Schwarzschild Ray Tracer

Authors:

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Background

- This project is a demonstrate the motion of light around a black hole as seen by an observer. Effectively a ray tracer, with curved rays or, rather, rays that propagate along a curved surface.
- The equations dictating motion across this surface are rather well known, the space-time around a black hole is described by the famous Schwarzschild Metric. While this is an interesting geometry, it isn’t exactly what we need for the ray-tracer.

Background

For this we want the optical metric, describing the space that light experiences around a black hole, as light (moving at the speed of light, necessarily) does not experience time. This metric can be expressed as a Lagrangian of the form:

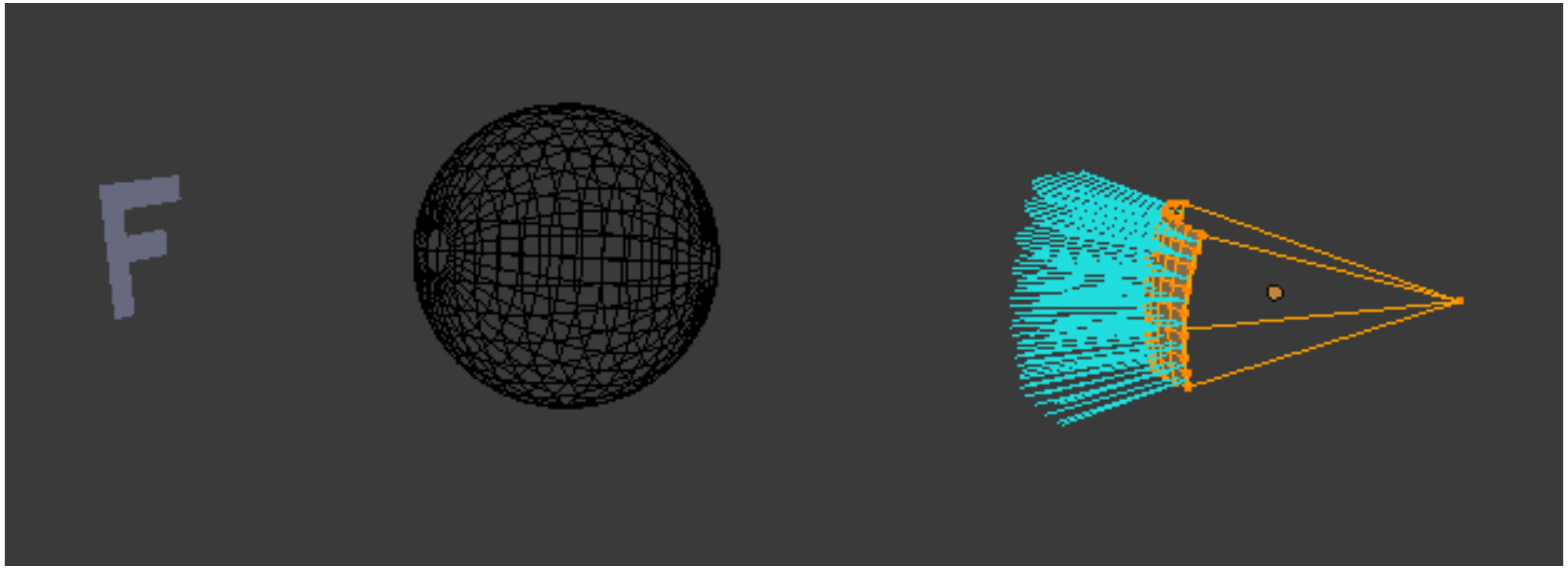
$$\mathcal{L} = \frac{r^2\dot{r}^2}{(r-2m)^2} + \frac{r^3}{r-2m}(\dot{\theta}^2 + \sin^2(\theta)$$

Which gives the equations of motion through the standard Lagrangian formulation.

Methods

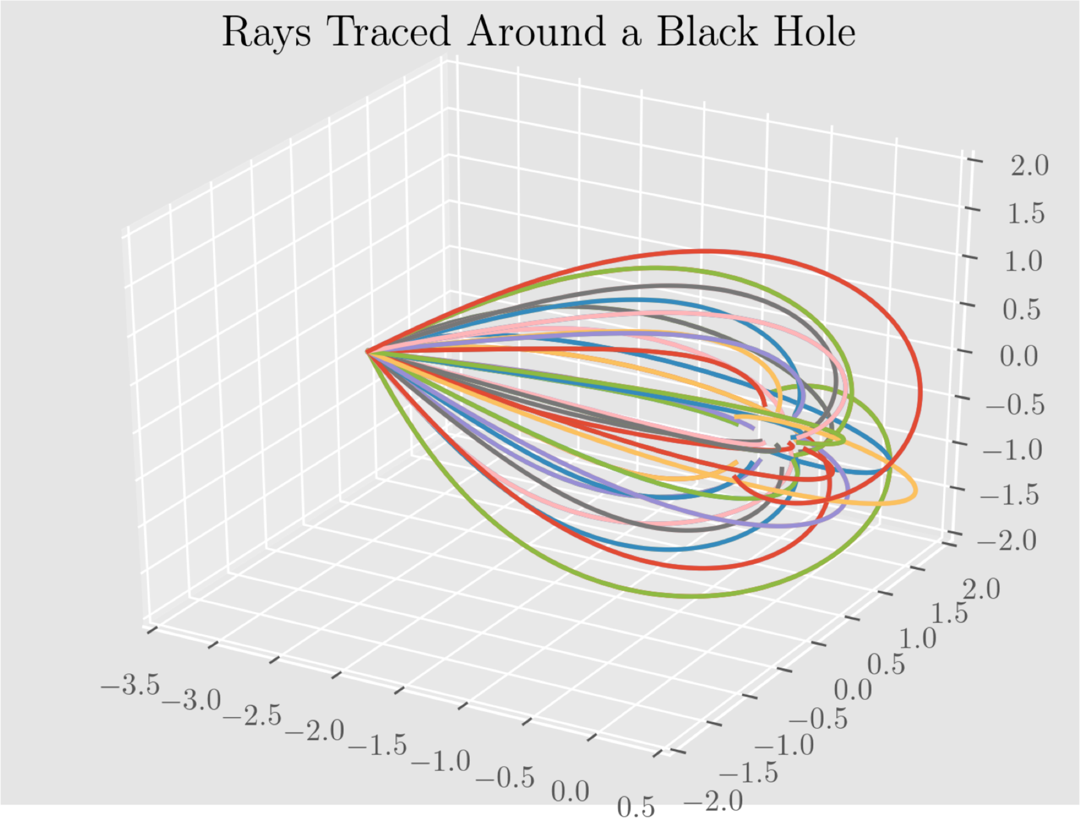
The concept behind creating this effect is rather simple, though the execution saw some difficulties. The idea is to create a spherical sector as the lens, and from rectangular segments of the lens, emit particles to follow their equations of motion until they encounter an obstacle, the black hole, or the boundaries of the allowed area

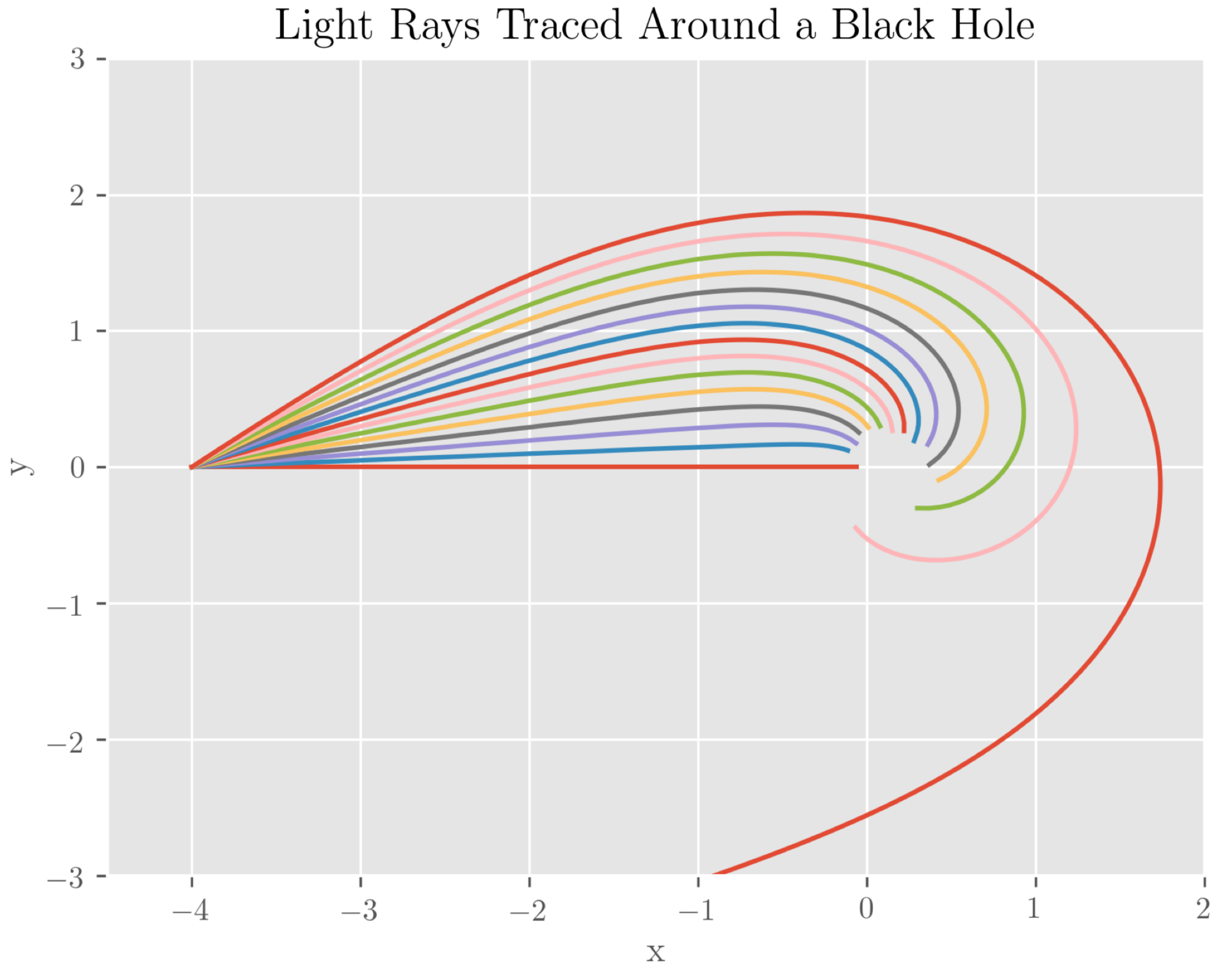
Because the equations of motion are solved through the Lagrangian formulas, they necessarily follow a conservation of energy, therefore the Verlet integrator was seen as appropriate

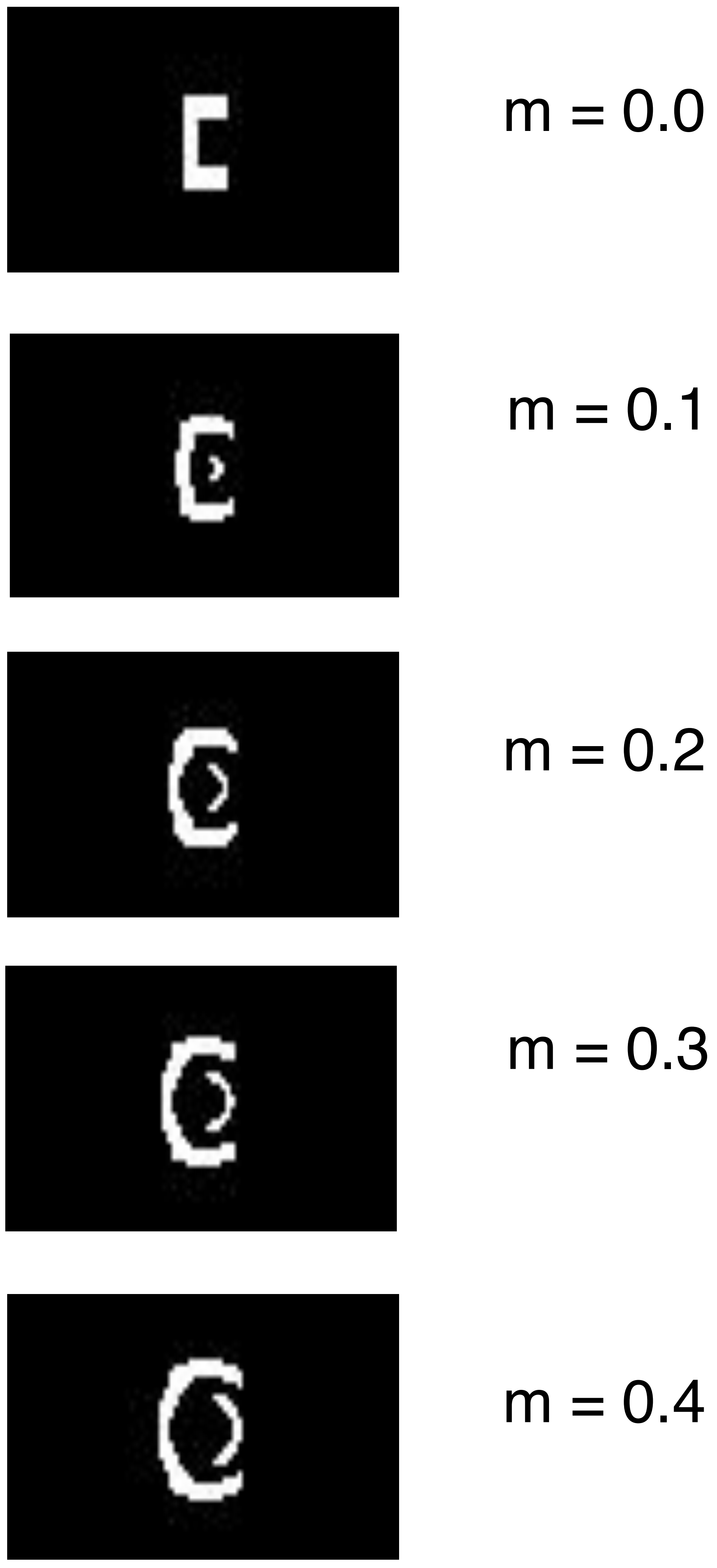


Pictured above is one scenario used to test the simulation. To the right, a camera faces the black hole in the center and projects rays of light from spherical segment of a lens. The number of rays is dependent on the resolution of the camera. To the left, an object is placed directly behind the black hole to test the distortion of the object as the black mass is increased

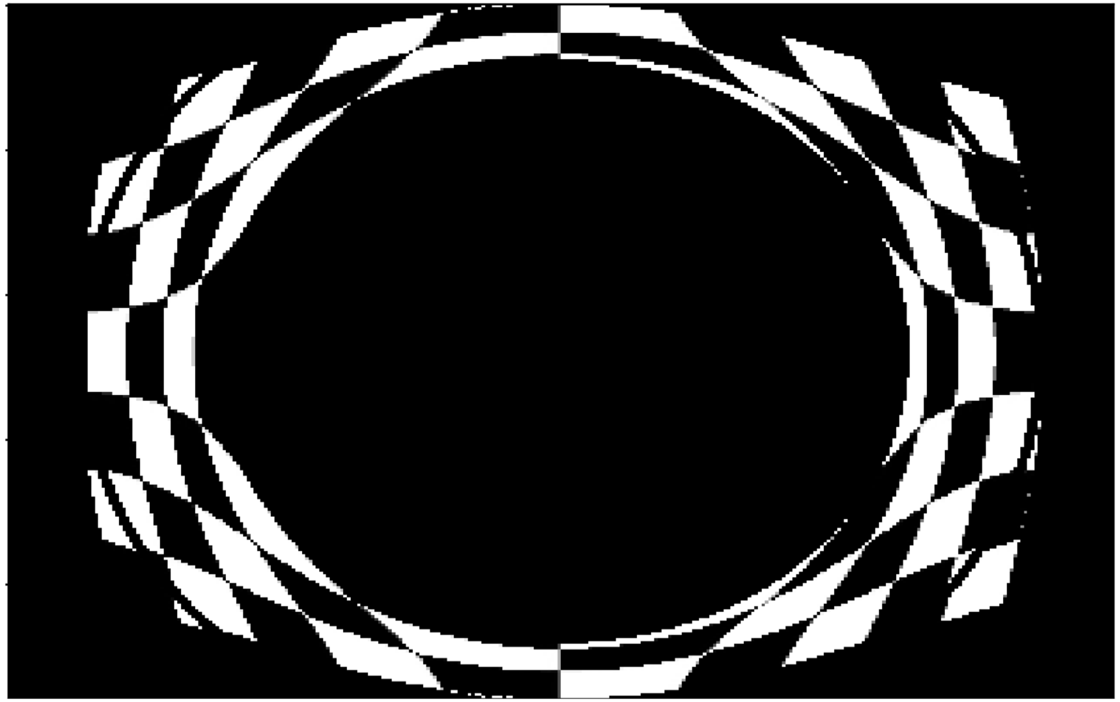
Results

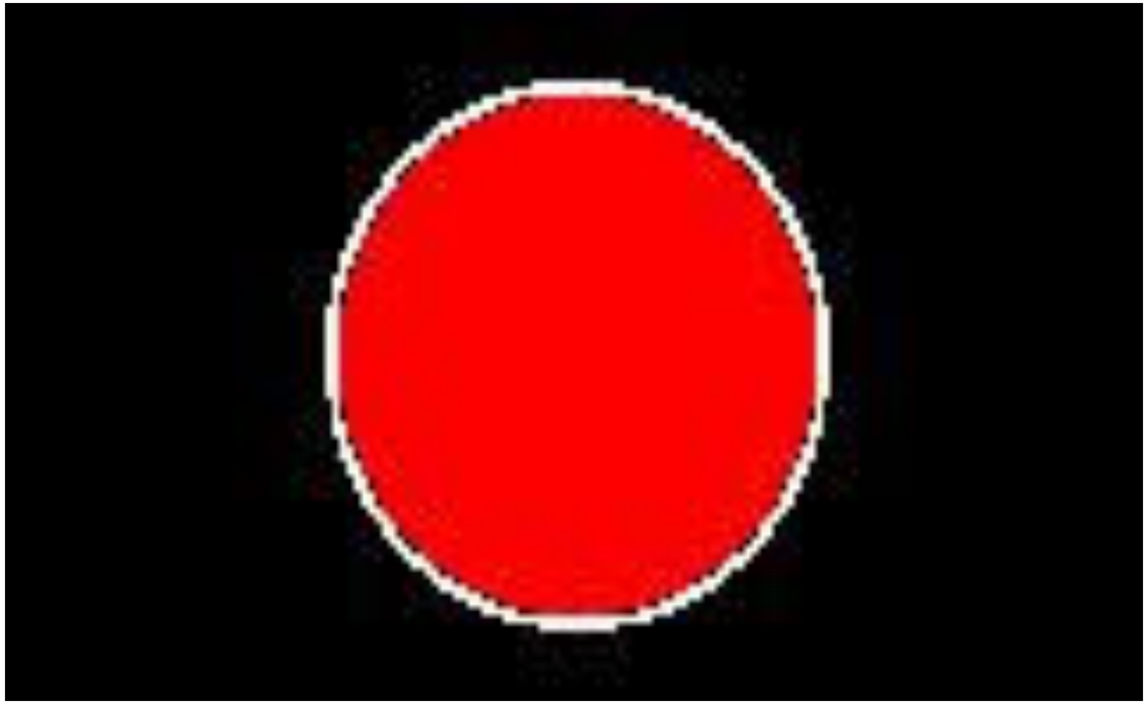

Rays Traced Around a Black Hole
−3.5 −3.0 −2.5 −2.0 −1.5 −1.0 −0.5 0.0 0.5
−2.0 −1.5 −1.0 −0.5 0.0 0.5 1.0 1.5 2.0
2.0 1.5 1.0 0.5 0.0 −0.5 −1.0 −1.5 −2.0


Light Rays Traced Around a Black Hole
y
3
2
1
0
−1
−2
−3
−4 −3 −2 −1 0 1 2
x


m = 0.0
m = 0.1
m = 0.2
m = 0.3
m = 0.4

Results





To the left, an checker board image is place directly behind a black hole to view the distortion of objects behind a massive object. To the right, an image is placed behind an observer and is viewed as a ring of white surrounding the black hole.

Summary and Future Work

- By numerically solving the equations of motion for light using the optical metric and the Verlet velocity integrator, we were able to accurately simulate the rays of light bending around a black hole.
- Additionally, we were able to generate images of a plane behind a black hole and behind an observer viewing the black hole.
- Future work will look to improve the ray-tracing algorithm to produce higher resolution images in shorter amounts of time. We also look to simulate a torus around the black hole due to an accretion disk.

References/Work

- Code Repository:
  - https://github.com/ASU-CompMethodsPhysics-PHY494/final-2018-doubularity
- Ray Tracing Algorithm Information:
  - http://rantonels.github.io/starless/