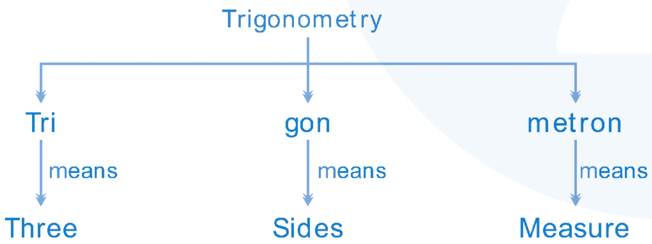
Introduction

This chapter is written for those students who were not so good in the secondary classes maths or who have not been in touch with the concept of trigonometry for the last few years. From this chapter, hardly a question is asked in the CAT examination directly, but based on the application of trigonometry, one or two questions are indirectly asked every year. In 'height and distance' and in geometry, we can find the use of the concept of trigonometry, which means that a CAT aspirant just refreshes the concept of trigonometry, and there is no need to dedicate much time for this topic.

Definition

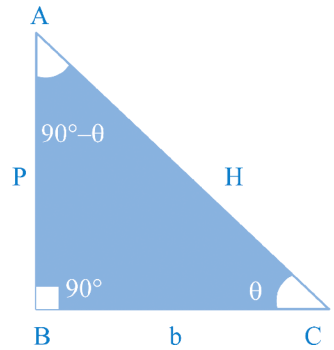
Trigonometry is the branch of mathematics that deals with the sides of a triangle and the angles of a triangle. The word 'trigonometry' has been taken from the Greek words 'tri', 'gon', and 'metron'.



Concept 1: Trigonometric Ratios

The ratio of the two sides of a right-angled triangle is called the trigonometric ratio. There are six trigonometric ratios.

Consider a right-angled triangle ABC.



AB = P = perpendicular

BC = b = base

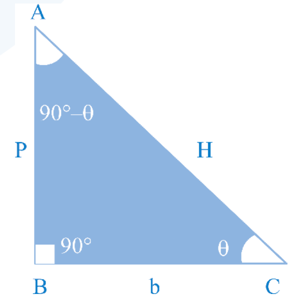
AC = H = hypotenuse

1. $\sin\theta = \frac{AB}{AC} = \frac{P}{H} \leftrightarrow \operatorname{cosec}\theta = \frac{H}{P}$
2. $\cos\theta = \frac{BC}{AC} = \frac{b}{H} \leftrightarrow \sec\theta = \frac{H}{b}$
3. $\tan\theta = \frac{AB}{BC} = \frac{P}{b} \leftrightarrow \cot\theta = \frac{b}{P}$

Moreover, we must know:

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$



Trigonometric Ratios of Some Important Angles

The trigonometric ratios of some specific angles, e.g., 0°, 30°, 45°, 60°, and 90° follow a special pattern so that we can easily remember the value of these specific angles; these values help solve the problems related to trigonometry.

Complementary angles

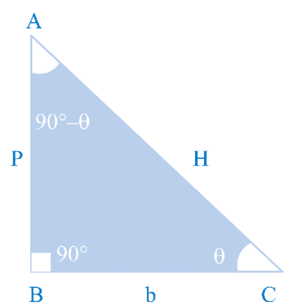
Suppose the sum of the two angles of a right-angled triangle is 90°. Then in the right-angled triangle, ABC, right-angled at B, ∠A and ∠C are complementary angles. The complementary angles are helpful in solving the complex problems related to trigonometric ratios.

Important table for the trigonometric ratios of the specific angles

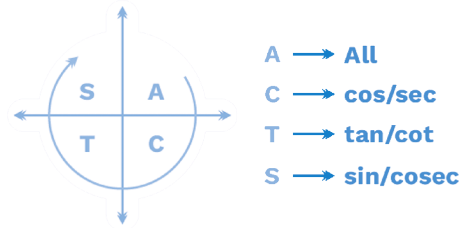
Angle θ ratios	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec}\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

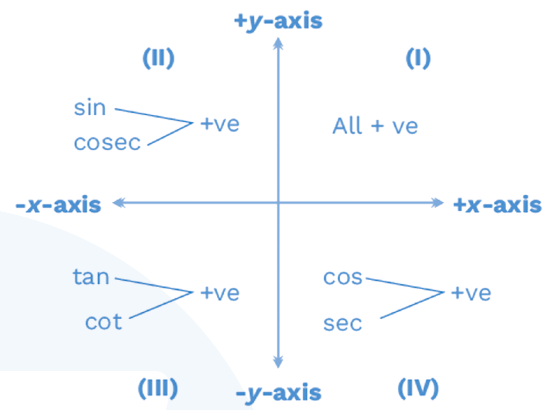
Trigonometric Ratios of Complementary Angles

1. $\sin(90-\theta) = \cos\theta$
2. $\cos(90-\theta) = \sin\theta$
3. $\tan(90-\theta) = \cot\theta$
4. $\operatorname{cosec}(90-\theta) = \sec\theta$
5. $\sec(90-\theta) = \operatorname{cosec}\theta$
6. $\cot(90-\theta) = \tan\theta$



Sign of all the Trigonometric Ratios





Note: Remember the word 'ACTS' in the clockwise direction, then you can easily memorise the sign of the trigonometric ratios in each quadrant.

Example 1:

Find the value of $\frac{5}{4}\sin^2 30° + 4\sin^2 90° - 5\tan^2 30° + \tan 30° \cot 60°$.

(A) $\frac{143}{48}$ (B) $\frac{48}{143}$

(C) $\frac{151}{7}$ (D) None of these

Solution: (A)

Since we know that

$$\sin 30° = \frac{1}{2},\ \sin 90° = 1,\ \tan 30° = \frac{1}{\sqrt{3}},$$

and $\cot 60° = \cot(90-30°) = \tan 30° = \frac{1}{\sqrt{3}}$

Therefore, $\frac{5}{4}\times\left(\frac{1}{2}\right)^2 + 4\times(1)^2 - 5\times\left(\frac{1}{\sqrt{3}}\right)^2 + \tan^2 30°$

$$= \frac{5}{16} + 4 - \frac{5}{3} + \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{69}{16} - \frac{5}{3} + \frac{1}{3}$$

$$= \frac{69}{16} - \frac{4}{3} = \frac{69 \times 3 - 4 \times 16}{48} = \frac{143}{48}$$

Hence, option (A) is the correct answer.

Example 2:

If $\tan\theta = \frac{3}{4}$, find the value of $\frac{3\sin\theta - 4\cos\theta}{3\sin\theta + 4\cos\theta}$.

(A) $-\frac{7}{25}$ (B) $-\frac{9}{25}$

(C) $-\frac{17}{25}$ (D) $-\frac{19}{25}$

Solution: (A)

Since $\tan\theta = \frac{3}{4}$ is given in the question,

$$\frac{3\sin\theta - 4\cos\theta}{3\sin\theta + 4\cos\theta} = \frac{\cos\theta\left(\frac{3\sin\theta}{\cos\theta} - 4\right)}{\cos\theta\left(\frac{3\sin\theta}{\cos\theta} + 4\right)}$$

$$= \frac{3\tan\theta - 4}{3\tan\theta + 4}$$

$$= \frac{3 \times \frac{3}{4} - 4}{3 \times \frac{3}{4} + 4} = \frac{\frac{9-16}{4}}{\frac{9+16}{4}}$$

$$= \frac{\frac{-7}{4}}{\frac{25}{4}} = \frac{-7}{25}$$

Hence, option (A) is the correct answer.

Example 3:

$\sin^2\theta \cdot \cot^2\theta + \cos^2\theta \cdot \tan^2\theta = ?$

(A) 0 (B) 3

(C) 1 (D) 2

Solution: (C)

Since we know that

$$\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}, \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$$

$\therefore$ $\sin^2\theta \cdot \cot^2\theta + \cos^2\theta \cdot \tan^2\theta$

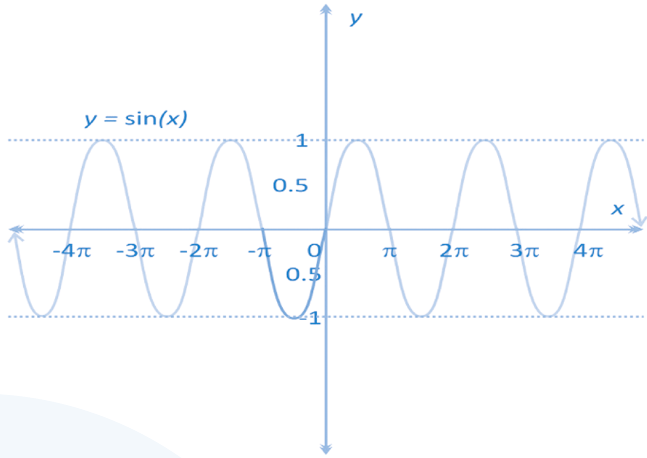
$$= \sin^2\theta \times \frac{\cos^2\theta}{\sin^2\theta} + \cos^2\theta \times \frac{\sin^2\theta}{\cos^2\theta}$$

$= \cos^2\theta + \sin^2\theta = 1$

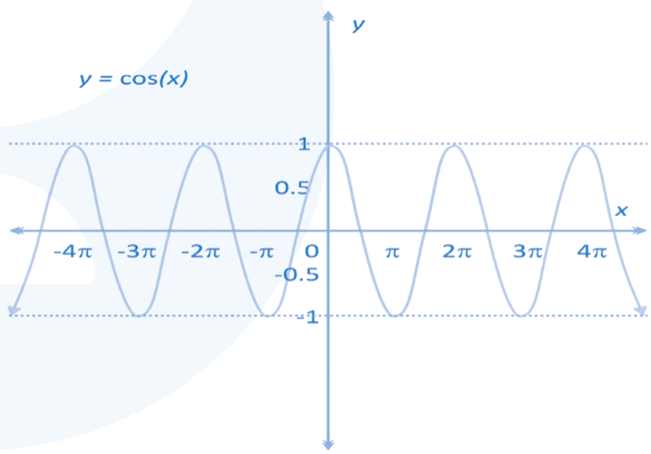
Hence, option (C) is the correct answer.

Range of Trigonometric Function or Ratios

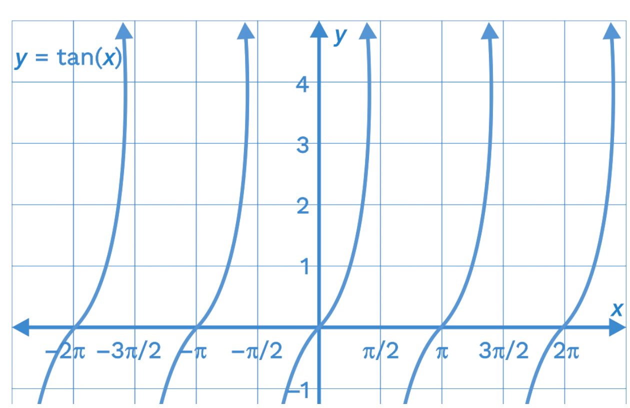
1. $-1 \le \sin\theta \le 1$ or $|\sin\theta| \le 1$



2. $-1 \le \cos\theta \le 1$ or $|\cos\theta| \le 1$



3. $-\infty \le \tan\theta \le \infty$



4. $-1 \le \text{cosec } \theta \le 1 \quad \Rightarrow \quad |\text{cosec } \theta| \ge 1$

5. $-1 \le \sec \theta \le 1 \quad \Rightarrow \quad |\sec \theta| \ge 1$

Trigonometric identities

- $\sin^2\theta + \cos^2\theta = 1$
- $\tan^2\theta + 1 = \sec^2\theta$
- $\cot^2\theta + 1 = \text{cosec}^2\theta$

Example 4:

Find the value of $\sin^2 5° + \sin^2 10° + \sin^2 15° + \cdots + \sin^2 90°$.

(A) $9\frac{1}{2}$ (B) $8\frac{1}{2}$

(C) $5\frac{1}{2}$ (D) $11\frac{1}{2}$

Solution: (A)

Since we know that $\sin^2\theta + \cos^2\theta = 1$

$\therefore \quad \sin^2 5° + \sin^2 10° + \sin^2 15° + \cdots + \sin^2 90°$

$= (\sin^2 5° + \sin^2 85°) + (\sin^2 10° + \sin^2 80°) + \cdots + \sin^2 45° + \sin^2 90°$

$= (\sin^2 5° + \sin^2(90 - 5°)) + (\sin^2 10° + \sin^2 (90 - 10°)) + \cdots + \sin^2 45° + \sin^2 90°$

$$= \underbrace{\left(\sin^2 5° + \cos^2 5°\right)}_{1} + \underbrace{\left(\sin^2 10° + \cos^2 10°\right)}_{1} + \cdots + \sin^2 45° + \sin^2 90°$$

$$= 8 \times 1 + \left(\frac{1}{\sqrt{2}}\right)^2 + 1 = 9\frac{1}{2}$$

Hence, option (A) is the correct answer.

Example 5:

Find the number of real roots of the equation $2\cos(x(x+1)) = 2^x + 2^{-x}$

(A) 1 (B) 0

(C) 2 (D) Infinite

Solution: (A)

As we know the maximum value of $\cos\theta = 1$ at $\theta = 0°$.

Therefore, the maximum value of $2\cos(x(x+1)) = 2 \times 1 = 2$.

Moreover, we know that $P + \frac{1}{P} \ge 2$

$\therefore \quad$ Minimum value of $2^x + \frac{1}{2^x} = 2$ at $x = 0$

$$2\cos((x+1)) = 2^x + \frac{1}{2^x} \text{ at } x = 0$$

Thus, there is only one possible case.

Example 6:

If $0° < \theta < \frac{\pi}{2}$, then $(\sin\theta + \cos\theta)$ is

(A) Greater than 1 (B) Less than 1

(C) Greater than 2 (D) Equal to 2

Solution: (A)

Let $y = \sin\theta + \cos\theta$

Now squaring both sides

$y^2 = (\sin\theta + \cos\theta)^2$

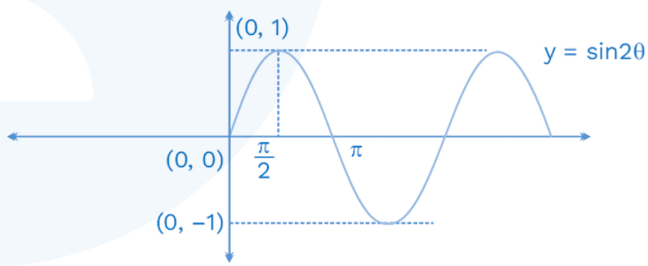
$y^2 = \underbrace{\sin^2\theta + \cos^2\theta}_{1} + 2\sin\theta\cos\theta$

$y^2 = 1 + 2\sin\theta\cos\theta$

Moreover, we know that $\sin 2\theta = 2\sin\theta\cos\theta$

Therefore, $y^2 = 1 + \sin 2\theta$

When θ lies from 0° to 90°, the value of $\sin 2\theta$ will always be less than 1.



Or $\sin 2\theta < 1$, but $\sin 2\theta > 0$

$y^2 = 1 + \sin^2 2\theta < 2$

$y^2 < 2$

or $y < \sqrt{2}$

Hence, the value $(\sin\theta + \cos\theta)$ is greater than 1, when 'θ' lies between 0° and 90°.

Example 7:

Find the value of log(tan 1°) + log(tan 2°) + ⋯ + log(tan 89°)

(A) 0 (B) 2

(C) 3 (D) 1

Solution: (A)

Log[tan 1° . tan 2°. tan 3° ⋯ tan 89°]

Since we know that angles 1° and 89° are complementary to each other.

And, tan 1° · tan 89° = tan 1° · tan(90 – 1°)

$= \tan 1° \cdot \cot 1° = \tan 1° \times \dfrac{1}{\tan 1°} = 1$

Similarly, tan 2° · tan 88° = tan 2° · tan(90 – 2°) = tan 2° × cot 2° = 1

Only one term, tan45°, will be left alone which will not pair with any term.

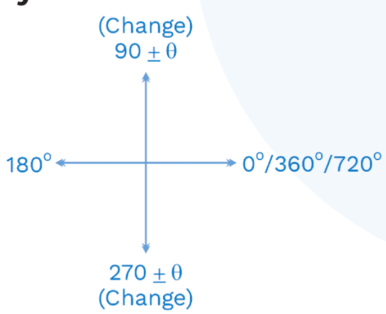
log[tan 1° × tan 2°... tan 45° ... tan 88° × tan 89°]

= log[tan45°] (since tan 45° = 1)

= log 1 = 0

Option (A) is the correct answer.

Periodicity



Some Important Formulae

1. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
2. $\sin(A - B) = \sin A \cos B - \cos A \sin B$
3. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
4. $\cos(A - B) = \cos A \cos B + \sin A \sin B$
5. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
6. $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$
7. $\sin 2A = 2\sin A \cos A$
8. $\cos 2A - \begin{cases} \cos^2 A - \sin^2 A \\ 2\cos^2 A - 1 \\ 1 - 2\sin^2 A \\ \dfrac{1 - \tan^2 A}{1 + \tan^2 A} \end{cases}$
9. $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$
10. $\sin(A + B) \cdot \sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$
11. $\cos(A + B) \cdot \cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$
12. $2 \sin A \cdot \cos B = \sin(A + B) + \sin(A - B)$
13. $2 \cos A \cdot \sin B = \sin(A + B) - \sin(A - B)$
14. $2 \cos A \cdot \cos B = \cos(A + B) + \cos(A - B)$
15. $2 \sin A \cdot \sin B = \cos(A - B) - \cos(A + B)$
16. $\cot(A + B) = \dfrac{\cot A . \cot B - 1}{\cot A + \cot B}$
17. $\cot(A - B) = \dfrac{\cot A . \cot B + 1}{\cot B - \cot A}$

Trigonometrical Ratios of Negative and Associated Angles

Angle θ Ratios	**$-\theta$**	**$90 - \theta$**	**$90 + \theta$**	**$180 - \theta$**	**$180 + \theta$**	**$360 - \theta$**	**$360 + \theta$**
$\sin\theta$	$\sin(-\theta) = -\sin\theta$	$\sin(90-\theta) = \cos\theta$	$\sin(90+\theta) = \cos\theta$	$\sin(180-\theta) = \sin\theta$	$\sin(180+\theta) = -\sin\theta$	$\sin(360-\theta) = -\sin\theta$	$\sin(360+\theta) = \sin\theta$
$\cos\theta$	$\cos(-\theta) = \cos\theta$	$\cos(90-\theta) = \sin\theta$	$\cos(90+\theta) = -\sin\theta$	$\cos(180-\theta) = -\cos\theta$	$\cos(180+\theta) = -\cos\theta$	$\cos(360-\theta) = +\cos\theta$	$\cos(360+\theta) = \cos\theta$
$\tan\theta$	$\tan(-\theta) = -\tan\theta$	$\tan(90-\theta) = \cot\theta$	$\tan(90+\theta) = -\cot\theta$	$\tan(180-\theta) = -\tan\theta$	$\tan(180+\theta) = \tan\theta$	$\tan(360-\theta) = -\tan\theta$	$\tan(360+\theta) = \tan\theta$

18. $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

19. $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$

20. $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

21. $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{D-C}{2}\right)$

Example 8:

If $\sec^2\theta = 2(1 - \tan^2\theta)$, find the value of $\sin 2\theta$.

(A) $\frac{1}{2}$ (B) $\sqrt{3}$

(C) $\frac{1}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2}$

Solution: (D)

Since we know that $\sec^2\theta = 1 + \tan^2\theta$

$\therefore\ 1 + \tan^2\theta = 2(1 - \tan^2\theta)$

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1}{2}$$

Moreover, we know that:

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

Therefore, $\cos 2\theta = \frac{1}{2}$

$\cos 2\theta = \cos 60°$ $\left[\text{since, } \cos 60° = \frac{1}{2}\right]$

$\therefore\ 2\theta = 60°$

$\theta = 30°$

Now, we have to find the value of $\sin 2\theta$

$$\sin 2\theta = \sin(2 \times 30°) = \sin 60° = \frac{\sqrt{3}}{2}$$

Or we can directly put the value of 2θ. No need to find the value of θ separately.

Example 9:

In ΔPQR, PR = 40 cm and $PQ = \frac{20}{\sqrt{3}}$ cm. PS is $\perp^r$ on the side QR from vertex P. $\angle RPS$ is two times the angle $\angle QPS$. If $\angle QSP$ = 30°, then find the ratio of QS:SR.

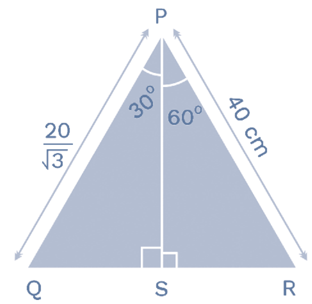
(A) 1:2 (B) 2:3

(C) 1:6 (D) 5:1

Solution: (C)

If $\angle QSP$ = 30°.

Then $\angle RPS$ = 60°



In right-angled ΔPSQ

$$\sin 30° = \frac{QS}{PQ} = \frac{QS}{\frac{20}{\sqrt{3}}}$$

$$\frac{1}{2} = \frac{QS}{\frac{20}{\sqrt{3}}}$$

$$QS = \frac{1}{2} \times \frac{20}{\sqrt{3}} = \frac{10}{\sqrt{3}}\text{ cm}$$

Moreover, in the right-angled triangle PSR:

$$\sin 60° = \frac{SR}{PR}$$

$$\frac{\sqrt{3}}{2} = \frac{SR}{40}$$

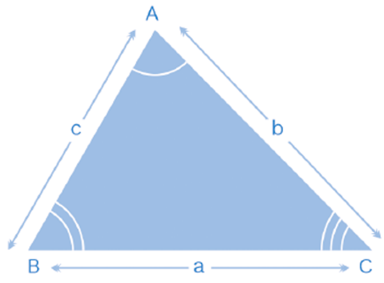
$$\text{Therefore, } QS:SR = \frac{10}{\sqrt{3}} : 20\sqrt{3} = 10 : 60 = 1 : 6$$

Hence, option (C) is the correct answer.

Law of 'sine': Application of Trigonometry

The law of sine states the ratio of side lengths of triangles to sin of their respective opposite angles is equal.

Note: The ratio of the sides and sin of the corresponding angle of a triangle is equal to the diameter of the circumcircle of the triangle.



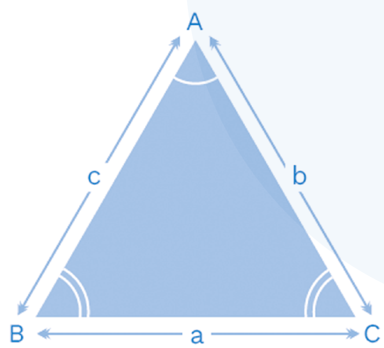
The sine law can be written in the form of

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

Law of Cosine Formula

The cosine law helps in establishing the relationship between the length of sides of a triangle and the cosine of its angles. The cosine law generally derives from the Pythagoras theorem.

The law of cosine states that the square of any one side of a triangle is equal to the difference between the sum of the squares of the other two sides and double the product of the other sides and the cosine angle included between them.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = c^2 + a^2 - 2ac\cos B,$$

or $c^2 = a^2 + b^2 - 2ab\cos C$

We can write the formula in other forms also:

$$*\cos A = \frac{b^2+c^2-a^2}{2bc}$$

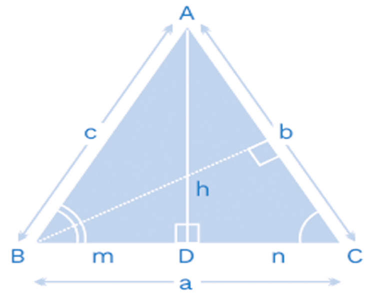
$$*\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$*\cos C = \frac{a^2+b^2-c^2}{2ab}$$

Application of Trigonometry

Triangles: Sine and Cosine Rule

Let a triangle ABC, in which AB = c unit, AC = b unit, and BC = a unit, and a perpendicular AD is drawn on BC from vertex A. AD divides the side BC into m:n.



In the right-angled ΔADC:

$$\sin C = \frac{AD}{AC} = \frac{h}{b}$$

$\therefore\quad h = b\sin C$ ...(i)

Now, again in right-angled ΔABD:

$$\sin B = \frac{AD}{AB} = \frac{h}{c}$$

$\therefore\quad h = c\sin B$ ...(ii)

By equating equations (i) and (ii), we get

$$b\sin C = c\sin B$$

$$\therefore\quad \frac{b}{\sin B} = \frac{c}{\sin C}$$

Similarly, if we draw $a \perp^r$ from vertex B on AC. Then we will get

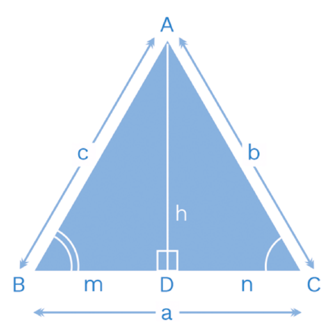
$$\frac{b}{\sin B} = \frac{c}{\sin C} = \frac{a}{\sin A}$$

or $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} \rightarrow$ (Result 1)

So, the above relation is known as the sine rule.

This rule is used to find the missing length or angles.

(Result 2):



In ΔADC:

$$\cos C = \frac{CD}{AC} = \frac{n}{b} \Rightarrow n = b\cos C$$

Again, in right-angled ΔADB:

$$\cos B = \frac{BD}{AB} = \frac{m}{c} \Rightarrow m = c\cos B$$

Therefore, $a = m + n = b \cos C + c \cos B$

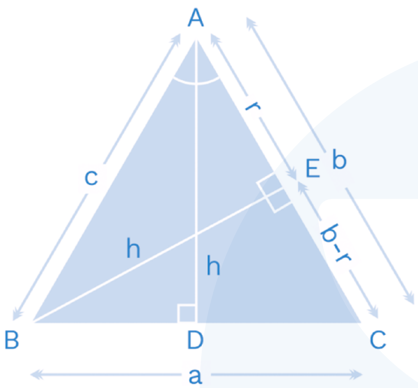
Cosine rule

Let in a ΔABC

EA = 'r' unit

And CE = $(b - r)$ unit

BE = h unit



Now, in right-angled ΔBEA:

$$\cos A = \frac{EA}{AB} = \frac{r}{c} \Rightarrow r = c\cos A$$

$$\sin A = \frac{BE}{AB} = \frac{h}{c} \Rightarrow h = c\sin A$$

Apply Pythagoras theorem in right-angled ΔBEC

$$a^2 = h^2 + (b - r)^2$$

Put the values of $h = c \sin A$ and $r = c \cos A$ in the above equation.

$$a^2 = (c\sin A)^2 + (b - c\cos A)^2$$

$$a^2 = c^2 \sin^2 A + b^2 + c^2 \cos^2 A - 2bc \cos A$$

$$a^2 = c^2 (\sin^2 A + \cos^2 A) + b^2 - 2bc \cos A$$

$$a^2 = c^2 + b^2 - 2bc \cos A$$

$$2bc \cos A = c^2 + b^2 - a^2$$

$$\cos A = \frac{c^2 + b^2 - a^2}{2bc}$$

This is known as the cosine rule.

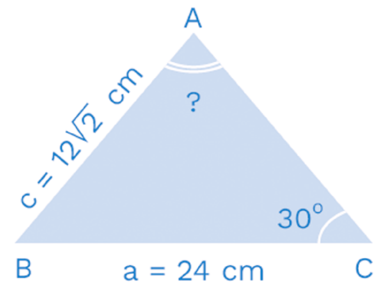
Cosine rule is used when adjacent lengths are given and included angle is also given, we can find the missing length of the 3rd side of a triangle.

Other forms of cosine rule

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}, \text{ also } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Example 10:

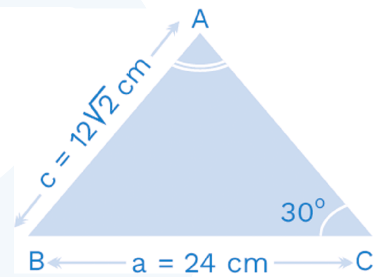
In a triangle ABC, a = 24 cm, c = $12\sqrt{2}$ cm and $\angle$C = 30°. Then find $\angle$A.



(A) 45° (B) 60°

(C) 35° (D) 120°

Solution: (A)



Use the sine rule here to find $\angle$A.

$$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{24}{\sin A} = \frac{12\sqrt{2}}{\sin 30^\circ}$$

$$\Rightarrow \frac{24}{\sin A} = \frac{12\sqrt{2}}{\frac{1}{2}}$$

$$\Rightarrow \sin A = \frac{1}{\sqrt{2}} \qquad \left[\text{since, } 45^\circ = \frac{1}{\sqrt{2}}\right]$$

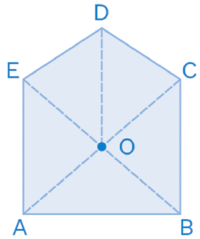
$\therefore$ $\angle$A = 45°

Hence, option (A) is the correct answer.

Polygons: Regular Polygon

If we want to find the different parts of a polygon we can use trigonometric ratios. But before going directly to the result, first consider a regular polygon (i.e., pentagon, hexagon, etc.)

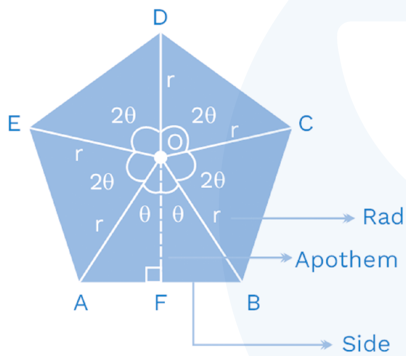
Let us consider the pentagon for finding different parts of it and the same will be applicable to every regular polygon.



Let 'O' be the centre of the polygon and OA, OB, OC, OD, and OE are the radii of the polygon because if you draw a circle passing through all vertices of the pentagon, then OA = OB = OC = OD = OE = r = radius.

AB will be the side of the polygon or pentagon.

An apothem is drawn from 'O' to 'F' on side AB.



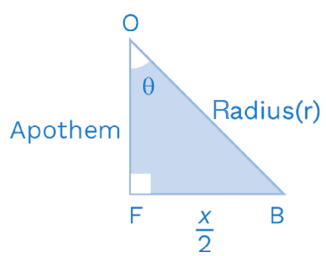
Apothem

It is a line drawn from the centre of any polygon to the mid-point of one of the sides.

Let the length of the side of the pentagon be x unit.

Then AF = FB = $\frac{x}{2}$ unit

Moreover,



Let $\angle FOB = \theta$, then also $\angle AOB = 2\theta$

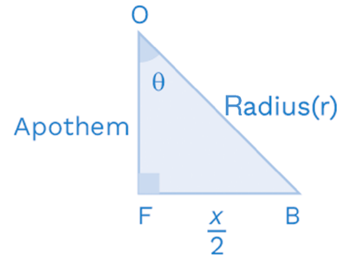
We know that the sum of all angles at the centre is 360°.

If 'n' angles are there, then:

$n \times 2\theta = 360°$

$$\theta = \frac{360°}{2n} = \frac{180°}{n}$$

$$\text{or, } \theta = \frac{\pi}{n}$$



$$\therefore \cos\left(\frac{\pi}{n}\right) = \cos\theta = \frac{\text{apothem}}{\text{radius}} = \frac{ap}{r}$$

$$\text{Moreover, } \sin\left(\frac{\pi}{n}\right) = \sin\theta = \left(\frac{\frac{x}{2}}{\text{radius}}\right)$$

$$\sin\left(\frac{\pi}{n}\right) = \frac{x}{2r}$$

Again, find $\tan\left(\frac{\pi}{n}\right)$

$$\therefore \tan\left(\frac{\pi}{n}\right) = \frac{\left(\frac{x}{2}\right)}{\text{apothem}}$$ (where n = no. of sides of a polygon)

By using the above ratios, we can easily find the side, radius and apothem, area, and perimeter of a polygon.

Area of a polygon = $\frac{1}{2} \times (n \times s) \times$ apothem

n → no. of sides of a polygon.

S → length of the side of a polygon.

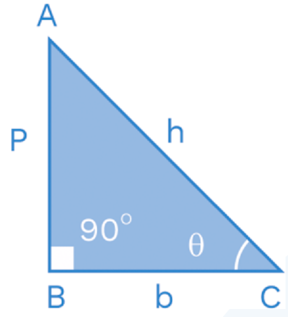
ns → is known as the perimeter of a polygon.

Height and Distance

Height and distance is one of those topics in which you are expected to know the basic concept of trigonometry. Height and distance comes in CAT and other MBA entrance examinations. Advanced concepts of trigonometry do not come in the CAT examination. But if you are preparing for other MBA entrance examinations like XAT, then you have to prepare for advanced concepts of trigonometry also.

If you remember the formula for three trigonometric ratios (sinθ, cosθ, and tanθ) and the values of sin, cos, and tan at 30°, 45°, and 60°, then you can easily solve any height- and distance-related problems.

Let a right-angled triangle be ΔABC.



1. $\sin\theta = \frac{AB}{AC} = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{P}{h}$

2. $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{b}{h}$

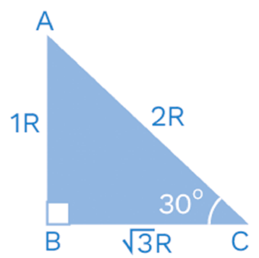
3. $\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{P}{B}$

Important values

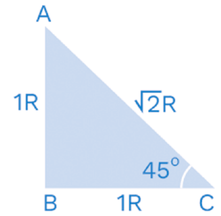
Angle θ ratios	30°	45°	60°
sinθ	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cosθ	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tanθ	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Some important trigonometric ratios

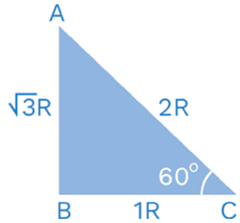
If θ = 30°



If θ = 45°



If θ = 60°

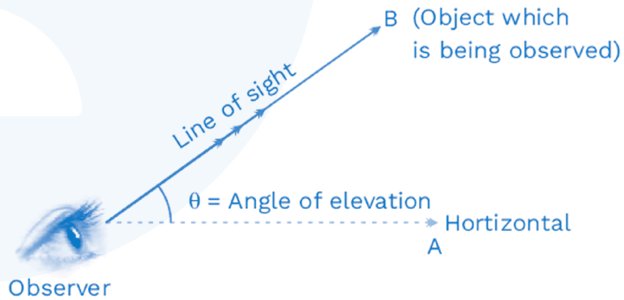


Three basic things will frequently come in height and distance problems.

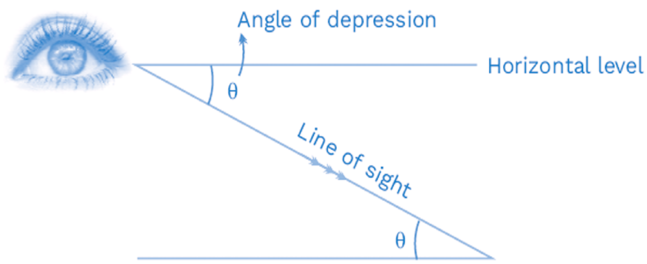
1. Line of sight
2. Angle of elevation
3. Angle of depression

1. **Line of sight:** It is the line drawn from the eye of the observer to the point in the object viewed by the observer.

2. **Angle of elevation:** The angle formed between the horizontal line and the line of sight joining a point or object which is being observed.



3. **Angle of depression:** The angle between the horizontal and the line of sight joining an observation point to an object below the horizontal level.



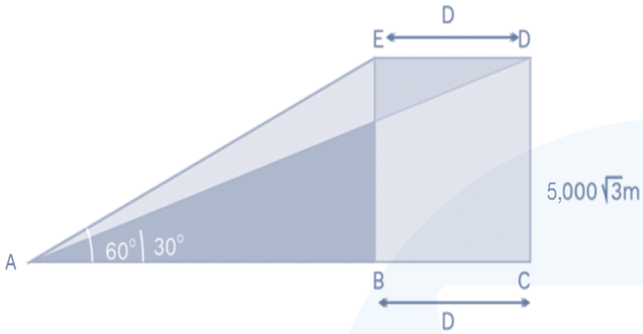
Example 11:

The angle of elevation of a flying bird from a point on the ground is 60°. After flying for 10 min, the angle of elevation at this instant becomes 30°. If the bird is flying horizontally at a constant height of $5000\sqrt{3}$, the distance travelled by the bird is:

(A) 10,000 m (B) 8,000 m

(C) 6,000 m (D) 9,000 m

Solution: (A)



Let the bird covers '*D*' distance in 10 min. Moreover, let's consider AB = *x* m.

In ΔADC:

$$\tan 30^\circ = \frac{DC}{AC} = \frac{5{,}000\sqrt{3}}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{5{,}000\sqrt{3}}{AC}$$

AC = 15,000 m

Now, again in ΔAEB:

$$\tan 60^\circ = \frac{5{,}000\sqrt{3}}{AB}$$

AB = 5,000 m [since $\tan 60^\circ = \sqrt{3}$]

∴ The distance travelled by the bird in 10 min

= AC − AB = 15,000 − 5,000 = 10,000 m

∴ BC = 10,000 m

Hence, option (A) is the correct answer.

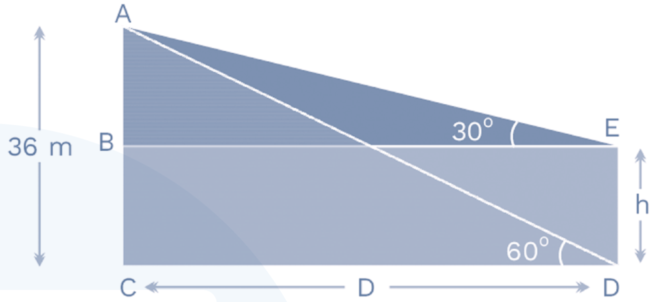
Example 12:

There are two temples, one on each bank of a river, just opposite to each other. One temple is 36 m high. From the top of this temple, the angle of depression of the top and foot of the other temple is 30° and 60°, respectively. Find the height of the other temple.

(A) 34 m (B) 25 m

(C) 24 m (D) 29 m

Solution: (C)



Let the distance between the two temples be *D* metres, and the height of the smaller temple be *h* metres.

In ΔACD:

$$\tan 60^\circ = \frac{AC}{CD} = \frac{36}{D}$$

$$\sqrt{3} = \frac{36}{D}$$

$$D = \frac{36}{\sqrt{3}} = \frac{36 \times \sqrt{3}}{3} = 12\sqrt{3} \text{ m}$$

Now, again in ΔABE:

$$\tan 30^\circ = \frac{AB}{BE} = \frac{36 - h}{12\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \frac{36 - h}{12\sqrt{3}}$$

12 = 36 − *h*

h = 24 m

∴ The height of the other temple is 24 m.

Option (C) is the correct answer.

Practice Exercise – 1

Level of Difficulty – 1

1. A car moves towards the foot of a tower at a certain constant speed. In 12 min, the elevation angle of the top of the tower changes from 30° to 45°. Find the time the car will take to reach the foot of the tower from the point where the angle of elevation was 45°.

(A) $6(\sqrt{3}+1)$ min

(B) $6(\sqrt{3}-1)$ min

(C) $4(\sqrt{3}+2)$ min

(D) $(2+\sqrt{3})$ min

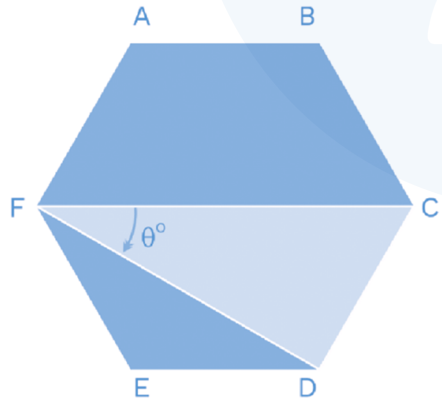
2. In a regular hexagon ABCDEF as shown below in the figure, find the value of tan2θ.

(A) $\frac{1}{\sqrt{3}}$

(B) $\sqrt{3}$

(C) $\frac{1}{2}$

(D) 1



3. If $\tan\theta = \frac{3}{4}$, find the value of $\left(\frac{1-\sin\theta}{1+\sin\theta}\right)$

(A) 1/4

(B) 3/4

(C) 7/8

(D) 5/7

4. In a right-angled triangle, the base is 'b' unit, the perpendicular is 'p' unit, and the hypotenuse is 'h' unit. Suppose it is known that 'P' and 'b' are positive integers. The square of hypotenuse can take all the values except one of the below. Find that value.

(A) 24

(B) 25

(C) 52

(D) 61

5. How many roots are present in the interval (0, π) of the equation $(\tan\theta + 1)(\tan\theta - 1)(1 + \tan^2\theta) - 2(\sec^2\theta - 1) + 2 = 0$?

(A) 2

(B) 3

(C) 4

(D) 1

Level of Difficulty – 2

6. Find the maximum and minimum value of 5 cos θ + 12 sin θ + 30.

(A) (17, –15)

(B) (18, –16)

(C) (43, 17)

(D) (17, 40)

7. If $\sin\theta + \sin^2\theta = 1$ and $a\cos^{12}\theta + b\cos^{10}\theta + c\cos^8\theta + d\cos^6\theta - 1 = 0$. Find the value of $(a + b) - (c - d)$.

(A) 2

(B) 3

(C) 4

(D) 1

8. Ritik saw an electric pole at an elevation of 30°. After walking for 2 min towards the electric pole, the angle of elevation changed to 60°. If Ritik walks at 5 m/sec, find the height of the electric pole (in m).

(A) $1{,}000\sqrt{3}$ m

(B) $500\sqrt{3}$ m

(C) $600\sqrt{2}$ m

(D) $300\sqrt{3}$ m

9. Arjun and Karan are 200 m from each other. Between them, there is a tree. Arjun found that the top of the tree was at $\theta°$, whereas Karan found that the top of the tree was at $\alpha°$. Find the height of the tree.

(A) $h = \dfrac{200}{\cot\theta + \cot\alpha}$

(B) $h = \dfrac{2}{\tan\alpha}$

(C) $h = \dfrac{5\cot\alpha}{\cot\alpha + \cot\theta}$

(D) $h = 9 \cot\alpha + 5 \tan\alpha$

10. A portion of a 60 m long tree is broken by the wind and the top of the tree touches the ground by making an angle of 30° with the ground. Find the height of the point where the tree is broken.

(A) 20 m
(B) 40 m
(C) 50 m
(D) 30 m

Level of Difficulty – 3

11. Given that $\operatorname{cosec}A - \sin A = 4$, find the value of $\operatorname{cosec}^6 A + \sin^6 A$.

12. A car, standing at the north of an electric pole, is moving at an angle of 30° with the top of the electric pole. Simultaneously, another car is standing at the east of the same electric pole, making an angle of 60° with the top of the electric pole. Find the shortest distance between these two cars. The height of the electric pole is 600 ft (Consider both the cars are of negligible dimensions.)

(A) 991 ft
(B) 1,795.4 ft
(C) 1,089 ft
(D) 1,095.45 ft

13. Arun and Varun are 20 km apart. They both see a bird flying in the sky, making an angle of 60° and 30°, respectively. Find the height at which the bird is flying.

(A) $5\sqrt{7}$ km

(B) $10\sqrt{3}$ km

(C) $5\sqrt{3}$ km

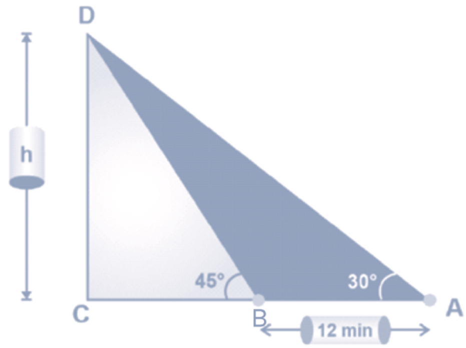
(D) both (B) and (C)

14. In a right-angled triangle, 'P' and 'b' are the perpendicular sides and 'h' is the hypotenuse. Find the minimum value of $\dfrac{h}{p} + \dfrac{h}{b}$.

15. Arun was playing on the ground when he observed two UFOs flying exactly one above the other. The angle of elevation of the lower UFO for Arun is 30°, whereas the angle of elevation for the higher UFO was 60°. If the difference between the heights of two UFOs was 300 m, calculate the height at which the higher UFO was flying.

(A) 450 m
(B) 460 m
(C) 900 m
(D) 1,000 m

1. (A)



Let the height of the tower be '*h*' unit.
Now, in ΔDCA

$$\tan 30° = \frac{h}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{AC}$$

$$AC = h\sqrt{3} \text{ unit} \qquad ...(i)$$

Again in ΔDCB:

$$\tan 45° = \frac{h}{BC}$$

$$BC = h \qquad ...(ii)$$

Therefore, AB = AC − BC

$$= h\sqrt{3} - h = h(\sqrt{3}-1) \text{ unit}$$

Since it is given in the question that the car covers AB distance in 12 min. But we have to find the time in which the car will cover the distance BC to reach the foot of the tower.

$$\therefore \quad h(\sqrt{3}-1)\text{unit} \rightarrow 12 \text{ min}$$

$$1 \text{ unit} \rightarrow \frac{12}{h\left(\sqrt{3}-1\right)} \text{ min}$$

$$\text{Therefore, } h \text{ unit} \rightarrow \frac{12}{h\left(\sqrt{3}-1\right)} \times h \text{ min}$$

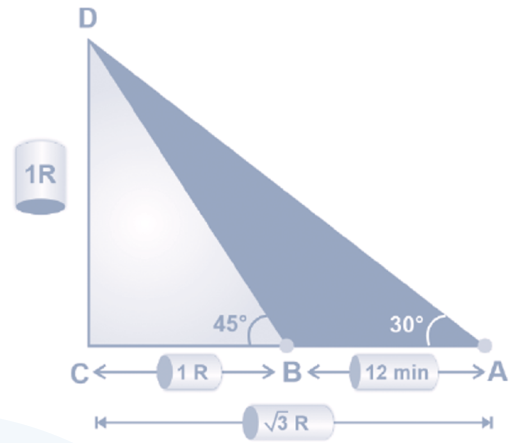
$$= \frac{12 \times (\sqrt{3}+1)}{(3-1)}$$

$$= \frac{12(\sqrt{3}+1)}{2}$$

$$= 6\left(\sqrt{3}+1\right) \text{ min}$$

Hence, option (A) is the correct answer.

Alternate solution



$$\therefore \quad AB = \sqrt{3}R - 1R = (\sqrt{3}-1)R$$

Moreover, it is given that:

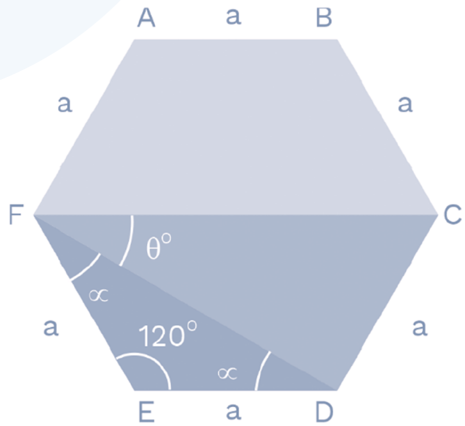
$$(\sqrt{3}-1)R \longrightarrow 12 \text{ min}$$

$$1R \longrightarrow \frac{12}{\sqrt{3}-1} \text{min}$$

$$= \frac{12(\sqrt{3}+1)}{(3-1)} = 6(\sqrt{3}+1) \text{ min}$$

2. (B)

All the sides of a regular hexagon are equal. Then ΔEFD is an isosceles Δ.



Since ∠FED = 120°

Then, ∝ = 30°

$$\text{Then, } \angle CFE = \frac{1}{2}\angle AFE$$

$\angle CFE = \frac{1}{2} \times 120° = 60°$

$\therefore \propto + \theta = 60°$

$30° + \theta = 60°$

$\theta = 30°$

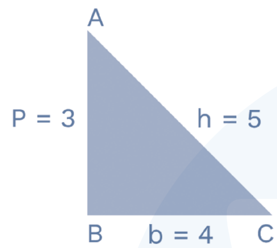
Hence, $\tan 2\theta = \tan(2 \times 30°)$

$= \tan 60° = \sqrt{3}$

Option (B) is the correct answer.

3. **(A)**

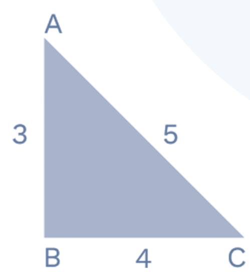
Since $\tan\theta = \frac{3^{\to p}}{4_{\to b}}$ (given)



By using the Pythagoras theorem or triplet, we can find the hypotenuse of the above triangle h = five units (or AC = 5 units).

Now, we have to find the value of $\left(\frac{1-\sin\theta}{1+\sin\theta}\right)$.

Since $\sin\theta = \frac{P}{H} = \frac{3}{5}$



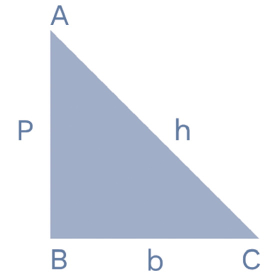
Put the value of $\sin\theta$ in the above equation.

$$= \frac{1-\sin\theta}{1+\sin\theta} = \frac{1-\frac{3}{5}}{1+\frac{3}{5}} = \frac{\frac{2}{5}}{\frac{8}{5}} = \frac{1}{4}$$

4. **(A)**

Since we know that in a right-angled triangle.

$h^2 = p^2 + b^2$



Moreover, it is given in the question that 'P' and 'b' are two positive integers; therefore, 'h^2' will be the sum of two perfect squares.

a) $24 = P^2 + b^2$ 24, can't be expressed as the sum of two perfect squares.

b) $h^2 = P^2 + b^2$

$25 = 3^2 + 4^2$

Therefore, 25 can be expressed as the sum of two perfect squares.

c) $52 = 6^2 + 4^2$

Hence, 52 is also expressed as the sum of two perfect squares.

d) $61 = 5^2 + 6^2$

Therefore 61, is also expressed as the sum of the two perfect squares.

Hence, option (A) is the correct answer.

5. **(A)**

Since we know that:

$(a + b)(a - b) = a^2 - b^2$

$(\tan^2\theta - 1)(\tan^2\theta + 1) - 2\tan^2\theta + 2 = 0$

$(\tan^4\theta - 1) - 2\tan^2\theta + 2 = 0$

$\tan^4\theta - 2\tan^2\theta + 1 = 0$

Let $\tan^2\theta = x$

$\therefore \quad x^2 - 2x + 1 = 0$

$(x - 1)^2 = 0$

$x = 1$

Since $\tan^2\theta = x$

$\therefore \quad \tan^2\theta = 1$

$\tan\theta = \pm 1$

When $\tan\theta = 1$

$\theta = 45°$ or $\frac{\pi}{4}$

When $\tan\theta = -1$

Then, $\theta = \frac{3\pi}{4}$ or $135°$

Hence, there are two values θ that satisfy the given equation in the interval $(0, \pi)$.

Hence, option (A) is the correct answer.

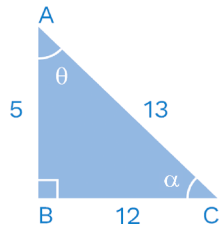
6. (A)

Since [5, 12, 13] is a Pythagorean triplet.

$\therefore$ If we divide and multiple by 13 in $5\cos\theta + 12\sin\theta$

$$= 13 \times \left(\frac{5}{13}\cos\theta + \frac{12}{13}\sin\theta\right)$$

Let there be another angle α from which



$$\sin\alpha = \frac{5}{13}$$

$$\cos\alpha = \frac{12}{13}$$

$\Rightarrow$ $13 \times [\sin\alpha\cos\theta + \cos\alpha\sin\theta]$

$\Rightarrow$ $13 \times [\sin(\alpha + \theta)]$

Since $[\sin(\alpha + \theta)]_{max} = 1$ and $[\sin(\alpha + \theta)]_{min} = -1$.

$\therefore$ Maximum value of the given equation $= 13 \times 1 + 30 = 43$.

Also minimum value of the given equation $= 13 \times (-1) + 30 = 17$.

Hence, option (C) is the correct answer.

7. (A)

Since $\sin\theta = 1 - \sin^2\theta$

$\sin\theta = \cos^2 q$

$\sin^2\theta = \cos^4 q$

$1 - \cos^2\theta = \cos^4 q$

$\cos^4\theta + \cos^2\theta = 1$

$(\cos^4\theta + \cos^2\theta)^3 = 1^3$

$\Rightarrow$ $(\cos^4\theta)^3 + \cos^6\theta + 3\cos^6\theta(\cos^4\theta + \cos^2\theta) = 1$

$\Rightarrow$ $\cos^{12}\theta + \cos^6\theta + 3\cos^{10}\theta + 3\cos^8\theta - 1 = 0$

On comparing with above equation

$a\cos^{12}\theta + b\cos^{10}\theta + c\cos^8\theta + \cos^6\theta - 1 = 0$

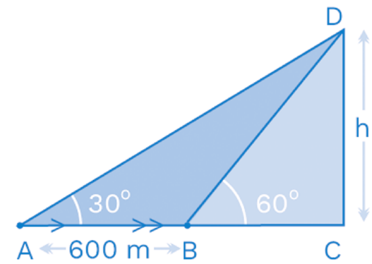
Moreover, $1\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + 1.\cos^6\theta - 1 = 0$

$\therefore$ $a = 1, b = 3, c = 3, d = 1$

$\therefore$ $(a + b) - (c - d) = (1 + 3) - (3 - 1) = 4 - 2 = 2$

Hence, option (A) is the correct answer.

8. (D)



Let the height of the electric pole is 'h' meters.

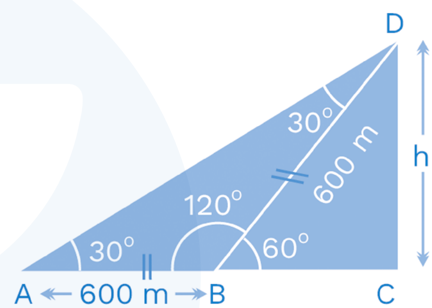
Distance travelled by Ritik in 2 min

= Speed × time = 5 m/sec × 120 sec

= 600 m

$\therefore$ AB = 600 m

By applying geometry in ΔABD



Now, in ΔBCD:

$$\sin 60° = \frac{h}{600}$$

$$\frac{\sqrt{3}}{2} = \frac{h}{600}$$

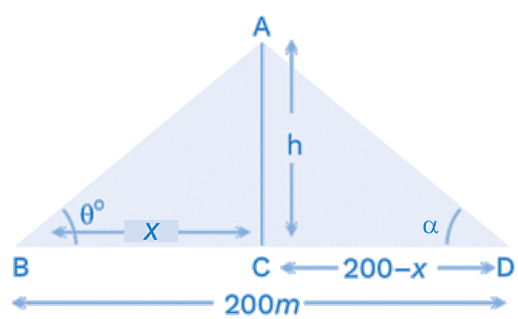
$$300\sqrt{3} = h$$

$$h = 300\sqrt{3}\text{ m}$$

$\therefore$ The height of the electric pole is $300\sqrt{3}$ m

Hence, option (D) is the correct answer.

9. (A)



Let the height of the tree is h m also BC = x m.
Moreover, suppose at point B Arjun is standing and Karan at point D is standing.
In ΔABC:

$$\tan\theta = \frac{h}{x} \quad \Rightarrow \quad x = h\cot\theta \qquad \text{...(i)}$$

Again, in ΔACD:

$$\tan\alpha° = \frac{AC}{CD}$$

$$\tan\alpha° = \frac{h}{200 - x} = \frac{h}{200 - x} \qquad \text{...(ii)}$$

Put the value of x = h cotθ in equations (ii) from (i)

$\therefore$ $200 - x = h \cot\alpha$

$200 - h\cot\theta = h\cot\alpha$

$h\cot\alpha + h\cot\theta = 200$

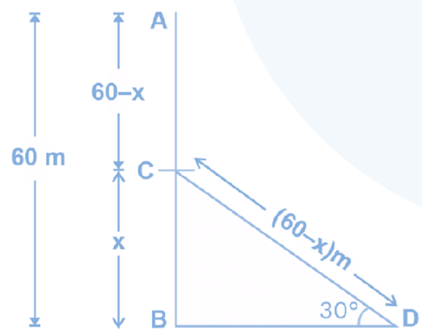
$h(\cot\alpha + \cot\theta) = 200$

$$h = \frac{200}{\cot\alpha + \cot\theta}$$

$\therefore$ The height of the tree is $\frac{200}{\cot\alpha + \cot\theta}$.

Hence, option (A) is the correct answer.

10. (A)



Let the height at which the tree is broken be x m from point 'B'.
In ΔCBD:

$$\sin 30° = \frac{BC}{CD}$$

$$\frac{1}{2} = \frac{x}{60 - x}$$

$60 - x = 2x$

$3x = 60$ m

$x = 20$ m

Hence, option (A) is the correct answer.

11. 5,778

Let cosec$A$ = $x \longrightarrow \sin A = \frac{1}{x}$

So, given $x - \frac{1}{x} = 4$ and we need to find $x^6 + \frac{1}{x^6}$

$$\Rightarrow x - \frac{1}{x} = 4$$

Square both sides

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 16 \longrightarrow x^2 + \frac{1}{x^2} - 2\times x \times \frac{1}{x} = 16$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 18$$

Cube both sides

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^3 = 18^3$$

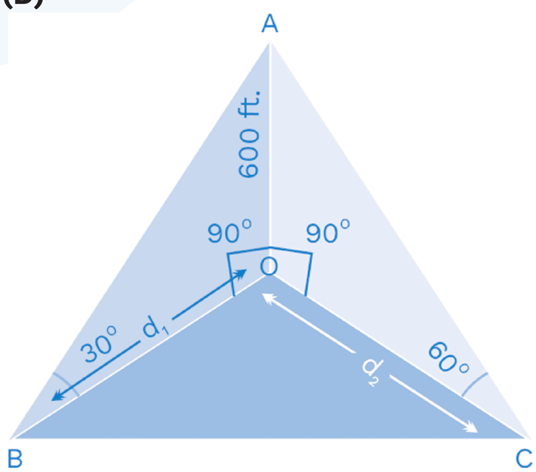
$$x^6 + \frac{1}{x^6} + 3\times x^2 \times \frac{1}{x^2}\left(x^2 + \frac{1}{x^2}\right) = 5,832$$

$$\downarrow$$

$$x^6 + \frac{1}{x^6} + 3\times 1\times(18) = 5,832$$

$$x^6 + \frac{1}{x^6} = 5,832 - 54 = 5,778$$

12. (D)



Let OA be an electric pole and at point B car 1 is standing and at point C 2nd car is standing. Moreover, assume the distance between the first car and second car from the base of the electric pole is d_1 and d_2 ft.

Now, in the right-angled ΔAOB:

$$\tan 30^\circ = \frac{AO}{OB}$$

$$\frac{1}{\sqrt{3}} = \frac{600}{d_1}$$

$$\Rightarrow d_1 = 600\sqrt{3} \text{ ft}$$

Again, in ΔAOC:

$$\tan 60^\circ = \frac{AO}{OC} = \frac{600}{d_2}$$

$$\sqrt{3} = \frac{600}{d_2}$$

$$d_2 = \frac{600}{\sqrt{3}} = \frac{600 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{600\sqrt{3}}{3} = 200\sqrt{3} \text{ ft}$$

$\therefore$ Shortest distance between the two cars.

$$= \sqrt{d_1^2 + d_2^2}$$

$$= \sqrt{(600\sqrt{3})^2 + (200\sqrt{3})^2}$$

$$= \sqrt{(36,000 \times 3) + 40,000 \times 3}$$

$$= 100\sqrt{108 + 12}$$

$$= 100\sqrt{120} = 1,095.45 \text{ ft}$$

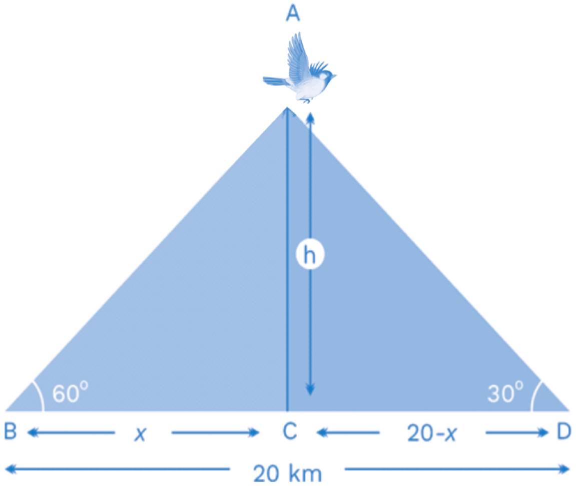
Therefore, option (D) is the correct answer.

13. (D)

In this type of question, two cases will form

Case 1:

When Arun and Varun are standing on the opposite side:



Suppose at 'h' height the bird is flying.

$$\therefore \quad \tan 30^\circ = \frac{h}{20 - x}$$

$$20 - x = h\sqrt{3}$$

$$x = (20 - h\sqrt{3})\text{km} \qquad \text{......(i)}$$

Again, $\tan 60^\circ = \dfrac{h}{x}$

$$\sqrt{3} = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}}$$

Put the value $x = \dfrac{h}{\sqrt{3}}$ in equation (i)

$$\therefore \quad x = (20 - h\sqrt{3})\text{km}$$

$$\frac{h}{\sqrt{3}} = (20 - h\sqrt{3})$$

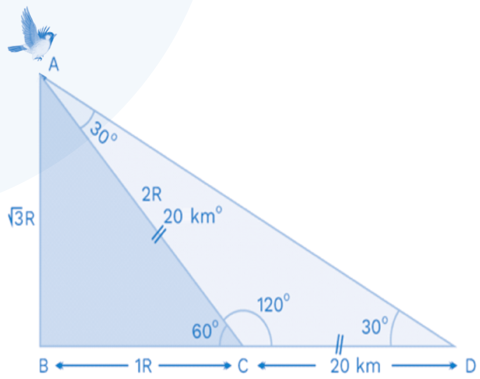
$$h = 20\sqrt{3} - 3h$$

$$4h = 20\sqrt{3}$$

$$h = 5\sqrt{3}\text{km}$$

Case 2:

When Arun and Varun are standing on the same side



By using geometry in ΔABD:

Since $\angle CAD = \angle ADC$

$\therefore$ $AC = CD$ (by property of an isosceles Δ)

Therefore, $2R \rightarrow 20$ km

$1R \rightarrow 10$ km

$\therefore$ $\sqrt{3}R \rightarrow 10\sqrt{3}$ km

$\therefore$ In this case, the height at which the bird is flying is $10\sqrt{3}$ km.

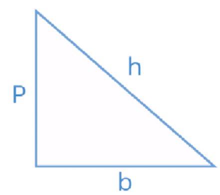
14. $2\sqrt{2}$

Since we know that AM ≥ GM

$$\therefore \frac{\frac{h}{P}+\frac{h}{b}}{2} \geq \sqrt{\frac{h}{P}\times\frac{h}{b}}$$

$$\frac{h}{p}+\frac{h}{b} \geq 2\sqrt{\frac{h^2}{Pb}}$$

$$\frac{h}{p}+\frac{h}{b} \geq 2\sqrt{\frac{P^2+b^2}{Pb}}$$



$\left[h^2 = P^2 + b^2\right]$

$$\frac{h}{p}+\frac{h}{b} \geq 2\sqrt{\frac{P}{b}+\frac{b}{p}}$$

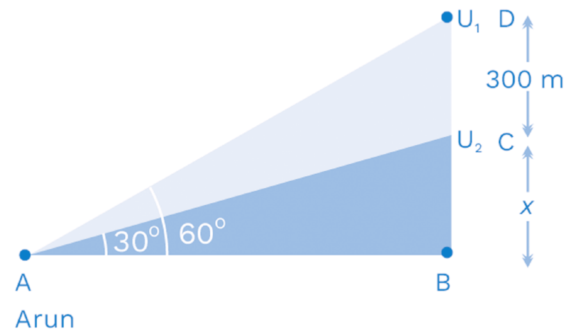
$$\frac{h}{p}+\frac{h}{b} \geq 2\sqrt{2}$$

Because $\left[X+\frac{1}{X}\geq 2\right]$

Therefore, $\frac{h}{p}+\frac{h}{b} \geq 2\sqrt{2}$

Hence, the minimum value of the expression $\left(\frac{h}{P}+\frac{h}{b}\right)$ is $2\sqrt{2}$

15. **(A)**



Let Arun be at point 'A' and distance BC = x.

In ΔABC:

$$\tan 30^\circ = \frac{BC}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{AB}$$

$$AB = x\sqrt{3} \qquad \text{......(i)}$$

$$\tan 60^\circ = \frac{300+x}{AB}$$

$$\sqrt{3} = \frac{300+x}{x\sqrt{3}}$$

$x \times 3 = 300 + x$

$3x = 300 + x$

$2x = 300$

$x = 150$ m

Therefore, the UFO1 is flying at height = $300 + x = 300 + 150 = 450$ m.

Hence, option (A) is the correct answer.

Practice Exercise – 2

Level of Difficulty – 1

1. Two towers of the same height stand on either side of a road 40 m wide. At a point on the road between the towers, the elevations of the towers are 60° and 30°. Find the approximate height of the towers.

(A) 19 m
(B) 17 m
(C) 15 m
(D) 13 m

2. A tree falls and rests against a vertical wall in a severe storm. The top of the tree is 20 ft above the foot of the wall. If now the bottom of the tree slides back again by 10 ft, then the tree lies flat on the ground with its top touching the wall. Find the height (length) of the tree.

(A) 25 ft
(B) 30 ft
(C) 50 ft
(D) 80 ft

3. A man walks 30 m towards a lamp post and notices that the angle of elevation of the top of the lamp post increases from 30° to 60°. Find the height of the post.

(A) $15\sqrt{3}$
(B) $15\left(\sqrt{3}+1\right)$
(C) $15\left(\sqrt{3}-1\right)$
(D) $15\left(2+\sqrt{3}\right)$

4. An aeroplane is flying at an altitude of 20 km above point P on the ground and its elevation from point R on the ground is 45°. It is flying horizontally away from point R and after 5 seconds it is directly above a point Q on the ground and the elevation of the airplane is now reduced by 15°. The speed of the aeroplane (in m/s) is:

(A) $4{,}000\sqrt{3}$
(B) $8{,}000(\sqrt{3}-2)$
(C) $4{,}000(\sqrt{3}-1)$
(D) $8{,}000(\sqrt{3}-\sqrt{2})$

5. A policeman observes from the top of a security tower a thief running away from the tower. The angle of depression from the top of the tower to the thief is 60° when the thief is 100 m from the tower. Twenty-five seconds later, the angle of depression becomes 45°. Find the speed with which the thief is running away?

(A) $4(\sqrt{3}+1)$ m/s
(B) $5(\sqrt{3}-1)$ m/s
(C) $6(\sqrt{3}+1)$ m/s
(D) $4(\sqrt{3}-1)$ m/s

6. Find the value of tan12° × tan24° × tan36° × tan60° × tan54° × tan66° × tan78°.

(A) 0
(B) $\frac{1}{\sqrt{3}}$
(C) $\sqrt{3}$
(D) 1

7. $\tan 50^\circ + \tan 70^\circ - \sqrt{3}\tan 50^\circ \tan 70^\circ$

(A) $\frac{1}{\sqrt{3}}$
(B) $\sqrt{3}$
(C) $-\sqrt{3}$
(D) $\frac{-1}{\sqrt{3}}$

8. Find the range of $\cos^2\theta + \sin^4\theta$.

(A) [0, 1]
(B) $\left[\frac{3}{4}, 1\right]$
(C) $\left[\frac{1}{2}, \frac{3}{4}\right]$
(D) [0, 2]

9. The sides of a right-angled triangle are in geometric progression. What is the ratio of the tan of acute angles of a triangle?

(A) $\dfrac{1+\sqrt{3}}{2}$

(B) $\dfrac{\sqrt{5}-1}{2}$

(C) $\dfrac{1-\sqrt{3}}{2}$

(D) $\dfrac{1+\sqrt{5}}{2}$

10. A man has a farmhouse in the shape of a triangle. The two adjacent sides of the farmhouse measure 180 and 200 cm and the angle between these two sides are 60°. Find the third side.

(A) 190.78 cm

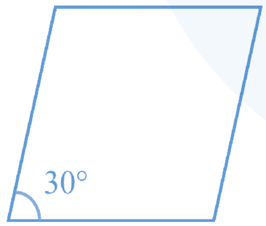
(B) 179.20 cm

(C) 145.20 cm

(D) 123.45 cm

Level of Difficulty – 2

11. The parallelogram shown in the figure has four sides of equal length. If the ratio of the length of the longer diagonal to the length of the shorter diagonal is of the form $a+\sqrt{b}$, then find the value of $2a+3b$.



12. If $\sin A + \cos A = 7x$, then find the value of $\sin^3 A + \cos^3 A$ is:

(A) $\dfrac{23x + 343x^3}{2}$

(B) $\dfrac{21x + 343x^3}{2}$

(C) $\dfrac{23x - 343x^3}{2}$

(D) $\dfrac{21x - 343x^3}{2}$

13. Ananya went to see two of her friends staying on different floors of the same building. When she was 60 m away from the building, she saw fire at the entrance. She started talking to her friends from there only. One of her friends was on the fourth floor, and the other was on the 8th floor. Ananya notices that the angles of elevation of the 4th and the 8th floor are 30° and 60°, respectively. Find the distance (in metres) between the fourth and the other floor.

(A) $20\sqrt{3}$

(B) $30\sqrt{3}$

(C) $40\sqrt{3}$

(D) $50\sqrt{3}$

14. $\cos\dfrac{\pi}{12}+\cos\dfrac{2\pi}{12}+\cos\dfrac{3\pi}{12}\ .....\cos\dfrac{10\pi}{12}+\cos\dfrac{11\pi}{12} = ?$

15. What is the maximum value of the expression $\sin^2 A + 12\sin A\cos A + 6\cos^2 A$ is?

16. A tall tree has its base at point 'P'. Three points, R, S, and T, are located at 8, 16, and 32 m, respectively, from 'P'. The elevation angle of the top of the tree from 'R' and 'T' are complementary angles. Find the angle of elevation (in degrees) of the tree's top from 'S'.

(A) 75°

(B) 30°

(C) 60°

(D) 45°

17. $\dfrac{\tan^2 37\frac{1}{2} - \tan^2 7\frac{1^\circ}{2}}{1-\tan^2 37\frac{1}{2}\tan^2 7\frac{1^\circ}{2}} =$ ________.

(A) 1

(B) $\sqrt{3}$

(C) $\dfrac{1}{\sqrt{3}}$

(D) $\sqrt{2}$

18. Find the value of sin10° sin30° sin50° sin70°, given that sin3*A* = 4sin*A* . sin(60° + *A*). sin(60° – *A*).

(A) $\frac{\sqrt{3}}{16}$

(B) $\frac{1}{8}$

(C) $\frac{1}{16}$

(D) $\frac{\sqrt{3}}{8}$

19. $\cos^{201} x + \sin^{201} x = 1; -\pi < x < \pi.$ How many values of x in the specified range satisfy the above equation?

20. If $x = \frac{\cos^2\theta}{1+\cot^2\theta + \sin^2\theta + \cos^2\theta\cot^2\theta}$, the maximum value of x is ________.

(A) $\frac{1}{8}$

(B) $\frac{1}{2}$

(C) $\frac{1}{6}$

(D) $\frac{1}{4}$

Level of Difficulty – 3

21. If $\frac{\sin\theta}{1+\cos\theta+\sin\theta} - k$, then $\frac{1-\cos\theta+\sin\theta}{1+\sin\theta}$ = is:

(A) k

(B) $2k$

(C) $\frac{1}{k}$

(D) $\frac{1}{2k}$

22. If sin*A* + sin²*A* + sin³*A* – 1 = 0 and cos⁶*A* + *a* cos⁴*A* + *b* cos²*A* – 4 = 0. Then *(a – b)/(a + b)* is equal to

(A) –3

(B) –2

(C) 1

(D) 3

23. In a triangle ABC, 4cos*A* + 5sin*B* = 6 and 5cos*B* + 4sin*A* = 5, find angles ACB which is an acute angle.

(A) 30°

(B) 45°

(C) 60°

(D) 75°

24. The angle of elevation of a tower from a point 20 m above a river is 30° and the angle of depression of the reflection of the tower in the river is 60°. Find the height of the tower.

(A) 60 m

(B) 30 m

(C) 40 m

(D) 50 m

25. Aisha is standing somewhere between the two towers A and B of her housing society. The angle of elevation of the top of towers A and B from her position is 30° and 60°, respectively. Then, she moved towards tower A and now the angle of elevation of the top of towers A and B from her new position changed to 45° each. Find the ratio of the height of tower A to the height of tower B.

(A) $\frac{\sqrt{3}-1}{\sqrt{3}}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\left(\sqrt{3}-1\right)$

(D) 1

26. A ladder of length 7.6 m is standing against a wall, and the difference between the wall and the foot of the ladder is 6.4 m. If the top of the ladder now slips by 1.2 m, then the foot of the ladder shifts by approximately:

(A) 0.4 m

(B) 0.6 m

(C) 0.8 m

(D) 1.2 m

27. The topmost point of a perfectly vertical tower is marked as ‘P’. The tower stands on level ground at point ‘S’. The points ‘Q’ and ‘R’ are somewhere between ‘P’ and ‘S’ on the tower. From a point ‘T’, located on the ground at a certain distance from the base of the tower, the points P, Q, and R are at angles of 60°, 45°, and 30°, respectively. Find the ratio of the (PQ + QR): (QR + ST).

(A) $5 : (\sqrt{3} + 1)$

(B) $2 : (\sqrt{3} + 1)$

(C) $2 : (2\sqrt{3} - 1)$

(D) $10 : (\sqrt{5} + 1)$

28. The angle of elevation of a balloon from the top of an 18-m high building situated on the bank of a river is found to be 30°. If the reflection of the balloon in the river is observed at an angle of depression of 60°, find the balloon’s height from the ground level.

(A) 18 m
(B) 27 m
(C) 36 m
(D) 45 m

29. A building has two windows that need repair. A ladder that is 40 m long is placed against a wall such that it just reaches the first window, which is 25 m high. The angle of elevation made by the first window from the foot of the ladder is 60°. The foot of the ladder is at point ‘D’ when the first window is repaired, and the foot of the ladder is pulled back up to the point ‘E’ so that the ladder can reach up to the second window (which is below the first window). The angle of elevation made by the second window is 30°. The approximate distance between the two points ‘D’ and E’ is:

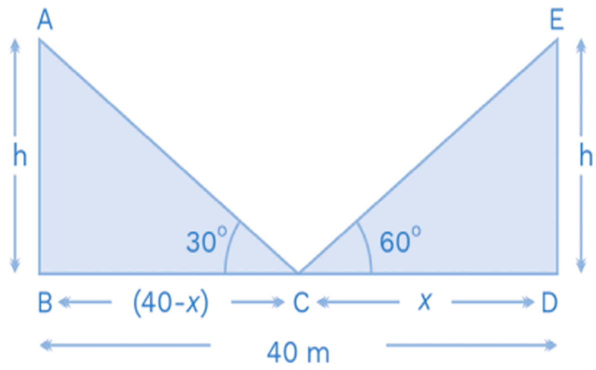
(A) 22 m
(B) 20 m
(C) 18 m
(D) 16 m

30. The value of $(\sec 35° \sec 70° + \tan 35° \tan 70°)^2 - (\sec 35° \tan 70° + \tan 35° \sec 70°)^2$ is:

Solutions

1. **(A)**

Let the height of the towers be '*h*' m.



In ΔABC:

$$\tan 30^\circ = \frac{h}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{40 - x}$$

$$40 - x = h\sqrt{3}$$

$$x = (40 - h\sqrt{3}) \text{ m} \qquad \text{....(i)}$$

Again, in ΔCDE:

$$\tan 60^\circ = \frac{DE}{CD}$$

$$\sqrt{3} = \frac{h}{X}$$

$$x = \frac{h}{\sqrt{3}} \qquad \text{....(ii)}$$

Put the value of $x = \frac{h}{\sqrt{3}}$ from equation (ii) in (i).

$$x = (40 - h\sqrt{3}) \text{ m}$$

$$\frac{h}{\sqrt{3}} = 40 - h\sqrt{3}$$

$$h = 40\sqrt{3} - 3h$$

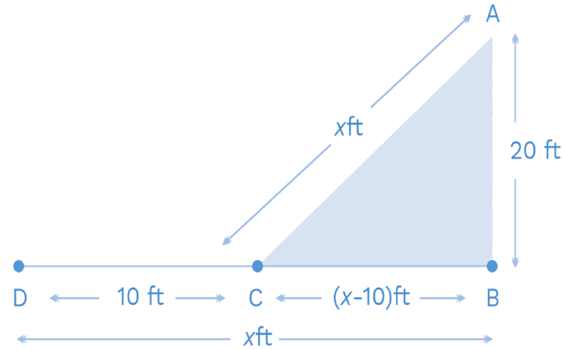
$$4h = 40\sqrt{3}$$

$$h = 10\sqrt{3} \text{ m}$$

Or h = 10 × 1.7320 = 17.3205 m, which is approximately equal to 17 m.

Hence, option (B) is the correct answer.

2. **(A)**



Let the height or length of the tree = x ft. When the tree slides back by 10 ft then it lies flat on the ground with its top touching the wall. Then BD will become the length of the tree.

Therefore, BC = (x – 10) ft

Now, in right-angled ΔABC:

$$x^2 = 20^2 + (x - 10)^2$$

$$x^2 = 400 + x^2 + 100 - 20x$$

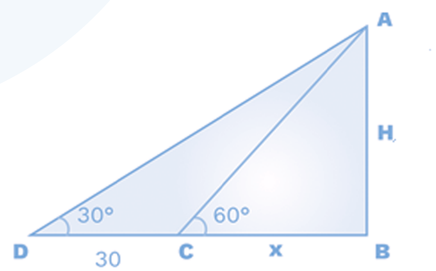
$20x$ = 500

x = 25 ft

Therefore, the height or length of the tree is 25 ft.

Hence, option (A) is the correct answer.

3. **(A)**



In triangle ABC

H/x = tan 60° or $H = x\sqrt{3}$ ...(i)

In triangle ABD

$H/(30 + x)$ = tan 30°

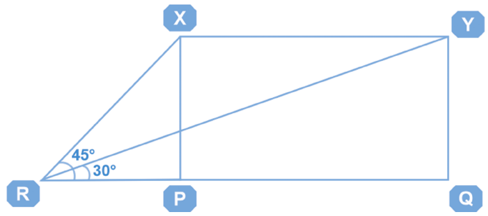
or $H = (30 + x) / \sqrt{3}$ ...(ii)

On solving equations (i) and (ii), we will get x = 15.

Putting the value of x = 15 in equation (i), we will get $H = 15\sqrt{3}$.

4. (C)

Let the aeroplane be at point X which is exactly 20 km above point P on the ground and then after 5 seconds let it be at point Y which is exactly 20 km above point Q on the ground, i.e., XP = YQ = 20 km.



Now, X will be at an angle of 45°, and Y will be at an angle of 30°. Let the speed of the airplane be '*a*' m/s.

Now, since angle XRP = 45°, therefore, XP = RP = 20 km = 20,000 m

Now, the distance moved by aeroplane in 5 seconds will be 5*a* m,

i.e., XY = PQ = 5*a*

Therefore, in triangle YRQ, angle YRQ = 30° and angle YQR = 90°.

tan 30° = YQ/RQ

i.e., $\frac{1}{\sqrt{3}} = \frac{20,000}{20,000 + 5a}$

$20,000 + 5a = 20,000\sqrt{3}$

$5a = 20,000(\sqrt{3} - 1)$

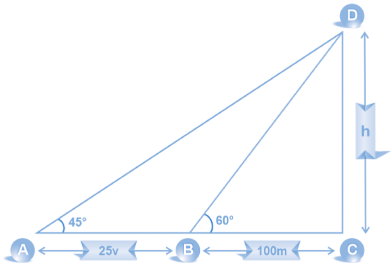
$a = 4,000(\sqrt{3} - 1)$

Thus, the speed of the aeroplane is $4,000(\sqrt{3} - 1)$ m/s.

Hence, option (C) is the correct answer.

5. (D)

Let '*h*' m be the height of the security tower and *v* m/s be the speed of the thief. The arrangement of the problem is shown in the following figure.



Distances AB = 25*v* m.

In triangle BCD,

tan60° = *h*/100

$\sqrt{3} = h/100$

$h = 100\sqrt{3}$ ...(i)

In triangle ACD,

tan45° = *h*/(25*v* + 100)

1 = *h*/(25*v* + 100)

25*v* + 100 = *h*

$25v + 100 = 100\sqrt{3}$... [by using equation (i)]

$v = 100(\sqrt{3} - 1)/25$

$v = 4(\sqrt{3} - 1)$

Therefore, the thief is running away at speed $4(\sqrt{3} - 1)$m/s.

Hence, option (D) is the correct answer.

6. (C)

tan78° = tan(90 – 12)° = cot12°= 1/tan12°

tan66° = tan(90 – 24)° = cot24°= 1/tan24°

tan54° = tan(90 – 36)° = cot36°= 1/tan36°

putting the values of tan78°, tan66°, and tan54° in the expression given in the question we will get

tan12° × tan24° × tan36° × tan60° × tan54° × tan66° × tan78° = tan60° = $\sqrt{3}$

Hence, option (C) is the correct answer.

7. (C)

$120^\circ = 50^\circ + 70^\circ \Rightarrow \tan 120^\circ = \tan(50^\circ + 70^\circ)$

$\Rightarrow -\sqrt{3} = \frac{\tan 50^\circ + \tan 70^\circ}{1 - \tan 50^\circ \tan 70^\circ}$

$\Rightarrow -\sqrt{3} + \sqrt{3}\tan 50^\circ \tan 70^\circ = \tan 50^\circ + \tan 70^\circ$

$\Rightarrow \tan 50^\circ + \tan 70^\circ - \sqrt{3}\tan 50^\circ \tan 70^\circ = -\sqrt{3}$

8. (B)

$\cos^2\theta + \sin^4\theta = 1 - \sin^2\theta + \sin^4\theta$

$= 1 - \sin^2\theta(1 - \sin^2\theta) = 1 - \sin^2\theta\cos^2\theta$

$= 1 - \frac{1}{4}4\sin^2\theta\cos^2\theta = 1 - \frac{1}{4}(\sin^2 2\theta)$

We know that $0 \le \sin^2 2\theta \le 1$

$$\therefore -\frac{1}{4} \le -\frac{1}{4}\sin^2 2\theta \le 0$$

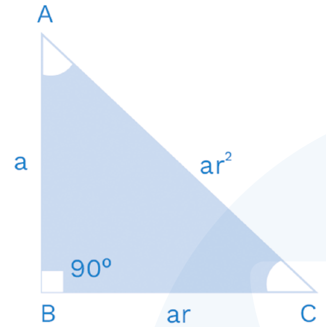
$$1-\frac{1}{4} \le 1-\frac{1}{4}\sin^2 2\theta \le 1-0$$

$$\frac{3}{4} \le \cos^2\theta + \sin^4\theta \le 1$$

$\therefore$ Range is $\left[\frac{3}{4}, 1\right]$

9. (D)

Let the sides of a right-angled triangle be a, ar, ar^2.



Since it is a right-angled triangle, use the Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$(ar^2)^2 = a^2 + (ar)^2$

$a^2r^4 = a^2 + a^2r^2$

$r^4 = 1 + r^2$

$r^4 - r^2 - 1 = 0$

$$r^2 = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\times 1\times(-1)}}{2\times 1}$$

$$r^2 = \frac{1\pm\sqrt{1+4}}{2}$$

$$r^2 = \frac{1\pm\sqrt{5}}{2}$$

Since $r^2 > 0$

$$r^2 = \frac{1\pm\sqrt{5}}{2}$$

Now, we have to find $\tan A$ and $\tan C$.

$\therefore \quad \tan A = \frac{BC}{AB} = \frac{ar}{a} = r$

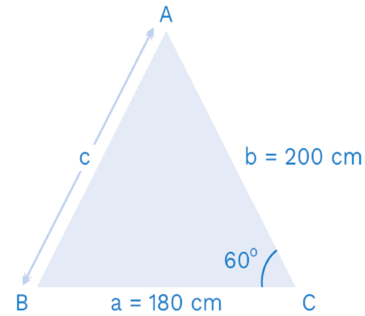
Moreover, $\tan C = \frac{AB}{BC} = \frac{a}{ar} = \frac{1}{r}$

Therefore, $\frac{\tan A}{\tan C} = \frac{r}{1/r} = r^2$

And $r^2 = \frac{1+\sqrt{5}}{2}$ (already we have found).

Hence, option (D) is the correct answer.

10. (A)



Let the third side be ‘C’. By using the co-sine rule,

We can find the third side.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos 60^\circ = \frac{180^2 + 200^2 - c^2}{2\times 180\times 200}$$

$$\frac{1}{2} = \frac{180^2 + 200^2 - c^2}{2\times 180\times 200}$$

$$\frac{1}{2} = \frac{72,400 - c^2}{72,000}$$

$36,000 = 72,400 - c^2$

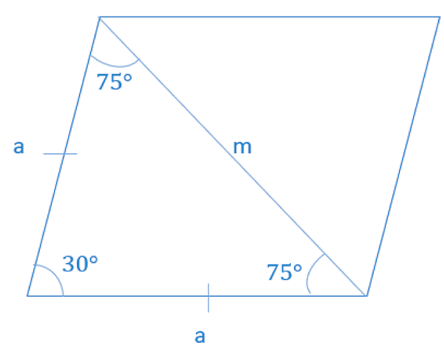
$c^2 = 36,400$

$c = \sqrt{36,400}$

$c \approx 190.78$ cm

Hence, third side of the triangle will be 190.78 cm (approx.).

11. 13

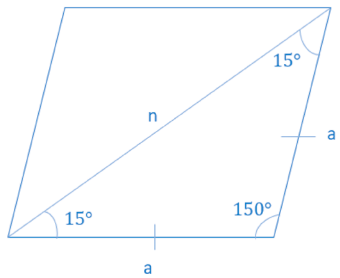


Using the sine rule,

$$\frac{a}{\sin 75^\circ} = \frac{m}{\sin 30^\circ}$$

$$\Rightarrow \frac{a}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{m}{\frac{1}{2}}$$

$$\Rightarrow m = \frac{\sqrt{2}a}{\sqrt{3}+1} \quad \text{...(i)}$$



Using the sine rule,

$$\frac{a}{\sin 15°} = \frac{n}{\sin 150°}$$

$$\Rightarrow \frac{a}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{n}{\frac{1}{2}}$$

$$\Rightarrow n = \frac{\sqrt{2}a}{\sqrt{3}-1} \quad \text{...(ii)}$$

From equations (i) and (ii) we get:

$$\frac{n}{m} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{\left(\sqrt{3}+1\right)^2}{2} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

Thus, $a+\sqrt{b} = 2+\sqrt{3} \quad \Rightarrow \quad a = 2$ and $b = 3$.

Therefore, $(2a+3b) = (2\times 2+3\times 3) = 13$.

12. (D)

We have:

$\sin A + \cos A = 7x$ ...(i)

Squaring equation (i) on both sides, we get:

$$\sin^2 A + \cos^2 A + 2\sin A\cos A = 49x^2$$

$$\Rightarrow 1 + 2\sin A\cos A = 49x^2$$

$$\Rightarrow 1+\sin 2A = 49x^2$$

$$\Rightarrow \sin 2A = 49x^2 - 1 \quad \text{...(ii)}$$

Now, $\sin^3 A + \cos^3 A = \left(\sin A + \cos A\right)^3$

$$-3\sin A\cos A\left(\sin A + \cos A\right)$$

$$= \left(\sin A + \cos A\right)^3 - \frac{3}{2}\sin 2A\left(\sin A + \cos A\right)$$

Using equations (i) and (ii), we get:

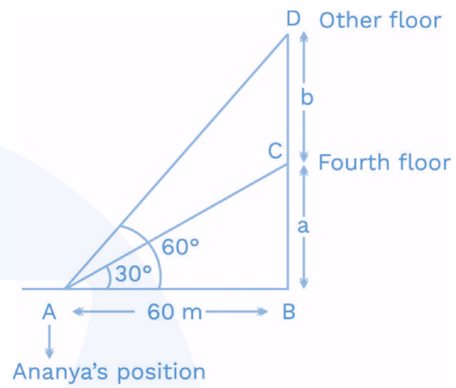
$$\sin^3 A + \cos^3 A = \left(7x\right)^3 - \frac{3}{2}\left(49x^2-1\right)\left(7x\right)$$

$$\Rightarrow \sin^3 A + \cos^3 A = 343x^3 - \frac{21x}{2}\left(49x^2-1\right)$$

$$\Rightarrow \sin^3 A + \cos^3 A = \frac{686x^3 - 1{,}029x^3 + 21x}{2}$$

$$\Rightarrow \sin^3 A + \cos^3 A = \frac{21x - 343x^3}{2}$$

13. (C)



AB = Distance between Ananya and building = 60 m

BC = a = Distance between fourth floor and ground

CD = b = Distance between eightth floor and 4th floor.

$\angle BAC = 30°$

$\angle BAD = 60°$

In ΔBAC:

$$\frac{BC}{AB} = \tan 30°$$

$$\Rightarrow \frac{a}{60} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow a = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow a = \frac{60\sqrt{3}}{3} = 20\sqrt{3} \quad \text{...(i)}$$

In ΔBAD:

$$\frac{BD}{AB} = \tan 60°$$

$$\Rightarrow \frac{a+b}{60} = \sqrt{3}$$

$\Rightarrow a + b = 60\sqrt{3}$

$\Rightarrow b = 60\sqrt{3} - 20\sqrt{3}$ .... [from equation (i)]

$\Rightarrow$ Distance between fourth floor and eighth floor = $40\sqrt{3}$.

14. 0

$$\cos\frac{\pi}{12} + \cos\frac{2\pi}{12} + \cos\frac{3\pi}{12} \ldots\ldots \cos\frac{10\pi}{12} + \cos\frac{11\pi}{12}$$

cos15° + cos30° + cos45° .... cos150° + cos165°

Now, cos 165° = cos (180 – 15)° = –cos 15°

cos 150° = cos (180 – 30°) = – cos 30°

Similarly, cos 135° = – cos 45°

cos 120° = – cos60°

cos 105° = – cos75°

Cos 15° + cos 30° + cos 45° + cos 60° + cos 75° + cos 90° – cos 75° – cos 60° – cos 45° – cos 30° – cos 15°

$\Rightarrow$ cos 90° = 0

15. 10

$$\sin^2 A + 12\sin A\cos A + 6\cos^2 A$$

$$= \frac{1-\cos 2A}{2} + \frac{12}{2}\cdot 2\sin A\cos A + 6\left(\frac{1+\cos 2A}{2}\right)$$

$$= \frac{1}{2}(1 - \cos 2A + 12\sin 2A + 6 + 6\cos 2A)$$

$$= \frac{1}{2}(7 + 12\sin 2A + 5\cos 2A).$$

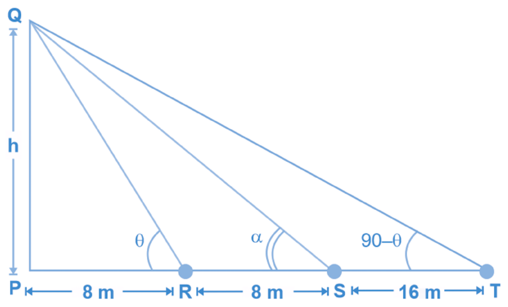
The maximum value of

$$12\sin 2A + 5\cos 2A = +\sqrt{12^2 + 5^2} = 13.$$

Maximum value of $\frac{1}{2}$

$$(7 + 12\sin 2A + 5\cos 2A) = \frac{1}{2}(7 + 13) = 10.$$

16. (D)



Let the height of the tree is 'h' m

In right-angled triangle PQR:

$$\tan\theta = \frac{PQ}{PR}$$

$$\tan\theta = \frac{h}{8} \quad \ldots(i)$$

Again in triangle QPT:

$$\tan(90 - \theta) = \frac{h}{32}$$

$$\cot\theta = \frac{h}{32} \quad \ldots(ii)$$

From equations (i) and (ii) we will get:

$$\tan\theta \times \cot\theta = \frac{h}{8} \times \frac{h}{32}$$

$$h^2 = 256$$

$$h = 16 \text{ cm}$$

Since we have to find the angle 'α'

In triangle QPR:

$$\tan\alpha = \frac{h}{8+8}$$

$$\tan\alpha = \frac{16}{16}$$

$\tan\alpha = 1$

$\tan\alpha = \tan 45°$

Thus, $\alpha = 45°$

Hence, option (D) is the correct answer.

17. (C)

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

$$\tan(A+B).\tan(A-B) = \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A\tan^2 B}$$

Put $A = 37\frac{1°}{2}; B = 7\frac{1°}{2}$

$$= \frac{\tan^2 37\frac{1°}{2} - \tan^2 7\frac{1°}{2}}{1 - \tan^2 37\frac{1°}{2}\tan^2 7\frac{1°}{2}}$$

$$= \tan\left(37\frac{1}{2} + 7\frac{1}{2}\right)\tan\left(37\frac{1}{2} - 7\frac{1}{2}\right)$$

$$= \tan 45° \tan 30° = 1.\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}}$$

18. (C)

$\sin 3A = 4 \sin A \sin(60° + A) \sin(60° - A)$

Put $A = 10°$

$\Rightarrow \sin 3(10°) = 4 \sin 10° \sin 70° \sin 50°$

$$\Rightarrow \frac{\sin 30°}{4} = \sin 10° \sin 50° \sin 70°$$

$$\Rightarrow \frac{1}{8} = \sin 10° \sin 50° 70°$$

$$\therefore \sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{8}\left(\frac{1}{2}\right) = \frac{1}{16}.$$

19. 2

Given equation is $\cos^{201} x + \sin^{201} x = 1$

The above equation is true only when $\cos x = 1$ or $\sin x = 1$, i.e., when $x = 0$ or $\pi/2$.

($\because$ 201 is odd and $-\pi < x < \pi$)

$\therefore$ The number of solutions is 2.

20. (C)

$$x = \frac{\cos^2\theta}{1+\cot^2\theta+\sin^2\theta+\cos^2\theta\cot^2\theta}$$

$$= \frac{\cos^2\theta\sin^2\theta}{\sin^2\theta+\cos^2\theta+\sin^4\theta+\cos^4\theta}$$

$$= \frac{\cos^2\theta\sin^2\theta}{1+\sin^4\theta+\cos^4\theta}$$

$$= \frac{\cos^2\theta\sin^2\theta}{1+(\sin^2\theta+\cos^2\theta)(-2\sin^2\theta\cos^2\theta)}$$

$$= \frac{\sin^2\theta\cos^2\theta}{2-2\sin^4\theta\cos^2\theta}$$

$$= \frac{4\sin^2\theta\cos^2\theta}{8-2(4\sin^2\theta\cos^2\theta)} = \frac{\sin^2 2\theta}{8-2\sin^2 2\theta}$$

when $\theta = \frac{\pi}{4}\left(2\theta = \frac{\pi}{\nu}\right)$, x has its maximum value which is $\frac{1}{6}$.

21. (B)

$$k = \frac{\sin\theta}{1+\cos\theta+\sin\theta} \times \frac{(1-\cos\theta+\sin\theta)}{(1-\cos\theta+\sin\theta)}$$

$$k = \frac{\sin\theta(1-\cos\theta+\sin\theta)}{(1+\sin\theta)^2-\cos^2\theta}$$

$$k = \frac{\sin\theta(1-\cos\theta+\sin\theta)}{1+2\sin\theta+\sin^2\theta-\cos^2\theta}$$

$$k = \frac{\sin\theta(1-\cos\theta+\sin\theta)}{2\sin\theta+\sin^2\theta+\left(1-\cos^2\theta\right)}$$

$$k = \frac{\sin\theta(1-\cos\theta+\sin\theta)}{2\sin\theta+2\sin^2\theta}$$

$$k = \frac{\sin\theta(1-\cos\theta+\sin\theta)}{2\sin\theta(1+\sin\theta)}$$

$$\frac{1-\cos\theta+\sin\theta}{1+\sin\theta} = 2k$$

22. (A)

$\sin A + \sin^2 A + \sin^3 A - 1 = 0$ and $\cos^6 A + a \cos^4 A + b \cos^2 A - 4 = 0$

$\sin A + \sin^3 A = 1 - \sin^2 A$

$\Rightarrow \sin A\,(1 + \sin^2 A) = \cos^2 A$

$\Rightarrow \sin^2 A\,\{1 + (1 - \cos^2 A)\}^2 = \cos^4 A$

$\Rightarrow (1 - \cos^2 A)(4 + \cos^4 A - 4\cos^2 A) = \cos^4 A$

$\Rightarrow 4 + \cos^4 A - 4\cos^2 A - 4\cos^2 A - \cos^6 A + 4\cos^4 A = \cos^4 A$

$\Rightarrow \cos^6 A - 4\cos^4 A + 8\cos^2 A = 4$

$\Rightarrow \cos^6 A - 4\cos^4 A + 8\cos^2 A - 4 = 0$

Comparing with $\cos^6 A + a\cos^4 A + b\cos^2 A - 4 = 0$

$\Rightarrow a = -4, b = 8$

Now, $(a - b)/(a + b) = (-4 - 8)/(-4 + 8) = -12/4 = -3$

Hence, $(a-b)/(a + b) = -3$.

23. (C)

$4\cos A + 5\sin B = 6 \xrightarrow{\text{Square}} 16\cos^2 A + 25\sin^2 B + 40 \cos A \sin B = 36$ ...(i)

$5\cos B + 4\sin A = 5 \xrightarrow{\text{Square}} 25\cos^2 B + 16\sin^2 A + 40 \cos B \sin A = 25$ ...(ii)

Add equations (i) and (ii)

$16(\cos^2 A + \sin^2 A) + 25(\sin^2 B + \cos^2 B) + 40(\sin A \cos B + \cos A \sin B) = 61$

$16 + 25 + 40 \sin(A + B) = 61$

$40 \sin(A + B) = 20$

$$\sin(A + B) = \frac{1}{2}$$

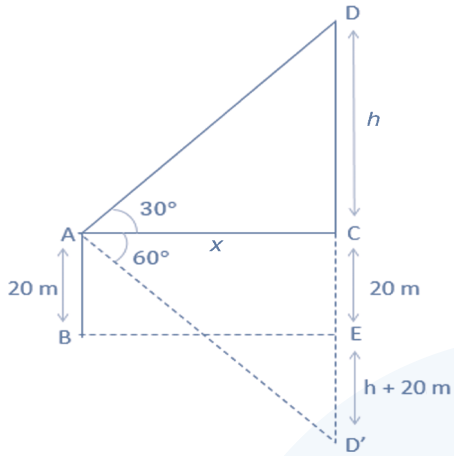
Given that ΔACB = acute, so $(A + B)$ would be obtuse

$\Rightarrow \sin(A + B) = \frac{1}{2} = \sin(120°)$ which means $A + B = 120°$

$\Rightarrow C = \angle ACB = 60°$

24. (C)

Let h be the height of the tower from above the point of observation at which the angle of elevation is 30°. The angle of depression of the shadow of the tower is 60°.



In ΔACD,

$\tan 30° = h/x$

$\Rightarrow 1/\sqrt{3} = h/x$

$\Rightarrow x = h\sqrt{3}$ ...(i)

In $\Delta ACD'$,

$\tan 60° = (20 + h + 20)/x$

$\Rightarrow \sqrt{3} = (40 + h)/x$

$\Rightarrow x = (40 + h)/\sqrt{3}$ ...(ii)

Using (i)

$\sqrt{3}h = (40 + h)/\sqrt{3}$

$\Rightarrow \sqrt{3} . \sqrt{3}h = 40 + h$

$\Rightarrow 3h - h = 40$

$\Rightarrow h = 20$ m

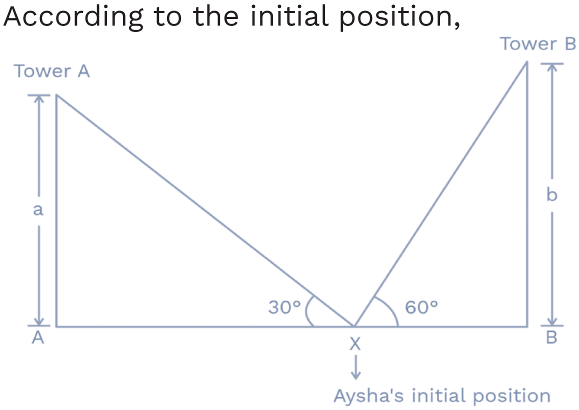
Height of tower = $h + 20 = 20 + 20 = 40$ m.

Hence, the height of the tower is 40 m.

25. (B)

Let's assume the height of tower A = a and the height of tower B = b

According to the initial position,



$$\frac{a}{AX} = \tan 30° = \frac{1}{\sqrt{3}}$$

$\Rightarrow AX = \sqrt{3}a$ ...(i)

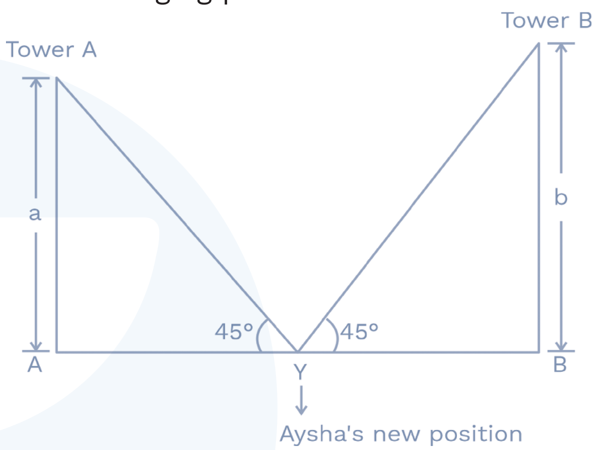
Moreover, $\dfrac{b}{BX} = \tan 60° = \sqrt{3}$

$BX = \dfrac{b}{\sqrt{3}}$ ...(ii)

Distance between towers = AX + BX

$= \sqrt{3}a + \dfrac{b}{\sqrt{3}}$ ...(iii)

After changing positions:



$$\frac{a}{AY} = \tan 45° = 1$$

$\Rightarrow AY = a$

Also $\dfrac{b}{BY} = \tan 45° = 1$

$\Rightarrow BY = b$

Distance between tower = AY + BY = $a + b$ ...(iv)

Distance between towers will not change.

Therefore, from equations (iii) and (iv):

$$\Rightarrow \sqrt{3}a + \frac{b}{\sqrt{3}} = a + b \Rightarrow \left(\sqrt{3} - 1\right)a = b - \frac{b}{\sqrt{3}}$$

$$\Rightarrow \left(\sqrt{3} - 1\right)a = \frac{\left(\sqrt{3} - 1\right)}{\sqrt{3}}b$$

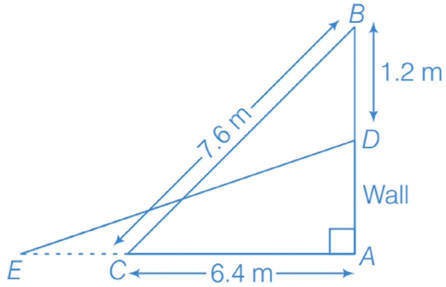
$$\Rightarrow \frac{a}{b} = \frac{1}{\sqrt{3}}$$

Hence, option (B) is the correct answer.

26. (B)

In right-angled ΔABC,

$BC^2 = AB^2 + CA^2$ (using Pythagoras theorem)



$$7.6^2 = (AD + BD)^2 + 6.4^2$$

$$7.6^2 = (AD + 1.2)^2 + 6.4^2$$

$$57.76 = (AD + 1.2)^2 + 40.96$$

$$(AD + 1.2)^2 = 57.76 - 40.96 = 16.8$$

AD + 1.2 = 4.1

AD = 2.9 m

Now, in right ΔDAE,

$$DE^2 = AD^2 + EA^2$$

$$7.6^2 = 2.9^2 + (6.4 + EC)^2$$

$$57.76 = 8.41 + (6.4 + EC)^2$$

$$49.35 = (6.4 + EC)^2$$

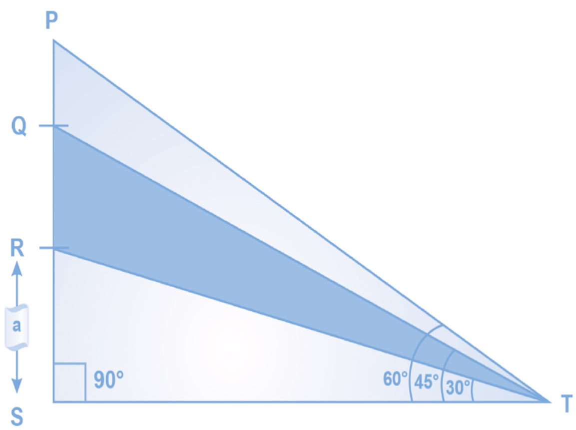
7 = 6.4 + EC

EC = 0.6 m.

Hence, option (B) is the correct answer.

27. (C)

Let the 'PS' be the tower and the length of the 'RS' be 'a'.



In right-angled D RST:

$$\tan 30° = \frac{RS}{ST}$$

$$\frac{1}{\sqrt{3}} = \frac{a}{ST}$$

$$ST = a\sqrt{3}$$

or $ST = \sqrt{3}\ a$ ...(i)

Again, in right-angled D QST:

$$\tan 45° = \frac{QS}{ST}$$

$$ST = QS = \sqrt{3}\,a$$

$\therefore$ QR = QS − RS

$$QR = \sqrt{3}\,a - a = (\sqrt{3} - 1)\,a$$

Similarly, in right-angled D PST:

$$\tan 60° = \frac{PS}{ST}$$

$$\sqrt{3} = \frac{PS}{\sqrt{3}\,a}$$

$$PS = 3a$$

$\therefore$ The length of PQ = 'PS' − 'QS' = $3a - \sqrt{3}\,a = (3 - \sqrt{3})\,a$

Now, we have to find the ratios of

$$(PQ + QR) : (QR + ST)$$

$$(3a - \sqrt{3}a + \sqrt{3}\,a - a) : (\sqrt{3}a - a + \sqrt{3}a)$$

$$2a : (2\sqrt{3}\,a - a)$$

$$2 : (2\sqrt{3} - 1)$$

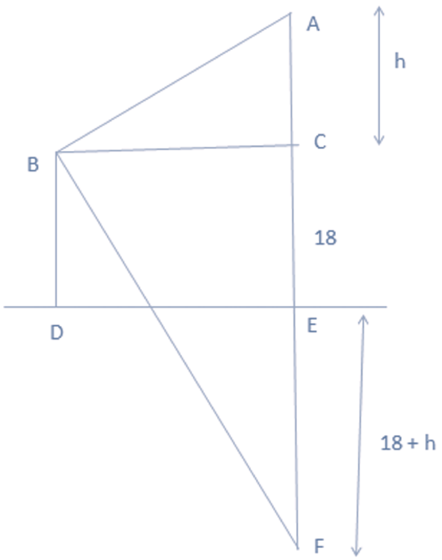
Hence, option (C) is the correct answer.

28. (C)

Let A be the balloon and F be its reflection.

Therefore, BD = CE is the height of the building = BD = (CE) = 18 m

Let AC = h m.



In ΔABC:

$$\frac{h}{BC} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow BC = \sqrt{3}h \text{ m} \qquad \text{...(i)}$$

In ΔBCF:

$$\frac{CF}{BC} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow BC = \frac{CF}{\sqrt{3}} = \frac{18 + 18 + h}{\sqrt{3}} \text{ m} \qquad \text{...(ii)}$$

From equations (i) and (ii), we get:

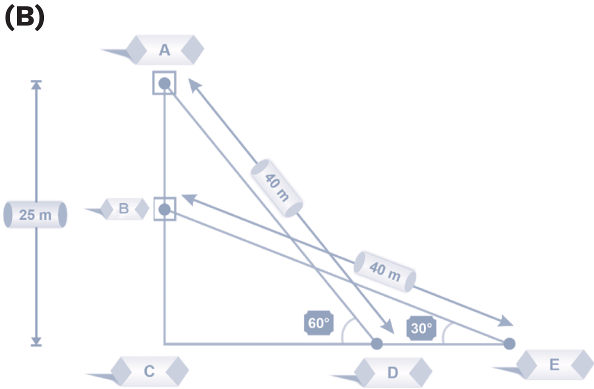
$$\sqrt{3}h = \frac{18 + 18 + h}{\sqrt{3}}$$

$$\Rightarrow 3h - h = 36$$

$$\Rightarrow h = 18 \text{ m}$$

Therefore, the height of the balloon from the ground = 18 + h = 36 m.

29. (B)



Let the first window be at 'A' and second window be at 'B'.

In ΔACD:

$$\tan 60^\circ = \frac{AC}{CD}$$

$$\sqrt{3} = \frac{25}{CD}$$

$$CD = \frac{25}{\sqrt{3}} \, m$$

Again in ΔBCE:

$$\cos 30^\circ = \frac{CE}{BE}$$

$$\frac{\sqrt{3}}{2} = \frac{CE}{40}$$

$$CE = 20\sqrt{3} \, m$$

Thus, the distance between the two points 'D' and 'E'

$$= CE - CD = 20\sqrt{3} - \frac{25}{\sqrt{3}} =$$

$$\frac{60 - 25}{\sqrt{3}} \text{ m} = \frac{35}{\sqrt{3}} \text{ m} = 20.207 \text{ m}$$

Hence, option (B) is the correct answer.

30. 1

$$\sec 35^\circ \sec 70^\circ + \tan 35^\circ \tan 70^\circ$$

$$= \frac{1}{\cos 35^\circ}\frac{1}{\cos 70^\circ} + \frac{\sin 35^\circ}{\cos 35^\circ}\frac{\sin 70^\circ}{\cos 70^\circ}$$

$$= \frac{1 + \sin 35^\circ \sin 70^\circ}{\cos 35^\circ \cos 70^\circ}\left(\sec 35^\circ \tan 70^\circ + \tan 35^\circ \sec 70^\circ\right)$$

$$= \frac{1}{\cos 35^\circ}\frac{\sin 70^\circ}{\cos 70^\circ} + \frac{\sin 35^\circ}{\cos 35^\circ}\frac{1}{\cos 70^\circ} = \frac{\sin 70^\circ + \sin 35^\circ}{\cos 35^\circ \cos 75^\circ}$$

$$GE = \left(\frac{1 + \sin 35^\circ \sin 70^\circ}{\cos 35^\circ \cos 70^\circ}\right)^2 - \left(\frac{\sin 70^\circ + \sin 35^\circ}{\cos 35^\circ \cos 70^\circ}\right)^2$$

$$= \left(1 + \sin^2 35^\circ \sin^2 70^\circ + 2\sin 35^\circ \sin 70^\circ\right)\frac{-\sin^2 70^\circ - \sin^2 35^\circ - 2\sin 35 \sin 70^\circ}{\cos^2 35^\circ \cos^2 70^\circ}$$

$$= \frac{1 + \sin^2 35^\circ \sin^2 70^\circ - \sin^2 70^\circ - \sin^2 35^\circ}{\cos^2 35^\circ \cos^2 70^\circ}$$

$$= \frac{\left(1\sin^2 70^\circ\right) - \sin^2 35^\circ\left(1 - \sin^2 70^\circ\right)}{\cos^2 35^\circ \cos^2 70^\circ}$$

$$= \frac{\left(1 - \sin^2 70^\circ\right)\left(1 - \sin^2 35^\circ\right)}{\cos^2 35^\circ \cos^2 70^\circ} = 1$$

Mind Map

TRIGONOMETRY MIND MAP

DEFINITION

Trigonometry is the branch of mathematics that deals with the sides of a Δ and angles of a Δ.

TRIGONOMETRIC RATIOS

1. $\sin\theta = \frac{P}{H}$
2. $\cos\theta = \frac{b}{H}$
3. $\tan\theta = \frac{P}{b}$
4. $\text{cosec}\,\theta = \frac{H}{P}$
5. $\sec\theta = \frac{H}{b}$
6. $\cot\theta = \frac{b}{P}$

VALUES OF THE SPECIAL ANGLES

Ratio \ θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined

SIGN OF ALL THE TRIGONOMETRIC

(II) Quadrant sin, cosec → +ve	(I) Quadrant All + ve
tan, cot → +ve (III) Quadrant	cos, sec → +ve (IV) Quadrant

Note : Remember word 'ACTS'

RANGE OF TRIGONOMETRIC RATIOS

(I) $-1 \le \sin\theta \le 1$

(II) $-1 \le \cos\theta \le 1$

(III) $-\infty < \tan\theta < \infty$

TRIGONOMETRIC IDENTITIES

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec\theta$
3. $1 + \cot^2\theta = \text{cosec}^2\theta$

POLYGON

Area of polygon $= \frac{1}{2} \times$ Perimeter of polygon $\times$ Apothem

SOME IMPORTANT FORMULAE

1. $\sin(A+B) = \sin A\cos B + \cos A\sin B$
2. $\sin(A-B) = \sin A\cos B - \cos A\sin B$
3. $\cos(A+B) = \cos A\cos B - \sin A\sin B$
4. $\cos(A-B) = \cos A\cos B + \sin A\sin B$
5. $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$
6. $\sin 2A = 2\sin A\cos A$
7. $\cos 2A$ → $\cos^2 A - \sin^2 A$; $2\cos^2 A - 1$; $1 - 2\sin^2 A$
8. $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

sin Rule -

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

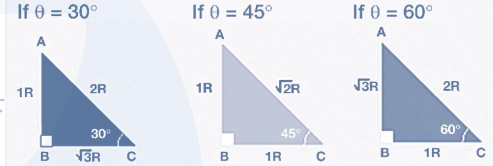
cosine Rule -

$$\cos A = \frac{c^2 + b^2 - a^2}{2bc}$$

HEIGHT AND DISTANCE

SOME IMPORTANT TRIGONOMETRIC RATIOS

If $\theta = 30°$ | If $\theta = 45°$ | If $\theta = 60°$



3 BASIC TERMS FREQUENTLY COME IN HEIGHT AND DISTANCE

(I) Line of sight

(II) Angle of elevation

(III) Angle of depression