Assignment 5
EE 6543 taught at UMSA
Adaptive Signal Processing
Due Date: Tuesday, July 2, 2019
Due Location: in the lecture
Ingeniería Electrónica
Universidad Mayor de San Andrés

Instructor: Brent Petersen Ph.D. P.Eng. b.petersen@ieee.org

No.	Mark
1	/5
2	/5
3	/5
Total	/15

Problem 1. ______ mark(s) / 5 mark(s)

Fig. 1 shows an inverse filtering application. The following are defined,

$$\underline{b_n} = \begin{bmatrix} b_n \\ b_{n-1} \\ b_{n-2} \\ \vdots \\ b_{n-(M-1)} \end{bmatrix}, \quad \underline{h} = \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ \vdots \\ h_{M-1} \end{bmatrix}, \quad \underline{u_n} = \begin{bmatrix} u_n \\ u_{n-1} \\ u_{n-2} \\ \vdots \\ u_{n-(N-1)} \end{bmatrix}, \quad \underline{w} = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \\ w_{N-1} \end{bmatrix},$$

$$s_n = \underline{h}^T \underline{b_n}, \qquad \widehat{d_n} = \underline{w}^T \underline{u_n}, \qquad \text{and} \qquad d_n = b_{n-D}.$$

The random processes b_n and η_n are wide-sense stationary. They have zero-mean. They are uncorrelated over time. They are mutually uncorrelated. The variance of b_n is σ_b^2. The variance of η_n is σ_η^2.

Specifically, $M = 2$, $N = 32$, $D = 16$, $h_0 = 1$, $h_1 = 1$, $\sigma_b^2 = 1$, and $\sigma_\eta^2 = 0.001$.

The autocorrelation matrix, R, is

$$R = E\left[\underline{u_n}\,\underline{u_n}^T\right]$$

where $E\,[]$ denotes the mathematical expectation over the random processes.

For R, what is the numerical value of the ratio of the maximum eigenvalue to the minimum eigenvalue of R? Show all your work.

If the only change is to make h_1 equal zero, what is the new eigenvalue ratio of R?

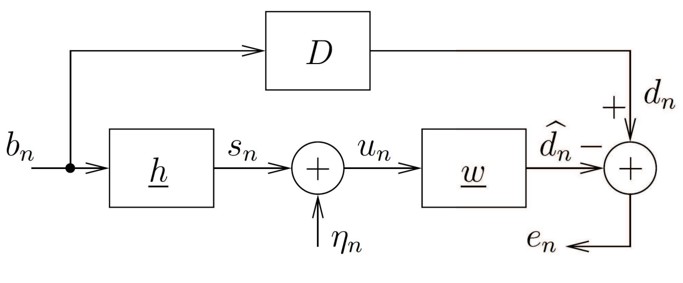

Figure 1: Inverse filtering application

Problem 2. ______ mark(s) / 5 mark(s)

The following is a brief statement of elements of the Levinson-Durbin algorithm for augmented normal equations.

Given

$$\begin{bmatrix} r(0) & r(1) & r(2) & & r(M) \\ r(1) & r(0) & r(1) & & r(M-1) \\ r(2) & r(1) & r(0) & & r(M-2) \\ & & & \ddots & \\ r(M) & r(M-1) & r(M-2) & & r(0) \end{bmatrix} \underline{a_M} = \begin{bmatrix} P_M \\ \\ \underline{0_M} \\ \\ \end{bmatrix}$$

where

$$\underline{r_m} = \begin{bmatrix} r(1) \\ r(2) \\ r(3) \\ \vdots \\ r(m) \end{bmatrix},$$

T denotes matrix transpose, B denotes a vector with elements taken backwards, then

$m = 0$:

$$\begin{aligned} \underline{a_0} &= [1] \\ P_0 &= r(0) \end{aligned}$$

$m = 1,\ 2,\ 3,\ \ldots,\ M$:

$$\begin{aligned} \Delta_{m-1} &= \underline{r_m^{BT}}\,\underline{a_{m-1}} \\ \Gamma_m &= -\frac{\Delta_{m-1}}{P_{m-1}} \\ \underline{a_m} &= \begin{bmatrix} \underline{a_{m-1}} \\ \\ 0 \end{bmatrix} + \Gamma_m \begin{bmatrix} 0 \\ \\ \underline{a_{m-1}^B} \end{bmatrix} \\ P_m &= P_{m-1}\left(1 - \Gamma_m^2\right). \end{aligned}$$

For this problem, the output of this is algorithm are the all the values of Δ_{m-1}, Γ_m, $\underline{a_m}$, and P_m.

In terms of M, how many additions and subtractions are required to evaluate the algorithm?

In terms of M, how many multiplications and divisions are required to evaluate the algorithm?

Operations involving zeros should not be counted. Multiplication by 1 should not be counted for this problem.

State the complexity of this algorithm in terms of $O(M)$ notation.

Problem 3. $\boxed{\text{\_\_\_\_\_\_ mark(s) / 5 mark(s)}}$

Given the autocorrelation matrix

$$R = \begin{bmatrix} 1 & 0.5 & 0.1 \\ 0.5 & 1 & 0.5 \\ 0.1 & 0.5 & 1 \end{bmatrix} \tag{1}$$

solve the Levinson-Durbin algorithm and show the reflection coefficients.