Black hole simulator

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Introduction

Black holes are a very well known and fascinating physical phenomenon. Space time around the singularity is warped to the point that beyond the event horizon, light cannot escape. As complicates as these objects are, the equations for light around them are not terribly difficult. Using a series of simple differential equations, the motion of light around a black hole can be solved, using a simplified raytracing algorithm it is possible for us to visualize this phenomenon rather than just understand it mathematically

Theory

Einstein's theory of General Relativity gives Einstein's field equation, to describe how spacetime is curved by gravitating bodies.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

The Schwarzchild solution is the most simple general solution to this formula, this solves the vacuum solution, in which all the mass is said to be at a single point and the rest of space to have no mass density. This solution can be shown to give the "proper time" experienced by a particle in space:

$$d\tau^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2m}{r}} - r^2 (d\theta^2 + \sin^2(\theta)\, d\varphi^2)$$

For simplicity, we use units such that the gravitational constant and speed of light are 1 ($G = c = 1$). At the speed of light, no time is experienced, this means that proper time is set to 0. Knowing this, we can re-arrange the metric to find the optical metric of the Schwarzchild solution:

$$dt^2 = \frac{dr^2}{\left(1 - \frac{2m}{r}\right)^2} + \frac{r^2}{1 - \frac{2m}{r}} (d\theta^2 + \sin^2(\theta)\, d\varphi^2)$$

Approach

It is a provable fact that, for a diagonal metric, geodesic path through a space can be solved by the Lagrangian method:

$$\mathcal{L} = g_{ij} \dot{x}^i \dot{x}^j$$

$$\frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^a}\right) = \frac{\partial \mathcal{L}}{\partial x^a}$$

In the case of light around a black hole, the Lagrangian takes the form:

$$\mathcal{L} = \frac{r^2 \dot{r}^2}{(r - 2m)^3} + \frac{r^3}{r - 2m} \left(\dot{\theta}^2 + \sin^2(\theta)\, \dot{\varphi}^2\right)$$

This result comes from the Schwarzchild solution for a black hole. By solving these equations we can trace the path of light around the black hole. Using raytracing algorithms, some borrowed from projects provided by github it is possible to use these equations to show what a black hole would look like in different situations.

Objectives

The objectives for this project will be to have a program that can return black and white png images representing certain scenarios of light curving around a black hole. The image types we will aim to create are:

1. Show a black hole on simple shape background
2. Show a black hole with a torus around it (similar to an accretion disk)
3. Attempt to loop light around the black hole to show objects behind the "camera"

To keep things simple, a black and white display is the primary goal of this project. Doing a colored display or even a somewhat realistic scenario for a black hole (for example: slightly realistic accretion disk) will be seen as stretch-goals. As another stretch-goal we may attempt to show the red-shifting of light as it travels further away from the center of gravity.