UNIT-3

Sentence

A number of words making a complete gramatical structure having a sense and meaning is called a sentence or assertion in logic.
This assertion may be of two types -
(i) Declarative
(ii) Non declarative

A proposition of a statement is a declarative sentence that is either true or false.

Question, exclamatory sentences and commands are not proposition.

The sun rises in the west.
(5,6) $\subset$ (6,7,5)
Do u speak hindi. — not proposition.
$4-x = 8$ ⊗.

Proposition is a simple statement denoted by letter P, Q, R and propositional variables have only two values true or false.

Compound proposition

A proposition consist of a single propositional variable or a single propositional constant is called atomic or primary or primitive proposition.
A proposition obtain from the combination of two or more propositions by some logical operator or connectives is known as compound composite or

molecular proposition.

Connectives

The word or symbols used to form compound proposition are called connectives.

Ex:
p: It is raining.
q: It is cold.

There are five basic connectives.

		symbol used	symbolic form	Connective word
1.	Negation:-	$\sim$ / $\neg$	$\sim p$	not
2.	Conjunction:-	$\wedge$	$p \wedge q$	And
3.	Disjunction:-	$\vee$	$p \vee q$	or
4.	Implication or conditional:-	$\rightarrow$ or $\Rightarrow$	$p \rightarrow q$ $p \Rightarrow q$	If — then
5.	Biconditional or equivalence	$\Leftrightarrow$ $\leftrightarrow$	$p \leftrightarrow q$ or $p \Leftrightarrow q$	If and only if.

Ex
p: Paris is in France.
$\sim p$: Paris is not in France.

1. p: Arun is healthy
q: He has blue eyes.
$p \wedge q$ =:
$p \wedge q \Rightarrow$ Arun is healthy and he has blue eyes.

2. $p: 2x+5=6$

$q: x \geq 2$

$p \wedge q$.

$p \wedge q$: $2x+5=6$ and $x \geq 2$

$p \vee q$: $2x+5=6$ or $x \geq 2$

1. Negation

p	~p
T	F
F	T

2. Conjunction ($\wedge$)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

3. Disjunction:-

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

4. Construct the truthtable for the compound proposition. (i) $p \wedge (\sim q \vee q)$

(ii) $\sim(p \vee q) \vee (\sim p \wedge \sim q)$

p	q	~q	$(\sim q \vee q)$	$p \wedge (\sim q \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

(ii) $\sim(p \vee q) \vee (\sim p \wedge \sim q)$

p	q	~p	~q	$\sim(p \vee q)$	$\sim p \wedge \sim q$	$(p \vee q) \vee (\sim p \wedge \sim q)$
T	T	F	F	F	F	F
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	T	T	T

Algebra of Proposition:

(1) Telumpotent law → $P \vee P \equiv P$ $\quad$ $P \wedge P \equiv P$ equivalent

(ii) Associative law → $(P \vee q) \vee r = P \vee (q \vee r)$

$(P \wedge q) \wedge r = P \wedge (q \wedge r)$

(iii) Commutative law:-

$$(P \vee q) \equiv (q \vee P)$$
$$(P \wedge q) \equiv (q \wedge P)$$

(iv) Distributive law:-

$$P \vee (q \wedge r) \equiv (P \vee q) \wedge (P \vee r)$$
$$P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$$

(v) Identity law:-

$$P \vee F = P \quad , \quad P \wedge T = P$$
$$P \vee T \equiv T \quad , \quad P \wedge F \equiv F$$

(vi) Complement law:- $\quad P \vee \sim P = T \quad\quad P \wedge \sim P \equiv F$

$$\sim T \equiv F \quad\quad \sim F \equiv T$$

(vii) Involution law:-

$$\sim(\sim P) = P$$

(viii) De morgan's law ⇒

$$\sim (P \wedge q) = \sim P \vee \sim q$$
$$\sim (P \vee q) \equiv \sim P \wedge \sim q$$

Q Prove it by truth table

$$p \vee (q \wedge r) \simeq (p \vee q) \wedge (p \vee r)$$

Solution

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$$

p	q	r	$q \wedge r$	A	$p \vee q$	$p \vee r$	B
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Conditional proposition

If p and q are proposition the compound proposition 'if p then q' is denoted by $\Rightarrow$ / $\rightarrow$ is called conditional proposition or implication and the connective is conditional connective and denote by $p \rightarrow q$.
The proposition 'p' is called antecedent or hypothesis and the proposition 'q' is called consequent or conclusion

Example

If $\underbrace{\text{tomorrow is sunday}}_{p}$ then $\underbrace{\text{today is saturday}}_{q}$.

If $\underbrace{\text{it rains}}_{p}$ then $\underbrace{\text{I will carry an umbrella}}_{q}$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

- The only circumstances under which the implication implies q $p \rightarrow q$ is false when p is true and q is false.

• **As** If earth is round then the earth travels ~~surround~~ around the sun. True

→ If Alexander Graham bell invented telephone then the tigers have wings. false.

→ If tigers have wings then RDX is dangerous. True.

Ques. Calculate truth table for $p \vee \sim q \Rightarrow p$

$q \Rightarrow p$

p	q	$\sim q$	$p \vee \sim q$	$q \to p$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	F

Question Calculate $\sim (p \wedge q) \vee r \Rightarrow \sim p$

$q \Rightarrow \sim p$

$q \Rightarrow \sim p$

p	q	r	$p \wedge q$	$\sim(p \wedge q)$	$\sim(p \wedge q) \vee r$	$\sim p$	$q \Rightarrow \sim p$
T	T	T	T	F	T	F	F
T	T	F	T	F	T	F	F
T	F	T	F	T	T	F	F
T	F	F	F	T	T	F	F
F	T	T	F	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Q) $p \Rightarrow q \equiv \sim p \vee r$

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Q) $p \Rightarrow q \equiv \sim p \vee q$

p	q	$p \vee q$	$\sim(p \vee q)$	$p \to q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T	F	T
T	F	T	F	F	F	F
F	T	T	F	T	T	T
F	F	F	T	~~F~~ T	T	T

p	q	conditional $p \Rightarrow q$	converse $q \Rightarrow p$	inverse $\sim p \Rightarrow \sim q$	contra positive $\sim q \Rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Q. Consider the statement P: It rains.
q: The crops will grow.

Conditional → If it rains then the crops will grow.
Converse → If the crops will grow then it rains.
Inverse → If It does not rain then the crops will not grow.
Contrapositive → If the crops will not grow then It does not rain.

Question Show that the contrapositive are logically equivalent to conditional statement.

p	q	$p \to q$ (conditional)	$\sim p$	$\sim q$	$\sim q \Rightarrow \sim p$ (contrapositive)
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

We can see that
contrapositive $(\sim q \Rightarrow \sim p) \equiv p \to q$ (conditional)
Hence proved conditional is logically equivalent to contrapositive.

Biconditional statement

If p and q are statement, then the compound statement p if and only if q, denoted by $p \Longleftrightarrow q$ or $p \leftrightarrow q$ is called a biconditional statement and the connective is if and only if. The biconditional statement can also be stated as p is a necessary and suficient condition for q.

Example

He swims if and only if the water is warm.
Sales of houses fall if and only if interest rate rises.

Truth table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Q) p: a new bike will be aquired.
q: additional funding is available.

$\longrightarrow$ A new bike will be aquired if and only if additional funding is available.

Q) Prove that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$	$p \leftrightarrow q$
T	T	T	T.	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Q Prove that $p \leftrightarrow q \equiv (p \vee q) \rightarrow (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	T	F	F	F
F	T	T	F	F	F
F	F	F	F	T	T

Q Negation of conjunction

$\sim(p \wedge q) = \sim p \vee \sim q$

p: It rains

q: It will sunny

$\sim(p \wedge q) =$ It will not rain or it will not sunny

Negation of disjunction

$\sim(p \vee q) = \sim p \wedge \sim q$

Q

p: $2+4=6$

q: $7<12$

$\sim(p \wedge q) = \sim p \vee \sim q$

= ~~two and four~~ will not make ~~six or~~ ~~seven is not less than twelve~~.

$\boxed{2+4 \neq 6 \text{ or } 7 \not< 12}$ Ans

$\sim(p \vee q) = \sim p \wedge \sim q$

= two and four will not make six and seven is not less than twelve.

$\boxed{2+4 \neq 6 \;\&\; 7 \not< 12}$ Ans

Q. $\sim(P \wedge q) = \sim P \vee \sim q$

P	q	$P \wedge q$	$\sim P$	$\sim q$	$\sim(P \wedge q)$	$\sim P \vee \sim q$
T	T	T	F	F	F	F
T	F	F	F	T	T	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T

Negation of Implication / conditional

$$P \Rightarrow q \equiv \sim P \vee q$$
$$\sim(P \Rightarrow q) = \sim(\sim P \vee q)$$
$$= \sim(\sim P) \wedge (\sim q)$$
$$\boxed{\sim(P \Rightarrow q) = (P \wedge \sim q)}$$

Truth table

P	q	$P \Rightarrow q$	$\sim q$	$(P \wedge \sim q)$	$\sim(P \Rightarrow q)$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	F	F

Q. Write the negation of the following statement–

(i) If it is raining then the game is cancel.
(ii) If he studies then he will pass the examination.

Negation

(i) ~~If~~ it is ~~not~~ raining ~~then~~ and the game is not cancel.
~~If~~ he ~~not~~ studies ~~then~~ and he will not pass the examination.

Negation of biconditional

$$\boxed{\sim(p \leftrightarrow q) \equiv \sim p \leftrightarrow q \ / \text{or } p \leftrightarrow \sim q}$$

Truthtable

p	q	$p \leftrightarrow q$	$\sim p$	$\sim q$	$\sim p \leftrightarrow q$	$p \leftrightarrow \sim q$	$p \wedge \sim q$
T	T	T	F	F	F	F	F
T	F	F	F	T	T	T	T
F	T	F	T	F	~~F~~ T	~~F~~ T	F
F	F	T	T	T	F	F	F

$\sim p \wedge q$	$(p \wedge \sim q) \vee (\sim p \wedge q)$
F	F
F	T
T	T
F	F

Q) He swims if and only if the water is warm.
The computer program is correct if and only if it produces the correct answer for all the questions

$\sim(p \leftrightarrow q) = \sim p \leftrightarrow q$

(i) p: He swims
q: The water is warm

$\sim p \leftrightarrow q \Rightarrow$ He does not swim if and only if the water is warm.

or

$p \leftrightarrow \sim q =$ He swims if and only if the water is not warm.

(ii) p: The computer program is correct.
q: It produces the correct answer for all the question.

$(p \leftrightarrow q)$

$\sim p \leftrightarrow q$ = The computer program is not correct if and only if it produces the correct answer for all the question.

or

$p \leftrightarrow \sim q$ The computer program is correct if and only if it not produces the correct answer for all the question.

Tautologies

A compound proposition that is always true for all possible truth ~~variables~~ table of its variables or in other word it contains only true ~~of its tr~~ in the last column of its truth table.

Contradiction

A compound proposition that is always false for all possible values or it only contains false in the last column of it's truth table is called contradiction.

Contingency

A proposition that is neither nor contradiction is called contingency.

Example

(i) The professor is either a women or a man – (Tautology).

(ii) People either like watching television or they don't like. (Tautology).

Q) $P \vee \sim P$ prove tautology.

P	$\sim P$	$P \vee \sim P$
T	F	T
F	T	T

It is a tautology

Q) $\sim(P \wedge q) \vee q$

P	q	$(P \wedge q)$	$\sim(P \wedge q)$	$\sim(P \wedge q) \vee q$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

for all different proposition output is true

It is a tautology.

Q) $P \Rightarrow (P \vee q)$

P	q	$P \vee q$	$P \Rightarrow (P \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

It is a tautology.

Q. $p \wedge (q \wedge \sim p)$

p	q	$\sim p$	$(q \wedge \sim p)$	$p \wedge (q \wedge \sim p)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

It is a contradiction.

Normal forms

By comparing truth tables we can determine two logical expressions p and q are equivalent. but it becomes very tedious when the number of variable increases. so there is a better method to transform the expression p and q to some standard form called normal forms or canonical form :

There are two types of normal form-

(i) Disjunctive normal form (DNF) — (Sum of Product) (V)

(ii) Conjunctive normal form (CNF) - (Product of sum) ($\wedge$)

DNF = $(A \wedge B) \vee (B \wedge C)$

CNF = $(A \vee B) \wedge (B \vee C)$

Procedure to obtain DNF from given logical expression

(i) Remove all conditional ($\rightarrow$) or biconditional ($\leftrightarrow$) by an equivalent logical expressions containing the connectives ($\wedge$, $\vee$) & $\sim$)

$$\boxed{p \rightarrow q = \sim p \vee q}$$

(ii) eliminate ($\sim$) (negation) before sums and products by using double negation($\sim$) or by using demorgan's law.

(iii) Apply the distributive law untill a sum of product (elementary) is obtained.

Q1 $p \wedge (p \Rightarrow q)$

$p \wedge (\sim p \vee q)$ (CNF)

Apply distributive law

$\cancel{a+(b \cdot c)} = (a+b) \cdot (a+c)$

$a \cdot (b+c) = (a \cdot b) + (a \cdot c)$

$= (p \wedge \sim p) \vee (p \wedge q)$

$\underline{F \vee (p \wedge q)}$ Ans

Q $(p \to q) \wedge (\sim p \wedge q)$

$(\sim p \vee q) \wedge (\sim p \wedge q)$

$\sim p \wedge (\sim p \wedge q) \vee (q \wedge (\sim p \wedge q))$

$(\sim p \wedge q) \vee (q \wedge \sim p)$

$(\sim p \wedge q) \vee (\sim p \wedge q)$

$=$

Q $\sim [p \Rightarrow (q \wedge r)]$

$\sim [\sim p \vee (q \wedge r)$

$= p \wedge \sim (q \wedge r)$

$= p \wedge (\sim q \vee \sim r)$

$(p \wedge \sim q) \vee (p \wedge \sim r)$

Q $\sim (p \vee q) \Leftrightarrow (p \wedge q)$

$\sim (\cdot p \vee q) \wedge (p \wedge q) \vee [\sim \sim (p \vee q) \wedge \sim (p \wedge q)$

Q2 $p \vee (\sim p \Rightarrow (q \vee (q \to \sim r)))$

$p \vee (\sim p \Rightarrow (q \vee (\sim q \vee \sim r)))$

$p \vee (\sim \sim p) \vee (q \vee (\sim q \vee \sim r)))$

$p \vee (p \vee (q \vee (\sim q \vee \sim r)))$

$p \vee p \vee (q \vee (\sim q \vee \sim r))$

$= p \vee (F \vee \sim r) = \boxed{p \vee \sim r}$ Ans

Q3 $p \Rightarrow ((p \to q) \wedge \sim (\sim q \vee \sim p)$

$(\sim p \wedge \sim q) \wedge (p \wedge q)$

($\vee$ - or, (+)
$\wedge$ ~~not~~ And.)

$p \Rightarrow (\sim p \vee q) \wedge (q \wedge p)$

$= \sim p \vee ((\sim p \vee q) \wedge (q \wedge p))$

$= \cancel{\sim p \vee q}$

$\cancel{\sim p} \vee$

Apply dist. law

$= \sim p \vee (\sim p \vee q) \wedge (\sim p \vee (\cdot p \wedge q))$

$= \sim p \vee (\sim p \vee q) \wedge (F \wedge q)$

$\sim p \vee (\sim p \wedge q)$

$[(\sim p \wedge \sim q) \wedge (p \wedge q)] \vee [(p \vee q) \wedge (\sim p \vee \sim q)$

$[(\sim p \wedge p) \wedge (\sim q \wedge q)] \vee [p \wedge (\sim p \vee \sim q) \vee (q \wedge (\sim p \vee \sim q)$

$(\cdot F \wedge F) \vee [(p \wedge \sim p) \vee (p \wedge \sim q) \vee (q \wedge \sim p) \vee (q \wedge \sim q)$

$F \vee [F \vee (p \wedge \sim q) \vee (q \wedge \sim p) \vee F]$

$= (p \wedge \sim q) \vee (q \wedge \sim p)$

$p \to q = \sim p \vee q$
$p \leftrightarrow q = (p \wedge q) \vee (\sim p \wedge \sim q)$

p	~p	p∧~p
T	F	F
F	T	F

CNF

Q) $p \wedge (p \Rightarrow q)$

CNF $\Rightarrow$ $p \wedge (\sim p \vee q)$

Applying distributive law

$\Rightarrow [p \wedge \sim p] \vee (p \wedge q)$

$F \vee (p \wedge q)$

$= (p \wedge q)$

$\Rightarrow$ $(p \wedge q)$ Ans

$(p \to q) \wedge (q \vee (p \wedge r))$

$(\sim p \vee q) \wedge (q \vee (p \wedge r))$

$(\sim p \vee q) \wedge (q \vee p) \wedge (q \vee r)$

Q) $[q \vee (p \wedge q)] \wedge \sim [(p \vee r) \wedge q]$

applying demorgans

$[q \vee (p \wedge q)] \wedge (\sim (p \vee r) \vee \sim q]$

$[q \vee (p \wedge q)] \wedge [(\sim p \wedge \sim r) \vee \sim q]$

apply distributive law

~~$[q \vee (p \wedge q)] \wedge (\sim p \wedge \sim r) \vee (\sim p \wedge \sim q)]$~~

apply distributive

$(q \vee p) \wedge (q \vee q] \wedge [\sim q \vee (\sim p \wedge \sim r)]$

$[(q \vee p) \wedge q] \wedge [(\sim q \vee \sim p) \wedge (\sim q \vee \sim r)]$ Ans

Logic of Proof

VIMP

(i) Valid arguments

an argument is a sequence of statements. all statements except the final one are called premises or assumption or hypothesis. The final statement is called the conclusion.

An argument is said to be logically valid if and only if the conjunction of the premises implies the conclusion that is if the premises are all true then the conclusion must also be true however if one of the premises is false so that the conjunction of all the premises is false then the conclusion may be either true or false.

Therefore to check the argument is valid or not, it is sufficient to check only those ~~rose~~ rows of the truth table for which all the premises are true and see the conclusion is always true there.

Critical row

Critical row are those rows in which all the premises are true and if in each critical row the conclusion is also true then the argument is valid

Premises

p	q	r	conclusion
T	T	T	T
T	T	T	F

If there is atleast one critical row in which the conclusion is false then the argument is invalid.

Qs check the validity $(p \Rightarrow q) \wedge (q \Rightarrow r) \equiv (p \Rightarrow r)$

Solution

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p\Rightarrow q)\wedge(q \Rightarrow r)$	$(p \Rightarrow r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	F	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

Argument is valid.

Q $p \vee q \Rightarrow \sim p \vee q$

p	q	$p \vee q$	$\sim p$	$\sim p \vee q$	$p \vee q \Rightarrow \sim p \vee q$
T	T	$\boxed{T}$	F	$\boxed{T}$	$\boxed{T}$
T	F	$\boxed{T}$	F	F (X)	F
F	T	T	T	T	$\boxed{T}$
F	F	F	T	T	T

Argument is valid.

In $p \vee q$ and $\sim p \vee q$ (Premises) ~~one is false~~

Q $p \vee q$ check

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

→ True

Argument is valid

Rule of inference	Tautological form	Name
(i) $p \mathbin{/} \therefore p \vee q$	$p \Rightarrow p \vee q$	Addition
(ii) $p \wedge q \mathbin{/} \therefore p$	$p \wedge q \Rightarrow p$	simplifications
(iii) p, q / $\therefore p \wedge q$	$p \wedge q \Rightarrow (p \wedge q)$	Conjunction
(iv) $p \Rightarrow q$, p / $\therefore q$	$(p \Rightarrow q) \wedge p \Rightarrow q$	modus ponens
(v) $p \Rightarrow q$, $\sim q$ / $\therefore \sim p$	$(p \Rightarrow q) \wedge \sim q \Rightarrow \sim p$	modus Tollens

Example :- If this no is divisible by 6 then it is divisible by 3

modus Tollens

It is not divisible by 3

This no is not divisible by 6.

	Rule of interference	Tautological form	Name
(vi)	$p \Rightarrow q$ $q \Rightarrow r$ $\therefore p \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$	Hypothetical syllogism
(vii)	$p \vee q$ $\sim p$ $\therefore q$	$(p \vee q) \wedge (\sim p) \Rightarrow q$	Disjunctive syllogism
(viii)	$p \Rightarrow q \wedge (r \Rightarrow s)$ $p \vee r$ $q \vee s$	$(p \Rightarrow q) \wedge (r \Rightarrow s) \wedge (p \vee r) \Rightarrow (q \vee s)$	Constructive Dilemma
(ix)	$p \Rightarrow q \wedge (r \Rightarrow s)$ $\sim q \vee \sim s$ $\therefore \sim p \vee \sim r$	$[(p \Rightarrow q) \wedge (r \Rightarrow s)] \wedge (\sim q \vee \sim s) \Rightarrow (\sim p \vee \sim r)$	Destructive Dilemma

Example

(i) Either A is not guilty or B is telling the truth.

(ii) B is not telling the truth.

conclusion — A is guilty.

Q

$$p \Rightarrow r$$
$$\sim p \Rightarrow q$$
$$q \Rightarrow s$$
$$\therefore \sim r \Rightarrow s$$

(1) $p \Rightarrow r$ -

(2) $\sim r \Rightarrow \sim p$ using Contrapositive in (1)

(3) $\sim p \Rightarrow q$

(4) $\sim r \Rightarrow q$ using Hypothetical syllogism in (2) & (3)

(5) $q \Rightarrow s$

(6) $\sim r \Rightarrow s$ by using Hypothetical syllogism in (4) & (5)

Q^n

$$\begin{array}{c} p \Rightarrow q \\ r \Rightarrow \sim q \\ \hline p \Rightarrow \sim r \end{array}$$

(1) $p \Rightarrow q$

(2) ~~$\sim q \Rightarrow \sim p$~~ ~~using contrapositive~~

~~$r \Rightarrow \sim q$~~

(2) $r \Rightarrow \sim q$ using contrapositive

$q \Rightarrow \sim r$

(3) $p \Rightarrow \sim r$ by using hypothetical syllogism in (1) and (2).

Q^n If the races are fixed or the casinos are crooked then the tourist trade ~~de~~ ~~with~~ decreases. If the tourist trade decreases then the policeforce will be happy. the police force ~~is~~ never happy therefore the races are not fixed.

check the validity of statement

p: The races are fixed.
q: The casinos are crooked
r: Tourist trade will be decreases.
s: Police force will be happy.

$(p \vee q) \Rightarrow r$ —— (i)
$r \Rightarrow s$ —— (ii)
$\sim s$ —— (iii)

$\therefore \sim p$

1. $p \wedge q \Rightarrow r$
2. $r \Rightarrow s$
3. $\sim s$
4. ~~(r ⇒ s) ∧ ∼s using modus tollens~~
5. $= p \vee q \Rightarrow s$ using hypothetical syllogism in 1 and 2
6. $\sim(p \vee q)$ using modus tollens in 5 and 3
7. $\sim p \wedge \sim q$ using demorgans in 6
8. $\sim p$ using simplifications in (7)

Q) If there was a ball game then travelling was difficult. If they arrived on time then travelling was not difficult. They arrived on time. Therefore there was no ball game.

p: there was a ball game.
q = travelling was difficult
r: they arrived on time

$p \rightarrow q$ — (i)
$r \rightarrow \sim q$ — (ii)
r — (iii)

$\therefore \sim p$

(1) $p \rightarrow q$
(2) $r \rightarrow \sim q$ apply contrapositive on (ii)
(3) $q \rightarrow \sim r$ apply hypothetical syllogism on (1) and (3)
(4) $p \rightarrow \sim r$
(5) r
(6) $\sim p$ using modus tollens on (4) and (5)

Hence proved

Q) Show that $R \wedge (P \vee q)$ is a valid conclusion from the premises

$P \vee q$ — (I)
$q \rightarrow r$ — (II)
$P \rightarrow M$ — (III)
$\sim M$ — (IV)

$\therefore R \wedge (P \vee q)$

(1) $P \vee q$
(2) $q \rightarrow r$
(3) ~~$P \vee q$~~ $P \rightarrow M$
(4) $\sim M$
(3) $\sim P$ apply modus tollens on (3) and (4)
(~~4~~) ~~$P \vee q$~~

(5) q apply disjunctive syllogism (1) and (3)

(6) R apply modus ponens on (11) and (5)

(7) $R \wedge (P \vee q)$ conjunction of (1) and (6)

Q. show that $\sim r$ is a valid conclusion from the premices

$$P \Rightarrow \sim q \quad -(i)$$
$$r \Rightarrow P \quad -(ii)$$
$$q \quad -(iii)$$

(4) $r \Rightarrow \sim q$ apply Hypothetical syllgism ~~modus tolens~~ on (1) and (2)

(5) $\sim r$ apply modus tollens on (4) and 3

Q

$$\sim t \Rightarrow \sim r \quad -(i)$$
$$\sim s \quad -(ii)$$
$$t \Rightarrow w \quad -iii$$
$$r \vee s \quad -iv$$
$$\overline{\quad w \quad}$$

~~apply conjunction on (i) and (ii)~~

~~$\sim t \Rightarrow \sim r \wedge t \Rightarrow w$~~

(iv) r apply disjunctive syllogism on (2) and 4

$r \Rightarrow t$ — 6 contrapositive on (1)

$t \rightarrow 7$ using (5) and (6) modus ponens

$t \Rightarrow w$ — (3)

$w \longrightarrow$ by using modus ponens on (3) and (7)

Q check the validity -

"If I get the job and work hard then I will get promoted. If I get promoted then I will be happy. I will not be happy. Therefore either I will not get the job or I will not work hard.

p : I get the job ~~and work hard~~. q = work hard

r : I will get promoted.

s : I will be happy

$$\begin{array}{l} \cancel{p \wedge q \Rightarrow r} \\ q \Rightarrow r \\ \sim r \\ \hline \therefore \sim p \vee \end{array}$$

$$\begin{array}{ll} p \wedge q \Rightarrow r & \text{— (i)} \\ r \Rightarrow s & \text{— (ii)} \\ \sim s & \text{— (iii)} \\ \hline \therefore \sim p \vee \sim q & \end{array}$$

(4) $\sim r$ ~~apply~~ modus tollens ~~on (2)~~ and 3

(4) $p \wedge q \Rightarrow s$ using hypothetical syllogism on (1) and (2)

(5) $\sim(p \wedge q)$ using modus tollens on (4) and (3)

~~$\sim p \wedge q$~~

(6) $\sim p \vee \sim q$ using demorgans on (5)

Hence proved

Predicate Calculus

x is smaller than 10

x is a boy.

Here x is a variable so it may be true or false and it depends upon the value of x. So in this type of case we can not use propositional logic. Here we use predicate calculus, and predicate calculs is extension of propositional logic.

Example

p: All the students from class X has height above 159 c.m.

q: Deepak is student of class X

$\therefore$ Deepak has height above 159 c.m

Predicate calculus is a generlaziation of propositional calculus. It contains all the components of propositional calculus including propositional variables and constants.

Part of a declarative sentence describing the properties of an object or relation among objects is called a predicate.

$$\underbrace{A}_{\text{Subject}} \ \underbrace{\text{is a bacholar.}}_{\text{Predicate}}$$

A is a boy.
B is a boy
C is a boy

x is a boy

A
B
C
Domain

(Predicate logic)

In this, Domain is known as universe of discourse.

So we can say subject present instance.

Quantifiers

Quantifier are words that represents quantities such as "some" or "All" and indicate how frequently a certain statement is true.
There are two types of quantifiers -

all students are of class X.
Some students are of classX.
Every students are of class X.

(i) Universal quantifier
(ii) Essential quantifier.

Universal quantifier

The Phrase " For all " is denoted by ($\forall$) is called universal quantifier.

for example

(i) All human beings are mortal.

(ii) Deepak is student of class x.
x — $P(x)$ $\Rightarrow x P(x)$

(iii) All students are of Class x.
$\forall x$ — $P(x)$ $\Rightarrow \forall x P(x)$

(iv) All mens are mortal.
$\forall x$ — $P(x)$ $\Rightarrow \forall x P(x)$

(v) $\forall x P(x)$ is true if $P(x)$ is true $\forall x (u)$ for every x in u

$\forall x P(x)$ is false if and only if $P(x)$ is false for atleast one x in u.

Essential quantifiers

The phrase " for some value " is denoted by ($\exists$) is called essential quantifier.

As "There exist x such that $x^2 = 5$"
$\exists(x)$ — $P(x)$

$P(x) = x^2 = 5$

Q $M(x)$: x is a man
$N(x)$: x is mortal
$A(x)$: x is integer
$B(x)$ Either +ve or -ve.
express the following using quantifiers.

(i) All mens are mortal
$\forall x \quad M(x) \Rightarrow N(x)$

(ii) An integer is either +ve or -ve
$\forall x \, A(x) \longrightarrow B(x)$

Q $A(x)$: x is a student.
$B(x)$: x is clever
$C(x)$: x is successful.
express using quantifiers.

- (i) Their exist a student.
$\exists(x) \; A(x)$

(ii) Some students ar clever.
$\exists(x) \; A(x) \wedge B(x) \quad \Rightarrow$

(iii) Some students are not successful.
$\exists(x) \; A(x) \wedge \sim C(x)$

$\sim$

$\wedge$

Q. Let z be the set of integer in the universe of discource and consider the statements $(\forall x \in z), x^2 = x$

$(\exists x \in z), x^2 = x$

find the truth value of each statement.

$(\forall x \in z)$ $x^2 = x$

$0^2 = 0$

$1^2 = 1$

$2^2 \neq 2$ so false

$(\exists x \in z)$ $x^2 = x$

$0^2 = 0$

$1^2 = 1$

$(-1)^2 = 1$ True

VIMP

Q. Let $D = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ determine the truth value of each of the following statement.

(i) $(\forall x \in D), x+4 < 15$ → True

(ii) $(\exists x \in D), x+4 = 10$ → True

(iii) $(\forall x \in D), x+4 \leq 10$ → False

(iv) $(\exists x \in D), x+4 > 15$ → false

Negation of quantified statements

" All students in the class have taken a cource in discrete mathematics".

Predicate calculus $\Rightarrow \forall x \; p(x)$

$$\boxed{\sim \forall x \; p(x) \equiv \exists x \sim p(x)}$$

The negation of universal statement ($\forall$) is logically equivalent to an essential (some or not) statement.

$$\boxed{\sim \exists x \; p(x) = \forall x \sim p(x)}$$

Qn Negate the statement for all real numbers if $x > 3$, then $x^2 > 9$

Solution

$$(\forall x \in R), \; x > 3 \rightarrow x^2 > 9$$

$$p(x) = x > 3$$

$$q(x) = x^2 > 9$$

$$\forall x \quad p(x) \rightarrow q(x)$$

$$\sim (\forall x \; p(x) \rightarrow q(x)) = \exists(x) \sim (p(x) \Rightarrow q(x))$$

$$= \exists(x) \sim (\sim (p(x) \vee q(x))$$

$$= \exists(x) \sim (\sim p(x) \vee q(x))$$

using demorgans $= \exists x \;\; p(x) \wedge q(x)$

Rules of inference

All the rule of inference for proposition formulas are also applicable for predicate calculus.

(i) Universal Instantiation — It is rule of inference that indicates the truth of a property in a particular case follow as a special instance of it's more general of universal truth

$\forall x\, P(x)$	- if true	where c is the
$\therefore P(c)$	→ then true	some element of the universe

(ii) Example

All students in the class understood logic.
Devid is a student in this class.
$\therefore$ Devid understand logic. is correct or not.

$\forall x \quad P(a)$

$P(x)$: x is a student in the class.
$q(x)$: x understand logic.
statement $\forall x\,(P(x) \Rightarrow q(x))$ -(i) $P(devid)$ -(2)
3 ⇐ $P(devid) \Rightarrow q(devid)$ ⇒ [by universal Instantation] from

apply modus ponens on 3 and 2

$$\begin{array}{l} P \to q \\ \;\; P \\ \hline \therefore q \end{array} = \begin{array}{l} P(devid) \Rightarrow q(devid) \\ \;\; P(devid) \\ \hline \therefore \text{devid understans logic} \end{array}$$

As Proved

(ii) **Universal generalization** This rule states that $\forall x\, P(x)$ is true. given the premises when $P(c)$ is true for any c.

$$\boxed{\begin{array}{l} \underline{P(c) \text{ for any } c} \\ \therefore\ \forall x\, P(x) \end{array}}$$

Q^n Justify that if $\forall x(P(x) \Rightarrow q(x))$ and $\forall x(q(x) \Rightarrow r(x))$ are true, then $\forall x(P(x) \Rightarrow r(x))$ is also true where the domain of all the quantifiers are same.

$\forall x(P(x) \Rightarrow q(x))$ — (i)
$\forall x(q(x) \Rightarrow r(x))$ — (ii)

So using hypothetical syllogism in (i) and (ii)

$\forall x(P(x) \Rightarrow r(x))$ Hence proved

Second method

$\forall x(P(x) \to q(x))$ — (i)
$\forall x(q(x) \to r(x))$ — (ii)
$P(c) \to q(c)$ — (iii) } by using universal
$q(c) \to r(c)$ — (iv) } instantiation

$P(c) \to r(c)$ — (V) by using Hypothetical syllogism on (iii) and (iv)

$\forall x(P(x) \to r(x))$ — (vi) by using universal generalization on (v)

hence proved

Essential Instantiation

It conclude that there is some element (c) for which P(c) is true when for $\exists x P(x)$ is true

$$\boxed{\begin{array}{l} \exists x\, P(x) \text{ is true} \\ \hline \therefore\ P(c) \text{ is true} \end{array}}$$

Essential Generalization

If there ~~exist~~ $x [\exists x P(x)]$ is true when for a paricular element $P(c)$ is true. That is if we know one element in c in the universe of discourse for which $p(c)$ is true, then $\exists x P(x)$ is also true.

Universal modus ponens

The rule of universal instantation can be combined with modus ponens to obtain the rule called universal modus ponens.

$$\begin{array}{ll} \forall x \text{ if } P(x) \longrightarrow q(x) & \\ P(a) & \text{for a particular element } (a) \\ \hline \therefore Q(a) & \end{array}$$

Universal modus tollens

The rule of universal can be combined with modus tollens to obtain the rule of universal modus tollens

$$\forall x \; P(x) \rightarrow Q(x)$$
$$\sim Q(a)$$
$$\therefore \sim P(a)$$

Question show that if $\exists x \, P(x) \wedge Q(x)$ is true then $\exists x \, P(x) \wedge \exists x \, Q(x)$ is also true

Soln

$\exists x \, P(x) \wedge Q(x)$ — (i)

$\therefore \exists x \, P(x) \wedge \exists x \, Q(x)$

$P(c) \wedge Q(c)$ — (ii) apply essential instantiation on (i)

$P(c)$ — (iii) apply simplification on (ii)

$\exists x \, P(x)$ — (iv) apply essential generalization on (iii)

$Q(c)$ — (v) again apply simplification on (ii)

$\exists x \, Q(x)$ — (vi) apply essential generalization on (v)

$\exists x \, P(x) \wedge \exists x \, Q(x)$ apply conjunction on (iv) and (vi)

Hence proved

Qn Prove the validity

some dogs are animals,

some cats are animal.

Therefore some dogs are cats.

Soln

$\exists P(x)$ = Some dogs are animal.

$\exists Q(x)$ = Some cats are animal.

forall = $\longrightarrow$

forsome Essential- and $\wedge$

$A(x)$: x is an animal

$D(x)$: x is a dog.

$C(x)$: x is a cat.

$\exists x\ D(x) \wedge A(x)$ — (i)

$\exists x\ C(x) \wedge A(x)$ — (ii)

$\therefore\ \exists x\ D(x) \wedge C(x)$

$D(c) \wedge A(c)$ — (iii) } apply ~~universal~~ essential instantiation

$\cancel{A(c)} \wedge C(c)$ — (iv)

$C(c) \wedge A(c)$

$\cancel{D(c) \wedge C(c)}$ } by ~~using hypothetical syllo.~~

$D(c)$ — (v) } simplification (iii), (iv)

$C(c)$ — (vi)

$\exists x\ D(x)$ — (vii) } apply essential generalization (v) and (vi)

$\exists x\ C(x)$ viii

~~$\exists x D(x) \wedge \exists x\ C(x)$ — (ix)~~ } ~~apply conjunction (vii) and (viii)~~

$D(c) \wedge C(c)$ — X ~~apply conjunction on (v) and (vi)~~

apply generalization on X

$\exists x\ (D(x) \wedge C(x))$

As

Q) check the validity

All graduates are educated.
John is a gradude

$\therefore$ John is educated.

$A(x)$: x is educated
$B(x)$: x is graduate

$\forall x\ B(x) \rightarrow A(x)$ —(i)
$B(J)$ —(ii) using universal modus ponens in (i) and (ii)

$\therefore$ ~~$B(C) \rightarrow A(C)$~~
$A(J)$

Ans

Hence proved

Q If a number is odd then its square is odd.
K is a particular number that is odd

$\therefore$ k^2 is odd.

Solⁿ

$A(x)$: ~~a number~~ x is odd
$B(x)$: x^2 is odd.

$\forall x\ A(x) \Rightarrow B(x)$ —(i)
$A(K)$ —(ii)

$Q(K)$ using universal modus ponens in (i) and (ii)

$= Q(K)$

Hence proved

Q^n Write the arguments using quantifiers and check it's validity.

All healthy people eat an apple per day.
Raju does not eat an apple per day.
Therefore Raju is not a healthy person.

$A(x)$ = ~~x are healthy people~~
$A(x)$ = x eat an apple per day
$B(x)$ = x is healthy people

$$\forall x \; B(x) \rightarrow A(x) \quad \text{—(i)}$$
$$\frac{\sim A(R) \quad \text{—(ii)}}{\sim B(R)}$$

using universal modus tollens in eq (i) and (ii)

$\sim B(R)$

Hence proved

Q^n All integers are irrational numbers.
Some integers are power of 2.
Therefore some irrational number are power of 2.

$A(x)$: x is an integer
$B(x)$: x is an irrational number
$D(x)$: x is power of 2.

$$\forall x \; A(x) \Rightarrow B(x) \quad \text{—(i)}$$
$$\frac{\exists x \; A(x) \wedge D(x) \quad \text{—(ii)}}{\therefore \; \exists x \; B(x) \wedge D(x)}$$

$A(c) \Rightarrow B(c)$ — (iii) By universal instantiation on (i)

$A(c) \wedge (D(c)$ — (iv) — essential instan on (ii)

$A(c)$ — (v) simplification on iv

$D(c)$ — (vi) again simplifi. iv

usi

Q^n All mamells are animals.
Some mamels are two leged.
Therefore some mamels are two leged.