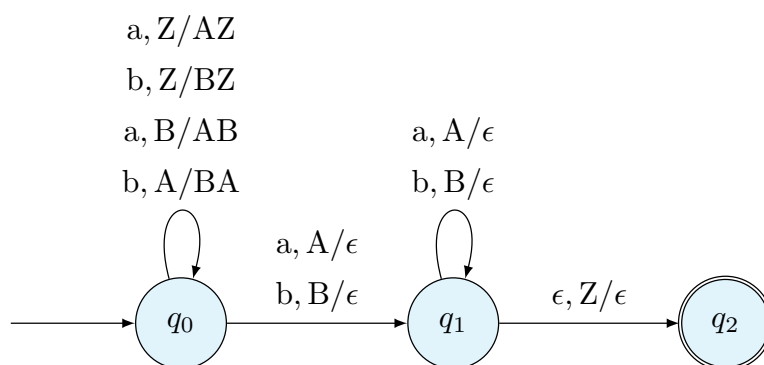# 「離散数学・オートマトン」演習問題 13 （解答例)

2020/1/19

1 プッシュダウンオートマトン

課題 1　以下のようなプッシュダウンオートマトン M を考える。

$$Q = \{q_0.q_1, q_2\}$$
$$\Sigma = \{a, b\}$$
$$\Gamma = \{A, B, Z\}$$
$$F = \{q_2\}$$

$$\delta\left(q_0, a, Z\right) = \left(q_0, AZ\right), \qquad \delta\left(q_0, b, Z\right) = \left(q_0, BZ\right),$$
$$\delta\left(q_0, a, B\right) = \left(q_0, AB\right), \qquad \delta\left(q_0, b, A\right) = \left(q_0, BA\right),$$
$$\delta\left(q_0, a, A\right) = \left(q_1, \epsilon\right), \qquad \delta\left(q_0, b, B\right) = \left(q_1, \epsilon\right),$$
$$\delta\left(q_1, b, B\right) = \left(q_1, \epsilon\right), \qquad \delta\left(q_1, a, A\right) = \left(q_1, \epsilon\right),$$
$$\delta\left(q_1, \epsilon, Z\right) = \left(q_2, \epsilon\right)$$

このとき、入力 ababbaba 及び babaabab に対する動作を示しなさい。

解答例

$$(q_0, \text{ababbaba}, Z) \vdash (q_0, \text{babbaba}, AZ)$$
$$\vdash (q_0, \text{abbaba}, BAZ)$$
$$\vdash (q_0, \text{bbaba}, ABAZ)$$
$$\vdash (q_0, \text{baba}, BABAZ)$$
$$\vdash (q_1, \text{aba}, ABAZ)$$
$$\vdash (q_1, \text{ba}, BAZ)$$
$$\vdash (q_1, \text{a}, AZ)$$
$$\vdash (q_1, \epsilon, Z)$$
$$\vdash (q_2, \epsilon, \epsilon)$$

$$(q_0, \text{babaabab}, Z) \vdash (q_0, \text{abaabab}, BZ)$$
$$\vdash (q_0, \text{baabab}, ABZ)$$
$$\vdash (q_0, \text{aabab}, BABZ)$$
$$\vdash (q_0, \text{abab}, ABABZ)$$
$$\vdash (q_1, \text{bab}, BABZ)$$
$$\vdash (q_1, \text{ab}, ABZ)$$
$$\vdash (q_1, \text{b}, BZ)$$
$$\vdash (q_1, \epsilon, Z)$$
$$\vdash (q_2, \epsilon, \epsilon)$$

課題 2　以下のようなプッシュダウンオートマトン M を考える。
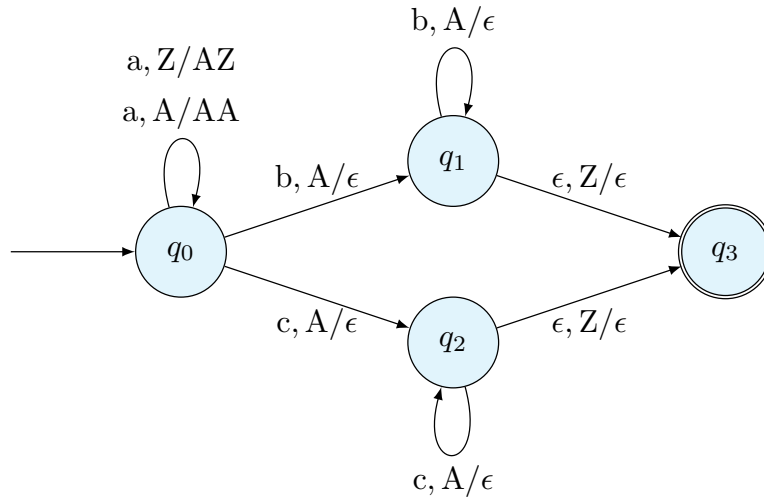
$$Q = \{q_0 . q_1, q_2, q_3\}$$
$$\Sigma = \{a, b, c\}$$
$$\Gamma = \{A, Z\}$$
$$F = \{q_3\}$$

$$\delta(q_0, a, Z) = (q_0, AZ), \qquad \delta(q_0, a, A) = (q_0, AA),$$
$$\delta(q_0, b, A) = (q_1, \epsilon), \qquad \delta(q_0, c, A) = (q_2, \epsilon),$$
$$\delta(q_1, b, A) = (q_1, \epsilon), \qquad \delta(q_2, c, A) = (q_2, \epsilon),$$
$$\delta(q_1, \epsilon, Z) = (q_3, \epsilon), \qquad \delta(q_2, \epsilon, Z) = (q_3, \epsilon)$$

このとき、入力 aaabbb 及び aacc に対する動作を示しなさい。

解答例

$$
\begin{aligned}
(q_0, aaabbb, Z) &\vdash (q_0, aabbb, AZ) \\
&\vdash (q_0, abbb, AAZ) \\
&\vdash (q_0, bbb, AAAZ) \\
&\vdash (q_1, bb, AAZ) \\
&\vdash (q_1, b, AZ) \\
&\vdash (q_1, \epsilon, Z) \\
&\vdash (q_3, \epsilon, \epsilon)
\end{aligned}
$$

$$(q_0, \text{aacc}, Z) \vdash (q_0, \text{acc}, AZ)$$
$$\vdash (q_0, \text{cc}, AAZ)$$
$$\vdash (q_2, \text{c}, AZ)$$
$$\vdash (q_2, \epsilon, Z)$$
$$\vdash (q_3, \epsilon, \epsilon)$$