$^nP_r = \frac{n!}{r}$ unit-1 Set $^nC_r = \frac{n!}{r!\,(n-r)!}$

Set

A set is any well defined collection of objects called the element ~~or~~ member of set.
These element may be anything it may be numbers paint in geomentry latters of alphabet etc.

Set are denoted by capital letter of alphabet.

Representation of a set

There are two ways for representing a set.
(i) Roster or tabulor
(ii) Rule method or set builder form

example

A = {1,2,3}
A = {x : x is any natural no}
B = {a, e}
B = {x : x is vowel of alphabet}

A = {a, b, c ----- }
B = {1, 2, 3, 4 --- }
C = ?
Z = Roster or tabular

Infinite set

Any set which have infinite number of element called infinite set.

Example

Set of Integer,
Set of natural number.

Q why { } is added in every set
Ans $1-1=0$
∴ If there is no common element in $A \cap B$ then we can write { } (empty set) contains no element.

Singleton set

Any set which have only one element called singulton set.

Ex →
$A = \{1\}$
$B = \{a\}$

Subset

Subset have all the properties of set. it is part of set. It is denoted by symbol ($\subseteq$)

$A \subseteq B$

where
B = { set of Alphabet }
A = { set of vowels }

$A = \{1, 2, 3\}$
$= (\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, (2,3), \{1,3\}, \{1,2,3\})$

$A = \{a, e, i, o, u\}$, $B = \{a - z\}$

Super set

Superset - If a is subset of B, then B is superset of A.

Universal set

universal set is non empty set which have all the set under consideration are subsets is called universal set.

Operation on Set

① Union

The union of two set A and B is denoted

$A = \{a, b\} \to (\{a\}, \{b\}, \{a, b\}\ \{\}/\phi)$

$B = \{b\} \to$ ~~since~~ $B \subseteq A$

$P = \{d\} \to$ now $P \not\subseteq A$ · so we can say that element must belongs ~~to~~ set so $d \notin A$

$\{\} \to$ by using contradiction property we can say that empty set contains not a single element that $\notin$ to A. so $\{\}$ is subset of A. we can not proof that the element $\notin$ in empty set that does not belongs to set A

by $(A \cup B)$ is the set of all element is belong to set A and set B.

$$A = \{x : x \in A \text{ or } x \in B\}$$

called $\Rightarrow$ x is such that x belongs to A and x belongs to B.

$2^+ = \{$ set of +ve int $\}$

$2^- = \{$ set of -ve int $\}$

$(2^+ \cup 2^-)$

Intersection — of 2 sets A & B is denoted by $A \cap B$ is the set of elements which belong to both A & B ie

$A \cap B = \{x : x \in A \ \& \ x \in B\}$.

if $A \cap B = \phi$ that is if A & B do not have any elements in common then A & B are disjoint or non intersecting.

Complement → The complement of set A is denoted by A' or A^c is the set of elements which belong to U but which do not belong to A ie $A' = \{x : x \in U \ \& \ x \notin A\}$

Relative complement → if there are 2 sets A & B the ~~the~~ relative complement of B with respect to A or simply difference of A & B is denoted by A−B is the set of elements which belong to A but which do not belong to B is the

$A - B = \{x : x \in A \ \& \ x \notin B\}$

$A = \{a, b, c\}$

$B = \{b, c, d, e\}$

$A - B = \{a\}$

$B - A = \{d, e\}$

$A - B = \{x : x \in A \ \& \ x \notin B\}$.

Symmetric difference

Symmetric difference of two sets A and B is denoted by $A \Delta B$ or $A \oplus B$ is the

triangle the symbol of delta   circle with plus sign inside

set of elements that belongs to set A or set B
but not from both A & B.

$$\boxed{A \oplus B = (A-B) \cup (B-A)}$$

Q→ $A = \{-3, 0, 1, 2\}$
$B = \{1, 2, 3, 4\}$

$A - B = \{-3, 0\}$
$B - A = \{3, 4\}$

$(A-B) \cup (B-A) = \{-3, 0, 3, 4\}$

Numbers

Real numbers

It is denoted by R. Every number except complex number $(a+ib)$ contain is the set of real numbers.
All the following types or numbers can also be thought as a real number.

Integers

It is represented by letter Z and integer is any number in the infinite set series.

$Z = \{-\infty \ldots \ldots -3, -2, -1, 0, 1, 2, 3 \ldots \infty\}$

$Z^+ = \{$ set of +ve int $\}$
$Z^- = \{$ set of -ve int $\}$

Integers are sometimes split into 3 subset z^+, z^- and 0. where z^+ is the set of +ve integers, z^- is the set of -ve integers. 0 is not included in these both sets.

Natural number -

The set of natural number is represented by the letter 'N'.

$$N = \{1, 2, 3 ---- \infty\}$$

This set is equivalent to set of +ve integers. so natural number is a +ve Integer.

Whole number

The set of whole numbers is represented by w. This set is equivalent to natural numbers with 0.

Rational number

It is represented by Q. Rational number that can be written as a ratio of two integers.

Irrational number

It is represented by I. Any real numbers that is not rational is called irrational number. These numbers can be written as decimal but not as a fraction.

Ex π, $\sqrt{2}$, $\sqrt{3}$

Q →

$A = \{1,2,3,4\}$
$B = \{2,4,6,8\}$
$C = \{3,4,5,6\}$

(i) $(A \cap B) \cap C = 4$
(ii) $(A \cup B) \cap C = \{3,4,6\}$
(iii) $A - B = \{1,3]$
(iv) $B - C = [2,8]$
(v) $A \oplus C = \{1,2,5,6\}$
(vi) $(A \oplus C) - C = [1,2]$
(vii) $A(C - A \oplus C) = [3,4]$
(viii) $A \oplus C \oplus C = [1,2,3,4]$

Algebra of sets

$x \in (A \cup A)$
$x \in A$ or $x \in A$

(1) Idempotent Law

$x \in (A \cup A) \subseteq (x \in A) = A \cup A \subseteq A$

$$\boxed{\begin{aligned} A \cup A &= A \\ A \cap A &= A \end{aligned}}$$

$x \in A$
$x \in A$ or $x \in A$
$x \in (A \cup A)$
$x \in A \subseteq x \in A \cup A$
$\Rightarrow A \subseteq A \cup A$

(2) Associative Law

$$\boxed{(A \cup B) \cup C = A \cup (B \cup C)}$$

(3) Commutative Law

$$\boxed{\begin{aligned} A \cup B &= B \cup A \\ A \cap B &= B \cap A \end{aligned}}$$

(4) Distributive Law

$$\boxed{\begin{aligned} A \cup (B \cap C) &= (A \cup B) \cap (A \cup C) \\ A \cap (B \cup C) &= (A \cap B) \cup (A \cap C) \end{aligned}}$$

(5) Involution Law

$$\boxed{(A')' = A}$$

$x \in (A')'$
$x \notin (A')$
$x \in A$

$x \in A$
$x \notin A'$
$x \in (A')'$

(6) Demorgans Law

$$\boxed{\begin{aligned} (A \cup B)' &= A' \cap B' \\ (A \cap B)' &= A' \cup B' \end{aligned}}$$

To prove - $A = B$ ① $A \subseteq B$ ② $B \subseteq A$ $\Rightarrow A = B$

Commutative Law

Prove that

$$A \cup B = B \cup A$$

So L.H.S

Let x be any element

$x \in A \cup B =$

$x \in (A \cup B)$

$x \in A$ or $x \in B$

$x \in B$ or $x \in A$

$x \in (A \cup B) = x \in (B \cup A)$ — (i)

or ~~$x \in B$~~ $\cup A$

or $A \cup B \subseteq B \cup A$ — ①

R.H.S

Let x be any element $\in B \cup A$

$x \in (B \cup A)$

$x \in (B \cup A)$ $x \in B$ or $x \in A$

$x \in A$ or $x \in B$

$x \in (B \cup A) = x \in (A \cup B)$ — (ii)

$\Rightarrow B \cup A \subseteq A \cup B$ — ⑪

From equation (i) and (ii)

$x \in A \cup B = B \cup A$

$A \cup B = B \cup A$

Hence proved

Prove Distributive Law

$\cap$ and

$\cup$ or

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Let x be any element $\in A \cup (B \cap C)$

$x \in A$ or $x \in (B \cap C)$

$x \in A$ or ($x \in B$ and $x \in C$)

($x \in A$ or $x \in B$) and ($x \in A$ or $x \in C$)

($x \in A \cup B$) $\cap$ ($x \in A \cup C$)

$x \in (A \cup B) \cap (A \cup C)$ — (i)

Hence.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proved

$x \in \{(A \cup B) \cap (A \cup C)\}$

$x \in (A \cup B)$ and $x \in (A \cup C)$

$\{x \in A$ or $x \in B\}$ and

($x \in A$ or $x \in C$)

$x \in A$ or ($x \in B$ and $x \in C$)

$x \in A \cup (x \in B$ and $x \in C)$

Prove Demorgan's Law

$$(A \cup B)' = A' \cap B'$$

because $x \notin$ to both A & B

Let x be any element $\in (A \cup B)' \Rightarrow x \notin (A \cup B)$

$x \notin A$ or $x \notin B$ $= x \notin A$ and $x \notin B$

$x \in A'$ and $x \in B'$ - $x \in A'$ and $x \in B'$

$x \in A' \cap B'$ $= x \in (A' \cap B')$

$(A \cup B)' = A' \cap B'$

Hence proved

we any to use subset property

if $x = y = ?$

then we have to prove

$x \subseteq y$ & $y \subseteq x$] then $x = y$

$x \in \bar{A} \cap \bar{B} = x \in \bar{A}$ and $x \in \bar{B} \Rightarrow x \notin A$ and $x \notin B$

$x \notin$ to both A & B $\therefore$

$x \notin (A \cup B) \Rightarrow x \in \overline{(A \cup B)}$

It does not belongs to $(A \cup B)$

Cardinal number in a set

The cardinal number of a set is the number of elements in that set and it is denoted by $n(A)$ or $|A|$.

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

$A-(B \cup C) = A.$

(I)

$A = \{a, b, c, d\}$

$B = \{a, e, f\}$

$A \cup B = \{a, b, c, d, e, f\}$

$n(A) = 4$, $n(B) = 3$

$n(A \cap B) = 1$

formula

$4 + 3 - 1$

$= \underline{6}$ Ans

Disjoint set

No element is common in set

$n(A \cup B) = n(A) + n(B)$

\#

$n(A-B) = n(A) - n(A \cap B)$

$n(B-A) = n(B) - n(A \cap B)$

Power set

If S is any set then the family of all the subsets of S is called the power set of S. and It is denoted by $P(S)$.

Callection of subsets $= 2^n$

$(A \cap B)' = A' \cup B'$

$= x \in (A \cap B)' = x \notin (A \cap B)$

$x \notin A$ and $x \notin B$

$x \in A'$ and $x \in B'$

$\Rightarrow \boxed{x \in (A' \cup B')}$

$x \in (A' \cup B')$

$x \in A'$ or $x \in B'$

$x \notin A$ or $x \notin B \rightarrow$ जो नहीं है

$x \notin (A \cap B) \rightarrow$ ∈ संबंधित है इसलिए -section में भी नहीं होगा।

$x \in (A \cap B)'$

Q. $A - B = A \cap B'$

$x(A-B) \Rightarrow x \in A$ & $x \notin B$

$x \in A$ and $x \in B'$

$x \in (A \cap B')$ — (i)

$A \cap B' \Rightarrow x \in (A \cap B')$

$x \in (A \cap B') = x \in A$ and $x \in B'$

$x \in A$ and $x \notin B \Rightarrow x \in (A - B)$ — (ii)

Q) If

$A_1 = \{1, 2, 3\}$

$A_2 = \{3, 4, 5\}$

$A_3 = \{3, 6, 7\}$

$A_4 = \{3, 8, 9\}$

$I = \{2, 3, 4\}$, then find the value of $\bigcup_{i=2}^{4} A_i$

Ans

$A_2 \cup A_3 \cup A_4 = \{3, 4, 5, 6, 7, 8, 9\}$

Q) Let

$A_n = \{i \in Z : i$ is divisible by $n\}$, where $n \in N$

then find.

(i) $A_3 \cap A_7$

$A_3 = \{3, 6, 9, 12, 15, 18, 21, 27, 30, 42 \text{ ---- }\}$

$A_7 = \{7, 14, 21, 28, 35, 42, 49, 56, 63, 70 \text{ ---- }\}$

$A_3 \cap A_7 = \{0, 21, 42, 63, 84 \text{ ---- }\}$

(ii) $A_3 \cup A_7$

$= \{3, 6, 9, 12, 7, 14, 15 \text{ ---- }\}$

Q A computer company must hire 20 programmers to handle system programming jobs. and 30 programmers for application programming. out of these five are expected to perform jobs of both types, how many programmers must be hired

Solution

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = 20 + 30 - 5$

$= 50 - 5 = \underline{45}$ Ans

(2) In a class of 25 students 12 have taken math 8 have taken mathematics but not biology. find the number of students who have taken math and biology and those who have taken biology but not mathematics.

$n(A) = \text{maths} = 12$

$n(A-B) = 8$

$n(A \cup B) = 25$

$n(A \cap B) = n(A) - n(A-B)$

$= 12 - 8 = 4$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$25 = 12 + n(B) - 4$

$n(B) = 25 - 8$

$= \underline{\underline{17}}$ Ans

Ordered pairs

An ordered pair is a pair of objects whose components occurs in a special order. It is written by listing two components in a specific order. In the ordered pair (a, b), a is called the first component and b is second component.

Cartesian Product

If a and b are two sets, then the cartesian product of a and b is denoted (A X B) is defined as $\{(a, b) : a \in A, b \in B\}$

Q

$A = \{a, b\}$

$B = \{2, 3\}$

find $(A \times B)$, $(B \times A)$

Solution

$(A \times B) = \{(a,2)\ (a,3)\ (b,2)\ (b,3)\}$

$(B \times A) = (2,a)\ (2,b)\ (3,a)\ (3,b)$

$(A \times A) = (a,a)\ (a,b),\ (b,a)\ (b,b)$

$(A \times B) \neq (B \times A)$

Distinguist between

ϕ = nullset

$\{\phi\}$ = singultonset that contains one element which is ϕ

$\{0\}$ = one element in a set which is 0

0 = number.

Q7.

$A = \{4, 5, 7, 8, 10\}$

$B = \{4, 5, 9\}$

$C = \{1, 4, 6, 9\}$

then verify $A \cap (B \cup B)$

Solution apply

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cap B = (4, 5)$

$A \cap C = (4)$

$(A \cap B) \cup (A \cap C) = \{4, 5\}$

$(B \cup C) = \{4, 5, 9, 1, 6\}$

$A \cap (B \cap C) = \{4, 5\}$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\{4, 5\} = \{4, 5\}$

Satisfied

Q) $A = \{1,2,3\}$ $B = \{4,5\}$

$C = \{1,2,3,4,5\}$

then prove that $(C \times B) - (A \times B) = (B \times B)$

Answer

$(C \times B) = (1,4)\ (1,5)\ (2,4)\ (2,5)\ (3,4)\ (3,5)$
$(4,4)\ (4,5)\ (5,4)\ (5,5)$

$(A \times B) = (1,4)\ (1,5)\ (2,4)\ (2,5)\ (3,4)\ (3,5)$

$(C \times B) - (A \times B) = (4,4)\ (4,5)\ (5,4)\ (5,5)$

$(B \times B) = (4,4)\ (4,5)\ (5,4)\ (5,5)$

hence

$(C \times B) - (A \times B) = B \times B$

Q) Prove that

$A - B = A \cap B'$

Answer

L.H.S

Let x be any element that belongs to $(A-B)$

$x \in (A-B)$

$x \in A$ and $x \notin B$

$x \in A$ and $x \in B'$

$x \in (A \cap B')$ —(i)

R.H.S

Let x be any element that belongs to $A \cap B'$

$x \in A$ and $x \in B'$

$x \in A$ and $x \notin B$

$x \in (A-B)$ —(ii)

from equation (i) and (ii)

$x \in (A \cap B') = (A-B)$

$A \cap B' = (A-B$

$(A-B) = A \cap B'$

Hence proved

Q) If $A_i = [0, i]$, where $i \in Z$
(i) $A_1 \cup A_2$ (ii) $A_3 \cap A_4$

Answer

$A_1 = [0,1]$
$A_2 = [0,2]$
$A_3 = [0,3]$
$A_4 = [0,4]$

$\longrightarrow A_1 \cup A_2 = [0,2]$
$A_3 \cap A_4 = [0,3]$

Q) $A = \{a, b, c, d\}$

subset $\{\phi\}$ $\{a\}$ $\{b\}$ $\{c\}$ $\{d\}$ $\{a,b\}$ $\{a,c\}$ $\{a,d\}$
$\{b,c\}$, $\{b,d\}$ $\{c,d\}$ $\{a,b,c\}$ $\{b,c,d\}$
$\{a,c,d\}$ $\{a,b,d\}$ $\{a,b,c,d\}$

Relation (R^n on sets)

Let A and B be two sets. a relation from A to B is subset of cartesian product of A and B ($A \times B$). suppose R is a relation from A to B, then R is a set of ordered pair (a, b) where

$\boxed{a \in A \ \& \ b \in B}$

Every such ordered pair is written as aRb and read as a is related to b.
If a is not related to b, then symbol must be $a \not R b$.

2^{mn}

Q) $A = \{1, 2, 5\}$
$B = \{2, 4\}$ If there is a relation of $x < y$ where x is element of set A and Y is element of set B. then find the relations.

$A \times B = \{1,2) (1,4) (2,2) (2,4) (5,2) (5,4)$
$ARB = \{1,2\} (1,4) (2,4)$

Domain & Range

The set $\{a \in A : (a,b)\} \in R$, for some $b \in B$ is called domain of R and it is denoted by Dom(R).
And
The set $\{b \in B : (a,b)\} \in R$, for some $a \in A$ is called range of R and denoted by Ran (R).

Q)
Let $A = \{2, 3, 4\}$, $B = \{3, 4, 5\}$ List the elements of each relation are defined below also find domain and range.
(i) There is a relation ARB, If $A < B$
(ii) Both number should be odd

(i) $A \times B = (2,3) (2,4) (2,5) (3,3) (3,4) (3,5)$
$(4,3) (4,4) (4,5)$
$A < B = (2,3) (2,4) (2,5) (3,4) (3,5) (4,5)$

(ii) – $(3,3) (3,5)$
domain = 3
range = (3, 5)

Q Let $S = \{x, y\}$ and S^2 is the set of all words of length of 2 then find.

(i) elements of S^2

(ii) The relation are on S^2 is defined by vRw means that the first letter in v is the same as the first/2nd letter in w. when v and w are in S^2.

Write R as a set of ordered pair.

Solution

(i) $S^2 = S \times S = \{(x,x)\ (x,y)\ (y,x)\ (y,y)\}$

(ii) first Pairs

$(x,x)\ (x,y)$	$\Rightarrow$	(xx, xy)
$(y,x)\ (y,y)$		(yx, yy)
$(y,y)\ (y,x)$		(yy, yx)
$(x,y)\ (y,x)$		(xy, yx) Ans

Related to 2nd letter

$(xx), (yx)$
(xy, xx)
(xy, yx)
(yx, xy)
(yx, yy)
(yy, xy) Ans

Q If $A = \{x, y, z\}$, $B = \{X, Y, Z\}$
$C = \{x, y\}$, $D = \{Y, Z\}$ are is a relation from A to B defined by $R = (x, X)\ (x, Y)\ (y, Z)$ and S is a relation from C to D defined by $(x, Y), (y, Z)$.

find $R \cup S$, $R \cap S$, $R - S$, R'

If $S = (x, y)$ and S^2 is the set of all the words of length 2 then find

(1) elements of S^2

$\Rightarrow$ $R \cup S \rightarrow \{(x, X)\ (x, y)\ (y, Z)\}$

$\Rightarrow$ $R \cap S \rightarrow \{(x, y)\ ,(y, Z)\}$

$\Rightarrow$ $R - S \rightarrow \{x, X\}$

$\Rightarrow$ $R' \rightarrow$

$A \times B = (x, X)\ (x, y)\ (x, z)\ (y, X)\ (y, y)\ (y, z)$
$(z, X)\ (z, y)\ (z, Z)$

$\Rightarrow R' = (A \times B) - R$
$= (x, Z)\ (y, X)\ (y, y)\ (z, X)\ (z, y)\ (z, z)$

DS

Types of Relation

(i) Inverse Relation

Inverse of R is denoted by R^{-1} is the relation from b to a which consist of those ordered pairs which when reverse belongs to R, that is

$$\boxed{R^{-1} = \{(b, a) : (a, b) \in R\}}$$

for example

$B = \{(x, y) : x \text{ is parent of } y\}$

Ans

$B^{-1} = \{(y, x) : y \text{ is child of } x\}$

i.

(2) $A = \{2, 3, 5\}$ $B = \{6, 8, 10\}$ and define a binary relation R from A to B as follows :-
$(x, y) \in A \times B$, $(x, y) \in R$: x divides y.

Ans

$A \times B = \{ (2,6), (2,8), (2,10)$
$(3,6), (3,8), (3,10)$
$(5,6), (5,8), (5,10) \}$

x divides y $= \{(2,6)\ (2,8)\ (2,10)\ (3,6)\ (5,10)\}$

Identity relation

A relation on a set (A) is denoted by I_A is defined as

$$\boxed{I_A = \{(x, x) : x \in A\}}$$

Properties of relation

(i) Reflexive Relation

A relation R on a set (A) is reflexive if aRa for every belongs $(a \in A)$.
Reflexive relation contains extra relation with Identity relation.

(ii) Irreflexive relation

A relation R on a set (A) is irreflexive if for every $a \in A$, $(a, a) \notin R$.

As $\{1, 2, 3, 4\} \Rightarrow \{ (1,2)\ (2,3)\ (4,3)\ (3,4)\ (4,3$ }
$(1,3)$ }

(iii) Nonreflexive relation

It is neither reflexive nor irreflexive relation. That is If $a \in A$ is true for some a and false for others.

Example

$A = \{a, b, c, d, e\}$

$R = \{(a,a)\ (a,b)\ (c,d)\ (d,e)\ (d,d)\ (b,b)\}$

Symmetric Relation

A relation R on a set A is symmetric if whenever $(a,b) \in R$ then $(b,a) \in R$.

As $R_1 = \{(1,1)\ (1,2)\ (1,3)\ (2,2)\ (2,1)\ (3,1)\}$

is symmetric relation.

$R_2 = \{(x,y) \in R^2 : x^2 + y^2 = 1\}$ $[y^2 + x^2 = 1]$

is symmetric.

Asymmetric relation

A relation R on a set A is asymmetric if whenever $(a,b) \in R$ then $(b,a) \notin R$.

for ~~a~~ $\neq b$

$A = \{1, 2, 3\}$

$A = \{(1,2)\ (1,3)\ (2,3)\}$

asymmetric relation

Antisymmetric relation

A relation R on a set A is antisymmetric if for all $(a,b) \in A$ then there is a relation aRb and bRa then $a = b$.

i.e. if $(x,y) \in R$ then $(y,x) \in R$ only if $x=y$ $\forall x,y \in A$.
eg - $A=\{1,2,3,4\}$ & $R=\{(1,2)(2,2)(2,1)\}$ is not anti symmetric because $(1,2)$ & $(2,1)$ both are present. Now $R=\{(1,1)(2,2)(1,4)\}$ is antisymmetric. because $(4,1)$ is not present

As→ $R=\{1,2,3\}$
$=\{(1,2),(2,3)\ (2,2)\ \ (3,1)\}$

As $R=(x,y) \in R^2 : x \le y$.
⇒ is an Antisymmetric relation when $(x=y)$.
since $x \le y$ and $y \le x$ implies that $x=y$.

Transitive relation

A relation R on a set A if whenever $(a,b) \in R$ and $(b,c) \in R$, than $(a,c) \in R$

Examples Parallel lines.

Q) Give example of each relation (R^n) which is
(i) which is reflexive or Transitive but not symmetric
(ii) Symmetric and Transitive but not reflexive.
(iii) Reflexive and symmetric but not transitive

Solution $A=\{1,2,3,4\}$
$=\{$ ~~(1,1,1) (2,2,2) (3,3,3) (4,4,4)~~ $\}$

Ans (i) $\{(1,1)\ (2,2)\ (3,3)\ (4,4)\ \ (1,3)\ (2,3)\ (1,2)\ \ (4,3)\}$

(ii) $\{(1,1)\ (1,2)\ (1,3)\ (2,3)\ (2,1)\ (3,1)\ (3,3),\ (3,2)\ \}$

(iii)

Equivalance relation

follows (i) Reflexivity (aRa)
(ii) Symmetric (aRb then bRa)
(iii) Transitive (aRb & bRc) ⇒ (aRc)

1. Consider the following relation on
$A = \{1, 2, 3, 4, 5, 6\}$
$R = \{(i, j) : |i - j| = 2\}$ is reflexive
Transitive and symmetric or equivalance
relation or not.

Solution $R = \{(1,3)\ (3,1)\ (2,4)\ (4,2)\ (5,3)\ (3,5)\ (6,4)\ (4,6)\}$
There is no reflexive relation.
so It is not equivalance relation.

Q) R is a relation in the set of integer Z
$R = \{(x, y) : x \in Z \ \& \ y \in Z,\ (x-y)$ is divisible by 6 $\}$.
Prove It is a equivalance relation.

Solution (i) operation is $(x-y)$
aRa.
$x - x = 0$, so 0 is divisible by 6
so It is a reflexive relation.

(ii) aRb then bRa
If $x - y$ is divisible by 6
then $-(y - x)$ is also divisible by 6
hence It is a symmetric relation.

$6\overline{)0}$

(iii) If $x - y$ is divisible by 6 — (I)
and $y - z$ is also divisible by 6 — (II)
So, Add Equ (I) & (II)
$(x - z)$ should also be divisible by 6

Hence It is a transitive relation.

Now we can say this relation is an equivalance relation.

Q A is set of non zero integers and let R be the relation on $A \times A$

$(a,b)\ R\ (c,d) \Leftrightarrow ad = bc$

Prove equivalance relation.

Solution

(i) Reflexive

$(a,b)\ R\ (a,b) \Rightarrow \quad ab = ba$

$ab = ab$

(ii) symmetric

$(c,d)\ R\ (a,b) = \quad cb = da$

$bc = ad$

$ad = bc$

(iii) Transitive

$(a,b)\ R\ (c,d) \Leftrightarrow ad = bc$ — (i)

$(c,d)\ R\ (e,f) \Leftrightarrow cf = de$ — (ii)

$(a,b)\ R\ (e,f) \Leftrightarrow af = be$ — (iii)

multiply (i) and (ii)

$a\cancel{d}\cancel{c}f = b\cancel{c}\cancel{d}e$

$af = be$

Proved → So Transitive relation.

Q) Let R be a relation in the set of integer (Z) defined by

$$R = \{(x, y) : x, y \in Z \text{ and } x - y \text{ is multiple of } 5\}$$

prove that this relation is an equivalance relation.

Solution: (i) Reflexive

(x, y)

$a \in a$.

$(a - a)$ is ~~divisible~~ multiple by 5

(ii) Symmetric

$\Rightarrow$ $a - b$ is ~~divisible~~ multiple by 5

$\Rightarrow$ ~~(b - a) is divisib~~ multiple $-(b - a)$ is also ~~divisible~~ by 5

(iii) Transitive

$(a - b)$ is multiple of 5

$(b - c)$ is multiple of 5

the

$(a - c)$ is also should be multiple of 5.

Proved. Hence set is an equivalance relation

Q) Show that

$$(x, y) R (a, b) \iff x^2 + y^2 = a^2 + b^2$$

is an equivalance relation.

(i) Reflexive

a R a

$$(x, y) R (x, y) \iff x^2 + y^2 = x^2 + y^2$$

(ii) Symmetric

$\cancel{(x,y) \cdot R\ (a,b)}$

$aRb = bRa$

$(a,b)\ R\ (x,y)$

$a^2+b^2 = x^2+y^2$ — (i)

(iii) Transitive

aRb & bRc

$\Rightarrow aRc$

$(a,b)\ R\ (e,f)$

$a^2+b^2 = e^2+f^2$

$x^2+y^2 = a^2+b^2$ — (ii)

from equation (i) and (iii)

$x^2+y^2 = e^2+f^2$

hence,

It is proved that this relation is an equivalance relation.

Q→ Consider the relation R from $(x \text{ to } Y)$ (x,Y) where $x = \{1,2,3\}$, $Y = \{7,8\}$ $R = \{(1,7),(2,7),(1,8),(3,8)\}$

find

(i) $R^{-1} = \{(7,1)\ (7,2)\ (8,1)\ (8,3)\}$

(ii) Complement of R (R')

$(x \times Y) = \{(1,7)\ (1,8)\ (2,7)\ (2,8)\ (3,7)\ (3,8)\}$

(ii) $R = \{(1,7)\ (2,7)\ (1,8)\ (3,8)\}$

$R' = (x \times Y) - R$

$= \{(2,8)\ (3,7)\}$

Ans

Let $\sim$ be the R^n on $A \times A$ defined as $(a,b) \sim (c,d)$ if $a+d = b+c$
Prove that $\sim$ is an equivalence R^n.

Q2 Define P on the set $R \times R$ of ordered pairs of real numbers as follows -
for all (a,b) P, $(c,d) \in R \times R$

$$(a,b) \; P \; (c,d) \Leftrightarrow a = c$$

then prove that it is an equivalance relation.

Solution

(i) **Reflexive** : $(a,b) \sim (a,b) = a+b = b+a = a+b = a+b$

(ii) Symmetric. $(a,b) \sim (c,d) \Rightarrow (c,d) \sim (a,b)$

$(a+d) = (b+c) = (c+b) = (d+a)$ — so reflexive

Transitive

$(a,b) \sim (c,d) \therefore a+d = b+c$ — (i)

$(c,d) \sim (e,f) = c+f = d+e$ — (ii)

Add equn (i) & (ii) $(a,b) \sim (e,f) = a+d+c+f = b+c+d+e$

$\boxed{a+f = b+e}$ proof.

If R is an equivalence R^n on A, then $P - T$ R^{-1} is also Equivalent R^n on A

Ans (1) . Let $x \in A$. Since R is a reflexive R^n $(x,x) \in R$

So $(x,x) \in R^{-1}$

So R is reflexive

(2) Symmetric R^n. Let $x,y \in R \Rightarrow (x,y) \in R \Rightarrow (y,x) \in R$

$\Rightarrow (y,x) \in R^{-1}$ and $(x,y) \in R^{-1}$

So $(y,x) \in R^{-1} \Rightarrow (x,y) \in R^{-1}$ $\therefore$ R^{-1} is symmetric

(3) Transitive $(x,y) \in R$ & $(y,z) \in R \Rightarrow (x,z) \in R$

$(y,x) \in R^{-1}$ & $(z,y) \in R^{-1} \Rightarrow$ ~~$(z,x) \in R^{-1}$~~ or $(z,y) \in R^{-1}$ & $(y,x) \in R^{-1}$

$= (z,y) \in R^{-1}$ & $(y,x) \in R^{-1} = (z,x) \in R^{-1}$

$\therefore$ R^{-1} is transitive

$\therefore$ R^{-1} is an equivalence R^n

Let R be a Relation defined as $R = \{(a,b) \in R^2 : (a-b) \le 3\}$ determine whether R is Reflexive, symmetric, antisymmetric and Transitive.

Let $R = (a,b) \in R^2 : (a-b) \le 3$

① Reflexive = $a - a = 0$ & $0 \le 3$

② Symmetric = $a - b \le 3$ then $(b-a) \ge 3$ so it is not symmetric

③ Anti Symm → $a - b \le 3 \Rightarrow b - a \ge 3$
It will possible only if $a = b$ so R is antisymmetric

④ Transitive → $a - b \le 3$ & $b - c \le 3$
$\Rightarrow a - c \le 6$.
So not transitive

Representation of relation

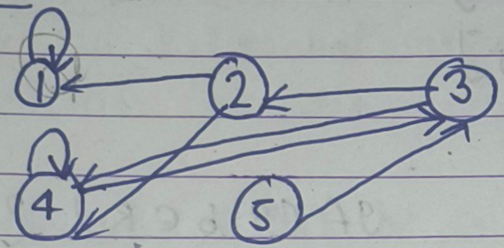
Node / vertex

Edge / Path

(i) $A = \{1, 2, 3, 4, 5\}$

$R_1 = \{(1,1)\ (2,1)\ (3,2)\ (4,3)\ (3,4)\ (4,4)\ (5,3)\ (2,4)\}$

Representation by graph



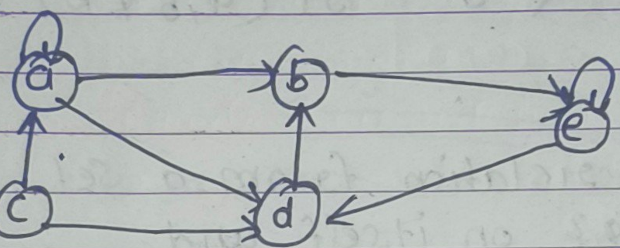
$a = 5$

$b = 6$

$5 - 6 = -1 \le 3$

$6 - 5 = 1 \le 3$

(ii)



$R_2 = \{(a,a)\ (a,b)\ (b,e)\ (e,d)\ (d,b)\ (c,d)\ (a,d)$
$(e,e)\ (c,a)\}$ Ans

for reflexive r^n There should be loop on every vertex/node.

A relation is symmetric if and only if for every edge between distinct vertices in the digraph (directed graph). There is an edge in the opposite direction so that (b,a) is in the relation whenever (a,b) exist.

A relation is Transitive if relation is as related that $(a,b)\ R\ (b,c)$ then (a,c) is also related.

Representation of relation by matrix

Let $A = \{a_1, a_2 \text{---} a_n\}$

$B = \{b_1, b_2 \text{---} b_n\}$

are finite sets containing m and n elements and let R be a relation from A to B, then R can be represented by mn matrix

$M_R = [m_{ij}]$

where

$$m_{ij} = \begin{cases} 1 & \text{if } (a, b \in R) \\ 0 & \text{if } (a, b \notin R) \end{cases}$$

Q. Let R be the relation from a set $A = \{1, 3, 4\}$ on itself and $R = \{(1,1)\ (1,3)\ (3,3)\ (4,4)\}$, then find the matrix of relation.

Solution

$$\begin{array}{c|ccc} & 1 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 \\ 3 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{array}$$

Q. Let $A = \{a_1, a_2, a_3\}$, $B = (b_1, b_2, b_3, b_4\}$ their relation is represented by the matrix find domain

$$M_R = \begin{array}{c|cccc} & b_1 & b_2 & b_3 & b_4 \\ \hline a_1 & 0 & 1 & 0 & 0 \\ a_2 & 1 & 0 & 1 & 1 \\ a_3 & 1 & 0 & 1 & 0 \end{array}$$

$R = \{(a_1 b_2)\ (a_2 b_1)\ (a_3 b_1)\ (a_2 b_3)\ (a_2 b_4)\ (a_3 b_3)\}$

domain (R) = $\{a_1, a_2, a_3\}$
Range (R) = $\{b_1, b_2, b_3, b_4\}$

Ans

Reflexive = If all the elements in the main diagonal of matrix is (1) then the relation is reflexive.

Symmetric = If the representative matrix of a relation is symmetric with the main diagonal that is $m_{ij} = m_{ji}$ for all the values of i and j.

or we can say that matrix $\boxed{(m_R) = m_R^T}$

Transitive → If and only if it has property if $m_{ij} = 1$
$m_{jk} = 1$
then
$m_{ik} = 1$

Q Let R be the matrix represented by

$$m_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

then show the relation in graphical form and also find R^{-1} and R^c.

$(I \times J) \Rightarrow \{(1,a) (1,b) (1,c) (2,a) (2,b) (2,c) (3,a) (3,b) (3,c)\}$

$R = \{(1,b) (1,c) (2,a) (2,c) (3,a) (3,b)\}$

(i) $\Rightarrow R^{-1} = \{(b,1) (c,1) (a,2) (c,2) (a,3) (b,3)\}$

(ii) $\Rightarrow R^c = \{(1,a) (2,b) (3,c)\}$

Composition of relation

Let A, B and C be the sets and R be a relation from A to B and S be a relation from B to C : $R \subseteq A \times B$ and $S \subseteq B \times C$ then the composition of R and S is denoted by (ROS).

It is the relation consisting of ordered pairs (a,c)

where $\boxed{\begin{matrix} a \in A \\ c \in C \end{matrix}}$ and for which there exist an element $b \in B$: $(a,b) \in R$ and $(b,c) \in S$.

$\boxed{ROS = (a,c)}$

Q $A = \{1, 2, 3\}$
$B = \{p, q, r\}$
$C = \{x, y, z\}$

S

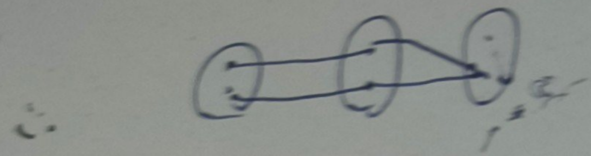
Let $R = \{(1,p)\ (1,r).\ (2,q)\ (3,q)$
$S = \{(p,y);\ (q,x)\ (r,z)\}$ compute ROS.

Solution $ROS = \{(1,y)\ (1,z), (2,x), (3,x)\}$

Q Let $R = \{(1,1)\ (2,1)\ (3,2)$ then compute R^2

$ROR = \{(1,1)\ (2,1)\ (3,2)\}$
$R = \{(1,1)\ (2,1)\ (3,2)$

$ROR = \{(1,1)\ (2,1)\ (3,1)\}$



Q Closure of relation (R^n)

Let R be a relation on set A. R may or may not have some property P such as reflexive, symmetric and transitive. If there is a relation S with property P containing R : S is ⊆ of every relation with P containing R then S is called closure of R with respect to P.

Q) $S = \{1, 2, 3, 4\}$
$R = \{(1,2) (2,1) (1,1) (2,2)\}$
find the reflexive closure of R.

Reflexive $R \cup I_A = \{(1,2) (2,1) (1,1) (2,2) (3,3) (4,4)\}$ **Ans**

Symmetric closure

$R \cup R^{-1} = \{(1,2) (2,1) (1,1) (2,2)\}$

Transitive closure

Transitive closure of R is denoted by R^+. Let a relation R is defined on A and A contains m elements so one never needs more than ~~one~~ m steps. To make a relation R transitive, we have to add all pairs of R^2, R^3 ——— R^m

$R^+ = R^2, R^3$ ---- R^m
$R^+ = R \cup R^2 \cup R^3$ --- $\cup R^m$.

where

$A = \{1, 2 \text{---} m\}$

Q. $R = \{(1,2)\ (2,3)\ (3,1)\}$
$A = \{1,2,3\}$

find reflexive, symmetric and transitive closure of R, using
(i) Composition of relation(R)
(ii) Composition of matrix (R^n)
(iii) Graphical representation of R

Solution

(i) Reflexive

$R \cup I_A = \{(1,2)\ (2,3)\ (3,1)\ (1,1)\ (2,2)\ (3,3)\}$

Symmetric

$R \cup R^{-1} = \{(1,2)\ (2,3)\ (2,1)\ (3,2)\ (3,1)\ (1,3)\}$

Transitive

$R \cup R^2 \cup R^3 =$

$R^2 = R \circ R = \{(1,2)\ (2,3)\ (3,1)\} \circ \{(1,2), (2,3)\ (3,1)\}$
$= \{(1,3)\ (2,1)\ (3,2)\}$

$R^3 = R^2 \circ R$
$= \{(1,3)\ (2,1)\ (3,2)\} \circ \{(1,2)\ (2,3)\ (3,1)\}$

$R^3 \Rightarrow \{(1,1)\ (2,2)\ (3,3)\}$

$R^4 = R^3 \circ R$
$= \{(1,1)\ (2,2)\ (3,3)\} \circ \{(1,2)\ (2,3)\ (3,1)\}$
$= \{(1,2)\ (2,3)\ (3,1)\} = R$

Transitive

$R \cup R^2 \cup R^3$
$= \{(1,2)\ (2,3)\ (3,1)\ (1,3)\ (2,1)\ (3,2)\ (1,1)\ (2,2)\ (3,3)\}$

(ii) <u>Composition of matrix (R^n)</u>

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 \end{array}$$

<u>Reflexive</u>

$$\forall I_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 1 \\ 3 & 1 & 0 & 1 \end{array}$$ Ans

<u>For symmetric</u>

$$m \vee m^T = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \end{array}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

OR operation ✓

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$ Ans

for transitive

$\vee$ = OR operation

$$m_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$0.0 + 1.0 + 0.1$

$0 + 0 + 0$

$= 0$

$$m_R^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Binary operation is performed

$$m_R^3 = m_R^2 \cdot m_R$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

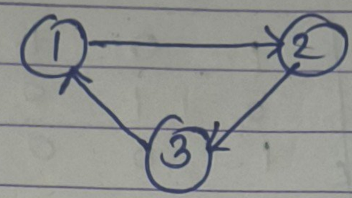
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$m_R \vee m_R^2 \vee m_R^3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

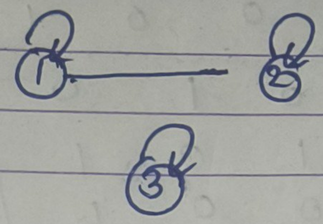
$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(iii) Graphical representation

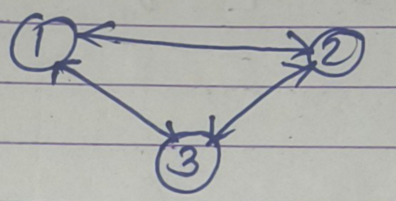
$R^h =$



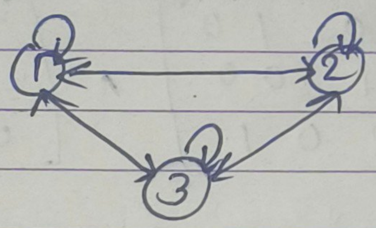
Reflexive



Symmetric



Transitive



Ans

Q) Let A, $A = \{1, 2, 3\}$ $B = \{b, c, d\}$, let R & S be the relation from A to B

$$m_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \qquad m_S = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

(i) find R^{-1} and S^{-1}

(ii) $(R \cap S) \circ R^{-1}$

(iii) $R \circ R^{-1} \cap S \circ R^{-1}$

(i) $R^{-1} \& S^{-1}$

$$m_R = m_R^T = R^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$m_S = m_S^T = S^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(ii) $(R \cap S) \circ R^{-1}$

$$(R \cap S) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \wedge \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Q. $R = (1,2)$

$$R_1 = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 1 & 1 \\ 2 & 1 & 0 \end{array} \quad , \quad R_2 = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 1 & 1 \\ 2 & 0 & 1 \end{array}$$

P.T $= R_1 \circ R_2 \neq R_2 \circ R_1$

$R_1 = \{ (1,1) (1,2) (2,1) \}$
$R_2 = \{ ((1,1) (1,2) (2,2) \}$

$R_1 \circ R_2 = \{ (1,1) (1,2) (2,1) (2,2) \}$

$R_2 \circ R_1 = \{ (1,1) (1,2) (2,1) \}$

$\underline{\underline{(R_1 \circ R_2) \neq (R_2 \circ R_1)}}$ PS

Set of natural no-s with divisibility as a R^n forms a Poset.

Ex: $a \leq b \iff a/b$

$P_1 = a \leq a$ ($\because a/a = 1$)

P_2: if $a \leq b$ & $b \leq a \Rightarrow a/b$ & $b/a \Rightarrow a = b$

P_3 = if $a \leq b$ & $b \leq c \Rightarrow a/b$ & $b/c \Rightarrow a/c$

non-empty set

Partial order set (Poset)

(P, R) — R is a binary operator

A Relation R on a set 'P' is called partial ordering if it is reflexive, antisymmetric & transitive.

It is Relation
$a \leq b$

(1) Reflexive

$a R a$ for all $a \in S$

Antisymmetric

$a R b$ & $b R a$

$a = b$

Transitive

$a R b$ & $b R c$ $\forall a, b, c \in P$

$a R c$

Q Show that the relation $\geq$ is a partial ordering on the set of integers 'Z'

$a \in z$

reflexive

$a R a$

$a \geq a$ Hence it follows the property of reflexive.

Antisymmetric

$a \geq b$ & $b \geq a$

$a \geq b$, Hence it follows the property of Antisymmetric.

Hence this is a poset.

Q Consider P(S) as the powerset than show that the relation $\subseteq$ is a partial ordering on the power set P(S).

Reflexive $ARA \Rightarrow A \subseteq A$ it follows the property of reflexive.

if $P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

$\Rightarrow (P(A), \leq)$. where $A \leq B$ if $A \subseteq B$

$S = \{\phi, \{a\}, \{b\}, \{a,b\}\}$ then $\{S, \subseteq\}$ is not Total order set (TOS)
Compare ϕ & a $\Rightarrow$ a is $\subseteq$ of $\{a\}$
but b is not $\subseteq$ of $\{b\}$ so it is not TOS.

Antisymmetric - $A \subseteq B$ & $B \subseteq A \Rightarrow A \subseteq B$ it followes the antisymmetric.

Transitive - $A \subseteq B$ & $B \subseteq C \Rightarrow A \subseteq C$ it followes transitive property.

Hence this is a poset.

Q Show that the set z^+ of all +ve integers under divisibility relation forms a poset.

Reflexive $\Rightarrow$ aRa a divisible by a it follows reflexive relation.

Antisymmetric $\Rightarrow$ $a \% b$ & $b \% a \Rightarrow a = b$, follows antisymmetric property.

Transitive $\Rightarrow$ a/b & $b/c \Rightarrow a/c$ follows a transitive property.

Hence this is a poset.

S is poset

notation of partial order set

Comparibility

show rian

The elements a R b of a poset $(S, \preceq)$ are ~~there~~ called comparable if $a \preceq b$ or $b \preceq a$ where a and b are elements of 'S' ~~such that~~ if neither $a \preceq b$ nor $b \preceq a$ means a and b are not related, so a and b are called "Incomparable".

Example In the poset $(z^+, /)$ the integers 4 and 2 are comparable but 3 and 5 are incomparable.

If $(S, \preceq)$ is a poset and every two element of S are comparable then S is called

Totally ordered set :- A poset {A, R) is called a totally order set if every pair of elements in A are comparable

eg ① A is any set of the int [A; /] is not Toset but It is poset

totally ordered or linearly ordered set. A totally ordered set is also called a chain.

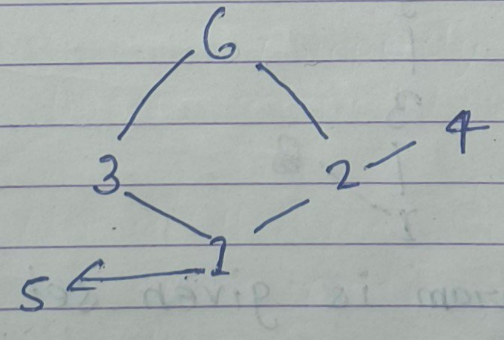
Hasse Diagram :-

A partial order $\preceq$ on a set X can be show by means of a diagram known as Hasse diagram of $(x, \preceq)$.

This gives a method of representing finite poset which worth well for posets with relativly few elements.

We show the elements of x by points and if y is an intermidiate successor of x, we take y at A higher level then x and Join x and y by a straight line A diagram form as above is known as Hasse diagram. Thus there will be No horizontal line in diagram of a poset.

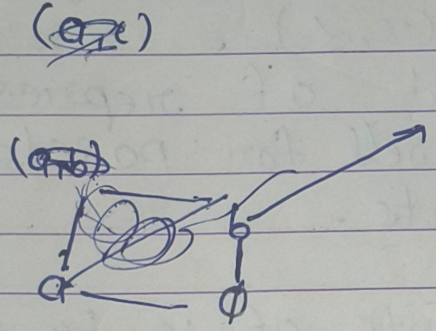
Ex Let $x = \{1, 2, 4, 5, 6\}$, then / is a partial order relation on x draw the Hasse diagram (X, /)

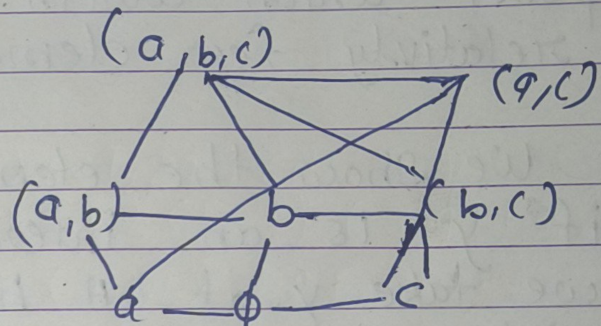


this called Hasse diagram.

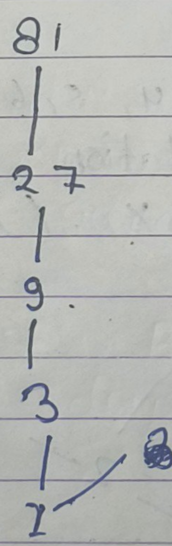
Q. Draw the Hasse diagram for the partial ordering $\{(A, B) \mid A \subseteq B\}$ on the powerset $P(S)$ where $S = \{a, b, c\}$

$P(S) = \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}$
$\{c, a\}$ $\{a, b, c\}$





Q. Let set $A = \{1, 3, 9, 27, 81\}$. draw the Hasse diagram of the poset $(A, /)$.



<u>Note</u> ⇒ Hasse diagram is given set is not unique always s.

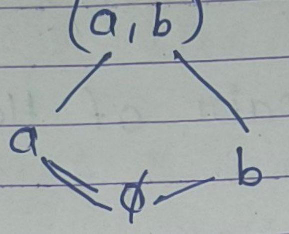
(1) $(A, \leq)$ where $A = \{3, 4, 12, 24, 48, 72\}$ & the R^n $\leq$ be such that $a \leq b$ if a divides b

(2) $B = \{2, 3, 4, 6, 12, 36, 48, 1\}$

(3) Let A be the set of factors of a particular +ve int m & let $\leq$ be the R^n "divides" ie $\leq = \{(x,y): x \in A, y \in A$ and $x/y\}$ draw Hasse dig for (1) $m = 12$, (2) $m = 30$ (3) $m = 45$

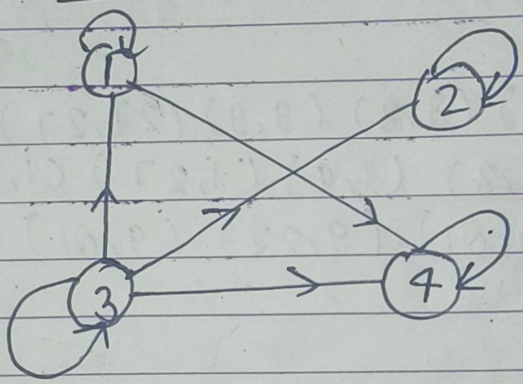
Q $S = \{a, b\}$ Create Hasse diagram.

(4) $(P_S, /)$ Power set $= \{\phi, \{a\} \{b\} \{a, b\}\}$



By directed graph :-

Constructing Hasse diagram from directed graph



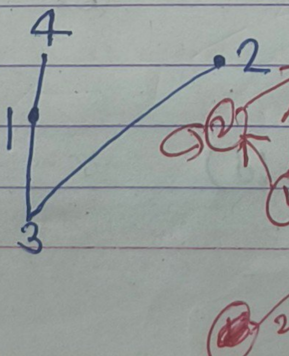
(1, 1)

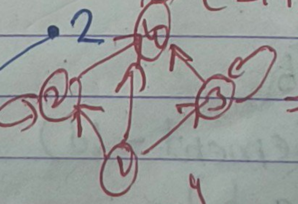
Some steps : making Hasse diagram

- Remove the loop at all vertexes.
- Remove all edge whose existance is applied by the transitive property.
- Arange all arrow pointing upword forward there terminal vertex.
- Remove all arrow

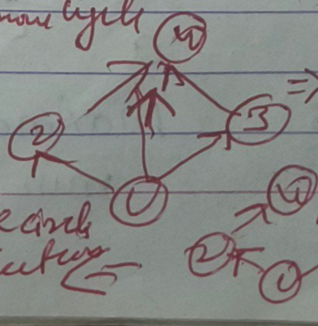
Q. Determine the Hasse dig of R^n R

$A = \{1, 2, 3, 4\}$ & $R = \{(1,1)(1,2)(2,2)(2,4)(1,3)(3,3)(3,4)(1,4)(4,4)\}$

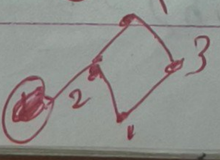




Remove cycle



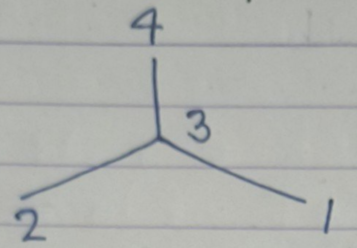
Remove transitive edge



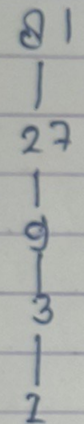
Replace circle & directions

Q) Find the relation by Hasse diagram in this not show reflexive but always ret. R with
(1,1) (2,2) (3,3) (4,4) (3,1) (1,4) (3,2) (3,4)

describe the ordered pair of Hasse diagram



(1,1) (2,2) (3,3) (4,4)
(1,3) (1,4) (3,4) (2,3)
(2,4)



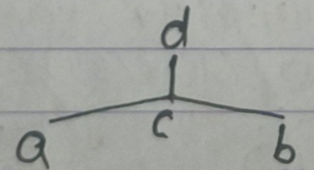
(1,1) (3,3) (9,9) (27,27) (81,81)
(1,3) (1,9) (1,27) (1,81) (3,9) (3,27)
(3,81) (9,27) (9,81) (27,81).

Special element in poset :-

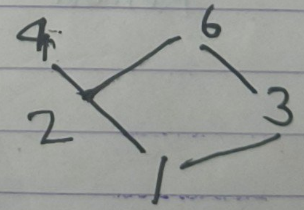
(a) Maximal element :- Top element of Hasse diagram is known as maximal element. It have no successor.

(b) minimal element :- Bottom element of Hasse diagram.

Ex (a)

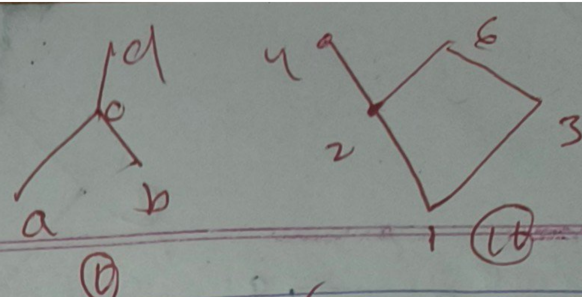


(b)



Ans maximal element = (d)
min = (a,b)

max = 4, 6
min = 1



(c) Greatest element

It is greater than every other element of the poset. It is unique if is exist.

d is greatest element — (i)
No greatest Element exist — (ii)

(d) Least element

It is less than any other element of the poset. & it is unique if it exist.

No least element in (i) fig
1 is least element in (ii) fig

Ex

{c} Ans from above diagram.

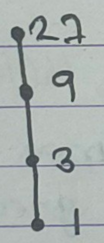
Greatest element = d

Least element = not exist b/c doesn't unique.

(d) Ans

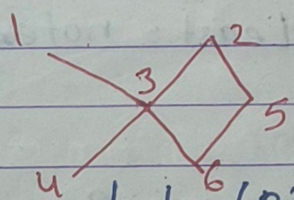
greatest element = does'nt exist b/o not unique.

least element = 1



min = 1 Least = 1

max = 27 Greatest = 27



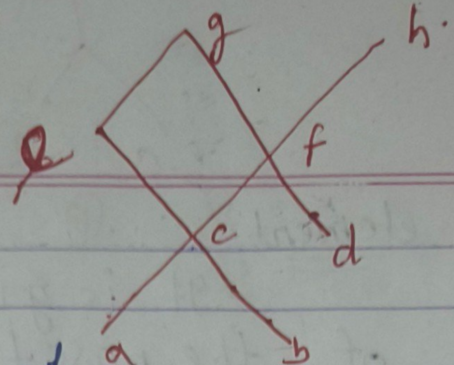
⟶ Max → 1, 2
min → 4, 6
Greatest — NO
Least — NO

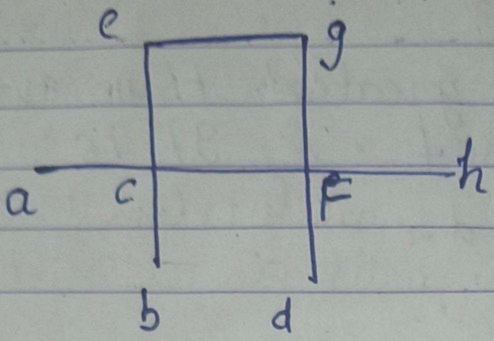
Upper bound

Let (B) be a subset of (A) and element $x \in A$ is in upperbound B if (x, y) ~~y~~

$(x, y) \in$ $y \in B$:

- common in both.
- and it's becomes for it's UB.





Find UB of {e, c}

(i) (e, c)

upper bound of c = {c, e, f, g, h}

upper bound of e = {e, g}

common in both

(e, c) = {e, g) Ans

Lower bound of c = {c, a, b}

e = {e c, a, b} = L-b of (e,c) = {a, b, c)

(ii) {f, d, c}

{f} = {f, g, h}

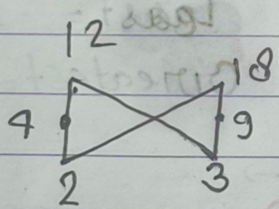
{d} = {d, f, g, h}

{c} = {c, e, f, g, h}

UB {f d c} = {f, g, h}

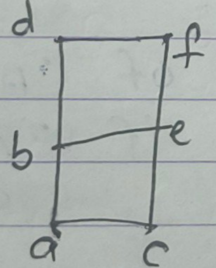
LB (f) = (f c, d, a, b) (d) = ( ) d

Lb (c) = (c, a, b) ⇒ Lb (f d c) = ϕ

① 

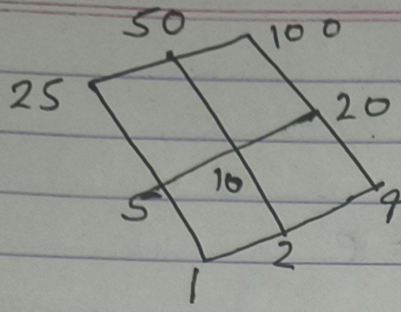
min = 2, 3 max = 12, 18

Least = not exist greatest = not exist

② 

max = Gr = f

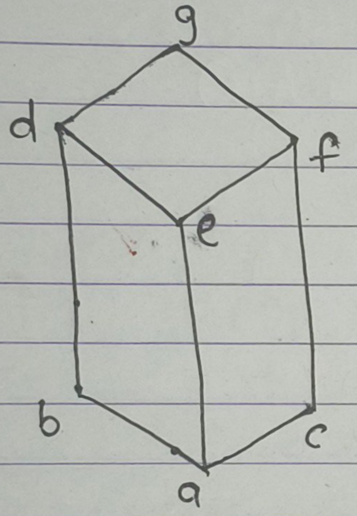
Least = min = g



find $\{5, 10\}$ UB and LB

UB = 10, 20, 50, 100
LB = 1, 5

(ii) $\{2, 4, 5, 10\}$ find LB, UB
UB = 20, 100
LB = 1

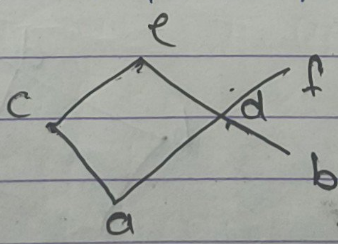


find B = (d, g) UB and LB
d = UB = d, g
g = UB = g
common UB = g Ans

d = LB = $\{d, e, f, c, b, a\}$
g = LB = $\{g, d, e, f, c, b, a\}$
low B $\{d, g\}$ = $\{d, e, f, c, b, a\}$ Ans

Least upperbound:- (supermum or / Join / $\vee$)

minimum element in upper bound



(1) $UB(B) = e$

$LB(B) = a$

Least upper bound $LUB(B) = e$

greatest upper bound $g \wedge B = a$

greatest Lower bound or ~~minimum~~ Infimum (or meet / $\wedge$):-

greatest or maximum element in lower bound.

(2) $B = \{a, b\}$

$UB(B) = a$

$LB(B) = \phi$

$LUB(B) = \{d\}$

$GLB(B) = \phi$

(3) $B = \{e, f\}$

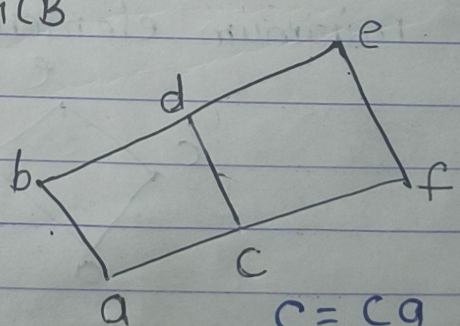
$UB(B) = \phi$

$LB(B) = d, a, b$

$LUB(B) = \phi$

$GLB(B) = d$

$B = \{a, c, f)$ find $UB(B)$ $LB(B)$, $LUB(B)$ and GLB



$UB(B) = (e, f)$

$LB(B) = \{c, a\}$

$LUB(B) = f$

$GLB(B) = a$

$C = Ca$

$a = a$

$F = a$

(18) $B = d, c$

$UB(B) = d, e$

$LB = c, a$

$LUB(B) = d$

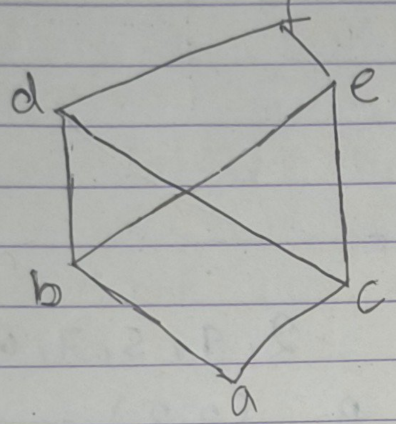
$GLB(B) = \{c\}$

$d = d, e$

$c = d, e, f$

$d = b, a, c$

$c = c, a$



$B = \{d, e\}$ f

$UB(B) =$ f

$LB(B) =$ b, a, c

LUB E

$GLB = \{\phi\}$ does not exist

b and c does not relate each other

(b, c)

$UB(B) =$ d, f, e

$LB(B) =$ a

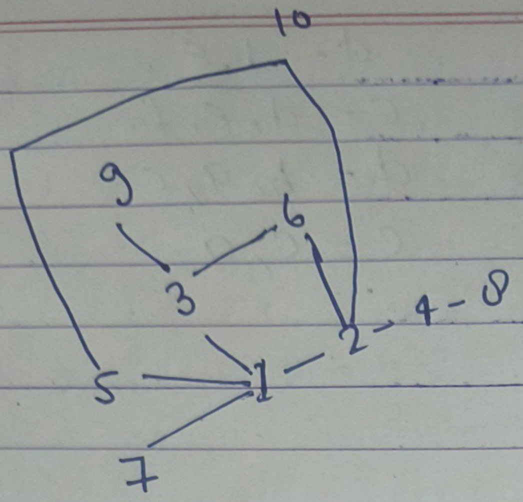
$LUB(B) =$ ϕ

$GLB(B) =$ a

Q. In the poset $A = \{1, 2, 3 \ldots 10\}$, / the subset $(2, 7)$ has UB LB GLB ULB exist or not.

$2 = 2, 4, 6, 8, 10$

$7 = 7$



2 = 2, 4, 8, 6, 10

7 = 7

UB = ϕ

LB = 1

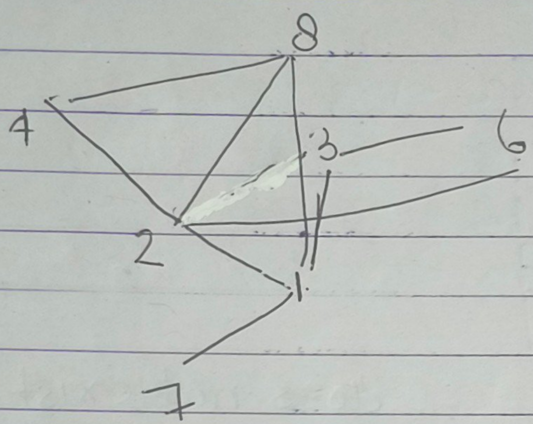
GLB = 1

LUB = ϕ

(1, 2, 3)

GLB = 1

LUB = 6



2, 4, 5, 7, 8

B = {2, 3}

= 3, 4, 5, 7, 8

UB = 4, 5, 7, 8

LB = ϕ

GLB = ϕ

LUB = does not exist.

4, 7, 8

B = {4, 6}

6, 8

UB = ϕ

LB =

LUB = 8

GLB =

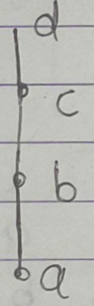
Lattice

A poset is called lattice if every 2 element subset of P or poset as both as both a least upperbound and a greatest upperbound. that is if LUB of (x, y) & GLB of (x, y) exist for every x & y in p in this case we denote LUB of (x), y $(x \vee y)$ and it read as join.

- GLB (x, y). meet of x and y.
- For lattice should be exist GLB and LUB.

(i) Denote (LUB) = $x \vee y = x + y$ join
(ii) Denote GLB) = $x \wedge y = x * y$ meet

Example



This is lattice or not.

A straight line such diagram is always lattice.

ULB Join

u.b(a) = (a, b, c, d)
u.b(c) = {c, d}
u.b of (a, c) = (c, d)
lub (a, c) = c

V	a	b	c	d
a	a	b	c	d
b	b	b	c	d
c	c	c	c	d
d	d	d	d	d

GLB join

∧	a	b	c	d
a	a	a	a	a
b	a	b	b	b
c	c	b	c	c
d	a	b	c	d

LUB join

∨	a	b	c	d	e
a	a	b	c	d	e
b	b	b	c	d	e
c	c	c	c	d	e
d	d	d	d	d	e
e	e	e	e	e	e

GLB

∧	a	b	c	d	e
a	a	a	a	a	a
b	c	b	b	b	b
c	d	e	c	c	c
d	a	b	c	d	d
e	a	b	c	d	e

#: LUB of a,bcdef doesnot exist lattice..

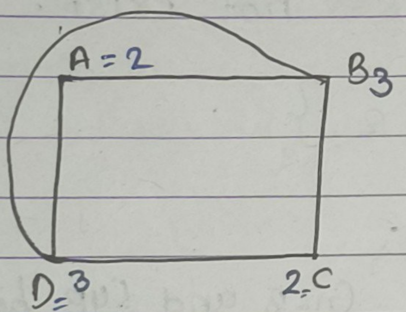
Isomorphism

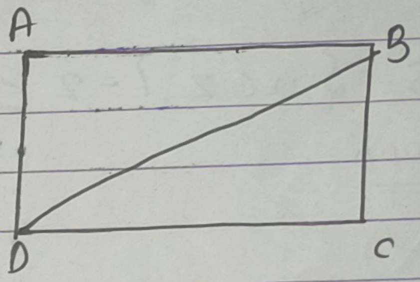
It is a concept that describes when two graphs are equivalent or can be made to look exactly the same by relabelling their vertices.

Any two graphs are isomorphism if they satisfy the following four conditions - (i) Their will be an equal number of vertices in the given graph.

(ii) There will be equal number of edges in given graph.

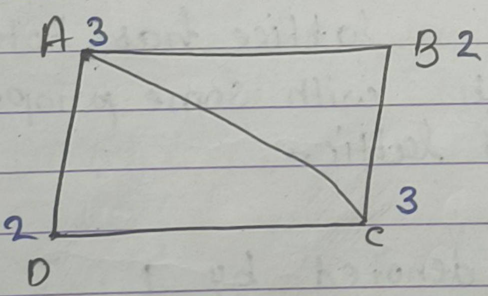
(iii) There will be equal amount of degree of sequence in the given graph.

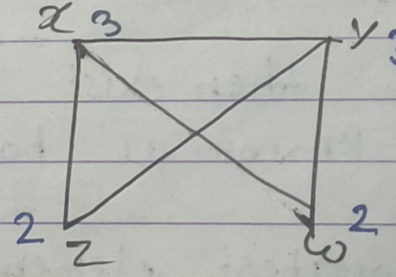




(iv) Maping should be exist between both graphs.

Prove isomorphism





(i) number of vertices $= 4 = 4$

(ii) number of edges $= 5 = 5$

(iii) degree $a = 3,\ b = 2,\ c = 3,\ D = 2$

$x = 3 \quad y = 3,\ z = 2,\ w = 2$

(iv) mapping

$a \to x$
$b \to w$
$c \to y$
$D \to z$

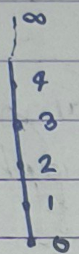
Hence proved

Types of Lattice

(1) Complete Lattice

A Lattice P is called complete if for every subset of P there should be GLB and LUB.

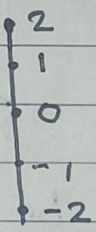
Q. $P = \{ x \in Z \mid x > 0 \}$ $(Z, \leq)$



This is not complete lattice because LUB does not exist.

Q. $S = \{ x \in Z \mid -2 \leq x \leq 2 \}$

graph



GLB and LUB both exist that's why it is a complete lattices

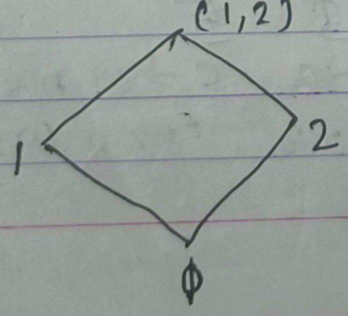
(2) Bounded Lattice

If a lattice has both GLB and LUB ~~then~~ exist then with some properties is known as bounded lattice

properties

(i) Greatest element is denoted by 1.

(ii) Least element is denoted by 0.



least $\phi \rightarrow 0$

greatest $(1,2) \rightarrow 1$

∧ ∨ join

last

(3) Isomorphic Lattice

Let L_1 and L_2 be two lattice and these lattices are isomorphic if there exist (one one onto) bijection from L_1 to L_2.

Properties one to one & onto

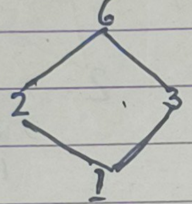
glB

$$\boxed{\begin{aligned} F(a \wedge b) &= f(a) \wedge f(b) \\ f(a \vee b) &= f(a) \vee f(b) \end{aligned}}$$

LUB

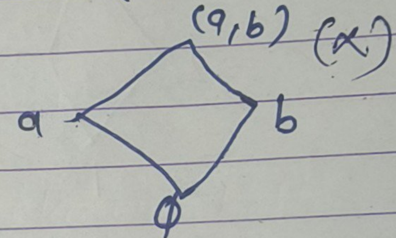
Q) Given $L = \{1, 2, 3, 6\}$ and $A = \{a, b\}$ then P.T the lattice $(L, /)$ & $(P(A), \leq)$ are isomorphic.

Hassy diagram (L)



$(P(A), \leq)$

$P(A) = \{\phi, a, b, ab\}$



suppose that

$f(1) = \phi$

$f(2) = a$

$f(3) = b$

$f(6) = (a, b)$

$a = 1, b = 2$

$f(1 \wedge b) = f(1) \wedge f(2)$

$f(1 \overset{gLB}{\wedge} 2) = f(1) \wedge f(2)$

$f(1) = \phi \wedge a$

$\boxed{\phi = \phi}$

glb

$f(2 \wedge 3) = 1 = f(a \wedge b) = \phi$

ϕ

$f(1 \wedge 3) = 1 = f(b \wedge \phi) = \phi$

$f(2 \wedge 6) = 2 = f(a, (x)) = a$

There is one to one fn.

Homomorphism Lattice

Let us consider two lattices $(L, \vee, \wedge)$ & $(m, \vee, \wedge)$ and for both (L and m) GLB and LUB exist.

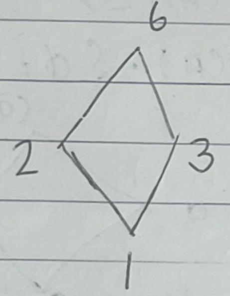
$F: L \to m$ is called lattice homomorphism if it satisfy two condition.

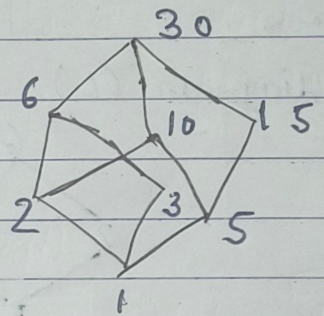
$$\boxed{\begin{aligned} f(a \wedge b) &= f(a) \wedge f(b) \quad \text{meet} \\ f(a \vee b) &= f(a) \vee f(b) \quad \text{Join} \end{aligned}}$$

$a, b \in L$ &

$\boxed{f(a), f(b) \in m}$

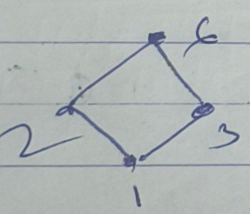
Q





Q Two lattices are $D_6 = \{1, 2, 3, 6\}$ and $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$ with divisibility relation. Show that there exist a homomorphism between $(D_6, /)$ & $(D_{30}, /)$

Solution suppose

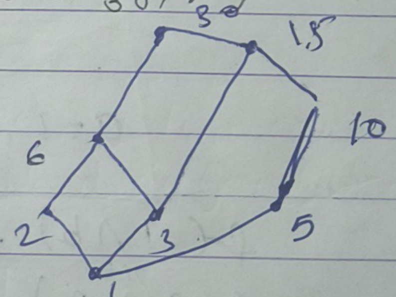


$f(1) = 1$

$f(2) = 6$

$f(3) = 15$

$f(6) = 30$



$\Rightarrow$ $f(a \wedge b) = f(a) \wedge f(b)$

$f(1 \wedge 2) = f(1) \wedge f(2)$

$f(1) = 1 \wedge 6$

$\boxed{1 = 1}$

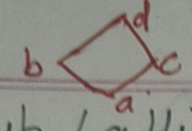
$f(2 \wedge 3) = f(2) \wedge f(3)$

$f(2) = 6 \wedge 15$

$\boxed{6 = 6}$

Complemented lattice → A lattice is said to b complemented if every element $\forall a \in L$ must have at least one complement (if it have L.U.b ~~l.g.b~~)

I. Maximal element & glb = minimal element



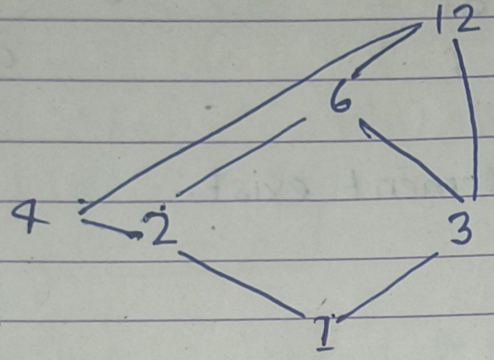
Complement of $b = c$ $\begin{bmatrix} L.U.b(b,c) = d \\ g.L.b(b,c) = a \end{bmatrix}$

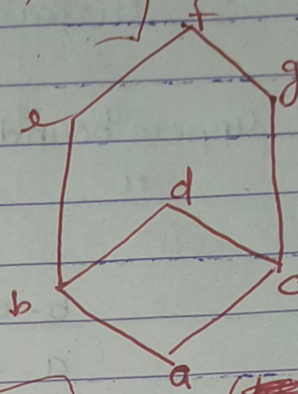
Complement of $c = b$

Complement of $d = a$ $\begin{bmatrix} U.b(a,d) = d \\ L.b(a,d) = a \end{bmatrix}$

Sub Lattice

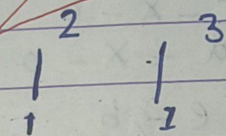
$L = \{1, 2, 3, 4, 6, 12\}$





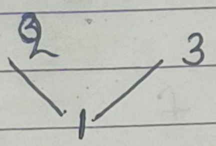
$a' = f$
$f' = a$
$b' = g$
$g' = b$
$c' = e$
$e' = g$
$e' = e$
d = has no complement

so it is not complemented



$L_1 = \{1, 2\}$
$L_2 = \{1, 3\}$ — L_1 and L_2 are sub lattice of lattice L.

$L_3 = \{1, 2, 3\}$



LUB does not exist.

Distributive lattice

If for any $a, b, c \in L$, There exist this

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

&

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

then the lattice is said to be distributive lattice

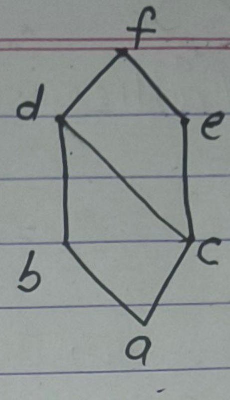
Or

A lattice L is said to be distributive lattice if every element in L has at most one complement That is 0 or 1.

~~UNIT-2~~

Q1

least element = a
Greatest element = f

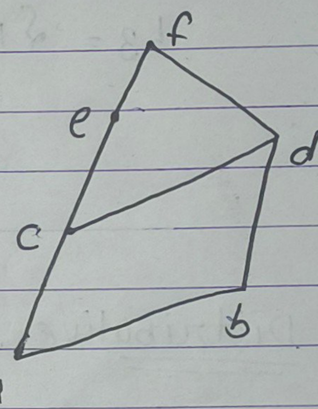


upper bound of c = df
" d = d f

b-e complement exist.
a-f
c - X - o
d - X - o
e - b
f - a

Hence this is a distributive lattice.

Q1



least element = a
Greatest element = f

c-b exist.
all the element's complement is either o or 1 and neither exist.
Hence this is a distributive lattice.

Modular lattice

A lattice is said to be moduler lattice if there exist

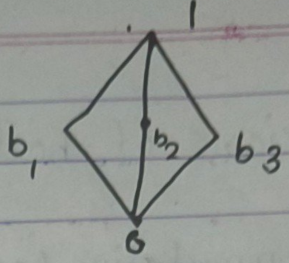
$$\boxed{a \vee (b \wedge c) = (a \vee b) \wedge c}$$

whenever a (preceed) $\preceq$ c for all $a, b, c \in L$

$\boxed{a \preceq c}$ a is related with c & a preceeds c

aRc

$a \leq c$ ~~~~ c should be upper from a



Q (i) suppose

$a = 0$
$b = b_1$
$c = b_2$

$0 \vee (b_1 \wedge b_2) = (0 \vee b_1) \wedge b_2$

$0 \vee (0) = b_1 \wedge b_2$

$\boxed{0 = 0}$

$b_3 R 1$ but $1 \not\leq b_3$

$1, b_2, b_3 \in L$

here $1 \leq b_3$

but here $b_3 \leq 1$ so this is not valid Relation

~~$b_1 \not\leq b_3$~~

(ii) $a = b_1$
$b = b_2$
$c = b_3$

$b_1 \vee (\underset{GLB}{b_2 \wedge b_3}) = (\underset{LUB}{b_1 \vee b_2}) \wedge b_3$

$\Rightarrow b_1 \vee 0 = 1 \wedge b_3$

$= \boxed{b_1 = b_3}$ $b_1 \not\leq b_3$

because $\boxed{b_1 \leq b_3}$

(iii) $a = 1$
$b = b_2$
$c = b_3$

$1 \vee (b_2 \wedge b_3) = (1 \vee b_2) \wedge b_3$

$1 \vee 0 = 1 \wedge b_3$

$= 1 = b_3$

L.H.S = R.H.S.

It is not a moduler lattice.

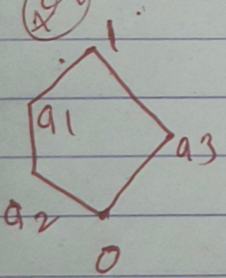
(iii) $b_2, b_3, 1 \in L$

$b_2 \vee (b_3 \wedge 1) = (b_2 \vee b_3) \wedge 1$

$b_2 \vee b_3 = 1 \wedge 1$

$1 = 1$

(iv)



$a = a_2$ $b = a_3$ $c = a_1$

$a_2 \vee (a_3 \wedge a_1) = (a_2 \vee a_3) \wedge a_1$ $(a \vee b) \wedge c$ here $a_2 \leq a_1$

$a \vee (b \wedge c)$

$= (1) \wedge a_1$

$a_2 \vee (0)$

$1 \wedge a_1 = a_1$

$a_2 \vee 0 = a_2$

$a_2 \neq a_1$ $\therefore$ It is not Moduler lattice