 + x^b$ generated via $g(x)$. $d_{\max} = 10$

$a(x)$	$b(x)$	$h(x)$
$1 + x^2 + x^3$	$1 + x^7$	$1 + x + x^2 + x^6 + x^7$
$1 + x^2 + x^3 + x^7 + x^9 + x^{10}$	$1 + x^{14}$	$1 + x + x^2 + x^6 + x^8 + x^9 + x^{13} + x^{14}$

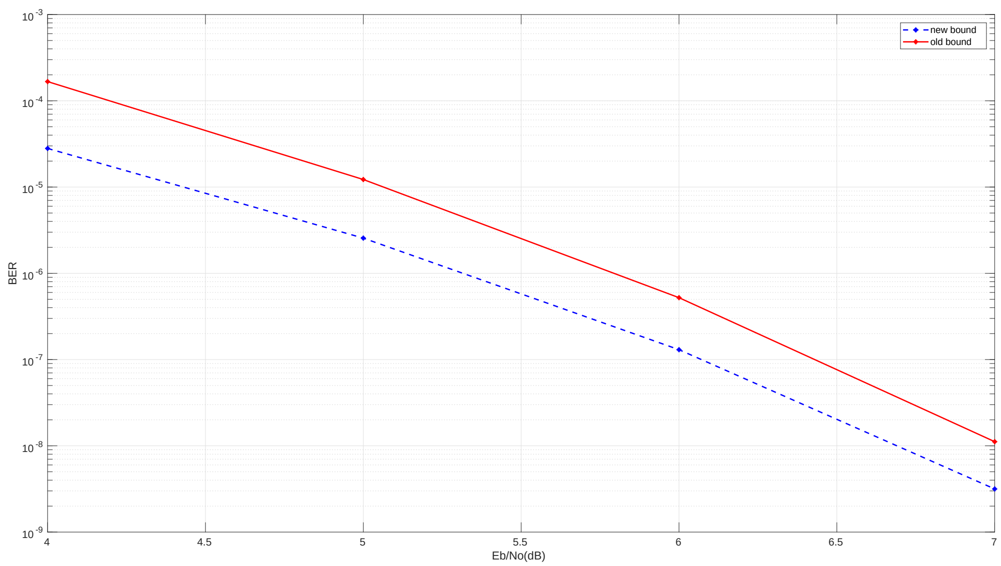

Figure 0-3: Old Bound vs New Bound for 23/35 RSC Code

0.2 List of Weight3 Parity-Check Sequences

It looks like $h(x)$ has a weight 3 Structure of the form $1 + x^a + x^b$ iff $f(x)$ has an odd number of terms ≥ 3.

1. $f(x)$ is a single primitive polynomial, eg $f(x) = 1 + x + x^2$.

Table 7: $f(x) = 1 + x + x^2$

$a(x)$	$h(x)$
1	$1 + x + x^2$
$1 + x + x^2$	$1 + x^2 + x^4$
$1 + x + x^3$	$1 + x^4 + x^5$
$1 + x^2 + x^3$	$1 + x + x^5$
$1 + x + x^2 + x^4 + x^5$	$1 + x^2 + x^7$
$1 + x + x^3 + x^4 + x^5$	$1 + x^5 + x^7$
$1 + x + x^3 + x^4 + x^6$	$1 + x^7 + x^8$

Table 8: $f(x) = 1 + x + x^4$

$a(x)$	$h(x)$
1	$1 + x + x^4$
$1 + x + x^2 + x^3 + x^5$	$1 + x^7 + x^9$
$1 + x + x^2 + x^3 + x^5 + x^7 + x^8$	$1 + x^{11} + x^{12}$
$1 + x + x^4$	$1 + x^2 + x^8$
$1 + x + x^2 + x^4 + x^6 + x^7 + x^{10}$	$1 + x^3 + x^{14}$
$1 + x + x^2 + x^3 + x^6$	$1 + x^5 + x^{10}$
$1 + x + x^2 + x^3 + x^5 + x^6 + x^9$	$1 + x^6 + x^{13}$
$1 + x + x^2 + x^3 + x^4 + x^6 + x^8 + x^9 + x^{12}$	$1 + x^4 + x^{16}$
$1 + x + x^2 + x^3 + x^5 + x^7 + x^9 + x^{10} + x^{13}$	$1 + x^8 + x^{17}$
$1 + x + x^2 + x^3 + x^5 + x^7 + x^8 + x^{11} + x^{14}$	$1 + x^{14} + x^{18}$

2. $f(x)$ is prime but not a primitive polynomial, eg $f(x) = 1 + x + x^2 + x^3 + x^4$ Could not find any parity-check bits with weight 3

3. $f(x)$ is made up of equal repeated polynomial roots, eg $f(x) = 1 + x^2 + x^4 = (1 + x + x^2)^2$.

Table 9: $f(x) = 1 + x^2 + x^4$

$a(x)$	$h(x)$
1	$1 + x^2 + x^4$
$1 + x^2 + x^4$	$1 + x^2 + x^4$
$1 + x^2 + x^6$	$1 + x^8 + x^{10}$
$1 + x^4 + x^6$	$1 + x^2 + x^{10}$
$1 + x^2 + x^4 + x^8 + x^{10}$	$1 + x^4 + x^{14}$
$1 + x^2 + x^6 + x^8 + x^{10}$	$1 + x^{10} + x^{14}$

4. $f(x)$ is made up of unique repeated polynomial roots.