UNIT- 4

Group ring theory

Group theory is one of the most important fundamental concept of modern algebra.
Groups have wide application in physical science and biological science particularly the study of crystal structure, configuration of molecules and structure of human genes.

Binary Operation

Let G be a non empty set then $G \times G = \{(a,b) : a \in G,\ b \in G\}$.
If ($f: G \times G \rightarrow G$) then f is said to be binary operation on G.
Thus a binary operation on G is a function that assign each ordered pairs of elements of G to an element of G.
The symbols $+, \cdot, *, o$ etc are used to denote binary operations on a set.

Therefore ($+$) will be a binary operation on G.
If and only if $a+b \in G$ for $\forall a,b \in G$ and $a+b$ is unique.
Therefore It follow the Closure property.
A particular binary operation $*$ on a given set is too characteristics the elements $a*b$ assign to each pair [(a,b)] by same property defines in term of a and b.

A binary operation on a set G is sometimes called composition in G. for a finite set a binary operation on the set can be defined by a table called composite table.

Composite table

$*$	a	b	c
a	a	a	b
b	b	a	c
c	c	a	b

Algebric structure

A non empty set together with one or more than one binary operations is called algebric structure.

As – $(R, +, \times)$, $Z(+, -)$

Laws of binary operation→

Associative Law

$$(a*b)*c = a*(b*c)$$

Commutative law = $a*b = b*a$

Identity element

An element e in a set S is called an Identity element with respect to binary operation $(*)$ for any element A in S.

$$a*e = e*a = a$$

If $a*e = a$ then e is called right Identity.

and If $e*a = a$ then e is called left Identity.

for the operation $(*)$

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Additive Identity = 0

Multiplicative Identity = 1

Inverse element

If there is a set S with Identity element e with respect to the binary operation $(*)$.

If corresponding to each element $a \in S$ there exist an element $b \in S$ such that $a*b = e$ or $b*a = e$

Prepare the composition table for multiplication on the element in the set $A = \{1, \omega, \omega^2\}$, where ω is a cube root of unity. Show that the multiplication satisfies the closure property, associative law, commutative law and one is inverse element. write down the multiplicative inverse of each element.

VIMP

Composite table

$\times$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	$\omega^3 = 1$
ω^2	ω^2	$\omega^3 = 1$	$\omega^4 = \omega \times \omega^3 = \omega$

all elements $\in$ set A ;

Hence it satisfies the closure property.

Associative law

$$a*b*c = a*(b*c)$$
$$(1\cdot\omega)\cdot\omega^2 = 1\cdot(\omega\cdot\omega^2)$$
$$(\omega\cdot\omega^2) = 1\cdot\omega^3$$
$$\omega^3 = 1\cdot 1$$
$$1 = 1$$

Commutative law

Since 1^{st}, 2^{nd} and 3^{rd} rows concite with 1^{st}, 2^{nd} and 3^{rd} columns respectively. So multiplication is commutative on S.

Identity	Inverse	multiplicative Inverse	
$a \times e = a$	$a \times b = e$	$\omega \times b = 1$	$1 \to 1$
$1 \times e = 1$	$1 \times b = 1$	$\omega \times \omega^2 = 1$	$\omega \to \omega^2$
$1 \times e = 1$	$1 \times 1 = 1$	$\omega \to \omega^2$	$\omega^2 \to \omega$
$e = 1$			

Groupoid

Let $(S, *)$ be an algebric structure in which S is a non empty set and $(*)$ is a binary operation on S. If S is closed with the operation $(*)$ then S is called groupoid.

Prove only closure property

Semigroup

An algebric structure $(S, *)$ is called semigroup if it follow the following conditions.

(i) Closure property (ii) Associative property.

Monoid

An algebric structure $(S, *)$ is monoid if it follows the following condition -

(i) Closure law (ii) Associative

(iii) There exist an Identity element.

Group

An algebric structure $(G, *)$ be an algebric structure where $(*)$ is a binary operation if it follow the following conditions -

(i) Closure law (ii) Associative law

(iii) Identity element should exist.

(iv) Inverse element - (for each $a \in G$, there exist an element a' must exist such that $a * a' = e$).

Abelian group

All group's property and also commutative law must be hold (first row = first column).

Qn Prove that the 4th root of unity $(1, -1, i, -i)$ form an Abelian multiplicative group.

(i) Composite table

$\times$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

→ same

So = 1

It follows the closure property

(ii) Associative law

$$(a*b)*c = a*(b*c)$$
$$(1\times -1)\times i = 1\times(-1\times i)$$
$$(-1)\times i = 1\times(-i)$$
$$-i = -i$$

It follows the associate law.

(iii) It follows the commutative law because first, 2nd, 3rd and 4th rows belongs to columns respectively.

(iv) Identity

$$a\times e = a$$
$$1\times e = 1$$
$$e = 1$$

(v) multiplicative Inverse

$$a\times b = e$$
$$1\times b = 1$$
$$1\to 1$$

$$-1\times i = 1$$
$$-1 = -1$$
$$i\times -i = 1$$
$$i\to -i$$

$$-i\to i$$

ps

Q^n Let the binary operation * defined on $S = \{a, b, c, d, e\}$ by means of composite table

*	a	b	c	d	e
a	a	b	c	b	d
b	b	c	a	e	c
c	c	a	b	b	a
d	d	e	b	e	d
e	d	b	a	d	e

Compute $c * d = b$

$b * b = c$

$(a * b) * (c) = b * c = a$

$[(a * c) * e] * a = a$

(ii) is * commutative = Rows are not equal to column.

Q^n Let z be the set of integers show that the operation * on z defined by

$a * b = a + b + 1$ for all $(a, b \in z)$ satisfies the closure property associative law and the comutative law. find the identity element. and what is inverse of integer.

(i) Closure

Since $(a$ and $b) \in z$,

$\therefore a + b + 1$ also $\in$ to z.

(ii) Commutative

$\boxed{a * b = a + b + 1}$

$\boxed{b * a = b + a + 1}$

(iii) Associative

Let the third element c that $\in z$

$(a * b) * c = a * (b * c)$

$(a + b + 1) * c = a * (b + c + 1)$

$z * c = a * z$

$z + c + 1 = a + z + 1$

$$a+b+1+c+1 = a+b+c+1+1$$
$$a+b+c+2 = a+b+c+2$$

Hence proved associative

Identity

$$a * e = a \quad \text{— formula}$$
$$a+e+1 = a$$
$$e = -1$$

Inverse element

$$a * a^{-1} = e$$
$$a * a^{-1} = e$$
$$a + a^{-1} + 1 = -1$$
$$a + a^{-1} = -2$$
$$a^{-1} = -2 - a$$
$$= -(a+2)$$

Qn Show that the set of all positive rational numbers (Q^{+}) forms an abelian group under the composition defined by

$$a * b = (ab)/2$$

(i) Closure

since for every element $(a,b) \in Q^{+}$, therefore $\frac{ab}{2}$ is also $\in Q^{+}$

so it follow closure property.

(ii) Associative

Let c be the third element $\in Q^+$.

$$(a*b)*c = a*(b*c)$$

$$\Rightarrow \left(\frac{ab}{2}\right)*c = a*\frac{bc}{2}$$

$$\frac{\frac{abc}{2}}{2} = \frac{\frac{abc}{2}}{2}$$

$$\frac{abc}{4} = \frac{abc}{4} \Rightarrow \frac{abc}{4} = \frac{abc}{4}$$

LHS = R.H.S. Hence this follows associative law.

(iii) Commutative

$$a*b = ab/2$$

So $b*a = \frac{ba}{2}$

$$\frac{ab}{2} = \frac{ba}{2}$$

L.H.S = R.H.S

Hence, It is commutative

(iv) Identity

$$a*e = a$$

$$\frac{ae}{2} = a$$

$$ae = 2a$$

$$e = 2$$

(v) Inverse

$$a*a^{-1} = e$$

$$a*a^{-1} = 2$$

$$\frac{aa^{-1}}{2} = 2$$

$$aa^{-1} = 4$$

$$a^{-1} = \frac{4}{a}$$

Q^n Let the $G = \{1, 2, 3, 4, 5\}$, then show that It is a group under the addition and multiplication % 6

Sol^n

$+_6$	1	2	3	4	5
1	1	3	4	5	0
2	3	4	5	0	1
3	4	5	0	1	2
4	5	0	1	2	3
5	0	1	2	3	4

In our composite table $0 \notin$ given set G — It does not follow the closure property.

So It is not a group.

$\times_6$	1	2	3	4	5
1	1	2	3	4	5
2	2	4	0	2	4
3	3	0	3	0	3
4	4	2	0	4	2
5	5	4	3	2	1

In our composite table $0 \notin G$, It does not follow the closure property.

Hence It is not a group.

Q^n Show that the matrices $\begin{bmatrix}1&0\\0&1\end{bmatrix}, \begin{bmatrix}-1&0\\0&1\end{bmatrix}, \begin{bmatrix}1&0\\0&-1\end{bmatrix}, \begin{bmatrix}-1&0\\0&-1\end{bmatrix}$

form a multiplicative Abelian group.

X	A	B	C	D
A	A	B	C	D
B	B	A	D	C
C	C	D	A	B
D	D	C	B	A

It follows the closure property.

(ii) Associative

$$(a*b)*c = a*(b*c)$$

$$\left(\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}-1&0\\0&1\end{bmatrix}\right)*\begin{bmatrix}1&0\\0&-1\end{bmatrix} = \begin{bmatrix}1&0\\0&1\end{bmatrix}\left(\begin{bmatrix}-1&0\\0&1\end{bmatrix}\begin{bmatrix}1&0\\0&-1\end{bmatrix}\right)$$

$$= \begin{bmatrix}-1&0\\0&1\end{bmatrix}\begin{bmatrix}1&0\\0&-1\end{bmatrix} = \begin{bmatrix}-1&0\\0&1\end{bmatrix}\begin{bmatrix}1&0\\0&-1\end{bmatrix}$$

$$= \begin{bmatrix}-1&0\\0&-1\end{bmatrix} = \begin{bmatrix}-1&0\\0&-1\end{bmatrix}$$

$$= D = D$$

so it follows the associative law.

(iii) Commutative

It in this 1st, 2nd, 3rd and 4th row is equal to 1st, 2nd, 3rd and 4th column, therefore it is commutative.

(iv) Identity

~~A*e = A~~ Identity e = A

Inverse

$A \times A^{-1} = e$ | $B \times B^{-1} = e$ | $C^{-1} = C$
$A \, A^{-1} = A$ | $B^{-1} = B$ | $D^{-1} = D$
$A^{-1} = A$

Therefore It is an Abelian / commutative group.

Qn Prove that the set {0,1,2,3,4} is a finite Abelian group of order 5 under addition % 5 as composition.

composite table

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

It follows the closure property.

Commutative

In this the 1st, 2nd, 3rd, 4th, and 5th row is equal to the 1st, 2nd, 3rd, 4th and 5th column, there It is a commutative.

Associative

$(A \times B) * C = A * (B \times C)$

$= (0 *_5 1) *_5 2 = 0 *_5 (1 *_5 2)$

$= 3 = 3$

So It follows the associativity.

$(B*C)*d = B*(C*d)$

$= (1*_6 2)*_6 3 = 1*_6(2*3)$

$\Rightarrow 3*3 = 1*_6(6)$

$6 = 6$

Hence It is Associative.

Identity

Identity $e = 0$

Inverse

$A \times A^{-1} = e$	$B \times B^{-1} = e$	$C^{-1} = 3$
$0 * A^{-1} = 0$	$1 \times B^{-1} = 0$	$D^{-1} = 2$
$A^{-1} = 0$	$B^{-1} = 4$	$E^{-1} = 1$

As

Qn Let $G = \{(a,b) / a,b \in R, a \neq 0\}$
Define a binary operation $(*)$ on G by $(a,b)*(c,d) = (ac, bc+d)$ for $\forall$ $(a,b),(c,d) \in G$, Then show that $(G,*)$ is a group.

(i) Closure property $\rightarrow$ $(a,b \in R)$ and $(c,d) \in R)$
So $(a,b)*(c,d) = (ac, bc+d)$ this also belongs to R.

(ii) Associative

Suppose third element $\cdot$ (e,f).

$[(a,b)*(c,d)]*(e,f) = (a,b)*[(c,d)*(e,f)]$

$= (ac, bc+d)*(e,f) = (a,b)*(ce, de+f]$

$\Rightarrow [ace, (bc+d)e+f] \Rightarrow ace, b(\cancel{de+f})+f\ \}de]$

$[ace, ebc + ed + f) = [ace, bce + f + de]$

$\therefore$ L.H.S = R.HS

Hence It follows the associativity

Commutative

$[(a,b) * (c,d)] = (ac, bc + d)$

So

$[(c,d) * (a,b)] = (ca, da + b)$

Identity

$a * e = a$

$(a,b) * (x,y) = (a,b)$

$= (ax, bx + y) = (a,b)$

$ax = a$

$x = 1$

$bx + y = b$

$y = 0 \Rightarrow (1,0)$ Identity element

Second method

$(a,b) * (e,0) = a$

$(ae, be + 0) = (a, 0)$

$ae = a$ $\quad$ $be = 0$

$e = 1$ $\quad$ $e = 0$

$e = (1,0)$

Inverse

$A \times A^{-1} = e$

$(a,b) * (A^{-1}, 0) = (1,0)$

$(aA^{-1}, bA^{-1}) = 1,0$

$(A^{-1} = \frac{1}{a}, 0)$

$(a,b) * (x,y) = (1,0)$

$ax, bx + y = (1,0)$

$ax = 1$ $\quad$ $bx + y = 0$

$x = 1/a$ $\quad$ $bx\frac{1}{a} = -y$

$y = -b/a$

Inverse

$A \times A^{-1} = e$

$(a,b) \times (y,z) = (1,0)$

$(ay,\ by+z) = (1,0)$

$y = \frac{1}{a}$, $z = \frac{-b}{a}$

$A^{-1} = \frac{1}{a}$, $B^{-1} = \frac{-b}{a}$

It follows the all property of group therefore so It is a group.

Prove

Q: The Inverse of every element in a group is unique.

Let a, Be any arbitrary element of a group G and
Let e, Be the Identity element and
b, c are two inverse of a

Prove

$ba = ab = e$ — (i) $\times c$
$ca = ac = e$ — (ii) $\times b$

$abc = ec$ — (iii)
$abc = eb$ — (iv)

$bac =$
$ec = c$
$b(ca)$
$be = b$
$c = b$

$\therefore ec = eb$
$\boxed{c = b}$

Hence proved the statement is true.

8935041095
priyanshu

(2) Left cancellation law — If $a, b,$ and c are in G

Prove = $a*b = a*c \Rightarrow b = c$

since $a^{-1} \in G$

$\therefore a^{-1}*a*b = a^{-1}*a*c$

$= e*b = e*c$

$= b = c$ Hence proved

(3) Prove that if $a^2 = a$, then $a = e$, where a is an element of group.

$a^2 = a$

$a \times a = a$

$a^{-1} \times a \times a = a \times a^{-1}$

$e a = e$

$\boxed{ae = a}$

$\boxed{ae = a}$

$= a = e$ Hence proved

Order of an element

The order of an element g in a group G is the smallest positive integers $\boxed{n : g^n = e}$

VIMP
Example $G = \{1, -1, i, -i\}$ be a multiple group. find the order of every element

$G = \{1, -1, i, -i\}$

$g^n = e$	$g^n = e$	$g^n = e$	$g^n = e$
$(1)^n = e$	$(-1)^n = e$	$(i)^n = 1$	$(-i)^n = 1$
$(1)^1 = 1$	$(-1)^2 = e$	$(i)^4 = 1$	$(-i)^4 = 1$
$n = 1$	$n = 2$	$n = 4$	$n = 4$

$\frac{7}{2}$ $\frac{9}{2}$ $\frac{+3}{2}$

Qn find the order of every element in the multiplicative group $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$

$e = a^6$

$(a)^n = e$	a^2	a^3	$(a^4)^1$
$(a)^6 = e$	$(a^2)^3 = e$	$(a^3)^2 = e$	$(a^4)^2 = a^8$
$n = 6$	$n = 3$	$n = 2$	$(a^4)^3 = (a)^{12} = (a^6)^2 = (e)^2$ $n = 2$

$(a^5)^1$

$(a^5)^6$

$(a^5)^6$

$n = 6$

$(a^6)^1 = e$

$n = 1$

Sub group

Let $(G, *)$ be a group and H (subgroup) is a subset of G then $(H, *)$ is said to be subgroup of G, if $(H, *)$ is also a group by itself.

Every set is a subset of itself, therefore if G is a group, then G itself is a subgroup of G.

$G = \{1, -1, i, -i\}$

$H = \{1, -1\}$

x	1	-1
1	1	-1
-1	-1	1

Now

$H = \{-1, i\}$

x	-1	i
-1	1	-i
i	-i	-1

It does not follows the closure property so it is not a subgroup or group.

Q^n Let $G = \{ \ldots\ 3^{-2}, 3^{-1}, 1, 3, 3^2 \ldots \}$ be the multiplicative group consisting of all integral powers of 3 and let $H = \{1, 3, 3^2, \ldots \}$ then H is subset of G, Prove H is subgroup (C) of G or not.

Solution

$$G = \{ \ldots\ 3^{-2}, 3^{-1}, 1, 3, 3^2 \ldots \}$$
$$H = \{1, 3, 3^2, \ldots \}$$

Closure

It follows the closure property because all the elements of H. is belongs to the set G so it has closure property.

Associative

$$a \cdot (b \cdot c) = (a \cdot (b) \cdot c)$$

elements $= 1, 3, 9$

$$= 1 \cdot (3 \cdot 9) = (1 \cdot (3)) \cdot 9)$$
$$= 27 = 27$$

Identity

$$a \cdot e = a$$

$a = 3$ $\quad 3 \cdot e = 3$

$$e = 1$$

Inverse

$$a a^{-1} = 1$$
$$= 3 a^{-1} = 1$$

$a^{-1} = \frac{1}{3} = 3^{-1}$ does not belongs to H

It is not Inverse, so It is not Subgroup

Q^n P.T. The additive group of even integers is a subgroup of additive group of all integers.

Sol^n

operation $= (G, +)$

$G = \{-\infty \; ---- \; -4, -3, -2, -1, 0, 1, 2, 3, 4 \; -- \; \infty\}$
$H = \{ \; --- \; -4, -2, 0, 2, 4, 6 \; -- \; \}$

Closure

The all elements of H is present in G so It follows the closure property.

Associative

$a+(b+c) = (a+b)+c$

$a = -4$ $\quad -4+(-2+0) = (-4-2)+0)$

$b = -2$ $\quad -4-2 = -6$

$c = 0$ $\quad -6 = -6$ $\quad$ It is associative.

Identity

$a+e = a$

$a = -4$ $\quad -4+e = -4$

$e = 0$ $\quad$ It has Identity element.

Inverse

$a+a^{-1} = e$ $\quad | \quad -2+a^{-1} = 0$

$-4+a^{-1} = e = 0$

$a^{-1} = 4$ $\quad | \quad a^{-1} = 2$

So (H) It is a subgroup of G

$G = \{1, 2, 3, 4\}$
$n = \{1, 2, 3\}$ $n = 2$
$a = 2$
4

Qh Multiplicative group.
$G = \{1, -1, i, -i\}$
$H = \{i, -i, -1\}$ Prove the subgroup.

Soln

$*$	i	$-i$	-1
i	-1	$+1$	$-i$
$-i$	$+1$	-1	i
-1	$-i$	i	1

$I \notin H$

Hence, It does not follows the closure property so It is not a subgroup of G.

(ii) $H = \{i, -i, +1\}$

	i	$-i$	1
i	-1	1	i
$-i$	$+1$	-1	$-i$
1	i	$-i$	1

It is not a subgroup.

Coset :-

Let H be a subgroup of a group G. and let $\{a \in G\}$, then the set $\{a * h : h \in H\}$ is called the left coset generated by (a and H) ~~(aH)~~ & denoted by aH. similarily the set $Ha = \{h * a, h \in H\}$ is called the right coset and is denoted by Ha. Here the element a is representative element.
Both aH and Ha are subsets of G.

Index of a subgroup in a group :-

If H is a subgroup of a group G then the number of distinct right or left cosets of H in G is called the Index of H ~~is~~ in G and is denoted by

$[G : H]$ or $[i_G(H)]$

Example

Let $G = \{ \ldots -3, -2, -1, 0, +1, +2, +3 \ldots \}$ be a additive group of integers and let $H = \{ \ldots -9, -6, -3, 0, 3, 6, 9 \ldots \}$ be a subgroup of G obtain on multiplying each element of G by 3. find the index of subgroup.

$G = \{ \ldots -3, -2, -1, 0, 1, 2, 3 \ldots \}$

$H = \{ \ldots -9, -6, -3, 0, 3, 6, 9 \ldots \}$

$H+0 = \{ \ldots -9, -6, -3, 0, 3, 6, 9 \ldots \}$

$H+1 = \{ \ldots -8, -5, -2, 1, 4, 7, 10 \ldots \}$

$H+2 = \{ \ldots -7, -4, -1, 2, 5, 8, \ldots \}$

$H+3 = \{ \ldots -6, -3, 0, 3, 6, 9 \ldots \}$

$H+4 = \{ \ldots -5, -2, 1, 4, 7, 10 \ldots \}$

$H+5 = \{ \ldots -7, -4, -1, 2, 5, 8 \ldots \}$

$H+3 = H+0$

$H+4 = H+1$

$H+5 = H+3$

So

$G = H \cup (H+1) \cup (H+2)$ Index = 3 Ans

$\ 1 \quad\quad 2 \quad\quad 3$

Order of group :-

The no of elements in a group is called the order of group.

The order of group (G) is denoted by $O(G)$.

ex: $G = \{0, 1, 2, 3, 4\}$

$O(G) = 5$

Imp

1. <u>Lagrange's theorem</u> :- The order of each subgroup of a finite group (G) is a divisor of the order of the group (G).

Proof :- let H be in any subgroup of order m of a finite group G of order n.

$$G = \{a_1, a_2, a_3 ------ a_n\}$$
$$H = \{h_1, h_2, h_3 ------ h_m\}$$

$$a_1 H = \{a_1 h_1, a_1 h_2, a_1 h_3 ------ a_1 h_m\}$$
$$a_2 H = \{a_2 h_1, a_2 h_2, a_2 h_3 ------ a_2 h_m\}$$
$$a_3 H = \{a_3 h_1, a_3 h_2, a_3 h_3 ------ a_3 h_m\}$$
$$a_K H = \{a_K h_1, a_K h_2, a_K h_3 \quad a_K h_m\}$$

total distinct left coset = K.

Hence the total no of elements of all coset is Km.

which is equal to total no of elements of group (G) $n = Km$.

$$G = Ha_1 \cup Ha_2$$

Hence, $O(G) = O(Ha_1) + O(Ha_2) + O(Ha_3)$

$$n = m + m + m + m$$
$$n = Km$$
$$K = \frac{n}{m}$$

or

$$O(a_1 H) = m$$
$$O(a_2 H) = m$$
$$O(a_3 H) = m$$

$$O(a_K H) = m$$
$$Km = n$$
$$\boxed{K = \frac{n}{m} = \frac{O(G)}{O(H)}}$$

Theorem:-

If G is a finite group of order n and $a \in G$ then $a^n = e$.

Let $o(a) = m$

since G is a finite group therefore the order of $a \in G$ is a divison of order of (G).

Let $o(a) = m \Rightarrow a^m = e$

$\therefore$ $n = mk$ (by lagrge's thearem)

$a^n = a^{mk}$ $\quad\quad\quad\quad$ $\because a^n = e$

$a = e^k$ $\quad\quad\quad\quad$ $a^m = e$

$= e$

Proved

2. Cyclic Group

A group G is called a cyclic group if for some belongs to G, every element of G is of the form a^n where n is some integer. The element a is then called generator of G.

Ex The multiplicative group $G = \{1, -1, i, -i\}$ is a cyclic group.

$(i)^1 = i$

$(i)^2 = -1$

$(i)^3 = (i^2)\, i = -i$

$(i)^4 = (i^2)(i^2)$

$= -1 \times -1 = 1$

$(-i)^1 = -i$
$(-i)^2 = 1$
$(-i)^3 = i$
$(-i)^4 = -1$

The generator is i and also $-i$ is generator.

2. Prove that the multiplicative group $(1, \omega, \omega^2)$ is a cyclic group.

$(\omega)^1 = \omega$
$(\omega)^2 = \omega^2$
$\omega^3 = 1$

$(\omega^2)^1 = \omega^2$
$(\omega^2)^2 = \omega^4 = \omega^3 . \omega = \omega$
$(\omega^2)^3 = \omega^6 = \omega^3 . \omega^3 = 1 \cdot 1 = 1$

3. The set of integer with respect to '+' that is $(Z, +)$ is a cyclic group with a generator 1.

$Z = \{1, 2, 3, 4, 5, 6, 7, \ldots\}$

$1^2 = 1 = 1$
$1^2 = 1+1 = 2$
$1^3 = 1+1+1 = 3$
$1^4 = 1+1+1+1 = 4$
$1^5 = 5$

4. Show that the group $G = \{1, 2, 3, 4, 5, 6\} \times_7$ is a cyclic group. How many generator are there.

$3^1 \bmod 7 = 3$
$3^2 \bmod 7 = 2$
$3^3 \bmod 7 = 6$
$3^4 \bmod 7 = 4$
$3^5 \bmod 7 = 5$
$3^6 \bmod 7 = 1$

3 is generator of this group.

$3 \times a^{-1} = 1$
$3^1 \times 3^5 = 1$
$3^6 \bmod 7 = 1$
$1 = 1$

$(5^1) \bmod 7 = 5$
$(5^2) \bmod 7 = 4$
$(5^3) \bmod 7 = 6$
$(5^4) \bmod 7 = 2$
$(5^5) \bmod 7 = 3$
$(5^6) \bmod 7 = 1$

5 is also generator.

5. $G = \{0, 1, 2, 3, 4, 5\} +_6$

$1^1 \bmod 6 = 1$
$1^2 \bmod 6 = 2 \bmod 6 = 2$
$1^3 \bmod 6 = 3 \bmod 6 = 3$
$1^4 \bmod 6 = 4 \bmod 6 = 4$
$1^5 \bmod 6 = 5 \bmod 6 = 5$

$a + e = a$
$1 + e = 1$
$e = 0$
$(1 + a^{-1}) = 0$
$(1+5) \bmod 6 = 0$
$6 \bmod 6 = 0$
1 inverse is 5

$2^1 \bmod 6 = 2$
$2^2 \bmod 6 = 2+2 = 4 \bmod 6 = 4$

$2^3 \bmod 6 = 2+2+2 = 6 \bmod 6$
$= 0$

$2^4 \bmod 6 = 2+2+2+2 = 8 \bmod 6 \Rightarrow 2$
$2^5 \bmod 6 = 2+2+2+2+2 = 10 \bmod 6 \Rightarrow 4$

The ~~gn.~~ generator is 2.

$5^1 \bmod 6 = 5$
$5^2 \bmod 6 = 10 \bmod 6 = 4$
$5^3 \bmod 6 = 5+5+5 \bmod 6 = 15 \bmod 6 \Rightarrow 3$
$5^4 \bmod 6 = 20 \bmod 6 = 2$
$5^5 \bmod 6 = 25 \bmod 6 = 1$
$5^6 \bmod 6 = 30 \bmod 6 = 0$

The generator is 5

(6). Find the generator of cyclic group.
$G = \{a, a^2, a^3, a^4 = e\}$

$(a^1) = a$
$(a)^2 = a^2$
$(a^3) = a^3$
$a^4 = e$

$a \times a^3 = e$
$a^4 = e$

$(a^3)^1 = a^3$
$(a^3)^2 = a^6 = a^4 \cdot a^2 \Rightarrow a^2$
$(a^3)^3 = a^9 = a^8 \cdot a^1 = a$
$(a^3)^4 = a^{12} = (a^4)^3 = a^8 \cdot a^4$.
$e \cdot a^4 = a^4$

Permutation Group :-

Let A be a finite set then a function $F : A \to A$ is set to be a permutation of A if

(i) F is one one.

(ii) F is onto.

$$F = \begin{pmatrix} a & b & c \\ f(a) & f(b) & f(c) \end{pmatrix}$$

example

$$f \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

2. Equality of 2 permutation.

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

$$g = \begin{pmatrix} 2 & 4 & 3 & 1 \\ 3 & 1 & 4 & 2 \end{pmatrix}$$

$$f \circ g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$$

$f(1) = 4$ $\quad$ $g(1) = 4$

$f(2) = 1$ $\quad$ $g(2) = 1$

$f(3) = 2$ $\quad$ $g(3) = 2$

$f(4) = 3$ $\quad$ $g(4) = 3$

$f(x) = g(x)$

3. Identity permutation:-

If each element of a permutation is replace by itself then it is called identity permutation. It is denoted by ~~E~~ I.

$$I = \begin{pmatrix} a & b & c \\ a & b & c \end{pmatrix}$$

4. Product of permutation or Composition of permutation

The product of two permutation f and g of same degree is denoted by fog means first perform f then perform g.

(i) $f = \begin{pmatrix} a_1 & a_2 & a_3 & \cdots & a_n \\ b_1 & b_2 & b_3 & \cdots & b_n \end{pmatrix}$

$g = \begin{pmatrix} b_1 & b_2 & b_3 & \cdots & b_n \\ c_1 & c_2 & c_3 & \cdots & c_n \end{pmatrix}$

$fog = \begin{pmatrix} a_1, & a_2, & a_3 & \cdots & a_n \\ c_1, & c_2, & c_3 & \cdots & c_n \end{pmatrix}$

(ii) find the product of two permutation and so that it is not permutative.

$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}, \quad g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$

$g = \begin{pmatrix} 2 & 1 & 4 & 3 \\ 2 & 3 & 4 & 1 \end{pmatrix}$

$fog = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$

$gof = g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$

$f = \begin{pmatrix} 3 & 2 & 1 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$

$gof = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$

$fog \neq gof$

3. $P_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

$P_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

$P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ Show the permutation multiplication is associative.

$$((P_1 \cdot P_2)P_3) = P_1 \cdot (P_2 \cdot P_3)$$

$$P_1 \cdot P_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

$$(P_1 \cdot P_2) \cdot P_3 = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

$$(P_1 \cdot P_2) \cdot P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

$$P_2 \cdot P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

$$(P_2 \cdot P_3) = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

$$P_1 \cdot (P_2 \cdot P_3) = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

$$(P_1 \cdot P_2) \cdot P_3 = P_1 \cdot (P_2 \cdot P_3)$$

Hence proved

3. Cyclic Permutation:-

A permutation which replace an object cyclically is called a cyclic permutation of degree n. if $f(a_1) = a_2$, $f(a_2) = a_3$ and $f(a_3) = a_4$ — $f(a_n) = a_1$, then the permutation is cyclic.

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

The no of elements permuted by a cycle is said to be its length and the disjoint cycles are those which have no common elements.

$$P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 6 & 5 & 3 \end{pmatrix}$$

$P_1 = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ length of cycle = 2

$P_2 = \begin{pmatrix} 3 & 4 & 6 \\ 4 & 6 & 3 \end{pmatrix}$ length of cycle = 3

$P_3 = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$ length of cycle = 1

4. $A = \{1, 2, 3, 4, 5\}$ (cyclic group)
$B = (2,3)(4,5)$ (cycle) find AB

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 2 & 5 & 4 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 3 & 4 & 5 & 1 \\ 3 & 2 & 5 & 4 & 1 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 5 & 4 & 1 \end{pmatrix}$$

5. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 2 & 4 & 3 & 1 \end{pmatrix}$

$(1,6)\ (2,5,3)\ (4)$

length of cycle

$(1,6) \rightarrow 2$

$(2,5,3) \rightarrow 3$

$(4) \rightarrow 1$

~~*~~** **Ring**

A Ring $(R, +, \cdot)$ is a set together with two binary operations that is addition and multiplication defined on R such that the following conditions must be satisfied.

1. $(a+b)+c = a+(b+c)$, $\forall\, a b c \in R$
2. $(a+b) = b+a$
3. There exist identity element over addition.
4. There exist inverse of element over additive group.
5. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
6. $a \cdot (b+c) = (a \cdot b) + (a \cdot c)$ (left distributive law)
7. $(b+c) \cdot a = ba + ca$ (Right distributive law)
8. $a \cdot b = b \cdot a$ (commutative Ring).

The algebric system $(R, +, \cdot)$ is called Ring if

(I) $(R, +)$ is an abelian group.
(II) $(R, \cdot)$ is a semigroup.
(III) The operation is distributive over the operation.

(1) for the set $A = \{0,1,2,3\}$ show that the module 4 system is a Ring.

(i) $(R, +)$ is an abelian.

$+_4$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

$\cdot_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

It follows the closure property.

(1) $(a+b)+c = a+(b+c)$

(i) $(0+1)+2 = 0+(1+2)$
$1+2 = 0+3$
$3 = 3$

(ii) $(1+2)+3 = 1+(2+3)$
$3+3 = 1+5$
$6 = 6$

It follows associative property.

(iii) $e = 0$. (iv) $a + a^{-1} = e$ | ⑧ $a = 1, a^{-1} = 3$

$0 + 0 = 0$ | $1 + 3 = 0$

$a = 0, a^{-1} = 0$ | $(4)_4 = 0$

(5) $a = 2.\ a^{-1} = 2$

$a = 3,\ a^{-1} = 1$

(2) $(R, \cdot)$ is a semigroup.

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$ | $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

$(0 \cdot 1) \cdot 2 = 0 \cdot (1 \cdot 2)$ | $(1 \cdot 2) \cdot 3 = 1 \cdot (2 \cdot 3)$

$0 \times 2 = 0 \times 2$ | $2 \cdot 3 = 1 \cdot 6$

$0 = 0$ | $6 = 6$

It follows the associative property.

(III) $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$

$a = 0$

$b = 1$

$c = 2$

$0 \cdot (1 + 2) = (0 \cdot 1) + (0 \cdot 2)$ | $a = 2, b = 3, c = 1$

$0 \cdot 3 = 0 + 0$ | $2 \cdot (3 + 1) = (2 \cdot 3) + (2 \cdot 1)$

$0 = 0$ | $2 \cdot 4 = 6 + 2,\ 8 = 8$

This is a Ring, because it follow all the property.

Note In a Ring $e \in R$ called an unit of identity element if $ea = ae = a\ \forall\ a \in R$.

An unit element of a ring R is an element of the semigroup $(R, \cdot)$ the unit of a ring is generally denoted by one.

1

A Ring ~~are~~ is called a ring with unity if it has an unit element.

Qn If $a, b, c \in R$ then prove that $a.(-b) = (-a).b$

Soln we know that

$$a \cdot 0 = 0$$
$$a \cdot (-b + b) = 0$$
$$a \cdot (-b) = -a \cdot b$$

Qn If R is a ring such that $a^2 = a \ \forall a \in R$ then prove that (I) $a + a = 0$

(II) $a + b = 0 \Rightarrow a = b$

Soln

$$a \in R$$
$$a + a \in R$$
$$(a+a)^2 = (a+a)$$
$$(a+a)(a+a) = (a+a)$$
$$a + a = 0$$

$$a + b = 0$$
$$\cancel{a} + b = a + \cancel{a}$$
$$b = a$$

Qn Prove that $a, b \in R$

$$(a+b)^2 = a^2 + ab + ba + b^2$$

we know that $(a+b)^2 = (a+b)(a+b)$

$= a(a+b) + b(a+b)$ distributive property
$= a^2 + ab + ba + b^2$

Types of Ring

1. Commutative Ring -:
If $(R, \cdot)$ is commutative. i.e. $(a \cdot b) = (b \cdot a)$

2. Ring with unity -: A ring $(R, +, \cdot)$ is ring with unity if $(R, \cdot)$ has identity element or unit element. i.e 1 or if a ring R has an identity element with respect to multiplication
$a \cdot 1 = a$

3. Ring without zero divisor :-
A ring $(R, +, \cdot)$. is called ring with zero divisor if $a \neq 0$, $b \neq 0$ and $ab \neq 0$ in $(R, \cdot)$

4. Ring with zero divisor :-
If $a \neq 0$, $b \neq 0$ but $ab = 0$ in $(R, \cdot)$
in ~~(...)~~ $(R, \cdot)$

* **Integral domain :-**
A commutative ring $(R, +, \cdot)$ is called integral domain if it is ~~called~~ without zero divisor.

field :-
A commutative ring $(R, +, \cdot)$ is field if every element has multiplicative inverse.

Qn proof $(Z, +, \cdot)$
It is a ring.
Ans $(Z, +)$ is Group, $(Z, \cdot)$.

g. $a \times b = e$

$ae = a$

$a a^{-1} = e$

1. Is it ring with zero divisor.
   It is a ring without zero divisor.

   Is it integral domain:
   yes, it is integral domain.

   Is it field.
   It is not field because multiplicative inverse does not exist

$$e = 1$$
$$a \times a^{-1} = e$$
$$2 \times \frac{1}{2} = e$$

Note :- Every finite integral domain is a field.

Q. Set of Real no.s is a field.

Qn Let $m = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ $a \in R$ and let $+, \cdot$ denotes usual matrix addition and multiplication is $(m, +, \cdot)$ is integral domain also check it is field or not.

$$A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \quad B = \begin{pmatrix} b & 0 \\ 0 & b \end{pmatrix} \quad C = \begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix}$$

+	~~A~~	~~B~~	C
~~A~~			
~~B~~			
C			

Soln

$$A+B = \begin{bmatrix} a+b & 0 \\ 0 & a+b \end{bmatrix} \in R$$

$$A+C = \begin{bmatrix} a+c & 0 \\ 0 & a+c \end{bmatrix} \in R$$

Closure property

$$(A+B)+C = \begin{bmatrix} a+b & 0 \\ 0 & a+b \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} a+b+c & 0 \\ 0 & a+b+c \end{bmatrix}$$

$$A+(B+C) = \begin{bmatrix} b+c & 0 \\ 0 & b+c \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$$

$$\begin{bmatrix} a+b+c & 0 \\ 0 & a+b+c \end{bmatrix}$$

$$\boxed{(A+B)+C = A+(B+C)}$$

Associative property

(iii) $a+e=a$

$$\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \quad , \quad e = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$(m,+)$ $\quad a * a^{-1} = e$

$$\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} + \begin{bmatrix} -a & 0 \\ 0 & -a \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$a^{-1} = \begin{bmatrix} -a & 0 \\ 0 & -a \end{bmatrix}$$

$(m,\cdot)$ $\quad a \times a^{-1} = e$

$$\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \times a^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$a^{-1} = \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & 1/a \end{bmatrix} \quad , \quad \frac{1}{a} \in R$$

Therefore inverse exist for every element so it is field.

$$a \neq 0 , \quad b \neq 0$$

$$ab \neq 0$$

So it is integral domain.

$$\text{permutation} = \frac{n!}{p!\,q!\,r!}$$

Combination

A combination is a choice of 'r' things from a set of n things without repeatition or replacement where order does not matter.

$$\boxed{{}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}}$$

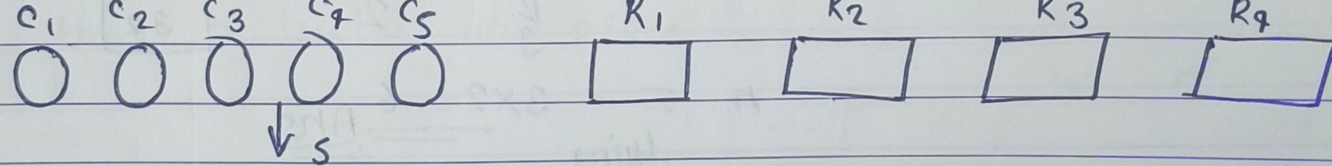
Qn find the number of permutations and combinations of $n=12$, $r=2$

$$ {}^{n}C_{r} = {}^{12}C_{2} = \frac{12!}{2!\,(10)!} = \frac{\overset{6}{\cancel{12}} \times 11 \times \cancel{10!}}{\cancel{2} \times \cancel{10!}} = 66 \text{ Ans}$$

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} = \frac{12!}{10!} = 12 \times 11 = 132 \text{ Ans}$$

OR Sum and Product rule

OR and AND Rule



Qn find the number of ways of choosing one figure.

Soln It means we have to choose one circle or one rectangle.

for circle — choices = 5

for rectangle — choices = 4

So

total choice = 5 + 4

= 9

Q. find the number of ways of choosing one circle and one rectangle

Soln for circle = choice = 5

Rectangle = choice = 4

circle and rectangle

total $= 5 \times 4 =$ 20 choices Ans

Qn If there are 10 boys and 10 girls, then

(i) find number of ways selecting one student

(ii) No. of ways selecting one boy and one girl.

(i) Ans = $10 + 10 = 20$

(ii) Ans = $10 \times 10 = 100$

Ans

Qn How many number of two digits can be formed with the digits (1, 2 and 3), If repeatition of digits is not allowed.

1, 2, 3

number of two digits = $\underset{3}{\_\_}\ \underset{2}{\_\_}$

12
13
21
23
31
32

$n = 3 \times 2 = 6$ Ans

Qn How many number ~~line~~ lying between 100 and 1000 can be formed with the digits (1, 2, 3, 4, 5) if the repeatition of digit is not allowed.

100 1000

101 → 999

$^nP_r = \begin{matrix} n=5 \\ r=3 \end{matrix} \longrightarrow$

$= \frac{5!}{2!} = 60$

$\underset{5}{\_\_}\ \underset{4}{\_\_}\ \underset{3}{\_\_}$

$= 5 \times 4 \times 3$

$= 60$ Ans

Qh How many 4 digits numbers are there with distinct digits

$0 \to 9$

digit $\to$ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

$\underline{\quad}$ $\underline{\quad}$ $\underline{\quad}$ $\underline{\quad}$

9 9 8 7

$= 9 \times 9 \times 8 \times 7$
$= 4536$ Ans

$^{n}P_{r} =$ $n = 10$, $r = 4$

$^{10}P_{4} = \frac{10!}{6!} =$ ~~$10 \times 9 \times 8 \times 7$~~

Qh In how many ways can be arrange 3 students from a group of 5 students to stand in line for a picture.

$c_1\ c_2\ c_3\ c_4\ c_5$

$c_1 c_2 c_3$, $c_1 c_3 c_4$, $c_1 c_3 c_5$

$c_1 c_4 c_5$, $c_1 c_2 c_5$

$c_1 c_2 c_4$

$\underline{\quad}$ $\underline{\quad}$ $\underline{\quad}$

5 4 3

$= 5 \times 4 \times 3 = 60$ Ans

Qh How many numbers are there between 100 and 1000 such that 7 is in the unit place

100 — 1000

101 999

$\to$ $\underline{\quad}$ $\underline{\quad}$ $\underline{\quad}$

9 10 1

$= 9 \times 10 \times 1 = 90$ Ans

permutation — permutation of things when some things are same then we use this formula.

$$\boxed{P \to \frac{n!}{p!\,q!\,r!}}$$

AS - BANANA

Qn find the number of 7 letters words can be formed using the word BENZENE

$n=7$
$B=1$
$E=3$
$N=2$
$Z=1$

Permutation $= \frac{n!}{B!\,E!\,N!\,Z!}$

$= \frac{7!}{3!\,2!} = \frac{7\times6\times5\times\cancel{4}^{2}\times3!}{\cancel{2}}$

$= \underline{\underline{420}}$ Ans

Qn Find the number of permutation of the letters of the word BANANA

Soln
$n=6$
$B=1$
$A=3$
$N=2$

$P = \frac{n!}{B!\,A!\,N!}$

$\Rightarrow \frac{6!}{3!\,2!} \quad \frac{6\times5\times\cancel{4}\,2}{\cancel{2}}$

$= \underline{60}$ Ans

Word Building Problems

Qn How many Permutations of the letters ABCDEFGH containing the string ABC.

Soln

ABC D E F G H
1

$= 6! = 6\times5\times4\times3\times2\times1 = \underline{\underline{720}}$ Ans

Qn find the number of permutations that can be formed of the letters of the word daughter

(i) Taken all the letters together $= 8!$

(ii) Begining with D $= 7!$.

(iii) Begining of D and ending with E $= 6!$

(iv) Vowels being always together $= 6! \times 3!$

(v) Not all vowels together $= 8! - (6! \times 3!)$

(vi) Vowels occupying even places. $\Rightarrow {}^4P_3 \times 5!$

Combination: In combination selection is made. The ordering of the selected object is immaterial

Qn If $A = \{1, 2, 3\}$. find the number of subset of set a taking two elements.

$$ {}^nC_r = \frac{3!}{2!} = \frac{3 \times 2!}{2!} = 3 \text{ Ans} $$

Qn A farmer buys 3 cows, 2 goats and 4 hens from a man who has 6 cows, 5 goats and 8 hens. find the number of choices that farmer has.

$$ \Rightarrow {}^6C_3 \times {}^5C_2 \times {}^8C_4 = \frac{\cancel{6} \times 5 \times 4}{3 \times \cancel{2}} \times \frac{5 \times \cancel{4}^2}{\cancel{2} \times 3!} \frac{\cancel{8}^2 \times 7 \times \cancel{6} \times 5}{\cancel{4} \times \cancel{3 \times 2} \times 1} $$

$20 \times 10 \times 70 = 14000$

Qn In how many ways 4 questions be selected from 7 question

$$ {}^7C_4 = \frac{7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}\, 3!} = 35 \text{ Ans} $$

Qn In how many ways can a committee of 5 teachers and 4 students be choosed from 9 teacher and 15 students

$$^9C_5 = \frac{9!}{5!\,4!} = \frac{9 \times \cancel{8}^{2} \times 7 \times \cancel{6}}{\cancel{4} \times \cancel{3} \times \cancel{2}} = \underline{126}$$

$$^{15}C_4 = \frac{15!}{4!\,11!} = \frac{15 \times \cancel{14}^{7} \times 13 \times \cancel{12}}{\cancel{4} \times \cancel{3} \times \cancel{2}} = \underline{1365}$$

$$^9C_5 \times {}^{15}C_4 = \underline{171,990}$$

Qn A box contains 8 blue socks and 6 red socks find the number of ways 2 socks can be dracon from the box if

(i) They can be of any color.

(ii) They must be of same color.

(i) $$^{14}C_2 = \frac{14!}{2!\,12!} = \frac{\cancel{14}^{7} \times 13}{\cancel{2}} = \underline{\underline{91}}$$

(ii) $$\cancel{14C_8} = {}^8C_2 \cancel{8}$$

$$= {}^6C_2 = {}^8C_2 + {}^6C_2$$

$$= \frac{\cancel{8}^{4} \times 7}{\cancel{2}} \quad \frac{\cancel{6}^{3} \times 5}{\cancel{2}}$$

$$28 + 15 = \underline{\underline{43}}$$

Qn A bag contains 6 white balls and 5 red balls. find the number of ways in which 4 balls can be drawn from the bag if

(i) They can be off any color $= \, ^{11}C_4$

(ii) two must be white and two red $= \underline{^6C_2 \times {}^5C_2}$

(iii) They must be off same color

$$= \underline{^6C_4 + {}^5C_4}$$