Chaos and Logistic Map : part2

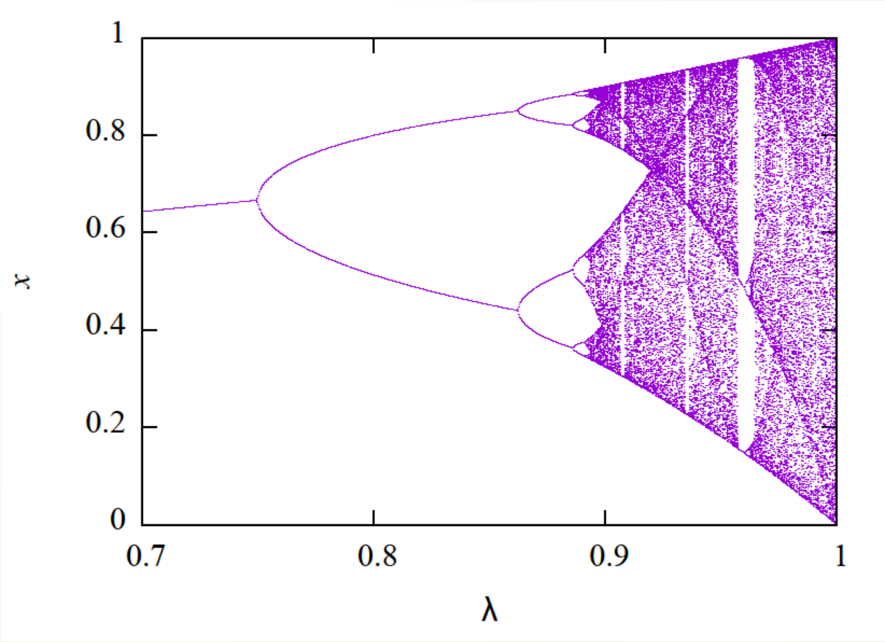
モデリングとシミュレーション特論

2019年度

只木進一

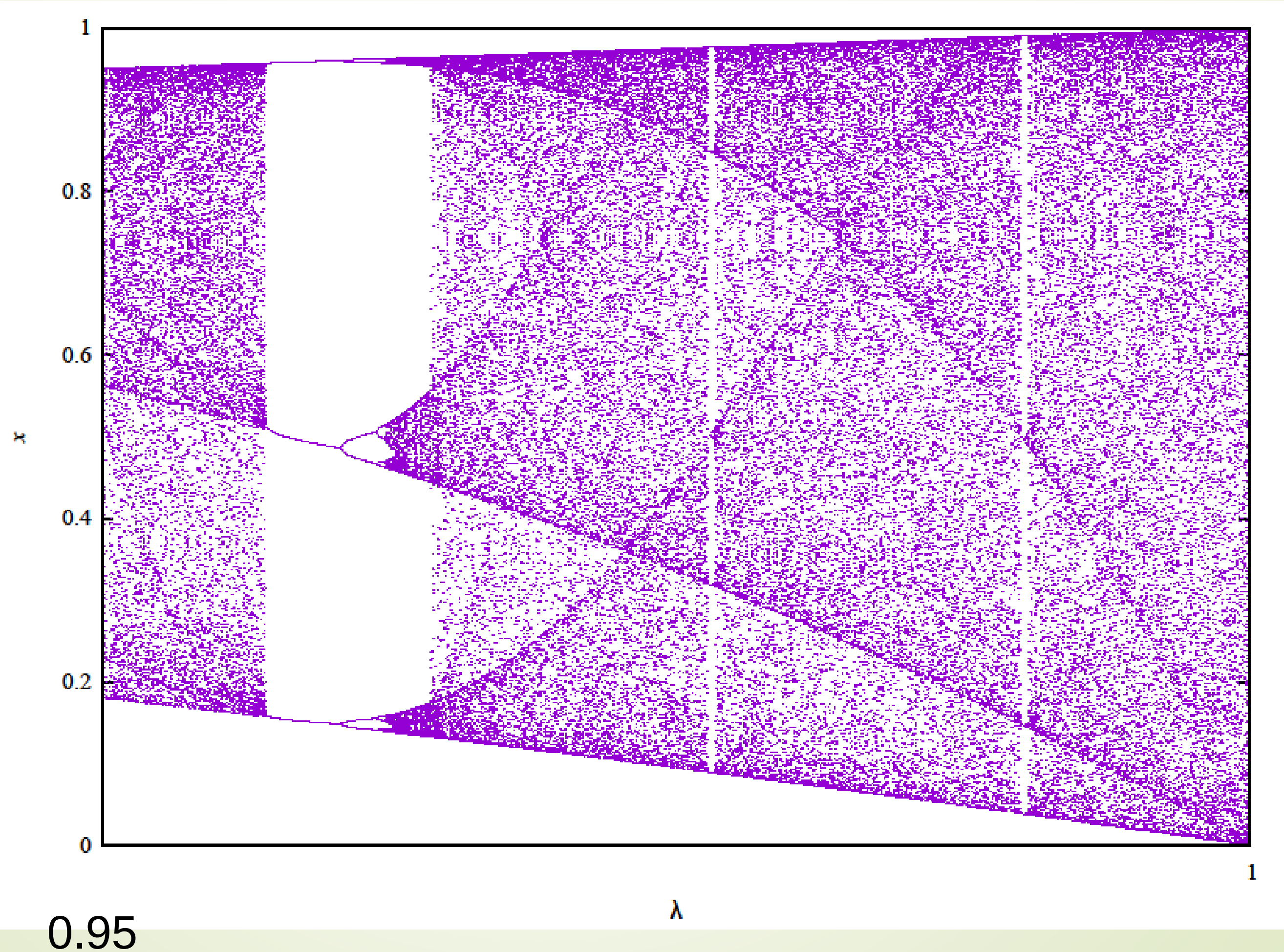
Period doubling to chaos

- Trajecctories are doubled repeatedly by increasing λ
- Period becomes infinite at $\lambda \approx 0.893$

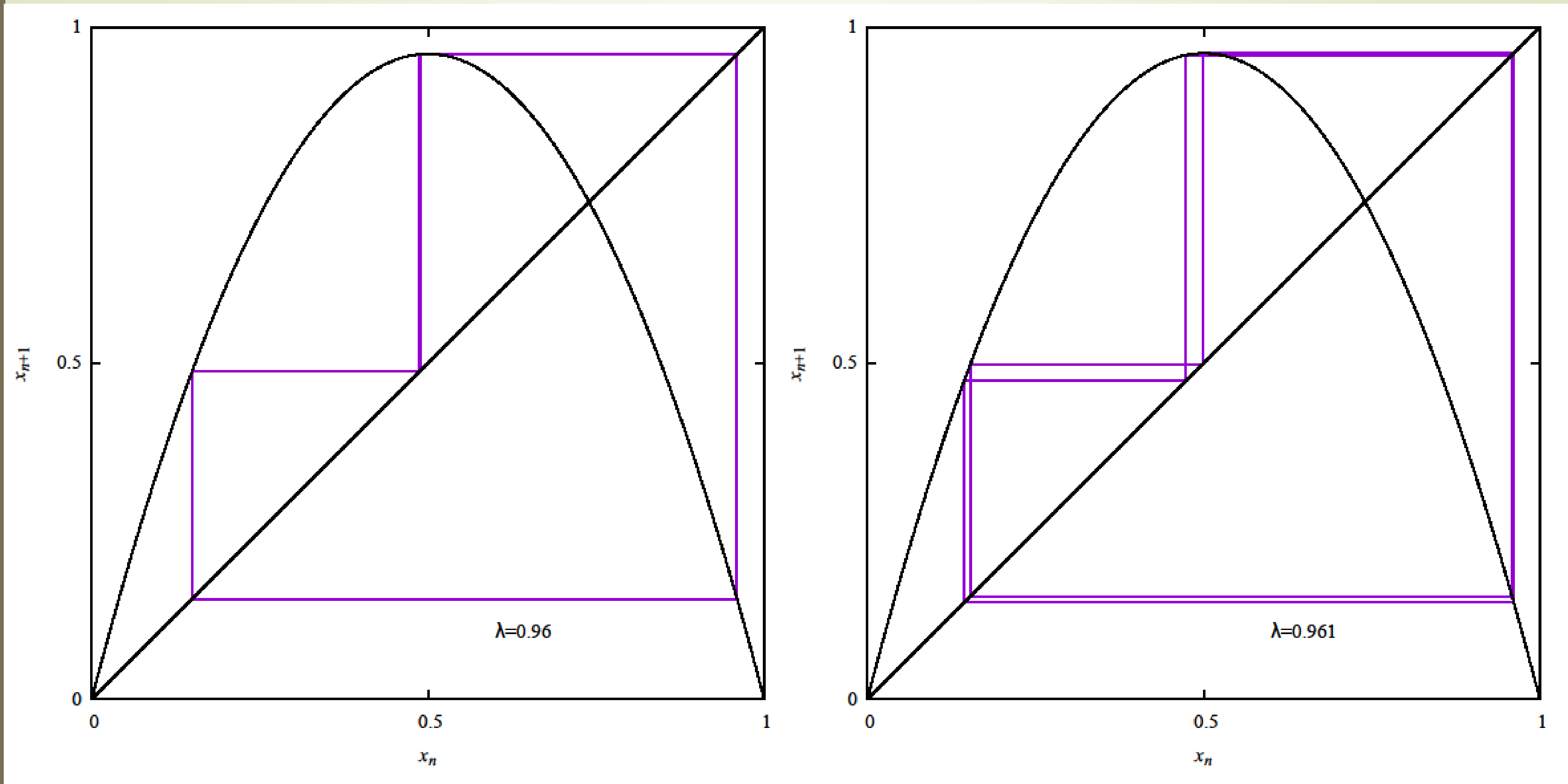


- For $\lambda > 0.893$, trajectories show band structure.
  - Not periodic, not random
  - Non-uniform density of trajectories

Period-3 region

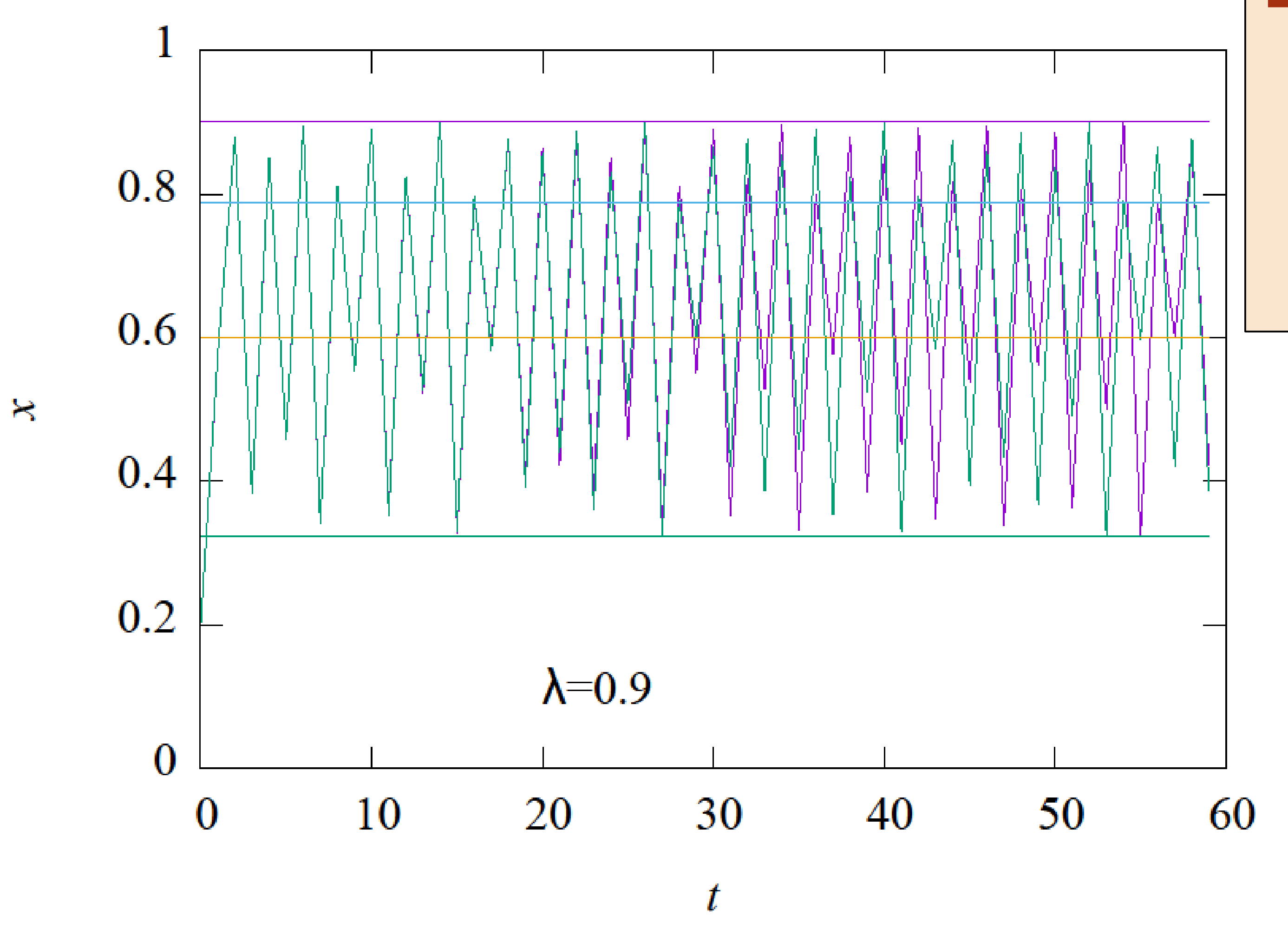


window around $\lambda \sim 0.96$



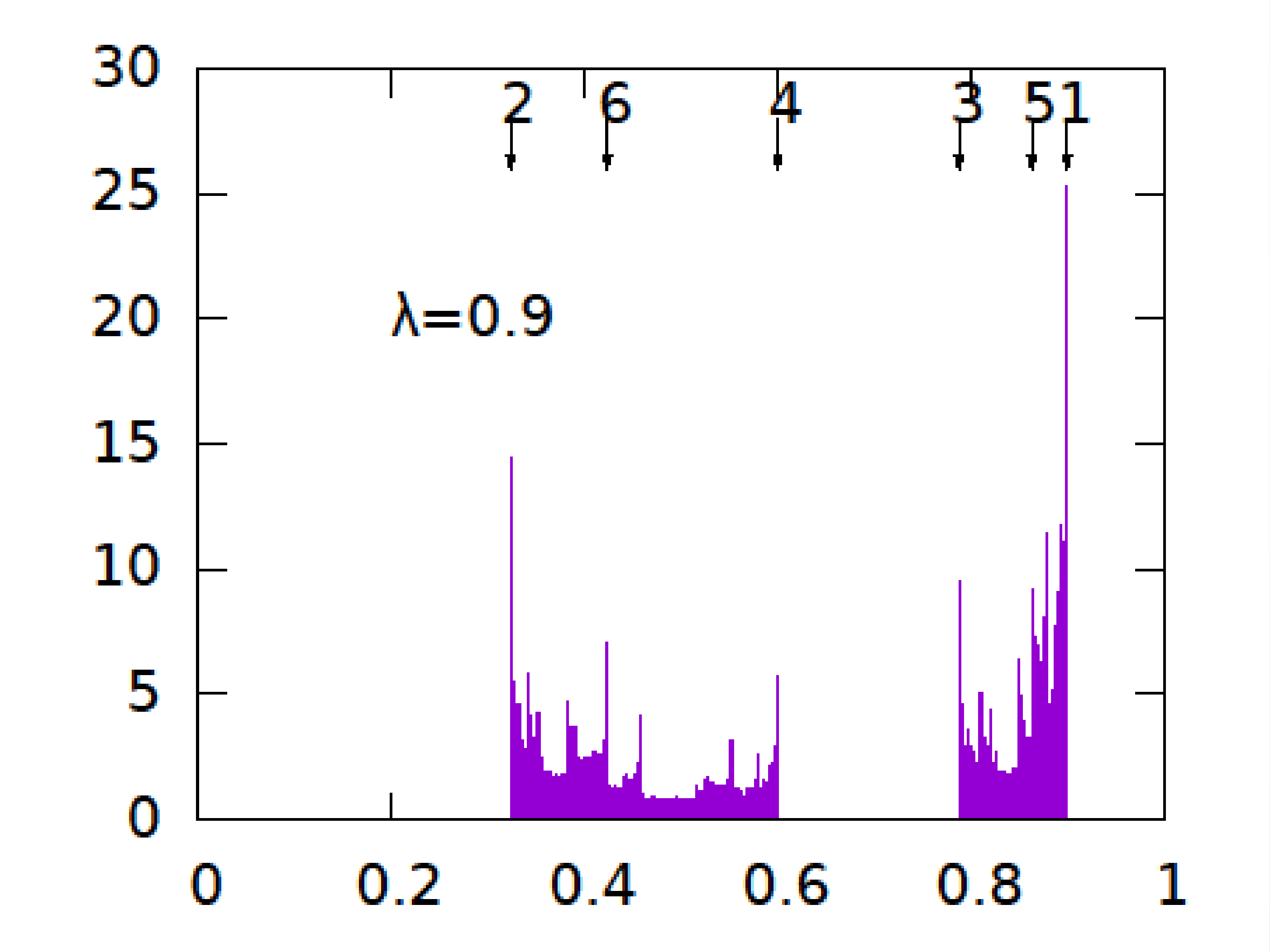
- Period-3 trajectories near $\lambda \sim 0.96$
  - Period doubling to period-6 trajectories

chaotic motions



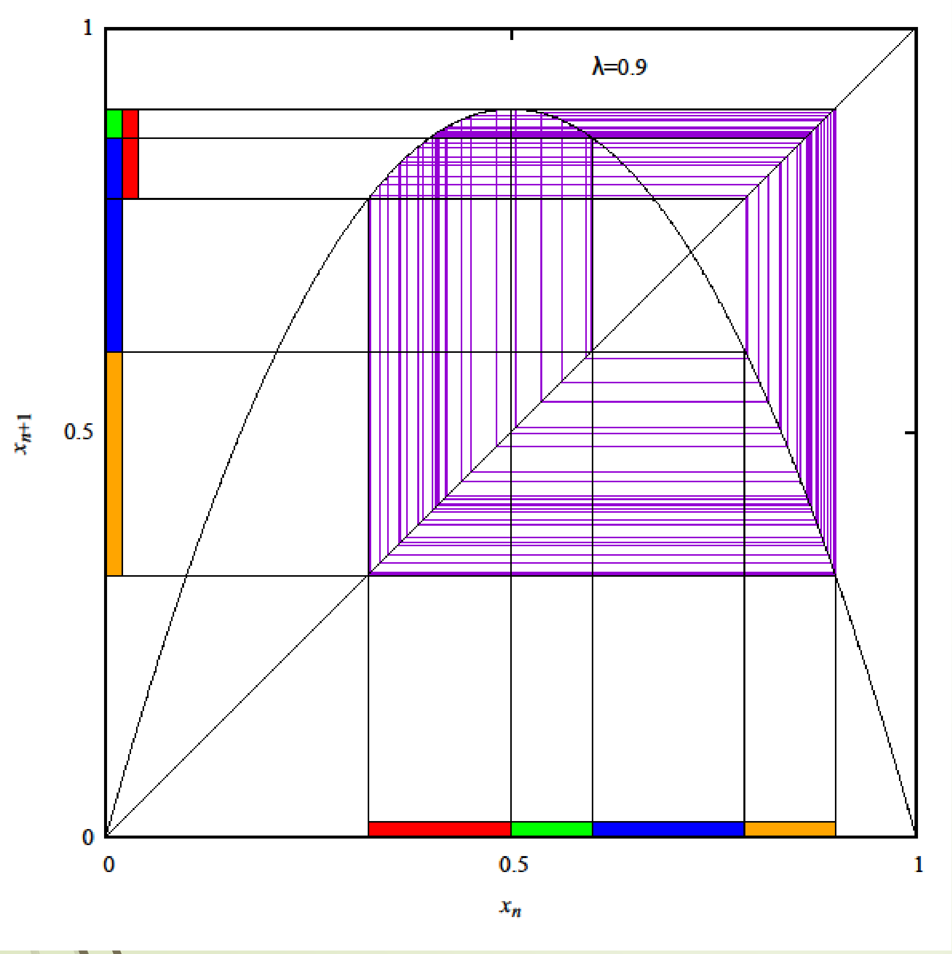
- small difference in initial values expands
- finally two trajectories seem to behave independently

Non-uniform density of trajectories



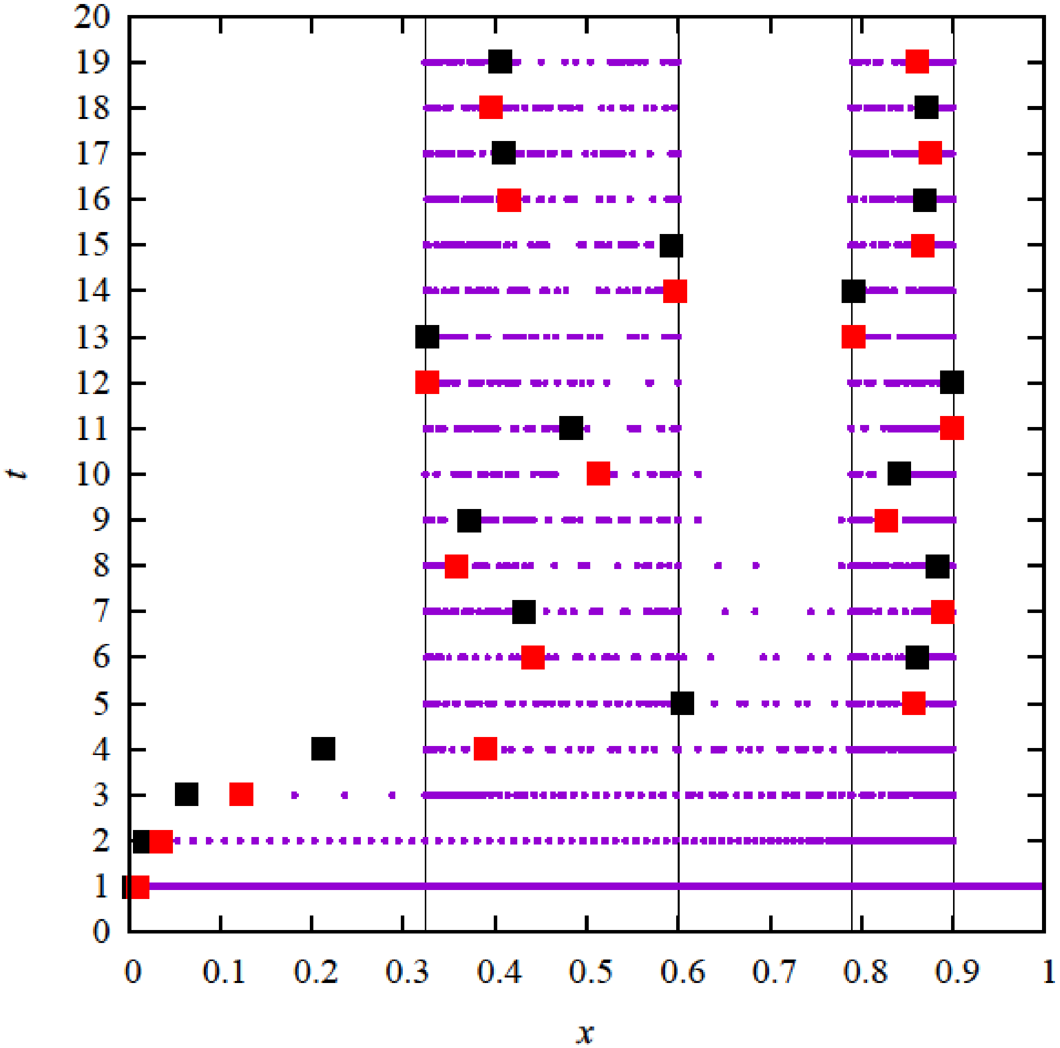
$$f_{\lambda}^{[k]}\left(\frac{1}{2}\right)$$

bands of trajectories



- Bands of trajectories are expended and folded.
- This is the origin of chaotic motion.

Uniform initial points are absorbed into two bands



Two point ■ ■, which are initially close each other, separate and behave almost independently.

Super-stable point

$$f_\lambda(x) = 4\lambda x(1-x)$$

$$f_\lambda{}'(x) = 4\lambda(1-2x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x} f_\lambda^{[2]}(x) = f_\lambda{}'\big(f_\lambda(x)\big) \cdot \frac{\mathrm{d}}{\mathrm{d}x} f_\lambda(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x} f_\lambda^{[n]}(x) = f_\lambda{}'\big(f_\lambda^{[n-1]}(x)\big) \cdot \frac{\mathrm{d}}{\mathrm{d}x} f_\lambda^{[n-1]}(x)$$

Super-stable point

$$f_\lambda'\left(\frac{1}{2}\right) = 4\lambda\left(1 - 2\frac{1}{2}\right) = 0$$

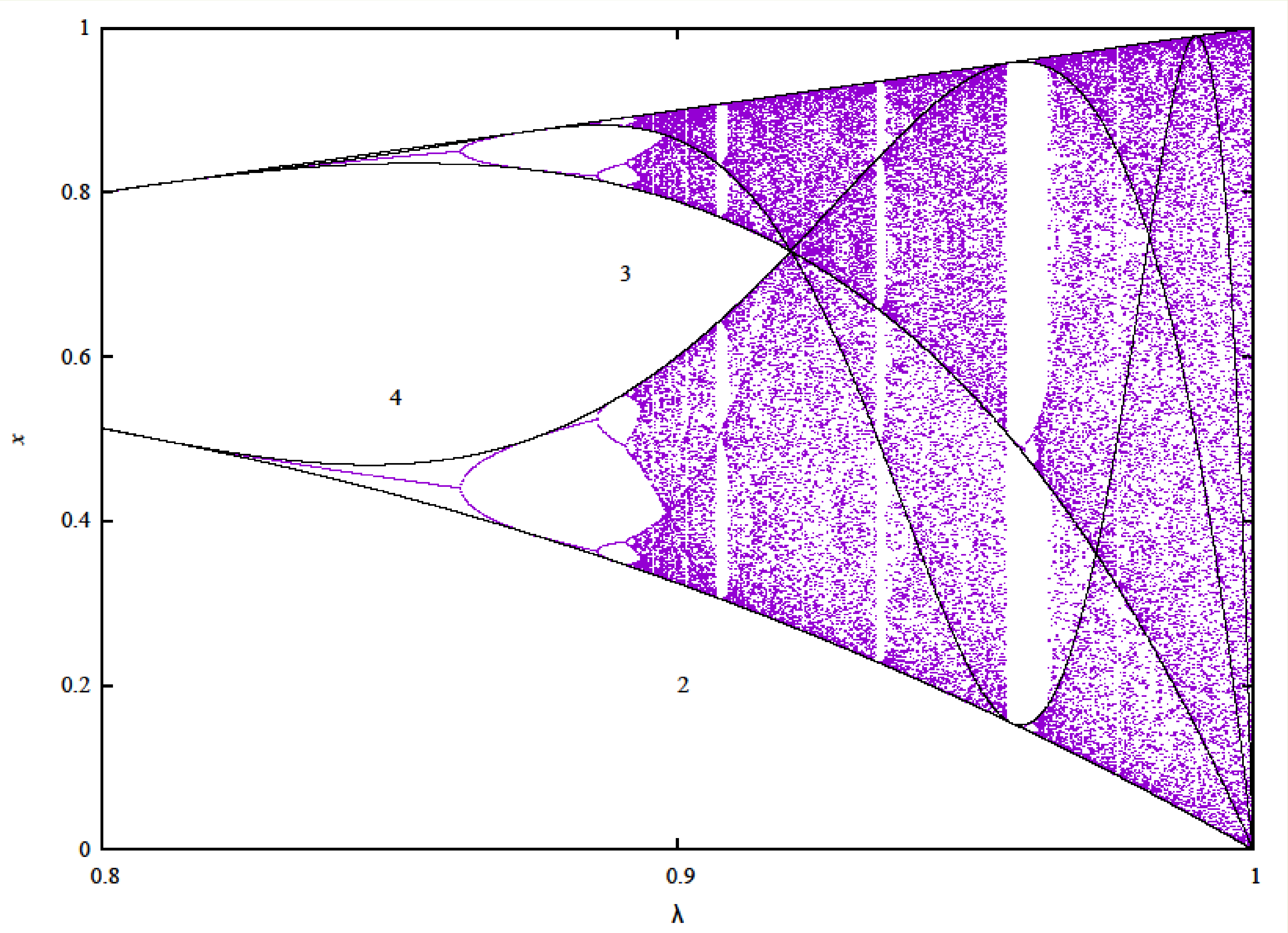
$$\left.\frac{\mathrm{d}}{\mathrm{d}x} f_\lambda^{[2]}(x)\right|_{x=x_0=1/2} = f_\lambda'(x_1)\cdot\left.\frac{\mathrm{d}}{\mathrm{d}x} f_\lambda(x)\right|_{x=x_0=1/2}$$

$$= f_\lambda'(x_1)\, f_\lambda'(x_0) = 0$$

$$\left.\frac{\mathrm{d}}{\mathrm{d}x} f_\lambda^{[n]}(x)\right|_{x=x_0=1/2} = f_\lambda'(x_{n-1})\cdot\left.\frac{\mathrm{d}}{\mathrm{d}x} f_\lambda^{[n-1]}(x)\right|_{x=x_0=1/2}$$

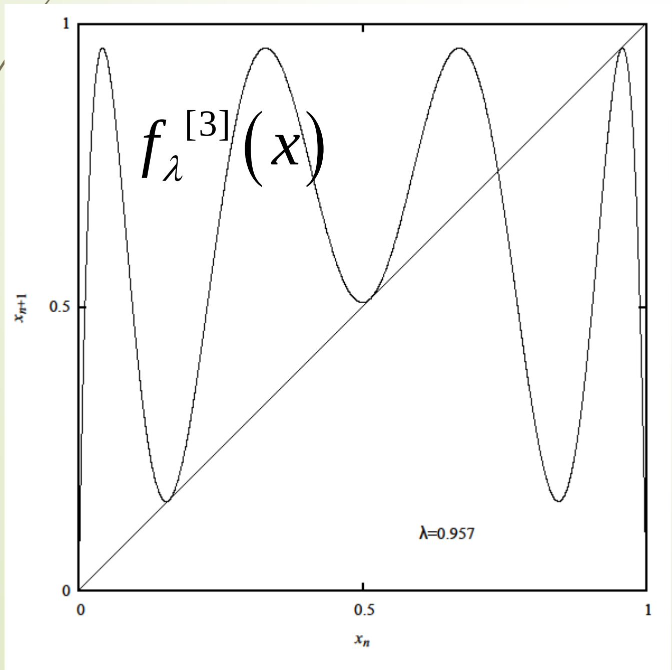
$$= \prod_{k=0}^{n-1} f_\lambda'(x_k) = 0$$

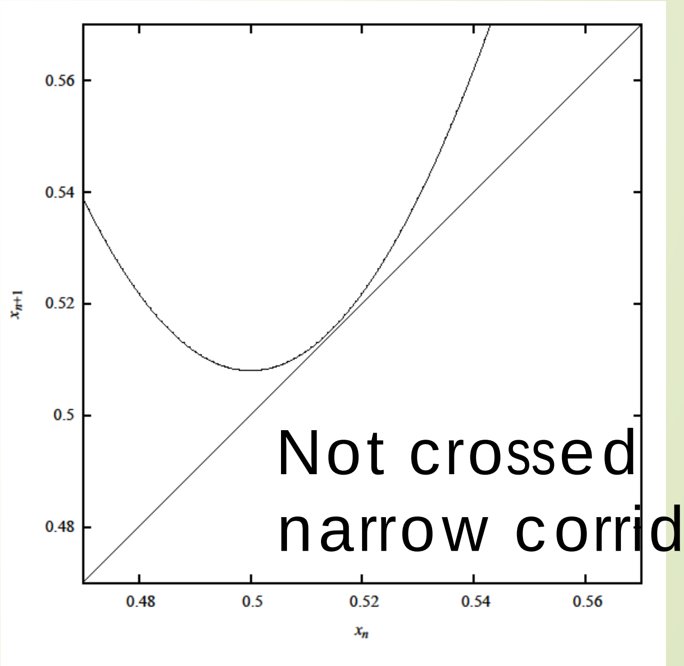
Trajectories of $x = 1/2$ are keys to understand band structure



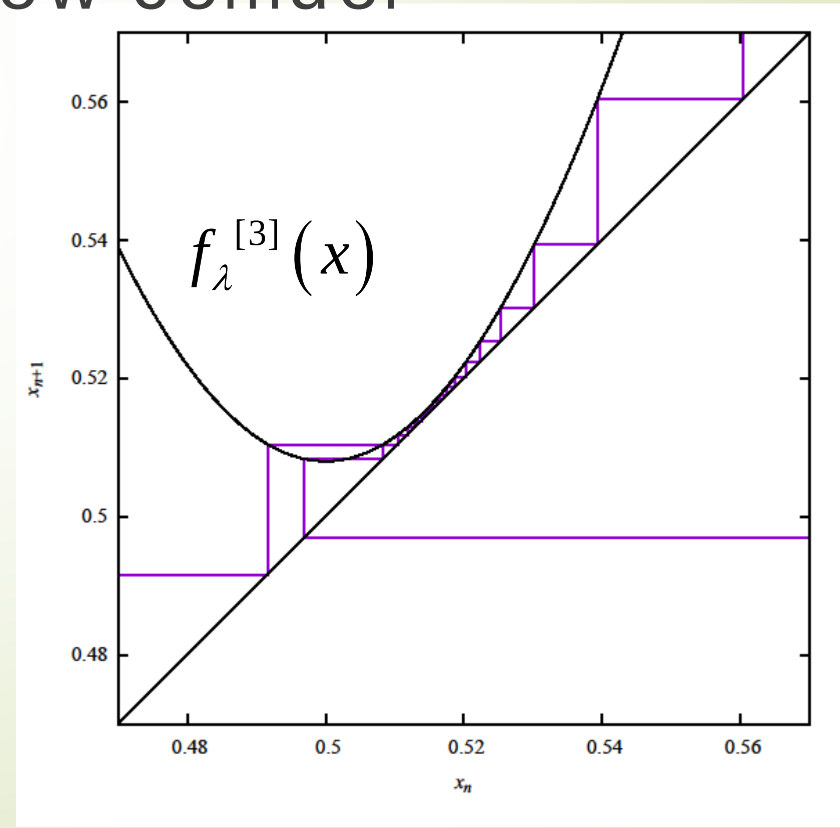
Tangent Bifurcation

- λ_C : period-3 trajectories emerges
- A little bit lower λ than λ_C





- Trajectories (per 3 times) stays long time at the narrow corridor



Intermittency

- After staying the narrow corridor, trajectories varies widely.