

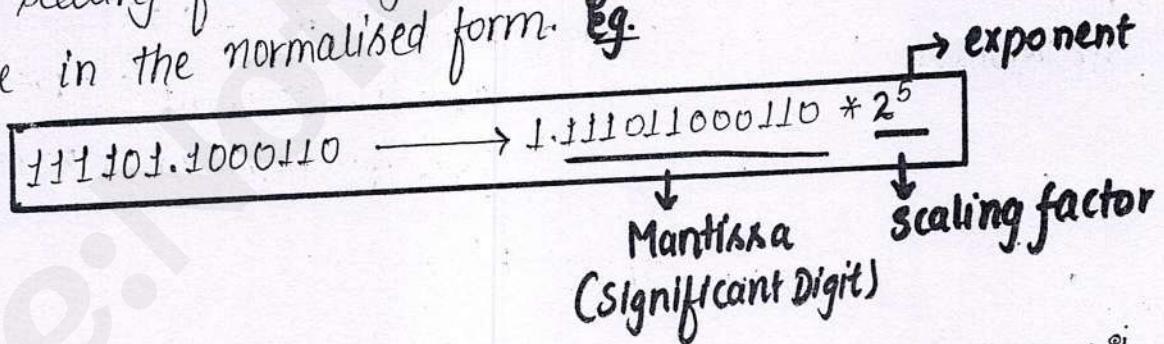
UNIT:-02

Arithmetic and Logic Unit

Floating Point Representation:

To accomodate very large integers, and very small fractions, a computer must be able to represent numbers and operate on them in such a way that the position of the binary point is variable and is automatically adjusted as computation proceeds. In this case, the binary point is set to float, and the numbers are called floating point numbers. The floating point representation has three fields :- sign, significant digits and exponent.

To represent the number in floating point format, first binary point is shifted to right of the first bit and the number is multiplied by the correct scaling factor to get the same value. The number is said to be in the normalised form. Eg.



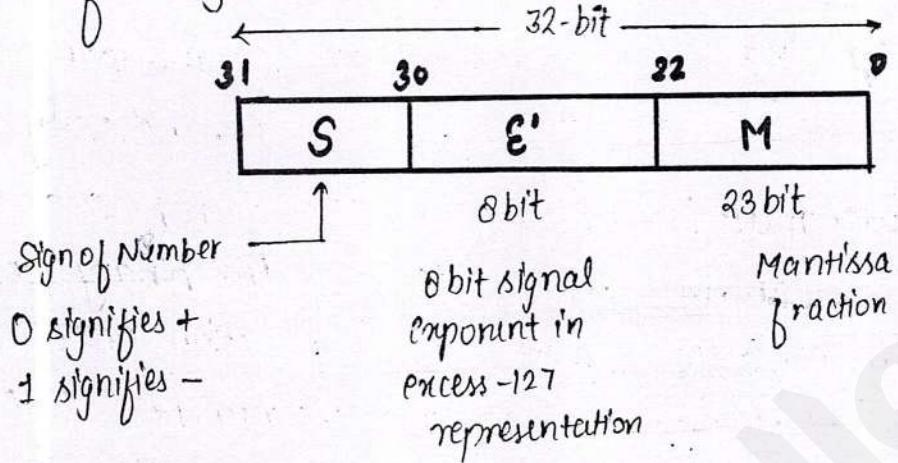
The standards for representing floating point numbers in 32 bits and 64 bits have been developed by the institute of electrical and electronics engineers (IEEE).

It is also known as IEEE 754 standards.

The 32-bit standard is called a single precision representation because it occupies a single 32-bit word. The 32-bits divided into three fields :- Field 1 sign, sign-bit (1 bit).
 Field 2 exponent (8-bit)
 Field 3 Mantissa (23-bit).

Instead of the sign exponent (E), the value actually stored in the Exponent field is $E' = E + \text{Base value}$.

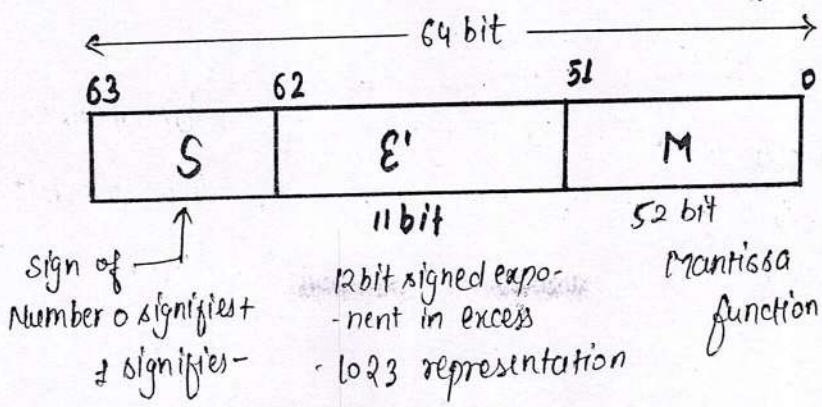
In 32-bit floating point system, base is 127. Hence $E' = E (\text{scaling}) + 127$ factor



$$E' = E + 127$$

32 bit Representation

- * The 64-bit representation is called a double precision representation because it occupies two 32-bit words. The 64-bits are divided into three fields:
 - field (1) sign - 1 bit
 - field (2) exponent \rightarrow 11 bit
 - field (3) Mantissa $= 52$ bit.
- * In double precision format, $E' = E + 1023$.



$$E' = E + 1023$$

* Represent $(1259.125)_{10}$ in single and double precision format.

Sol:

2	1259	1
2	629	1
2	314	0
2	157	1
2	78	0
2	39	1
2	19	1
2	9	1
2	4	0
2	2	0
2	1	0

$$0.125 \times 2 = 0.250 \quad 0$$

$$0.250 \times 2 = 0.500 \quad 0$$

$$0.500 \times 2 = 1.000 \quad 1$$

$$= (10011101011.001)_2$$

$$= 1.0011101011001 \times 2^{10}$$

Single Precision Format

$$E' = E + 127$$

$$E' = 10 + 127$$

$$\boxed{E' = 137}$$

2	137	1
2	68	0
2	34	0
2	17	1
2	8	0
2	4	0
2	2	0
2	1	0
2	0	1

$\Rightarrow 10001001$

31	30	22	0
0	10001001	001110101001	-----0

Double Precision Format

$$E' = E + 1023$$

$$E' = 10 + 1023$$

$$\boxed{E' = 1033}$$

63	62	52	0
0	10000001001	001110101001	F00FF0F0F0F0F0F0

$$E' = 1033$$

$$E' = 100000001001$$

② $(-309.1875)_{10}$

$$309 = 100110101$$

$$0.1875 \times 2 = 0.3750 \quad 0$$

$$0.3750 \times 2 = 0.7500 \quad 0$$

$$0.7500 \times 2 = 1.5000 \quad 1$$

$$= -(100110101.001)_2$$

$$= -1.00110101001 \times 2^8$$

Single Precision

$$E' = E + 127$$

$$E' = 8 + 127$$

$$\boxed{E' = 135}$$

$$= (10000111)_2$$

31	30	22	0
S	E'	M	
1	10000111	00110101001	0

Double Precision

$$E' = E + 1023$$

$$E' = 8 + 1023$$

$$\boxed{E' = 1031} = (010000000111)_2$$

63	52	51	0
S	E'	M	
1	100000000111	00110101001	0

③ $(-79.1258)_{10}$

$$79 = (1001111)_2$$

$$0.1258 \times 2 = 0.2516 \quad 0$$

$$0.2516 \times 2 = 0.5032 \quad 0$$

$$0.5032 \times 2 = 1.0064 \quad 1$$

$$(1001111.001)_2 = 1.001111001 \times 2^6$$

Single Precision

$$E' = E + 127$$

$$E' = 6 + 127$$

$$\boxed{E' = 133}$$

$$= (10000101)_2$$

31	30	22	0
1	100000101	001111001	0

Double Precision

$$E' = E + 1023$$

$$E' = 1029 = (100000000101)_2$$

63	62	51	0
1	10000000101	001111001	0

④ $(1460.125)_{10}$

$$1460 = (10110110100)$$

$$= (10110110100.001)_2$$

$$= (1.0110110100001 \times 2^{10})$$

$$.125 \times 2 = .250 \quad 0$$

$$0.250 \times 2 = .500 \quad 0$$

$$0.500 \times 2 = 1.00 \quad 1$$

Single Precision

$$E' = E + 127$$

$$E' = 10 + 127$$

$$\boxed{E' = 137} = (10001001)_2$$

31	30	22	0
0	10001001	0110110100001	0

Double Precision

$$E' = E + 1023 = 10 + 1023$$

$$\boxed{E' = 1033} = (10000001001)_2$$

63	62	51	0
0	10000001001	0110110100001	0

$$\textcircled{B} \quad (-1523.012)_{10}$$

$$= (1011110011, 0000)_2$$

$$= 1.011110011 \times 2^{10}$$

$$E = 10$$

$$0.012 \times 2 = 0.024$$

$$0.024 \times 2 = 0.048$$

$$0.048 \times 2 = 0.096$$

$$0.096 \times 2 = 0.192$$

$$0.192 \times 2 = 0.384$$

Single Precision

$$E' = E + 127$$

$$E' = 10 + 127$$

$$E' = 137 = (10001001)_2$$

1	10001001	0111100111100100
---	----------	------------------

Double Precision

$$E' = E + 1023$$

$$E' = 1033 = (10000001001)_2$$

1	10000001001	0011000111100100
---	-------------	------------------

$$\textcircled{6} \quad (1486.125)_{10}$$

$$= (10111001110.001)_2$$

$$= 1.0111001110001 \times 2^{10}$$

$$E = 10$$

Single Precision

$$E' = E + 127$$

$$E' = 10 + 127$$

$$E' = 137 = (10001001)_2$$

31	30	32
0	10001001	0111001110001

Double Precision

$$E' = E + 1023$$

$$E' = 10 + 1023$$

$$E' = 1033 = (100000001001)_2$$

63	62	51	0
0	100000001001	011100111000	0

⑦ $(1243.725)_{10}$

$$(10011011011.1011)_2$$

$$= 1.00110110111011 \times 2^{10}$$

$$E = 10$$

Single Precision

$$E' = E + 127$$

$$E' = 10 + 127$$

$$E' = 137 = (100010001)_2$$

0	10001001	0011011011011	0
---	----------	---------------	---

Double Precision

$$E' = E + 1023$$

$$E' = 10 + 1023$$

$$E' = 1033 = (100000001001)_2$$

0	10000001001	0011011011011	0
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⑧ $(-176.375)_{10}$

$$= (101100000.011)_2$$

$$= 1.0110000011 \times 2^7$$

$$E = 7$$

Single Precision

$$E' = E + 127$$

$$E' = 7 + 127$$

$E' = 134$	= $(10000110)_2$
31 30 22 1 10000110 0110000011---0	0

Double Precision

$$E' = E + 1023$$

$$E' = 7 + 1023$$

$E' = 1030$	= $(100000000110)_2$
63 62 51 1 100000000110 01100000011---0	0

$$\textcircled{2} \quad (-1976.453)_{10}$$

$$= (111101110000.0111)_2$$

$$= 1.111011100000111 \times 2^{10}$$

$$[E = 10]$$

Single Precision

$$E' = E + 127$$

$$E' = 10 + 127$$

$E' = 137$	= $(10001001)_2$
31 30 22 1 10001001 11101110000111---0	0

Double Precision

$$E' = E + 1023 = 10 + 1023$$

$E' = 1033$	= $(100000001001)_2$
63 62 51 1 100000001001 11101110000111---0	0

Parallel Adder

A single full-adder is capable of adding two one-bit numbers and an input carry. In order to add binary numbers with more than one bit, additional full-adders must be employed.

A n-bit, parallel adder can be constructed using number of full adder circuits connected in parallel.

Figure shows the block diagram of n-bit parallel adder using n-number of full adder circuits connected in cascade i.e., the carry output of each adder is connected to the carry input of the next higher-order adder.

It should be noted that either a half adder can be used for least significant position or the carry input of a full adder is made 0 because there is no carry into the least significant bit position.

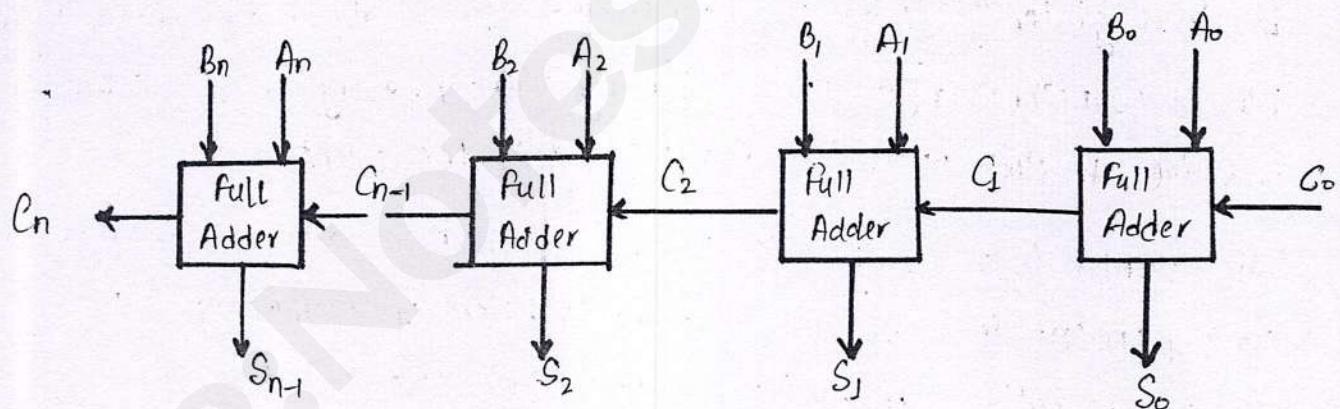


Fig. 2.6.11 - Block diagram of n-bit parallel adder

Parallel Subtractor

The subtraction of binary numbers can be done most conveniently by means of complements.

The subtraction $A - B$ can be done by taking 2's complement of B and adding it to A . The 2's complement can be obtained by taking the 1's complement and adding one to the least significant pair of bits.

The 1's complement can be implemented with inverters and a one can be added to the sum through the input carry to get 2's complement as shown in the fig.

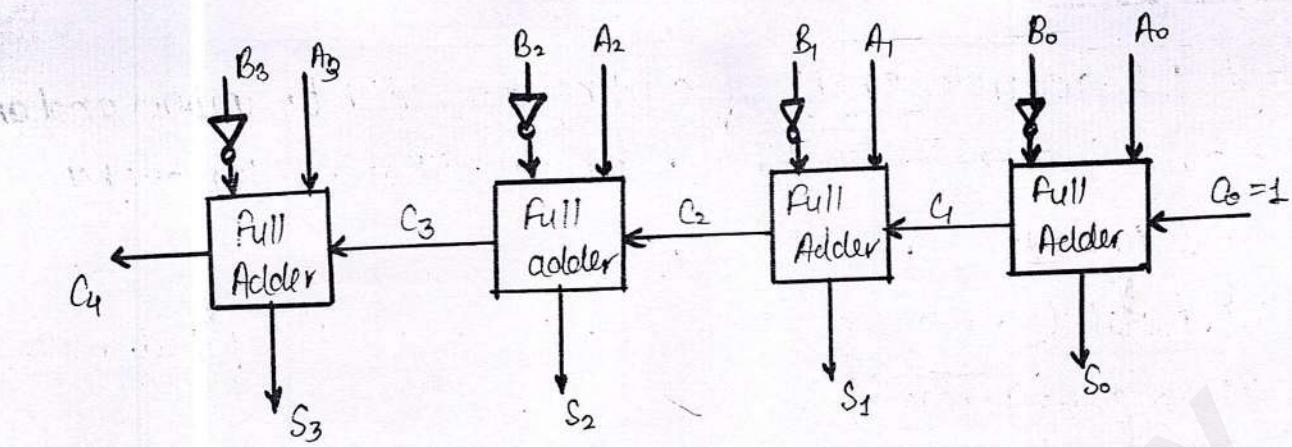
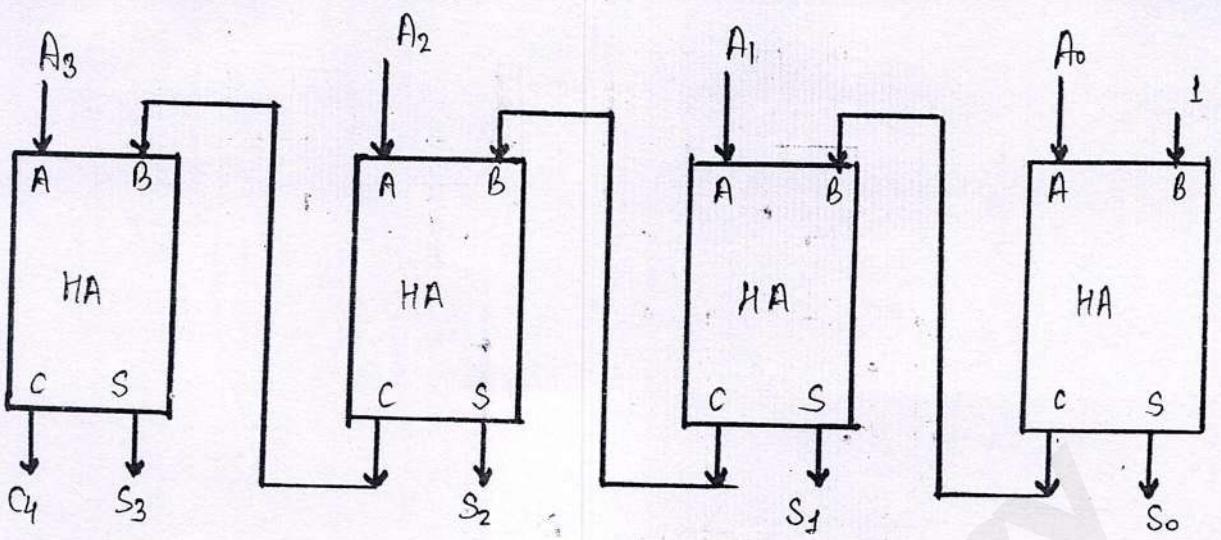


Fig. 4-bit parallel subtractor

Binary Incrementor :

We have seen that the increment micro-operation adds one to a number in register. This micro-operation can be easily implemented with a binary counter. In a binary counter, every time the count enable is active, the clock pulse transition increments the count.

The counter is a sequential circuit. The Fig. shows a combinational circuit to implement increment micro-operation. Here, one of the inputs to the least significant half-adder (HA) is connected to logic 1 and the other input is connected to the least significant bit of the number to be incremented. The output carry from lower order half adder is connected to the one of the inputs of the next higher order half adder. Such circuits get the four bit number from bits A₀ through A₃, adds one to it, and generates the incremented output in S₀ through S₃. If the number to be incremented is 1111, the output carry C₄ is generated and we get 0000 at S₀ through S₃.



- Fig: 2.6.17. 4-bit Incrementor using combinational circuit

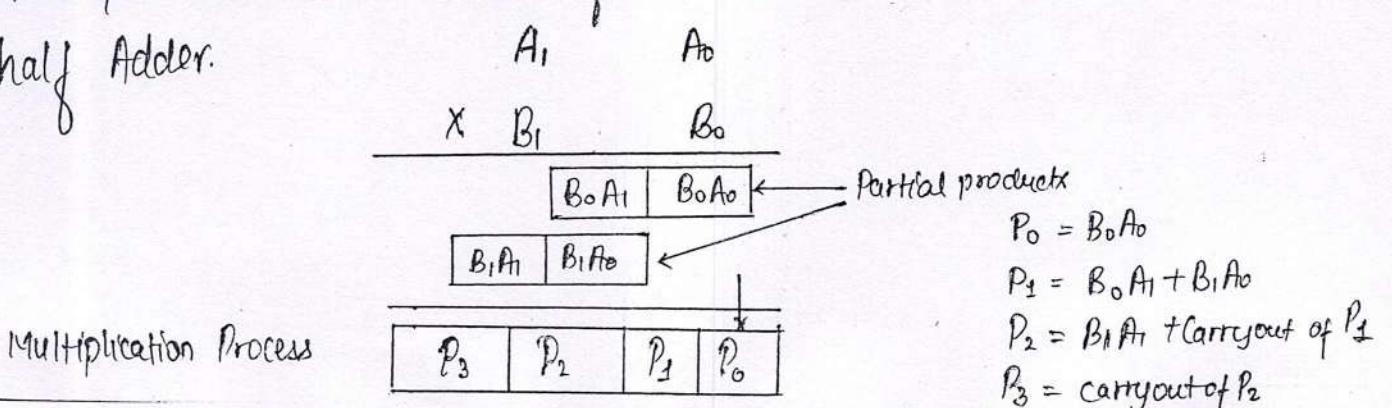
Array Multiplier: The multiplication process of binary number is similar to decimal nos. Actually binary multiplication is simple than decimal multiplication because it involves 0 and 1 only.

for multiplication in binary nos., it uses n-shifts and ADDS to multiply n-bit binary number.

The combinational logic circuit implemented to perform such multiplication is called combinational multiplier or array multiplier. Let us generalize the multiplication process for 2×2 multiplier for two unsigned 2-bit nos. - multiplicand $A = A_1 A_0$ and multiplier $B = B_1 B_0$.

The multiplication process involves multiplication of 2-bit nos. and addition of 2-bit nos.

The multiplication of two bits can be implemented using 2 input AND gate whereas addition of 2 bits can be implemented using half adder.



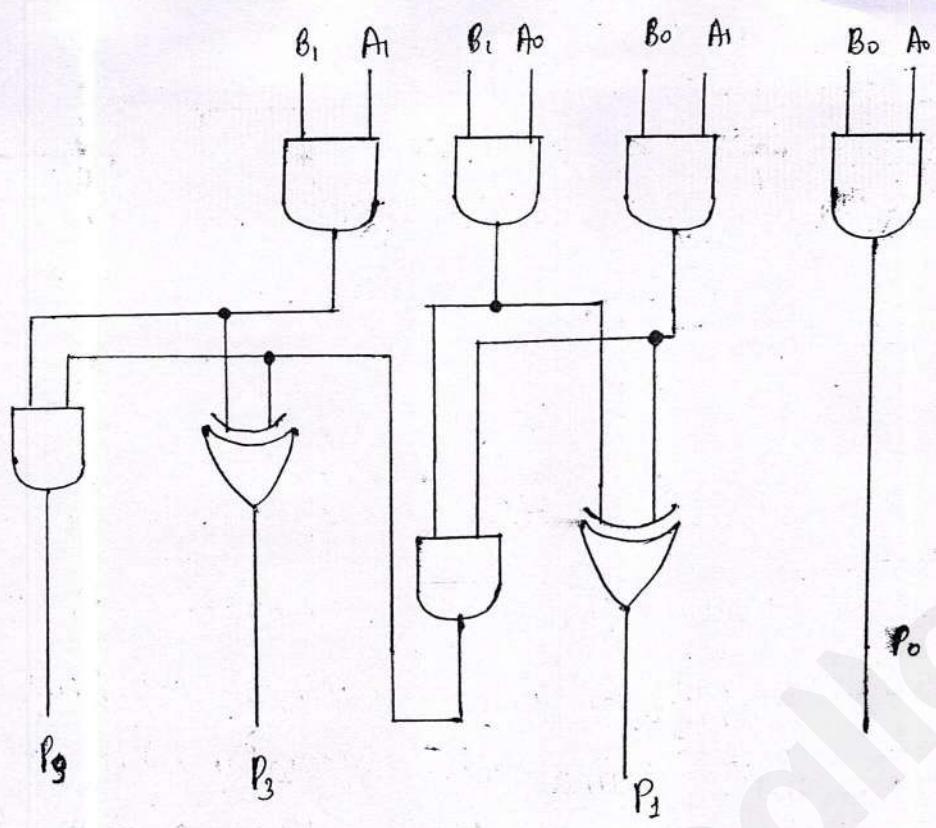
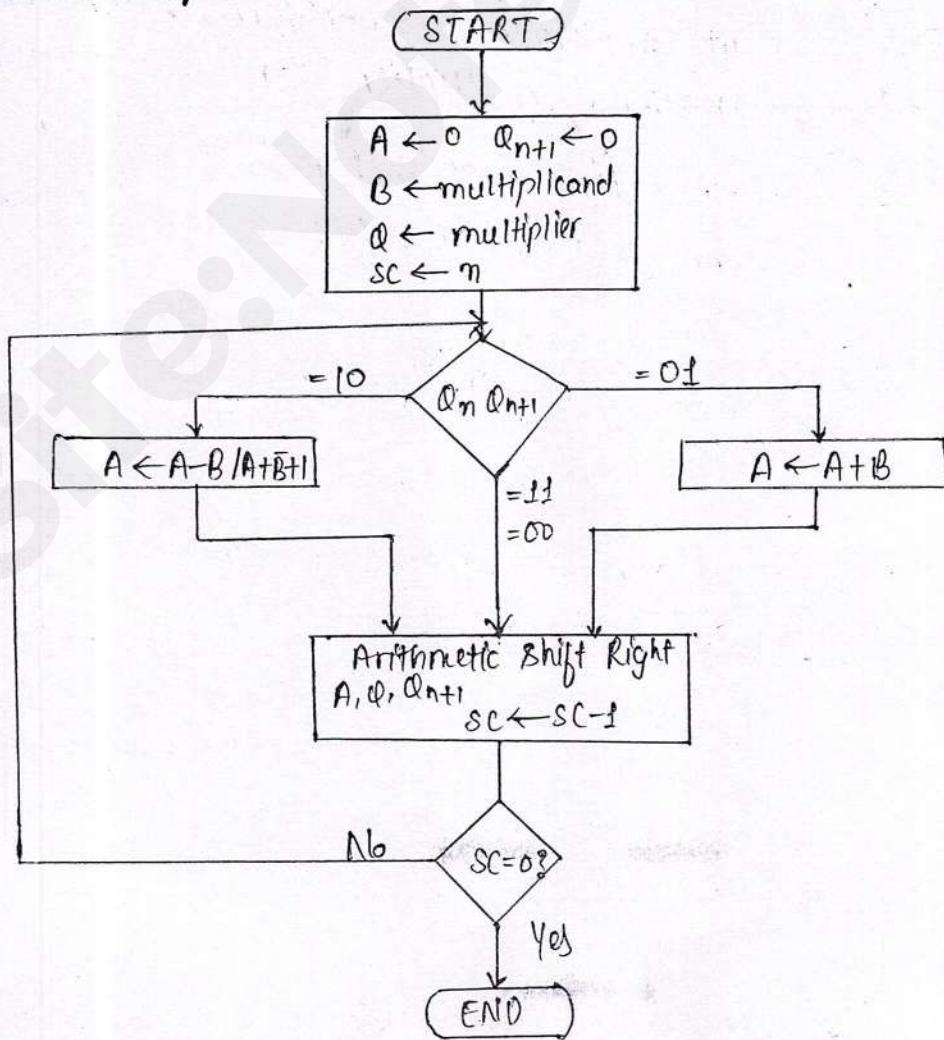


Fig 2.10.2 2x2 bit combinational array multiplier

Booth Multiplication



Algorithm:-

Step 1:- Load $A = 0$; $Q_{n+1} = 0$
 B = multiplicand
 Q = multiplier
 SC = n

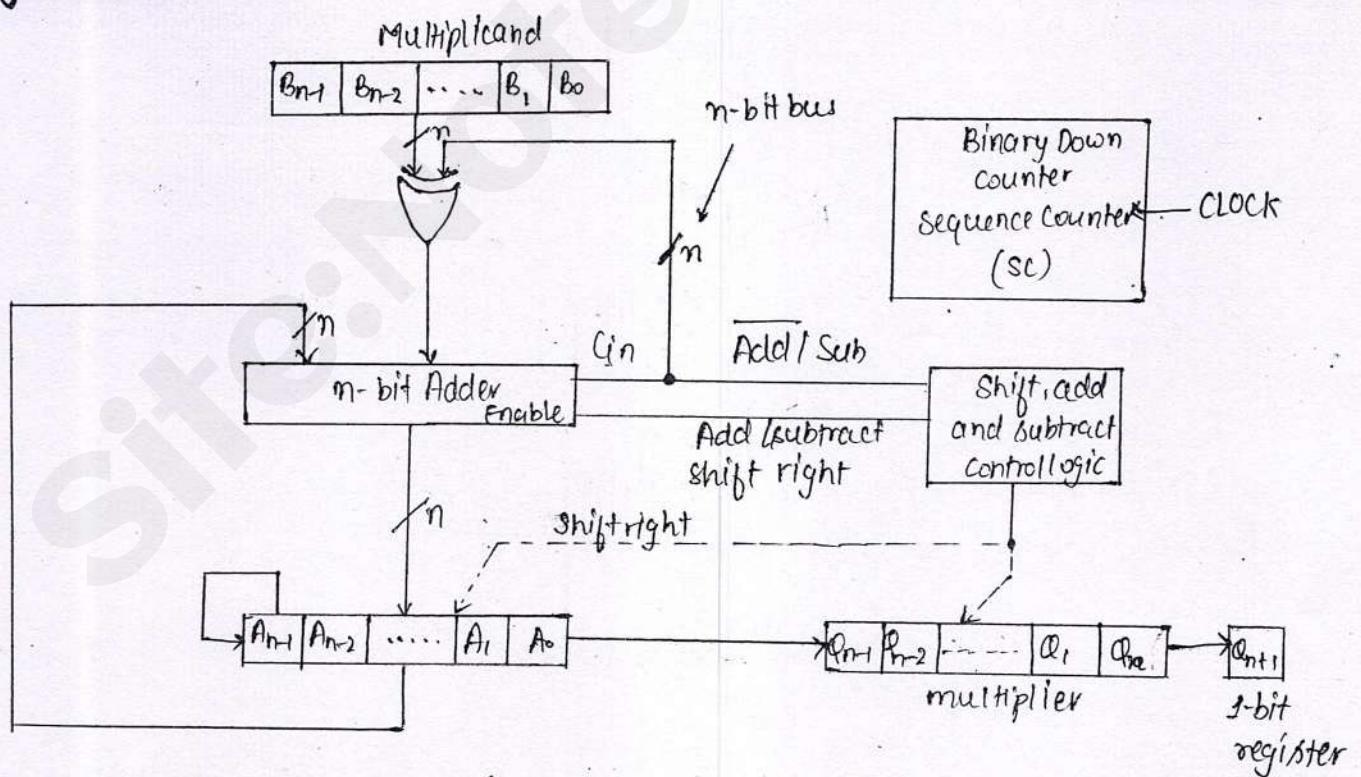
Step 2:- Check the status of $Q_n Q_{n+1}$.
 If $Q_n Q_{n+1} = 10$ perform $A \leftarrow A - B$
 If $Q_n Q_{n+1} = 01$ perform $A \leftarrow A + B$

Step 3:- Arithmetic shift right: A, Q_n, Q_{n+1} .

Step 4:- Decrement sequence counter if not zero, repeat step 2 through 4.

Step 5:- Stop.

Algorithm



- Initial settings: $A \leftarrow 0$ and $Q_{n+1} = 0$

\therefore Hardware implementation of signed binary multiplication

① Evaluate

(i) $9 * 17$

$$B = 9 = 001001 \quad \bar{B} + 1 = 110111$$

$$Q = 17 = 010001$$

Q_n	Q_{n+1}	$B = 001001$	A	Q	Q_{n+1}	SC
		$\bar{B} + 1 = 110111$				
		Initial	000000	010001	0	110
1	0	$A \leftarrow A + \bar{B} + 1$	$\frac{110111}{110111}$			
		$Ashr(A, Q, Q_{n+1})$	111011	101000	1	101
0	1	$A \leftarrow A + B$	$\frac{001001}{000100}$			
		$Ashr(A, Q, Q_{n+1})$	000010	010100	0	100
0	0	$Ashr(A, Q, Q_{n+1})$	000001	001010	0	011
0	0	$Ashr(A, Q, Q_{n+1})$	000000	100101	0	010
1	0	$A \leftarrow A + \bar{B} + 1$	$\frac{110111}{110111}$			
		$Ashr(A, Q, Q_{n+1})$	111011	110010	1	001
0	1	$A \leftarrow A + B$	$\frac{001001}{000100}$			
		$Ashr(A, Q, Q_{n+1})$	000010	011001	0	000

$$B * Q = 000010011001$$

② $-13 * 16$

$$B = -13 = 110011 \Rightarrow \bar{B} + 1 = 001101$$

$$Q = 16 = 010000$$

$$+ 13 = 001101$$

\downarrow 2's complement

$$= 110010 + 1$$

$$-13 = 110011$$

Q_n	Q_{n+1}	$B = 110011$	A	Q	Q_{n+1}	SC
		$\bar{B} + 1 = 001101$				
		Initial	0000000	0100000	0	110
0	0	$Ashr(A, Q, Q_{n+1})$	0000000	0010000	0	101
0	0	$Ashr(A, Q, Q_{n+1})$	0000000	0000100	0	100
0	0	$Ashr(A, Q, Q_{n+1})$	0000000	0000100	0	011
0	0	$Ashr(A, Q, Q_{n+1})$	0000000	0000001	0	010
1	0	$A \leftarrow A + \bar{B} + 1$	$\begin{array}{r} 001101 \\ 001101 \\ \hline 001101 \end{array}$			
		$Ashr(VA, Q, Q_{n+1})$	0000110	1000000	1	001
0	1	$A \leftarrow A + B$	$\begin{array}{r} 110011 \\ 111001 \\ \hline 111100 \end{array}$			
		$Ashr(A, Q, Q_{n+1})$	1111000	1100000	0	000

$$B * Q = 111100110000$$

③ $18 * -14$

$$B = 18 = 010010$$

$$Q = -14 = 110010$$

$$14 = 001110$$

$$\begin{array}{c} \downarrow \\ \text{2's complement} \\ = 110001 + 1 \\ = 110010 \end{array}$$

Q_n	Q_{n+1}	$B = 010010$	A	Q	Q_{n+1}	SC
		$B + 1 = 101110$	Initial	0000000	110010	0
0	0	$Ashr(A, Q, Q_{n+1})$	0000000	011001	0	101
1	0	$A \leftarrow A + \bar{B} + 1$	$\begin{array}{r} 01110 \\ 101110 \end{array}$			
		$Ashr(A, Q, Q_{n+1})$	110111	0011100	1	100
0	1	$A \leftarrow A + B$	$\begin{array}{r} 010010 \\ 001001 \end{array}$			
		$Ashr(A, Q, Q_{n+1})$	0000100	100110	0	011
0	0	$Ashr(A, Q, Q_{n+1})$	0000010	010011	0	010
1	0	$A \leftarrow A + \bar{B} + 1$	$\begin{array}{r} 101110 \\ 1100000 \end{array}$			
		$Ashr(A, Q, Q_{n+1})$	1110000	001001	1	001
1	1	$Ashr(A, Q, Q_{n+1})$	1111000	0001000	1	000

$$B * Q = 111000000100$$

④ -12 * -11

$$B = -12 = 10100 \quad \overline{B} + i = 01011 + 1 = 01100$$

$$Q = -11 = 10101$$

$$+12 = 01100$$

$$+11 = 01011$$

↓ 2's comp.

$$110011 + 1$$

↓ 2's comp.

$$10100 + 1$$

$$-12 = 10100$$

$$-11 = 10101$$

A_n	Q_{n+1}	$B = 10100$	A	Q	Q_{n+1}	SC
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$$\overline{B} + i = 01100$$

Initial

$$000000$$

$$10101$$

$$0$$

$$101$$

1	0	$A \leftarrow A + \overline{B} + 1$	$\frac{01100}{01100}$			
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$$\text{Ashr}(A, Q, Q_{n+1}) \quad 00110 \quad 01010 \quad 1 \quad 100$$

0	1	$A \leftarrow A + B$	$\frac{10100}{11010}$			
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$$\text{Ashr}(A, Q, Q_{n+1}) \quad 11101 \quad 00101 \quad 0 \quad 011$$

1	0	$A \leftarrow A + \overline{B} + 1$	$\frac{01100}{01001}$			
---	---	-------------------------------------	-----------------------	--	--	--

$$\text{Ashr}(A, Q, Q_{n+1}) \quad 00100 \quad 10010 \quad 1 \quad 010$$

0	1	$A \leftarrow A + B$	$\frac{10100}{11000}$			
---	---	----------------------	-----------------------	--	--	--

$$\text{Ashr}(A, Q, Q_{n+1}) \quad 11100 \quad 01001 \quad 0 \quad 001$$

1	0	$A \leftarrow A + \overline{B} + 1$	$\frac{01100}{01000}$			
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$$\text{Ashr}(A, Q, Q_{n+1}) \quad 00100 \quad 00100 \quad 1 \quad 000$$

$$B * Q = 0010000100$$

Look Ahead Carry Adder

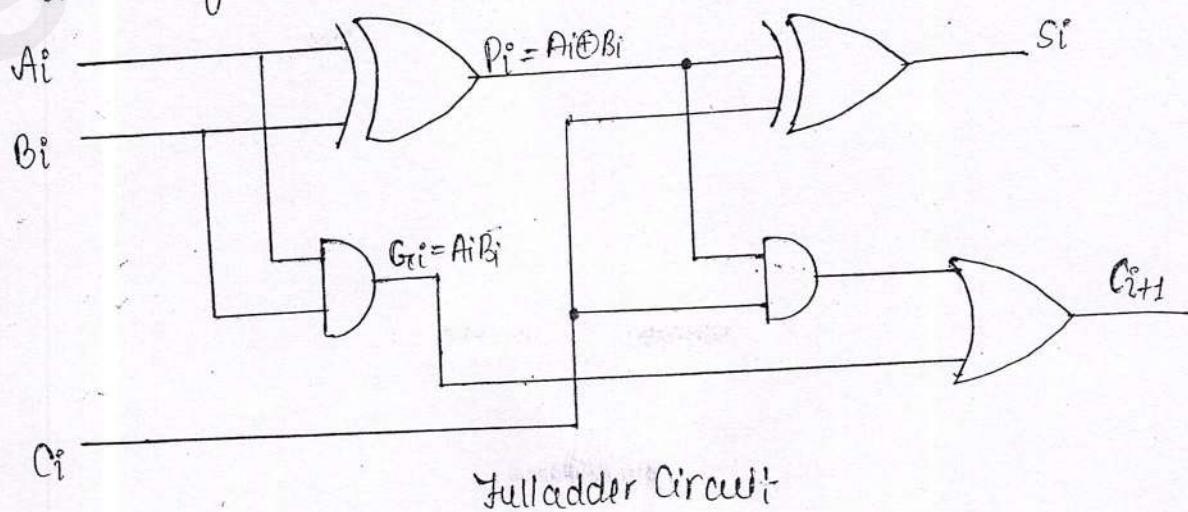
(CLA)

↳ Carry Look Ahead.

- * It is also known as fast adder / high speed adder.
- * In ripple carry adder, the sum and carry outputs of any stage cannot be produced until the input carry occurs, this leads to a time delay in the addition process.
- * This delay is known as carry propagation delay.
- * Considering the example:

$$\begin{array}{r}
 0101 \\
 + 0011 \\
 \hline
 1000
 \end{array}$$

- * Addition of the LSB position produces a carry into the second position.
- * This carry, when added to the bits of the second position produces a carry for the third position. The latter carry when added to the bits of third position, produces a carry to the last position.
- * One method of speeding up of this process by eliminating interstage carry delay is called look ahead carry addition.
- * It uses two functions - carry generate (G_i) and carry propagate (P_i).
- * Consider the full adder which is designed with the help of two half adder



- * We define two functions carry generate and carry propagate as follows:

$$\begin{aligned} P_i &= A_i \oplus B_i \\ G_i &= A_i B_i \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \rightarrow \textcircled{1}$$

- * The output sum and carry can be expressed as:

$$\begin{aligned} S_i &= P_i \oplus C_i \\ C_{i+1} &= G_i + P_i C_i \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \rightarrow \textcircled{2}$$

- * G_i is called carry generate and it produce a carry when both A_i and B_i are 1, regardless of the input carry.
- * P_i is called carry propagate because it is term associated with propagation of carry G_i to C_{i+1} .
- * For example, 4 stage carry look ahead adder can be defined as follows.
- * For initial condition, $C_i = C_0$, which is equal to zero.

Put $i=0$ in eqⁿ $\textcircled{2}$

$$C_{i+1} = G_i + P_i C_i \quad \left. \begin{array}{l} \\ \end{array} \right\} \rightarrow \textcircled{3}$$

$$i=0; \quad C_1 = G_0 + P_0 C_0$$

where $G_0 = A_0 B_0$ and $P_0 = A_0 \oplus B_0$ and $C_0 = 0$

$$\text{Hence, } C_1 = G_0 + P_0 C_{in}$$

now put $i=1$ in eqⁿ $\textcircled{2}$

$$i=1 \quad C_2 = G_1 + P_1 C_1$$

$$C_2 = G_1 + P_1 (G_0 + P_0 C_{in})$$

$$C_2 = G_1 + P_1 G_0 + P_1 P_0 C_{in} \rightarrow \textcircled{4}$$

now put $i=2$ in eqⁿ $\textcircled{2}$

$$C_3 = G_2 + P_2 C_2$$

$$C_3 = G_2 + P_2 (G_1 + P_1 G_0 + P_1 P_0 C_{in})$$

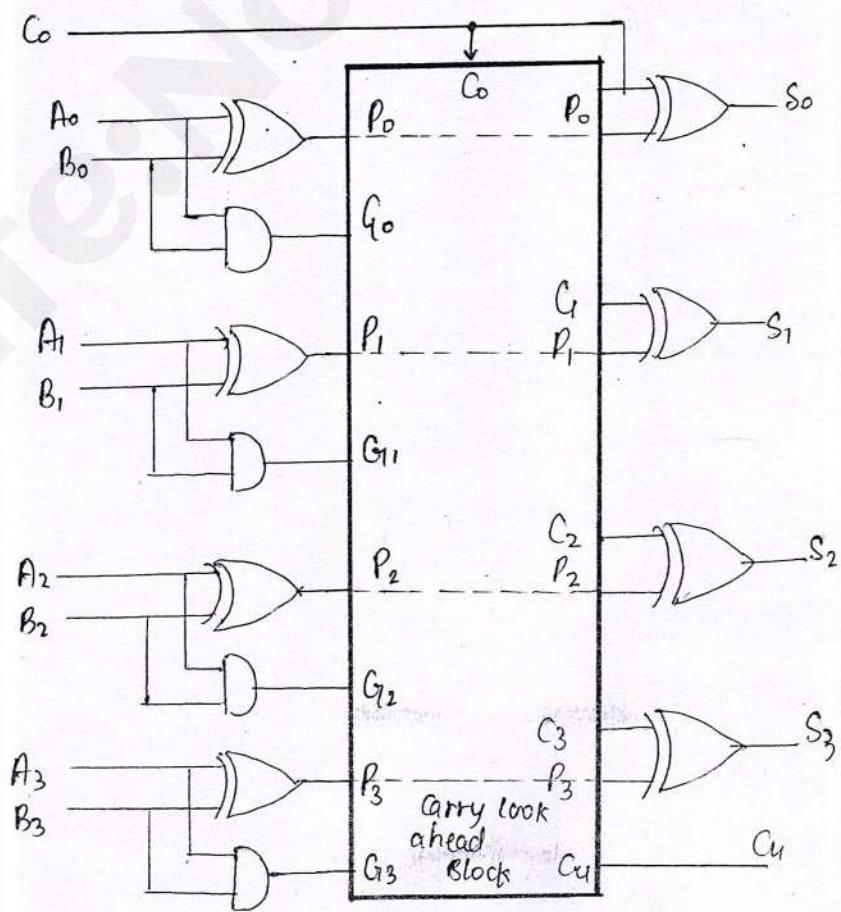
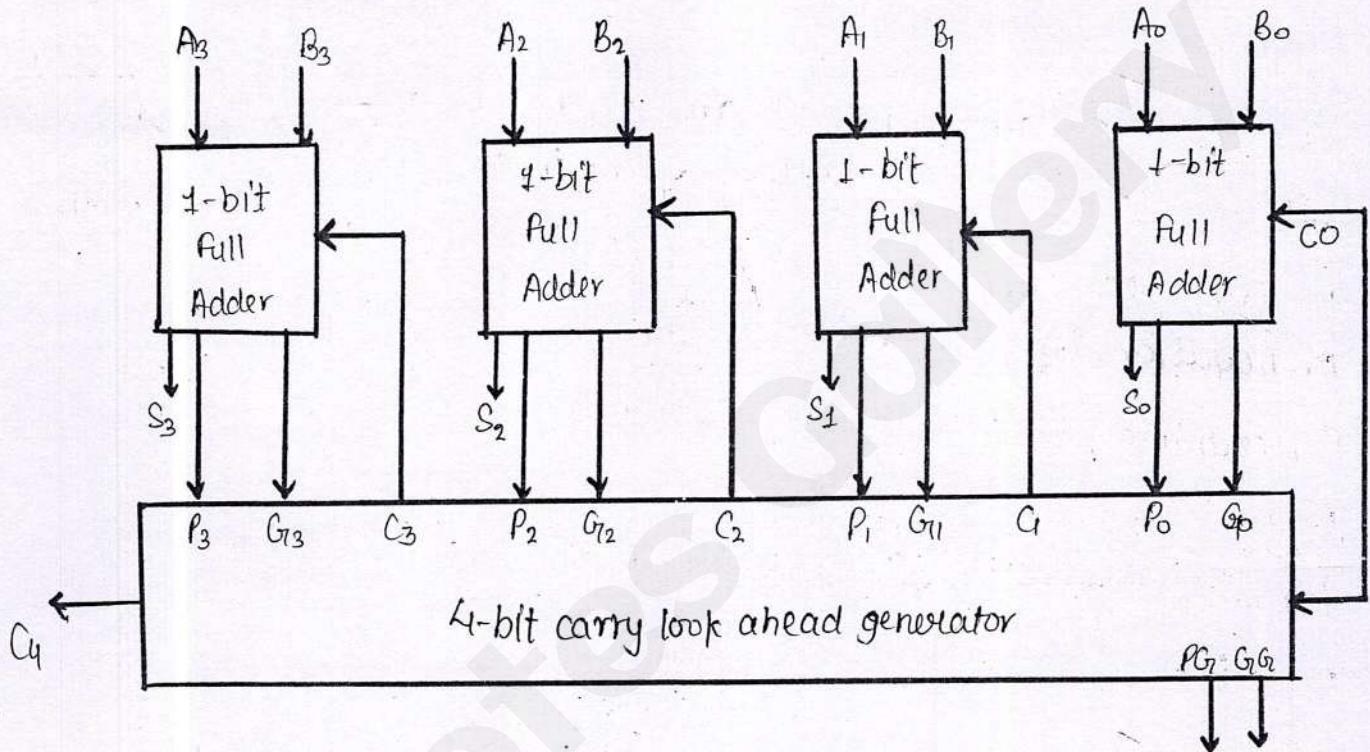
$$C_3 = G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_{in} \rightarrow \textcircled{5}$$

now part i=3 eqⁿ ②

$$C_4 = G_{13} + P_3 C_3$$

$$C_4 = G_{13} + P_3 (G_{12} + P_2 G_{11} + P_2 P_1 G_{10} + P_2 P_1 P_0 C_{in})$$

$$C_4 = G_{13} + P_3 G_{12} + P_3 P_2 G_{11} + P_3 P_2 P_1 G_{10} + P_3 P_2 P_1 P_0 C_{in} \quad \leftarrow ⑥$$

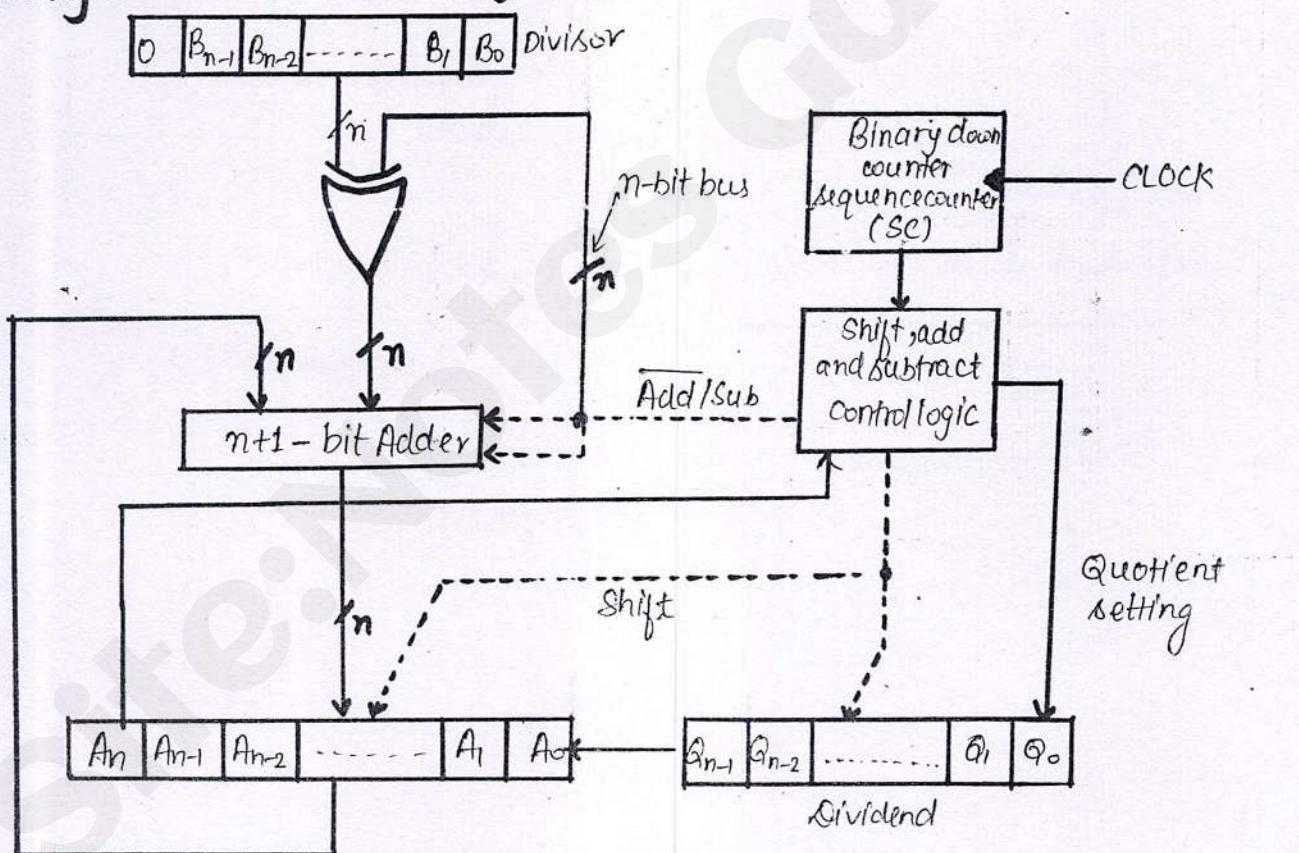


Booth's Division Algorithm

The division process for binary number is similar to decimal number. In division process, first the bits of the dividend are examined from left to right until the set of bits examined represents a no. of number greater than or equal to the divisor. until this condition occurs, 0's are placed in the ~~as~~ quotient. from left to right when the condition is satisfied one is placed in the quotient and the divisor is subtracted from the partial dividend.

This result is referred as partial remainder.

Restoring Booth's Division Algorithm



Hardware to implement binary division

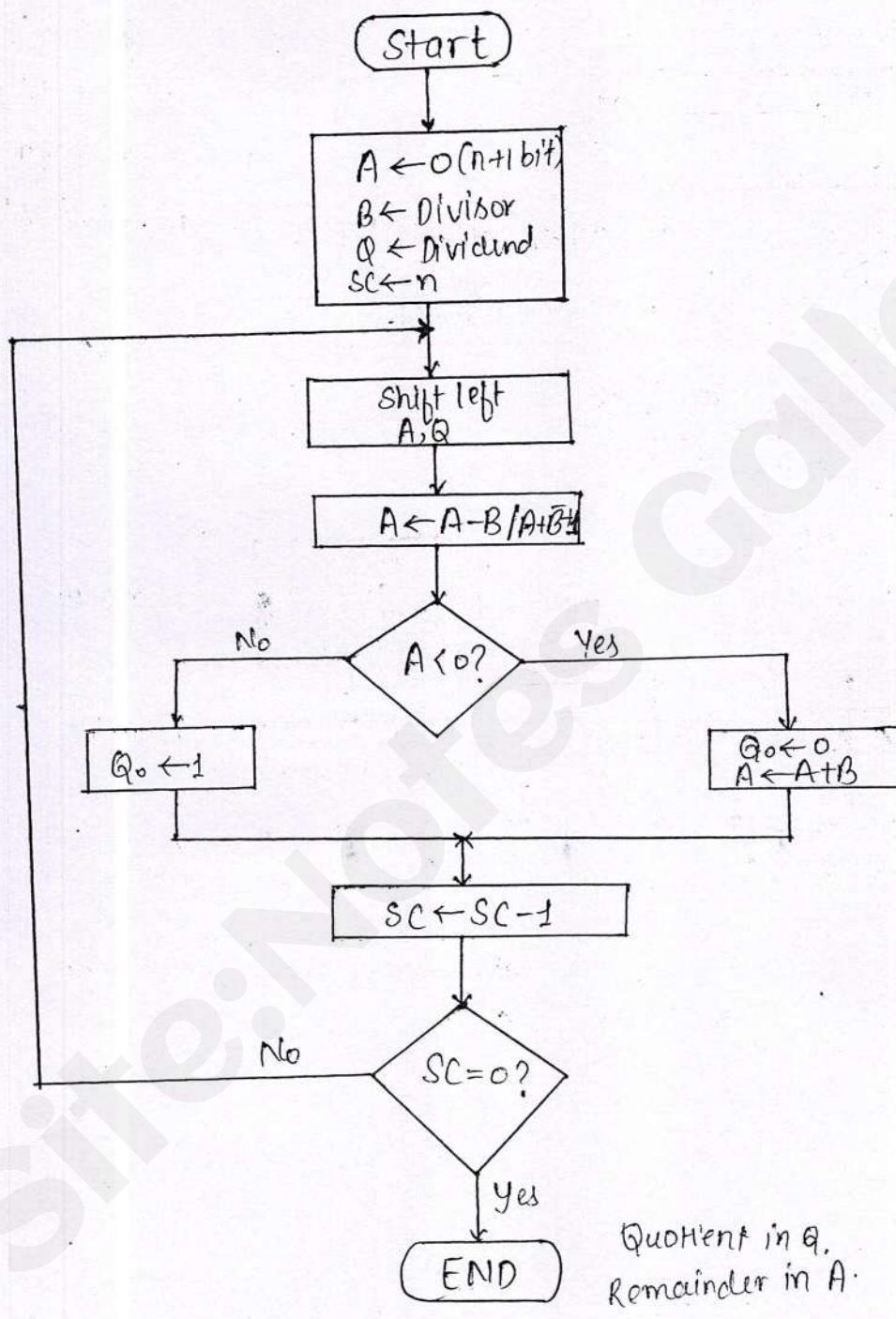
Division operation steps:

1. shift A and Q left one binary operation position.
2. subtract divisor (i.e. add 2's complement of divisor (B)) from A and place answer back in A ($A \leftarrow A - B$)
3. if the sign bit of A is 1, set Q_0 to 0 and add divisor back to A

(that is, restore A); otherwise, set Q_0 to 1.

4. Repeat steps 1, 2, and 3 n times.

A flowchart for division operation is as shown in fig. 2.11.3



Quotient in Q.
Remainder in A.

∴ Flowchart for restoring division algorithm

Perform the division of the following nos using restoring division algorithm,

i. Dividend = 1010 and Divisor = 0011

Divisor(B) = 0011

Dividend (A) = 1010

$B = 0011$

$\bar{B}+1 = 11101$

Operation	A	Q	SC
Initial	00000	1010	100
Shift left(A, Q)	<u>00001</u> 11101	0100	
$A \leftarrow A + \bar{B} + 1$	<u>11110</u>		
Set $Q_0 \leftarrow 0$	<u>0.0 0 1 1</u>		
$A \leftarrow A + B$	0000 1		011
Shift left(A, Q)	00010 11101	1000	
$A \leftarrow A + \bar{B} + 1$	<u>11111</u>		
Set $Q_0 \leftarrow 0$	<u>00 0 1 1</u>		
$A \leftarrow A + B$	00010		010
Shift left(A, Q)	00101 11101	0001	
$A \leftarrow A + \bar{B} + 1$	<u>00010</u>		
Set $Q_0 \leftarrow 1$			001
Shift left(A, Q)	00100 11101	0011	
$A \leftarrow A + \bar{B} + 1$	<u>00001</u>		000

Answ

Quotient = 0011

Remainder = 00001

(ii) $17 \div 3$

$$B = 3 = 000011$$

$$Q = 17 = 10001$$

$$B = 000011$$

$$\bar{B} + 1 = 111101$$

Operation	A	Q	SC
Initial	000000	10001	101
Shift Left (A, Q)	000001	100010	
$A \leftarrow A + \bar{B} + 1$	$\begin{array}{r} 111101 \\ \hline \textcircled{1} 11110 \end{array}$		
Set $Q_0 \leftarrow 0$	$\begin{array}{r} 000011 \\ \hline 000001 \end{array}$		100
$A \leftarrow A + B$			
Shift left (A, Q)	000010	00100	
$A \leftarrow A + \bar{B} + 1$	$\begin{array}{r} 111101 \\ \hline \textcircled{1} 11111 \end{array}$		
Set $Q_0 \leftarrow 0$	$\begin{array}{r} 000011 \\ \hline 000010 \end{array}$		01100
Set $Q_0 \leftarrow 1$			
Shift left (A, Q)	000100	01001	
$A \leftarrow A + \bar{B} + 1$	$\begin{array}{r} 111101 \\ \hline \textcircled{0} 00001 \end{array}$		010
Set $Q_0 \leftarrow 1$			
Shift left (A, Q)	000010	10010	
$A \leftarrow A + \bar{B} + 1$	$\begin{array}{r} 111101 \\ \hline \textcircled{0} 11111 \end{array}$		
Set $Q_0 \leftarrow 0$	$\begin{array}{r} 000011 \\ \hline 000010 \end{array}$		001
$A \leftarrow A + B$			

Shift Left(A,Q)

$A + \bar{B} + 1$

000101 00101

$$\begin{array}{r} 11110 \\ \hline \textcircled{0} 00010 \end{array}$$

000

Set $Q_0 \leftarrow 1$

$17/3 = 000010 \Rightarrow$ Quotient

00101 \Rightarrow Remainder

(iii) $1101/101$

$Q = 1101$

$B = 00101$

$\bar{B} + 1 = 11011$

Operation.

A

Q

SC

Initial

00000

1101

100

Shift Left(A,Q)

00001

1010

$A \leftarrow A + \bar{B} + 1$

$$\begin{array}{r} 11011 \\ \hline \textcircled{0} 1100 \end{array}$$

011

Set $Q_0 \leftarrow 0$

$A \leftarrow A + B$

$$\begin{array}{r} 00101 \\ \hline 00001 \end{array}$$

Shift Left(A,Q)

00011

0100

$A \leftarrow A + \bar{B} + 1$

$$\begin{array}{r} 11011 \\ \hline \textcircled{0} 1110 \end{array}$$

010

Set $Q_0 \leftarrow 0$

$A \leftarrow A + B$

$$\begin{array}{r} 00101 \\ \hline 00011 \end{array}$$

Shift Left(A,Q)

00110

1001

$A \leftarrow A + \bar{B} + 1$

$$\begin{array}{r} 11011 \\ \hline \textcircled{0} 001 \end{array}$$

001

Set $Q_0 \leftarrow 1$

shift left (A, Q)

$$A \leftarrow A + \bar{B} + 1$$

$$\text{Set } Q_0 \leftarrow 0$$

$$A \leftarrow A + B$$

00011

$$\begin{array}{r} 11011 \\ \hline 11110 \\ 00101 \\ \hline 00011 \end{array}$$

0010

000

Remainder \Rightarrow 00011

Quotient = 0010

(iv) 1000 / 111

$$Q = 1000$$

$$B = 00111$$

$$\bar{B} + 1 = 11001$$

Operation	A	Q	SC
initial	00000	1000	100
shift left (A, Q)	00001	0000	
$A \leftarrow A + \bar{B} + 1$	$\begin{array}{r} 11001 \\ \hline 01010 \end{array}$		
$Set Q_0 \leftarrow 0$	$\begin{array}{r} 00111 \\ \hline 00001 \end{array}$		011
$A \leftarrow A + B$			
shift left (A, Q)	00010	0000	
$A \leftarrow A + \bar{B} + 1$	$\begin{array}{r} 11001 \\ \hline 01011 \end{array}$		
$Set Q_0 \leftarrow 0$	$\begin{array}{r} 00111 \\ \hline 00010 \end{array}$		010
$A \leftarrow A + B$			
shift left (A, Q)	00100	0000	
$A \leftarrow A + \bar{B} + 1$	$\begin{array}{r} 11001 \\ \hline 01101 \end{array}$		010
$Set Q_0 \leftarrow 0$			

$A \leftarrow A+B$

$$\begin{array}{r} 11101 \\ 00111 \\ \hline 00100 \end{array}$$

shiftLeft(A, Q)

$A \leftarrow A+\bar{B}+1$

$$01000$$

$$00001$$

$$0010$$

Set $Q_0 \leftarrow 1$

$A \leftarrow A+B$

$$\begin{array}{r} 11001 \\ 00001 \\ \hline \end{array}$$

$$000$$

Shift Left(A, Q)

$A \leftarrow A+\bar{B}+1$

$$00010$$

$$0010$$

Set $Q_0 \leftarrow 0$

$A \leftarrow A+B$

$$\begin{array}{r} 11001 \\ 01011 \\ \hline \end{array}$$

$$0750$$

$$\begin{array}{r} 00111 \\ 00010 \\ \hline \end{array}$$

$$\text{Quotient} = 00010$$

$$\text{Remainder} = 00001$$

Non-Restoring Division

Consider the sequence of operations that takes place after the subtraction operation in the restoring algorithm.

If A is positive

Shift left and subtract

divisor $\rightarrow 2A-B$

If A is negative

Restore $\rightarrow A+B$

Shift left and subtract

divisor $\rightarrow 2(A+B)-B$

$$= 2A+B$$

Looping at the above operations that takes place after the subtraction operation !!

Looping at the above operations, we can write following steps for non-restoring algorithm.

Step 1: If the sign of A is 0, shift A and Q left one bit position and subtract divisor from A, otherwise, shift A and Q left and add divisor to A. If the sign of A is 0, set Q_0 to 1, otherwise, set Q_0 to 0.

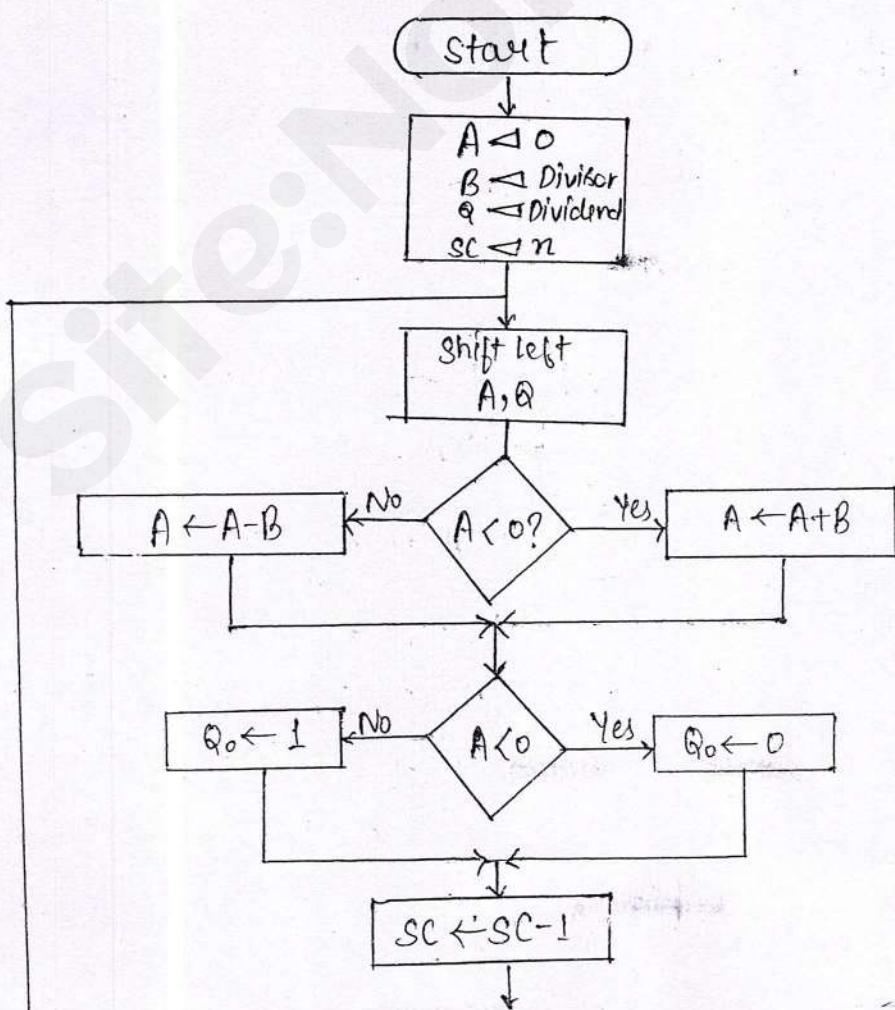
Step 2: Repeat steps 1 and 2 for n times.

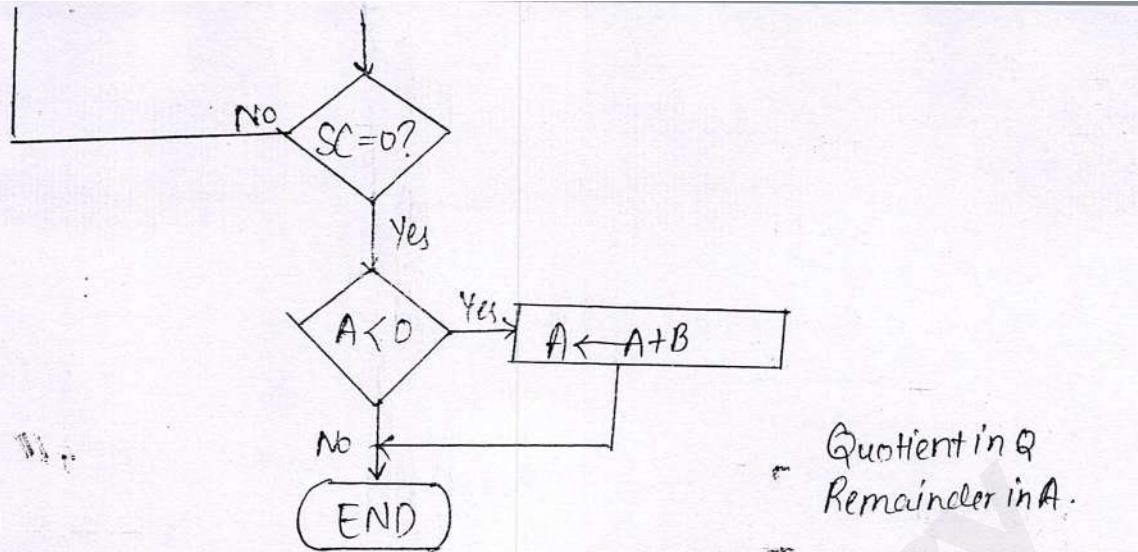
Step 3: if the sign of A is 1, add divisor to A.

Note: Step 3 is required to leave the proper positive remainder in A at the end of n cycles.

A flowchart for non-restoring division operation is as shown in

Fig 2.11.10





(i) $1101 / 11$

$$Q = 1101$$

$$B = 0011$$

$$B+1 = 00011$$

$$B+1 = 11101$$

Operation	A	Q	SC
Initial	00000	1101	100
Shiftleft A, Q	00001	1010	
$A \leftarrow A + B + 1$	$\begin{array}{r} 11101 \\ \hline 11110 \end{array}$		011
Set $Q_0 \leftarrow 0$			
ShiftLeft(A, Q)	11101	0101	
$A + B$	$\begin{array}{r} 00011 \\ \hline 00000 \end{array}$		
Set $Q_0 \leftarrow 1$			
Shiftleft(A, Q)	00000	1010	
$A \leftarrow A + B + 1$	$\begin{array}{r} 11101 \\ \hline 11101 \end{array}$		001
Set $Q_0 \leftarrow 0$			
Shift left (A, Q)	11011	0100	
$A \leftarrow A + B$	$\begin{array}{r} 00011 \\ \hline 11110 \end{array}$		000
$A + B$	$\begin{array}{r} 00011 \\ \hline 00001 \end{array}$		
		$Q = 100$	
		$R = 00001$	

(ii) 17/3

$$Q = 17 = 10001$$

$$B = 3 = 00011$$

$$B = 000011$$

$$\bar{B} + 1 = 111101$$

Operation	A	Q	SC
initial	000000	10001	101
Shift left (A, Q) A $\leftarrow A + \bar{B} + 1$ Set $Q_0 \leftarrow 0$	$ \begin{array}{r} 000001 \\ 111101 \\ \hline 111110 \end{array} $	00010	
Shift left (A, Q) A $\leftarrow A + B$ Set $Q_0 \leftarrow 0$	$ \begin{array}{r} 111100 \\ 000011 \\ \hline 111111 \end{array} $	00100	011
Shift left (A, Q) A $\leftarrow A + B$ Set $Q_0 \leftarrow 1$	$ \begin{array}{r} 111110 \\ 000011 \\ \hline 000001 \end{array} $	01001	010
Shift left (A, Q) A $\leftarrow A + \bar{B} + 1$ Set $Q_0 \leftarrow 0$	$ \begin{array}{r} 000010 \\ 111101 \\ \hline 111111 \end{array} $	10010	001
Shift left (A, Q) A $\leftarrow A + B$ Set $Q_0 \leftarrow 1$	$ \begin{array}{r} 111111 \\ 000011 \\ \hline 000010 \end{array} $	00101	000

(iii) 19/4

$$Q = 19 = 10011$$

$$B = 00100$$

$$B = 000100$$

$$\bar{B} + 1 = 1110111 + 1$$

$$\bar{B} + 1 = 111100$$

Operation	A	Q	SC
Initial	000000	10011	101
Shift left (A, Q)	$\begin{array}{r} 000001 \\ - \\ 111100 \end{array}$	00110	
$A \leftarrow A + \bar{B} + 1$	$\begin{array}{r} \\ 111101 \end{array}$		100
Set $Q_0 \leftarrow 0$			
Shift left (A, Q)	$\begin{array}{r} 111010 \\ - \\ 000100 \end{array}$	01100	
$A \leftarrow A + B$	$\begin{array}{r} \\ 111110 \end{array}$		011
Set $Q_0 \leftarrow 0$			
Shift left (A, Q)	$\begin{array}{r} 111100 \\ - \\ 000100 \end{array}$	11001	
$A \leftarrow A + B$	$\begin{array}{r} \\ 000000 \end{array}$		010
Set $Q_0 \leftarrow 1$			
Shift left (A, Q)	$\begin{array}{r} 000001 \\ - \\ 111100 \end{array}$	10010	
$A \leftarrow A + \bar{B} + 1$	$\begin{array}{r} \\ 111101 \end{array}$		001
Set $Q_0 \leftarrow 0$			
Shift left (A, Q)	$\begin{array}{r} 111011 \\ - \\ 000100 \end{array}$	100100	
$A \leftarrow A + B$	$\begin{array}{r} \\ 111111 \end{array}$		000
Set $Q_0 \leftarrow 0$			

$A + B$

$$\begin{array}{r} 000100 \\ - \\ 000011 \end{array}$$

Comparison b/w restoring and non-restoring algorithm.

Restoring

Needs restoring of register A if the result of subtraction is negative.

In each cycle content of register A is first shifted left and then divisor is subtracted from it.

Does not need restoring of remainder.

Slower algorithm

Non-restoring

Does not need restoring

In each case cycle content of register A is first shifted left and then division is added or subtracted with the contents of register A depending on the sign of A.

Needs restoring of remainder if remainder is negative

Faster algorithm