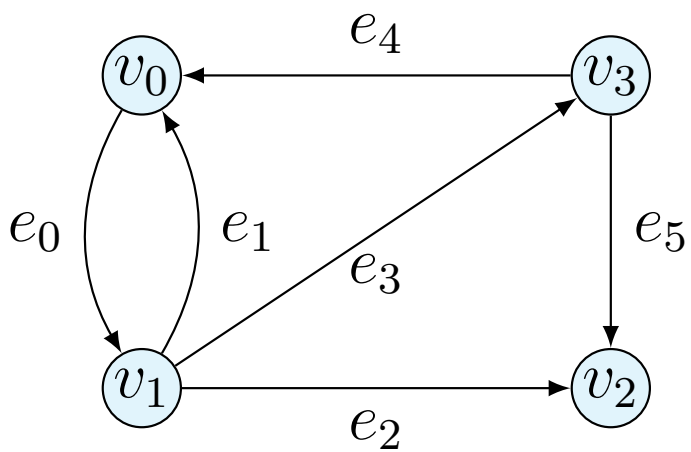
グラフの探索

離散数学・オートマトン
2024 年後期
佐賀大学理工学部　只木進一

有向グラフと探索

指定した頂点から、各頂点への経路を調べる

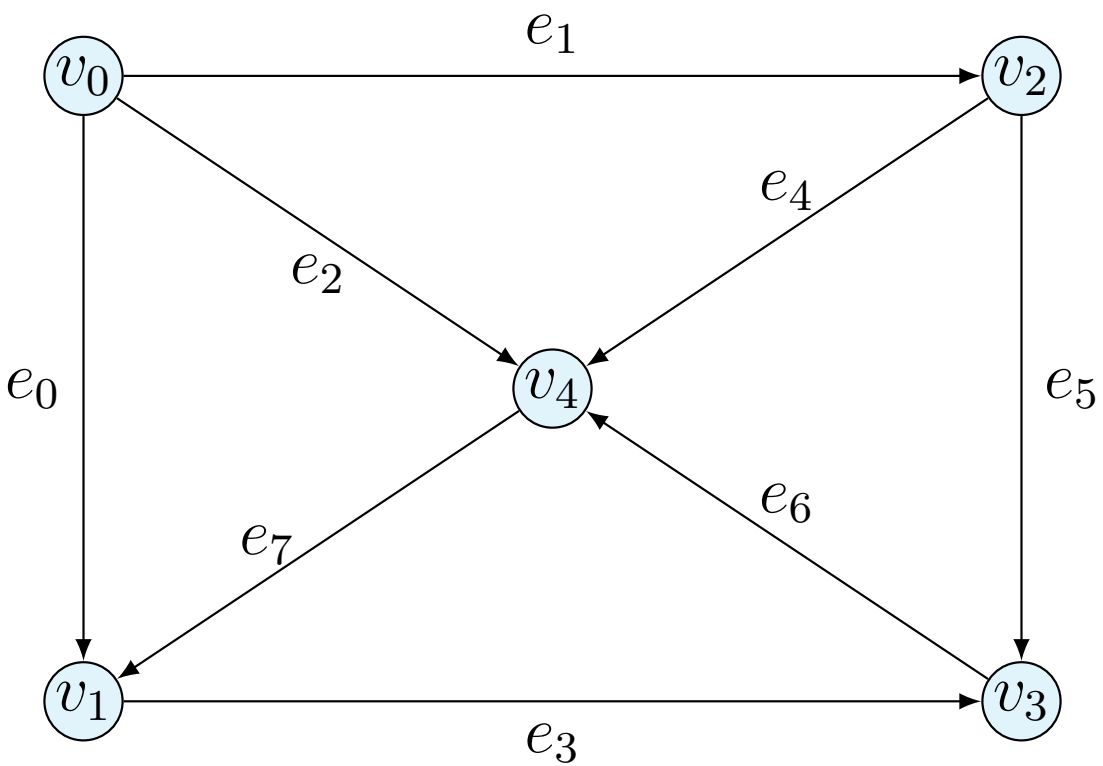


適切なアルゴリズムを作る

深さ優先探索 DFS: Depth-First Search

- 出発点を定める
- たどれる限り、辺をたどる: 再帰的アルゴリズム
  - それ以上進めなくなるまで
  - 新たな点が無くなるまで
- 道を戻って、分岐可能な頂点から、別の辺をたどる
- 結果としてできる木 (spanning tree) は、深いものができる

例 1.1:



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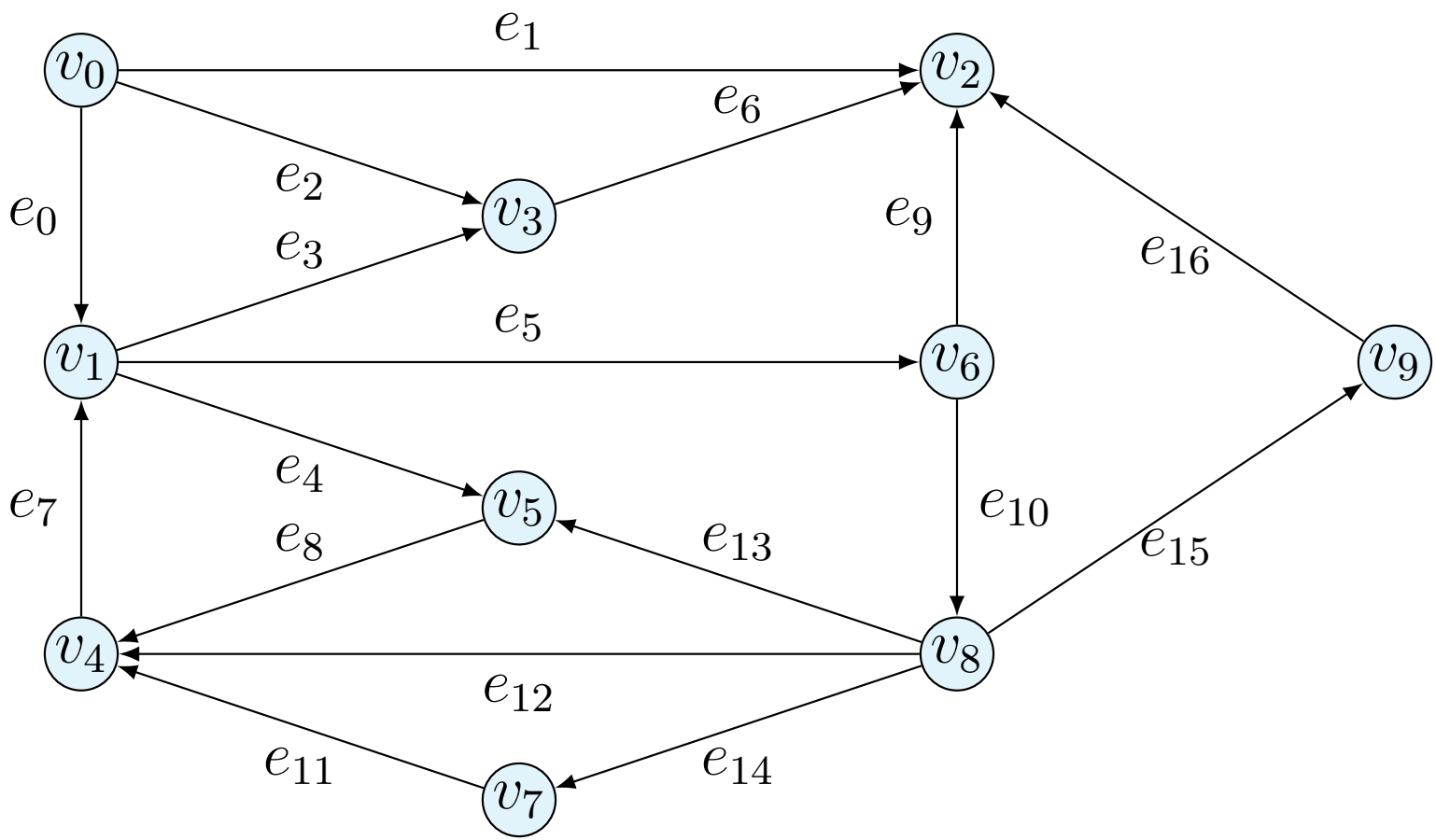
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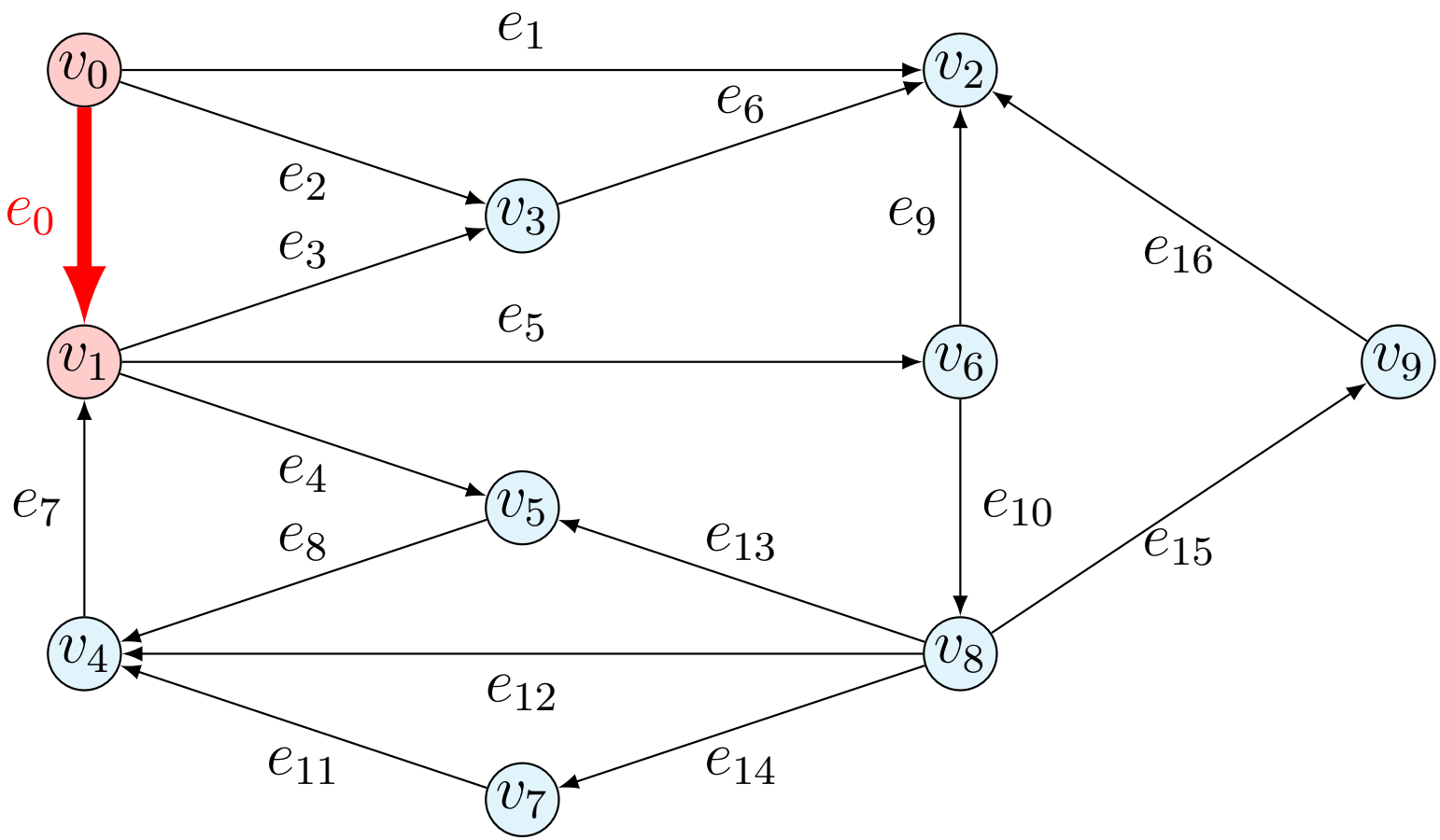
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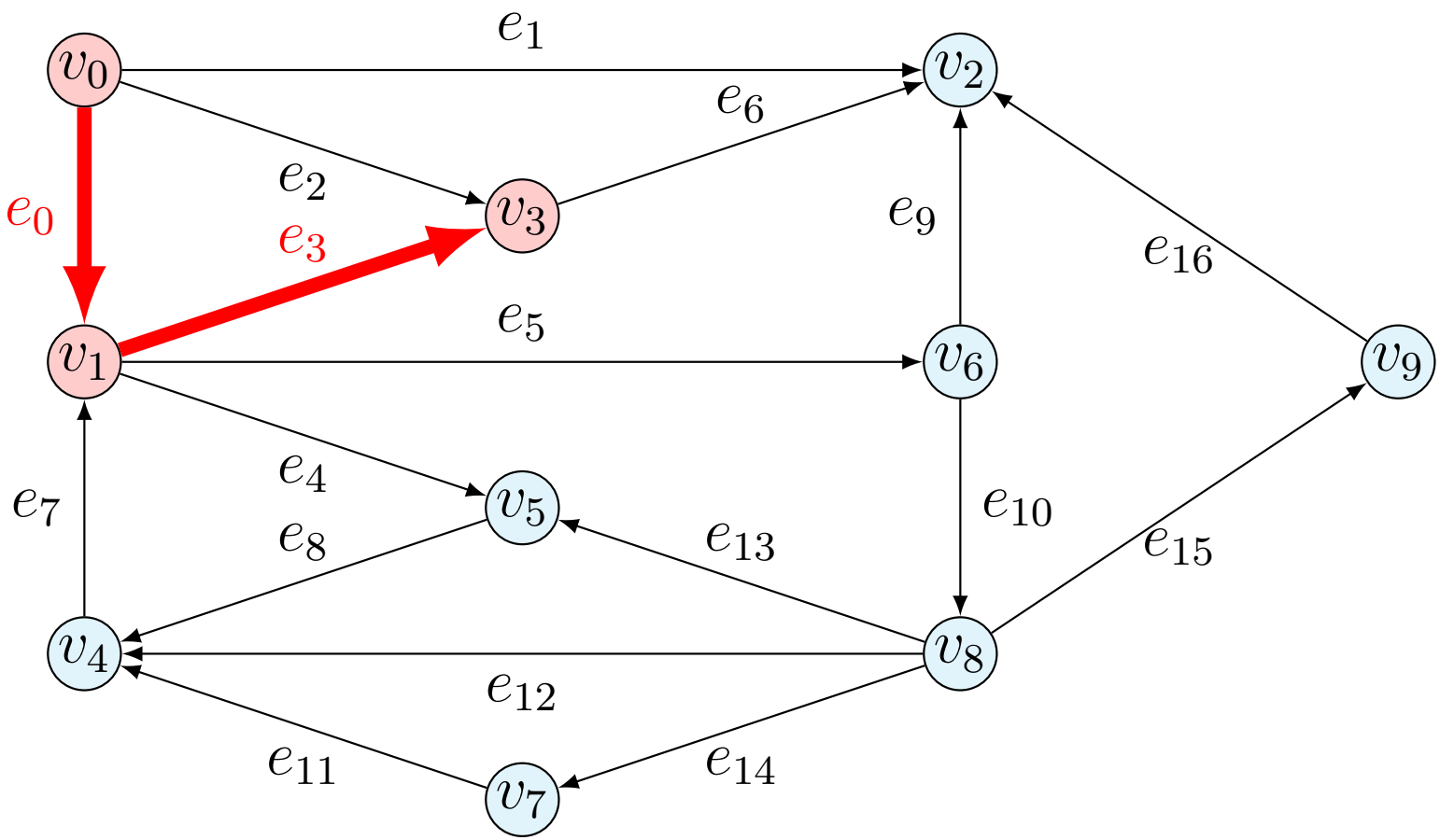
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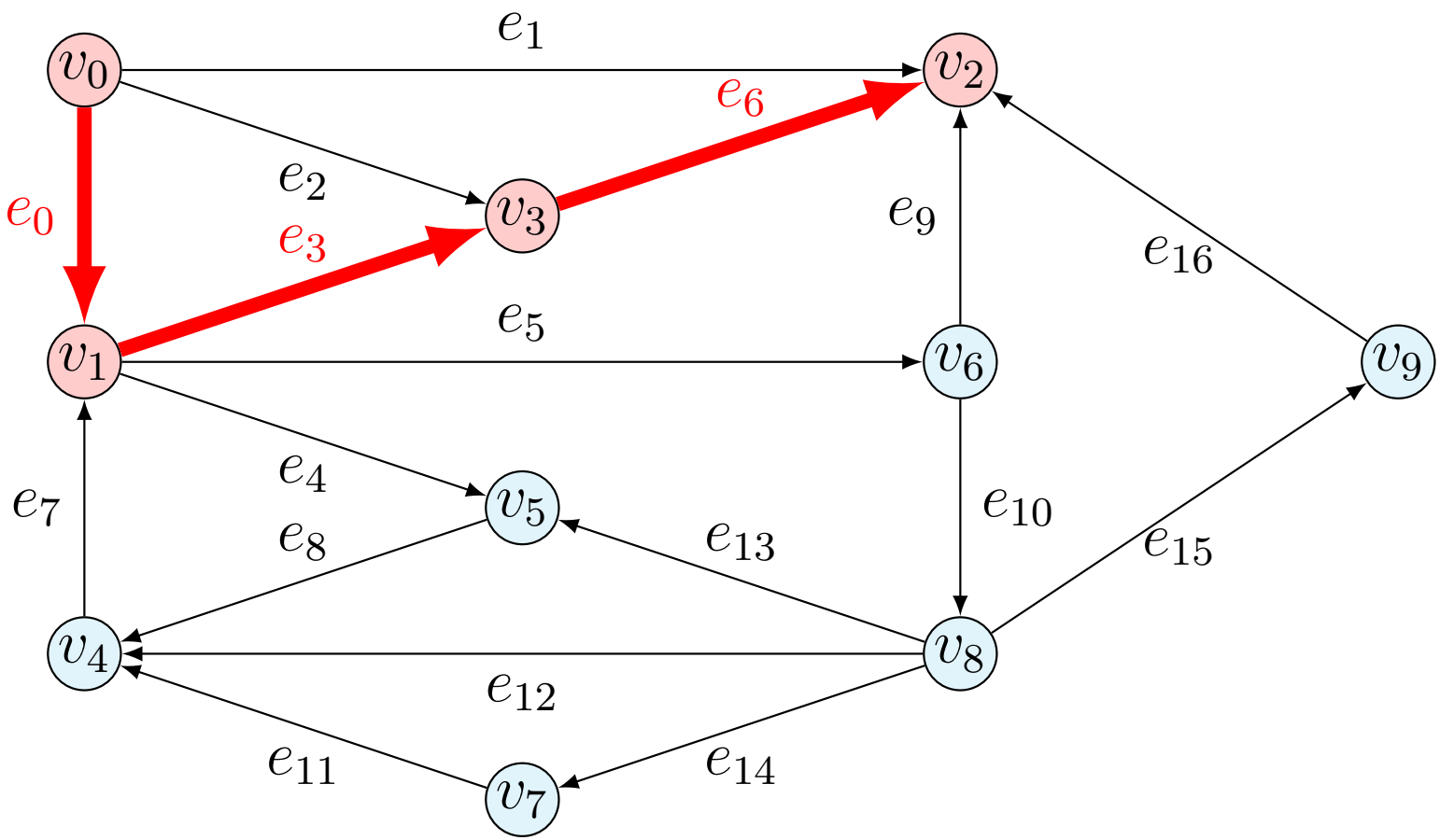
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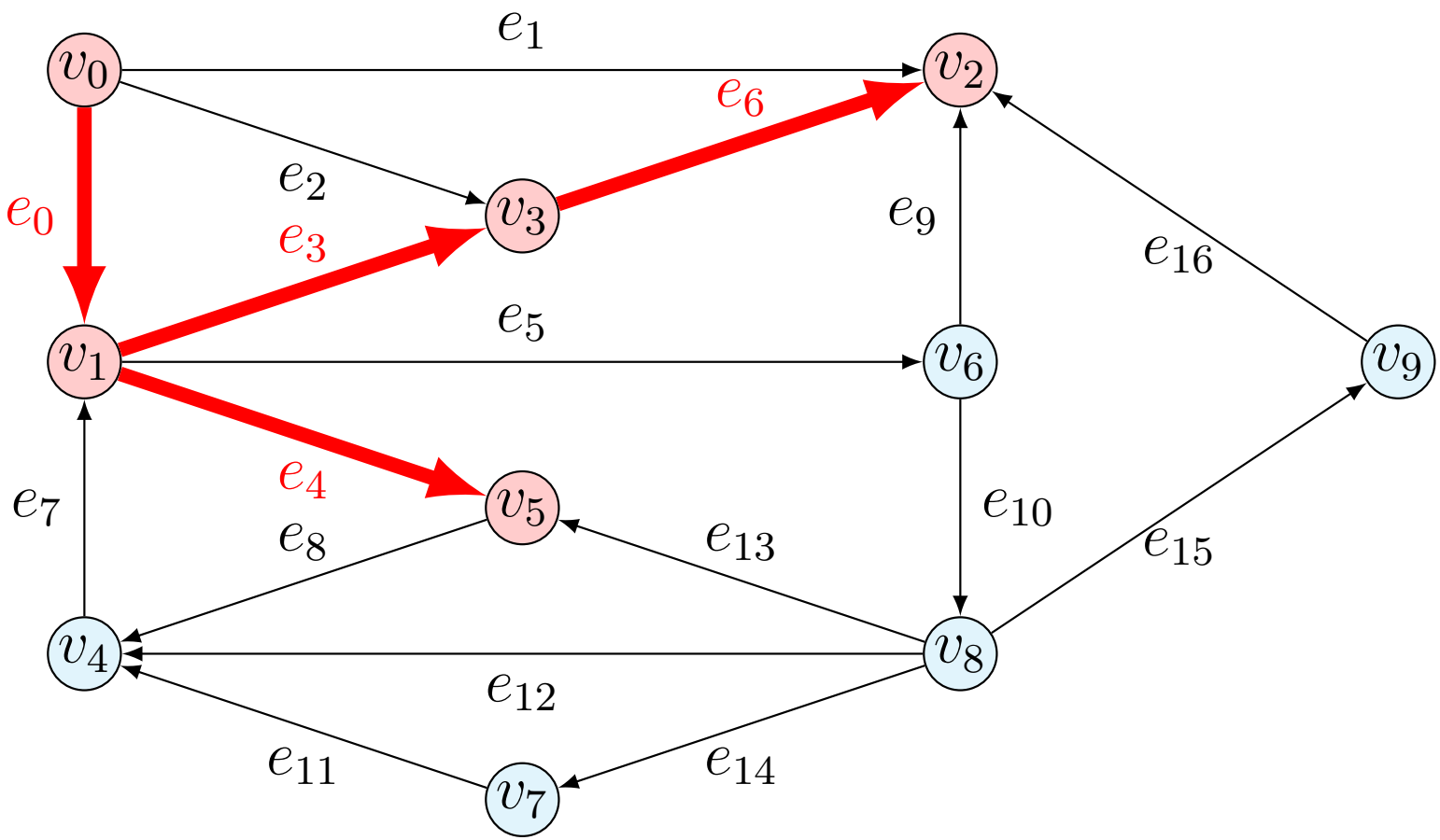
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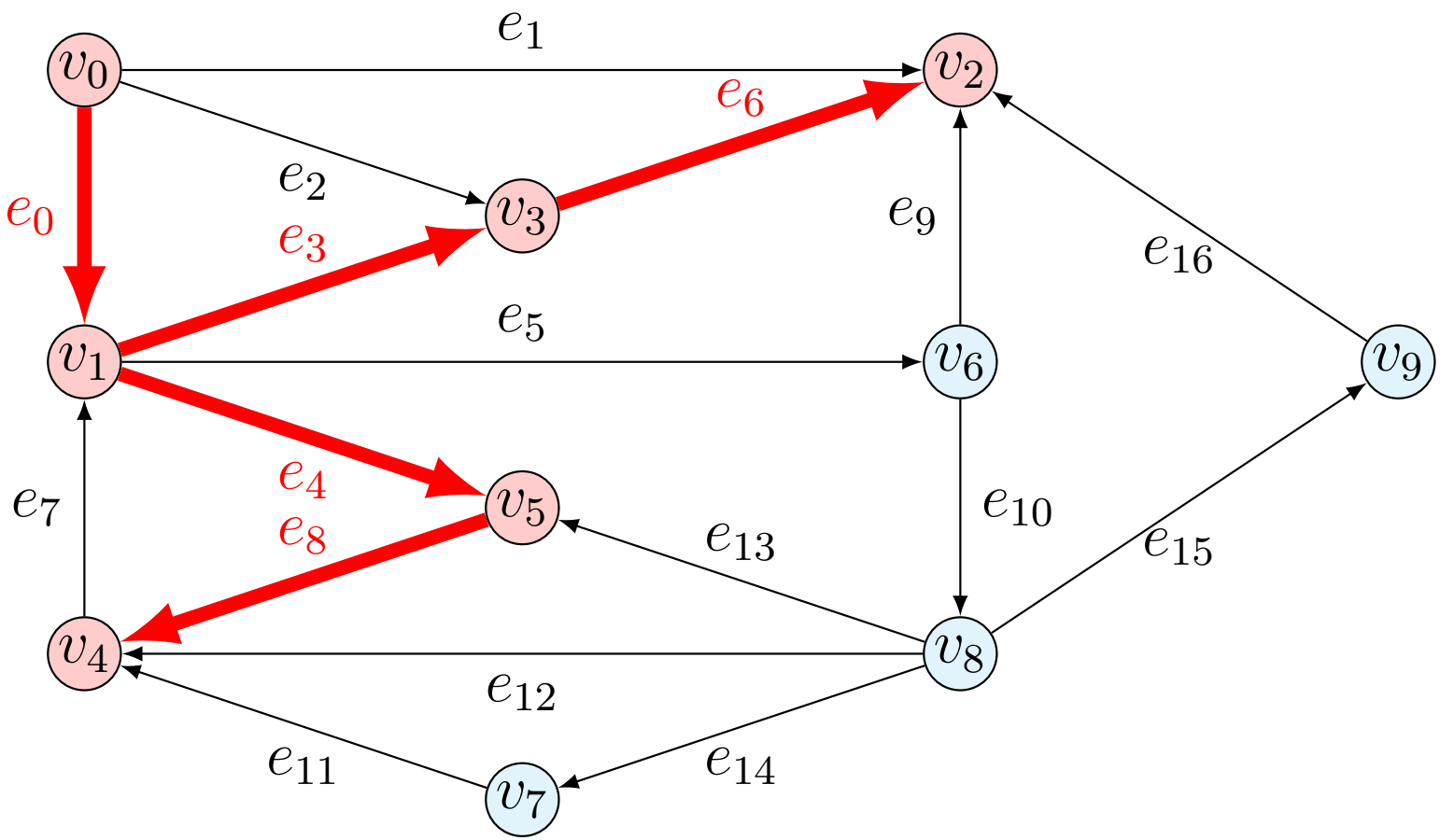
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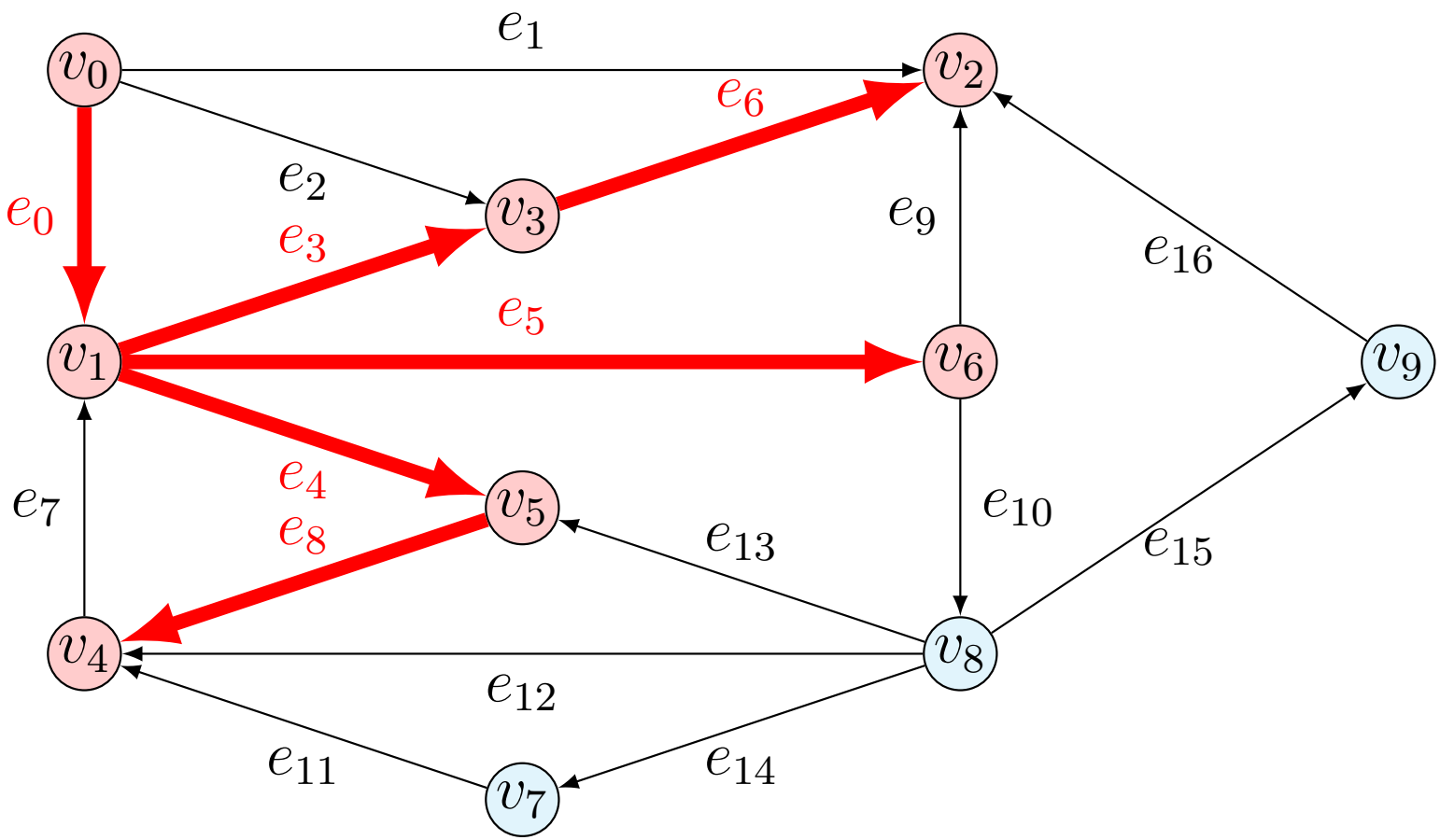
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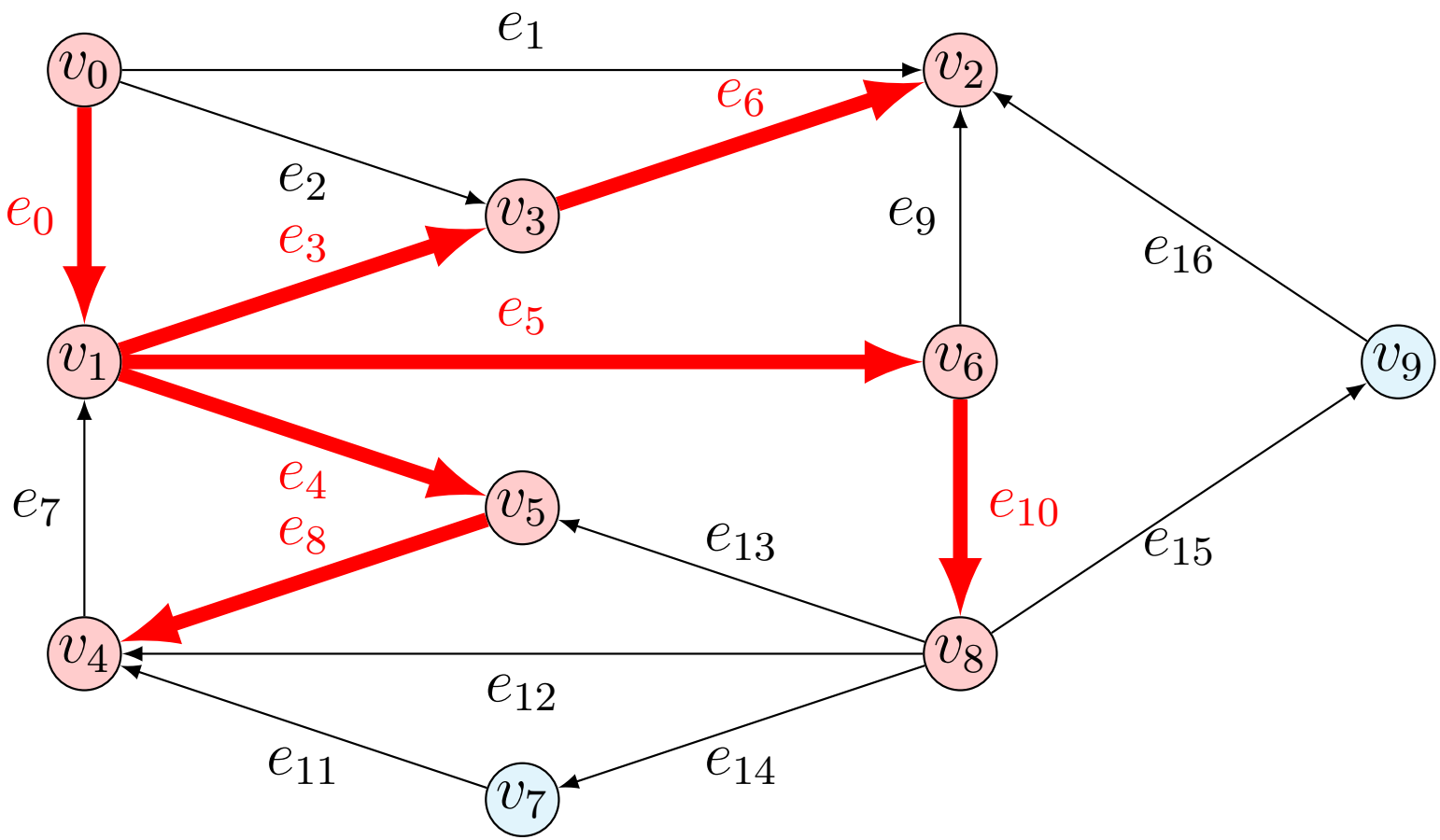
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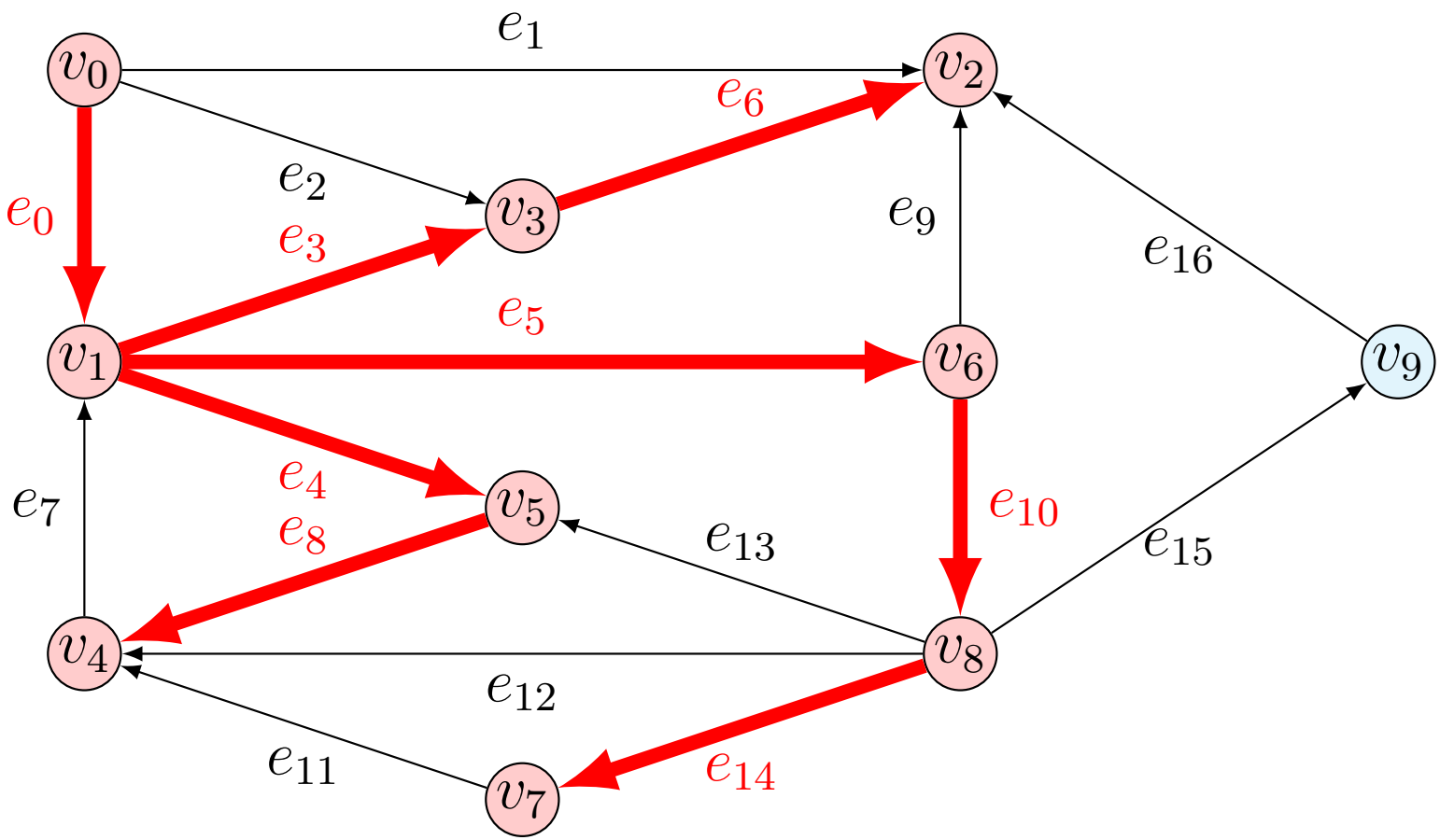
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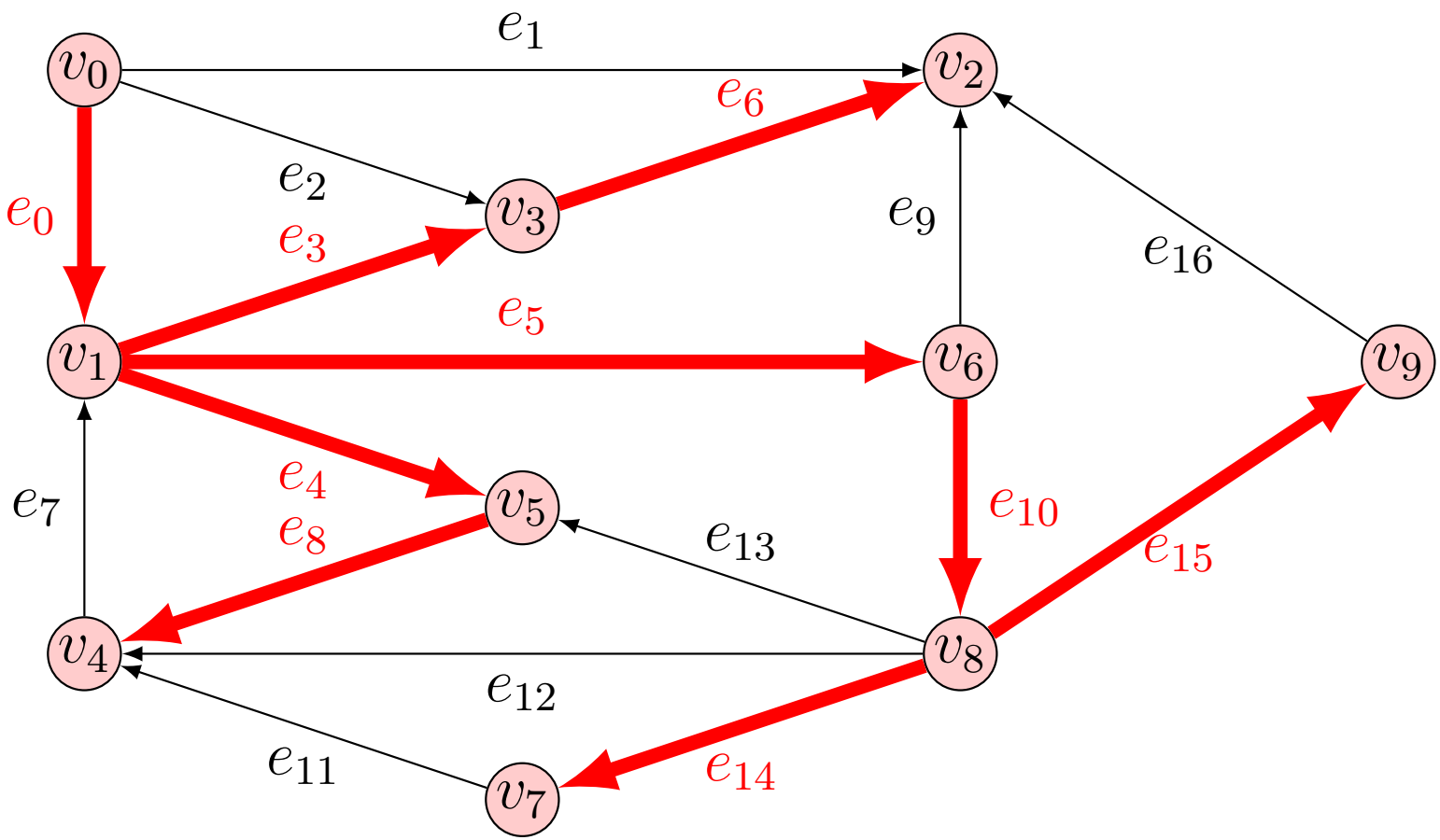
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- L: 到達した頂点のリスト: 初期値 $L = \{v_0\}$
- A: 使用する辺のリスト: 初期値 $A = \emptyset$

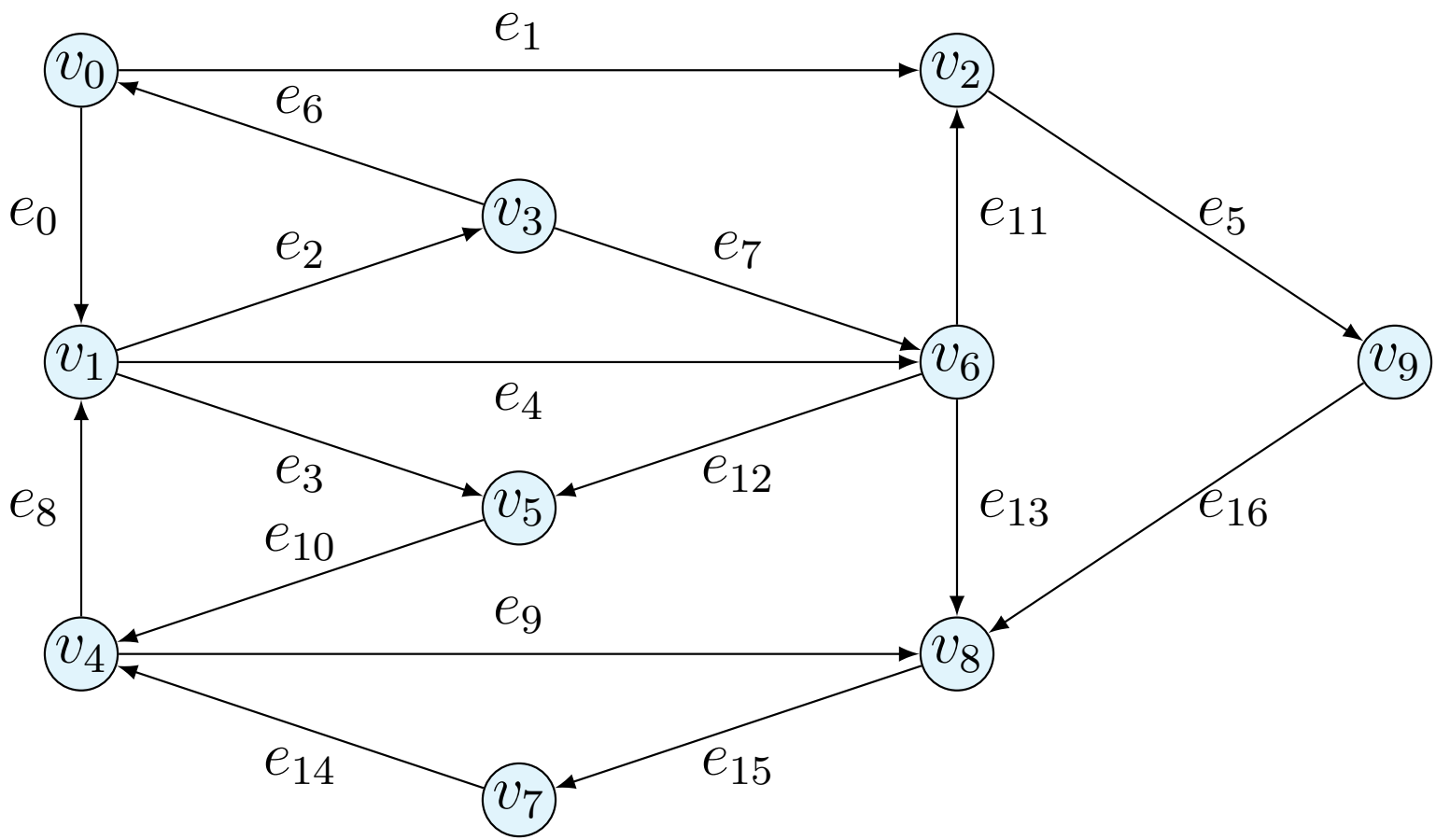
Algorithm 1 DFA アルゴリズム

procedure $\textsc{search}(v, L, A)$
  for all $e \in \delta^+ v$ **do** ▷ $//v$ から出る全ての辺
    $w = \partial^- e$
    if $w \notin L$ **then**
      $L.\mathsf{append}(w)$
      $A.\mathsf{append}(e)$
      $\textsc{search}(w, L, A)$
    end if
  end for
end procedure

実行状況

$$
\begin{aligned}
(v_0, [v_0], []) &\to (v_1, [v_0, v_1], [e_0]) \\
&\to (v_3, [v_0, v_1, v_3], [e_0, e_3]) \\
&\to (v_2, [v_0, v_1, v_3, v_2], [e_0, e_3, e_6]) \\
&\to (v_5, [v_0, v_1, v_3, v_2, v_5], [e_0, e_3, e_6, e_4]) \\
&\to (v_4, [v_0, v_1, v_3, v_2, v_5, v_4], [e_0, e_3, e_6, e_4, e_8]) \\
&\to (v_6, [v_0, v_1, v_3, v_2, v_5, v_4, v_6], [e_0, e_3, e_6, e_4, e_8, e_5]) \\
&\to (v_8, [v_0, v_1, v_3, v_2, v_5, v_4, v_6, v_8], [e_0, e_3, e_6, e_4, e_8, e_5, e_{10}]) \\
&\to (v_7, [v_0, v_1, v_3, v_2, v_5, v_4, v_6, v_8, v_7], [e_0, e_3, e_6, e_4, e_8, e_5, e_{10}, e_{14}]) \\
&\to (v_9, [v_0, v_1, v_3, v_2, v_5, v_4, v_6, v_8, v_7, v_9], [e_0, e_3, e_6, e_4, e_8, e_5, e_{10}, e_{14}, e_{15}])
\end{aligned}
$$

例 2.1: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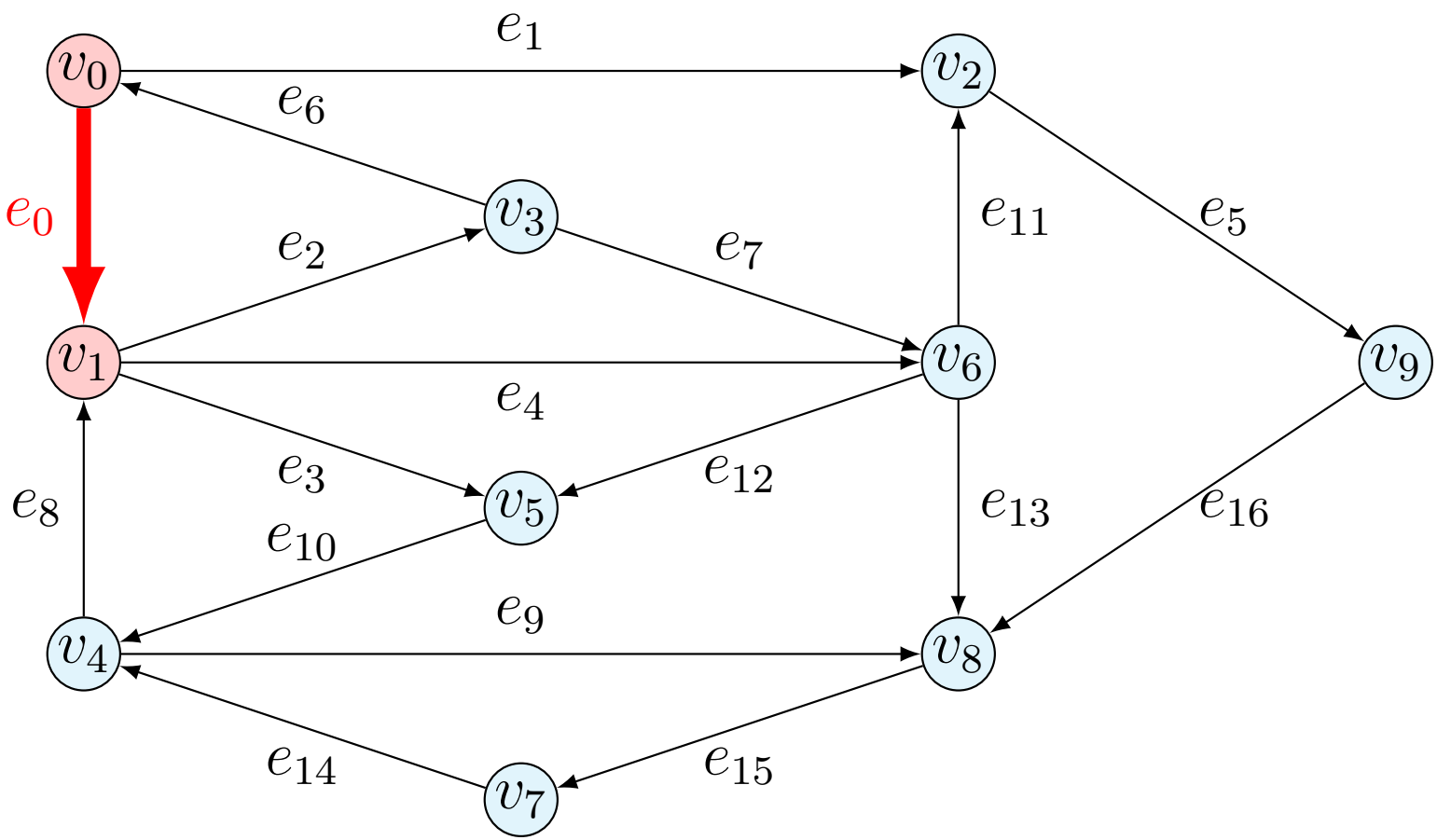
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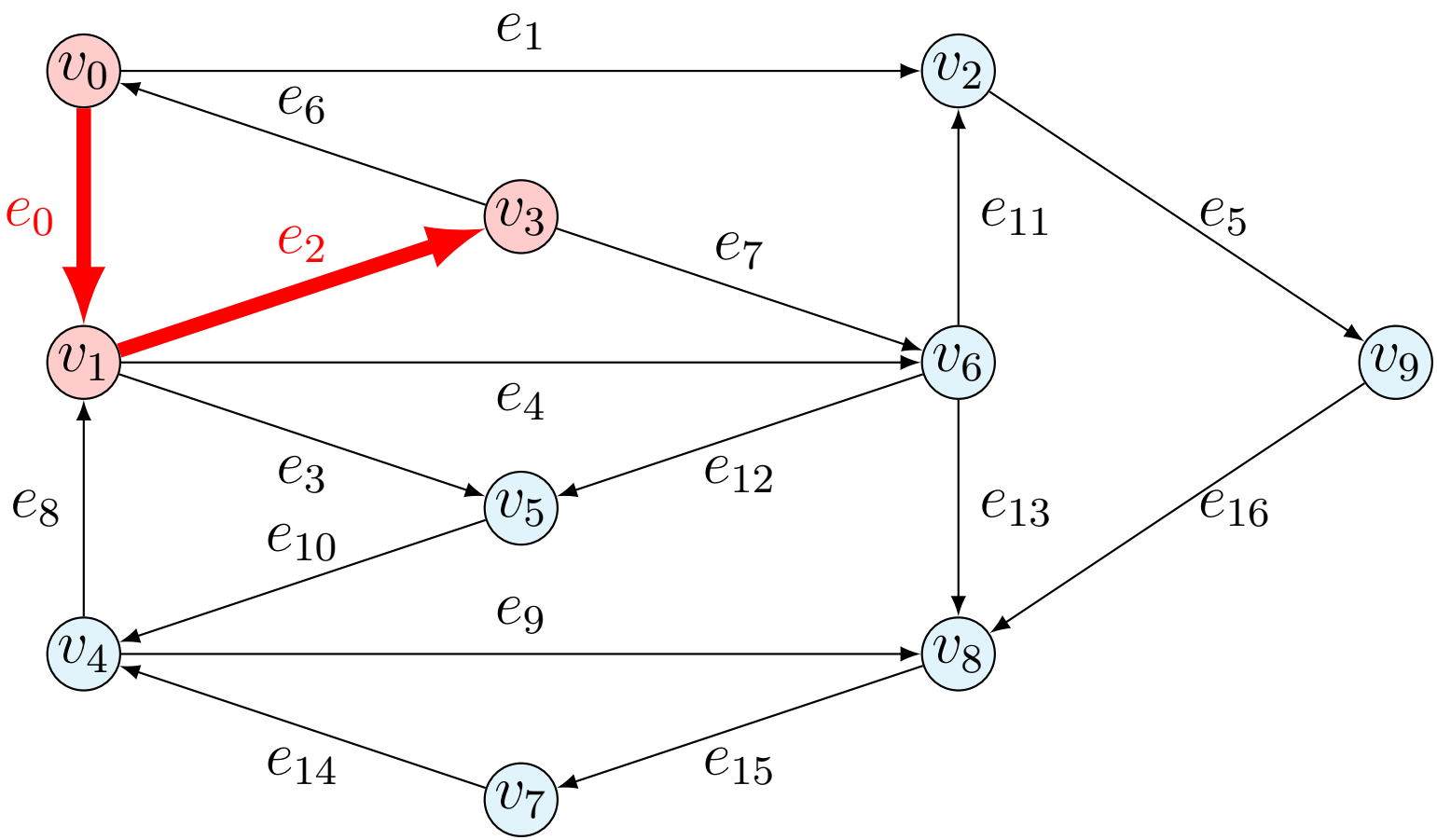
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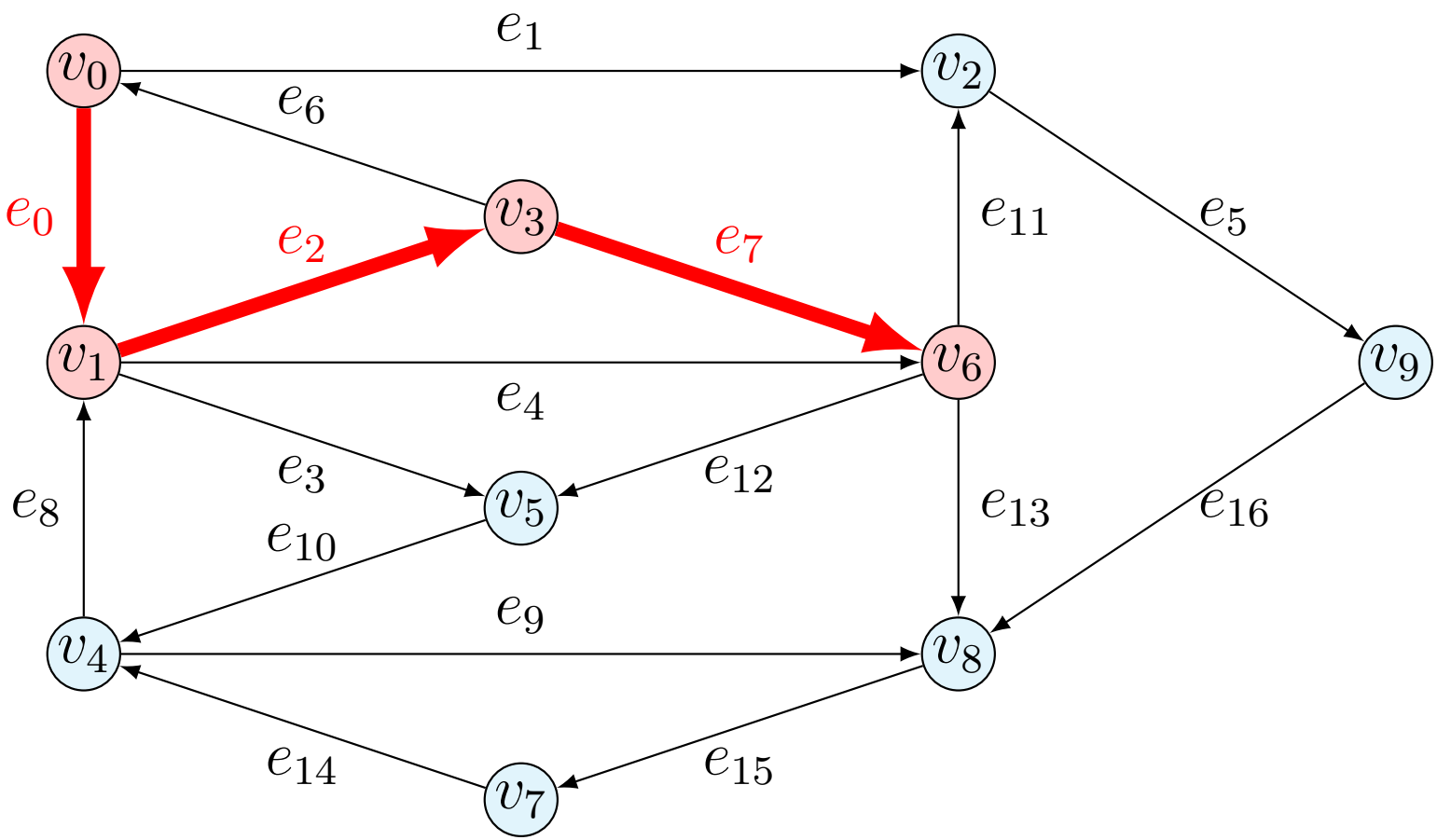
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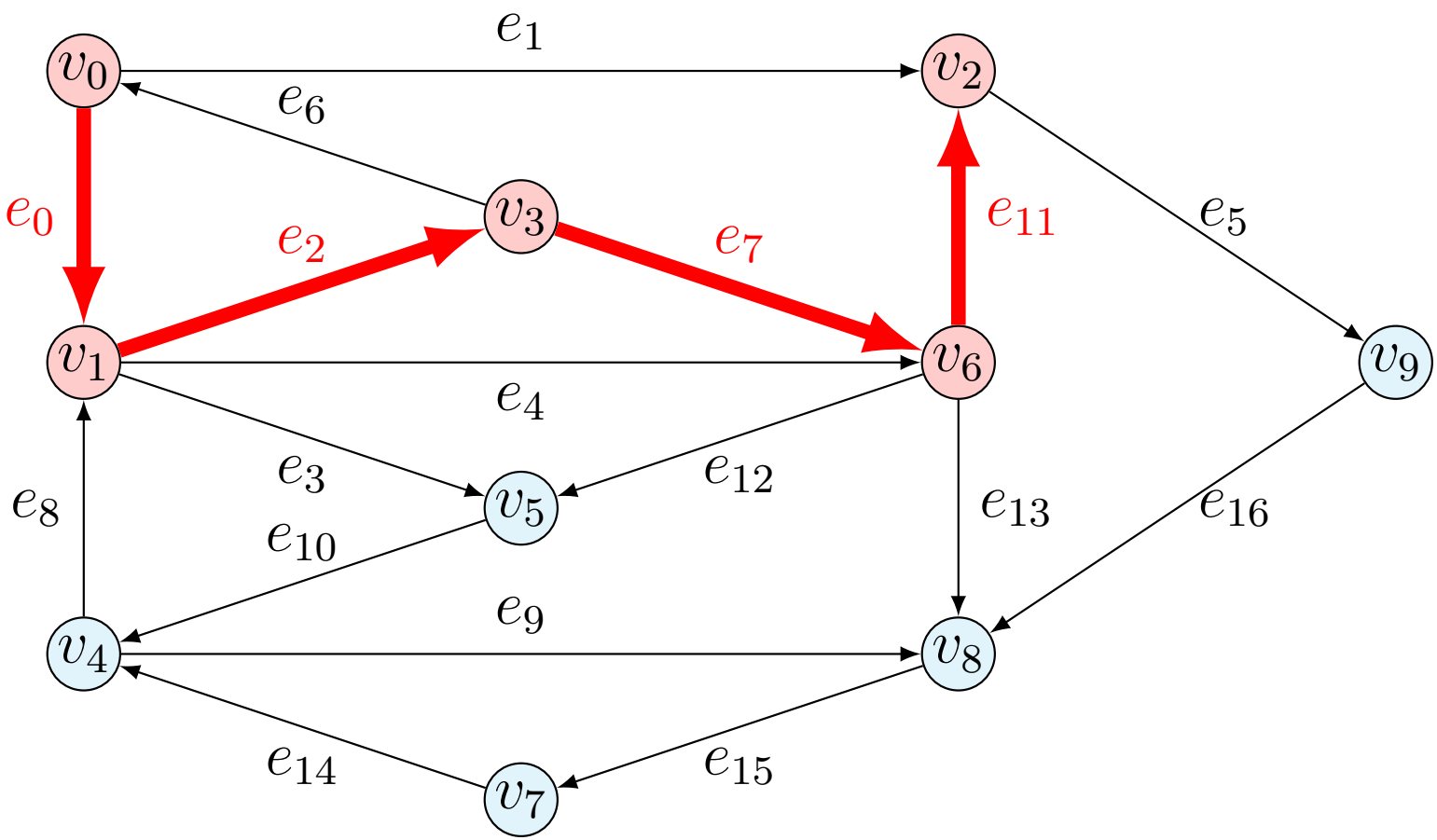
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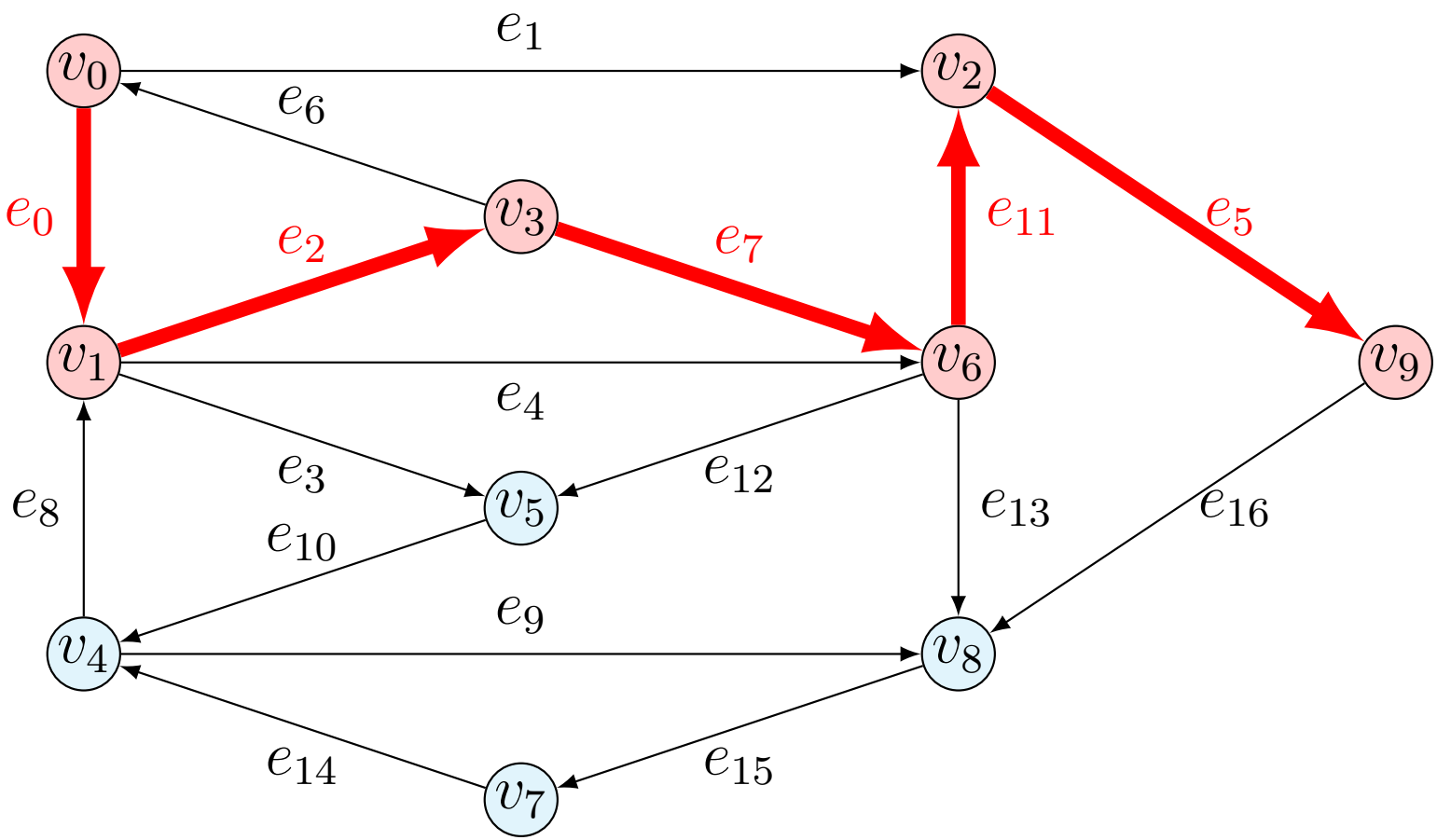
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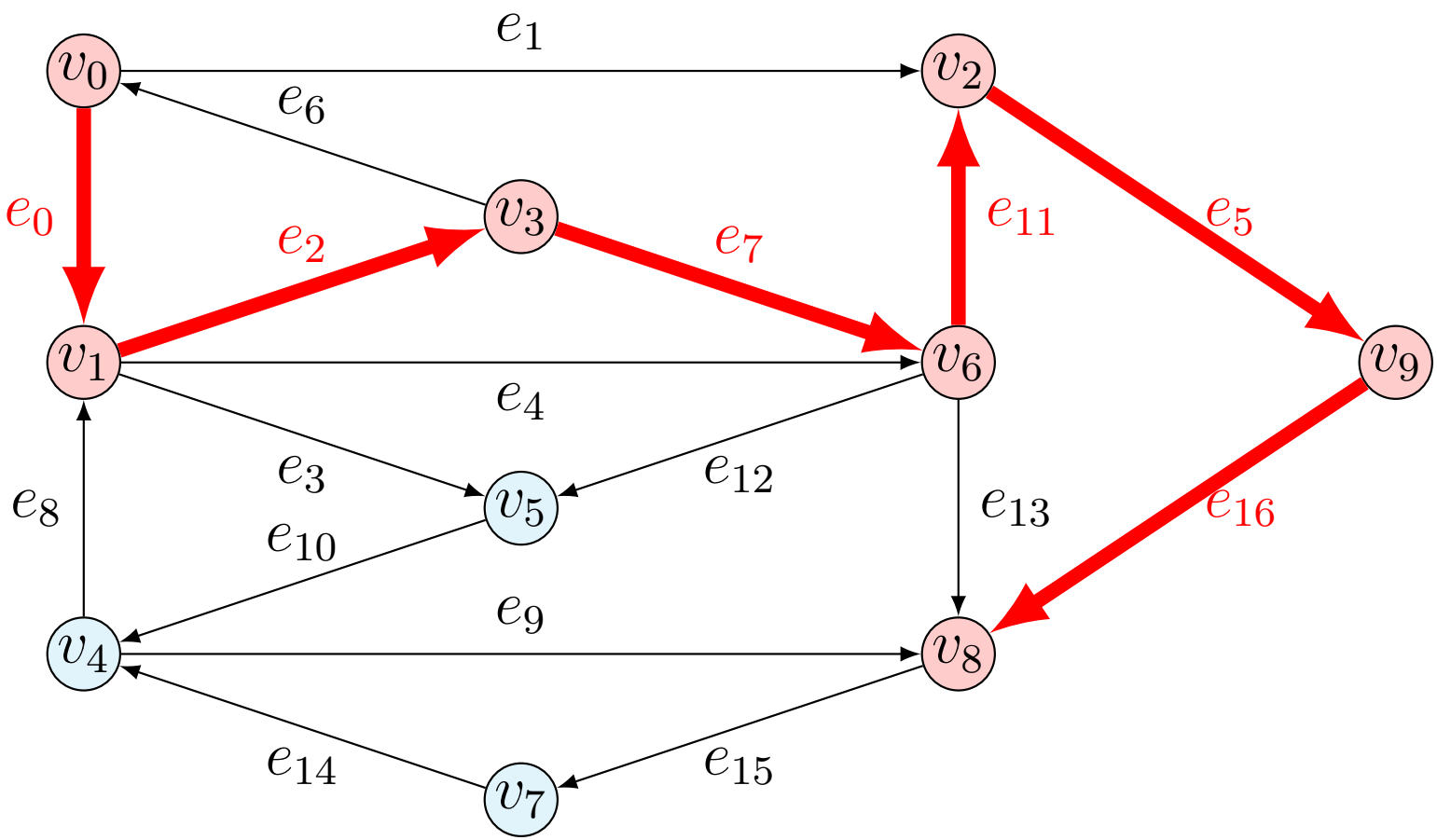
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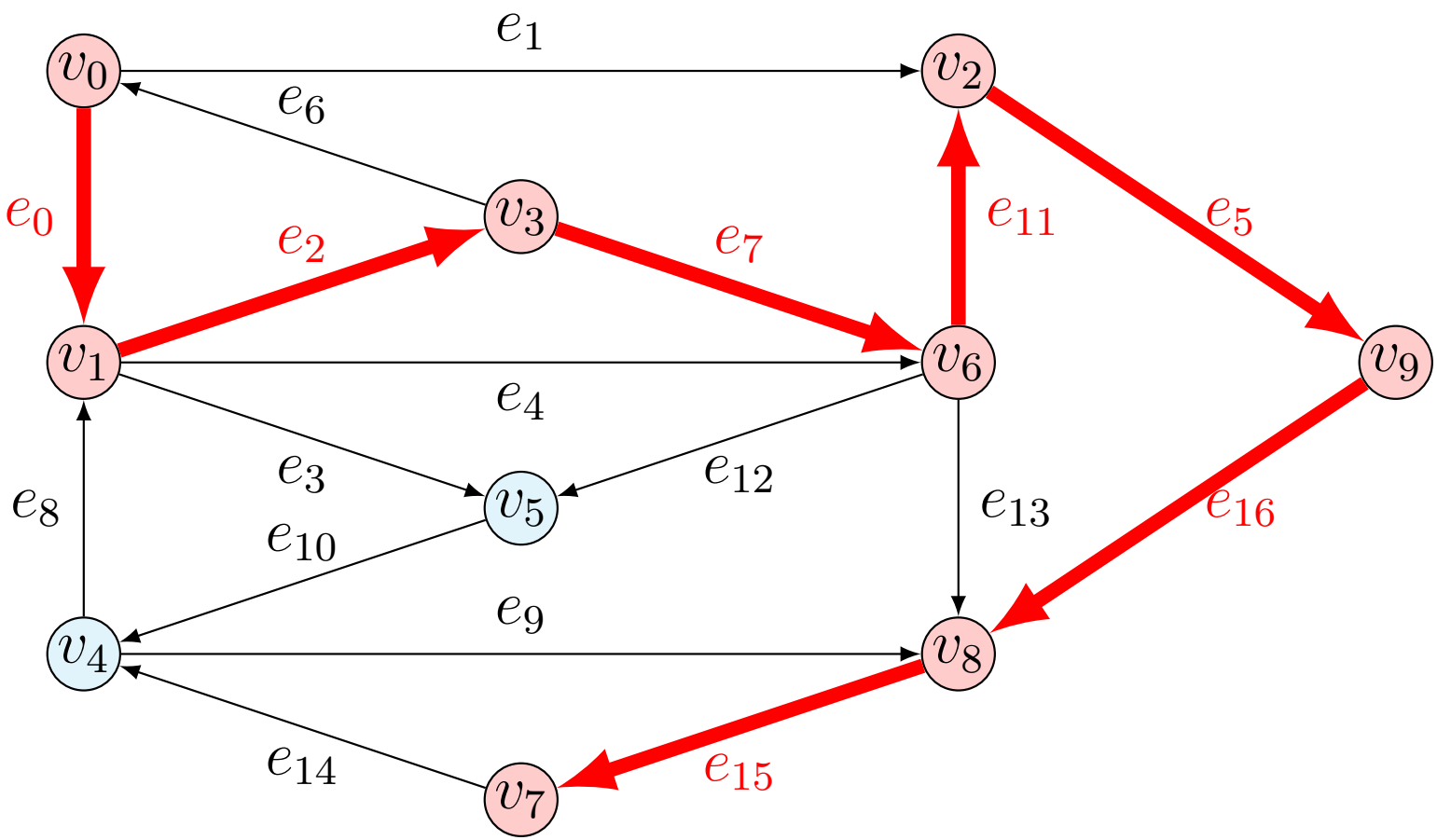
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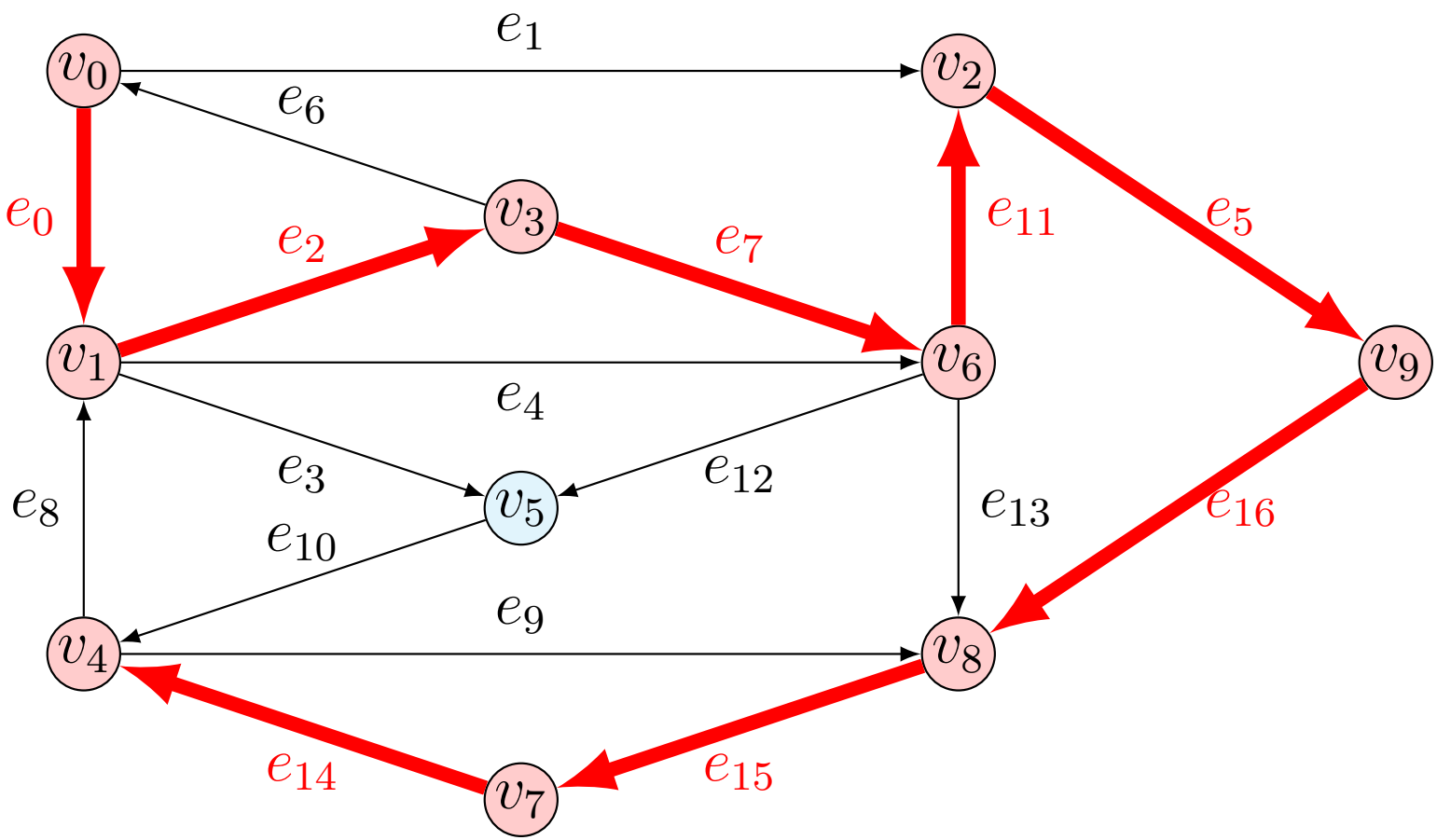
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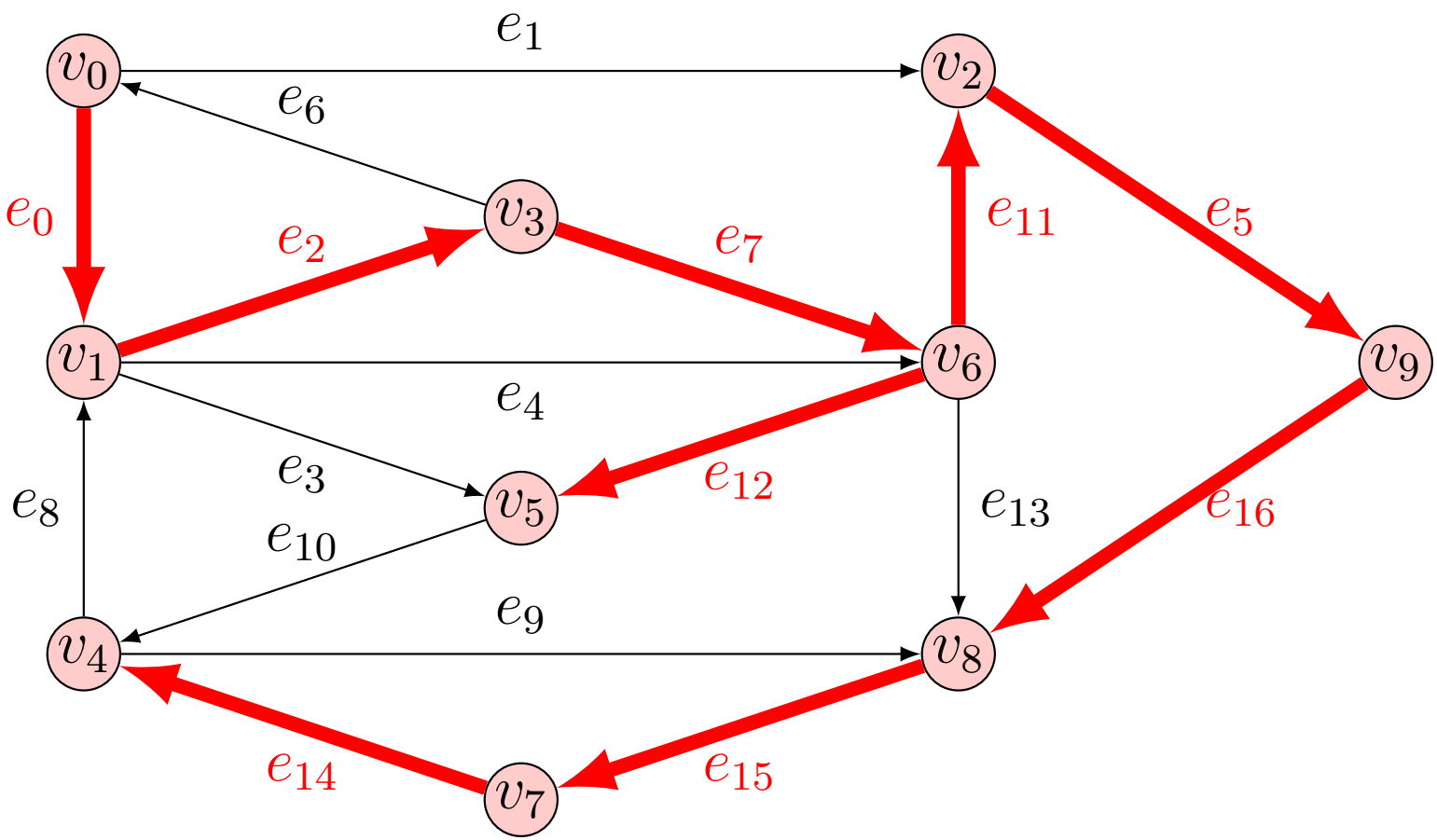
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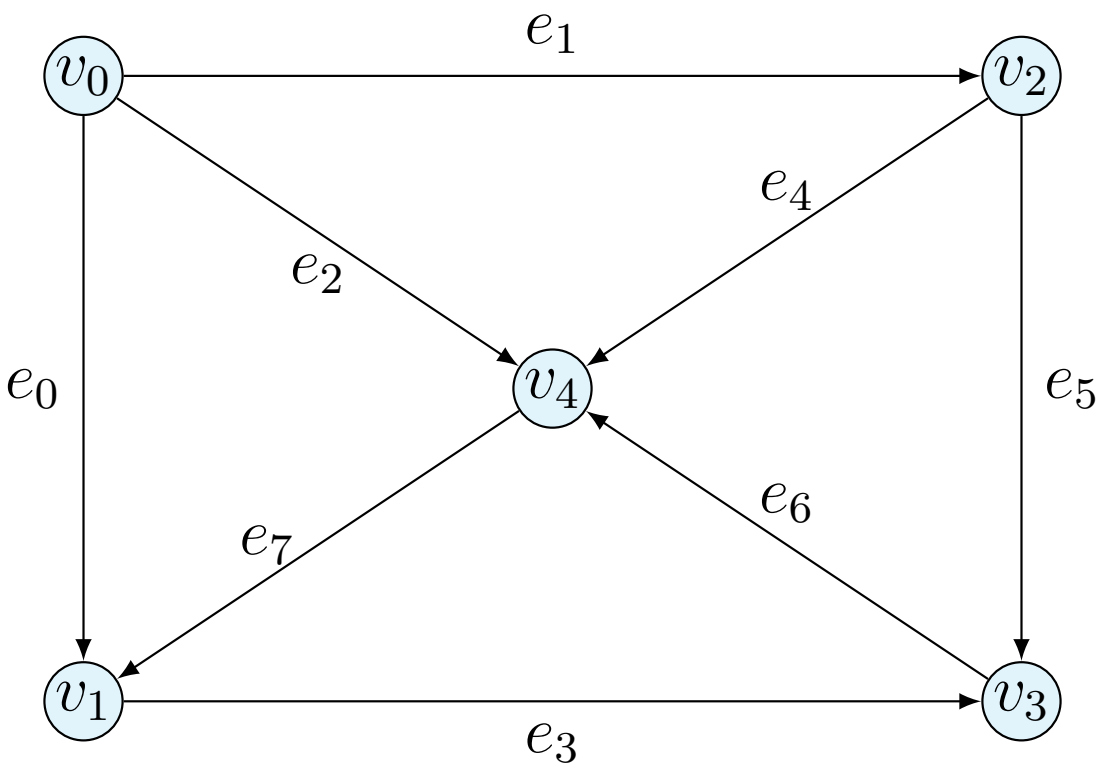
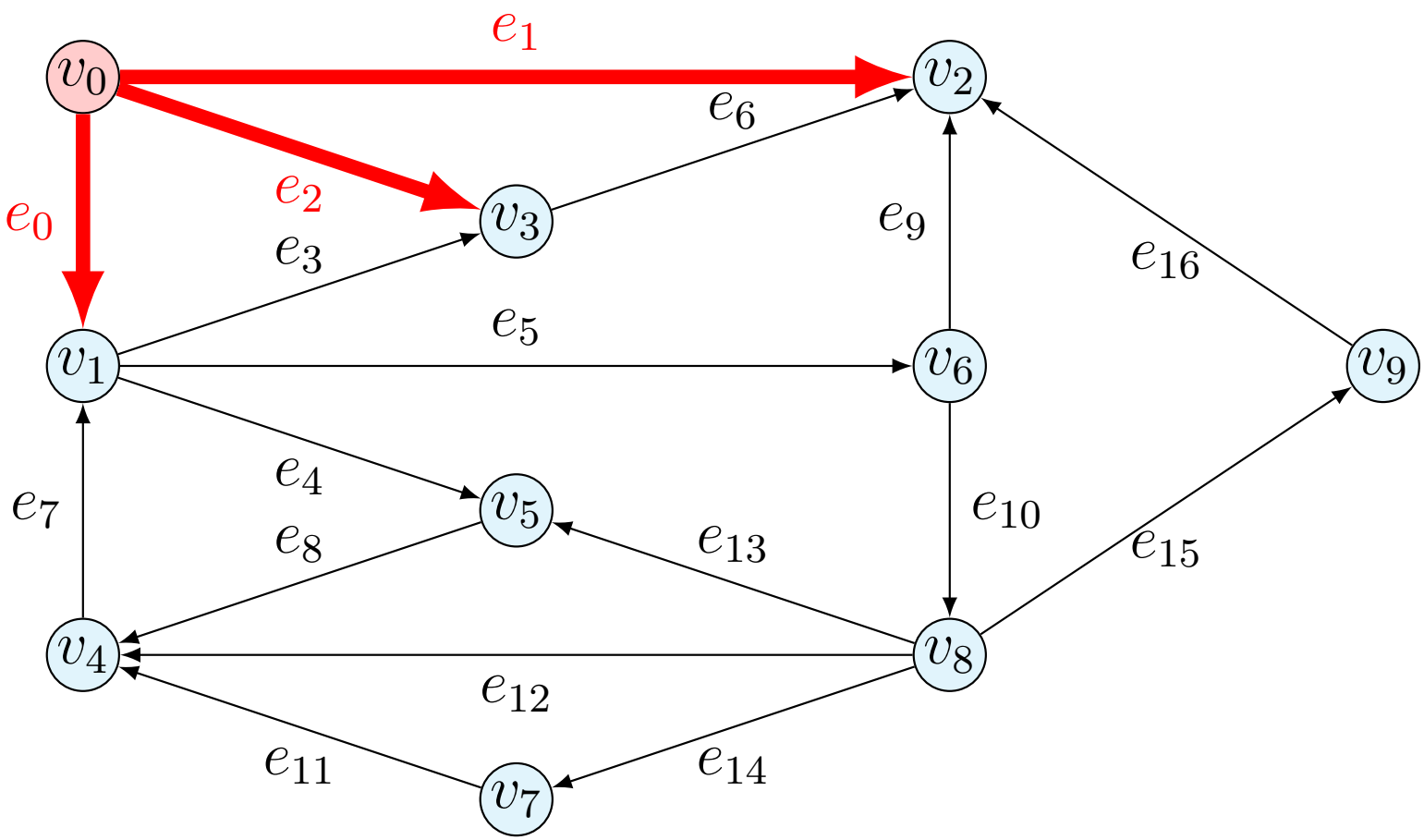
幅優先探索 BFS: Breadth-First Search

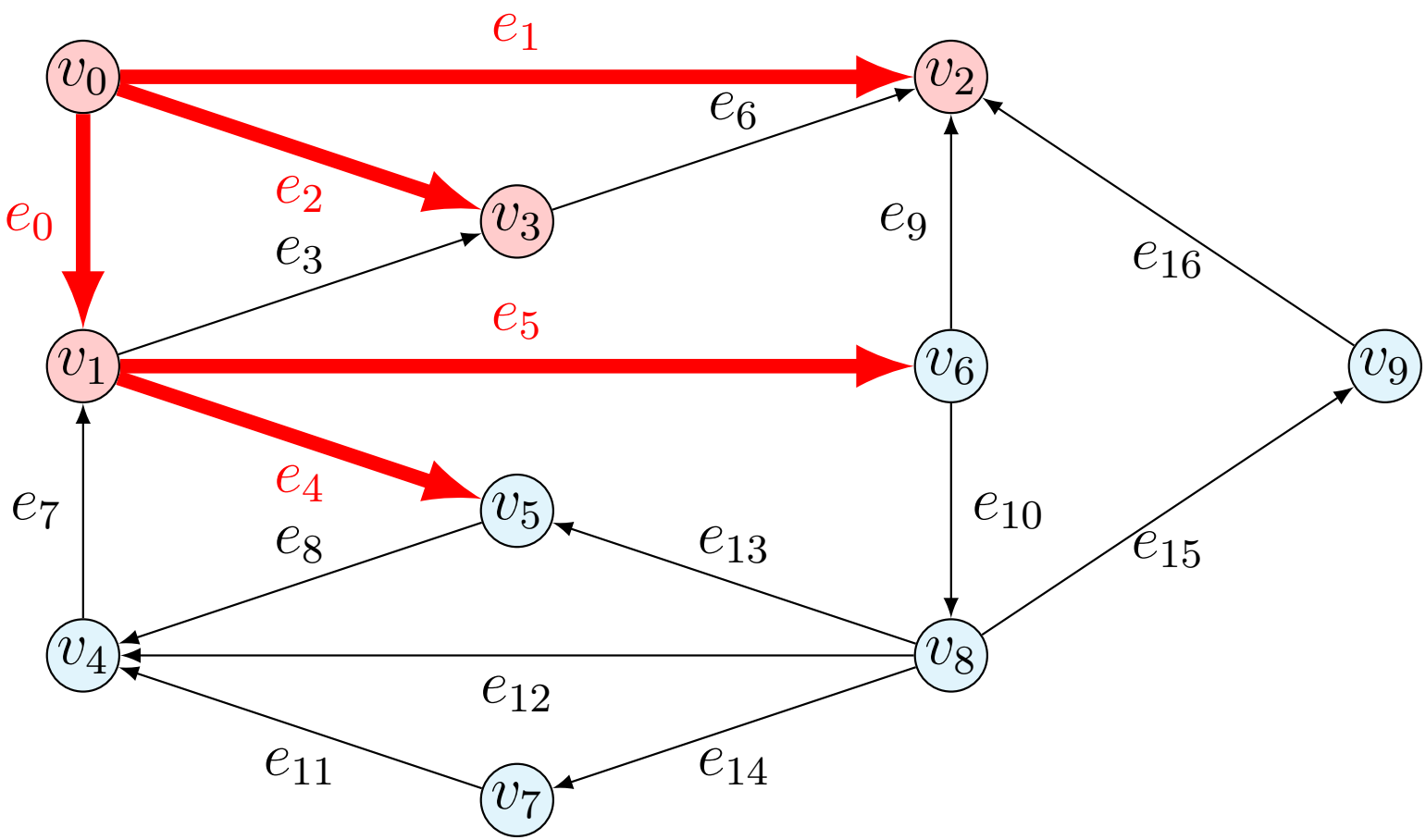
- 出発点を定める。この点の集合を S_0 とする。
- 新たな頂点がなくなるまで繰り返す
  - S_{i-1} の各点の隣接頂点のうち、未調査の点の集合を S_i とする
- 結果としてできる木 (spanning tree) は、浅いものができる

例 3.1:



例 1.2 に対する BFS の結果

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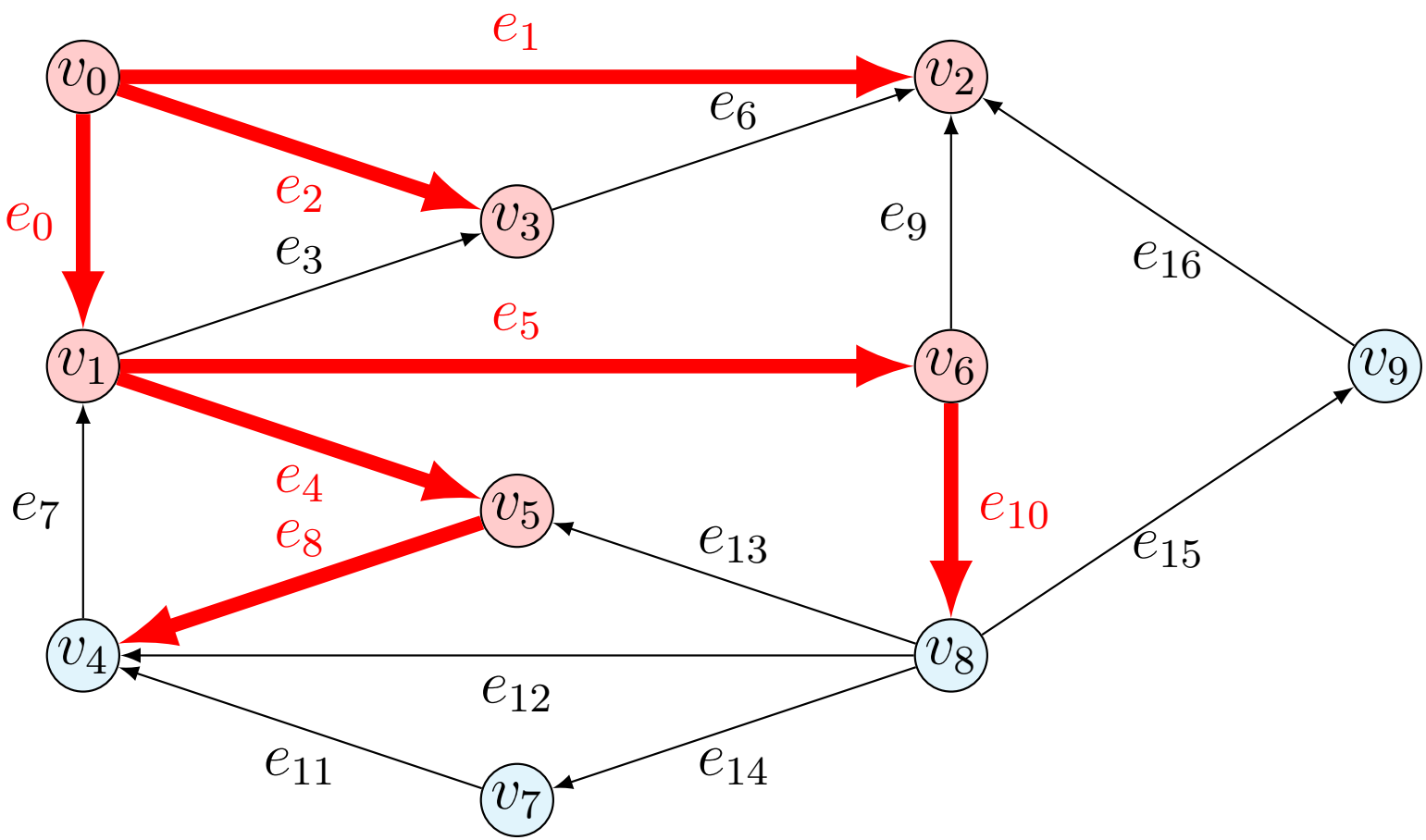
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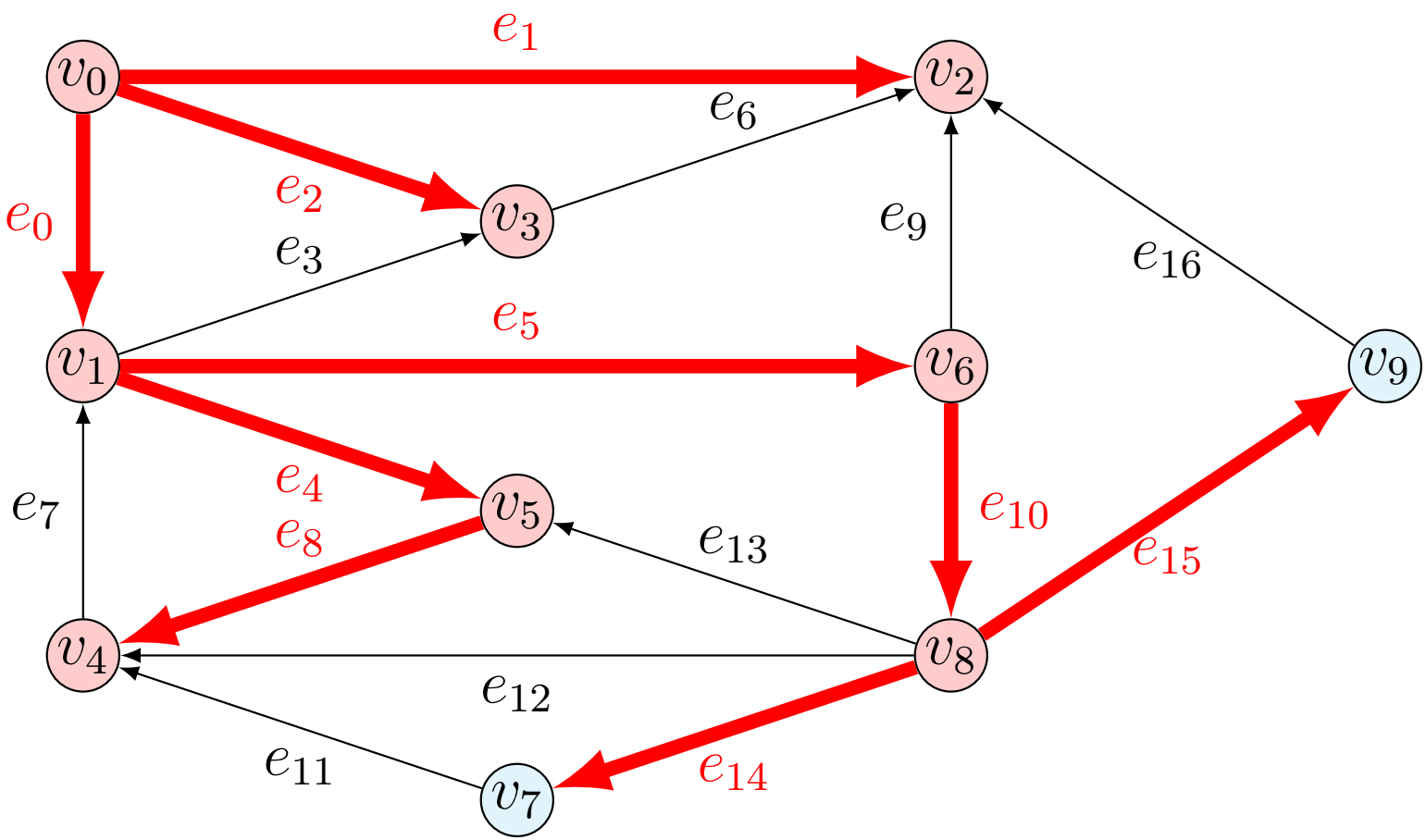
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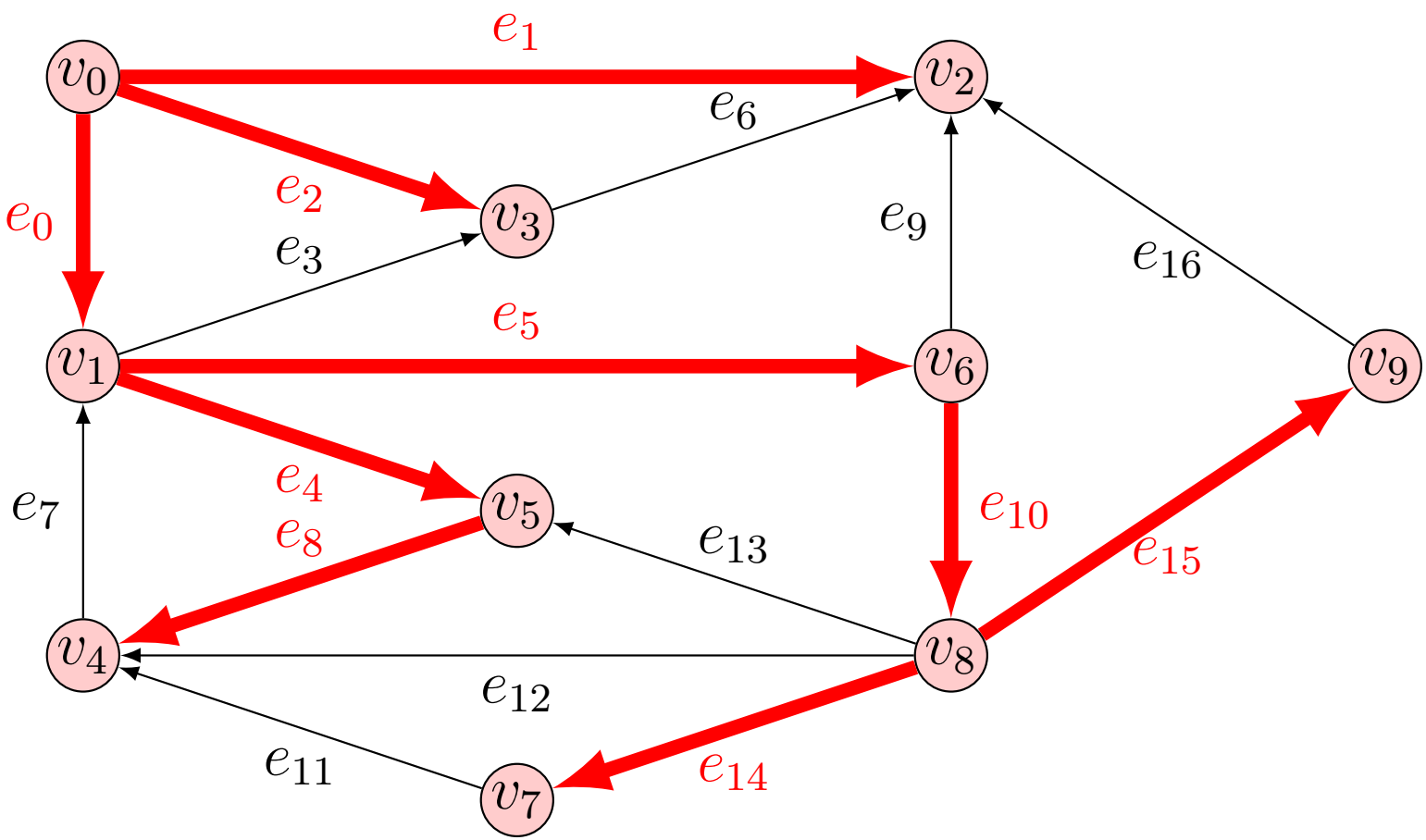
e_{15}
e_{16}


v_0
v_1
v_2
v_3
v_4
v_5
v_6
v_7
v_8
v_9
e_0
e_1
e_2
e_3
e_4
e_5
e_6
e_7
e_8
e_9
e_{10}
e_{11}
e_{12}
e_{13}
e_{14}
e_{15}
e_{16}

Algorithm 2 BFS アルゴリズム

```
L = ∅                                          ▷ 到達済み頂点のリスト
A = ∅                                          ▷ 探索木の辺のリスト
Q = [(Null, r)]                                ▷ 調査すべき辺の待ち行列
while Q ≠ ∅ do
    (s, v) = e = Q.poke()                      ▷ 待ち行列の先頭要素を取り出す
    if v ∉ L then
        if s ≠ Null then
            A.append(e)
        end if
        for all e ∈ δ+ v do
            if e ∉ Q then
                Q.push(e)
            end if
        end for
        L.append(v)
    end if
end while
```

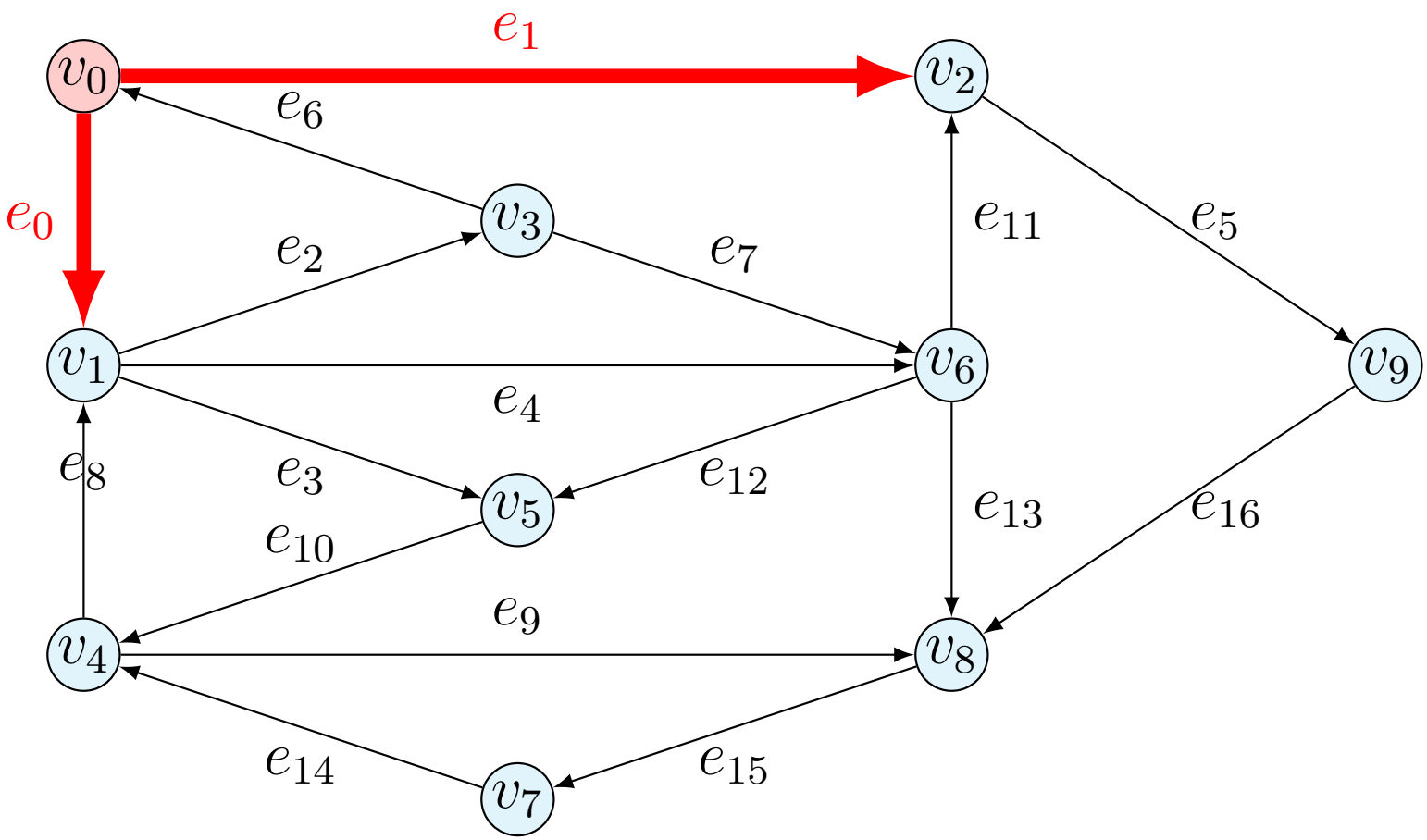
待ち行列: Queue

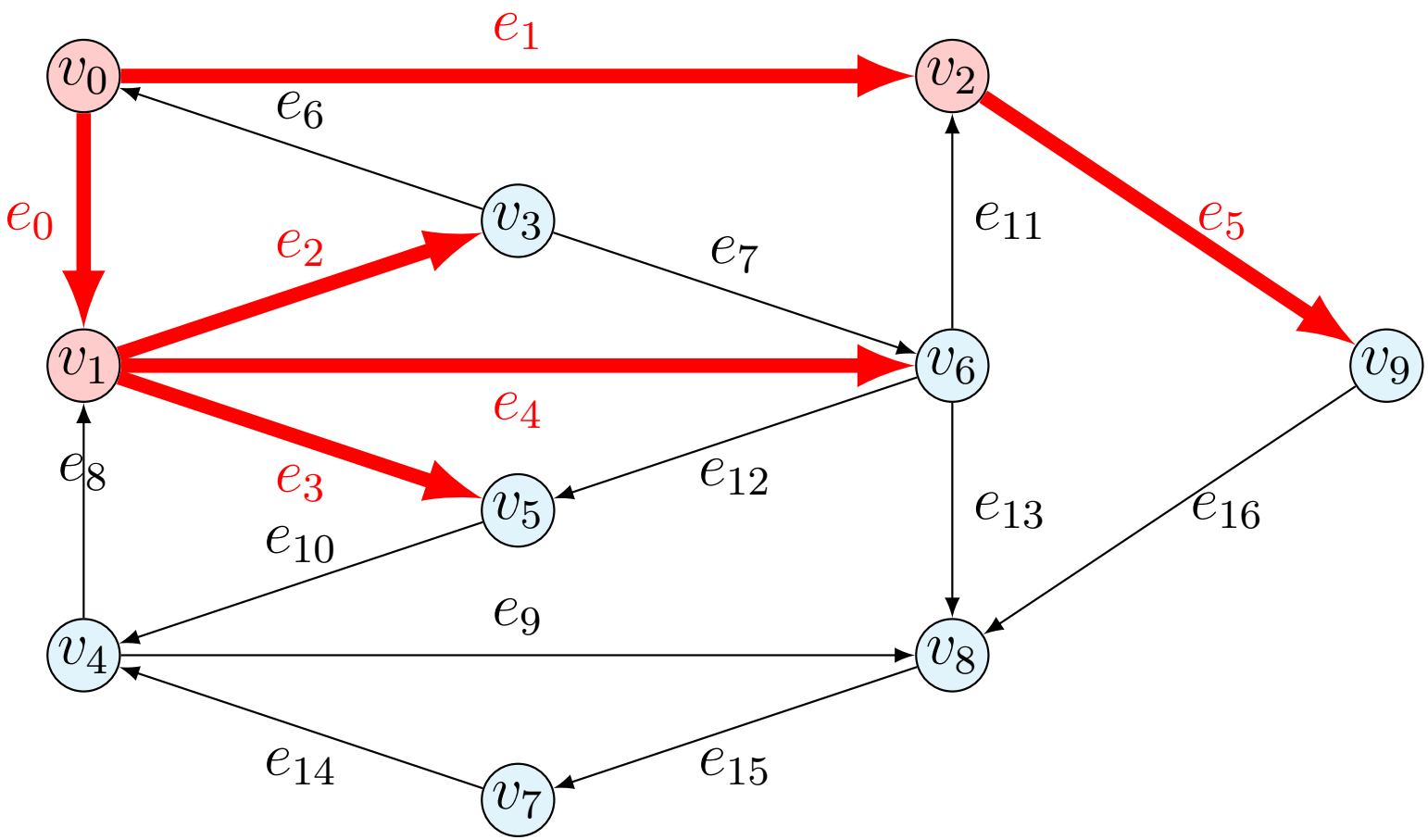
- リストの一種
- 末尾から要素を追加
- 先頭から要素を削除
- First-In-First-Out

例 1.2: 探索の状況

	現在の頂点	L	Q
0		$\emptyset$	$[, v_0]$
1	v_0	$[v_0]$	$[e_0, e_1, e_2]$
2	v_1	$[v_0, v_1]$	$[e_1, e_2, e_4, e_5]$
3	v_2	$[v_0, v_1, v_2]$	$[e_2, e_4, e_5]$
4	v_3	$[v_0, v_1, v_2, v_3]$	$[e_4, e_5]$
5	v_5	$[v_0, v_1, v_2, v_3, v_5]$	$[e_5, e_8]$
6	v_6	$[v_0, v_1, v_2, v_3, v_5, v_6]$	$[e_8, e_{10}]$
7	v_4	$[v_0, v_1, v_2, v_3, v_5, v_6, v_4]$	$[e_{10}]$
8	v_8	$[v_0, v_1, v_2, v_3, v_5, v_6, v_4, v_8]$	$[e_{14}, e_{15}]$
9	v_7	$[v_0, v_1, v_2, v_3, v_5, v_6, v_4, v_8, v_7]$	$[e_{15}]$
10	v_9	$[v_0, v_1, v_2, v_3, v_5, v_6, v_4, v_8, v_7, v_9]$	$[]$

例 2.1: 結果


e_1
v_0
v_2
e_6
e_0
v_3
e_2
e_7
e_{11}
e_5
v_1
v_6
v_9
e_4
e_8
e_3
e_{12}
v_5
e_{13}
e_{16}
e_{10}
e_9
v_4
v_8
e_{14}
e_{15}
v_7


v_0
v_1
v_2
v_3
v_4
v_5
v_6
v_7
v_8
v_9
e_0
e_1
e_2
e_3
e_4
e_5
e_6
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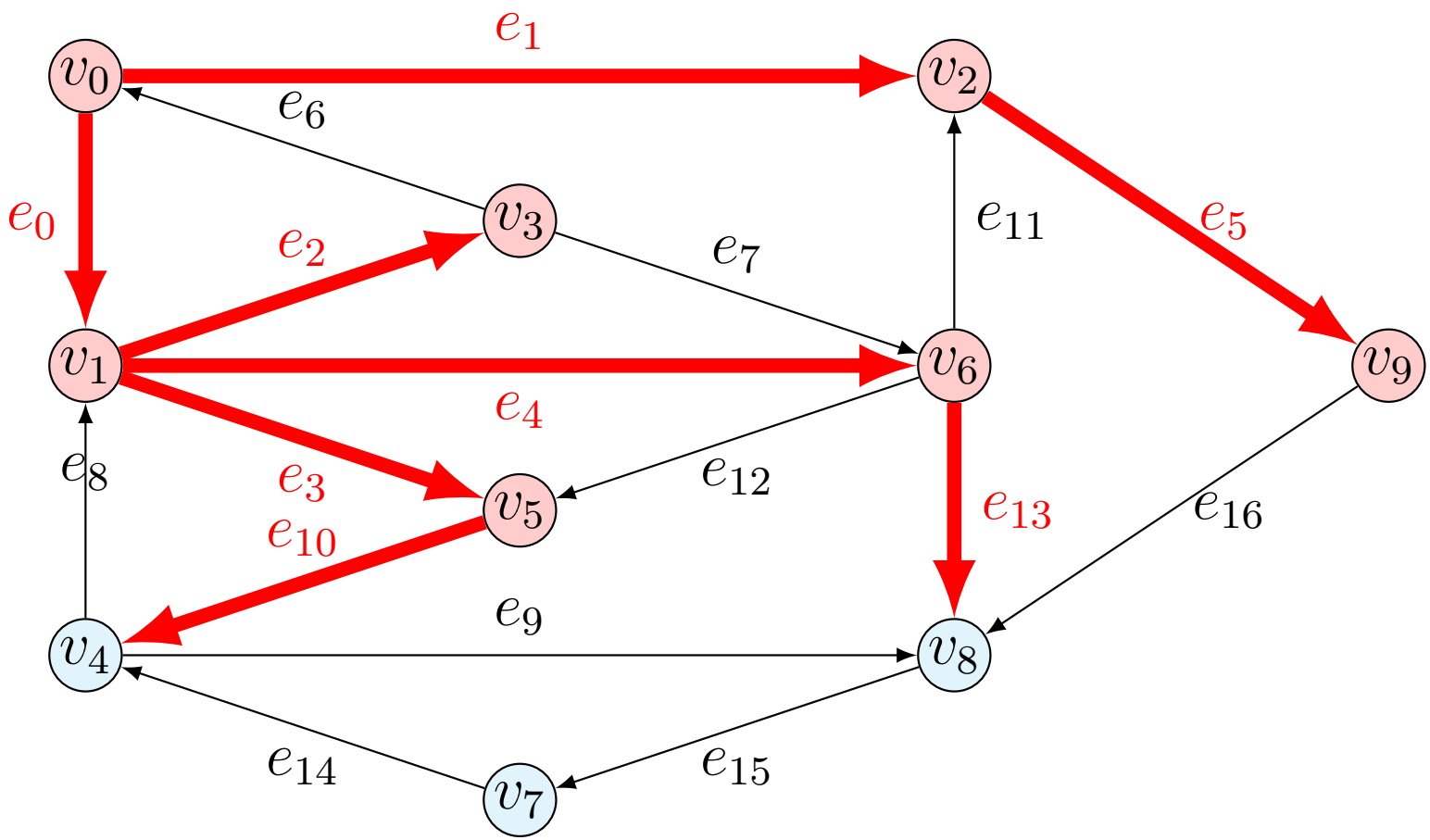
e_9
e_{10}
e_{11}
e_{12}
e_{13}
e_{14}
e_{15}
e_{16}


v_0
v_1
v_2
v_3
v_4
v_5
v_6
v_7
v_8
v_9
e_0
e_1
e_2
e_3
e_4
e_5
e_6
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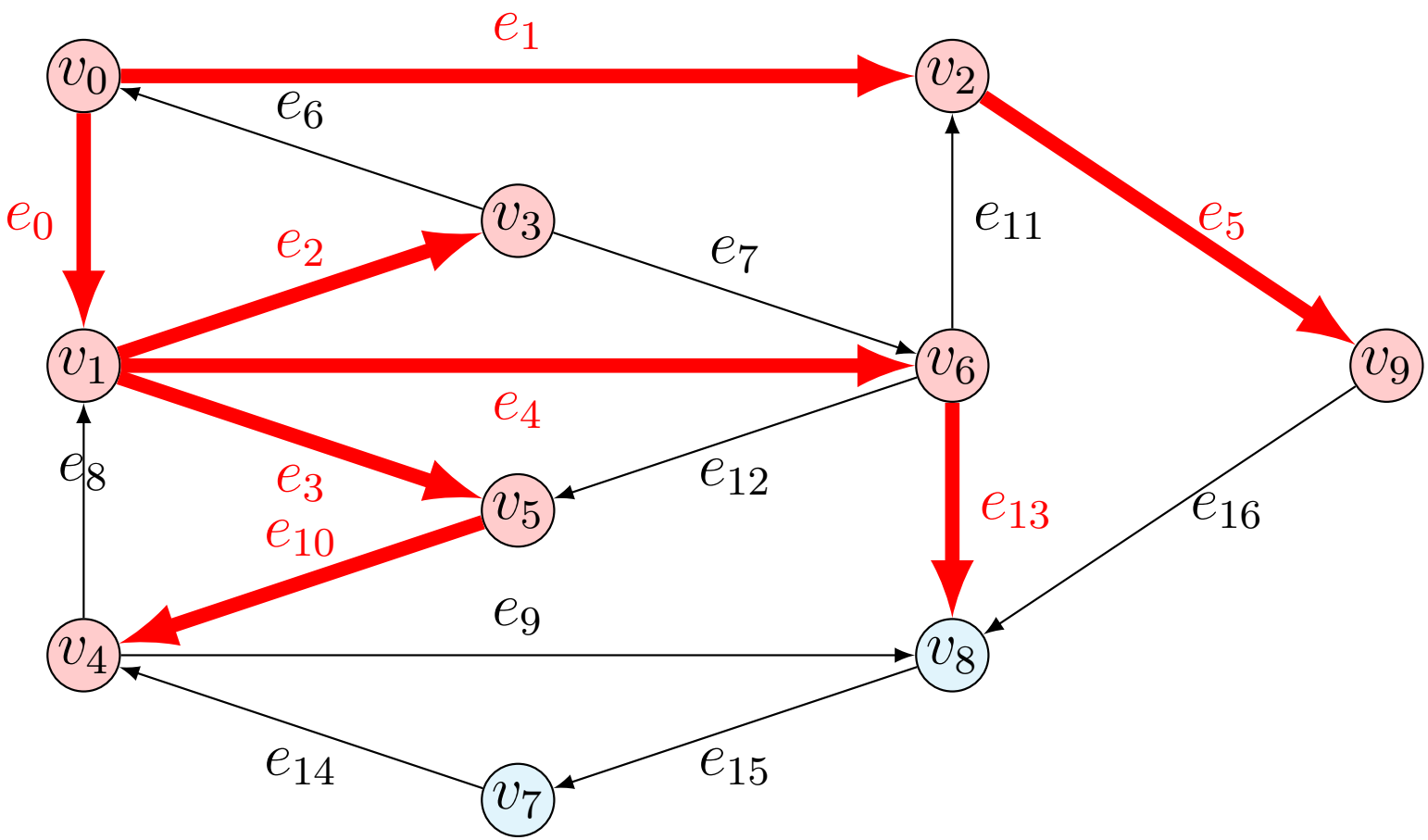
e_{12}
e_{13}
e_{14}
e_{15}
e_{16}


v_0
v_1
v_2
v_3
v_4
v_5
v_6
v_7
v_8
v_9
e_0
e_1
e_2
e_3
e_4
e_5
e_6
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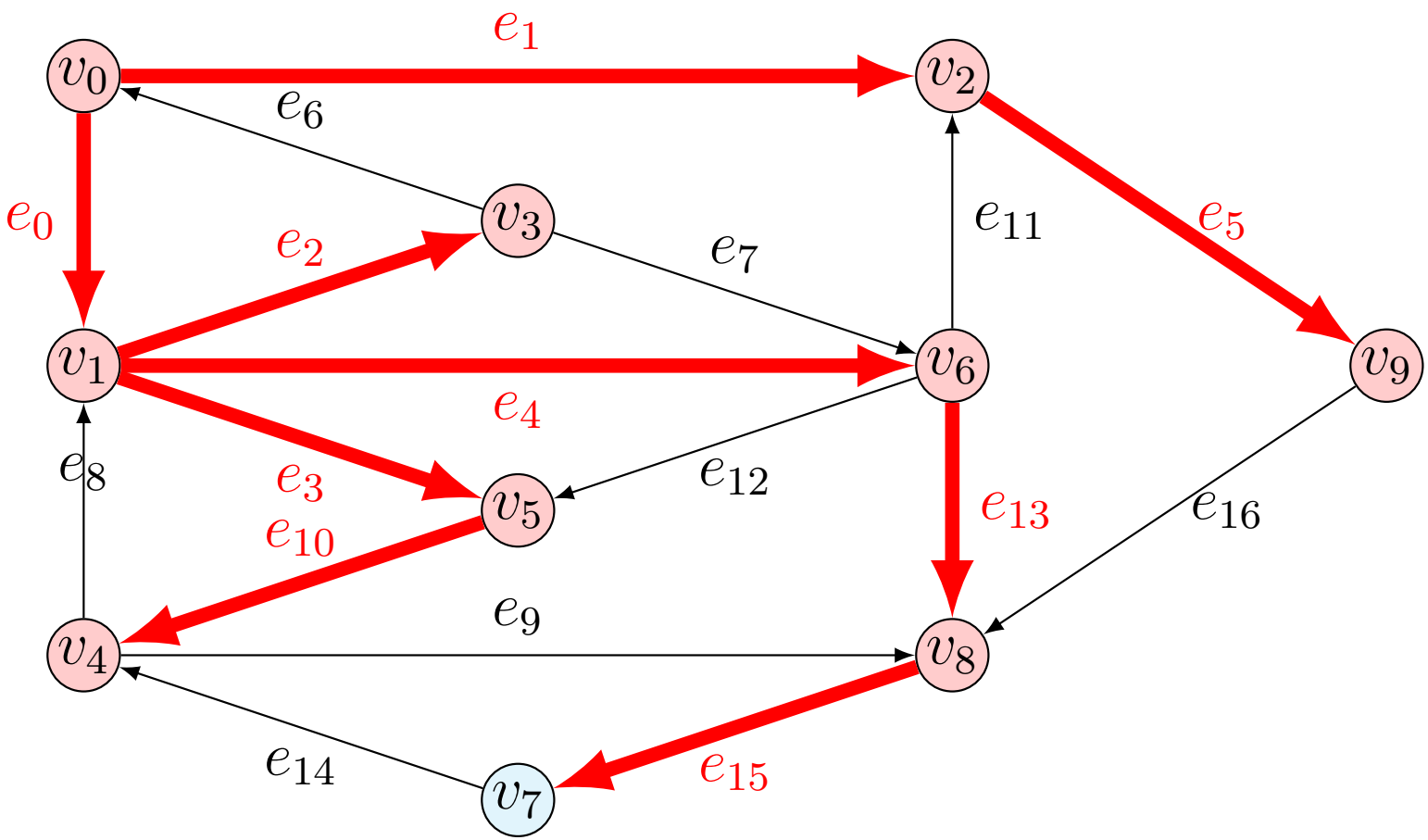
e_9
e_{10}
e_{11}
e_{12}
e_{13}
e_{14}
e_{15}
e_{16}


v_0
v_1
v_2
v_3
v_4
v_5
v_6
v_7
v_8
v_9
e_0
e_1
e_2
e_3
e_4
e_5
e_6
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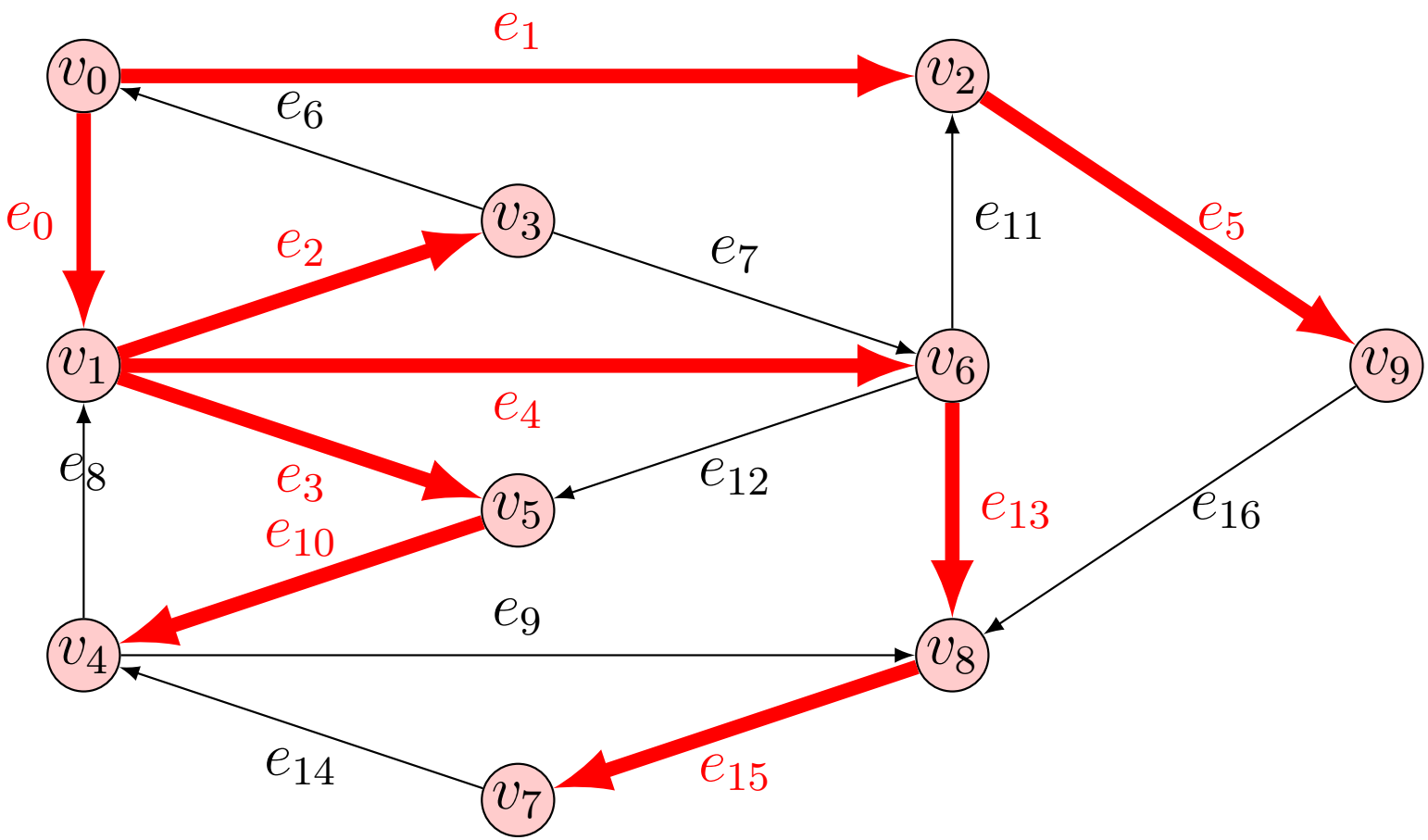
e_8
e_9
e_{10}
e_{11}
e_{12}
e_{13}
e_{14}
e_{15}
e_{16}


v_0
v_1
v_2
v_3
v_4
v_5
v_6
v_7
v_8
v_9
e_0
e_1
e_2
e_3
e_4
e_5
e_6
e_7
e_8
e_9
e_{10}
e_{11}
e_{12}
e_{13}
e_{14}
e_{15}
e_{16}

例 2.1: 探索の状況

	現在の頂点	L	Q
0		$\emptyset$	$[, v_0]$
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5	v_5	$[v_0, v_1, v_2, v_3, v_5]$	$[e_4, e_5, e_{10}]$
6	v_6	$[v_0, v_1, v_2, v_3, v_5, v_6]$	$[e_5, e_{10}, e_{13}]$
7	v_9	$[v_0, v_1, v_2, v_3, v_5, v_6, v_9]$	$[e_{10}, e_{13}]$
8	v_4	$[v_0, v_1, v_2, v_3, v_5, v_6, v_9, v_4]$	$[e_{13}]$
9	v_8	$[v_0, v_1, v_2, v_3, v_5, v_6, v_9, v_4, v_8]$	$[e_{15}]$
10	v_7	$[v_0, v_1, v_2, v_3, v_5, v_6, v_9, v_4, v_8, v_7]$	$[]$