3 Time, Speed, and Distance

Introduction

The concept of time, speed, and distance forms the major chunk of arithmetic. Two to three questions are asked on this topic in CAT and other management entrance exams every year.

$$\text{Speed} = \frac{\text{Distance}\,(d)}{\text{Time}\,(t)}$$

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Time} = \frac{\text{Distance}\,(d)}{\text{Time}\,(t)}$$

Units of Speed

1. $\text{Speed} = \dfrac{\text{Distance} \rightarrow [\text{in metres}]}{\text{Time} \rightarrow [\text{in seconds}]}$

If the distance is measured in meters and time is measured in seconds, then the unit of speed will be in m/s.

2. $\text{Speed} = \dfrac{\text{Distance} \rightarrow \text{km}}{\text{Time} \rightarrow \text{hr}}$

If the distance is measured in kilometres and time is measured in hours, then the unit of speed will be km/hr.

1 km = 1,000 m.

Conversion

1. $1 \text{ km/hr} = \dfrac{1 \text{ km}}{1 \text{ hour}} = \dfrac{1{,}000 \text{ m}}{60 \times 60 \text{ sec}} = \dfrac{5}{18} \text{ m/s}$

If the unit of km/hr has to be converted to m/s, we have to multiply it with 5/18.

$$72 \text{ km/hr} \xrightarrow{\times\frac{5}{18}} 72 \times \frac{5}{18} = 20 \text{ m/s}$$

Hence, the speed of car is 20 m/s.

$$1 \text{ m/s} = \frac{\frac{1}{1{,}000}}{\frac{1}{(60 \times 60)}} = \frac{36}{10} = \frac{18}{5} = 3.6 \text{ km/hr}$$

Hence, if we want to convert a speed given in m/s to km/hr, we have to multiply with $\frac{18}{5}$.

2. For example, if a car is moving at 45 m/s, then find its speed in km/hr.

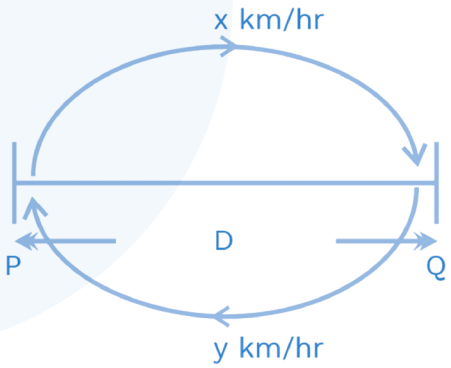
$$45 \text{ km/hr} \xrightarrow{\times\frac{18}{5}} 45 \times \frac{18}{5} = 162 \text{ km/hr}$$

Concept 1: Average Speed

Average speed is the average or mean value of the speed of a body over a period of time. The formula for the average speed becomes necessary because the speed of a moving body is not constant and varies across a period of time.

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

Case 1:



Let us suppose a person is going from point P to Q with the speed of *x* km/hr and returns from point Q to P with the speed of *y* km/hr. Then we have to find the average speed of his whole journey.

We know that

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}.$$

Since total distance from P to Q and Q to P = D + D = 2D.

$$\text{Time taken from P to Q, } (t_1) = \frac{D}{x}$$

Again, time taken from Q to P, $(t_2) = \frac{D}{y}$

Average speed $= \frac{2D}{\left[\frac{D}{x} + \frac{D}{y}\right]} = \frac{2xy}{x+y}$

Note: Average speed is the harmonic mean of the given two speeds whenever a person covers the same distance twice with different speeds.

Harmonic mean $= \frac{2xy}{x+y}$

Case 2:

When three equal distances are covered with three different speeds of x km/hr, y km/hr, and z km/hr, then the average speed of the whole journey $= \frac{3xyz}{xy + yz + zx}$

Example 1:

Find the approximate average speed, when a person travels the first 20 km at 5 km/hr and next 40 km at 10 km/hr and a further 60 km at 30 km/hr, respectively.

(A) 12 km/hr (B) 18 km/hr

(C) 36 km/hr (D) 9 km/hr

Solution: (A)

Average speed $= \frac{\text{Total distance}}{\text{Total time taken}}$

$= \frac{20 + 40 + 60}{\frac{20}{5} + \frac{40}{10} + \frac{60}{30}} = \frac{120}{4 + 4 + 2} = \frac{120}{10}$

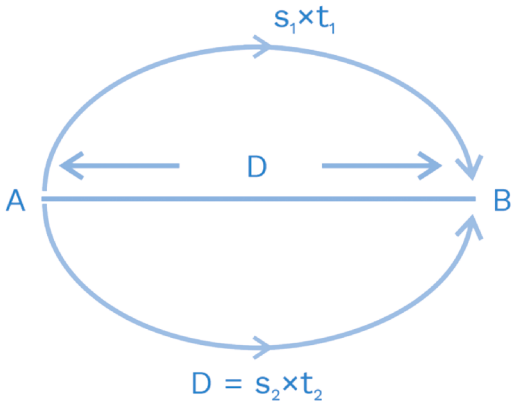
Average speed = 12 km/hr

Concept 2: When Distance Remains Constant

Speed $= \frac{\text{Distance}}{\text{Time}}$

Speed is inversely proportional to the time whenever the distance is constant.

When two persons start moving from city A, towards city B, which is at a distance D km, with different speeds of S_1 and S_2 and time taken by them is t_1 and t_2, respectively, to reach from city A to city B.



Distance covered by the first person $(D) = S_1 \times t_1$ ....(i)

Distance covered by the second person $(D) = S_2 \times t_2$ ....(ii)

Since distance is the same in both cases, therefore, $D = S_1 \times t_1 = S_2 \times t_2$

$\frac{S_1}{S_2} = \frac{t_1}{t_2}$

Speed $\propto \frac{1}{\text{Time}}$

Hence, if distance remains constant, speed becomes inversely proportional to time.

Example 2:

Manish wants to go from Bangalore to Mysore. The distance between the two cities is 192 km. If he decreases his speed by 8 kmph, then he would take 16 hours more than the time required if he had travelled at 150% of his usual speed. What is the time taken (in hours) to travel between the two cities at the usual speed?

Solution: 12

Let the usual speed = s kmph

So, in the first case, he travels at speed $(s - 8)$ kmph, and in the second case he travels at speed 1.5s kmph.

According to the question

$\frac{192}{s-8} - \frac{192}{1.5s} = 16$

$s^2 - 12s - 64 = 0$

or, $s^2 - 16s + 4s - 64 = 0$

or, $s(s - 16) + 4(s - 16) = 0$

or, $(s + 4)(s - 16) = 0$

i.e. $s = 16$ kmph

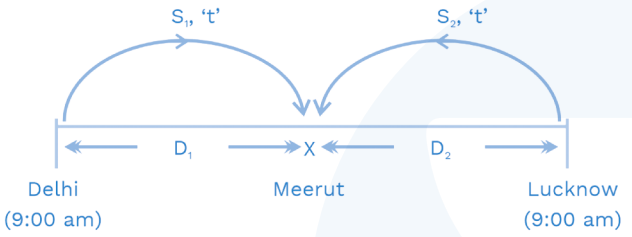
Time taken = 192/16 = 12 hours.

Concept 3: When Time Remains Constant

Distance is directly proportional to speed when time is the same.

For example,

One train starts from Delhi at 9:00 am at speed S_1 towards Lucknow and at the same time another train starts from Lucknow with speed S_2 towards Delhi. Both the trains met after t hours at Meerut.



Distance between Delhi to Meerut $(D_1) = S_1 \times t$ ...(i)

Distance between Lucknow to Meerut $(D_2) = S_2 \times t$ ...(ii)

If we divide equation (i) by (ii), we get

$$\frac{D_1 = S_1 \times t}{D_2 = S_2 \times t}$$

$$\frac{D_1}{D_2} = \frac{S_1}{S_2}$$

Therefore, distance $\propto$ Speed (if time remains constant).

Concept 4: When Speed Remains Constant

$D_1 = S \times t_1$ ...(i)

$D_2 = S \times t_2$ ...(ii)

Dividing equations (i) ÷ (ii), we get

$$\frac{D_1}{D_2} = \frac{S \times t_1}{S \times t_2}$$

$$\frac{D_1}{D_2} = \frac{t_1}{t_2}$$

If speed remains constant, then the distance ratio becomes equal to the time ratio or distance is directly proportional to the time.

Concept 5: Relative Speed

When two bodies are moving simultaneously with speeds S_1 and S_2, the speed of any object or body, when observed from the other object's perspective, is called the relative speed. Here assume $S_1 > S_2$.

- When two objects are moving in the same direction, relative speed = $S_1 - S_2$.
- When two objects are moving in the opposite direction, relative speed = $S_1 + S_2$.

Note: Relative speed is usually considered when one has to find the time taken to meet or catch (between two objects), and it can be found as follows:

$$\text{Time is taken to meet/catch} = \frac{\text{Initial distance separating them}}{\text{Relative speed } (S_1 \pm S_2)}$$

Opposite direction

Relative speed = $(S_1 + S_2)$ km/hr

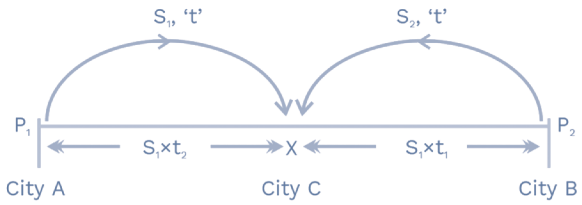
Same direction

Relative speed = $(S_1 - S_2)$ km/hr

Concept 6: Meeting Concept

Let's assume that P_1 and P_2 are two persons who start from city A and city B, respectively, at the same time, with speeds of S_1 km/hr and S_2 km/hr, towards their destination of city B and city A, respectively. After t hours,

they meet each other at city C. After meeting, they take t_1 and t_2 hours, respectively, to reach their destination.



Since we know that time is constant, the distance ratio always equals the speed ratio.

$$\frac{AC}{BC} = \frac{S_1}{S_2}$$

Now, distance AC is travelled by P_2 in t_2 hours with speed S_2 and distance BC is travelled by P_1 in t_1 hours with speed S_1. So, AC = $S_2 \times t_2$ and BC = $S_1 \times t_1$

$$\frac{S_2 \times t_2}{S_1 \times t_1} = \frac{S_1}{S_2}$$

$$\frac{S_1^2}{S_2^2} = \frac{t_2}{t_1}$$

$$\frac{S_1}{S_2} = \sqrt{\frac{t_2}{t_1}}$$

After how much time (t) they meet in the above concept?

Now, distance AC is travelled by P_2 in t_2 with speed S_2, and the same distance is covered by P_1 in t hours with speed S_1

So, distance AC = $S_2 \times t_2 = S_1 \times t$

$$t = \frac{S_2 \times t_2}{S_1}$$

Use, $\frac{S_1}{S_2} = \sqrt{\frac{t_2}{t_1}}$ in the above equation

$$t = \frac{S_2 \times t_2}{S_1} = \sqrt{\frac{t_1}{t_2}} \times t_2$$

$$t = \sqrt{t_1 t_2}$$

By this formula, we can find the meeting time if t_1 and t_2 (time taken by both the person after crossing each other to reach their destination) is given in the question.

Total distance between cities A and B

Total distance = AC + BC

$D = S_2 \times t_2 + S_1 \times t_1$

Example 3:

Two trains, Lucknow Mail and Delhi Express, start at the same time from stations Lucknow and Delhi, respectively, towards each other. After passing each other, they take 8 hours 32 minutes and 14 hours 42 minutes to reach Delhi and Lucknow, respectively. If Lucknow Mail is moving at the speed of 105 km/hr, what is the speed of the Delhi Express?

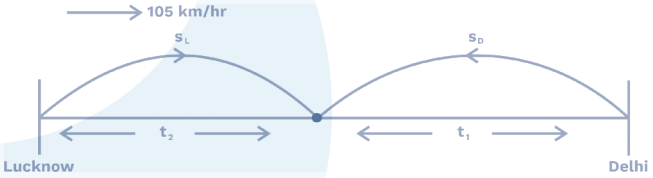
(A) 80 km/hr

(B) 90 km/hr

(C) 45 km/hr

(D) 70 km/hr

Solution: (A)



Since t_1 = 8 hr 32 min = 8 × 60 + 32 = 480 + 32 = 512 minutes

t_2 = 14 hr 42 min = 14 × 60 + 42 = 840 + 42 = 882 minutes

$$\frac{S_L}{S_D} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{105}{S} = \sqrt{\frac{882}{512}}$$

$$\frac{105}{S_D} = \sqrt{\frac{441}{256}}$$

$$\frac{105}{S_D} = \frac{21}{16}$$

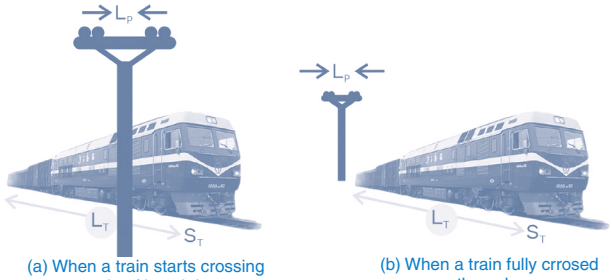
S_D = 80 km/hr

Therefore, the speed of Delhi Express is 80 km/hr.

Concept 7: Trains

Case 1:

When a train passes a pole or a stationary man



(a) When a train starts crossing a pole of length Lp.

(b) When a train fully crrosed the pole.

In this case, the total distance travelled by train = $L_T + L_P$

Since L_P is negligible in comparison to the length of the train.

Hence, $L_P = 0$, which we are considering.

Total distance travelled by train = $D = L_P + L_T$ = $0 + L_T = L_T$

Therefore, in this case, when a train passes a pole or a man, the length of the pole or man is negligible compared to the length of the train.

$\therefore \quad L_P = 0, L_M = 0 \quad (L_M \rightarrow$ Length of the man, $L_P \rightarrow$ Length of the pole).

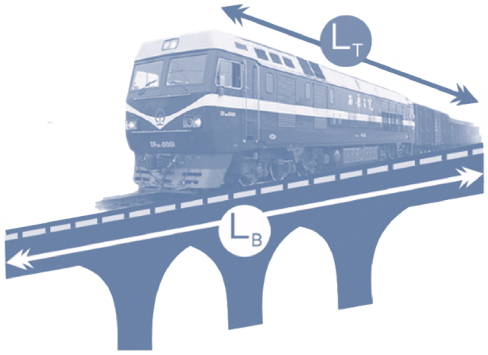
Hence, the train will cover its own length.

Time taken by the train to cross the pole:

$$(t) = \frac{L_T}{S_T}$$

Case 2:

When a train crosses a bridge or a platform or a tunnel

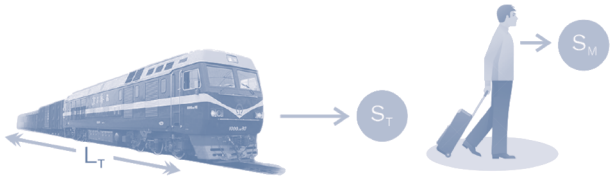


- Total distance covered by the train while crossing a platform or bridge or tunnel = $(D) = L_T$ (length of train) + L_B (length of bridge).
- Time taken by train to cross the bridge/ platform = $\frac{L_T + L_B}{S_T}$

Case 3:

Scenario 1

When a train passes a moving man



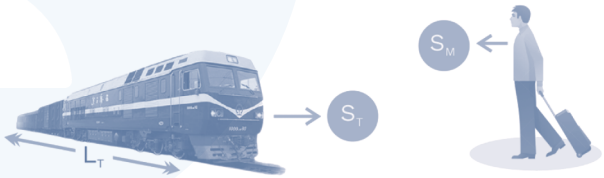
- Total distance travelled by train while crossing a moving man $(D) = L_T + L_M$
- Since $L_M = 0$, then, $D = L_T$

Relative speed = S_T (Speed of train) $-$ S_M (Speed of man)

$\therefore$ Time is taken by train to cross a moving man = $\frac{L_T}{\text{Relative speed}}$

Scenario 2

In the opposite direction



- Total distance travelled by train while crossing a moving man in the opposite direction $(D) = L_T + L_M$. Since, $L_M = 0$, $D = L_T$
- Time taken by train to cross a moving man in the opposite direction $(t) = \frac{L_T}{S_T + S_M}$

$S_T + S_M \rightarrow$ Relative speed

Case 4:

Scenario 1

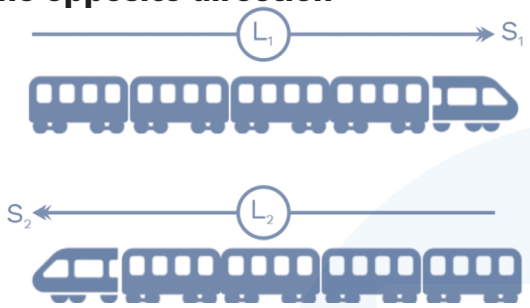
When a train passes another train

If $S_1 > S_2$

- Total distance travelled by the faster train to cross the slower train
  (D) = Length of the faster train + Length of the slower train
  $D = L_1 + L_2$
- Time taken by the faster train to cross the slower train (t) = $\frac{L_1 + L_2}{S_1 - S_2}$

Scenario 2

In the opposite direction



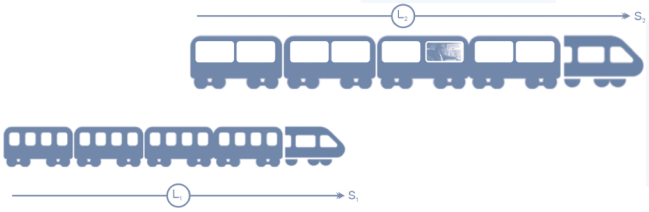
The relative speed of the train when they are moving in opposite directions = $S_1 + S_2$.

Time taken to cross each other (t) = $\frac{L_1 + L_2}{S_1 - S_2}$

Case 5:

Scenario 1

When a train passes a man sitting in another moving train



If $S_1 > S_2$

Relative speed (RS) = $S_1 - S_2$

Total distance travelled by the faster train to cross a man sitting in the slower train

$D = L_1 + L_M \quad (L_M = 0)$

$D = L_1$

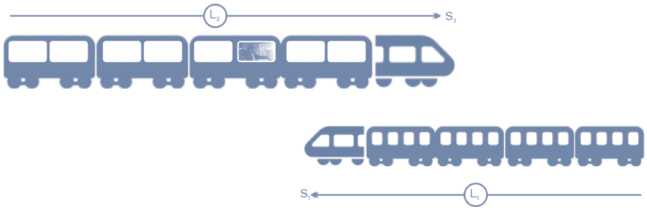
Since the faster train is crossing a man sitting in a slower train, we have to neglect the length or thickness of the man.

Note: In this case, the length of the slower train will not come into consideration.

Time taken (t) = $\frac{L_1}{S_1 - S_2}$

Scenario 2

In the opposite direction



- Total distance covered by the faster train to cross a man sitting in a slower train = $L_1 + L_M$ ($L_M = 0$)
  $D = L_1$
- Time taken to cross the man sitting in slower train by faster train
  (t) = $\frac{L_1}{S_1 + S_2}$
  Since relative speed (RS) = $S_1 + S_2$

Example 4:

A train with a speed of 108 km/hr crosses a 1,000 m long platform in 40 sec. Find the length of the train.

(A) 200 m (B) 400 m

(C) 300 m (D) 250 m

Solution: (A)

Speed of the train = 108 km/hr

Speed of the train in m/s = $108 \times \frac{5}{18} = 30$

= 30 m/s.

Time taken to cross the platform (t) = $\frac{L_T + L_P}{S_T}$

$40 = \frac{1,000 + L_T}{30}$

$\therefore$ Length of the train is 200 m.

Example 5:

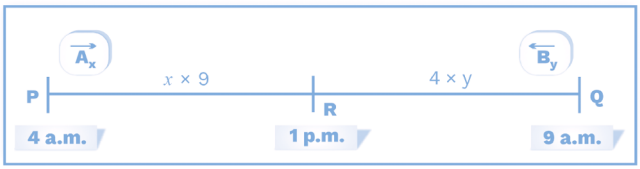
Two trains, A and B, started from two stations, P and Q, at 4 a.m. and 9 a.m., respectively, moving towards each other's starting point. They meet at station R at 1 p.m. on the same day. After the two trains meet, train A reaches Q and train B reaches P simultaneously. If the distance between the two stations is 800 km, then at what time did train A reach station Q?

(A) 5 p.m. (B) 7 p.m.

(C) 9 p.m. (D) Data insufficient

Solution: (B)

Let's assume the speed of train A = x km/hr and train B = y km/hr, respectively.



Distance travelled by train A till 1 p.m. = (x × 9) km.

Distance travelled by train Q till 1 p.m. = (4 × y) km.

Now according to the question, after the trains meet after some time (let's assume T hrs) to reach their respective distance.

$$T = \frac{\text{Distance QR}}{\text{Distance by train A}} = \frac{\text{Distance PR}}{\text{Distance by train B}}$$

$$\text{Speed of train A} \qquad \text{Speed of train B}$$

$$T = \frac{4y}{x} = \frac{9x}{y} \Rightarrow \frac{x^2}{y^2} = \frac{4}{9}$$

$$\Rightarrow \frac{x}{y} = \frac{2}{3}$$

Take $x = 2k$ and $y = 3k$

So, time taken by train A to reach Q from R.

$$= T = \frac{4y}{x} = \frac{4 \times 3k}{2k} = 6 \text{ hr.}$$

Train A will reach Q at 7 p.m.

Hence, option (B) is the correct answer.

Concept 8: Boats and Streams

u = Speed of boat or man in still water.

v = Speed of current or river.

Downstream speed (D)

If a boat or man is rowing in the same direction in which the river is flowing, then this effective speed is known as downstream speed. It is denoted by D.

Downstream speed = Speed of boat + Speed of current or river

$D = u + v$

Upstream speed (U)

If the boat or man is rowing in the opposite direction in which the river is flowing, then this effective speed is known as upstream speed. It is denoted by U.

Upstream speed = $U = u - v$ (only if $u > v$)

Speed of boat (u) = $\frac{D+U}{2}$ and Speed of Current (v) = $\frac{D-U}{2}$

Example 6:

A boat takes 2 hours less time to travel 240 km downstream than to travel the same distance upstream. If the speed of the stream is 20 km/hr, then find the speed of the boat.

(A) 10.56 km/hr

(B) $20\sqrt{5}$ km / hr

(C) $20\sqrt{13}$ km / hr

(D) $5\sqrt{3}$ km / hr

Solution: (C)

Let the speed of the boat is u km/hr.

Speed of stream = 20 km/hr (given).

Therefore, downstream speed (D_s) = (u + 20) km/hr.

Upstream speed (U) = (U – 20) km/hr

Now, according to questions:

$$\frac{240}{U-20} - \frac{240}{U+20} = 2$$

Time taken in upstream

Time taken in downstream

$40 \times 120 = U^2 - 400$

$U^2 = 4{,}800 + 400$

$U^2 = 5{,}200$

$U = \sqrt{5{,}200}$

$= \pm 10\sqrt{52}$

$= \pm 20\sqrt{13}$

Speed is not considered negative.

Therefore, speed of the boat (U) = $20\sqrt{13}$ km/hr.

Concept 9: Escalators

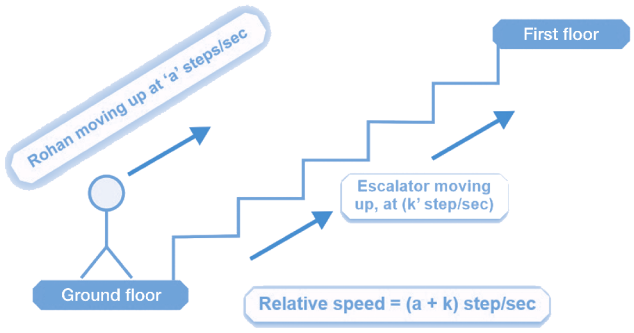
The concept of escalators is similar to the concept of boats and streams. In escalators-based questions

Distance travelled = Number of steps visible on the escalator

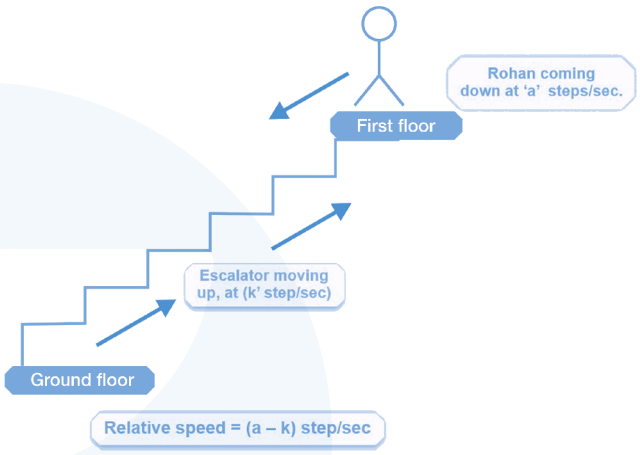
Speed = steps/sec

Let's assume that an escalator is moving up from the ground floor to the first floor in a shopping mall. The speed of the escalator is k steps/sec. Also, assume the total number of steps of the escalator is D. A person named Rohan is using the same escalator to move up and get down. Rohan's speed is a steps/sec.

1. If Rohan is climbing up on the moving up escalator, then effective speed = $(a + k)$ steps/sec.



2. If Rohan is coming down on the moving up escalator, then effective speed = $(a - k)$ steps/sec.



3. If Rohan does not have his own speed (he just stands on the moving up escalator), then the time taken by him to reach the first floor from the ground floor

= Total steps on the escalator/Speed of the escalator = (D/k) sec.

4. If Rohan is climbing up (with the speed of a steps/sec) on the moving up the escalator, then time taken by him to reach the first floor from the ground floor

= Total steps on the escalator/effective speed = $D/(a + k)$ sec.

5. If Rohan is coming down (with the speed of a steps/sec) on the moving up escalator, then time taken by him to reach the first floor from the ground floor

= Total steps on the escalator/effective speed = $D/(a - k)$ sec.

Example 7:

Let's assume that an escalator moves up from the ground floor to the first floor in a shopping mall. Speed of the escalator is 1 step/sec. Also, assume the total number of steps of the escalator is 120. A person named Rohan is using the same escalator to move up and get down. Rohan speed is 3 steps/sec.

1. If Rohan is climbing up on the moving up escalator, then effective speed = (3 + 1) steps/sec.
2. If Rohan is coming down on the moving up escalator, then effective speed = (3 – 1) steps/sec.
3. If Rohan does not have his own speed (he just stands on the moving up escalator), then the time taken by him to reach the first floor from the ground floor.

   = Total steps on the escalator/Speed of the escalator = (120/1) = 120 sec.
4. If Rohan is climbing up (with the speed of a steps/sec) on the moving up escalator, then time taken by him to reach the first floor from the ground floor

   = Total steps on the escalator/effective speed = 120/(3 + 1) = 30 sec
5. If Rohan is coming down (with the speed of *a* steps/sec) on the moving up escalator, then the time taken by him to reach the first floor from the ground floor.

   = Total steps on the escalator/effective speed = 120/(3 – 1) = 60 sec.

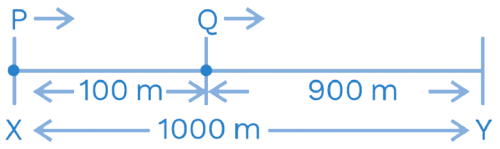
Concept 10: Linear Races

Terminology

- **Dead heat**
  When the runners reach the finishing line or end point simultaneously.
- **Head start**

Head start in distance

If there is a race of 1 km between P and Q, and P has given a head start of 100 m to Q, then



So, in this race, P has to run 1,000 m and Q has to run only 900 m.

Head start in time

Suppose two points are given X and Y, and the distance between them is 1 km. There are two people, P and Q, where P is a faster runner, and he gives a head start of 10 seconds to Q.

Head start distance = 10 sec × Speed of Q.

So, if the race ended in a dead heat, then Q has to run for *t* seconds and P has to run for (*t* – 10) sec.

Q → *t* sec

P → (*t* – 10) sec

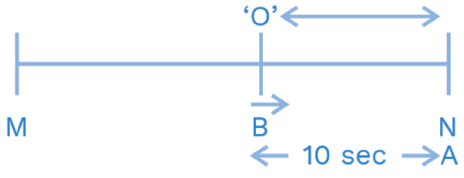
Hence, both runs for the same distance.

Therefore, distance *XY* = Speed of P × (*t* – 10) = Speed of Q × *t*.

Beat Time and Beat Distance:

Beat in time

Suppose two points M and N are 1 km apart. The two runners A and B are such that A is the faster runner. Suppose A reaches point N 10 seconds before B reaches point N. So, we can say that A has beaten B by 10 seconds. We have to find the distance between A and B while B is at O point and A is at N point.



So, that distance between O and N will be known as 'beat distance'.

Beat distance = 10 seconds × speed of B

Therefore, time taken by A to complete the race is 10 seconds less than the time taken by B to complete the race.

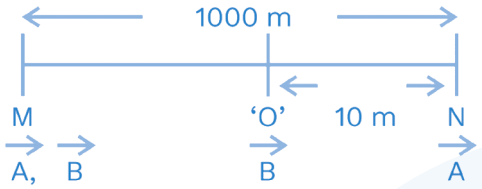
Let B → t sec

∴ A → (t – 10) sec

Now the equation can be formed as:

Total distance = t × speed of B = (t-10) × speed of A

Beat by distance



In this case, A beats B by 10 m distance. So, here the beat distance is 10 m.

→ Beat distance = 10 m

Since they are running for the same time.

A runs 1,000 m distance in the same time when B runs only 990 m distance.

Therefore

A → 1,000 m

B → (1,000 – 10) m = 990 m

Time is the same for both the runners; hence

$$\text{time}\,(t) = \frac{1{,}000\text{ m}}{\text{Speed A}} = \frac{990\text{ m}}{\text{Speed of B}}$$

$$\frac{\text{Speed of A}}{\text{Speed of B}} = \frac{1{,}000\text{ m}}{990\text{ m}}$$

Thus, whenever the time is constant, the ratio of the speeds = ratio of the respective distances covered by runners.

Example 8:

In a race of 1,520 m, A beats B by 190 m. In another race of 1,750 m, B beats C by 250 m. By what distance A will beat C in a race of 1,280 m? (Assume that the speed of a runner does not change in different races.)

(A) 300 m (B) 320 m

(C) 360 m (D) 380 m

Solution: (B)

When A covers 1,520 m, B will cover 1,330 (1,520 – 190) m at the same time, so the ratio of their speeds = ratio of the distances travelled = 1,520 : 1,330 = 8 : 7.

When B covers 1750 m, C will cover 1,500 m (1,750 – 250) at the same time, so the ratio of their speeds = ratio of the distances travelled = 1,750 : 1500 = 7 : 6.

So, the ratio of speeds of A, B, and C = 8 : 7 : 6.

Now, when A will cover 8 m, C will cover 6 m at the same time.

So, when A will cover 1,280 (8 × 160) m, C will cover 960 (6 × 160) m.

Hence, in a race of 1,280 m, A will beat C by 320 m.

Example 9:

In a 400 m race, Ussain Bolt beats Kipchoge by 8 m and Kipchoge beats Justin Gatlin by 20 m. What head start should be given to Justin by Ussain Bolt in a 200 m race, so that they finish the race at the same time?

(A) 15.2 m (B) 14.4 m

(C) 13.8 m (D) 17 m

Solution: (C)

Know that ratio of distance = Ratio of speed (t = constant)

$$\frac{\text{Distance covered by Bolt}}{\text{Distance covered by Kipchoge}} = \frac{400}{392} = \frac{50}{49}$$

$$\text{Also, } \frac{\text{Distance covered by Kipchoge}}{\text{Distance covered by Justin}} = \frac{400}{380} = \frac{20}{19}$$

Therefore, the ratio between Bolt and Justin can be expressed as:

$$\frac{\text{Distance covered by Bolt}}{\text{Distance covered by Justin}} = \frac{50}{49} \times \frac{20}{19} = \frac{1{,}000}{931}$$

Since we need to find the relation for 200 m race, we can do it by dividing the nominator and the denominator by 5:

$$\frac{\text{Distance covered by Bolt}}{\text{Distance covered by Justin}} = \frac{\left(\frac{1{,}000}{5}\right)}{\left(\frac{931}{5}\right)} = \frac{200}{186.2}$$

Now, when Bolt covers 200 m, Justin covers 186.2 m.

Therefore, Bolt should give a head start of = 200 – 186.2 = 13.8 m

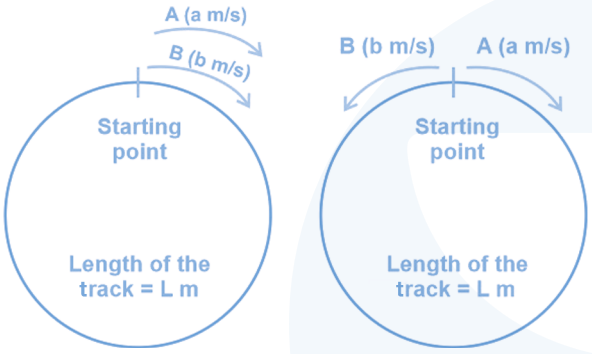
Hence, option (C) is the correct answer.

Concept 11: Circular Races

Case 1a:

When will the two runners meet at the starting point for the first time?

Let's assume that two runners A and B are running around a circular track of length *L* metres with speeds of *a* m/s and *b* m/s, respectively.



Time taken to complete one round of track = Length of the track/Speed of the runner.

So, the time taken by A and B to complete one round of the track will be (*L*/*a*) sec and (*L*/*b*) sec, respectively.

Time taken to meet for the 1st time at the starting point = LCM (*L*/*a*, *L*/*b*).

Note: Time taken to meet at starting point does not depend on the runners' direction. It will be the same whether the runners are running in the same direction or the opposite direction.

Example 10:

If two runners A and B are running around a circular track (in the same or opposite direction) of length 720 m with speeds of 10 m/s and 6 m/s, respectively, then

Time taken by A to complete one round (to reach the starting point) = 720/10 = 72 sec.

Time taken by B to complete one round (to reach the starting point) = 720/6 = 120 sec.

Time taken to meet for the first time at the starting point = LCM (72, 120) = 360 sec.

Time taken to meet for the second time at the starting point = (360 × 2) sec.

Time taken to meet for the *N*th time at the starting point = (360 × *N*) sec.

Case 1b:

When will the three or more runners meet at the starting point for the first time?

Let's assume that 3 runners A, B, and C are running around a circular track of length *L* metres with speeds of *a* m/s, b m/s, and *c* m/s, respectively.

So, the time taken by A, B, and C to complete one round of the track will be (*L*/*a*) sec, (*L*/*b*) sec, and (*L*/*c*) sec, respectively.

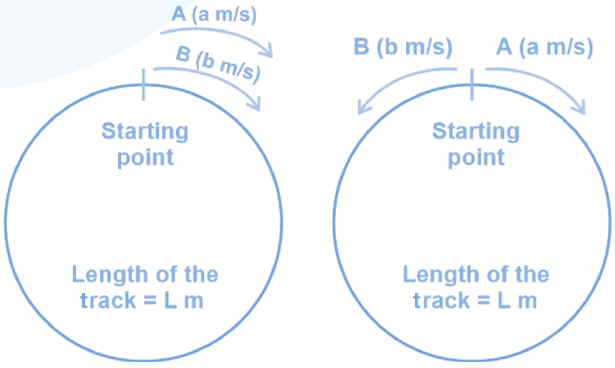
Time taken to meet for the 1st time at the starting point = LCM (*L*/*a*, *L*/*b*, *L*/*c*).

Similarly, when *N* runners *A*, *B*, *C*, *D*, ..., *N* are running around a circular track of length L metres with speeds of *a* m/s, *b* m/s, and *c* m/s, *d* m/s, ..., *n* m/s, respectively.

Time taken to meet for the 1st time at the starting point = LCM (*L*/*a*, *L*/*b*, *L*/*c*. *L*/*d*,..., *L*/*n*).

Case 2a:

When the two runners will meet for the first time on the circular track (not necessarily at the starting point)?



Let's assume that two runners A and B are running around a circular track of length *L* metres with speeds of *a* m/s and *b* m/s, respectively. (Assume *a* > *b*)

Time taken to meet for the 1st time on the track = Length of the track/relative speed

- When they are moving in the same direction, time taken = $L/(a - b)$.
- When they are moving in opposite directions, time taken = $L/(a + b)$.

Example 11:

If two runners A and B are running around a circular track of length 720 m with speeds of 10 m/s and 6 m/s, respectively, then time taken to meet for the 1st time on track (same direction) = 720/(10 – 6) = 180 sec.

Time taken to meet for the second time on track (same direction) = (180 × 2) sec.

Time taken to meet for the *N*th time on track (same direction) = (180 × *N*) sec.

Time taken to meet for the 1st time on track (opposite direction) = 720/(10 + 6) = 45 sec.

Time taken to meet for the 2nd time on track (opposite direction) = (45 × 2) sec.

Time taken to meet for the *N*th time on track (opposite direction) = (45 × *N*) sec.

Case 2b:

When the three runners will meet for the first time on the circular track (not necessarily at the starting point)?

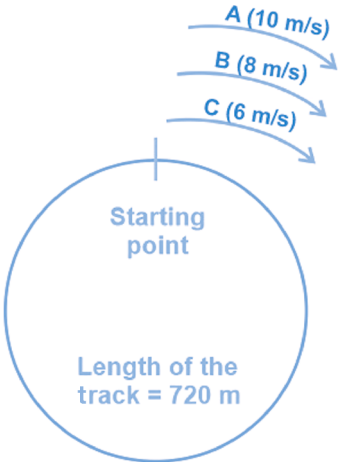
Let's assume that three runners, A, B, and C, are running around a circular track of length *L* meters with speeds of *a* m/s, *b* m/s, and *c* m/s, respectively.

Time taken to meet for the first time on track = LCM (time taken by any two pairs).

Time taken to meet for the first time on track = LCM (time taken by A and B, time taken by B and C) or LCM (time taken by A and B, time taken by A and C) or LCM (time taken by A and C, time taken by B and C).

Example 12:

If three runners, A, B, and C, are running (all in the same direction) around a circular track of length 720 m with speeds of 10 m/s, 8 m/s, and 6 m/s, respectively, then



Time taken by A and B to meet for the 1st time on track = *L*/relative speed = 720/(10 – 8) = 360 sec.

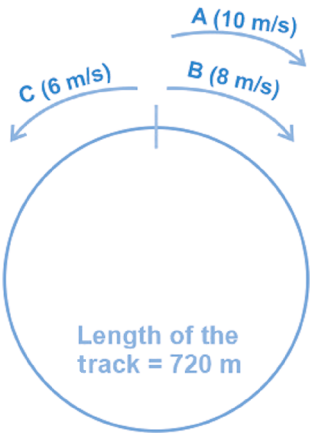
Time taken by A and C to meet for the 1st time on track = *L*/relative speed = 720/(10 – 6) = 180 sec.

Time taken by B and C to meet for the 1st time on track = *L*/relative speed = 720/(8 – 6) = 360 sec.

Time taken to meet for the first time on track = LCM (360, 360) or LCM (360, 180) or LCM (120, 360) = 360 sec.

Example 13:

If three runners, A, B, and C, are running (A and B are running in the clockwise direction while C is running in the anticlockwise direction) around a circular track of length 720 m with speeds of 10 m/s, 8 m/s, and 6 m/s, respectively, then



Time taken by A and B to meet for the 1st time on track = *L*/relative speed = 720/(10 – 8) = 360 sec.

Time taken by A and C to meet for the 1st time on track = L/relative speed = 720/(10 + 6) = 45 sec.

Time taken by B and C to meet for the 1st time on track = L/relative speed = 720/(8 + 6) = (360/7) sec.

Time taken to meet for the 1st time on track

= LCM (360, 45) or LCM (360, 360/7) or LCM (45, 360/7) = 360 sec.

Case 3:

At how many distinct points on the track will the two runners meet?

Number of distinct meeting points = Time taken to meet for the first time at the starting point/Time taken to meet for first time on the track

Example 14:

If two runners A and B are running (in the same direction) around a circular track of length 720 m with speeds of 10 m/s and 6 m/s, respectively, then time taken by A to complete one round(to reach the starting point) = 720/10 = 72 sec.

Time taken by B to complete one round (to reach the starting point) = 720/6 = 120 sec.

Time taken to meet for the 1st time at the starting point = LCM (72, 120) = 360 sec.

Time taken to meet for the 1st time on track = 720/(10 – 6) = 180 sec.

Number of distinct meeting points = 360/180 = 2.

Example 15:

If two runners A and B are running (in opposite directions) around a circular track of length 720 m with speeds of 10 m/s and 6 m/s, respectively, then time taken by A to complete one round (to reach the starting point) = 720/10 = 72 sec.

Time taken by B to complete one round (to reach the starting point) = 720/6 = 120 sec.

Time taken to meet for the 1st time at the starting point = LCM (72, 120) = 360 sec.

Time taken to meet for the 1st time on track = 720/(10 + 6) = 45 sec.

Number of distinct meeting points = 360/45 = 8.

Important short-cut

If 2 runners are running on a circular track with speeds in the ratio $P : Q$ ($P > Q$), then

Number of distinct meeting points (in the same direction) = $P - Q$

Number of distinct meeting points (in the opposite direction) = $P + Q$

In our examples of two runners, A and B are running around a circular track of length 720 m with speeds of 10 m/s and 6 m/s, respectively, the ratio of speeds = 10 : 6 = 5 : 3.

Number of distinct meeting points (in the same direction) = 5 – 3 = 2.

Number of distinct meeting points (in the opposite direction) = 5 + 3 = 8.

Example 16:

Anand, Bakshi, Rakesh, and Omprakash are four friends running along a circular path. Anand and Omprakash are running in the anticlockwise direction, while Bakshi and Rakesh are running in the clockwise direction. If the ratio of speeds of Anand, Bakshi, Rakesh, and Omprakash is 1 : 3 : 7 : 13, then find at how many distinct points will Anand meet with any other runner, if they are running indefinitely.

(A) 24 (B) 18

(C) 16 (D) 15

Solution: (C)

Speed =	↑A:	↓B:	↓R:	↑O
	1:	3:	7:	13

Since we know that two persons have their speeds in the ratio $a : b$:

- If travel in the opposite direction = They meet at total $(a + b)$ distinct points
- If travel in the same direction = They will meet at total $|a - b|$ distinct points

$$A \to B\left(4 \text{ points}\right)\frac{3L}{4}, \frac{2L}{4}, \frac{L}{4}$$

$$A \to R(8 \text{ points}) \frac{7L}{8}, \frac{6L}{8}, \frac{5L}{8} \frac{4L}{8} \frac{3L}{8}, \frac{2L}{8} \frac{L}{8}$$

$$A \to O(12 \text{ points}) = L, \quad \frac{11L}{12}, \frac{10L}{12}, \frac{9L}{12}, \frac{8L}{12},$$

$$\frac{7L}{12}, \frac{6L}{12}, \frac{5L}{12}, \frac{4L}{12}, \frac{3L}{12}, \frac{2L}{12}, \frac{L}{12}$$

Total points = 16 points

Hence, option (C) is the correct answer.

Concept 12: Clocks

The clock's dial is a complete circle having a total angle of 360° around the centre of the clock. The dial is divided into 60 equal divisions (minutes as we usually say).

Now, 60 minutes of the dial represents 360°. So, 1 minute of the dial will represent 6°, which means 1-minute gap between the hands of the clock will represent an angle of 6° between them.

N minutes gap between the hands of the clock = angle of (N × 6) ° between the hands of the clock.

Let's understand this with the help of some examples.

At 5 o'clock, you can see that the gap between the hands of the clock is 25 minutes (or 35 minutes), so the angle between the hands of the clock will be (25 × 6)° = 150° or (35 × 6)° = 210°.

At 8 o'clock, you can see that the gap between the hands of the clock is 20 minutes (or 40 minutes), so the angle between the hands of the clock will be (20 × 6)° = 120° or (40 × 6)° = 240°.

(At any point in time, two angles exist between the hands of the clock. For example, at 8 pm the angle between the hands of the clock is either 120° or (360 - 120)° = 240°, but in order to simplify the explanation, we generally use angles that are less than 180°).

Speed of Clock's Hands

As you can see in the above images that in 1 hr time period (60 minutes) from 12 o'clock to 1 o'clock, the minute hand completes one full circle (360°), while the hour hand moves from 12 to 1.

So, the minute hand moves 360° in 60 minutes. Thus, its speed is 6° per minute.

The hour hand moves 5 minutes or (5 × 6)° = 30° in 60 minutes. Thus, its speed is 0.5° per minute.

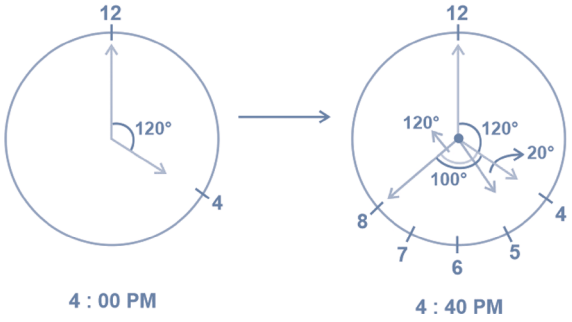
As both hands always move in the same direction (clockwise direction).

The relative speed = 6 – 0.5 = 5.5° per minute.

Example 17:

Find the angle between the hands of the clock at 4:40 p.m.

Solution: 100°



At 4 p.m., the gap between the hands of the clock is 20 minutes. So, the angle between the hands of the clock at 4 p.m. = 20 × 6 = 120°

Now from 4 to 4:40 (duration of 40 minutes)

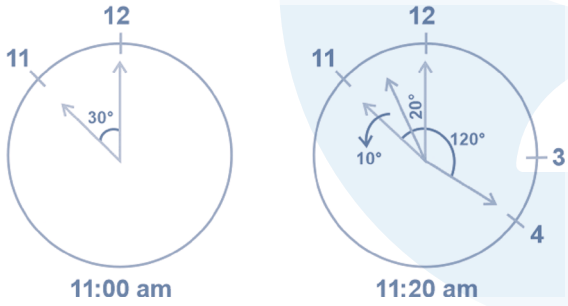
The minute hand moves 40 × 6 = 240° clockwise and the hour hand moves 40 × (0.5) = 20° clockwise, as shown in the figure above.

So, the angle between the hands of the clock at 4:40 pm is 100°.

Example 18:

Find the angle between the hands of the clock at 11:20 a.m.

Solution: 140°



At 11:00 a.m., the gap between the hands of the clock is 5 minutes. So, the angle between the hands of the clock at 11:00 a.m. = 5 × 6 = 30°.

Now from 11:00 am to 11: 20 am (duration of 20 minutes).

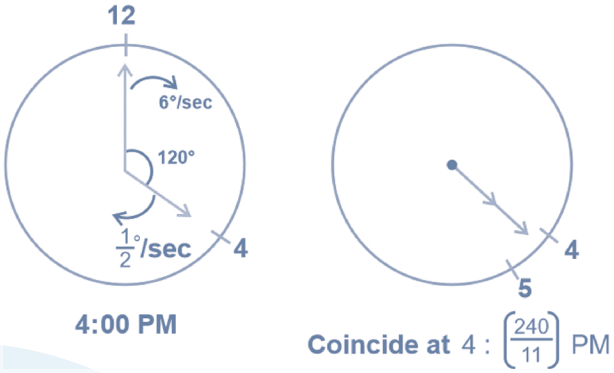
The minute hand moves 20 × 6 = 120° clockwise and the hour hand moves 20 × (0.5) = 10° as shown in the figure above.

So, the angle between the hands of the clock at 11:20 p.m. is 140°.

Example 19:

At what time between 4 p.m. and 5 p.m. will the hands of the clock coincide with each other?

Solution: 4 240/11 min



As you can see in the image, at 4 p.m., the gap between the hands of the clock is 20 minutes, which means the angle between the hands of the clock will be (20 × 6)° = 120°.

Now, as the clock starts from 4 p.m., the angle will start reducing (from 120°) every minute until it becomes zero (coincides).

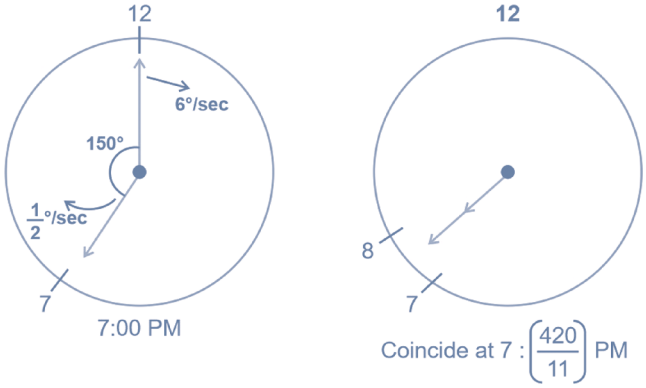
Time taken for angle to reduce from 120° to 0° = Distance/relative speed = 120/(5.5) = 240/11 minutes

Thus, 240/11 minutes past 4 p.m., the hands of the clock will coincide with each other. Also, after coinciding the gap between the hands will start increasing and they will not further coincide till 5 p.m.

Example 20:

At what time between 7 p.m. and 8 p.m. will the hands of the clock coincide with each other?

Solution: 7 420/11 minutes



As you can see in the image above, at 7 p.m., the gap between the hands of the clock is 25 minutes, which means the angle between the hands of the clock will be (25 × 6)° = 150°.

Now as the clock starts from 7 p.m., the angle will start increasing (from 150°) every minute and we need to find the time at which the angle becomes 360° (or 0°).

Time taken for angle to increase from 150° to 360° = Distance/relative speed = 210/(5.5) = 420/11 minutes.

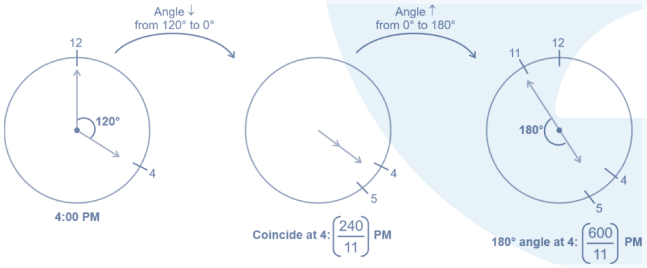
Thus, 420/11 minutes past 7 p.m., the hands of the clock will coincide with each other.

Note: Hands of the clock coincide once every hour in general, but they coincide 11 times in 12 hours. As between 11 a.m. and 1 p.m. (2 hours duration), there is only one coincide at noon and there will be 10 coincides in the remaining 10 hours.

Example 21:

At what time between 4 p.m. and 5 p.m. will the hands of the clock are opposite (angle of 180°) to each other?.

Solution: 4 600/11 minutes



As you can see in the image above, at 4 pm, the gap between the hands of the clock is 20 minutes, which means the angle between the hands of the clock will be (20 × 6)° = 120°.

Now as the clock starts from 4 pm, the angle will start reducing (from 120°) every minute till it becomes zero.

Time taken for angle to reduce from 120° to 0° = Distance/relative speed = 120/(5.5) = 240/11 minutes.

Now after coinciding, the angle between the hands of the clock will again start increasing and we need to find the time at which the angle becomes 180°.

Time taken for angle to increase from 0° to 180° = Distance/relative speed = 180/(5.5) = 360/11 minutes.

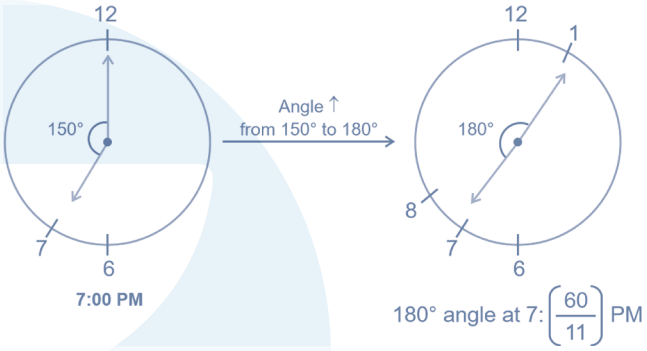
Total time = 240/11 + 360/11 = 600/11 minutes.

So, 600/11 minutes past 4 p.m., the hands of the clock will be opposite to each other other.

Example 22:

At what time between 7 p.m. and 8 p.m. will the hands of the clock be opposite (angle of 180°) to each other?

Solution: 4 600/11 minutes



As you can see in the image above, at 7 p.m., the gap between the hands of the clock is 25 minutes, which means the angle between the hands of the clock will be (25 × 6) ° = 150°.

As the clock starts from 7 p.m., the angle will start increasing (from 150°) every minute until it becomes 180°.

Time taken for angle to increase from 150° to 180° = Distance/relative speed = 30/(5.5) = 60/11 minutes

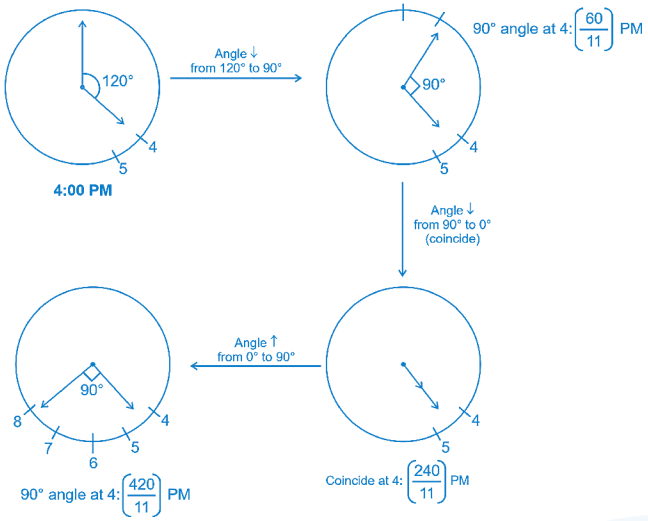
So, 60/11 minutes past 7 p.m. the hands of the clock will be opposite each other.

Note: Hands of the clock are opposite to each other at once every hour in general, but they are opposite to each other at 11 times in 12 hours. As between 5 p.m. and 7 p.m. (2 hours duration) there is only one time (at 6 p.m.) at which hands of clocks are opposite to each other.

Example 23:

At what time between 4 p.m. and 5 p.m. will the hands of the clock be at a right angle (90° angle)?

Solution: 4 60/11 minutes and 4 420/11 minutes



As you can see in the image, at 4 p.m., the gap between the hands of the clock is 20 minutes, which means the angle between the hands of the clock will be (20 × 6)° = 120°.

Now as the clock starts from 4 p.m., the angle will start reducing (from 120°) every minute, and we need to find the time at which the angle becomes 90°.

Time taken for the angle to reduce from 120° to 90° = Distance/relative speed = 30/(5.5) = 60/11 minutes, and this will be the first right angle.

Now after this right angle, the angle will further start reducing till it becomes zero (coincide).

Time taken for angle to reduce from 90° to 0° = Distance/relative speed = 90/(5.5) = 180/11 minutes.

After coinciding, the angle between the hands of the clock will again start increasing, and we need to find the time at which the angle becomes 90°.

Time taken for angle to increase from 0° to 90° = Distance/relative speed = 90/(5.5) = 180/11 minutes.

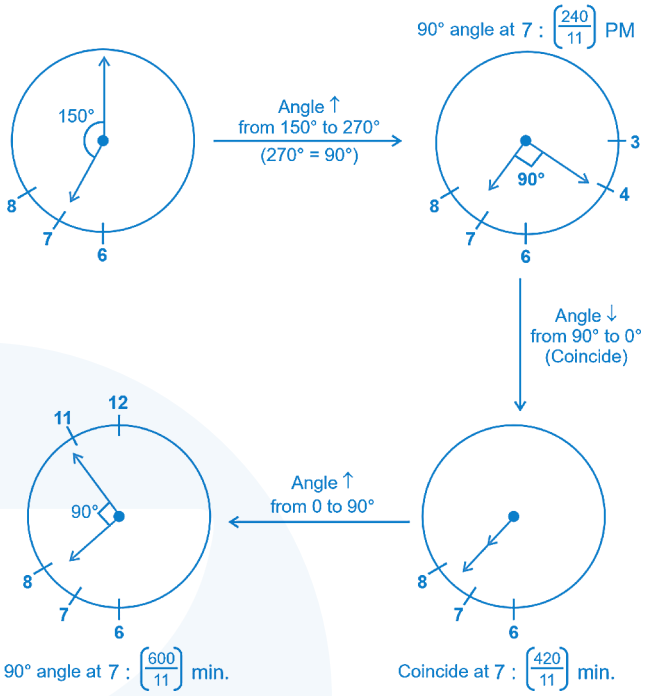
Total time for the second right angle = 60/11 + 180/11 + 180/11 = 420/11 minutes

So, 420/11 minutes past 4 p.m., the second right angle will be formed.

Example 24:

At what time between 7 p.m. and 8 p.m. will the hands of the clock be at a right angle (90° angle)?

Solution: 7 240/11 minutes and 7 600/11 minutes



As you can see in the image, at 7 p.m., the gap between the hands of the clock is 25 minutes, which means the angle between the hands of the clock will be (25 × 6) ° = 150°.

Now as the clock starts from 7 p.m., the angle will start increasing (from 150°) every minute and we need to find the time at which the angle becomes 270° (or 90°).

Time taken for the angle to increase from 150° to 270° = Distance/relative speed = 120/(5.5) = 240/11 minutes and this will be the first right angle.

Now after this right angle, the angle will further start increasing till it becomes 360° (or 0°) (coincide).

Time taken for angle to increase from 270° to 360° = Distance/relative speed = 90/(5.5) = 180/11 minutes

After coinciding, the angle between the hands of the clock will again start increasing, and we need to find the time at which the angle becomes 90°.

Time taken for angle to increase from 0° to 90° = Distance/relative speed = 90/(5.5) = 180/11 minutes.

Total time for the second right angle = 240/11 + 180/11 + 180/11 = 600/11 minutes.

So, 600/11 minutes past 7 p.m. the second right angle will be formed.

Note: Hands of the clock are at a right angle twice every hour in general, but they are at a right angle 22 times in 12 hours. As between 2 p.m. and 4 p.m. (2 hours duration) there are only three right angles and also between 8 p.m. and 10 p.m. (2 hours duration) there are only three right angles.

Example 25:

Shubham started drinking between 2 o'clock and 3 o'clock. When he finished drinking between 6 o'clock and 7 o'clock, he found the hour and the minute hands positions to be the same as those of the minute hand and hour hand, respectively, when he had started drinking. How long does Shubham spend in drinking?

(A) $160\frac{6}{13}$ minutes

(B) $161\frac{7}{13}$ minutes

(C) $221\frac{7}{13}$ minutes

(D) $222\frac{6}{13}$ minutes

Solution: (C)

We know that, in 1 minute, the minute hand rotates 6° and the hour hand rotates $\left(\frac{1}{2}\right)^\circ$.

Let Shubham start drinking at x minutes past 2 o'clock.

Hence, the minute hand will move = $6x°$

The hour hand will move = $60^\circ + \left(\frac{x}{2}\right)^\circ$

Let Shubham finish drinking y minutes past 6 o'clock.

Now, the hour hand will move =

Also, $60° + 4 \times (30)° + \frac{y}{2}$

$12x = 360° + y$ ...(i)

And, the minute hand will move = $6y°$

Also, $60 + \frac{x}{2} = 6y$

$12y = 120 + x$ ...(ii)

Here, (1) — (2)

$12(x - y) = 240 + (y - x)$

or $13(x - y) = 240$

$x - y = \frac{240}{13}$ min

Now, time spent by Shubham in drinking

$= 4 \text{ hr} - (x - y)$

$= \left[240 - \frac{240}{13}\right]$ min

$= 240 - \left(18\frac{6}{13}\right)$ min

$= 221\frac{7}{13}$ min

Hence, option (C) is the correct answer.

Practice Exercise – 1

Level of Difficulty – 1

1. A boy goes from home to school at a speed of 30 km/hr and comes back from school to home at a speed of 40 km/hr; it took him 10.5 hours in the entire journey. Find the distance from home to school.
(A) 160 km
(B) 200 km
(C) 180 km
(D) 185 km

2. Two trains 100 m and 120 m long are running in the same direction with the speed of 72 km/hr and 54 km/hr, respectively. How much time will the first train (100 m length) take to cross the other?
(A) 44 sec
(B) 30 sec
(C) 50 sec
(D) 20 sec

3. A boat has to travel 240 km in 8 hours, moving downstream. It is given that the speed of the stream is 20 km/hr. Find the speed of the boat.
(A) 10 km/hr
(B) 15 km/hr
(C) 20 km/hr
(D) 5 km/hr

4. When Pragya travels from her home at 16 km/hr speed, she reaches 40 minutes late to meet her boyfriend Kaalia at his home. While if she travels at 20 km/hr speed, she reaches 20 minutes early. Find the distance (in km) between Pragya's home and her boyfriend Kaalia's home.
(A) 72
(B) 75
(C) 76
(D) 80

5. In covering a distance of 180 km, Sonu takes 4 hours more than Monu. If Sonu doubles his speed, then he would take 1 hour less than Monu to cover the same distance. Sonu's speed (in km/hr) is:
(A) 36
(B) 30
(C) 24
(D) 18

Level of Difficulty – 2

6. Chennai Express starts from Banaras on Monday at 7:30 a.m. and reaches Chennai on Tuesday at 3:45 a.m. While Banaras Express starts from Chennai on Monday at 11:45 a.m. and reaches Banaras on Tuesday at 2 p.m. Find at what time both the trains crossed each other.
(A) 07:47 p.m., Monday
(B) 11:45 p.m., Monday
(C) 02:45 a.m., Tuesday
(D) 08:47 p.m., Monday

7. On moving up the escalator, Anish takes 147 steps to reach the bottom from the top, while his wife Richa takes 35 steps to reach the top from the bottom. If the time taken by Anish and Richa is 49 sec and 7 sec, respectively, then find the total number of steps of the escalator when it is not moving.
(A) 100
(B) 98
(C) 50
(D) 49

8. In a kilometre race, A beats B by 200 m. In a 500 m race, B beats C by 50 m. In a 3,600 m race, if A beats C by 56 sec, how long would B take to race (in sec) a distance of 1,500 m?
(A) 75
(B) 90
(C) 120
(D) 150

9. A lady walks up an ascending escalator at a speed of 4 steps per second and reaches the top in 40 steps. If she walks up the ascending escalator at a speed of 8 steps per second, she reaches the top in 60 steps. Find the number of steps on the stationary escalator.

(A) 90 steps
(B) 120 steps
(C) 150 steps
(D) 180 steps

10. The distance from P to Q is 120 km. Monu and Sonu start from P simultaneously and move towards Q. Monu takes 4 hours more than Sonu to reach B. Moreover, the time taken by Monu to cover 80 km is 1 hr less than the time taken by Sonu to reach Q from P. What is the speed of Monu in km/hr?

(A) 6 km/hr
(B) 8 km/hr
(C) 10 km/hr
(D) 12 km/hr

Level of Difficulty – 3

11. Anuj and Manuj are running on different circular tracks. When Anuj completes 6 rounds of his track, Manuj completes 5 rounds of his track. If Anuj and Manuj exchange their tracks, then the time taken by both of them to complete two rounds is the same. Find the square of the ratios of their speeds.

(A) $\frac{6}{5}$
(B) $\frac{5}{4}$
(C) $\frac{7}{8}$
(D) $\frac{5}{6}$

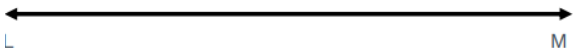
12. Rohan completes a journey in such a way that he starts at a certain initial speed, and then after every half an hour, he doubles his speed and reaches his destination at the scheduled time. If he completed his journey, travelling (entire trip) at his initial speed, he would have taken $\left(\frac{21}{2}\right)$ times the scheduled time. Find the scheduled travelling time (in hours) if the total distance travelled is 189 km.

13. Three athletes, Aman, Baman, and Chaman, are running around a circular track of radius 175 m. Aman started 2 sec after Baman, but ran 1 m/s faster than Baman. Baman started 2 sec after Chaman, but his speed is 1 m/s faster than Chaman. If Aman won the race and the speeds of Aman Baman and Chaman are positive integers, then what could be the maximum speed of Chaman?

14. Two bus and truck wheels have radii of 30 cm and 40 cm, respectively. While traveling a certain distance, each wheel of the bus required 2,000 more revolutions than each wheel of the truck. If the truck travelled this distance in 1 hr, then its approximate speed in km/hr was? $(\text{Use } \pi = 3.14)$

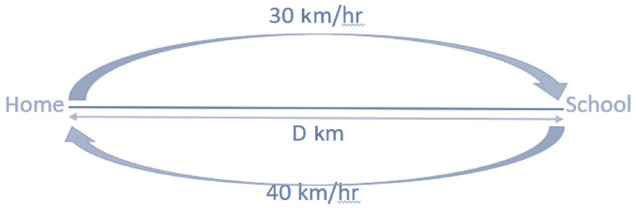
(A) 15
(B) 50
(C) 25
(D) 20

15. Runners P and Q started running simultaneously towards each other from the two ends of the linear track LM, and the ratio of their speeds is 7 : 4. Every time they meet, they interchange their speeds and also reverse their directions. At how many distinct points on the track do they meet each other if they run continuously between the two ends of the track LM?



Solutions

1. (C)



As distance is the same, the ratio of time taken by him would be inverse of the ratio of his speeds.

	Going	:	Returning
Ratio of speed	3	:	4
Ratio of time	4	:	3

Let's assume time taken while going from home to school is $4K$ and while returning back from school to home is $3K$.

Total time = $7K = \frac{21}{2}$ or $K = 1.5$.

Distance = D = Speed × time = $30 \times 4K$ = $30 \times 4 \times 1.5 = 180$ km.

2. (A)

Relative speed = $(72 - 54)$ km/ hr = 18 km/hr

Converting it into m/s;

$18 \text{ km/hr} = 18 \times \frac{5}{18} = 5 \text{ m/s}$

Total distance covered by the first train to cross the other = Length of the first train + Length of the second train = 100 + 120 = 220 m.

∴ Time taken by the first train to cross the other (t) = $\frac{220 \text{ m}}{5 \text{ m/s}}$, t = 44 sec

3. (A)

Since downstream (D_S) = Speed of boat + Speed of stream = $u + v$

Also, downstream speed

$$(D_S) = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$$D_S = \frac{240}{8} = 30 \text{ km/hr}$$

Therefore, $D_S = u + v$

$30 = u + 20$

Given that U = 10 km/hr

Hence, the speed of the boat is 10 km/hr.

4. (D)

Let the actual time taken by Pragya to reach her boyfriend's house be t minutes. We know that speed is inversely proportional to time when distance is constant.

Therefore, ATQ, $\frac{16}{20} = \frac{t-20}{t+40}$

t = 260 units

Now, time taken with speed 16 km/hr = $(t + 40) = (260 + 40) = 300$ minutes.

Therefore, distance = $16 \times \frac{300}{60} = 80$ km

Hence, option (D) is correct.

5. (D)

Let's assume the distance = D, and the original speeds of Sonu and Monu be A km/hr and B km/hr, respectively. Also assume the time taken by Monu to cover the distance is T hr.

$D = 180 = B \times T = A \times (T + 4)$ ...(i)

Also, $D = 180 = 2A \times (T - 1)$ ...(ii)

From equations (i) and (ii)

$A \times (T + 4) = 2A \times (T - 1)$ solving which we will get T = 6 hr.

Put the value of T in (2), we will get A = 18 km/hr.

Hence, option (D) is the correct answer.

6. (D)

Here time taken by both the trains is

Chennai Express = $T_{CE} = \frac{81}{4}$ hr

Banaras Express = $T_{BE} = \frac{105}{4}$ hr

For simplicity, let distance be the LCM of time taken by trains

$$\text{Distance} = \text{LCM}\left(\frac{18}{4}, \frac{105}{4}\right)$$

$$= \frac{\text{LCM}(81, 105)}{\text{HCF}(4, 4)} = \frac{2835}{4} \text{ km}$$

Now, the speed of Chennai Express,

$S_{CE} = \frac{2,835}{4} \times \frac{4}{81} = 35 \text{ km/hr}$

Speed of Banaras Express,

$S_{BE} = \frac{2,835}{4} \times \frac{4}{105} = 27 \text{ km/hr}$

Now, from 7:30 am to 11:45 am, the distance travelled by Chennai Express:

$D_{CE} = S_{CE} \times T_{CE} = 35 \times \frac{17}{4} = \frac{595}{4}$ km

Therefore, the remaining distance

$= \frac{2,835}{4} - \frac{595}{4} = \frac{2,240}{4} = 560$ km

Time taken to cross each other after 11:45 a.m.

$= \frac{560}{(35+27)} = \frac{560}{62} = 9\frac{1}{31} \approx 9$ hr 02 minutes (approx.)

Therefore, the required time is 8:47 p.m. Monday.

Hence, option (D) is the correct answer.

7. (D)

Let the speed of the escalator be *e* steps/sec.

Then, according to the question,

$\overbrace{147 - 49e}^{\text{Anish}} = \overbrace{35 + 7e}^{\text{Richa}}$ (Equating total number of steps)

112 = 56e

E = 2 steps/sec

Therefore, total number of steps = 147 – 49 × 2 = 147 – 98 = 49.

Hence, option (D) is the correct answer.

8. (A)

By the time A runs 1,000 m, B will run 800 m.

Similarly, when B runs 500 m, C will run 450 m.

Ratio of speeds of A and B = Ratio of distance travelled = 1,000:800 = 5:4.

Ratio of speeds of B and C = Ratio of distance travelled = 5,000:450 = 10:9.

So, ratio of speeds of A, B, and C = 25:20:18.

Let's assume the speed of *A* = 25*x*, speed of *C* = 18*x*, and speed of *B* = 20*x*.

Now, (3,600/18*x*) – (3,600/25*x*) = 56

Solving which we will get *x* = 1

Speed of B = 20*x* = 20 × 1 = 20 m/s

Time taken by B to travel 1500 m = 1,500/20 = 75 sec.

9. (B)

Let the speed of the escalator be *e* steps/sec.

Time is taken by a lady to cover 40 steps $\frac{40}{4} = 10$ sec.

∴ Distance covered by escalator in 10 sec = 10*e*.

∴ Total steps on the escalator = (10*e* + 40) ...(i)

Now, according to the second condition:

The time is taken by the lady to cover 60 steps

$\frac{60}{8} = 7.5$ sec

∴ Distance covered by escalator = 7.5*e*

Thus, the length of the escalator = (7.5*e* + 60) ...(ii)

If we solve equations (i) and (ii), we will get:

10*e* + 40 = 7.5*e* + 60

2.5*e* = 20

e = 8

Therefore, the total steps on the escalator = 10*e* + 40 = 120 steps.

Hence, option (B) is the correct answer.

10. (B)

Let's assume the speeds of Monu and Sonu be M km/hr and S km/hr, respectively.

Let's assume the time taken by Sonu to cover 120 km = *K* hr.

So, time taken by Monu to cover 120 km = (*K* + 4) hr.

120 km = *M* × (*K* + 4) = *S* × *K* ...(i)

Also, 80 km = *M* × (*K* – 1)

Or

120 km = 1. 5 × *M* × (*K* – 1) ...(ii)

From equations (i) and (ii)

M x (*K* + 4) = 1. 5 × *M* × (*K* – 1)

Solving which we will get *K* = 11 hr

From equation (i), 120 km = *M* × (*K* + 4) = *M* × 15

M = 8 km/hr

Hence, option (B) is the correct answer.

11. (A)

Let the radii of the Anuj's track and Manuj's track be r_1 and r_2.

Anuj completes 6 rounds at the same time ss Manuj completes five rounds of his track.

Therefore, $\frac{\text{Speed of Anuj}}{\text{Speed of Manuj}} = \frac{6 \times 2\pi r_1}{5 \times 2\pi r_2}$

$= \frac{6r_1}{5r_2}$ ...(i)

Again, when they interchange their tracks, as given in the question that after interchanging the track, both take the same time to complete two rounds of their tracks.

$\frac{\text{Speed of Anuj}}{\text{Speed of Manuj}} = \frac{2 \times 2\pi r_2}{2 \times 2\pi r_1} = \frac{r_2}{r_1}$ ...(ii)

Now, if we solve equations (i) and (ii), we will get

$\frac{6r_1}{5r_2} = \frac{r_2}{r_1}$

$\frac{r_1^2}{r_2^2} = \frac{5}{6}$

$\frac{r_1}{r_2} = \sqrt{\frac{5}{6}}$

Now, if we use the value of $\frac{r_1}{r_2} = \sqrt{\frac{5}{6}}$ in equation (ii), then we will get

$\frac{\text{Speed of Anuj}}{\text{Speed of Manuj}} = \sqrt{\frac{6}{5}}$

But, we have to find the square of the ratio of the speeds of Anuj and Manuj.

$\frac{(\text{Speed of Anuj})^2}{(\text{Speed of Manuj})^2} = \left(\sqrt{\frac{6}{5}}\right)^2 = \frac{6}{5}$

Hence, option (A) is the correct answer.

12. 3

Let the initial speed of Rohan be v kmph and assume he takes n hours to complete his journey.

∴ Time taken by Rohan while travelling at his initial speed = $\left(\frac{189}{v}\right)$ hours.

By the problem,

$\left(\frac{189}{v}\right) = \left(\frac{21}{2}\right)n$

$\Rightarrow v = \left(\frac{18}{n}\right)$ ...(i)

According to the question, Rohan doubled his speed after every $\left(\frac{1}{2}\right)$ hour. So, in n hours, there are $(2n)$ intervals of $\left(\frac{1}{2}\right)$ an hour.

We know distance = speed × time

$\Rightarrow 189 = v\left(\frac{1}{2}\right) + 2^1 \cdot v \cdot \left(\frac{1}{2}\right) + 2^2 \cdot v\left(\frac{1}{2}\right)$

$+...+ 2^{2n-1} \cdot v\left(\frac{1}{2}\right)$

$\Rightarrow 189 = v\left(\frac{1}{2}\right)(1 + 2^1 + 2^2 +...+ 2^{2n-1})$

$\Rightarrow 189 = \frac{v}{2}\left[1 \cdot \frac{(2^{2n} - 1)}{(2-1)}\right]$ (Since this is a GP series whose sum = $\frac{a(r^n - 1)}{(r-1)}$

Putting $v = \left(\frac{18}{n}\right)$ is the above equation, we get $189 = \frac{18}{2n}\left[\frac{2^{2n} - 1}{1}\right]$

$\Rightarrow \left(\frac{189}{9}\right) n = (2^{2n} - 1)$

$\Rightarrow 21n = (2^{2n} - 1)$

By the approach of hit and trial method, we get $n = 3$.
Hence, the scheduled time to reach the destination is 3 hours.

13. 22

If the speed of Chaman is n m/s, then the speed of Baman will be $(n + 1)$ m/s and that of Aman will be $(n + 2)$ m/s.
Even after starting 4 sec late w.r.t. Chaman, Aman still won the race
Time taken by Aman is less than time taken by Chaman by more than 4 sec.
Length of track

$$= 2 \times \frac{22}{7} \times 175 = 50 \times 22 = 1,100 \text{ m}$$

Now, $\frac{1,100}{n} - \frac{1,100}{n+2} > 4$

$$\frac{550}{n \times (n+2)} > 1$$

If we put $n = 22$ it satisfies the above inequality but at 23 it fails.
Thus, the maximum speed of Chaman can be 22 m/s.

14. (A)

The distance travelled by bus in one revolution

$= 2\pi r_1$
$= 2\pi \times 30$ cm
$= 60\pi$ cm

Also, the distance travelled by truck in one revolution

$= 2\pi r_2$
$= 2\pi \times 40$ cm
$= 80\pi$ cm

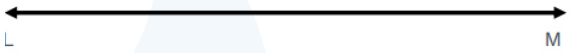
Let each wheel of the truck take n revolutions to cover a certain distance.
∴ The total revolution has been taken by the wheel of the bus to cover the same certain distance = $(n + 2{,}000)$ revolution.
Now, $(n + 2{,}000) \times 60\pi = n \times 80\pi$
$3n + 6{,}000 = 4n$
$n = 6{,}000$
Therefore, the distance travelled by the truck $= n \times 2\pi r_2$
$= n \times 2 \times \pi \times 40 = 80\pi \times 6{,}000$ cm
$= 4,80,000\pi$ cm $= 4,800\pi$ m $= 4.8\pi$ km

$$\therefore \text{Speed of the truck} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$$= \frac{4.8\pi \text{ km}}{1 \text{ hr}}$$

$= 4.8\pi$ km/hr $= 4.8 \times 3.14$ km/hr
$= 15.072$ km/hr ≈ 15 km/hr.
Hence, option (A) is the correct answer.

15. 11



This is similar to the case of circular motion, where two runners are running in the opposite direction with their speeds in a ratio 7 : 4.
So, the number of distinct points on the track where they will meet = 7 + 4 = 11.
Note: If two persons are moving in the opposite direction on a circular track with their speeds in the ratio of A : B, then the number of distinct points where they will meet on the circular track = A + B.

Practice Exercise – 2

Level of Difficulty – 1

1. If a man travels at $\frac{3}{5}$ of his usual speed to travel a certain distance, he takes $\frac{7}{2}$ hours more than usual time. Find the usual time.

(A) $5\frac{1}{4}$ hours

(B) $4\frac{1}{4}$ hours

(C) $5\frac{1}{2}$ hours

(D) $4\frac{1}{2}$ hours

2. The distance between Shubham's house and his girlfriend Nora's house is 72 km. One day he increases his speed by 6 km/hr from the usual speed, which results in taking him 2 hours less than the usual time to reach his girlfriend's house. Find his increased speed (in m/s).

(A) 8 m/s

(B) 6 m/s

(C) 4 m/s

(D) None of these

3. X and Y are running on a circular track of length 180 m, in the same direction at the speed of 20 m/s and 18 m/s, respectively. When will X and Y meet for the first time and at how many distinct points on the circular track will they meet?

(A) 30 sec and 2 points

(B) 60 sec and 1 point

(C) 90 sec and 2 points

(D) 90 sec and 1 point

4. Two trains cross each other in 20 seconds when running in the opposite direction along parallel tracks. The faster train is 150 m long and crosses a lamp post in 15 sec. If the speed of the second train is 18 km/hr less than the faster train (first train), what is its length (in m)?

(A) 140 m

(B) 150 m

(C) 160 m

(D) 180 m

5. A and B both start walking simultaneously from the point P towards point Q. The ratio of the speeds of A and B is 13:8. After reaching Q, A starts walking back towards P and meets B. If PQ = 840 m, then find the distance of their meeting point from point Q (in m).

(A) 200

(B) 640

(C) 760

(D) 780

6. In a race of 945 m, A beats B by 105 m. In another race of 1,360 m B beats C by 170 m. By what distance A will beat C in a race of 3,240 m? (Assume that the speed of a runner does not change in different races.)

(A) 600 m

(B) 700 m

(C) 720 m

(D) 760 m

7. A running track is in the form of a regular pentagon. A runner covers each side of this pentagonal track with different speeds of 8, 12, 15, 18, and 24 km/h. The average speed K of the runner during the entire journey (in km/hr) lies in which of the following range?

(A) $12 < K < 13$

(B) $13 < K < 14$

(C) $14 < K < 15$

(D) $15 < K < 16$

8. One day Ashu drove at two-thirds of his usual speed and reached his office 10 minutes late. The next day, he drove at his usual speed for 10 minutes, but had to stop for 5 minutes because the railway track had been closed. By what percentage should Ashu increase his speed to reach his office on time?

(A) 50%

(B) 100%

(C) 150%
(D) 200%

9. Ram and Lakhan both are climbing on a moving escalator that is going up. Ram takes 72 steps to reach the top, but Lakhan takes 60 steps to reach the top. Ram can take four steps in a second, while Lakhan can take three steps in a second. Calculate the total number of steps on the escalator.
(A) 360
(B) 240
(C) 180
(D) 120

10. P, Q, R, and S are four friends running on a circular track of circumference 200 km in the anticlockwise direction. Speeds of P, Q, R, and S are 8, 12, 20, and 28 km/h, respectively. What is the difference between the total distance travelled by Q and S, when P, Q, R, and S all together are meeting for the first time at the starting point?
(A) 640 km
(B) 720 km
(C) 800 km
(D) None of these

Level of Difficulty – 2

11. In a 900-m race, Abhishek should give Mudit a head start of 55 seconds and Mudit should give Vikas a head start of 80 seconds, for all of them to finish the race simultaneously. What is the speed of Abhishek if the difference between speeds of Abhishek and Vikas is 15 m/s?
(A) 15 m/s
(B) 18 m/s
(C) 20 m/s
(D) 24 m/s

12. Keerti usually takes 10 hours to ride from Agra to New Delhi. One day her car had a technical issue at Greater Noida, so she has to stop for 20 minutes to resolve the issue. After that, she increases her speed by 25% and reaches New Delhi 20 minutes before her schedule. What is the ratio of the distance between Agra to Greater Noida to Greater Noida to New Delhi?
(A) 3:4
(B) 4:3
(C) 2:1
(D) 1:2

13. Two trains are traveling in opposite directions with speeds of 60 and 45 km/hr, respectively. They take 15 seconds to cross each other. If the two trains are travelling in the same direction, then a person sitting in the faster-moving train would have overtaken the slower train in 40 seconds. Find the length of the trains (in m).
(A) $\frac{1625}{6}$ m, $\frac{500}{3}$ m
(B) $\frac{825}{3}$ m, $\frac{125}{3}$ m
(C) $\frac{119}{3}$ m, $\frac{800}{3}$ m
(D) $\frac{700}{3}$ m, 600 m

14. Two horses 'a' and 'b' start from a point '*x*' on a circle simultaneously with 'a' moving in the clockwise direction and 'b' in the anticlockwise direction. They meet for the first time at 4 p.m., when 'a' has covered 80% of the track. If 'a' returns to '*x*' at 4:40 p.m., then 'b' returns to '*x*' at:
(A) 12:20 a.m.
(B) 2:20 a.m.
(C) 3:00 a.m.
(D) 3:20 a.m.

15. In a mile race, Arjun can be given a start of 240 m by Bhisma. If Bhisma can give Karna a start of 25 m in a 200-m race, then who out of Arjun and Karna will win a race of 7 miles and what will be the final lead taken by the winner over the loser? (1 mile = 1600 m)
(A) Arjun, $\frac{3}{10}$ mile
(B) Arjun, $\frac{1}{5}$ mile

(C) Karna, $\frac{1}{5}$ mile

(D) Karna, $\frac{3}{10}$ mile

16. Five friends, A, B, C, D, and E, are running on a circular track with their speeds in the ratio 1 : 3 : 5 : 7 : 9, respectively. 'A', 'C', and 'E' are running in an anticlockwise direction, while 'B' and 'D' are running in a clockwise direction. Find at how many distinct points will 'A' meet with any other runner if they all run indefinitely?

(A) 8
(B) 10
(C) 12
(D) 24

17. Trains P and Q travel at 180 and 144 km/hr, respectively. Points A and B are marked on the railway track such that AB is a straight line. Time taken by train P to completely cross a stationary pole is 12 seconds. Time taken by trains Q and P to completely cross the line AB from the instant the front end of respective trains cross point A is 40 and 30 sec, respectively. What is the length of the train Q?

(A) 500 m
(B) 600 m
(C) 700 m
(D) 800 m

18. Ram went from Delhi to Shimla via Chandigarh by car. The distance from Delhi to Chandigarh is 4/3 times the distance from Chandigarh to Shimla. The average speed from Delhi to Chandigarh was 1.5 times that from Chandigarh to Shimla. If the average speed for the entire journey was 77 km/hr, what was the average speed from Chandigarh to Shimla (in km/hr)?

(A) 51
(B) 56
(C) 68
(D) 72

19. If two trains A and B leave from their respective stations P and Q at 2 p.m. moving towards each other, they meet at 3:40 p.m. Instead, if train B leaves station Q at 2 p.m. and train A leaves station P at 2:23 p.m., they meet at 4:54 p.m.. Find the ratio of the speeds of train B and train A.

(A) 14 : 9
(B) 9 : 14
(C) 5 : 8
(D) 8 : 5

20. A man is travelling from A to B in a car whose tank's capacity is 22 L, with the reserve level at $\frac{3}{11}^{th}$ of the tank capacity. He fills up the tank and after travelling for 192 km, finds that the fuel is at the reserve level. He fills 4 L more and just manages to cover the remaining distance by using up all the fuel. What is the distance between A and B?

(A) 120 km
(B) 309 km
(C) 322 km
(D) 312 km

Level of Difficulty – 3

21. Rohan can row a boat on still water at a speed of 11 km/hr. However, it takes him 7 hr more to row the boat 36 km upstream than downstream on a given river. One day, Rohan rows the boat on the given river from P to Q, which is *N* km (an integer) upstream from P, then he rows back to P immediately. If he takes at least 9 hr to complete this round trip, what is the minimum value of *N*?

(A) 29
(B) 30
(C) 31
(D) 32

22. A man travels by a motorboat down the river to a temple and back with the speed of the river unchanged. If he doubles the speed of the motorboat, then his total time for the round trip gets reduced by

80%. Find the ratio of the original speed of the motorboat to the speed of the river.

(A) $\sqrt{5} : \sqrt{3}$

(B) $\sqrt{7} : \sqrt{3}$

(C) $\sqrt{6} : \sqrt{5}$

(D) $\sqrt{3} : \sqrt{2}$

23. In a 1.5-km race, Dhoni gives Hardik a head start of 150 m and beats him by 25 seconds. If Dhoni gives Hardik a head start of 40 seconds, then Hardik beats Dhoni by 150 m. Find the ratio of Dhoni's speed to Hardik's speed.

(A) 7 : 4

(B) 19 : 14

(C) 35 : 19

(D) 35 : 22

24. Time taken by a person to reach the first floor from the ground floor on a moving up escalator is 12 seconds less than the time taken when the escalator is turned off. Again, the time taken by the same person to reach the ground floor from the first floor on the same moving up escalator is 48 sec more compared to when the escalator is turned off. If the speed of the person and escalator are in the ratio 5 : 3, respectively, find by what percentage the person should increase his speed to reach the first floor from the ground floor in 20 seconds on a turned-off escalator.

(A) 80%

(B) 60%

(C) 40%

(D) 20%

25. In a race of x km, Rajesh beats Ankit by 25 km and Sahil by 55 km. Also, Ankit beats Sahil by 40 km. Find x.

26. Anand starts from Bombay in his boat with a speed of 9 km/hr and Bimal starts from Pune in his boat with a speed of 11 km/hr. As soon as they reach their destination, they will turn back and start moving towards their initial departure station. The speed of the current is 5 km/hr, and the flow of the current is towards Pune. If the distance between Bombay and Pune is 1,680 km, then what is the distance of their second meeting point from Pune?

(A) 848 km

(B) 872 km

(C) 832 km

(D) 880 km

27. A man can row a certain distance upstream in 15 minutes more than he could row in still water. Also, he can row the same distance downstream in 10 minutes less than he could row it in still water. How long will he take to row down the same distance with the stream?

(A) 50 minutes

(B) 60 minutes

(C) 40 minutes

(D) 70 minutes

28. At his usual speed, Nitin can travel 36 km downstream in a river in 18 hours less than what he takes to travel the same distance upstream. If he doubles the speed of rowing, then he takes 2 hours less time in the downstream trip than what the upstream trip would take. Find the speed of the stream?

(A) $\frac{20}{3}$ km / hr

(B) $\frac{17}{5}$ km / hr

(C) 9.2 km/hr

(D) 9 km/hr

29. Two trains, Amritsar Express and Bhatinda Express, started from Pathankot towards Kolkata at 10 a.m., and Bengal Kranti train is travelling from Kolkata to Pathankot at 55 km/hr. The sum of speeds of Amritsar express and Bhatinda express was 150 km/hr. At 4 p.m., Amritsar Express met Bengal Kranti. Exactly after 1 hour, Bengal Kranti met Bhatinda Express. After this Amritsar Express and Bhatinda Express

interchanged their speeds. At what time the Bhatinda Express will overtake the Amritsar express?

(A) 10:40 a.m.
(B) 11:15 a.m.
(C) 11:40 a.m.
(D) Midnight

30. At his usual rowing rate, Rohan can travel 20 km downstream in a certain river in 2 hours less than it takes him to travel the same distance upstream. But if the speed of the stream becomes twice and his rowing speed remains the same, then downstream 20 km would take 6.4 hours less than the upstream 20 km. Find the value of Rohan's rowing speed (in km/hr).

Solutions

1. (A)

When speed is $\frac{3}{5}$ of usual speed, then the time taken will be $\frac{5}{3}$ times the usual time.

Let usual time = t hours

Therefore, $\frac{5}{3}t = t + \frac{7}{2}$

$\Rightarrow \frac{2}{3}t = \frac{7}{2}$

$\Rightarrow t = \frac{21}{4} = 5\frac{1}{4}$ hours

Hence, option (A) is the correct answer.

2. (D)

Let's assume the usual speed of Shubham is 'x' km/hr and the usual time taken to reach his girlfriend's house with this speed is y hr.

Now, according to the question

$x \times y = 72$ ...(i)

and $(x + 6)(y - 2) = 72$

$\Rightarrow xy - 2x + 6y - 12 = 72$

put $xy = 72$ from equation (i)

$6y - 2x = 12$

$\Rightarrow 6\left(\frac{72}{x}\right) - 2x = 12$

$\Rightarrow 2x^2 + 12x - 432 = 0$

$\Rightarrow x^2 + 6x - 216 = 0$

on solving, we will get $x = 12$ or (-18)

Therefore, the usual speed of Shubham = 12 km/hr

Increased speed = (12 + 6) km/hr

= 18 km/hr = $18 \times \frac{5}{18}$ m/s = 5 m/s

Hence, option (D) is the correct answer.

3. (D)

Length of track = 180 m

Speed of X = 20 m/s

Speed of Y = 18 m/s

They will meet first time at = 180/(20 – 18) = 90 sec.

If 2 persons are moving in the same direction on a circular track with their speeds in the ratio A : B, (A > B), then the number of distinct points where they will meet on the circular track = (A – B).

So, in this question, speed ratio is 20 : 18 = 10 : 9

Thus, they will meet at (10 – 9) = 1 point

Hence, option (D) is the correct answer.

4. (B)

Let the faster train and slower train length be L_1 m and L_2 m.

L_1 = 150 m is given

Since the faster train crosses a lamp post in 15 sec.

$\therefore$ Speed of the faster train

$= \frac{L_1}{15\,\text{sec}} = \frac{150\,\text{m}}{15} = 10$ m/s

Thus, the speed of the slower train

$= \left[10 - \left(18 \times \frac{5}{18}\right)\right]$ m/s = 5 m/s

Also, it is given in the question that the two trains cross each other in 20 sec.

$\therefore$ Relative speed = $(S_1 + S_2)$ m/s = (10 + 5) m/s = 15 m/s

Therefore, $\frac{L_1 + L_2}{20\,\text{sec}} = 15$m/s

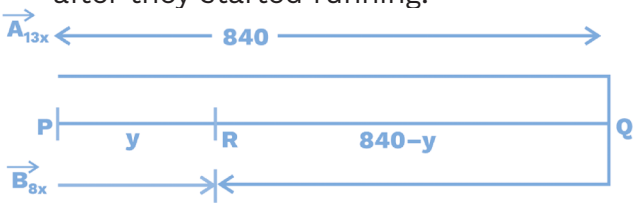
$\Rightarrow 150 + L_2 = 300$ m

$\Rightarrow L_2 = 150$m

Hence, option (B) is the correct answer.

5. (A)

Let's assume the speed of $A = 13x$ m/s and speed of $B = 8x$ m/s and R is the point where they meet for the first time after they started running.



Let's suppose they meet at a distance of y metres from point P.

Now when they meet, the time is the same.

Time $= \frac{D_A}{S_A} = \frac{D_B}{S_B}$

$= \frac{840+840-y}{13x} = \frac{y}{8x}$

$\Rightarrow 8 \times 1{,}680 - 8y = 13y$

$\Rightarrow 21y = 8 \times 1{,}680$

$\Rightarrow y = 640$

Hence, they meet at a distance of 640 m from point P and (840 – 640) = 200 m from Q.

Hence, option (A) is the correct answer.

6. (C)

When A cover 945 m, B will cover (945 – 105) = 840 m at the same time, so the ratio of their speeds would be 945 : 840 = 9 : 8.

When B cover 1,360 m, C will cover (1,360 – 170) = 1,190 m at the same time, so the ratio of their speeds would be = 1,360 : 1190 = 8 : 7.

So, the ratio of speeds of A, B, and C = 9 : 8 : 7.

Now, when A will cover 9 m, C will cover 7 m at the same time.

So, when A will cover (9 × 360 = 3,240) m, C will cover (7 × 360 = 2,520) m

Thus, in a race of 3,240 m, A will beat C by 720 m.

Hence, option (C) is the correct answer.

7. (B)

Let's assume each side of pentagonal track = LCM (8, 12, 15, 18, and 24) = 360 km

Now the time taken by runner to run 360 km each with the speeds 8, 12, 15, 18, and 24 km/hr will be 45, 30, 24, 20, and 15 hours, respectively.

So, average speed $= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{360 \times 5}{(45+30+24+20+15)} = \frac{1{,}800}{134} = 13.43$

approximately.

Hence, option (B) is the correct answer.

8. (B)

Let Ashu's usual speed be 'x' km/min and distance between his home and office be 'd' km and the usual time taken be 't' minutes.

On the first day:

When speed $= \frac{2}{3}x$, time taken $= \frac{3}{2}t$

Now, $\frac{3}{2}t - t = 10$

$\Rightarrow t = 20$ minutes (normal time)

Next day:

For 10 minutes he drove at usual speed.

Distance covered in 20 minutes at usual speed $x = d$

$\Rightarrow$ Distance covered in 10 minutes at usual speed $x = \frac{d}{2}$

Distance left $= \frac{d}{2}$ to be covered in next 10 minutes. But out of 10 minutes, 5 minutes were wasted. So, time left = 5 minutes.

At usual speed, distance covered in 5 minutes $= \frac{d}{4}$

But Ashu has to cover $\frac{d}{2}$ in 5 minutes, which is possible only if he doubles his speed (= $2x$).

Percentage increase in speed should be = 100%.

Hence, option (B) is the correct answer.

9. (C)

Let the speed of the escalator = y steps/sec

Ram's speed = 4 steps/sec

Lakhan's speed = 3 steps/sec

As Ram is taking 72 steps of his own to reach to the top,

Time taken by Ram $= \frac{72 \text{ steps}}{4 \text{ steps/sec}}$

= 18 sec

Similarly, time taken by Lakhan

$= \frac{60 \text{ steps}}{3 \text{ steps / sec}} = 20$ sec

Now distance = Total steps =

$\overbrace{S_1 \times T_1}^{\text{Ram}} = \overbrace{S_2 \times T_2}^{\text{Lakhan}}$

Total steps = $(4 + y) \times 18 = (3 + y) \times 20$

$\Rightarrow 72 + 18y = 60 + 20y$

$\Rightarrow 2y = 12$

$\Rightarrow y = 6$

Total steps = $(3 + y) \times 20 = 9 \times 20 = 180$

Hence, option (C) is the correct answer.

10. (C)

As we know that P, Q, R, and S have to meet at the starting point; therefore, time to meet them can be given by

$t = \text{LCM}(t_P, t_Q, t_R, t_S)$

$= \text{LCM}\left(\frac{200}{8}, \frac{200}{12}, \frac{200}{20}, \frac{200}{28}\right)$

$= \frac{\text{LCM}(200, 200, 200, 200)}{\text{HCF}(8, 12, 20, 28)} = \frac{200}{4} = 50 \text{ hr}$

Now, total distance travelled by Q in 50 hours = $12 \times 50 = 600$ km.

Similarly, total distance travelled by S in 50 hours = $28 \times 50 = 1{,}400$ km.

Therefore, the required difference = $1{,}400 - 600 = 800$ km.

Hence, option (C) is the correct answer.

11. (C)

Let the speed of Abhishek be 's' m/s and the time taken by him to finish race be 't' seconds.

Now,

	Abhishek	Mudit	Vikas
Time	t	$t + 55$	$t + 135$
Speed	s		$s - 15$

Since all completed the race

$s \times t = 900$ ...(i)

$(s - 15)(t - 135) = 900$ ...(ii)

From equations (i) and (ii)

$s \times t = (s - 15)(t - 135)$

$s \times t = s \times t - 15t + 135s - 15 \times 135$

$t = 9s - 135$.

Putting this in equation (i):

$s(9s - 135) = 900$

$s(s - 15) = 100$

Therefore, $S = 20$ m/s.

Hence, option (C) is the correct answer.

12. (C)

25% increase means increase of 1 unit over 4 units

Let the usual speed = $4x$ km/hr

Increased speed = $5x$ km/hr

Total distance = speed × time = $4x \times 10 = 40x$ km.

Time taken by Keerti if she travels the whole distance at increased speed

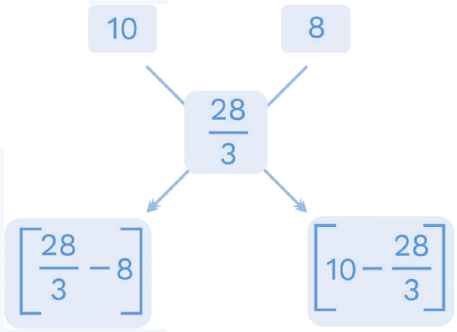
Distance/Speed = $40x/5x = 8$ hours

Now, the average time is taken by her to cover the whole distance

= 10 hr – 20 min – 20 min = 9 hr 20 min

$= \frac{28}{3}$ hr

By using the rule of allegation, we get.



$\frac{\text{Distance from agra to Greater Noida}}{\text{Distance from Greater Noida to New Delhi}} = \frac{\frac{28}{3} - 8}{10 - \frac{28}{3}}$

$= \frac{\frac{4}{3}}{\frac{2}{3}} = \frac{2}{1}$

Hence, the required ratio will be 2 : 1.

Hence, option (C) is the correct answer.

13. (A)

Let the length of the faster and slower trains be L_1 and L_2.

When the two trains are traveling in the opposite direction, they cross each other in 15 sec.

Relative speed = $(S_1 + S_2) = (45 + 60) \times \frac{5}{18}$

$= \frac{105}{18} \times 5 = \frac{175}{6}$ m/s

Time = Distance/Speed = $\frac{L_1 + L_2}{\frac{175}{6}} = 15$

$\Rightarrow L_1 + L_2 = 15 \times \frac{175}{6} = 437.5$ m

The person sitting in the faster train would have overtaken the slower train when the train was moving in the same direction in 40 seconds.

Therefore, relative speed (in the same direction) = $S_1 - S_2 = 60 - 45 = 15$ km/hr

$= 15 \times \frac{5}{18} = \frac{75}{18}$ m/s $= \frac{25}{6}$ m/s

So, $\frac{L_2}{\frac{25}{6}} = 40$ sec

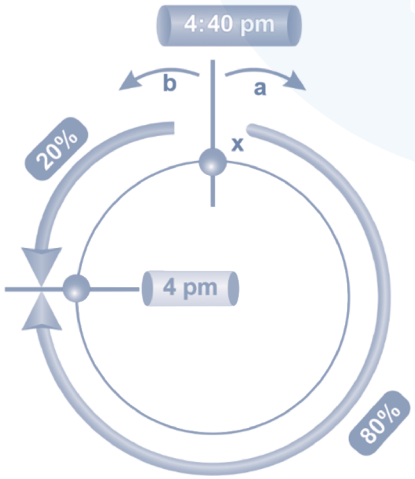
$\Rightarrow L_2 = 40 \times \frac{25}{6} = \frac{20 \times 25}{3} = \frac{500}{3}$ m.

Since, $L_1 + L_2 = 437.5$, hence, $L_1 = 437.5 - \frac{500}{3}$

$= \frac{812.5}{3} = \frac{1625}{6}$ m

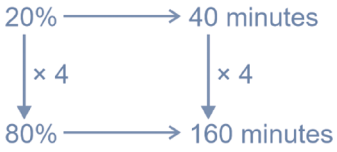
Hence, option (A) is the correct answer.

14. (D)



When horses 'a' and 'b' met for the first time at 4 pm, horse 'a' had covered 80% of the whole track and horse 'b' had covered 20% of the track.

It is given that horse 'a' returns to point *x* at 4:40 pm. Therefore, horse 'a' has covered 20% of the track in 40 minutes.



Therefore,

Horse 'b' covered 20% of the track when horse 'a' covered 80% of the track.

Thus, horse 'b' covers 20% of the track in 160 minutes.

As 'b' had covered 20% of the track till the first meeting and now he has to cover 80% of the track more to reach at 'x', thus, horse 'b' will return to 'x' at = 4:40 pm + 640 min = 3:20 a.m.

Hence, option (D) is the correct answer.

15. (C)

Arjun was given a start of 240 m by Bhisma.

Here time is constant.

Therefore, ratio of distance covered = ratio of speeds.

$\frac{\text{Bhisma}}{\text{Arjun}} = \frac{1600}{1360} = \frac{20}{17}$

Similarly, when Bhisma gives a start of 25 m to Karna, then the ratio can be given by

$\frac{\text{Bhisma}}{\text{Karna}} = \frac{200}{175} = \frac{8}{7}$

Now,

Bhisma	:	Arjun	:	Karna
2×(20)	:	2×(17)		
5×(8)	:		:	5×(7)
40	:	34	:	35

Let's assume the speeds of Bhisma, Arjun, and Karna be 40, 34, and 35 m/s. So, in 1 sec, Karna will take 1 m lead over Arjun.

Now time taken by Karna to cover 7 miles = (7 x 1,600)/35 = 320 sec
Thus in 320 sec, Karna will take a lead of 320 m or 1/5 mile over Arjun.
Hence, option (C) is the correct answer.

16. (A)

We should know that if two objects are moving in a circular track with their speeds in the ratio a : b, then the number of distinct points at which they meet when they run indefinitely can be given by:
a) When they move in the opposite direction, the number of distinct points = $a + b$ and, b) When they move in the same direction, the number of distinct points = $|a - b|$.
Now, in this case 'A' has to meet with any other runner:
$\uparrow A : B \downarrow = 1 : 3 \Rightarrow$ 4 distinct points
$\uparrow A : C \uparrow = 1 : 5 \Rightarrow$ 4 distinct points
$\uparrow A : D \downarrow = 1 : 7 \Rightarrow$ 8 distinct points
$\uparrow A : E \uparrow = 1 : 9 \Rightarrow$ 8 distinct points

$$A \to B\left(4\text{points}\right) \Rightarrow L^*, \frac{3L^*}{4}, \frac{2L^*}{4}, \frac{L^*}{4}$$

$$A \to C\left(4\text{points}\right) \Rightarrow L, \frac{3L}{4}, \frac{2L}{4}, \frac{L}{4}$$

$$A \to D\left(8\text{points}\right) \Rightarrow L, \frac{7L^*}{8}, \frac{6L}{8}, \frac{5L^*}{8}, \frac{4L}{8}, \frac{3L^*}{8}, \frac{2L}{8}, \frac{L^*}{8}$$

$$A \to E\left(8\text{points}\right) \Rightarrow L, \frac{7L}{8}, \frac{6L}{8}, \frac{5L}{8}, \frac{4L}{8}, \frac{3L}{8}, \frac{2L}{8}, \frac{L}{8}$$

All-star (*) marked points in the above explanation are the distinct points.
Hence, option (A) is the correct answer.

17. (C)

Speed of train P = 180 km/hr = 180 × 5/18 = 50 m/s.
Train P crosses a stationery pole in 12 seconds,
So, length of the train P = 50 × 12 = 600 m.
Now train P completely crossed the line segment AB in 30 sec, so the distance travelled by train P to cross AB = length of train P + length of line segment AB
AB + 600 = 50 m/s × 30 sec = 1,500 m
$\Rightarrow$ AB = 900 m
Similarly, train Q completely crossed the line segment AB in 40 seconds, so the distance travelled by train Q to cross AB = length of train Q + length of line segment AB = length of train Q + 900
900 + length of train Q = 144 × 5/18 × 40
$\Rightarrow$ length of train Q = 700 m
Hence, option (C) is the correct answer.

18. (C)

Let's assume the distance between Delhi and Chandigarh = $4x$
Hence, the distance between Chandigarh and Shimla = $3x$

Let's assume the speed from Chandigarh to Shimla = y km/hr.
So, speed from Delhi to Chandigarh = 1.5 y km/hr.
Average speed for the journey = $\frac{\text{Total Distance}}{\text{Total Time}}$

$$\Rightarrow \frac{7x}{\frac{4x}{1.5y} + \frac{3x}{y}} = 84$$

$$\Rightarrow \frac{7}{\frac{8}{3y} + \frac{3}{y}} = \frac{7}{\frac{1}{y}\left(\frac{8}{3} + 3\right)} = \frac{7}{\frac{1}{y} \times \frac{17}{3}} = 84$$

$$\Rightarrow \frac{7 \times 3y}{17} = 84 \Rightarrow y = 68$$

Hence, option (C) is the correct answer.

19. (B)

Let speed of train A and train B be 'a' m/min and 'b' m/min.
Since, trains are meeting at 3:40 p.m. after they departed simultaneously at 2 p.m.
Therefore, total distance covered by both the trains together in 100 min = 100 $(a + b)$. ...(i)

If train 'A' leaves at 2:23 p.m., then the distance covered by B in this time interval = 23b.

Now, the total distance covered by both the trains together in 91 min = 91 (a + b) ...(ii)

From equations (i) and (ii)

Total distance = 100a + 100b = 23b + 91a + 91b

$\Rightarrow 9a = 14b$

$\Rightarrow \frac{a}{b} = \frac{14}{9}$

Thus, the ratio of speed of train B to train A = 9 : 14.

Hence, option (B) is the correct answer.

20. (D)

Given: The capacity of the tank = 22 L

The reserve level $\frac{3}{11}$th of the tank capacity which is equal to $\frac{66}{11}$ L.

$\therefore$ He travels 192 km with $\left(22 - \frac{66}{11}\right)$ L

$\therefore$ The mileage of a car = $\frac{192}{\left(22 - \frac{66}{11}\right)}$

$= \frac{192}{16} = 12$ kmpL

He fills 4 more litres and covers the remaining distance.

$\therefore$ The distance covered after refilling the tank = $\left(4 + \frac{66}{11}\right) \times 12$

= 10 × 12 = 120 km.

$\therefore$ The total distance between A and B = 192 + 120 = 312 km

Hence, option (D) is the correct answer.

21. (B)

Let's assume the speed of the river = x km/hr

According to the questions

$\frac{36}{11 - x} - \frac{36}{11 + x} = 7$

$\Rightarrow \frac{36(11 + x - 11 + x)}{121 - x^2} = 7$

$\Rightarrow 36(2x) = 847 - 7x^2$

$\Rightarrow 7x^2 + 72x - 847 = 0$

$\Rightarrow 7x^2 + 121x - 49x - 847 = 0$

$\Rightarrow x\,(x + 121) - 7(7x + 121)$

$\Rightarrow (7x + 121)\,(x - 7) = 0$

$\Rightarrow x = 7$

Now, according to the question

$\frac{N}{11 + 7} + \frac{N}{11 - 7} \geq 9$

$\Rightarrow \frac{N}{18} + \frac{N}{4} \geq 9$

$\Rightarrow \frac{2N + 9N}{36} \geq 9$

$\Rightarrow \frac{11N}{36} \geq 9$

$\Rightarrow 11N \geq 324$

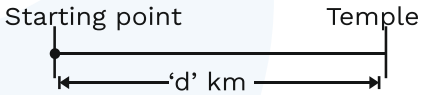
$\Rightarrow N \geq 29.45$

So, the minimum value of N = 30

Hence, option (B) is the correct answer.

22. (D)

Let the initial speed of the motorboat is u km/hr and the speed of the river is v km/hr and the one-way distance is d km.



According to condition 1

$\frac{d}{u + v} + \frac{d}{u - v} = t$ (i)

Now according to the second condition:

$\frac{d}{2u + v} + \frac{d}{2u - v} = t \times 20\%$

$\Rightarrow \frac{d}{2u + v} + \frac{d}{2u - v} = \frac{t}{5}$

$\Rightarrow \frac{5d}{2u + v} + \frac{5d}{2u - v} = t$ (ii)

Now equating equations (i) and (ii)

$\Rightarrow \frac{d}{u + v} + \frac{d}{u - v} = \frac{5d}{2u + v} + \frac{5d}{2u - v}$

$\Rightarrow \frac{1}{u + v} + \frac{1}{u - v} = \frac{5}{2u + v} + \frac{5}{2u - v}$

$\Rightarrow \frac{u - v + u + v}{u^2 - v^2} = \frac{5(2u - v + 2u + v)}{4u^2 - v^2}$

$\Rightarrow \frac{2u}{u^2 - v^2} = \frac{5(4u)}{4u^2 - v^2}$

$$\Rightarrow \frac{1}{u^2 - v^2} = \frac{10}{4u^2 - v^2}$$
$$\Rightarrow 4u^2 - v^2 = 10u^2 - 10v^2$$
$$\Rightarrow 6u^2 = 9v^2$$
$$\Rightarrow \frac{u^2}{v^2} = \frac{9}{6}$$
$$\Rightarrow \frac{u}{v} = \sqrt{\frac{9}{6}}$$
$$\Rightarrow \frac{u}{v} = \sqrt{\frac{3}{2}}$$
$$\Rightarrow \frac{u}{v} = \frac{\sqrt{3}}{\sqrt{2}}$$

Hence, option (D) is the correct answer.

23. (D)

For simplicity, we can represent the information as follows:

	Case I		Case II	
	Dhoni	Hardik	Dhoni	Hardik
Distance	1,500	1,350	1,350	1,500
Time	t_1	$t_1 + 25$	t_2	$t_2 + 40$

Here, speed of Dhoni is $= \frac{1,500}{t_1} = \frac{1,350}{t_2}$

$$\text{or } t_1 = \frac{10}{9} t_2$$

Also, speed of Hardik is

$$= \frac{1,350}{t_1 + 25} = \frac{1,500}{t_2 + 40}$$

$$\text{or } \frac{9}{t_1 + 25} = \frac{10}{t_2 + 40}$$

$$\text{or } 9t_2 + 360 = 10t_1 + 250$$

$$\text{or } 9t_2 + 360 = 10 \times \left(\frac{10}{9} t_2\right) + 250$$

$$\text{or } \frac{19}{9} t_2 = 110 \text{ or } t_2 = \frac{990}{19}$$

Now, ratio of speed $= \frac{\text{Speed of Dhoni}}{\text{Speed of Hardik}}$

$$= \frac{\left(\frac{1,500}{t_1}\right)}{\left(\frac{1,500}{t_2 + 40}\right)} = \frac{t_2 + 40}{t_1} = \frac{\left(\frac{990}{19}\right) + 40}{\left(\frac{10}{9} \times \frac{990}{19}\right)} = \frac{1,750}{1,100}$$

$$= \frac{35}{22}$$

Hence, option (D) is the correct answer.

24. (B)

Let's assume the speed of the person and the escalator be $5K$ steps/sec and $3K$ steps/sec respectively, and the total steps on the escalator be D.

According to the first statement

$$\frac{D}{(5K)} - \frac{D}{5K + 3K} = 12$$

$$\Rightarrow \frac{D}{5K} - \frac{D}{8K} = 12 \quad \ldots(i)$$

Also, according to the second statement

$$\frac{D}{(5K - 3K)} - \frac{D}{5K} = 48$$

$$\Rightarrow \frac{D}{2K} - \frac{D}{5K} = 48 \quad \ldots(ii)$$

On adding equations (ii) + (i)

$$\frac{D}{2K} - \frac{D}{8K} = 60$$

$$\Rightarrow \frac{D(4 - 1)}{8K} = 60$$

$$\Rightarrow D = 160K$$

Now to cover a distance of 160 K steps in 20 seconds on a turned-off escalator the person's speed must be $\frac{160K}{20} = 8K$ steps / sec

Person's original speed = $5K$ step/sec hence, percent increase in speed should be $= \frac{3K}{5K} \times 100 = 60\%$

Hence, option (B) is the correct answer.

25. 100

Distance of x km is given.

Case 1:

Now, when Rajesh finished all of x km, Ankit has finished $(x - 25)$ km and Sahil finished only $(x - 55)$ km.

x km. ⟶ by Rajesh

(x-25) km ⟶ by Ankit

(x-55) km ⟶ by Sahil

Case 2:

When Ankit finished × km, Sahil finished only $(x - 40)$ km.

x km. ⟶ by Ankit

(x - 40)km ⟶ by Sahil

Ratio of speeds of Rajesh and Ankit $= \frac{x}{x-25}$

Ratio of speeds of Rajesh and Sahil $= \frac{x}{x-55}$

Ratio of speeds of Ankit and Sahil $= \frac{x}{x-40}$

(Ratio of speeds of Rajesh and Ankit) × (Ratio of speed of Ankit and Sahil)

= (Ratio of speeds of Rajesh and Sahil)

$\Rightarrow \frac{x}{x-25} \cdot \frac{x}{x-40} = \frac{x}{x-55}$

$\Rightarrow \frac{x}{x-25} = \frac{x-40}{x-55}$

We know that if $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{b} = \frac{c-a}{d-b}$

$\Rightarrow \frac{x}{x-25} = \frac{x-40-x}{x-55-x+25}$

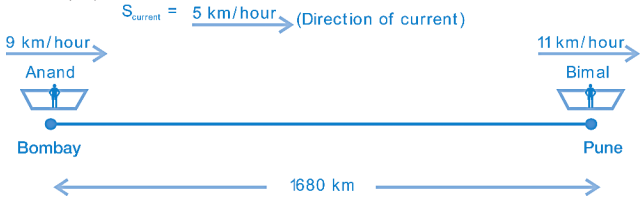
$\Rightarrow \frac{x}{x-25} = \frac{-40}{-30}$

$\Rightarrow 3x = 4(x - 25)$

$\Rightarrow 3x = 4x - 100$

$\Rightarrow x = 100$ km.

26. (A)



For the first meeting

Anand's downstream speed = (9 + 5) = 14 km/hr.

Bimal's upstream speed = (11 – 5) = 6 km/hr.

Therefore, time taken for the first meeting $= \frac{1,680}{20} = 84$ hr .

Distance travelled by Anand = 14 × 84 = 1,176 km.

Hence, Distance travelled by Bimal = 1,680 – 1,176 = 504 km.

Now, time taken to reach Pune for Anand $= \frac{504}{14} = 36$hr

Time taken to reach Bombay for Bimal $= \frac{1,176}{6} = 196$hr

Time difference = 196 – 36 = 160 hr

In this time period, Anand will start from Pune and travel to Bombay.

Therefore, distance covered by Anand = (9 – 5) × 160 = 4 × 160 = 640 km

Now, distance between them = 1,680 – 640 = 1,040 km.

For the second meeting

Now, this is the moment when Bimal will start from Bombay towards Pune.

Distance between Anand and Bimal = 1,040 km.

Speed of Anand = (9 – 5) = 4 km/hr

Speed of Bimal = (11 + 5) = 16 km/hr

Therefore, time taken for second meeting $= \frac{1,040}{(16+4)} = \frac{1,040}{20} = 52$ hr

Distance travelled by Bimal in this time period = 16 × 52 = 832 km

Therefore, distance of the second meeting point from Pune = 1,680 – 832 = 848 km.

Hence, option (A) is the correct answer.

27. (A)

Let's assume the speed of the boat in still water is B and the speed of stream is R and the normal time taken by the boat to travel the given distance in still water = t minutes

	Downstream	Upstream	In still water
Time	$(t - 10)$ min	$(t + 15)$ min	t min
Speed	$B + R$	$B - R$	B

As we can see that speeds are in arithmetic progression, so time would be in harmonic progression.

So, the time required in still water.

= harmonic mean of ($t + 15$ min and $t - 10$ min)

$$\Rightarrow t = \frac{2 \times (t+15) \times (t-10)}{(t+15) + (t-10)}$$

$$\Rightarrow t \times (2t + 5) = 2t^2 + 10t - 300$$

$$\Rightarrow 2t^2 + 5t = 2t^2 + 10t - 300$$

$$\Rightarrow 5t = 300$$

$$\Rightarrow t = 60$$

Thus, the time needed to row down the same distance $= (t - 10) = (60 - 10) = 50$ minutes.

Hence, option (A) is the correct answer.

28. (A)

Let the usual speed of Nitin is u km/hr and the speed of current or stream is v km/hr.

So, according to the conditions given in the question

$$\frac{36}{u-v} - \frac{36}{u+v} = 18 \quad \text{...(i)}$$

Again, if Nitin doubles his speed, he takes 2 hours less.

$$\frac{36}{2u-v} - \frac{36}{2u+v} = 2 \quad \text{...(ii)}$$

Let $u = mv$

Then, the first equation becomes

$$\frac{36}{mv-v} - \frac{36}{mv+v} = 18$$

$$\Rightarrow \frac{1}{m-1} - \frac{1}{m+1} = \frac{v}{2}$$

$$\Rightarrow \frac{m+1-m+1}{m^2-1} = \frac{v}{2}$$

$$\Rightarrow \frac{2}{m^2-1} = \frac{v}{2} \quad \text{...(iii)}$$

Similarly, the second equation will become

$$\frac{36}{2mv-v} - \frac{36}{2mv+v} = 2$$

$$\Rightarrow \frac{1}{2m-1} - \frac{1}{2m+1} = \frac{v}{18}$$

$$\Rightarrow \frac{2m+1-2m+1}{4m^2-1} = \frac{v}{18}$$

$$\Rightarrow \frac{2}{4m^2-1} = \frac{v}{18} \quad \text{...(iv)}$$

Now, we have to divide equation (iii) by (iv), and we will get

$$\frac{\frac{2}{m^2-1}}{\frac{2}{4m^2-1}} = \frac{\frac{v}{2}}{\frac{v}{18}}$$

$$\Rightarrow \frac{4m^2-1}{m^2-1} = 9$$

$$\Rightarrow 4m^2 - 1 = 9m^2 - 9$$

$$\Rightarrow 5m^2 = 8$$

$$\Rightarrow m^2 = \frac{8}{5}$$

$$\Rightarrow m = \sqrt{\frac{8}{5}}$$

Now put the value of $m = \sqrt{\frac{8}{5}}$ in equation (iii)

$$\frac{2}{\left(\sqrt{\frac{8}{5}}\right)^2 - 1} = \frac{v}{2}$$

$$\Rightarrow \frac{4}{\frac{8}{5} - 1} = \frac{4 \times 5}{3} = \frac{20}{3} \quad \text{...(v)}$$

Hence, the speed of the stream is $\frac{20}{3}$ km / hr.

Hence, option (A) is the correct answer.

29. (D)

Let Amritsar express and Bhatinda express travel with speeds x km/hr and y km/hr

So, $x + y = 150$ (i)

Now, Amritsar express met Bengal Kranti at 4 p.m.

So, Amritsar express is 6x km away from Pathankot at 4 p.m.
And Bhatinda express and Bengal Kranti meet at 5 p.m.
So, Bhatinda express is 7y km away from Pathankot at 5 p.m.
Difference between those 2 points is 55 km as in 1 hr Bengal Kranti travelled 55 km.
So, $6x - 7y = 55$ (ii)
Solving equations (i) and (ii), we get
x = 85 km/hr and y = 65 km/hr.
In these 7 hours, Amritsar Express travelled 595 km and Bhatinda Express has travelled 455 km.
Therefore, their relative distance is 140 km and their relative speed is 20 km/hr.
So, Bhatinda Express will take next 7 hour to overtake Amritsar Express.
They will meet at 12 am, i.e., midnight.
Hence, option (D) is the correct answer.

30. 7.5

Let's assume Rohan's rowing speed = A km/hr and stream's usual flow speed = B km/hr

First case: $\frac{20}{A-B} - \frac{20}{A+B} = 2$

Simplifying the above equation, we will get $B = (A^2 - B^2)/20$ (i)

Second case: $\frac{20}{A-2B} - \frac{20}{A+2B} = 6.4$

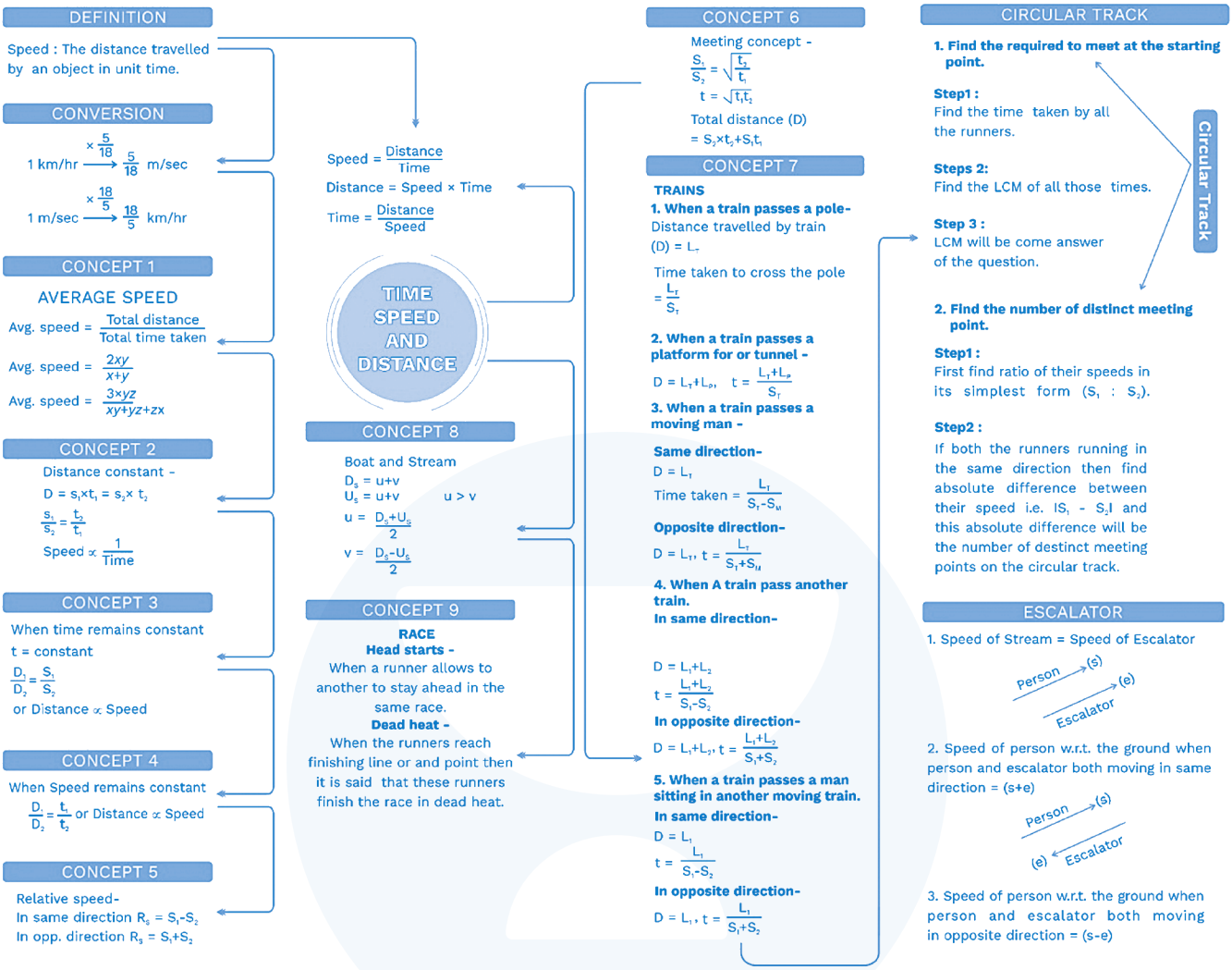
Simplifying the above equation, we will get $B = [6.4\ (A^2 - 4B^2)]/80$ (ii)

On equating (i) and (ii) and after simplification, we will get $A : B = 3 : 1$.

Now taking A = 3K and $B = K$ and putting the values of A and B in equation (i), we will get $B = K = 2.5$.

Thus, Rohan's rowing speed = $A = 3K = 3 \times 2.5 = 7.5$ km/hr.

Mind Map


DEFINITION
Speed : The distance travelled by an object in unit time.
CONVERSION
1 km/hr × 5/18 → 5/18 m/sec
1 m/sec × 18/5 → 18/5 km/hr
CONCEPT 1
AVERAGE SPEED
Avg. speed = Total distance / Total time taken
Avg. speed = 2xy/(x+y)
Avg. speed = 3×yz/(xy+yz+zx)
CONCEPT 2
Distance constant -
D = s1xt1 = s2x t2
s1/s2 = t2/t1
Speed ∝ 1/Time
CONCEPT 3
When time remains constant
t = constant
D1/D2 = S1/S2
or Distance ∝ Speed
CONCEPT 4
When Speed remains constant
D1/D2 = t1/t2 or Distance ∝ Speed
CONCEPT 5
Relative speed-
In same direction Rs = S1-S2
In opp. direction Rs = S1+S2
Speed = Distance / Time
Distance = Speed × Time
Time = Distance / Speed
TIME SPEED AND DISTANCE
CONCEPT 8
Boat and Stream
Ds = u+v
Us = u+v u > v
u = (Ds+Us)/2
v = (Ds-Us)/2
CONCEPT 9
RACE
Head starts -
When a runner allows to another to stay ahead in the same race.
Dead heat -
When the runners reach finishing line or and point then it is said that these runners finish the race in dead heat.
CONCEPT 6
Meeting concept -
S1/S2 = √(t2/t1)
t = √(t1t2)
Total distance (D)
= S2xt2+S1t1
CONCEPT 7
TRAINS
1. When a train passes a pole-
Distance travelled by train (D) = LT
Time taken to cross the pole = LT/ST
2. When a train passes a platform for or tunnel -
D = LT+LP, t = (LT+LP)/ST
3. When a train passes a moving man -
Same direction-
D = LT
Time taken = LT/(ST-SM)
Opposite direction-
D = LT, t = LT/(ST+SM)
4. When A train pass another train.
In same direction-
D = L1+L2
t = (L1+L2)/(S1-S2)
In opposite direction-
D = L1+L2, t = (L1+L2)/(S1+S2)
5. When a train passes a man sitting in another moving train.
In same direction-
D = L1
t = L1/(S1-S2)
In opposite direction-
D = L1, t = L1/(S1+S2)
CIRCULAR TRACK
1. Find the required to meet at the starting point.
Step1 :
Find the time taken by all the runners.
Steps 2:
Find the LCM of all those times.
Step 3 :
LCM will be come answer of the question.
Circular Track
2. Find the number of distinct meeting point.
Step1 :
First find ratio of their speeds in its simplest form (S1 : S2).
Step2 :
If both the runners running in the same direction then find absolute difference between their speed i.e. IS1 - S2I and this absolute difference will be the number of destinct meeting points on the circular track.
ESCALATOR
1. Speed of Stream = Speed of Escalator
Person (s) (e) Escalator
2. Speed of person w.r.t. the ground when person and escalator both moving in same direction = (s+e)
Person (s) (e) Escalator
3. Speed of person w.r.t. the ground when person and escalator both moving in opposite direction = (s-e)