1 Solve graphically

Find two positive numbers that minimize the sum of twice the first number plus second if product of two is 288.

- Assume that the first and second numbers are a and b, respectively.
- Using the given information, we already know:

$$\begin{aligned} a &> 0 \\ b &> 0 \\ ab &= 288 \\ b &= \frac{288}{a} \\ f(x) &= 2a + b \\ f(x) &= 2a + \frac{288}{a} \end{aligned}$$

and we need to find out the number a that minimizes the function:

$$f(x) = 2a + \frac{288}{a}$$

We can tell from the below graph that f(x) has a local minimum of $(12, 48)$ and since $a > 0$ in this case, it is also the global minimum of $f(x)$.

$$a = 12$$

$$b = \frac{288}{a}$$

$$b = \frac{288}{12}$$

$$b = 24$$

The two positive numbers the minimize the sum of twice the first number plus the second number if the product of the two is 288 is: $a = 12, b = 24$
ANSWER: $a = 12, b = 24$

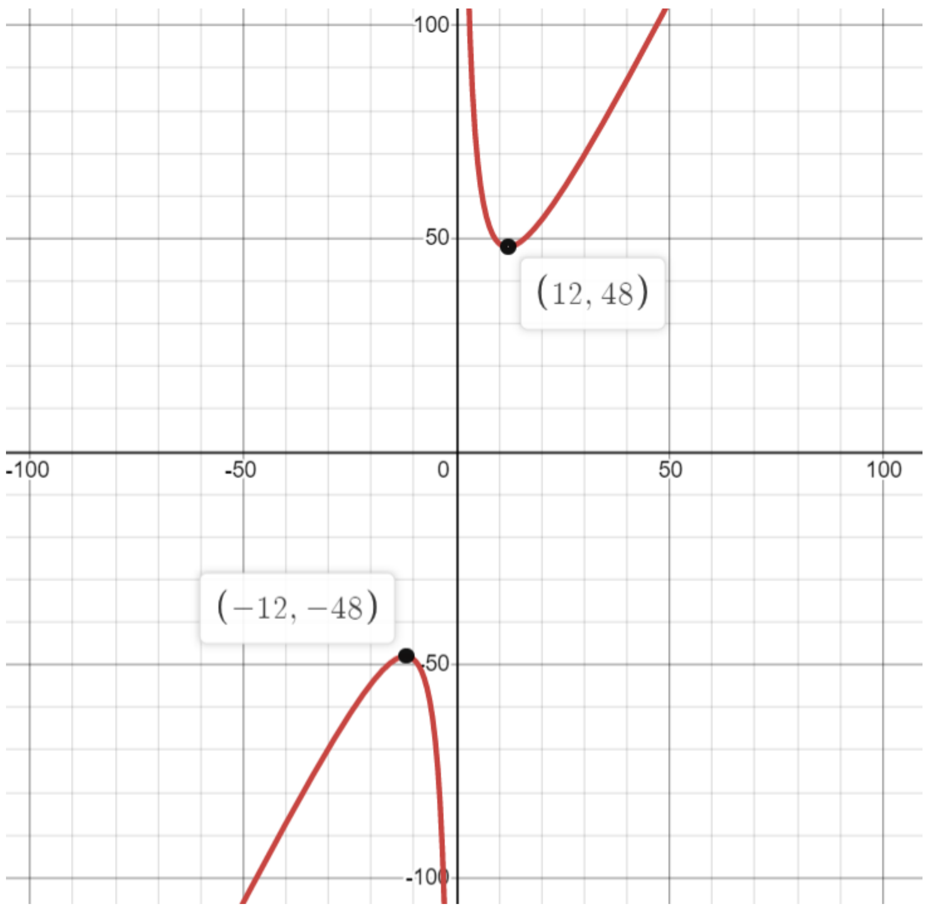

Figure 1: Graph of $f(x) = 2a + \frac{288}{a}$

2 Solve algebraically

Find two positive numbers whose product is 750 and for which the sum of one and 10 times the other is a minimum.

- Assume that the first and second numbers are a and b, respectively.
- Using the given information, we already know:

$$\begin{aligned} a &> 0 \\ b &> 0 \\ ab &= 750 \\ b &= \frac{750}{a} \\ f(x) &= a + 10b \\ f(x) &= a + \frac{7500}{a} \end{aligned}$$

To algebraically determine the minimum in this case, we have to find which point on the function does its tangent line have a slope of zero. Although this sounds fairly complicated, this point is found by finding the derivative of the function and setting it equal to zero since the derivative $\frac{d}{dx}$ represents rate of change at any given point and we're looking for a slope of zero at a particular point.

$$
\begin{aligned}
f(a) &= a + 7500a^{-1} \\
\frac{d}{da}f(a) &= \frac{d}{da}(a) + 7500 * \frac{d}{da}(a^{-1}) \\
\frac{d}{da}f(a) &= 1 + 7500 * (-a^{-2}) \\
\frac{d}{da}f(a) &= 1 + \frac{-7500}{a^2} \\
\frac{d}{da}f(a) &= 0 \\
1 + \frac{-7500}{a^2} &= 0 \\
\frac{-7500}{a^2} &= -1 \\
-7500 &= (-a)^2 \\
a &= 50\sqrt{3} \\
a &= -50\sqrt{3}[rejected]
\end{aligned}
$$

$$
\begin{aligned}
b &= \frac{750}{50\sqrt{3}} \\
b &= 5\sqrt{3}
\end{aligned}
$$

The answer to the question, "What two positive numbers whose product is 750 and for which the sum of one and 10 times the other is a minimum?" is: $a = 50\sqrt{3}, b = 5\sqrt{3}$.