EE 4323 – Industrial Control Systems

Module 6: Frequency Response & Stability (Nyquist Criterion, Gain Margin)

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Overview

- Introduction
- Bode Plots
- Nyquist Plots
- Linear System Stability via the Nyquist Criterion
- Comparison of the commands `margin`, `nyquist`, `newnyq`

Introduction

- The <u>frequency domain</u> $W(j\omega)$ is very important in control theory:
  - Performance specifications (bandwidth, gain margin, phase margin)
  - Stability conditions
  - Loop shaping via lead, lag, lead-lag compensation
- The basic thread from <u>modelling</u> to <u>frequency response</u> is:
  1. We usually start with a nonlinear model: $\dot{x} = f(x, u)$; $y = h(x, u)$
  2. Then we determine operating point(s): $u \equiv u_0 \to x \equiv x_0$
  3. Then generate linearized model(s): $\dot{\delta x} = A\,\delta x + B\,\delta u$; $\delta y = C\,\delta x + D\,\delta u$, with $\delta x = x - x_0$, $\delta u = u - u_0$
  4. Transform into Laplace variables, $\Delta X = \mathcal{L}(\delta x)$, $\Delta U = \mathcal{L}(\delta u)$, $\Delta Y = \mathcal{L}(\delta y)$: $(sI - A)\,\Delta X = B\,\Delta U$, $\Delta Y = C\,\Delta X + D\,\Delta U$; $\underline{\Delta Y = [\,C\,(sI - A)^{-1}\,B + D\,]\,\Delta U}$
  5. Finally, obtain the transfer function:

$$W(s) = \frac{\mathcal{L}(\delta y)}{\mathcal{L}(\delta u)} = \frac{p(s)}{q(s)} \tag{1}$$

$$= C\,(sI - A)^{-1}B + D \tag{2}$$

Introduction (cont'd)

- Nyquist plots are most easily obtained from Bode plots, so we start with Bode plots:

- Bode plots are built up from the "very low frequency" term and first- and second-order factors:

$$W(s) = W_{LF}(s) \cdot \frac{(1+T_1 s)(1+T_2 s)\ldots(1+2\zeta_1 s/\omega_{n1} + s^2/\omega_{n1}^2)\ldots}{(1+T_k s)(1+T_{k+1} s)\ldots(1+2\zeta_m s/\omega_{n,m} + s^2/\omega_{n,m}^2)\ldots} \tag{3}$$

  where $W_{LF}(s)$ may be a constant K; or have zeroes at $s = 0$, $W_{LF}(s) = K\,s^q$; or have poles, in which case q is negative; $W_{LF}(s)$ defines the behaviour of $W(j\omega)$ for very low frequencies – *the remaining factors have a unity leading coefficient*, so they are essentially 1 at low frequencies.

- We proceed by looking at the behaviour of various factors; then, since we use log scales in the Bode plot, we add the results to obtain the overall frequency response (magnitude and phase)

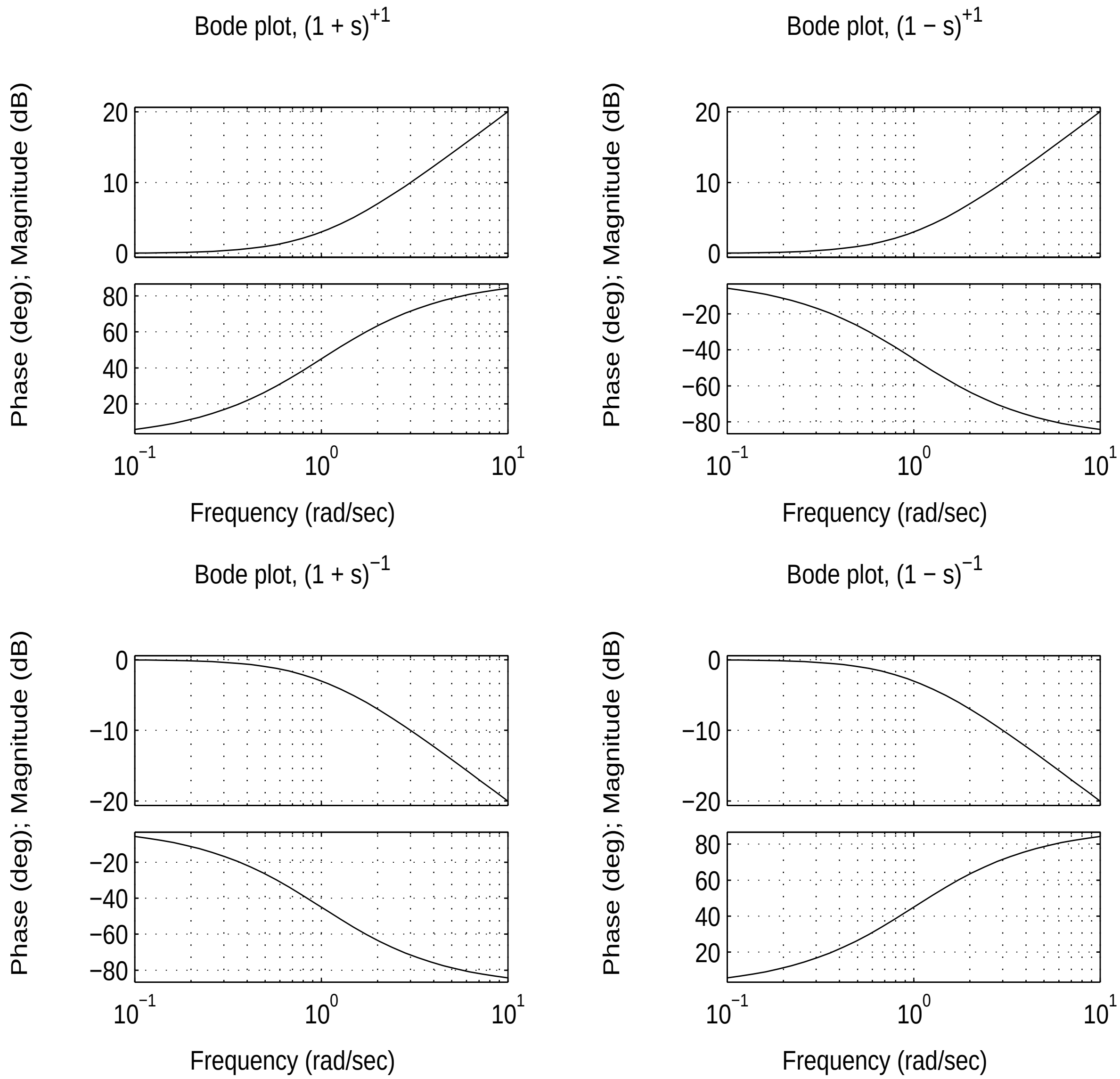
- Dealing with $W_{LF}(s)$:

  – $W_{LF}(s) = K$ – just add $20 \times \log_{10} |\,K\,|$ dB to the magnitude, the phase is 0 ($K > 0$) or $\pm$ 180 deg ($K < 0$)

  – $W_{LF}(s) = K\,s^q$ – for magnitude draw a line passing through the point $\omega = 1$, $|\,W(j\omega)\,|_{dB} = 20\,\log_{10} |\,K\,|$ with slope $20\,q$ dB/decade, and for phase add $90\,q$ degrees (plus $\pm$ 180 deg if $K < 0$); q may be positive or negative for this rule

Dealing with First-Order Factors

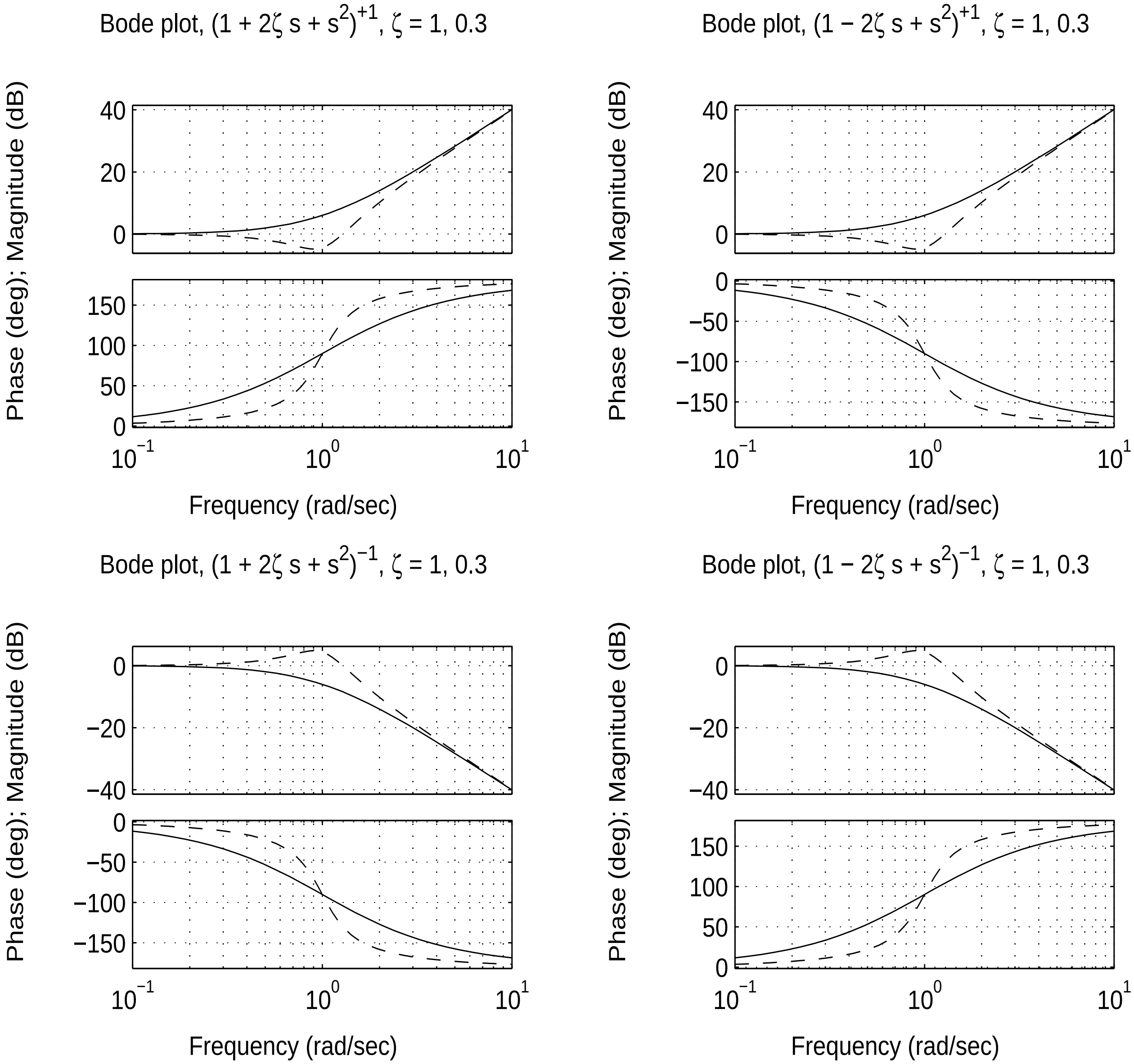
- ... and for first-order factors, $(1 + T_k s)$



- Note that the break frequency $\omega_k = 1/T_k$ is normalized to unity; shift these templates to the desired break frequency
- The asymptotic behaviour is 0 dB and 0 deg at low frequencies, $\pm$ 20 dB/decade and $\pm$ 90 degrees at high frequencies,

Dealing with Second-Order Factors

- ... and finally for second-order factors, $(1+2\zeta_1 s/\omega_{n1}+s^2/\omega_{n1}^2)$

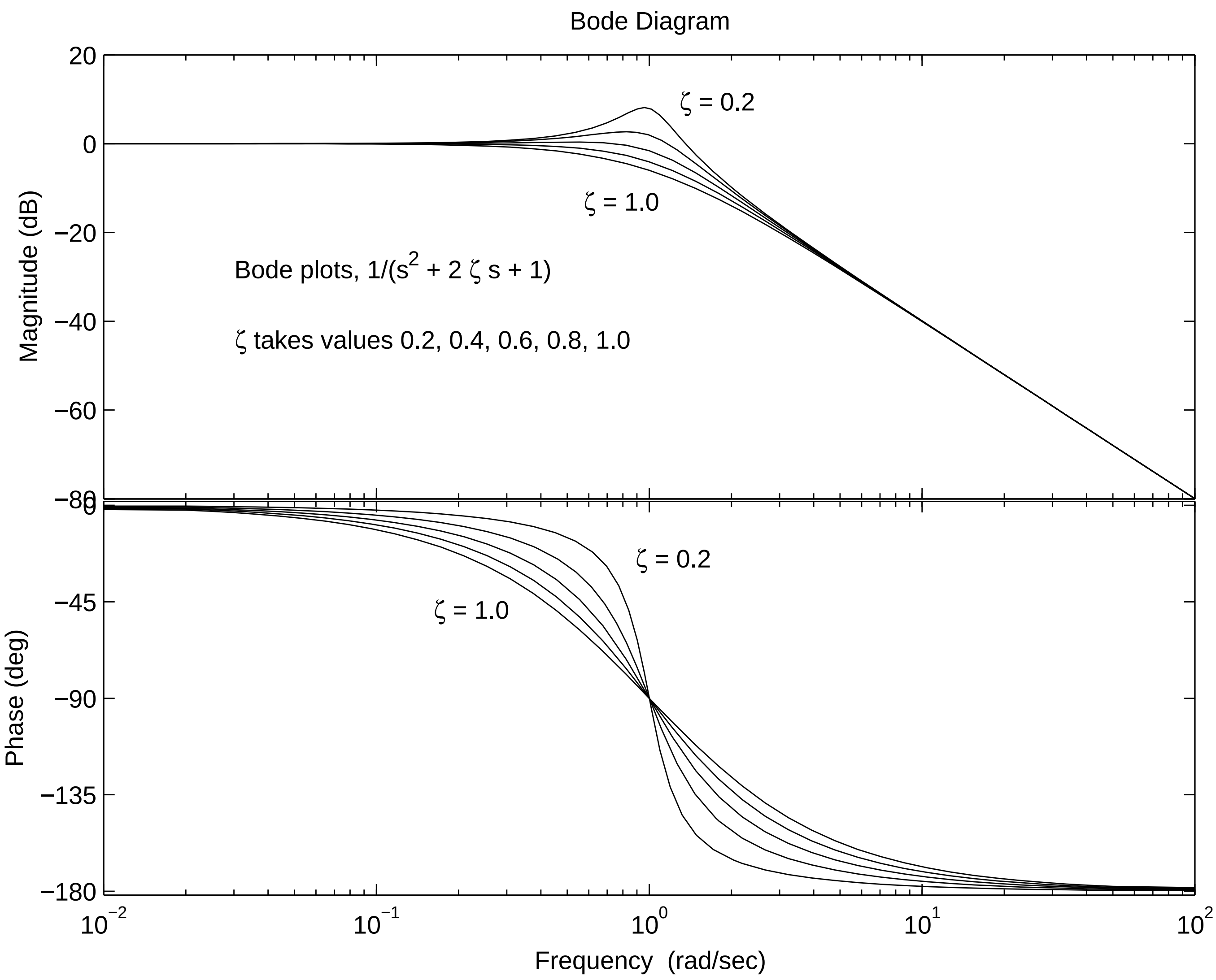


- Note that the break frequency ω_n is normalized to unity; shift these templates to the desired break frequency
- The asymptotic behaviour is 0 dB and 0 deg at low frequencies, $\pm$ 40 dB/decade and $\pm$ 180 degrees at high frequencies

Finally: Check the high-frequency behaviour against $W_\infty = K_\infty/s^{n-m}$; m = order(numerator), n = order(denominator)!

Second-Order Factors, Effect of Damping

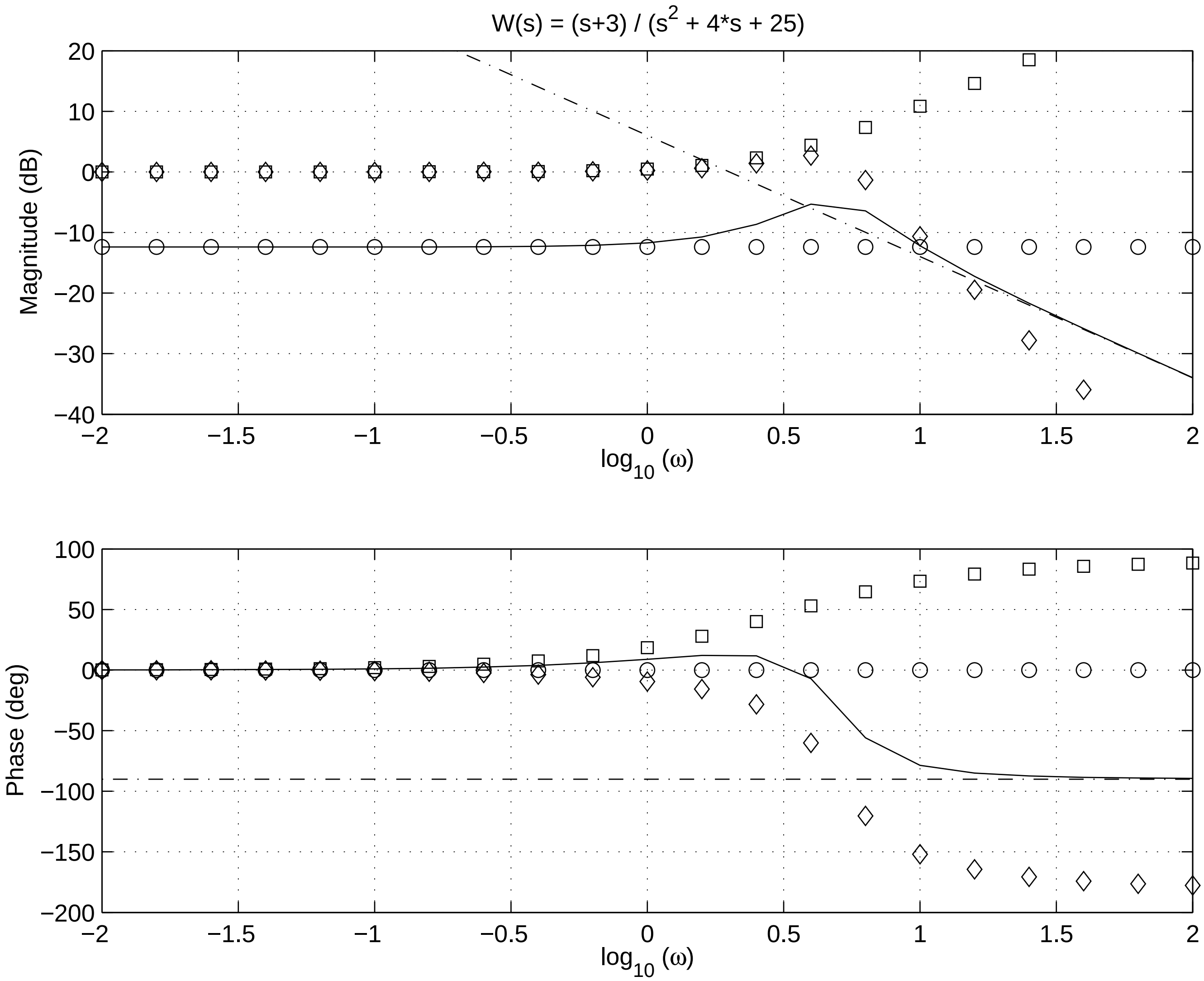
Unfortunately, the damping ζ makes a lot of difference; for the factor $1/(s^2 + 2\zeta s + 1)$ we have the following:



Bode Plot, Simple Example

Example 1: Given a stable plant, $W_1(s) = \dfrac{2(s+3)}{s^2+4s+25}$

1. First put $W_1(s)$ in standard form: $W_1(s) = 0.24 \cdot \dfrac{(1+s/3)}{(1+0.16s+s^2/25)}$
2. Then, $W_{LF} = 0.24 \rightarrow -12.4$ dB, 0 deg for all ω
3. Draw the individual templates for each factor: $\circ$ for W_{LF}, $\square$ for $(1+s/3)$, $\diamond$ for $(1+0.16s+s^2/25)$; notice $\zeta = 0.4$
4. Add magnitude & phase to obtain the final plot (solid curve):

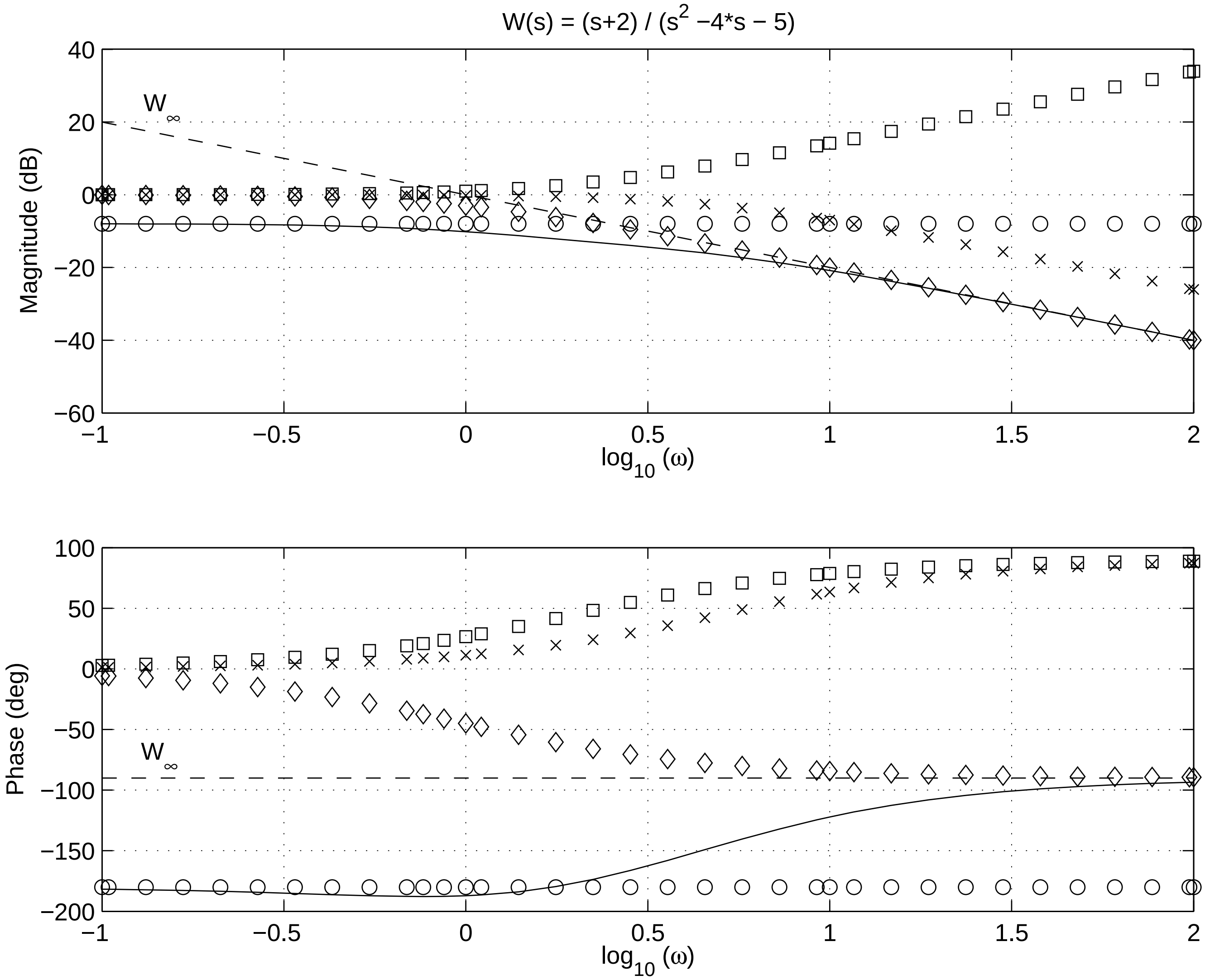


5. Finally, **check against** $W_\infty = 2/s$ **(dash-dot curve)**

Bode Plot, Unstable Plant

Example 2: Given an unstable plant, $W_2(s) = \dfrac{s+2}{s^2 - 4s - 5}$

1. First put $W_2(s)$ in standard form: $W_2(s) = -0.4 \cdot \dfrac{(1 + s/2)}{(1 - s/5)(1 + s)}$
2. Then, $W_{LF} = -0.4 \rightarrow -\ 8$ dB, – 180 deg for all ω
3. Draw the individual templates for each factor: $\circ$ for W_{LF}, $\square$ for numerator, $\times$ for $(1 - s/5)$, $\diamond$ for $(1 + s)$
4. Add magnitude & phase to obtain the final plot (solid curve):



Finally, **check against $W_\infty = 1/s$ (dashed curve)**

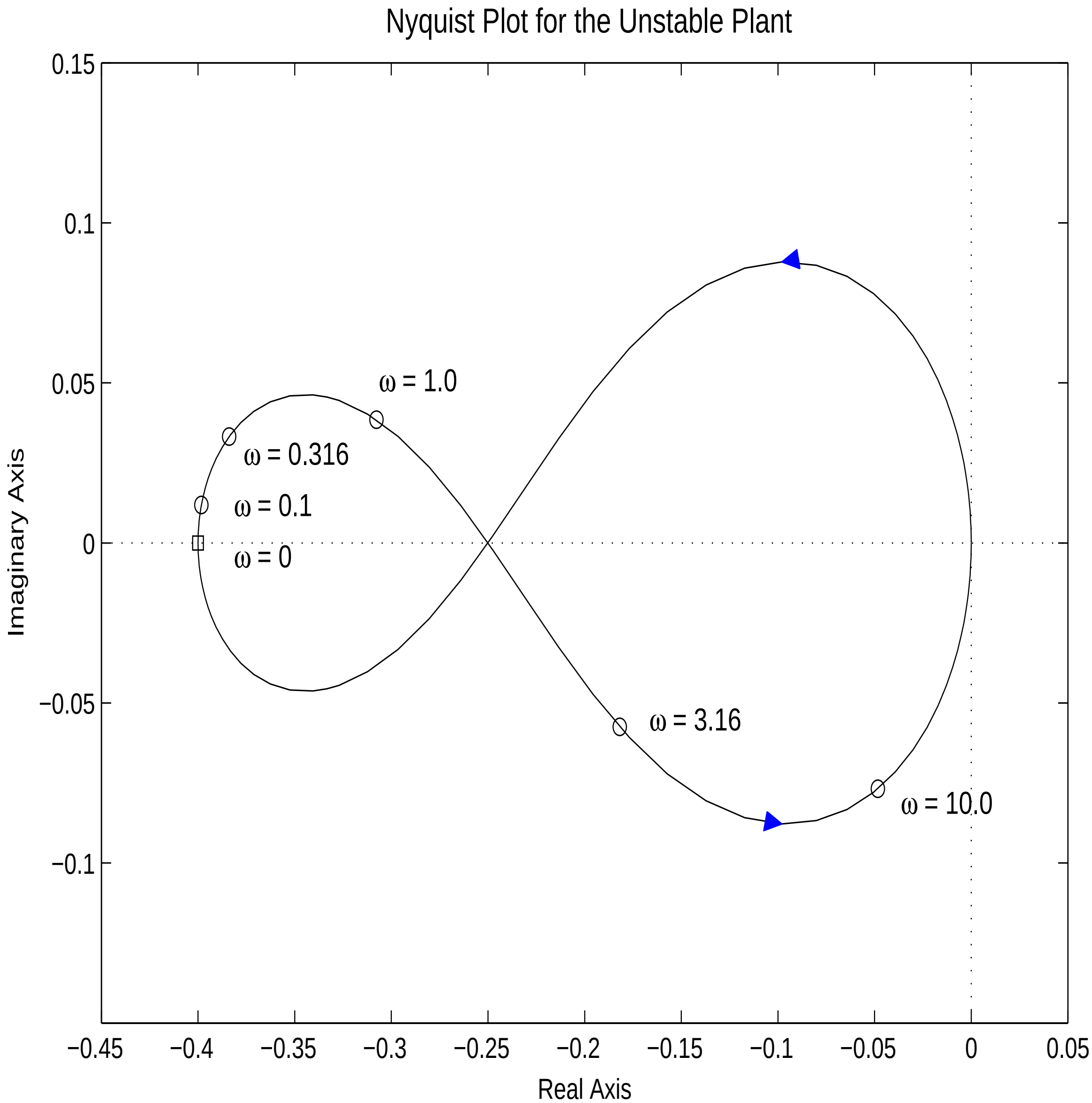
Bode to Nyquist Plot Process

The following data is taken from the preceeding Bode plot:

```
Table to create a Nyquist plot:

   w       dB       deg    mag

 0.1    -7.99   -181.7  0.398

 0.316  -8.283  -184.9  0.385

 1.0   -10.17   -187.1  0.310

 3.16  -14.39   -162.5  0.19

 10.   -20.8    -122.2  0.091
```

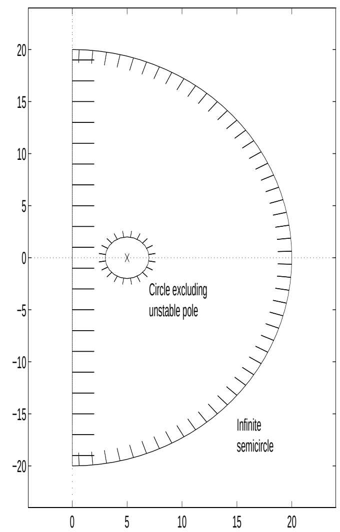
Basic Nyquist Plot from Bode Data

Again, given the unstable plant $W_2(s) = \dfrac{s+2}{s^2-4s-5}$ the data from the preceeding table can be used directly to generate the Nyquist plot:

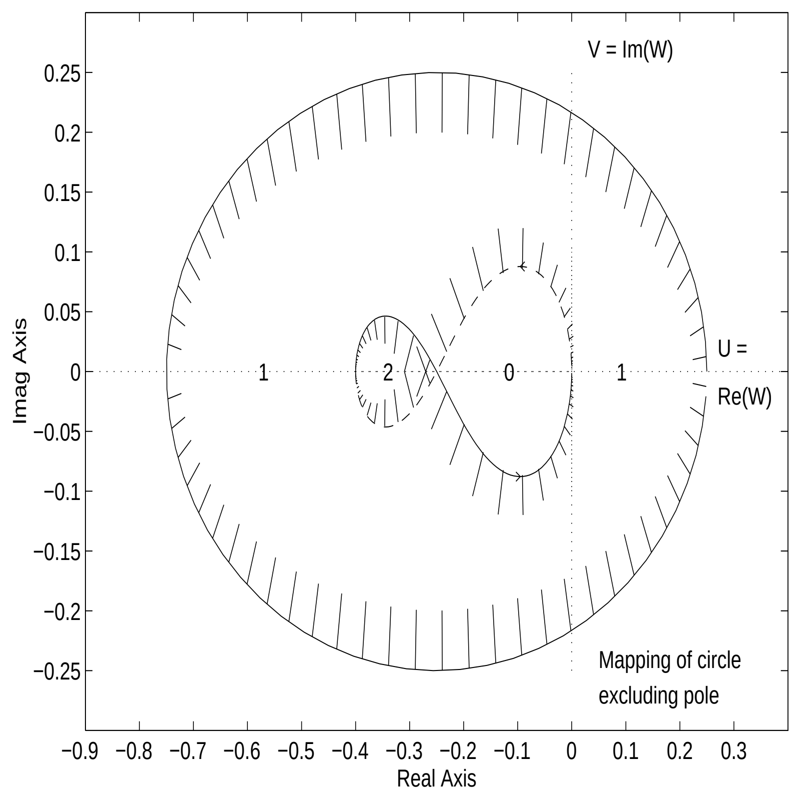


Nyquist Plot and Criterion

Example 2 (Cont'd): Given an unstable plant, $W_2(s) = \dfrac{s+2}{s^2-4s-5}$



s-plane region mapped

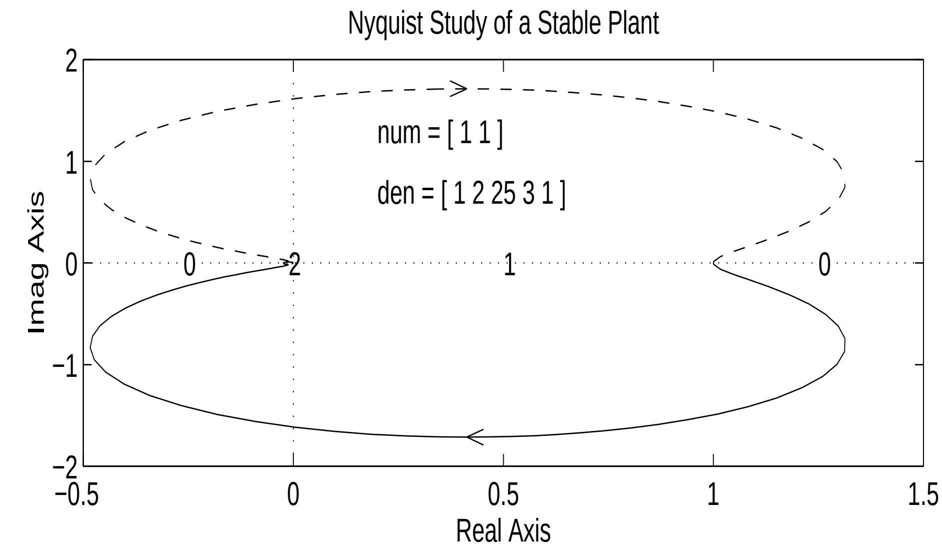


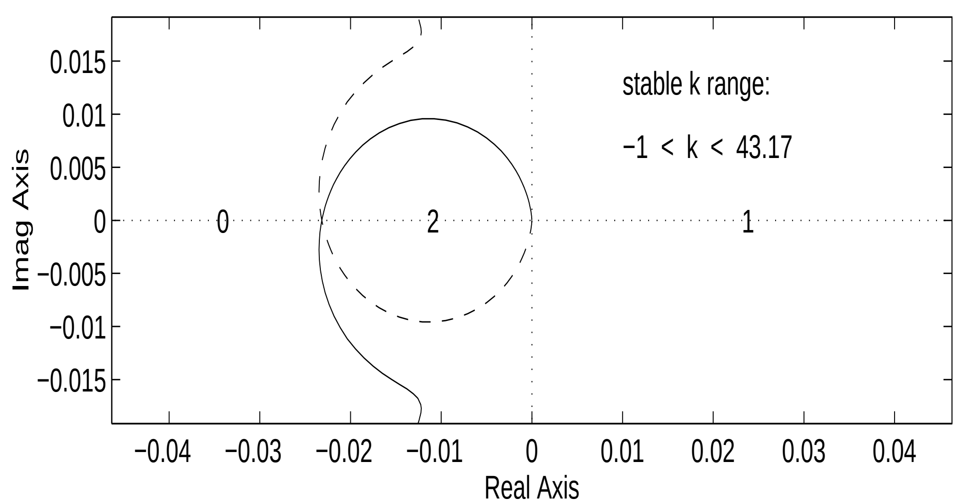
$W_2(s)$-map for the Nyquist criterion

A New MATLAB Nyquist Tool

Another example: Consider a simple stable plant:

$$W_3(s) = \frac{s+1}{s^4 + 2s^3 + 25s^2 + 3s + 1} \tag{4}$$





The report that **`newnyq`** provides is:

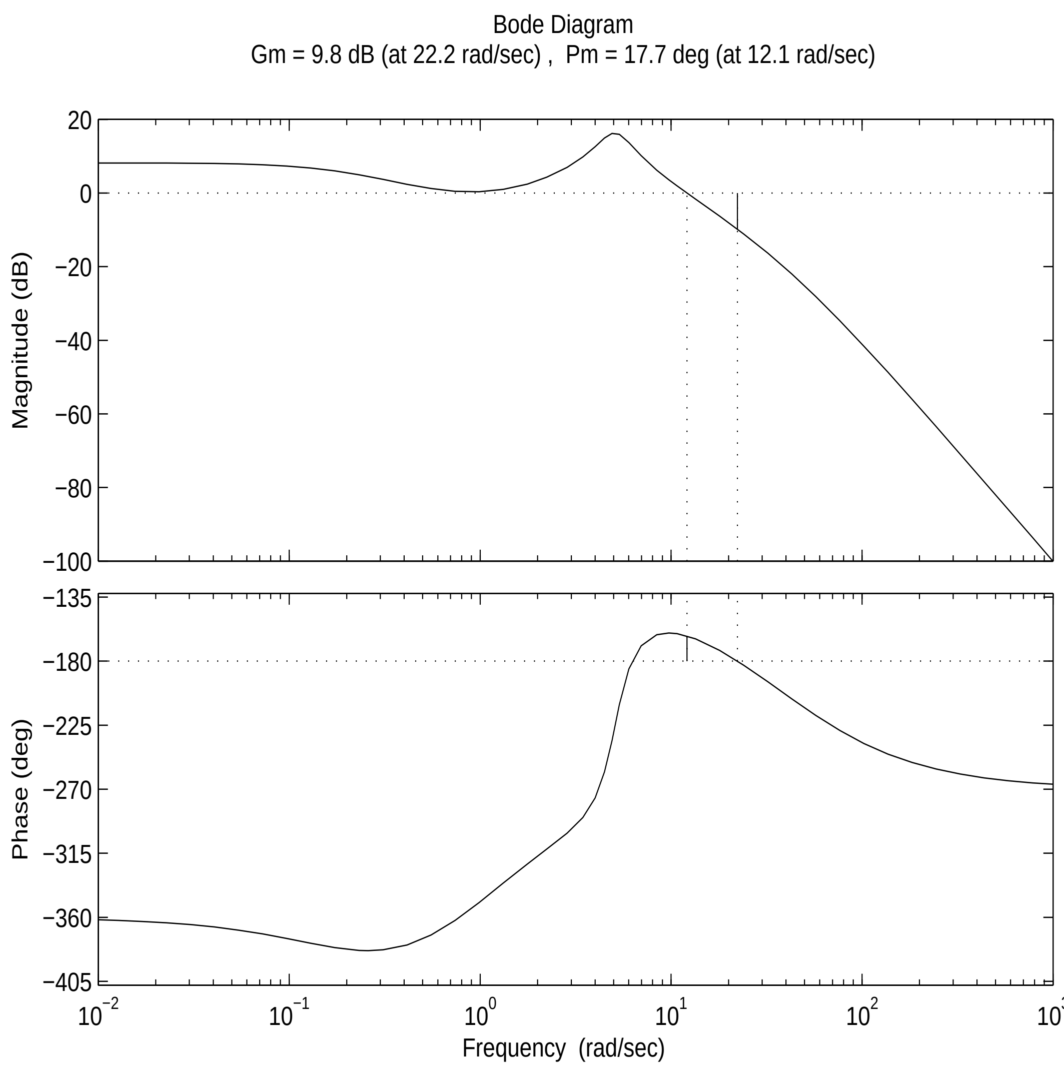
```
>> newnyq(num,den)

stable k range
-1  <  k  <  43.17
```

Comparison of `nyquist`, `newnyq` and `margin`

Final example: Consider a high-order unstable plant:

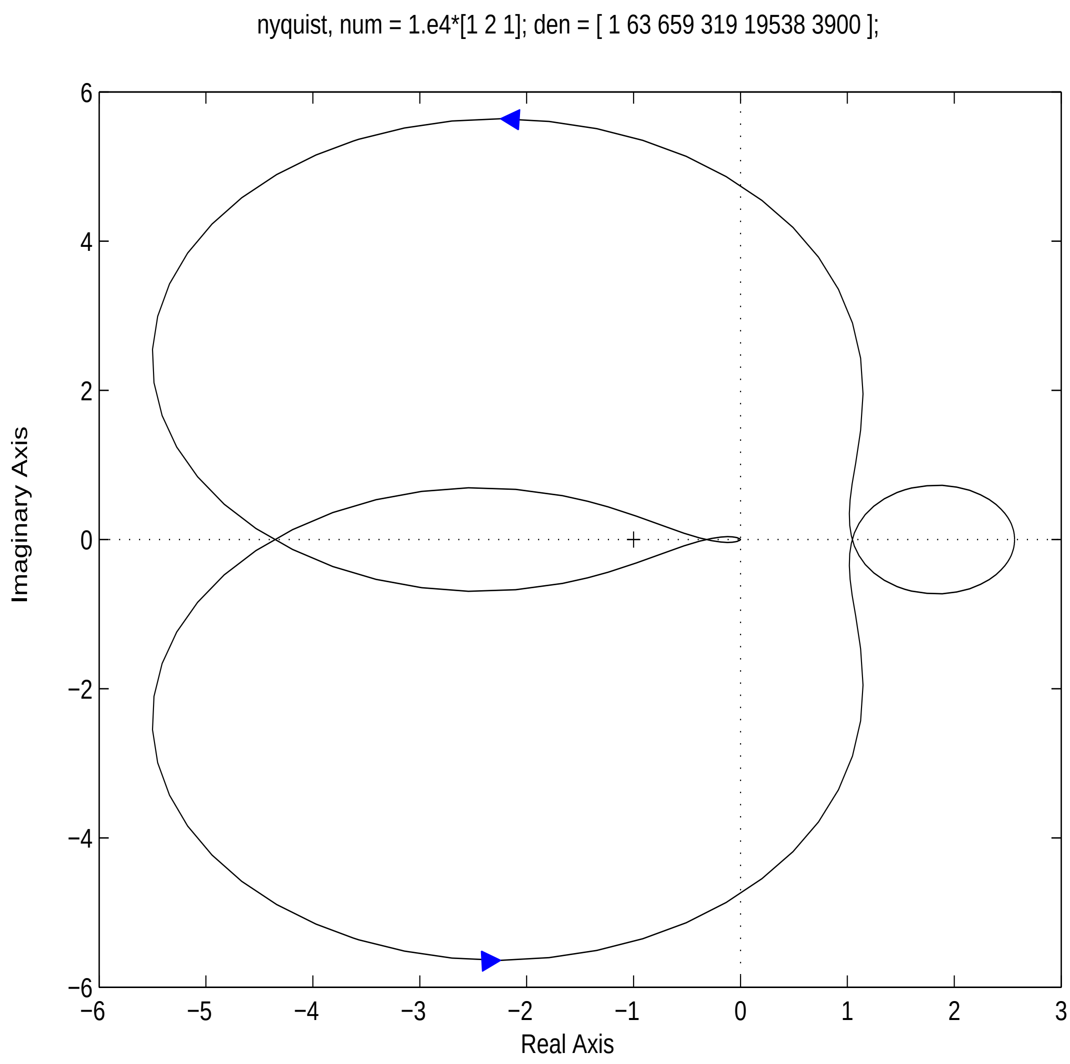
$$W_4(s) = 10^4 \cdot \frac{s^2 + 2s + 1}{s^5 + 63s^4 + 659s^3 + 319s^2 + 19538s + 3900} \quad (5)$$

```
num = 1.e4*[1 2 1];
den = [ 1 63 659 319 19538 3900 ];
margin(num,den);
title('num = 1.e4*[1 2 1]; den = [ 1 63 659 319 19538 3900 ];');
```



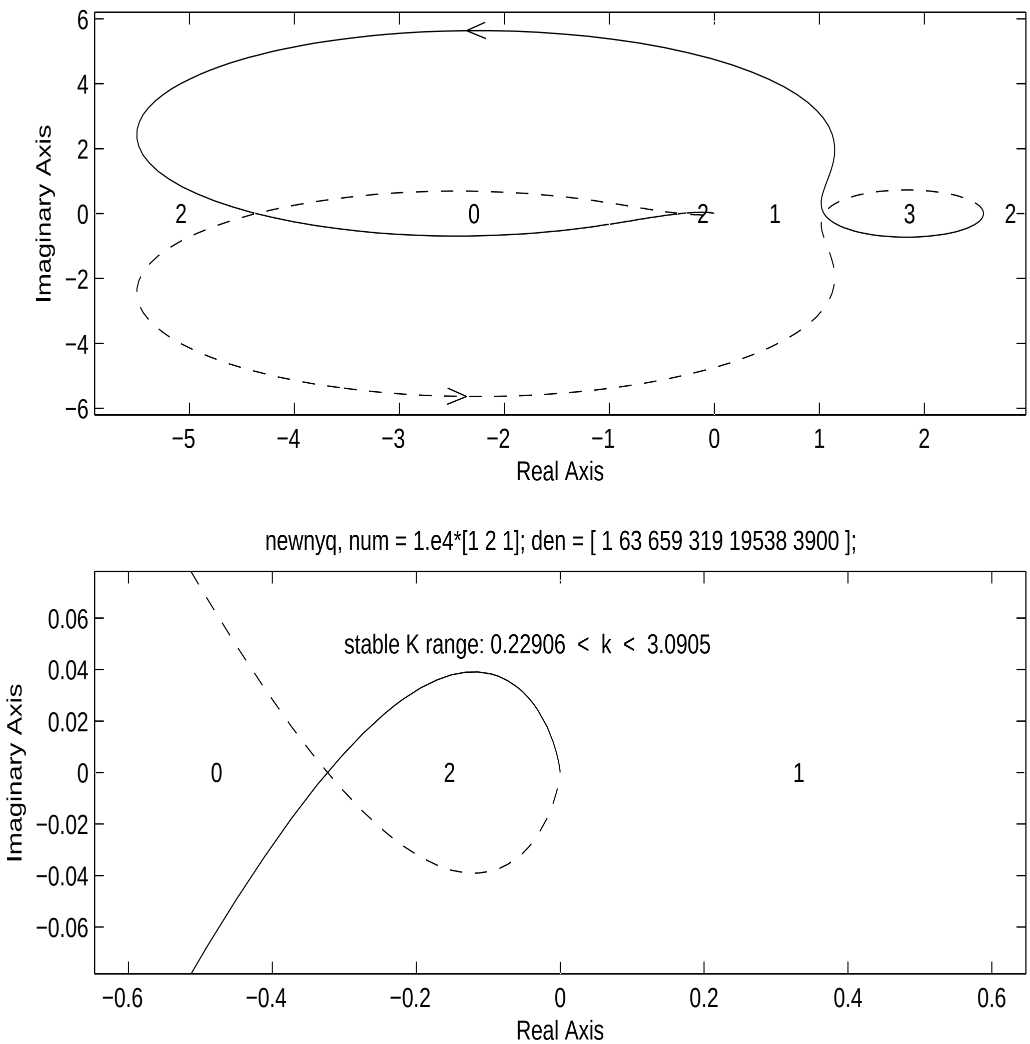
Comparison of `nyquist`, `newnyq` and `margin` (Cont'd)

```
num = 1.e4*[1 2 1];
den = [ 1 63 659 319 19538 3900 ];
nyquist(num,den);
title('nyquist, num = 1.e4*[1 2 1]; den = [ 1 63 659 319 19538 3900 ];')
```



Comparison of `nyquist`, `newnyq` and `margin` (Cont'd)

```
num = 1.e4*[1 2 1];
den = [ 1 63 659 319 19538 3900 ];
newnyq(num,den);
title('newnyq, num = 1.e4*[1 2 1]; den = [ 1 63 659 319 19538 3900 ];');
```



The report that **newnyq** provides is:

```
stable k range
0.22906  <  k  <  3.0905
```

The Nyquist Criterion – Why it is so Important in Practice

- You do not need a precise analytic model – just $W(j\omega)$ (although it is important to keep in mind the usual context starting from modelling and linearization)

- Empirical data (amplitude and phase data taken in the lab) is directly useful without a need to assume system order and perform curve fitting

- You have a direct graphical interpretation of the impact of uncertainty – just draw a **band** instead of a precise locus of $W(j\omega)$; this is especially useful in dealing with empirical data