Assignment # 3 – Due Jul 29, 2020 before Noon

1. Biologists cultured 3 types of bacteria in one test tube using 3 different foods. By using 3 types of food (A,B,C) each day, each type of bacteria needs food as shown in the table

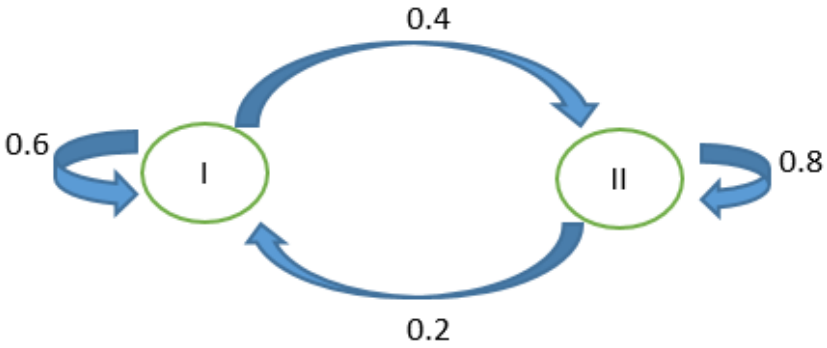
	Type 1	Type 2	Type 3
Food A	2	2	4
Food B	1	2	0
Food C	1	3	1

   Each day, there are 2300, 800 and 1500 units of Food A, B, and C, respectively. Determine how many bacteria of each type should be in the same test tube in order to eat all of food.

2. Let x and λ be an eigenvector and its corresponding eigenvalue of A.
   Find the eigenvectors and eigenvalues of A^3, A^{-1} and $A^{-1} - I$. Justify your answer.

3. Find a formula for A^k where
$$A = \begin{bmatrix} 2a - b & b - a \\ 2a - 2b & 2b - a \end{bmatrix}.$$
   Using the obtain result to find A^k where $a = b = -1,\ k = 100$.

4. Particle jumps between locations I and II with initial particle at locations I and II given by $u0 = [1\ \ 0]^T$ and the given probalility as showed in the figure



   (a) What is the particle distribution for $t = 1$?

   (b) What is the particle distribution at $t = k$?

   (c) Does the steady state exist? if it does, find it.