4. Energy Spectrum and Dissipation Spectrum

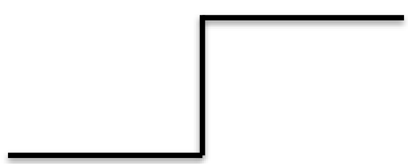
- Physical Meaning of Energy Spectrum

Comment:

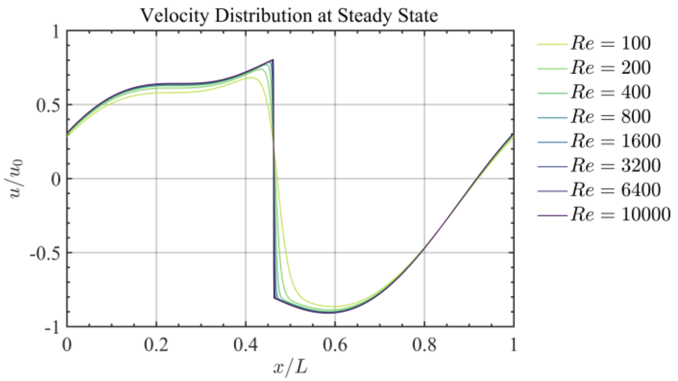
❑ With the time evolution, the burgers turbulence exhibits an inertial range wherein low wavenumber flow component transfer energy to the higher ones until to dissipate in the dissipative range, similar to that of real turbulence;

❑ The energy spectrum shows an inertial sub-range have a -2 slope, which is different from the real turbulence (Kolmogorov's -5/3 law);

Proof: The velocity distribution at steady state is



Heaviside function $H(x)$



Use $u(x) \approx 1 - 2H(x)$ to replace $u(x)$

Fourier transform of Heaviside function is $\mathcal{F}[H(x)] = \frac{1}{ik} + \pi\delta(k)$

Energy spectrum $E(k) = \sum|\tilde{u}(k,t)|^2$

Consequently, $E(k) \propto k^{-2}$

4. Energy Spectrum and Dissipation Spectrum

- Physical Meaning of Energy Spectrum

Comment:

❑ The dissipation spectrum shows an inertial sub-range have a 0 slope, which is consistent with energy spectrum because

$$D(k) = \mathcal{F}\left[\nu \frac{\partial^2 u}{\partial x^2}\right] = \nu(-ik)^2 \mathcal{F}[u^2] = -2\nu k^2 E(k) \propto k^0$$

❑ With the time evolution, it can be seen that the inertial sub-range evolves toward the correct slope –2;

❑ As the Reynolds number increases, the inertial sub-range extends to higher wavenumbers;

❑ Dynamics of the small scales in Burgers turbulence is similar but also different from that of the Navier–Stokes because small flow scales are shocks of thickness proportional to the viscosity [2] and are essentially not stochastic [3].

[2] J.M. Burgers, Adv. Appl. Mech. 1 (1948).
[3] A. Das, R. Moser, Phys. Fluids 14 (2002) 14.