Asignment 5

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1 Problem 1

The autocorrelation matrix R is given by:

$$R = E[\underline{u_n} \underline{u_n}^T]$$

and the channel is $h = [h_0 \ h_1]^T$. For N = 32 and because $\underline{h}$ has only 2 elements in it, we can define our R matrix as follows:

$$R = \begin{bmatrix} r(0) & r(1) & 0 & \cdots & 0 & 0 \\ r(1) & r(0) & r(1) & 0 & \cdots & 0 \\ \vdots & & & & & \vdots \\ 0 & \cdots & 0 & r(1) & r(0) & r(1) \\ 0 & 0 & \cdots & 0 & r(1) & r(0) \end{bmatrix} \tag{1}$$

$$r(0) = \sigma_\eta^2 + \sigma_b^2(h_0^2 + h_1^2) \tag{2}$$

$$r(1) = \sigma_b(h_0 h_1) \tag{3}$$

the following MATLAB code was used for building the R matrix and obtain the ratio of eigenvalues:

```
% build and analyze the R matrix
sigma_n2 = 0.001;
sigma_b2 = 1;
h_0 = 1;
h_1 = 1;
h = [h_0 ; h_1];
N = 32;

r_0 = sigma_n2 + sigma_b2 * (h_0^2 + h_1^2);
r_1 = sigma_b2 * h_0 * h_1;

R = eye(N) * r_0;
r1_diag = eye(N-1) * r_1;
up_diag = [ zeros(N-1,1) r1_diag ; zeros(N,1)'];

bot_diag = [zeros(N,1)'; r1_diag zeros(N-1,1) ];
```

```
17
18 R = R + up_diag + bot_diag;
19
20 lambda_i = eig(R);
21 ratio = max(lambda_i) / min(lambda_i)
```

Listing 1: Code for Problem 1

for $\underline{h} = [1\ 1]^T$ the ratio between the maximum eigenvalue and the minimum eigenvalue is:

$$ratio = \frac{\lambda_{max}}{\lambda_{min}} = 396.9652$$

if we make $h_1 = 0$ (from Eq. 3) we have $r(1) = 0$, then R is a diagonal matrix with $r(0)$ all over its diagonal, then, all the eigenvalues are the same, hence, the ratio would be 1. Running the code we can see this occurs in MATLAB as well:

$$ratio = \frac{\lambda_{max}}{\lambda_{min}} = 1$$

2 Problem 2

3 Problem 3