# 「離散数学・オートマトン」演習問題 13 (解答例)

## 2022/1/18

### 1 プッシュダウンオートマトン

**課題1** 以下のようなプッシュダウンオートマトン M を考える。

a, 
$$Z/AZ$$
  
b,  $Z/BZ$   
a,  $B/AB$   
a,  $A/\epsilon$   
b,  $A/BA$   
b,  $B/\epsilon$   
 $q_0$   
 $q_1$   
 $e, Z/\epsilon$   
 $q_2$ 

$$Q = \{q_0.q_1, q_2\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{A, B, Z\}$$

$$F = \{q_2\}$$

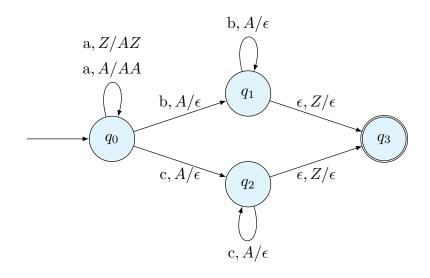
$$\begin{split} \delta \left( q_0, \mathbf{a}, Z \right) &= \left( q_0, AZ \right), \\ \delta \left( q_0, \mathbf{a}, B \right) &= \left( q_0, AB \right), \\ \delta \left( q_0, \mathbf{a}, A \right) &= \left( q_1, \epsilon \right), \\ \delta \left( q_1, \mathbf{b}, B \right) &= \left( q_1, \epsilon \right), \\ \delta \left( q_1, \mathbf{b}, B \right) &= \left( q_1, \epsilon \right), \\ \delta \left( q_1, \mathbf{c}, Z \right) &= \left( q_2, \epsilon \right) \end{split} \qquad \qquad \delta \left( q_0, \mathbf{b}, Z \right) &= \left( q_0, BZ \right), \\ \delta \left( q_0, \mathbf{b}, A \right) &= \left( q_0, BA \right), \\ \delta \left( q_0, \mathbf{b}, B \right) &= \left( q_1, \epsilon \right), \\ \delta \left( q_1, \mathbf{a}, A \right) &= \left( q_1, \epsilon \right), \\ \delta \left( q_1, \mathbf{c}, Z \right) &= \left( q_2, \epsilon \right) \end{split}$$

このとき、入力 ababbaba 及び babaabab に対する動作を示しなさい。

#### 解答例

```
(q_0, ababbaba, Z) \vdash (q_0, babbaba, AZ)
                          \vdash (q_0, abbaba, BAZ)
                          \vdash (q_0, bbaba, ABAZ)
                          \vdash (q_0, \text{baba}, BABAZ)
                          \vdash (q_1, aba, ABAZ)
                          \vdash (q_1, ba, BAZ)
                          \vdash (q_1, \mathbf{a}, AZ)
                          \vdash (q_1, \epsilon, Z)
                          \vdash (q_2, \epsilon, \epsilon)
(q_0, babaabab, Z) \vdash (q_0, abaabab, BZ)
                          \vdash (q_0, baabab, ABZ)
                          \vdash (q_0, aabab, BABZ)
                          \vdash (q_0, abab, ABABZ)
                          \vdash (q_1, \text{bab}, BABZ)
                          \vdash (q_1, ab, ABZ)
                          \vdash (q_1, b, BZ)
                          \vdash (q_1, \epsilon, Z)
                          \vdash (q_2, \epsilon, \epsilon)
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**課題2** 以下のようなプッシュダウンオートマトン M を考える。



$$Q = \{q_0.q_1, q_2, q_3\}$$

$$\Sigma = \{a, b, c\}$$

$$\Gamma = \{A, Z\}$$

$$F = \{q_3\}$$

$$\begin{split} \delta \left( q_{0}, \mathbf{a}, Z \right) &= \left( q_{0}, AZ \right), & \delta \left( q_{0}, \mathbf{a}, A \right) &= \left( q_{0}, AA \right), \\ \delta \left( q_{0}, \mathbf{b}, A \right) &= \left( q_{1}, \epsilon \right), & \delta \left( q_{0}, \mathbf{c}, A \right) &= \left( q_{2}, \epsilon \right), \\ \delta \left( q_{1}, \mathbf{b}, A \right) &= \left( q_{1}, \epsilon \right), & \delta \left( q_{2}, \mathbf{c}, A \right) &= \left( q_{2}, \epsilon \right), \\ \delta \left( q_{1}, \epsilon, Z \right) &= \left( q_{3}, \epsilon \right), & \delta \left( q_{2}, \epsilon, Z \right) &= \left( q_{3}, \epsilon \right) \end{split}$$

このとき、入力 aaabbb 及び aacc に対する動作を示しなさい。

#### 解答例

$$(q_0, aabbb, Z) \vdash (q_0, aabbb, AZ)$$

$$\vdash (q_0, abbb, AAZ)$$

$$\vdash (q_0, bbb, AAAZ)$$

$$\vdash (q_1, bb, AAZ)$$

$$\vdash (q_1, b, AZ)$$

$$\vdash (q_1, \epsilon, Z)$$

$$\vdash (q_3, \epsilon, \epsilon)$$

$$(q_0, aacc, Z) \vdash (q_0, acc, AZ)$$

$$\vdash (q_0, cc, AAZ)$$

$$\vdash (q_2, c, AZ)$$

$$\vdash (q_2, \epsilon, Z)$$

$$\vdash (q_3, \epsilon, \epsilon)$$