EE 4323 – Industrial Control Systems

Module 6: Control System Design Basics

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Overview

- Introduction
- Control Problems
- Closed-loop Specifications
- Translation to Open-loop Specifications
  - Frequency-domain
  - Time-domain
- This will set the scene for:
  - Frequency-domain Design
  - Root Locus Design

References:

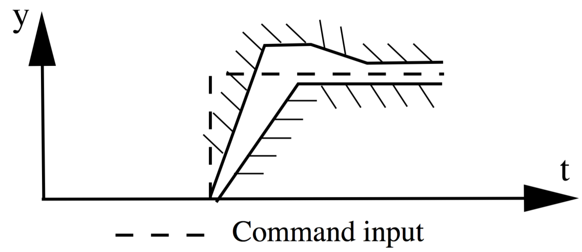
- N. S. Nise, *Control Systems Engineering*, 4^{th} Ed., John Wiley & Sons, 2004 (EE3323 textbook; advanced topics useful).
- P. H. Lewis and C. Yang, *Basic Control Systems Engineering*, Prentice-Hall, 1997.

- Many other good, basic control texts exist ...

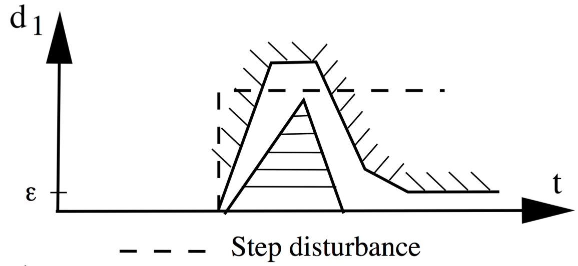
Standard 1-DOF Control Problem

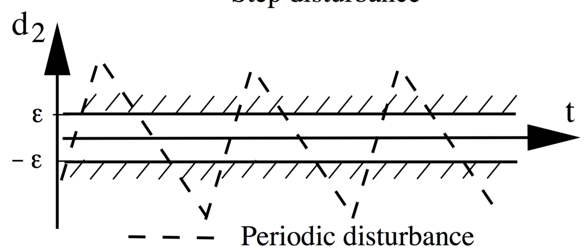
Given:

- Servo specs ("speed", overshoot, "settling", steady-state)

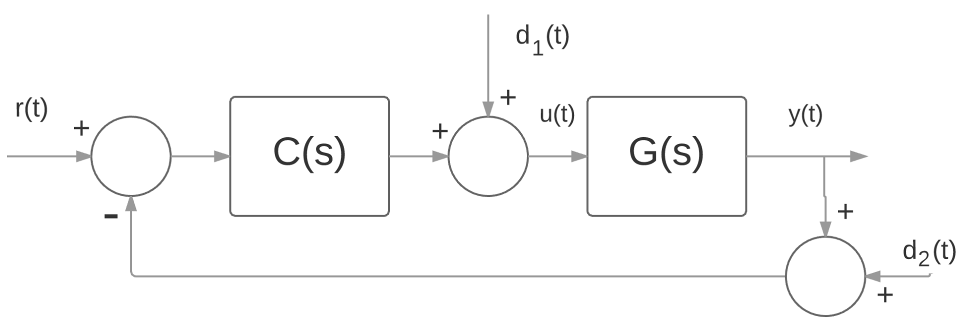


- Disturbance-rejection specs (transient, steady-state)





Find: Regulator $C(s)$ that satisfies the specifications...

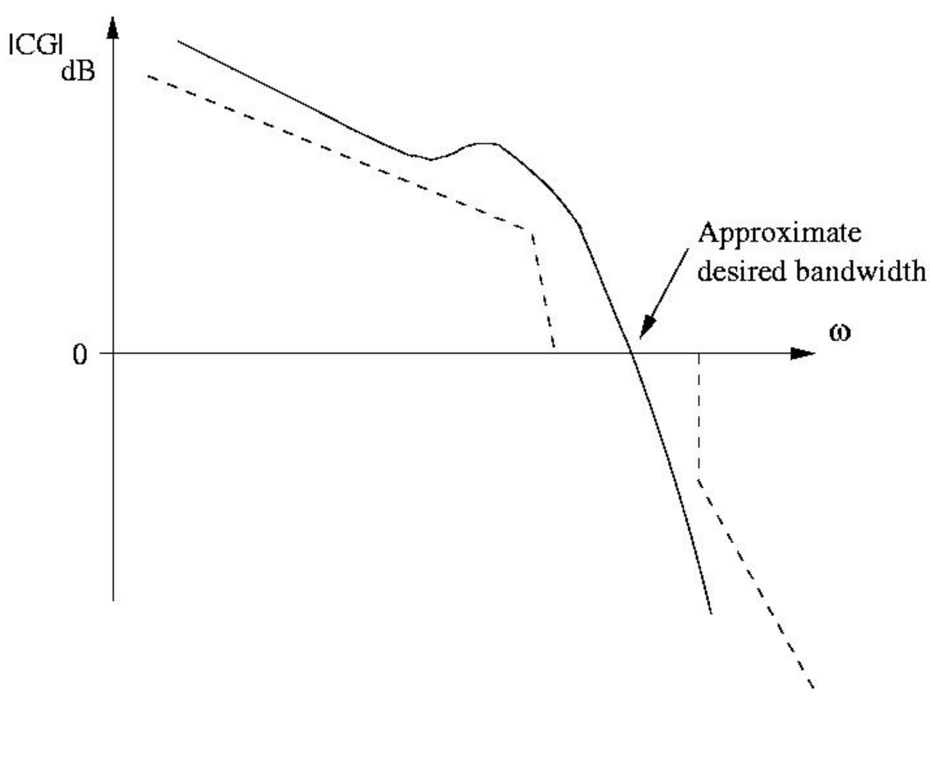


$$Y(s) = \frac{1}{1+CG}D_2(s) + \frac{G}{1+CG}D_1(s) + \frac{CG}{1+CG}R(s) \qquad (1)$$

This is the **standard one-degree-of-freedom (1-DOF) control problem.** Compromises / trade-offs have to be made. For frequencies where $C \cdot G$ is large we're good!

Open-Loop Design

- Translate closed-loop specs into specs for the open loop $C(s)G(s)$
  - Cross-over frequency ω_n
  - Gain margin M_G
  - Phase margin M_ϕ
  - Number of free integrators $(C \cdot G_{LF})$
  - M-circle design to meet closed-loop specs
- The Bode Problem: Given specs in the open loop frequency domain for $C(s)G(s)$ find $C(s)$ such that these specs are satisfied.
- These specifications show that $|\,C(j\omega)G(j\omega)\,|$ should be large over the frequency range (bandwidth) of interest and **small** for high frequencies



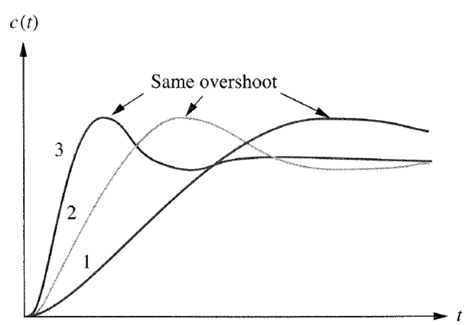
Closed- / Open-loop Translations

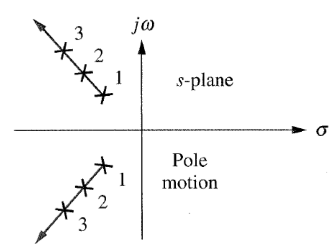
- Steady-state tracking requirements $\rightarrow$ number of open-loop free integrators; let $CG_{LF} = K_{LF}/s^q$ (recall the Bode plot convention) and define *Steady-State Error Coefficients:*

Type $= q$	$R(s)$: $1/s$	$1/s^2$	$1/s^3$	Error Coefficient
0	$\frac{1}{1+K_p}$	∞	∞	$K_p = \lim_{s \to 0} CG(s) = K_{LF}$
1	0	$\frac{1}{K_v}$	∞	$K_v = \lim_{s \to 0} sCG(s) = K_{LF}$
2	0	0	$\frac{1}{K_a}$	$K_a = \lim_{s \to 0} s^2CG(s) = K_{LF}$

see derivations, next page

- Transient response specs $\rightarrow$ "dominant poles" $\rightarrow$ ω_n, ζ





...be careful – make sure the poles are *dominant*

- Closed-loop frequency-response spec $\rightarrow$ "M-Circles" on the Nyquist plot of $G(s)$ (see two pages hence)

- Disturbance-rejection specs $\rightarrow$ $|1+C(j\omega)G(j\omega)| > D_R$ (a large value of D_R yields good rejection) $\rightarrow$ circles on the Nyquist plot centered on -1

Steady-state Tracking Error

Sample derivations (based on the final value theorem); note that $E(s) = R(s)/(1 + CG(s))$ and $e_{ss} = \lim_{s \to 0} \{sE(s)\}$:

- Type = 0, $R(s) = 1/s$ (tracking a step input):

$$e_{ss} = \lim_{s \to 0} \{s \cdot \frac{1}{s} \cdot \frac{1}{(1 + CG(s))}\} = \frac{1}{1 + K_p} \tag{2}$$

- Type = 0, $R(s) = 1/s^2$ (tracking a ramp input):

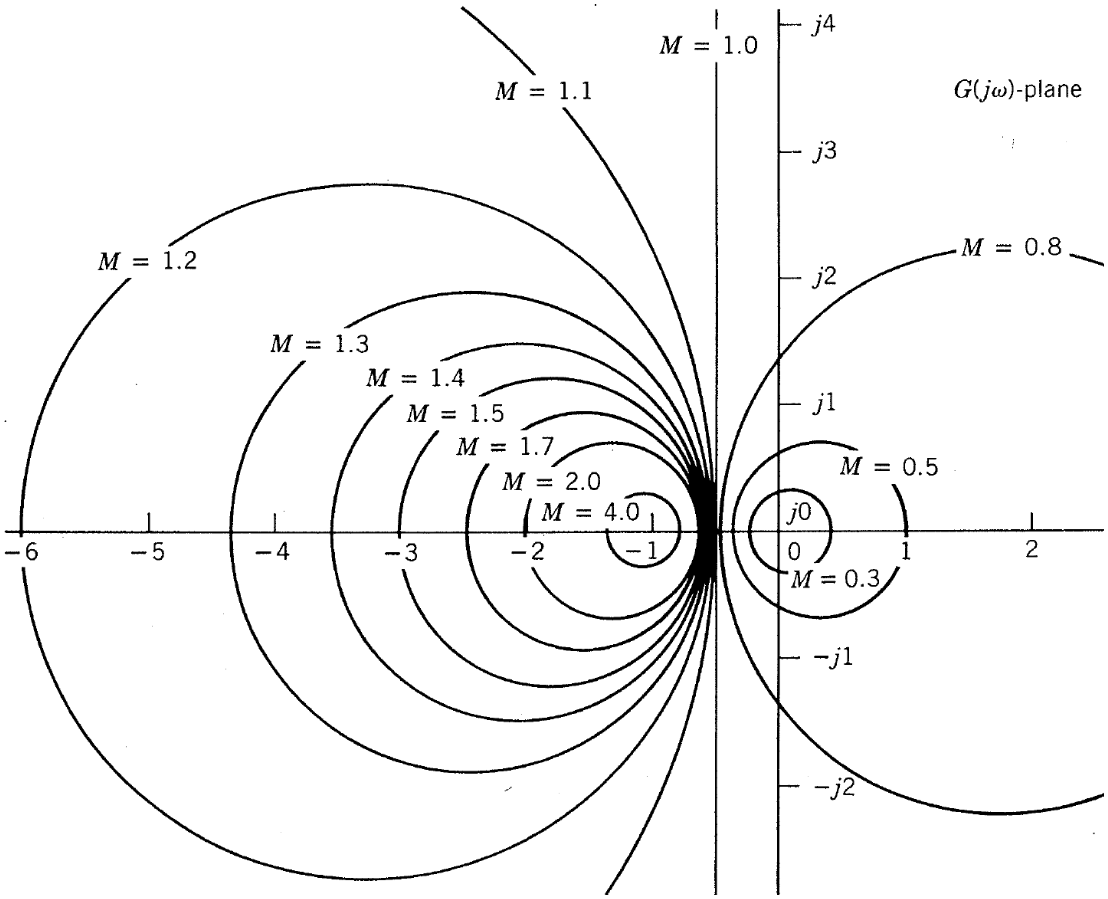
$$e_{ss} = \lim_{s \to 0} \{s \cdot \frac{1}{s^2} \cdot \frac{1}{(1 + CG(s))}\} = \infty \tag{3}$$

- Type = 1, $R(s) = 1/s^2$ (tracking a ramp input):

$$e_{ss} = \lim_{s \to 0} \{s \cdot \frac{1}{s^2} \cdot \frac{1}{(1 + CG(s))}\} \tag{4}$$

$$= \lim_{s \to 0} \{\frac{1}{(s + K_{LF})}\} = \frac{1}{K_v} \tag{5}$$

M-Circles



The closer the Nyquist plot of $C(j\omega)G(j\omega)$ comes to the point -1, the more resonsant the closed-loop frequency response will be.

M-Circles (Cont'd)

Example: $G(s) = 200(s+5)/[s(s+1)(s^2+20s+200)]$

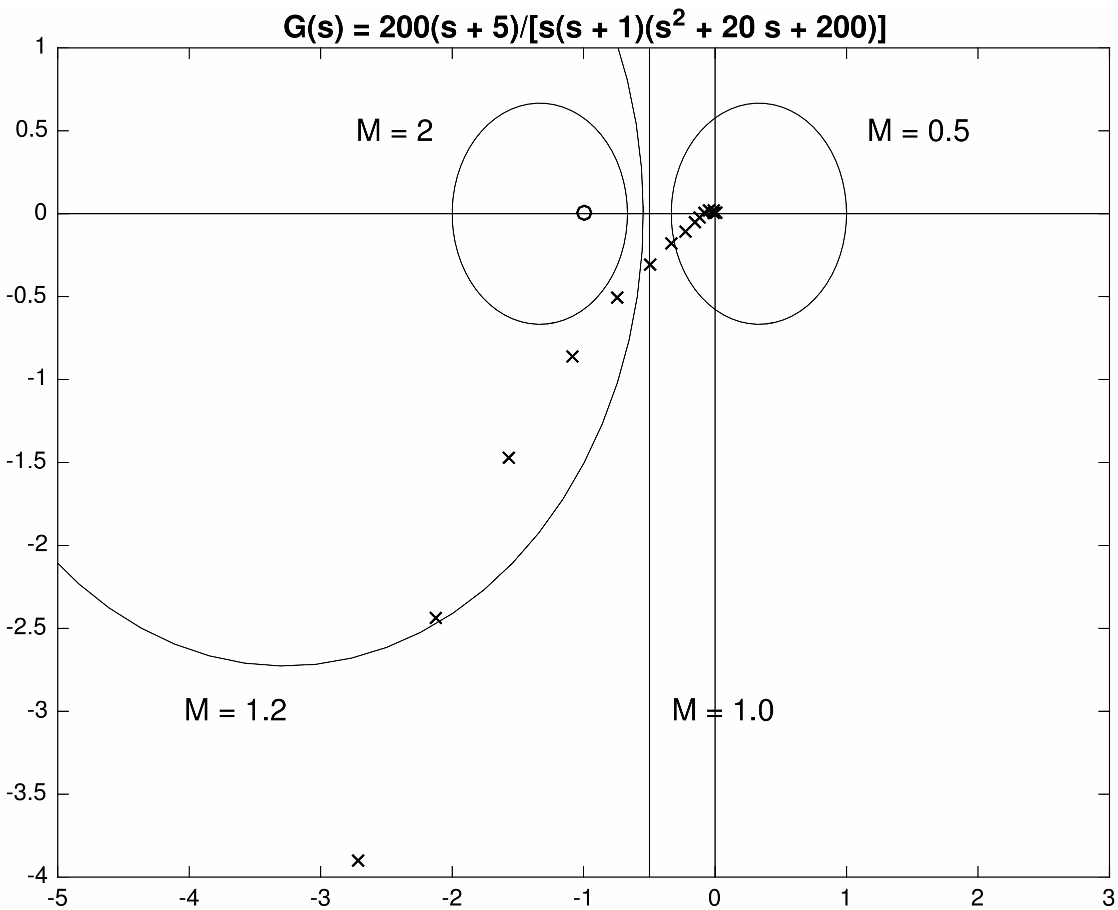


Figure 1: M-Circles →Closed-loop Magnitude Response

```
num = 200*[1 5];
den = poly( [-1 -10+10*j -10-10*j 0 ]);
ww = logspace(-.2,2,10);
[re,im,w] = nyquist(num,den,ww);
plot(re,im,'x');
mcircle(2);
mcircle(1.1);
mcircle(1);
mcircle(.5);
< et cetera >
```