Proving the Parity Weight Equation for RTZ Inputs Via Polynomials

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December 18, 2020

0.1 Parity Weight Equation for W2RTZs given the 5/7 RSC code

Theorem 1. The parity weight equation for the 5/7 RSC code's W2RTZ is given by

$$w_p^{(2)} = 2\alpha + 2 \tag{0-1}$$

Proof. In polynomial for, the parity-bit sequence $h(x)$ for any RSC code is given by

$$h(x) = f(x)g^{-1}(x)b(x) \tag{0-2}$$

where $f(x) = 1 + x^2$ and $b(x)$ is the message input and if it is an RTZ input and it can be written as

$$b(x) = a(x)g(x)$$

We then have

$$\begin{aligned} h(x) =& f(x)g^{-1}(x)a(x)g(x) \\ =& f(x)a(x) \end{aligned} \tag{0-3}$$

Specifically, if it is a W2RTZ input for the 5/7 RSC code, $b(x)$ has the general for

$$b(x) = 1 + x^{3\alpha}$$

. Given $g(x)$ and $b(x)$, it is possible to find $a(x)$ in its general form as it relates to W2RTZs simply by dividing $b(x)$ by $g(x) = 1 + x + x^2$ Then, we have

$$\begin{aligned} a(x) =& x^{3\alpha-2} + x^{3\alpha-3} + x^{3\alpha-5} + x^{3\alpha-6} + \cdots \\ =& \sum_{\alpha=1}^{i} x^{3(\alpha-1)+1} + x^{3(\alpha-1)} \end{aligned} \tag{0-4}$$

Fixing (0-4) into (0-3) we have

$$\begin{aligned} h(x) =& f(x)\Big[\sum_{\alpha=1}^{i} x^{3(\alpha-1)+1} + x^{3(\alpha-1)}\Big] \\ =& 1 + x^2\Big[\sum_{\alpha=1}^{i} x^{3(\alpha-1)+1} + x^{3(\alpha-1)}\Big] \\ =& \sum_{\alpha=1}^{i} x^{3(\alpha-1)+1} + x^{3(\alpha-1)} + \sum_{\alpha=1}^{i} x^{3\alpha} + x^{3\alpha-1} \end{aligned} \tag{0-5}$$

$\square$