RISK MANAGEMENT COACH

Exercise Solution 2.2

Formula [2.26] provides a quadratic polynomial approximation $p_2(\boldsymbol{x})$ for the function $f(\boldsymbol{x})$:

$$p_2(\boldsymbol{x}) = f\left(\boldsymbol{x}^{[0]}\right) + \nabla f\left(\boldsymbol{x}^{[0]}\right)'\left(\boldsymbol{x} - \boldsymbol{x}^{[0]}\right) + \frac{1}{2}\left(\boldsymbol{x} - \boldsymbol{x}^{[0]}\right)' \nabla^2 f\left(\boldsymbol{x}^{[0]}\right)\left(\boldsymbol{x} - \boldsymbol{x}^{[0]}\right) \qquad [2.26]$$

We have

$$\nabla f(x_1, x_2) = \begin{pmatrix} \dfrac{\partial f}{\partial x_1} \\ \dfrac{\partial f}{\partial x_2} \end{pmatrix} = \begin{pmatrix} e^{x_2} \\ x_1 e^{x_2} \end{pmatrix} \qquad [s1]$$

and

$$\nabla^2 f(x_1, x_2) = \begin{pmatrix} \dfrac{\partial^2 f}{\partial x_1^2} & \dfrac{\partial^2 f}{\partial x_1 \partial x_2} \\ \dfrac{\partial^2 f}{\partial x_2 \partial x_1} & \dfrac{\partial^2 f}{\partial x_2^2} \end{pmatrix} = \begin{pmatrix} 0 & e^{x_2} \\ e^{x_2} & x_1 e^{x_2} \end{pmatrix} \qquad [s2]$$

Valuing f, ∇f, and $\nabla^2 f$ at $\boldsymbol{x}^{[0]} = (0, 0)$, we obtain

$$f(0,0) = 0 \qquad [s3]$$

$$\nabla f(0,0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad [s4]$$

$$\nabla^2 f(0,0) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad [s5]$$

Substituting these into [2.26], our result is

$$p_2(x_1, x_2) = 0 + \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} x_1 & x_2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad [s6]$$

$$= x_1 + x_1 x_2 \qquad [s7]$$