UNIT-2 Boolean Algebra

- Axioms of boolean algebra:

① Commutative law → $a+b = b+a$
$a \cdot b = b \cdot a$

② Distributive law → $a+(b \cdot c) = (a+b) \cdot (a+c)$
$a \cdot (b+c) = (a \cdot b) + (a \cdot c$

③ Identity law → $a+0 = a$
$a \cdot 1 = a$

Note
$+$ → or gate
$\cdot$ → AND gate

④ Complement law → $a+\bar{a} = 1$
$a \cdot \bar{a} = 0$

⑤ Idempotent law → $a+a = a$
$a \cdot a = a$

⑥ Boundedness law → $a+1 = 1$
$a \cdot 0 = 0$

⑦ Absorption law → $a+(a \cdot b) = a$
$a \cdot (a+b) = a$

⑧ Associative law → $a+(b+c) = (a+b)+c$
$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

⑨ De-morgan's law: → $(a+b)' = a' \cdot b'$
$(a \cdot b)' = a'+b'$

Qn: prove that $a + a = a$.

Solution

~~Talking R1~~

Taking RHS:

$a = a + 0$

$a = a + (a \cdot a')$

$a = (a + a) \cdot (a + a')$

$a = a + a$

Hence proved

a

$a = a + 0$

$= a + (a \cdot a')$

$= a(1 + a')$

$a + 0 = a + 0$

Qn: prove that $a \cdot a = a$

Soln

Taking RHS

$a = a \cdot 1$

$a = a \cdot (a + a')$

$a = a \cdot a + (a \cdot a')$

$a = a \cdot a + 0$

Hence proved

Qn: prove that $a + 1 = 1$

Soln

Taking LHS

$a + 1 = 1$

$a + (a + a') = 1$ by complement law

$(a + a) + a' = 1$ by associative law

$a + a' = 1$ by idempotent law

$1 = 1$ by complement law

Hence proved.

Qn: prove that $a + (a \cdot b) = a$

Soln

Taking LHS →

$a + (a \cdot b) = a$

$(a \cdot 1) + (a \cdot b) = a$

$a(1 + b) = a$

$a \cdot 1 = a$ by boundedness law.

$a = a$

Hence proved

Q $a \cdot (a+b) = a$

Sol^n Taking LHS:

$a \cdot (a+b)$

$(a \cdot a) + (a \cdot b)$ – Distributive

$a + (a \cdot b)$ Idempotent

$a((1) + (b))$

$a \cdot 1$ boundedness

$= \underline{a}$ Ans Hence proved

Q^n $A + BC = (A+B)(A+C)$

Sol^n Taking RHS

$(A+B) \cdot (A+C)$

$\Rightarrow (A+B) \cdot A + (A+B) \cdot C \Rightarrow A \cdot A + A \cdot B + A \cdot C + BC$

$\Rightarrow A + A \cdot B + A \cdot C + B \cdot C$

$\Rightarrow A(1+B+C) + B \cdot C \Rightarrow A(1+z) + BC$

$= A + BC = LHS$

Hence proved

Q^n Simplify $C(B+C)(A+B+C)$

Sol^n

$(CB + CC)(A+B+C) = (C(B+1))(A+B+C)$

$\Rightarrow C(A+B+C) = CA + CB + CC$

$= CA + CB + C$

$= AC + C(B+1)$

$= AC + C = C(A+1)$

$= \underline{C}$ Ans

Q^n $A + B(A+B) + A(A'+B)$

Sol^n

$\Rightarrow A + AB + BB + AA' + AB$

$\Rightarrow A + AB + B + 0 + AB \Rightarrow A + B + AB$

$= A + B(1+A) \Rightarrow \underline{A+B}$ Ans

Q^n $xy + x'z + yz$

Sol^n

$xy + yz + x'z$

multiply with $(x+x')$ in (yz)

$\Rightarrow xy + x'z + yz(x+x')$

$\Rightarrow xy + x'z + xyz + x'yz$

$\Rightarrow xy + xyz + x'z + x'yz$

$= xy(1+z) + x'z(1+y)$

$= \underline{xy + x'z}$ Ans

$\underline{Q^n}$ $(AB'C' + AB'C + ABC + ABC')(A+B)$

$\underline{Sol^n}$ $= AAB'C' + AAB'C + AABC + AABC' + ABB'C + AB'BC + ABBC + ABBC')$

$= (AB'C' + AB'C + ABC + ABC' + 0 + 0 + ABC + ABC)$

$\Rightarrow (AB'C' + AB'C + ABC + ABC') =$

$= AB'(C'+C) + AB(C+C')$

$= AB' + AB \Rightarrow A(B'+B)$

$\Rightarrow \underline{A}$ ~~AS~~

$\underline{Q^n}$ $A \oplus B \oplus AB = A+B$

$\underline{Sol^n}$ $= \underbrace{(A'B + B'A)}_{Let = C} \oplus AB = A+B$

$\Rightarrow C \oplus AB = A+B$

$\Rightarrow C'AB + C(AB)' = A+B$

$\Rightarrow (A'B + B'A)'AB + (A'B + B'A)(A'+B')$

$\Rightarrow ((A'B)' \cdot (B'A)')AB + (A'B + B'A)(A'+B')$

$\Rightarrow ((A+B') \cdot (B+A'))AB + A'A'B + A'B'A + B'BA' + B'B'A$

$\Rightarrow (AB + AA' + BB' + A'B')AB + A'B + 0 + 0 + B'A$

$\Rightarrow (AB + 0 + 0 + A'B')AB + A'B + B'A$

$= AB \cdot AB + AB \cdot A'B' + A'B + B'A$

$= AB + 0 + A'B + B'A$

$= AB + A'B + AB'$

$= A(B+B') + A'B$

$= A + A'B$ $\rightarrow$ Distributive law

$= (A+A') \cdot (A+B)$

$\Rightarrow 1 \cdot (A+B)$

$\Rightarrow \underline{(A+B)}$ Ans

Sum of product

A boolean expression E is said to be in a sum of product form if E is a sum of two or more products of variables. (complemented or uncomplemented)

$$\boxed{E = xy + x'y + xy'}$$

Product of sum

A boolean expression E is said to be in a product of sum form if it consist of several sum terms logically multiplied.

$$\boxed{E = (x+y)(x'+y)(x'+y')}$$

Min Term

A min term of n variables is a product of n literals in which each variable appears exactly once either in true or complemented form but not both.

for example - The list of all the min term of the two variables x and y are. 2^n

Combination

x	y	
0	0	$x'y'$
0	1	$x'y$
1	0	xy'
1	1	xy

x	y	z	
0	0	0	$x'y'z'$
0	0	1	$x'y'z$
0	1	0	$x'yz'$
0	1	1	$x'yz$
1	0	0	$xy'z'$
1	0	1	$xy'z$
1	1	0	xyz'
1	1	1	xyz

Max Term

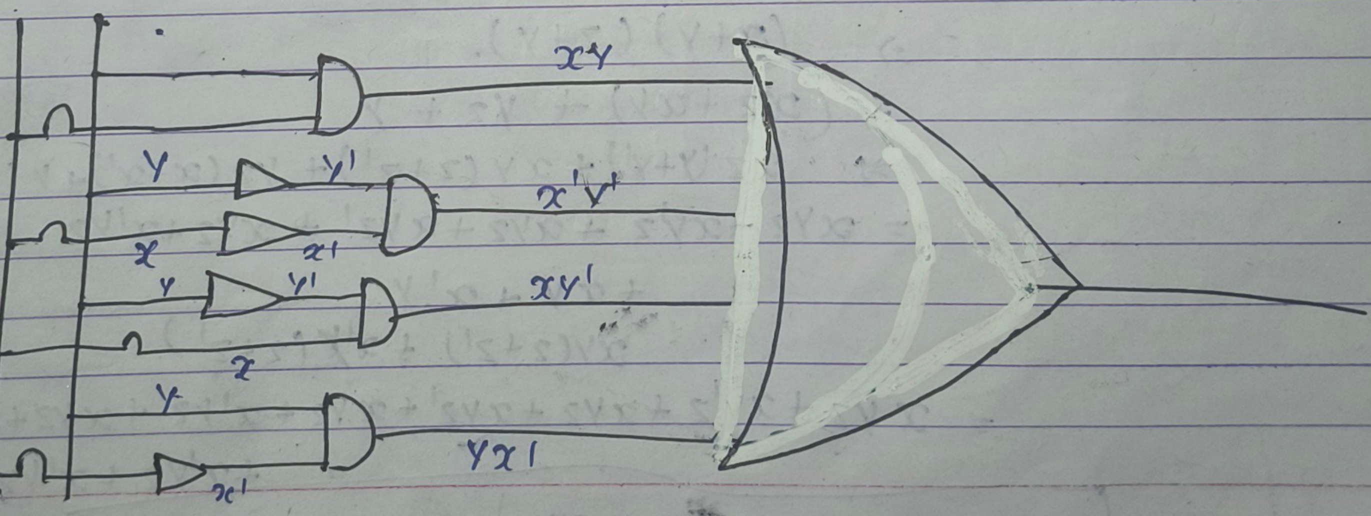
A max term of n variables is a sum of n literals in which each variable appears exactly once either in true or complemented form but not both.

x	y	
0	0	$x'+y'$
0	1	$x'+y$
1	0	$x+y'$
1	1	$x+y$

x	y	Z	
0	0	0	$x'+y'+z'$
0	0	1	$x'+y'+z$
0	1	0	$x'+y+z'$
0	1	1	$x'+y+z$
1	0	0	$x+y'+z'$
1	0	1	$x+y'+z$
1	1	0	$x+y+z'$
1	1	1	$x+y+z$

When a boolean function 'F( )' is written as sum of min terms is refered as min term expansion or disjunctive normal form. It is also known as canonical sum of products or standard sum of product.

$$\boxed{x'y'+x'y+xy'+xy}$$



of max term

when a boolean function (f) is written as product then it is refered as max term expansion or conjunctive normal form or canonical product of sum or standard product of sum.

Question Express the boolean function $f(x,y,z) = x+y'z$ in a sum of min term.

$$f(x,y,z) = x + y'z$$
$$= x(y+y') + y'(z)(x+x')$$
$$= xy + xy' + xy'z + x'y'z$$
$$= xy(z+z') + xy'(z+z') + xy'z + x'y'z$$
$$= xyz + xyz' + xy'z + xy'z' + xy'z + x'y'z$$
$$= \boxed{xyz + xy'z + xyz' + xy'z' + x'y'z}$$

DS

Question: Convert into DNF $f(x,y,z) = (x+y)(z'y')'$
sum of product

$(a \cdot b)' = a' + b'$
$(z' \cdot y')' = z'' + y''$
$= z+y$

$f(x,y,z) = (x+y)(z'y')'$
demorgans — $(z'y')' = (z+y)$

$$\Rightarrow (x+y)(z+y)$$
$$\Rightarrow (xz + xy) + yz + y$$
$$\Rightarrow xz(y+y') + xy(z+z') + yz(x+x') + y(x+x')$$
$$= xyz + xy'z + xyz + xyz' + xyz + x'yz$$
$$+ xy + x'y$$
$$xy(z+z') + x'y(z+z')$$
$$= xyz + xy'z + xyz + xyz' + xyz + x'yz + xyz + xyz'$$
$$+ x'yz + x'yz'$$

$$\boxed{f = xyz + xy'z + xyz' + x'yz + x'yz'}$$

DS

$$= \boxed{111 + 101 + 110 + 011 + 010}$$

Qn Represent $(x+y+z)(xy+x'z)'$ in DNF i.e sum of product.

$\Rightarrow (x+y+z)(xy+x'z)'$

$\Rightarrow (x+y+z)((xy)' \cdot (x'z)')$

$\Rightarrow (x+y+z)((x'+y') \cdot (x+z'))$

$\Rightarrow (x+y+z)(x \cdot x' + x'z' + xy' + y'z')$

$\Rightarrow (x+y+z)(x'z' + xy' + y'z')$

$\Rightarrow (xx'z' + xxy' + xy'z' + x'yz' + xyy' + yy'z' + x'zz'$
$+ xy'z + y'z'z)$

$\Rightarrow (xy' + xy'z' + x'yz + x'z + xy'z)$

$\Rightarrow (xy'(z+z') + xy'z' + x'yz + x'z(y+y') + xy'z)$

$\Rightarrow (xy'z + xy'z' + xy'z' + x'yz + x'yz + x'y'z + xy'z)$

$\Rightarrow (xy'z + xy'z' + x'yz' + x'y'z + xy'z)$ AS

Q Convert $f(a,b,c) = ab + a'c$ into CNF i.e product of sum.

$f(a,b,c) = ab + a'c$

apply distributive law:

$\Rightarrow (ab+a') \cdot (ab+c)$

$\Rightarrow (\bar{a} + ab) \cdot (c + ab)$

apply distributive law:

$\Rightarrow (a'+a) \cdot (a'+b) \cdot (c+a) \cdot (c+b)$ # this is CNF form product of sums.

$\Rightarrow (a'+b+cc') \cdot (c+a+bb') \cdot (c+b+aa')$

AS

Q) Convert $xy' + xy + xz$ in CNF form:

$$xy' + xy + xz$$
$$\Rightarrow x(y' + y + z)$$
$$x(z+1)$$
$$x$$
$$= x + yy' + zz'$$
$$= (x+y)\cdot(x+y') + (z+z')$$
$$= x + yy' + zz'$$
$$= (x+y)\cdot(x+y') + (z+z')$$
$$= (x+y+z)\cdot(x+y+z')\cdot(x+y'+z)\cdot(x+y'+z')$$
$$= (1+1+1)\cdot(1+1+0)\cdot(1+0+1)\cdot(1+0+0)$$

Truth table

x	y	z	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Q. $f(x, y, z) = (x \cdot \bar{y} + x \cdot z)' + z'$ convert into CNF (product of sum)

(DNF = sum of product)

<u>Solution</u>

$$= (x \cdot \bar{y} + x \cdot z)' + z'$$

using demorgans

$$= (x \cdot y')' \cdot (x \cdot z)' + z'$$

again demorgans

$$= (x' + y) \cdot (\bar{x} + \bar{z}) + z'$$

$$= z' + (x' + y) \cdot (\bar{x} + \bar{z})$$

using distributive $a + b \cdot c = (a+b)(a+c)$

$$= (z' + x' + y) \cdot (z' + \bar{x} + \bar{z})$$

$$= (x' + y + \bar{z}) \cdot (x' + z')$$

$$= (x' + y + z') \cdot (x' + z' + y \cdot y')$$ again distributive

$$= (x' + y + z') : (x' + z' + y)(x' + z' + y')$$

$$= (x' + y + z') \cdot (x' + y + z')(x' + z' + y')$$ Ans

K-map - Karnough map

It is a graphical technique which provides simple procedure for simplification of Boolean expression of two, three or four variables. It can also be extended to functions of 5, 6 and more variables.

K-map is a diagram made up of a number of squares if the expression contains n variables the map will have 2^n squares. Each square represents a min term and ~~one~~ 1 is written in the corresponding square for the present minterm. and 0 is written in the square which corresponds to the minterm not present in the expression.

How to solve SOP

2 Variables

Question $AB' + A'B'$

A \ B	0 (B')	1 (B)
A' 0	1	0
A 1	1	0

pair

$= B'$ Ans

$A'B + AB'$

Q2

A \ B	0 (B')	1 (B)
$\bar{A}$ 0		1
A 1	1	

No pair

$= A'B + AB'$ Ans

Q3 $AB' + A'B + A'B'$

A \ B	0 (B')	1 (B)
A' 0	1	1
A 1	1	

$= A' + B'$ Ans

$AB' + A'(B+B')$
$AB' + A'$
$A' + AB'$
$(A' + A) \cdot (A' + B')$
$1 \cdot (A' + B')$
$= A' + B'$

3 Variables

Q = $ABC' + ABC$

A \ BC	00	01	11	10
A' 0	0	1	3	2
A 1	4	5	1 (7)	1 (6)

Ans = AB

Q1 $A'B'C' + AB'C'$

A \ BC	00	01	11	10
A' 0	1 (0)	1	3	2
A 1	1 (4)	5	7	6

Ans = $B'C'$ ✓

Q $A'B'C' + A'BC' + ABC' + AB'C$

A \ BC	00	01	11	10
A' 0	1 (0)	1	3	1 (2)
A 1	4	1 (5)	7	1 (6)

BC'

$\Rightarrow BC' + A'C' + AB'C$ ✓ $\cancel{A'C'}$.

Q) $X = A'B'C' + A'B'C + A'BC + A'BC' + AB'C + ABC$

A \ BC	00	01	11	10
A' 0	1 (0)	1 (1)	1 (3)	1 (2)
A 1	(4)	1 (5)	1 (7)	(6)

$A'B'$
$+ A'B + AC$
$A'1 + AC$
$A'.$

$\Rightarrow$ $\underline{A' + C}$ Ans

4 variables

Q) $X = A'B'CD + AB'CD' + AB'C'D' + ABCD'$

AB \ CD	00	01	11	10
00	0	1	1 (3)	2
01	4	5	7	6
11	12	13	15	1 (14)
10	1 (8)	9	11	1 (10)

Ans = $\underline{A'B'CD + ACD' + AB'D'}$

Q) $X = A'BC'D' + ABC'D' + A'BCD' + ABCD'$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1 (0)	1	3	1 (2)
$\bar{A}B$ 01	1 (4)	5	7	1 (6)
AB 11	1 (12)	13	15	1 (14)
$A\bar{B}$ 10	1 (8)	9	11	1 (10)

Ans = $\underline{BD'} = \bar{D}$

Q) $X = A'B'C'D' + AB'C'D' + A'B'CD' + AB'CD'$

$A'B'D' + AB'D'$

$= B'D'(A' + A) = \underline{B'D'}$ As

Q) $F(ABCD) = \Sigma(0,1,2,3,4,5,6,7,8,9,11)$

$= A' + B'C' + B'D$

$= \underline{A' + B'C' + B'D}$ As

AB \ CD	00	01	11	10
$\bar{A}$ 00	1 (0)	1 (1)	1 (3)	1 (2)
01	1 (4)	1 (5)	1 (7)	1 (6)
11	12	13	15	14
10	1 (8)	1 (9)	1 (11)	10

How to solve POS

In POS we use 0 in place of 1.

$\bar{\ } = 1$

nonbar = 0 in POS (SOP ka bar karke POS ho jata hai).

Σ = SOP

π = POS

$(\bar A \bar B \bar C)' = \boxed{A+B+C}$ (x')

$0 - 1$

$4 -$

AB \ CD	00	01	11	10
00	0	0 1	0 3	0 2
01	0 4	5	7	6
11	0 12	13	15	14
10	8	0 9	11	10

$B\bar C\bar D$ $\bar B \bar C D$ $\bar A \bar B C$

$\bar B + C + D$ $B + C + \bar D$ $A + B + \bar C$

POS

$\Rightarrow (B'+C+D)\cdot(B+C+D')\cdot(AB+C') = (B'+C+D)\cdot(B+C+D')\cdot(A+B+C')$

Q) $F(ABC) = \pi(0,3,6,7)$

A \ BC	00	01	11	10
0	0 0	1	2	0 3
1	4	5	0 7	0 6

$= (A+D)\cdot(A'+C)$

$= (A+C)\cdot(A'+B')$

A \ BC	00	01	11	10
0	0 0	1	0 3	2
1	4	5	0 7	0 6

ABC

~~ABC~~: $(A+B+C)\cdot(B'+C')(A'+B')$ POS

$P+P'QR'+(Q+R)' \Rightarrow P+P'QR'+Q'R'$

$\Rightarrow P+R'(P'Q+Q') \Rightarrow P+\cancel{P'QR'}\,R'(P'Q+Q') = P+R'[(Q'+P'Q)]$

$\Rightarrow P+R'[(Q'+R')\cdot(Q'+Q)] \Rightarrow P+R'[Q'+R']$

$P+R'Q'+R'R' = P+Q'R'+R' = P+R'(Q'+1)$

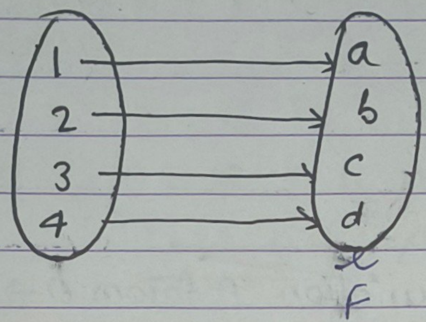
$= \boxed{P+R'}$

$x\cdot(y'\cdot z)'$ in DNF $\Rightarrow x\cdot(y+\bar z) = xy+x\bar z = (xy\ \cancel{+yz})\cdot(\cancel{xy+xz})$

~~$(y+x)\cdot(y+y)\cdot(z+x)(z+y)$~~   ~~$xy(z+\bar z)+x\bar z(x+x)$~~ $xy(z+\bar z)+x\bar z(y+\bar y)$

Function

function is a special case of relation.
Let A and B two non empty sets and R be a relation from A to B then R may not relate an element of A to an element of B or It may relate an element of a to more than 1 element of B.

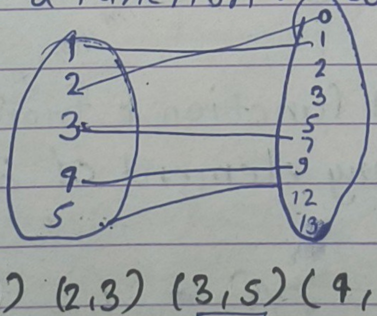


F: A → B

There may be some element of set B which are not associated to any element of set A.
each element of set A must be associated to one and only one element of set B.
If F is a function from A to B then. (F:A → B) then A is called domain of F denoted by dom(F) and its members are the first coordinate of the ordered pair ∈ F. and the set B is called the codomain.

Example

Let set A = {1,2,3,4,5} , B = {0,1,2,3,5,7,9,12,13}
and F = [(1,1) (2,0) (3,7) (4,9) (5,12)] is function or not
Ans → It is a function. all elements are related



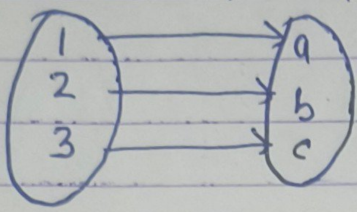
Q F = (1,1) (2,3) (3,5) (4,9) (3,7)
→ 3 has two images, so it is not a function.

Types of function

1. One to one function

A function from $A \to B$ is one to one or injective if for all elements x_1, x_2 in A: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

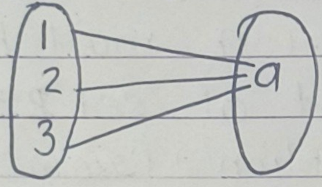
Example



2. Many to one function

A function f from $A \to B$ is said to be many one if and only if two or more elements of A have same image in B.

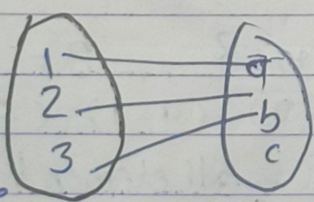
Example



3. Into function

A function F from A to B is called into function if and only if there exist at least one element in B which is not the image of any element in A.

Example

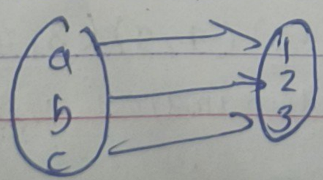


surjective

4. Onto function

A function F from A to B is onto or surjective if every element of B is the image of some element in A

Example

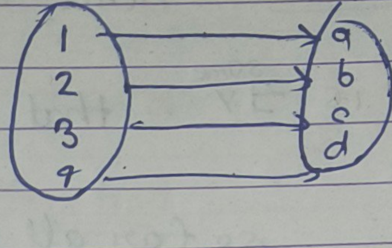


← No element is vacent

5. **Bijective function**

A function from A to B is said to be bijective or one to one correspondence if it is both injective and surjective both one to one and one to.

Example



Q Let $f(x) = x^2$ in any real number then prove that it is many one function.

Solution $f(x) = x^2$

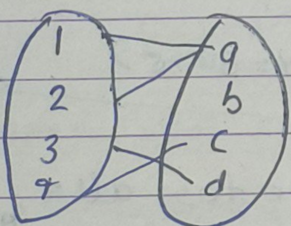
for $f(1) = 1$

$f(-1) = 1$ so it is many one function.

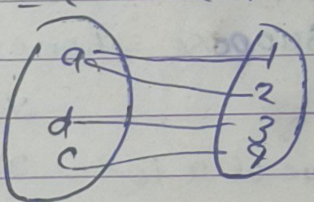
Q Let $A = \{1,2,3,4\}$, $B = (a,b,c,d)$ and

$f = \{(1,a)(2,a)(3,d)(4,c)$ then show that f is a function and f^{-1} is also a function.

F



$F^{-1} = \{(a,1)(a,2)(d,3)(c,4)\}$



F^{-1} is not a function.

Q $F: R \to R$ then Prove that $f(x) = x^2$ is one one function

$f(x) = x^2$

$f(x_1) = x_1^2$, $f(x_2) = x_2^2$

$x_1^2 = x_2^2$

$x_1^2 - x_2^2 = 0$

$(x_1 + x_2)(x_1 - x_2) = 0$

$x_1 + x_2 = 0$ $\quad$ $x_1 - x_2 = 0$

$\boxed{x_1 = -x_2}$ $\quad$ $x_1 = x_2$ — two values of x_1

Hence we say that it is not a one one function.

Q $f: R \to R$ then P.T $f(x) = 2x - 3$ is bijective.

Solution oneone $f(x) = 2x-3$

$f(x_1) = 2x_1 - 3$, $f(x_2) = 2x_2 - 3$

$2x_1 - 3 = 2x_2 - 3$

$x_1 = x_2$ Hence it is a one one function.

onto

suppose there is $\exists y$ same that $\in R$ ($\exists y \in R: y = 2x-3$)

$y = 2x - 3$

$x = \frac{y+3}{2}$ so for all x, the some y exist that's why it is a onto function.

So it is a bijective function.

Hence proved

Q $f: N \to N$ then P.T $f(x) = 3x+5$ is bijective

Solution oneone $f(x) = 3x+5$

$f(x_1) = 3x_1 + 5$, $f(x_2) = 3x_2 + 5$

$x_1 = x_2$ Proved It is one one function.

onto

suppose there is $\exists y \in R: y = 3x+5$)

$y = 3x+5$

There is value of x is Real number $x = \frac{y-5}{3}$ so for all x, the some 'y' exist

So for natural number → it is not onto function.

Hence proved it is not bijective function.

So range for $\exists x \notin N$. It $\in R$;.

$f: z^+ \to z^+$ then P.T $f(x) = x^2$ It is one one but not onto

one one $f(x) = x^2$

$f(x_1) = x_1^2$, $f(x_2) = x_2^2$

$x_1^2 - x_2^2 = 0$

$(x_1 - x_2)(x_1 + x_2) = 0$

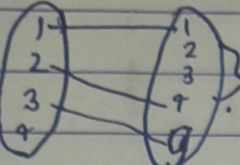
$x_1 = x_2$ It is one one function.

(-3) $\quad x^2 = y$

$9 = 3^2 = y$

$(9)\ (-3)^2 =$

onto $\quad$ Suppose there is $\exists y \in R : y = x^2$



empty

$y = x^2$

$x = \sqrt{y}$ $\quad$ So It is not onto function

$\sqrt{9}$ $\quad$ ~~fog = 9~~

~~$x = \sqrt{9}$~~ $= 3$ $\quad$ for $x = 3$ & -3

$x =$ ~~...~~ $\quad y = 9$.

~~Q Com~~

Composition of function

$f : A \to B$ & $g : B \to C$ then the composition of f and g is denoted by gof and read as g of f that results a new function from $A \to C$ and is given by

$gof(x) \,/\, g(f(x))$ for $\forall x$ in A.

$Fog(x) \,/\, f(g(x))$ for $\forall x$ in A.

Q $\quad A = \{1, 2, 3\}$

$B = \{a, b\}$

$C = \{r, s\}$ and $f : A \to B$ defined by $f(1) = a$

$f(2) = a$, $f(3) = b$ and $g : B \to C$ defined

by $g(a) = s$, $g(b) = r$, then find $gof(1)$, $gof(2)$ $gof(3)$.

Solution

$gof(1) = g(f(1)) = g(a) = s$

$gof(2) = g(f(2)) = g(a) = s$

$gof(3) = g(f(3)) = g(b) = r$ Ans

Q $\quad f : R - R$

$f(x) = 2x - 3$ find f^{-1}

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Q $f(x) = x^2+1$
$g(x) = 2x+1$
Prove that $fog = gof$

$gof = gof$
$= g(f(x))$
$= g(x^2+1)$
$= 2(x^2+1)+1 = 2x^2+2+1$
$= (2x^2+3)$

$fog = f(g(x))$
$= f(2x+1)$
$= (2x+1)^2+1$
$= (2x+1)^2+1$
$fog \neq gof$

Q $A = \{1,2,3\}$
$f : A \to A$
$f = \{(1,2)(2,1)(3,3)\}$
f^2, f^3

$f^2 = f(f(x))$
$= f(f(1)) = f(2) = 1$
$f(1) = 2$ $f(f(2)) = f(1) = 2$
$f(2) = 1$
$f(3) = 3$ $f(f(3)) = f(3) = 3$

$f^3 = f(f(f(x)))$
$x = 1$ $f(f(f(1)))$
$= f(f(2)) = f(1) = 2$

if $x = 2$

$$f(f(f(2))) = f(f(1)) = f(2) = 1$$

if $x = 3$

$$f(f(f(3))) = f(f(3)) = f(3) = 3$$

Q) $f : R \to R$ and $f(x) = x^2$ and $g : R \to R$. $g(x) = x+3$ then prove that $fog = gof$

$$\begin{aligned} fog &= f(g(x)) \\ &= f(x+3) \\ &= (x+3)^2 \end{aligned}$$

$$\begin{aligned} gof &= g(f(x)) \\ &= g(x^2) \\ &= x^2+3 \end{aligned}$$

$$fog \neq gof$$

Q) $f(x) = x^3 - 4x$, $g(x) = \frac{1}{x^2+1}$, $h(x) = x^4$

then find (i) $fogoh$

$hogof$

$fogoh$

$$\begin{aligned} f(g(h(x))) &= f(g(x^4)) \\ & f\left(\frac{1}{x^8+1}\right) \\ &= \left(\frac{1}{x^8+1}\right)^3 - 4\left(\frac{1}{x^8+1}\right) \\ &= \frac{(x^8+1)^2 - 4}{x^8+1} = x^{16}+1+2x^8-4 \\ &= \frac{x^{16}+2x^8-3}{x^8+1} \end{aligned}$$

hogof

$h(g(f(x)))$

$h(g(x^3-4x))$

$h\left(\frac{1}{(x^3-4x)^2+1}\right)$

$= h\left(\frac{1}{(x^3-4x)^2+1}\right)$

$= \left(\frac{1}{(x^3-4x)^2+1}\right)^4$

$f(x) = \sqrt{x}$

$g(x) = 3x+1$