EE 6383 Nonlinear Control Systems
Homework 2 – Lyapunov's Direct Method

Asst. date: 21 May 2019 – Due date: 24 May 2019

Department of Electrical & Computer Engineering
University of New Brunswick

In this assignment, we will investigate the theory and application of Lyapunov's Direct Method and work with Absolute Stability criteria.

1. Lyapunov's Direct Method

   (a) Consider the Lyapunov function candidate $x^T P x$ where $P = P^T > 0$, and explain **fully and rigorously** why it satisfies the requirements of LaSalle's version of the Lyapunov stability theorem.

   (b) Given a mechanical system with the nonlinear model as follows:

   $$M\ddot{x} + g(x)\dot{x} + f(x) = 0 \tag{1}$$

   (note that the friction depends on **position**, it is not nonlinear in $\dot{x}$!), use the Lyapunov function candidate

   $$V(x, \dot{x}) = \frac{1}{2}M\dot{x}^2 + \int_0^x f(\xi)d\xi \tag{2}$$

   (similar to the V used in the proof of the Popov criterion) to obtain the **least restrictive conditions possible** on $f(x)$ and $g(x)$ such that you can guarantee UASIL by the Lasalle version of the Lyapunov theorem.

2. Absolute Stability Criteria: Given $W(s)$ defined by:

```
num = poly([-4+j*15 -4-j*15 -20]);
den = 150*poly([-1+j*5 -1-j*5 -.1+j -.1-j])
```

   Note you should NOT use the `newnyq`, `popov` and `circle` commands in answering the questions below.

   (a) Obtain the Nyquist plot for the above $W(s)$, look it over; hand in a hardcopy.

   (b) Produce a Popov plot for this $W(s)$ using the standard `nyquist` and `plot` commands; hand in a hardcopy. Use `help nyquist` to discover how to get the Nyquist plot data you need to manipulate (scale) to produce this plot.

   (c) Determine all possible ranges of a linear gain K such that $W/(1 + KW)$ is asymptotically stable.

   (d) Use the Nyquist plot from (a) to determine the best ranges for stability of $g(y, t)$ you can obtain for (i) [0, Gmax] (ii) [Gmin, 0] (iii) [Gmin < 0 , Gmax > 0] (use a compass; note that the vertical and horizontal axes must have the same scale!

   (e) Apply the `axis` command after you create the Popov plot in (b) (to get a convenient scale), then report on the best ranges for stability of $f(y)$ you can obtain for [0, Fmax]