

1 Speech Recognition using CDHMMs

Lab Objective: *Understand how speech recognition via CDHMMs works, and implement a simplified speech recognition system.*

Continuous Density Hidden Markov Models

Some of the most powerful applications of Hidden Markov Models, speech and voice recognition, result from allowing the observation space to be continuous instead of discrete. These are called Continuous Density Hidden Markov Models (CDHMMs), and they have two standard formulations: Gaussian HMMs and Gaussian Mixture Model HMMs (GMMHMMs). The former is a special case of the latter, so we will just discuss GMMHMMs in this lab.

In order to understand GMMHMMs, we need to be familiar with a particular continuous, multivariate distribution called a *mixture of Gaussians*. A mixture of Gaussians is a distribution composed of several Gaussian distributions with corresponding weights. Such a distribution is parameterized by the number of mixture components K (where each component is a Gaussian distribution), the dimension M of the normal distributions involved, a collection of component weights $\{c_1, \dots, c_K\}$ that are nonnegative and sum to 1, and a collection of mean and covariance parameters $\{(\mu_1, \Sigma_1), \dots, (\mu_K, \Sigma_K)\}$ for each Gaussian component. To sample from a mixture of Gaussians, one first chooses the mixture component i according to the probability weights $\{c_1, \dots, c_K\}$, and then one samples from the normal distribution $\mathcal{N}(\mu_k, \Sigma_k)$. The probability density function for a mixture of Gaussians is given by

$$p(\mathbf{z}|\theta) = \sum_{k=1}^K c_k N(\mathbf{z}; \mu_k, \Sigma_k),$$

where $N(\cdot; \mu_k, \Sigma_k)$ denotes the probability density function for the normal distribution $\mathcal{N}(\mu_k, \Sigma_k)$. See Figure 1.1 for the plot of such a density curve. Note that a mixture of Gaussians with just one mixture component reduces to a simple normal distribution, and so a GMMHMM with just one mixture component is simply a Gaussian HMM.

Similar to discrete HMMs, GMMHMMs seek to model a hidden state sequence $\{X_1, \dots, X_T\}$ and a corresponding observation sequence $\{Z_1, \dots, Z_T\}$ where T is the number of time steps or number of observations. The major difference is that each observation \mathbf{z}_t is a real-valued vector of length M distributed according to a mixture of Gaussians with K components. The parameters for such a model include the initial state distribution π and the state transition matrix A (just as with dis-

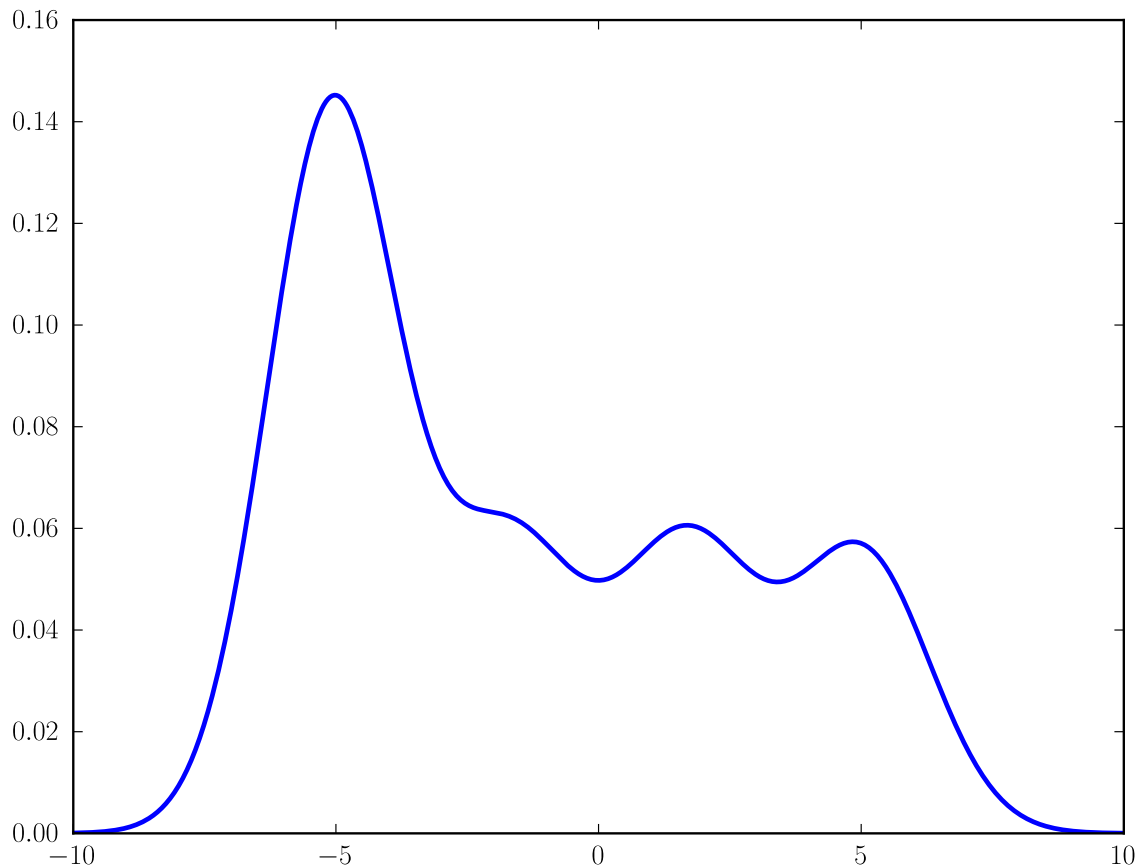


Figure 1.1: The probability density function of a mixture of Gaussians with four components.

create HMMs). Additionally, for each state $i = 1, \dots, N$, we have component weights $\{c_{i,1}, \dots, c_{i,K}\}$, component means $\{\mu_{i,1}, \dots, \mu_{i,K}\}$, and component covariance matrices $\{\Sigma_{i,1}, \dots, \Sigma_{i,K}\}$.

Let's define a full GMMHMM with $N = 3$ states, components of dimension $M = 2$, and $K = 5$ components.

```
>>> import numpy as np

# NxN transition matrix
>>> A = np.array([[.3, .3, .4], [.2, .3, .5], [.3, .2, .5]])

# NxK collection of component weights
>>> weights = np.array([[.3, .2, .1, .2, .2], [.1, .3, .3, .2, .1],
                        [.1, .3, .2, .1, .3]])

# NxKxM collection of component means
>>> means = np.array([np.floor(np.random.uniform(-100, 100, size = (5, 4)))
                        for i in range(3)])

# NxKx(MxM) collection of component covariance matrices
```

```
>>> covars = np.array([[np.floor(np.random.uniform(1, 20))*np.eye(4)
                        for i in range(5)] for j in range(3)])

# (N,) ndarray initial state distribution
>>> pi = np.array([.15, .15, .7])

# Save the model parameters
>>> gmm = [A, weights, means, covars, pi]
```

Once we have a GMMHMM, we can randomly choose the first state based on the initial state distribution π . Now we can iteratively sample from our GMMHMM as follows:

- Randomly select a GMM Gaussian component according to the probability weights of the current state.
- Sample from the selected GMM Gaussian component using the corresponding mean and covariance matrix.
- Obtain the next state using the transition matrix A .

```
# Steps to randomly sample from GMMHMM

# Choose initial state
>>> state = np.argmax(np.random.multinomial(1, pi))

# Randomly select component using the probability weights of current state
>>> sample_component = np.argmax(np.random.multinomial(1, weights[state,:]))

# Sample an observation from the selected GMM Gaussian component
>>> sample = np.random.multivariate_normal(means[state, sample_component, :],
                                           covars[state, sample_component, :, :])

# Obtain the next state using the transition matrix
>>> state = np.argmax(np.random.multinomial(1, A[:, state]))
```

Figure 1.2 shows an observation sequence generated from a GMMHMM with two mixture component and two states.

Problem 1. Write a function which accepts a GMMHMM in the format above as well as an integer T , and which simulates the GMMHMM process, generating T different observations. Do so by implementing the following function declaration.

```
def sample_gmmhmm(gmmhmm, T):
    """
    Simulate sampling from a GMMHMM.

    Returns
    -----
```

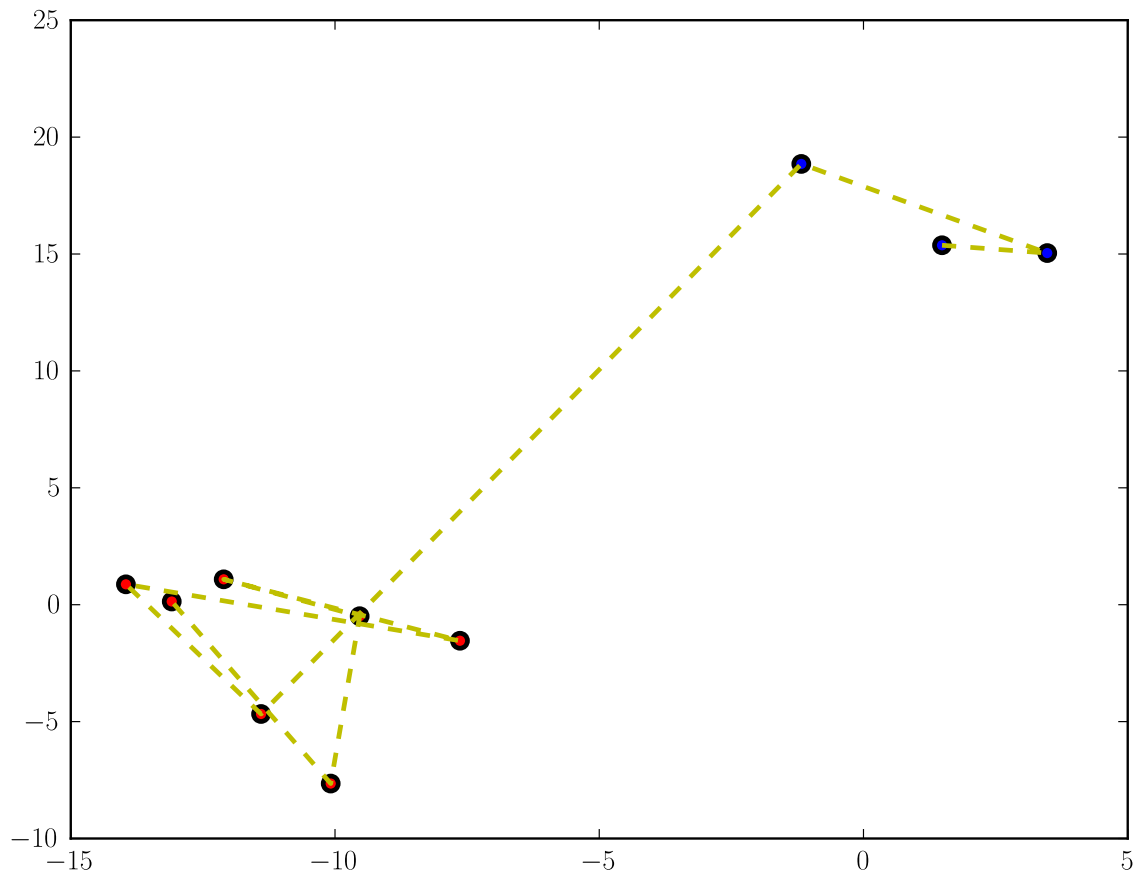


Figure 1.2: An observation sequence generated from a GMMHMM with two mixture components and two states. The observations (points in the plane) are shown as solid dots, the color indicating from which state they were generated. The connecting dotted lines indicate the sequential order of the observations.

```
states : ndarray of shape (T,)
        The sequence of states
obs : ndarray of shape (T, M)
        The generated observations
"""
pass
```

Test your function by running it on the gmmhmm given in the example, with $T = 900$. Use `sklearn.decomposition.PCA` with 2 components to plot the observations in two-dimensional space. Color the observations by state. How many clusters do you see?

The classic problems for which we normally use discrete observation HMMs can also be solved by using CDHMMs, though with continuous observations it is much more difficult to keep things numerically stable. We will not have you implement any of the three problems for CDHMMs yourself; instead, you will use a stable module we will provide for you. Note, however, that the techniques for

solving these problems are still based on the forward-backward algorithm; the implementation may be trickier, but the mathematical ideas are virtually the same as those for discrete HMMs.

Speech Recognition and Hidden Markov Models

Hidden Markov Models are the basis of modern speech recognition systems. However, a fair amount of signal processing must precede the HMM stage, and there are other components of speech recognition, such as language models, that we will not address in this lab.

The basic signal processing and HMM stages of the speech recognition system that we develop in this lab can be summarized as follows: The audio to be processed is divided into small frames of approximately 30 ms. These are short enough that we can treat the signal as being constant over these intervals. We can then take this framed signal and, through a series of transformations, represent it by mel-frequency cepstral coefficients (MFCCs), keeping only the first M (say $M = 10$). Viewing these MFCCs as continuous observations in \mathbb{R}^M , we can train a GMMHMM on sequences of MFCCs for a given word, spoken multiple times. Doing this for several words, we have a collection of GMMHMMs, one for each word. Given a new speech signal, after framing and decomposing it into its MFCC array, we can score the signal against each GMMHMM, returning the word whose GMMHMM scored the highest.

Industrial-grade speech recognition systems do not train a GMMHMM for each word in a vocabulary (that would be ludicrous for a large vocabulary), but rather on *phonemes*, or distinct sounds. The English language has 44 phonemes, yielding 44 different GMMHMMs. As you could imagine, this greatly facilitates the problem of speech recognition. Each and every word can be represented by some combination of these 44 distinct sounds. By correctly classifying a signal by its phonemes, we can determine what word was spoken. Doing so is beyond the scope of this lab, so we will simply train GMMHMMs on five words/phrases: biology, mathematics, political science, psychology, and statistics.

Problem 2. The `Samples` folder contains 30 recordings for each of the words/phrases mathematics, biology, political science, psychology, and statistics. Remove the files that end in 00 (eg. `Biology00.wav`). These audio samples are 2 seconds in duration, recorded at a rate of 44100 samples per second, with samples stored as 16-bit signed integers in WAV format. Load the recordings into Python using `scipy.io.wavfile.read`.

Extract the MFCCs from each sample using code from the file `MFCC.py`. Store the MFCCs for each word in a separate list. You should have five lists, each containing 30 MFCC arrays, corresponding to each of the five words under consideration.

To load and extract, use the following code:

```
>>> samplerate, data = wavfile.read(file)      # Load wavfile
>>> model = MFCC.extract(data, show = False)  # Extract MFCC
```

For a specific word, given enough distinct samples of that word (decomposed into MFCCs), we can train a GMMHMM. Recall, however, that the training procedure does not always produce a very effective model, as it can get stuck in a poor local minimum. To combat this, we will train 10 GMMHMMs for each word (using a different random initialization of the parameters each time) and keep the model with the highest log-likelihood.

For training, we will use the python package called `hmmlearn`, as this is a stable implementation of GMMHMM algorithms. To facilitate random restarts, we need a function to provide initializations for the initial state distribution and the transition matrix.

Let `data` be a list of arrays, where each array is the output of the MFCC extraction for a speech sample. Using a function `initialize()` that returns a random initial state distribution and row-stochastic transition matrix, we can train a GMMHMM with 5 states and 3 mixture components and view its score as follows:

```
>>> import numpy as np # Import packages
>>> from hmmlearn import hmm

>>> data = train_samples[word] # Reshape data for hmmlearn's fit method
>>> lengths = [data[0].shape[0]] * len(data)
>>> data_collected = np.vstack(data)

# Get probabilities and transition matrices
>>> startprob, transmat = initialize(5)
>>> model = hmm.GMMHMM(n_components=5, covariance_type="diag",
                        init_params="mc") # Initialize model
>>> model.startprob_ = startprob # Set probabilities and transition matrices
>>> model.transmat_ = transmat

>>> model.fit(data_collected) # Fit the model
>>> model.monitor_.history[-1] # Check the score (log-likelihood)
```

Problem 3. Randomly split each list of MFCCs into a training set of 20 samples and a test set of the remaining 10 samples. Using the training sets, train a GMMHMM on each of the words from the previous problem with at least 10 random restarts (reinitializing and creating a new model). Use `n_components=5` and `initialize(5)` to train your GMMHMMs with 5 states. Save the best model for each word (the one with the highest log-likelihood).

Given a trained model, we would like to compute the score of a new sample. Letting `obs` be an array of MFCCs for a speech sample we do this as follows:

```
>>> score = model.score(obs)
```

We classify a new speech sample by scoring it against each of the five trained GMMHMMs, and returning the word corresponding to the GMMHMM with the highest score.

Problem 4. Find the classification accuracies of the five GMMHMMs, and return a dictionary where the words/phrases are the keys, and the values are the percent accuracies.

To find the accuracies of your GMMHMMs, implement the following. Iterate through each sample of each test set, and score each model on that sample using `model.score(sample)`. The predicted class for that sample is the class of the model that yielded the highest score.

The accuracy of each GMMHMM model is the percentage of correct predicted classes. For

example, the accuracy of the mathematics model is the number of samples in the mathematics test set whose predicted class was mathematics, divided by the number of samples in the mathematics test set (which should be 10).