REMOTE SENSING DATA ACQUISITION

WAVES ELECTROMAGNETIC / ACOUSTIC

PLAYERS TARGET (CAN BE THE SOURCE ITSELF E.G.: STUDY OF THE SUN, WHALE, EARTH)
SOURCE
SENSOR
PHYSICAL MEDIUM

SENSORS PASSIVE (+) DON'T NEED TO EMIT RADIATION → LOW POWER
(−) NEED AN EXTERNAL SOURCE, CAN ONLY USE THE AVAILABLE PERTURBATION

ACTIVE (+) DON'T DEPEND ON EXTERNAL SOURCES, IT IS POSSIBLE TO MEASURE THE TIME OF FLIGHT
(−) HIGH POWER CONSUMPTION

N.B.: SENSORS ARE ALSO CLASSIFIED ACCORDING THE USED BAND

SONAR (SOUND NAVIGATION AND RANGING)

USES ACOUSTIC WAVES IN WATER, CAN BE ACTIVE OR PASSIVE

APPLICATIONS: NAVIGATION, ARMY, BATHYMETRY, FISHING

ECHO SOUNDING (BATHYMETRY): THE SOUND WAVE IS DIRECTED VERTICALLY TO MEASURE THE DEPTH

SIDE SCAN SONAR: SONAR WITH TWO MAIN LATERAL LOBES IN OPPOSITE DIRECTIONS USED TO STUDY THE SHAPE OF THE SEAFLOOR SURFACE AND/OR OBJECTS ON THE SEAFLOOR. IT USES ULTRASONIC IMPULSES (VERY SHORT PULSES) THE TIME OF FLIGHT OF THE PULSES REVEALS THE DISTANCE FROM THE TARGET $d = \frac{1}{2} v t$

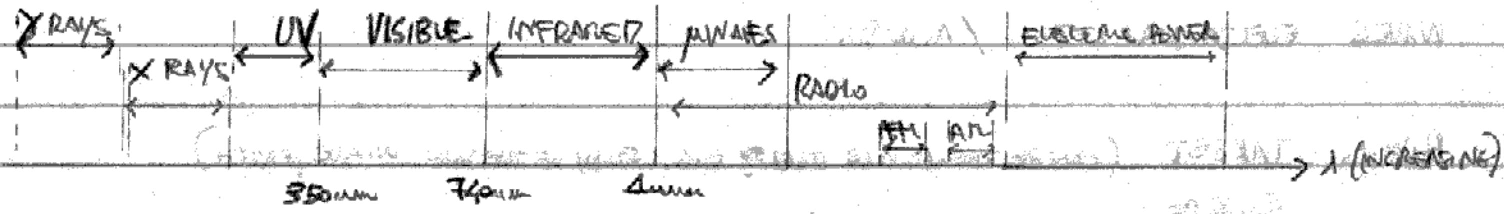
(E.G.: 2° HORIZONTAL AND 20° VERTICAL BEAMWIDTH. TRADEOFF BETWEEN RESOLUTION AND ACHIEVABLE DISTANCE AT 60-500 KHZ RESOLUTION OF 20-50 CM ACHIEVED WITH 0,1 ms PULSES. MAX DISTANCE ~ 400 m)

SODAR (SONIC DETECTION AND RANGING)

USES ULTRASONIC WAVES IN THE ATMOSPHERE

APPLICATION: METEOROLOGY

ELECTROMAGNETIC SPECTRUM



SPECTRUM STUDIED IN DIFFERENT SUBBANDS:

- DIFFERENT FREQUENCIES RADIATED BY SOURCES
- DIFFERENT TYPE OF INTERACTION WITH MATTER IN DIFFERENT WAVELENGTHS
- DIFFERENT BEHAVIOR OF THE ATMOSPHERE AT DIFFERENT FREQUENCIES
- DIFFERENT LEVEL OF EARTH RADIATION IN DIFFERENT SUBBANDS

ATMOSPHERE: OPAQUE FOR γ,X-RAYS, STRONG ABSORPTION IN RANGE 0.1 - 1 mm (INFRARED - μWAVES)

EARTH: IN THE THERMAL INFRARED AND MICROWAVE RANGE THE EMITTED INTENSITY IS MORE THAN THE REFLECTED ONE

SPECTRUM SUBBANDS: GAMMA RAYS AND X RAYS
OPTICAL FREQUENCIES (VISIBLE AND NEAR INFRARED)
THERMAL INFRARED
μWAVES
RADIO WAVES

PASSIVE SENSORS

GAMMA RAY SPECTROMETERS: MEASURE THE EMISSION OF γ RAYS FROM THE EARTH SURFACE DUE TO NUCLEAR DECAY OF RADIOACTIVE NATURAL MATERIALS (POTASSIUM, URANIUM, THORIUM). SERIOUS LIMITS TO ACHIEVABLE DISTANCE AND DEEPNESS DUE TO STRONG ABSORPTION BY ROCK AND ATMOSPHERE. (MINERAL EXPLORATION)

CAMERAS: BASED ON FILM OR DIGITAL (CCD) (SOMETIMES ANALOG IS BETTER! E.G. FOR PRECISE CONTOURS). WAVELENGTH RANGE $400nm \leq \lambda \leq 900nm$, USED W AERIAL PHOTOGRAPHY. (PHOTOGRAMMETRY, MAP CREATION)

MULTISPECTRAL/HYPERSPECTRAL SENSORS: SENSE EM WAVE COMING FROM DIFFERENT POINTS OF THE SURFACE (USUALLY REFLECTED LIGHT IN THE OPTICAL BAND) IN DIFFERENT SUBBANDS AT THE SAME TIME
MULTISPECTRAL = SOME TENS OF BANDS, HYPERSPECTRAL: FROM FEW HUNDREDS TO MANY THOUSANDS
(ENVIRONMENTAL MONITORING)

TERMOGRAPHS: SIMILAR TO MULTISPECTRAL SENSORS BUT SENSITIVE TO WAVELENGTHS FROM 10 TO 14 μm WHICH ARE RELATED TO THE TEMPERATURE OF THE TARGET OBJECT (WEATHER FORECAST, FIRES, CLIMATE MONITORING)

RADIOMETER: GROUND/NEAR-UNDERGROUND SOIL RADIATES IN WAVELENGTHS 1 cm – 1 m. INTENSITY DEPENDS ON SOIL COMPOSITION AND HUMIDITY (MINERAL EXPLORATION, MAP CREATION, HUMIDITY MAPS)

ACTIVE SENSORS

LIDAR (LIGHT DETECTION AND RANGING): SENSOR THAT EMITS LASER BEAM IN THE ATMOSPHERE AND SENSES THE RETURN (IR BAND). TWO TYPES: AIRBORNE OR SATELLITE LIDAR AND GROUND BASED LIDAR.
AIRBORNE LIDAR: USED TO MEASURE ELEVATION DATA (TOPOGRAPHY) OR POSITION AND SHAPE OF OBJECTS ON THE GROUND. TIME OF FLIGHT IS CONVERTED INTO A DISTANCE (GENERATION 3D MODELS, TOPOGRAPHIC MAPS)
GROUND BASED LIDAR: ACQUIRES INFORMATION ON THE ATMOSPHERE BASED ON THE INTENSITY OF RETURNS FROM DIFFERENT ATMOSPHERE LAYERS.

RADAR ALTIMETER: ACQUIRES ELEVATION MEASURES ALONG A SATELLITE TRACK. EM IN RANGE BETWEEN 1-5 CM, PRECISION OF 2-4 CM. NOTHING TO DO WITH LIDAR! THEY MEASURE ELEVATION VARIATIONS ON A LARGE SCALE.

IMAGING RADAR: MANY PULSES, CAN CONSTRUCT A 2D IMAGE. WAVELENGTHS IN RANGE BETWEEN 1CM - 1M
DIFFERENT λ SENSE DIFFERENT CHARACTERISTICS EG: HUMIDITY, CONDUCTIVITY, SIZES, ROUGHNESS.
WAVELENGTHS THAT PENETRATE CLOUDS ALLOW USE IN ANY WEATHER CONDITION (CLASSIFICATION, MAP CREATION, GROUND MOVES)

CHARACTERISTICS

RADIOMETRIC RANGE: INTERVAL OF VALUES OVER WHICH THE SYSTEM GIVES ACCURATE MEASUREMENTS
RADIOMETRIC RESOLUTION: MINIMAL VARIATION DETECTED BY THE SENSOR

SPATIAL COVERAGE: SIZE OF THE STUDIED AREA OR THE AREA OF INTEREST
SWATH: WIDTH OF LAND STRIP COVERED DURING ONE PASS OF A SATELLITE OR AIRPLANE (TENS TO THOUSANDS KM)
FIELD OF VIEW (FOV): FIELD OF VIEW IN ANGULAR MEASURE FROM WHICH THE SWATH CAN BE DERIVED ON NOTHING
GEOMETRIC RESOLUTION: ABILITY TO DISTINGUISH DETAILS
INSTANTANEOUS FIELD OF VIEW (IFOV): MINIMAL ANGLE BETWEEN TWO DISTINGUISHABLE ELEMENTS
GROUND RESOLUTION: SIZE OF THE AREA PROJECTED ON THE GROUND BY A SOLID ANGLE OF THE SIZE OF THE IFOV

SPECTRAL SUB-BANDS: # OF SUBBANDS AND THEIR LOCATION
SPECTRAL RESOLUTION: WIDTH OF EACH SUB BAND. IN MULTISPECTRAL SENSORS THEY HAVE ALMOST THE SAME WIDTH

TEMPORAL TEMPORAL RESOLUTION

PLATFORMS

AIRPLANES: ELEVATION FROM FEW HUNDREDS TO 3-10'000 m

(+) FLEXIBLE CHOICE OF ALTITUDE AND RESOLUTION

VERY USEFUL FOR LOCAL HIGH RESOLUTION ACQUISITIONS

RELATIVELY LOW COST FOR SPECIFIC SMALL AREAS

(−) DIFFICULT TO USE OVER LARGE AREAS

DIFFICULT TO USE FOR LONG TERM MONITORING (E.G. 10 YEARS)

SATELLITES: ELEVATION FROM FEW HUNDREDS TO TENS OF THOUSANDS KILOMETERS

(+) ALLOW CONSTANT VERY LARGE SCALE ACQUISITIONS, BOTH IN TIME AND IN AREA

IN LONG TERM THE SYSTEM BECOMES CHEAP, SINCE IT CAN ACQUIRE DATA WITH VIRTUALLY NO COSTS

(−) TOO EXPENSIVE FOR SHORT TERM MISSIONS

LIMITED BY PHYSICAL CONSTRAINTS IMPOSED BY ORBITS

NOT APPROPRIATE FOR SMALL AREAS REMOTE SENSING

SATELLITE STRUCTURE

COMMUNICATION SUB-SYSTEM: COMPOSED OF TRANSCEIVERS AND ANTENNAS
SENDS IMAGES AND INFORMATION TO EARTH
RECEIVES CONTROL COMMANDS FROM THE EARTH

MISSION PAYLOAD: THE SET OF SENSORS AND ACQUISITION COMPONENTS (CAMERAS, RADARS, ... DEPENDING ON THE MISSION)
DIFFERENT PARTNERS OFTEN SHARE THE PAYLOAD OF THE SAME SATELLITE

COMMAND AND DATA HANDLING: IN CHARGE OF HANDLING THE SENSORS AND THE STORAGE/PROCESSING OF THE COLLECTED DATA

ATTITUDE AND ORBIT CONTROL SUBSYSTEM: MONITORS POSITION AND ORIENTATION OF THE SATELLITE, PERFORMS THE ACTIONS TO MAINTAIN OR CHANGE POSITION/POINTING DIRECTION (REACTION WHEELS, GYROSCOPES, ...)

POWER SUPPLY: SOLAR PANEL TO DERIVE ENERGY FROM THE SUNLIGHT AND BATTERY FOR STORING ENERGY

THERMAL CONTROL: MONITORS AND CONTROL THE TEMPERATURE OF CRITICAL COMPONENTS, CRUCIAL FOR EXAMPLE FOR SENSING IN THE THERMAL INFRARED

ORBITS

ORBIT CHARACTERISTICS

ORBITAL ECCENTRICITY

INCLINATION: ANGLE BETWEEN THE ORBITAL PLANE AND THE EQUATORIAL PLANE ; $>90°$ RETROGRADE ORBIT, $<90°$ PROGRADE ORBIT

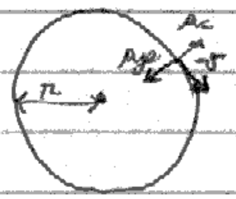
ALTITUDE (FOR CIRCULAR ORBITS): DISTANCE OF THE ORBIT FROM THE EARTH SURFACE (400 - 36000 KM)

PERIOD: TIME REQUIRED FOR ONE REVOLUTION AROUND THE EARTH (80' - 24 h)

REVISIT TIME: TIME REQUIRED FOR THE SATELLITE TO OBSERVE AGAIN THE SAME POINT ON THE EARTH (DAYS)

(GEOSTATIONARY ORBIT: 35786 KM / SUN SYNCHRONOUS ORBIT: 705 KM / GPS: 20200 KM
LAGRANGIAN POINTS: POINTS THAT ORBIT THE SUN MAINTAINING FIXED ANGLES WITH THE EARTH. USEFUL FOR STEREO VISION OF THE SUN.)

CIRCULAR ORBIT



$A_g = \frac{GM}{r^2}$ = GRAVITATIONAL ACCELERATION, M = MASS, G = GRAVITATIONAL CONSTANT

$A_c = \frac{v^2}{r}$ = CENTRIPETAL ACCELERATION

EQUATING THE VALUES:

$\boxed{v = \sqrt{\frac{GM}{r}}}$ = SPEED

$\boxed{T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}} = \sqrt{\frac{4\pi^2 r^3}{g R_E^2}}}$ = ORBITAL PERIOD

CIRCULAR ORBITS

LOW EARTH ORBITS (LEO): ALTITUDE RANGE 180 KM - 2000 KM
VERY HIGH RESOLUTION
SMALL PERIODS AND HIGH SPEED

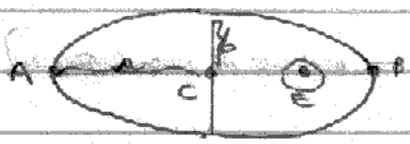
MEDIUM EARTH ORBITS (MEO): ALTITUDE RANGE 2000 - 35786 KM (MOST USED 20000 KM)
PERIOD 2-24 h (USUALLY 12 h)

GEOSTATIONARY ORBIT: 35786 KM

HIGH EARTH ORBITS (HEO): ABOVE GEOSTATIONARY

GEOSTATIONARY ORBITS: THE SATELLITE ORBITS THE EARTH WITH THE SAME PERIOD OF THE EARTH ROTATION SO THE SATELLITE STAYS IN A FIXED POINT IN THE EARTH REFERENCE FRAME. ZERO INCLINATION ≡ EQUATORIAL ORBIT. (WEATHER AND TELECOMMUNICATION SATELLITES)

ELLIPTICAL ORBITS:

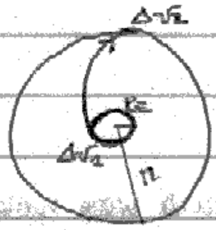


$e = \overline{CE}/\overline{CA}$ ≡ ECCENTRICITY ($e \in [0,1)$)

$b^2 = a^2(1-e^2)$, $T = 2\pi\sqrt{\frac{a^3}{GM}}$ ≡ PERIOD

$r^2 \frac{d\vartheta}{dt}$ = CONSTANT ≡ CONSERVATION OF THE ANGULAR MOMENTUM

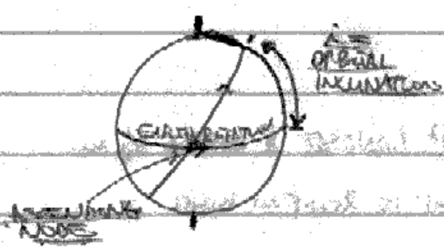
SATELLITE LAUNCH: AN INITIAL BOOST BRINGS THE SATELLITE ON AN ELLIPTICAL ORBIT



$$\Delta v_1 = \sqrt{\frac{2r\,GM}{R_E(r+R_E)}}$$

THEN WITH A CORRECTION THE SPEED IS INCREASED TO PUT THE SATELLITE ON A CIRCULAR ORBIT $\Delta v_2 = \sqrt{\frac{GM}{r}} - \sqrt{\frac{2r\,GM}{R_E(r+R_E)}}$

NEAR POLAR ORBITS: A ORBIT IN WHICH A SATELLITE PASSES ABOVE NEARLY BOTH POLES



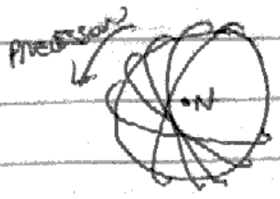
INCLINATION i IS NEAR 90°: $i < 90°$ PROGRADE ORBIT

$i > 90°$ RETROGRADE ORBIT

DUE TO EARTH'S ROTATION THE ASCENDING NODE MOVES ROUND THE EQUATOR (~12-16 REVOLUTIONS PER DAY)

PRECESSION: THE EARTH IS NOT A PERFECT SPHERE ($R_e \cong 6378$ km, $R_p \cong 6356$ km)
THIS CAUSES TWO TYPES OF PRECESSION

NODAL PRECESSION: THE ASCENDING NODE ROTATES WITH A CONSTANT SPEED, THAT IS, THE ORBITAL PLANE CHANGES ITS DIRECTION WITH A ROTATION AROUND THE EARTH AXIS. IMPORTANT FOR SUN SYNCHRONOUS ORBITS



$$\Omega_p = -\frac{3 J_2 \sqrt{GM}\, R_e^2\, a^{-7/2} \cos i}{2(1-e^2)^2}$$

(SPEED OF ASCENDING NODE ROTATION)

APSIDAL PRECESSION: THE ELLIPSE AXES ROTATE WITH A CONSTANT SPEED IN ORBITAL PLANE
IMPORTANT FOR ELLIPTIC ORBITS (E.G. MOLNIYA) (REGARDS ELLIPTIC ORBITS)

$$\omega_p = -\frac{3 J_2 \sqrt{GM}\, R_e^2\, a^{-7/2} (1-5\cos^2 i)}{2(1-e^2)^2}$$

SUN SYNCHRONOUS ORBITS
WE EXPLOIT NODAL PRECESSION TO MAKE THE ORBITAL PLANE ROTATE WITH THE SAME PERIOD WITH WHICH THE EARTH ROTATES THE SUN. THESE ORBITS ENSURE A CONSTANT ILLUMINATION DURING THE YEAR.
(A PROGRADE PRECESSION IS NEEDED: $\Omega_p > 0$ THUS i MUST BE $> 90°$, THE ORBIT ITSELF IS RETROGRADE)
(IN THIS WAY THE SATELLITE VISITS EACH POINT OF THE EARTH SURFACE ALWAYS AT THE SAME TIME)

DEATH OF A SATELLITE
WHEN IT FINISHES THE FUEL TO CORRECT ORBIT. AT EACH REVOLUTION LANDSAT 5 LOSES 1/2 m THAT IS 5 m PER DAY. IN THE ATMOSPHERE IS BURNT. IF THE ORBIT IS 800 km HIGH IT TAKES ~100 YEARS TO FALL. WITH A LOW ORBIT (E.G. MILITARY) 180 km IT TAKES A FEW DAYS

N.B. TO AVOID APSIDAL PRECESSION $(1-5\cos^2 i)=0 \Rightarrow i \cong 63.4°$ OR $i \cong 116.6°$

MAXWELL'S EQUATIONS IN FREE SPACE

THEY DESCRIBE THE MUTUAL INTERACTION BETWEEN ELECTRIC AND MAGNETIC FIELDS

$$\boxed{\begin{aligned} \nabla\cdot E &= 0 \\ \nabla\cdot B &= 0 \\ \nabla\times E &= -\frac{\partial B}{\partial t} \\ \nabla\times B &= \varepsilon_0\mu_0\frac{\partial E}{\partial t}\end{aligned}}$$

ε_0 = ELECTRICAL PERMITTIVITY OF FREE SPACE
μ_0 = ELECTRICAL PERMEABILITY OF FREE SPACE

$$\begin{cases} \nabla\times(\nabla\times E) = \nabla\times\left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla\times B) \\ \nabla\times(\nabla\times E) = \nabla(\nabla\cdot E) - \nabla^2 E \end{cases} \Rightarrow \boxed{\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2} = \nabla^2 E}$$

WAVE EQUATIONS

I CAN DO THE SAME FOR B

$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$ = SPEED OF LIGHT IN VACUUM $\Rightarrow$ $\boxed{\varepsilon_0\mu_0\frac{\partial^2 B}{\partial t^2} = \nabla^2 B}$

WAVE EQUATIONS IN CARTESIAN COORDINATES:

$$\frac{1}{c^2}\cdot\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2}$$

$$\frac{1}{c^2}\cdot\frac{\partial^2 B}{\partial t^2} = \frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2} + \frac{\partial^2 B}{\partial z^2}$$

I ASSUME THAT THE FIELDS ONLY DEPEND ON Z (CONSTANT OVER x-y PLANE):

$$E = E(z,t) \qquad B = B(z,t)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\frac{1}{c^2}\cdot\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial z^2} \qquad \frac{1}{c^2}\cdot\frac{\partial^2 B}{\partial t^2} = \frac{\partial^2 B}{\partial z^2}$$

BUT ALSO: $\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t}$ $\qquad \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = \varepsilon_0\mu_0\frac{\partial E_z}{\partial t}$

N.B.: $\nabla\times E = \hat{u}_x\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \hat{u}_y\left(-\frac{\partial E_z}{\partial x} + \frac{\partial E_x}{\partial z}\right) + \hat{u}_z\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$

BUT THE FIELDS DON'T VARY ALONG x AND y, HENCE:

$$\boxed{\frac{\partial E_z}{\partial t} = 0} \qquad \boxed{\frac{\partial B_z}{\partial t} = 0}$$

THE COMPONENTS ALONG Z DIRECTION ARE CONSTANT IN TIME, WE ASSUME $\varnothing$

CONSIDER THE WAVE EQUATION EVALUATED FOR THE x COMPONENT:

$$\frac{1}{c^2}\cdot\frac{\partial^2 E_x}{\partial t^2} = \frac{\partial^2 E_x}{\partial z^2}$$

ITS SOLUTION IS OF THE TYPE:

$$E_x(z,t) = f(z-ct) + g(z+ct)$$

THIS MUST EXIST WITH A MAGNETIC FIELD: I EVALUATE $\nabla \times E = -\frac{\partial B}{\partial t}$ ALONG $\hat{y}$

$$-\cancel{\frac{\partial E_z}{\partial x}} + \frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$ (WE DON'T HAVE EZ COMPONENT, ONLY Ex)

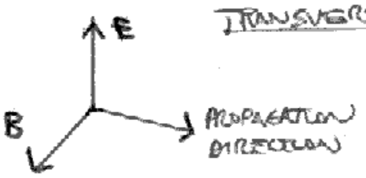
$$\frac{\partial B_y}{\partial t} = -\frac{\partial}{\partial z}(E_x(z,t)) = -\frac{\partial}{\partial z}\Big(f(z-ct)+g(z+ct)\Big) = \frac{1}{c}\cdot\frac{\partial f(z-ct)}{\partial t} - \frac{1}{c}\cdot\frac{\partial g(z+ct)}{\partial t}$$

INTEGRATING IN TIME WE OBTAIN:

$$\boxed{\begin{aligned} E_x(z,t) &= f(z-ct) + g(z+ct) \\ B_y(z,t) &= \frac{1}{c} f(z-ct) + \frac{1}{c} g(z+ct) \end{aligned}}$$ ← THE BACKWARD WAVE FOR B HAS OPPOSITE SIGN!

PLANE WAVES IN FREE SPACE

CONSIDER THE FORWARD WAVE. E AND B VECTORS ARE ORTHOGONAL TO EACH OTHER AND TO THE DIRECTION OF PROPAGATION (RIGHT HANDED BASIS). FOR THIS REASON WE CALL THEM TRANSVERSE WAVES. A PLANE WAVE CARRIES SOME POWER:



$$\boxed{S = \frac{1}{\mu_0} E \times B} \equiv \text{POYNTING VECTOR}$$

$|S|$ = INSTANTANEOUS POWER THAT TRANSVERSES THE SURFACE

THE IRRADIANCE IS THE POWER THAT TRANSVERSES A UNIT AREA ORTHOGONAL TO THE PROPAGATION DIRECTION

N.B.: THIS SOLUTION SEEN ABOVE DOESN'T DESCRIBE THE E.M. WAVES GENERATED BY AN ANTENNA BUT AT LARGE DISTANCES WE CAN APPROXIMATE IT WITH A PLANE WAVE.

WE CAN THINK OF AN ELECTROMAGNETIC WAVEFORM AS A SUM OF HARMONICS (FOURIER SERIES)

IF THE PROPAGATION IS NOT IN VACUUM THE SPEED OF DIFFERENT COMPONENTS CAN BE DIFFERENT:

$$\omega = \omega(k), \quad k = \frac{2\pi}{\lambda} \equiv \text{WAVE NUMBER}$$

EXAMPLE:

$$\begin{cases} E_x = E_0 \cos(\omega t - kz) \\ E_y = 0 \\ E_z = 0 \end{cases} \qquad \begin{cases} B_x = 0 \\ B_y = \frac{E_0}{c}\cos(\omega t - kz) \\ B_z = 0 \end{cases} \qquad c = \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

POWER FOR A SINUSOIDAL WAVE:

$$I = \frac{E_0^2}{2Z_0}, \quad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\,[\Omega]$$

POLARIZATION

$$E_x = E_{0x} \cos(\omega t - kz - \phi_x)$$
$$E_y = E_{0y} \cos(\omega t - kz - \phi_y)$$

LINEAR $\phi_x - \phi_y = 0, \pi, -\pi$

CIRCULAR $E_{0x} = E_{0y}$

RIGHT HANDED $\phi_y - \phi_x = \pi/2$

LEFT HANDED $\phi_y - \phi_x = -\pi/2$

STOKES VECTOR

$$S_0 = \langle E_{0x}^2 \rangle + \langle E_{0y}^2 \rangle$$
$$S_1 = \langle E_{0x}^2 \rangle - \langle E_{0y}^2 \rangle$$
$$S_2 = \langle 2 E_{0x} E_{0y} \cos(\phi_x - \phi_y) \rangle$$
$$S_3 = \langle 2 E_{0x} E_{0y} \sin(\phi_x - \phi_y) \rangle$$

DEGREE OF POLARIZATION

$$DOP = \sqrt{\frac{S_1^2 + S_2^2 + S_3^2}{S_0^2}}$$

IRRADIANCE $F = \frac{S_0}{2 Z_0}$

NO WAVE IS REALLY MONOCHROMATIC, WE CAN FOCUS THEN ON PASS-BAND WAVES;

$$E(z,t) = E_0(z,t)\, e^{j(k_0 z - \omega_0 t)} + E_0^*(z,t)\, e^{-j(k_0 z - \omega_0 t)}$$

$E_0(z,t)$ IS A SLOWLY VARYING FUNCTION OF z AND t
WE COULD WRITE $E(z,t)$ ALSO IN TERMS OF SLOWLY VARYING VALUES OF ϕ_x, ϕ_y

COHERENCE TIME: MAXIMUM SIZE OF A TIME INTERVAL OVER WHICH THE PHASES CAN BE CONSIDERED CONSTANT

GROUP VELOCITY DISPERSION

IF THE MEDIUM IS NOT VACUUM THEN $\omega = \omega(k)$ AND DIFFERENT HARMONIC COMPONENTS WILL TRAVEL WITH DIFFERENT SPEEDS. N.B. IN VACUUM PHASE VELOCITY = GROUP VELOCITY

PHASE VELOCITY $\boxed{v_p = \frac{\omega_0}{k_0}}$

GROUP VELOCITY $\boxed{v_g = \left.\frac{\partial \omega}{\partial k}\right|_{\omega=\omega_0}}$ = VELOCITY OF THE ENVELOPE

THE ENVELOPE BROADENS IF THE SECOND DERIVATIVE IS NOT ZERO $\left(-\left.\frac{\partial^2 \omega}{\partial k^2}\right|_{\omega=\omega_0} \right)$
THAT MEANS THAT THERE IS NO UNIQUE VELOCITY FOR THE PULSE

DOPPLER EFFECT

SHIFT OF THE PERCEIVED FREQUENCY DUE TO THE RELATIVE MOTION BETWEEN SOURCE AND TARGET.

APPROXIMATION $\omega' = \omega\left(1 + \frac{v\cos(\theta)}{c}\right)$

(EXAMPLE: $v = 7$ km/s, $\theta = 10°$, $\nu = 5$ GHz → SHIFT OF ~ 115 kHz)

RADIOMETRIC QUANTITIES

RADIANT ENERGY Q_e: TOTAL AMOUNT OF ENERGY CARRIED BY THE E.M. WAVE [J]

RADIANT FLUX (OR RADIANT POWER): RADIANT ENERGY PER UNIT TIME $\boxed{\phi = \frac{dQ_e}{dt}}$ [W]
N.B.: IMPORTANT BECAUSE IT REPRESENTS A UNIT INDEPENDENT FROM THE DIRECTIONALITY OF THE SOURCE

SPECTRAL POWER ϕ: RADIANT POWER PER WAVELENGTH OR HERTZ [W/m], [W/Hz] $\boxed{\phi(\lambda) = \frac{\partial\phi}{\partial\lambda}}$

RADIANT INTENSITY I: POWER RADIATED BY A POINT SOURCE IN A GIVEN DIRECTION PER SOLID ANGLE [W/sr] $\boxed{I = \frac{\partial\phi}{\partial\Omega}}$

EXCITANCE M: RADIANT POWER EMITTED PER UNIT SOURCE AREA $\left[\frac{W}{m^2}\right]$, $\left[\frac{W}{Hz}\right]$ $\boxed{M = \frac{\partial\phi}{\partial A}}$

SPECTRAL RADIANT EXITANCE: $\boxed{M(\lambda) = \frac{\partial M}{\partial\lambda}}$

FOR EXTENDED SOURCES

IRRADIANCE E: RADIANT INCIDENT POWER PER UNIT AREA $\left[\frac{W}{m^2}\right]$ $\boxed{E = \frac{\partial\phi}{\partial A}}$ FOR ILLUMINATED OBJECTS

RADIANCE L: RADIANT POWER EMITTED IN ONE DIRECTION PER SOLID ANGLE AND UNIT AREA PROJECTED IN THE PLANE ORTHOGONAL TO THE CONSIDERED DIRECTION $\left[\frac{W}{sr\, m^2}\right]$

$$L = \frac{\partial^2\phi}{\partial\Omega\,\partial A\cos(\theta)}$$

SPECTRAL RADIANCE: RADIANCE PER WAVELENGTH $\left[\frac{W}{sr\, m^3}\right]$ $\boxed{L(\lambda) = \frac{\partial L}{\partial\lambda}}$

FOR EXTENDED SOURCES

N.B.: AN E.M. WAVE IS ALWAYS COMPOSED OF A NUMBER OF ELEMENTARY QUANTITIES CALLED PHOTONS (QUANTIZATION). EVERY E.M. WAVE IS A STREAM OF A HUGE N° OF THESE ELEMENTARY WAVE PACKETS, EACH WITH ENERGY $E = \frac{hc}{\lambda} = h\nu$

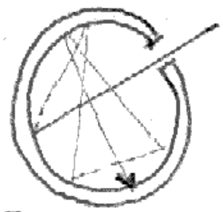
THIS IS USEFUL TO BETTER UNDERSTAND COHERENCE AND COLLIMATION...

BLACK BODY RADIATION

ANY BODY WITH A $T > 0K$ EMITS RADIATION. THE BODY ABSORBS ENERGY IN THE FORM OF ELECTROMAGNETIC WAVES/HEAT AND THIS CAUSES THERMAL AGITATION OF ITS MOLECULES, THAT IS RESPONSIBLE FOR THE E.M. ENERGY RADIATED.

IF THE TEMPERATURE IS KEPT CONSTANT, A DYNAMIC THERMAL EQUILIBRIUM IS SET SUCH THAT THE EMITTED ENERGY PRECISELY COMPENSATES THE ABSORBED ONE.

THE COLOR OF THE LIGHT EMITTED FROM A HOT BODY DEPENDS BOTH ON THE TEMPERATURE AND ON THE CHARACTERISTICS OF THE BODY.



CAVITY WITH A SMALL HOLE; INCOMING LIGHT CANNOT BE REFLECTED OUT AND IT IS ABSORBED. THE ENERGY IS EMITTED AFTERWARDS AS A RADIATION. IT IS EXPERIMENTALLY PROVED THAT THE COLOR ONLY DEPENDS ON THE TEMPERATURE

RADIATION LAWS

THE EMITTED RADIATION IS NOT MONOCHROMATIC AND IT THUS CONTAINS DIFFERENT WAVELENGTHS (COLORS) IN DIFFERENT AMOUNTS, WE CAN DESCRIBE THIS USING SPECTRAL RADIANCE

PLANK'S LAW $\boxed{L(\lambda;T) = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}}$

h = PLANK CONSTANT
k = BOLTZMANN CONSTANT
L = SPECTRAL RADIANCE

N.B.: THIS LAW WAS MADE BY ASSUMING THAT THE ENERGY COULD ONLY BE EMITTED IN DISCRETE QUANTITIES, MULTIPLES OF THE QUANTUM ENERGY VALUE $E = hc/\lambda$

STEFAN LAW $\boxed{M = \sigma T^4}$

M = EXITANCE
σ = STEFAN CONSTANT
(TOTAL EMITTED RADIATION FROM A BLACKBODY)

WIEN'S DISPLACEMENT LAW $\boxed{\lambda_{MAX} = b/T}$

b = CONSTANT (DOMINANT WAVELENGTH OF A BLACKBODY)

RAYLEIGH-JEANS APPROXIMATION FOR LARGE WAVELENGTHS WE CAN SEE THAT THE FOLLOWING CONDITION HOLDS $\frac{hc}{\lambda kT} \ll 1$ AND SO $\boxed{L(\lambda;T) \approx \frac{2ckT}{\lambda^4}}$

(TAYLOR EXPANSION $e^x \approx 1 + x$)

N.B.: AT ROOM TEMPERATURE THE APPROXIMATION HOLDS IN THE MICROWAVE AND RADIO REGION

REAL BODIES

ALL BODIES ARE NON-IDEAL AND THE PLANCK'S MODEL DOES NOT DESCRIBE THE RADIATION EXACTLY. WE INTRODUCE THEN A CORRECTION COEFFICIENT, DIMENSIONLESS, CALLED <u>EMISSIVITY $\varepsilon(\lambda)$</u>

$$L(\lambda) = \varepsilon(\lambda) \cdot \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} = \varepsilon(\lambda) \cdot L(\lambda, T)$$

N.B.: $\varepsilon(\lambda)$ IS NOT PERMITTIVITY!!!

IF $\varepsilon(\lambda)$ CAN BE CONSIDERED CONSTANT OVER THE RANGE OF WAVELENGTHS OF INTEREST ONE USUALLY SPEAKS OF <u>GRAY BODY</u>

<u>BRIGHTNESS TEMPERATURE T_b</u>: IS THE TEMPERATURE OF AN IDEAL BLACK BODY THAT WOULD HAVE THE SAME SPECTRAL RADIANCE AS THE CONSIDERED BODY, AT THE SAME WAVELENGTH

$$\varepsilon(\lambda) \cdot \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT_b}} - 1}$$

USING THE RAYLEIGH-JEANS APPROXIMATION WE HAVE $\boxed{T_b \approx \varepsilon(\lambda) T}$

SOLAR RADIATION

SUN CAN BE CONSIDERED AS A GRAY BODY WITH A TEMPERATURE AROUND 5800K AND EMISSIVITY $\varepsilon \approx 0.99$. (OR 6000K BLACK BODY)

THE RADIUS IS ABOUT $R_S = 6.96 \times 10^8$ m AND THE DISTANCE FROM THE EARTH IS $d = 1.5 \times 10^{11}$ m

$$M = \varepsilon \sigma T^4 \approx 6.35 \times 10^7 \left[\frac{W}{m^2}\right] \quad \text{(EXITANCE)}$$

$$\phi = 4\pi R_S^2 \cdot M \approx 3.87 \times 10^{26} \,[W] \quad \text{(TOTAL RADIANT FLUX)}$$

$$E = \frac{\phi}{4\pi d^2} \approx 1.37 \times 10^3 \,[W/m^2] \quad \text{(IRRADIANCE OF THE EARTH)}$$

TERRESTRIAL RADIATION

THE PEAK EMISSION OF A T=300K BLACK BODY IS AROUND $\lambda = 10.3\ \mu m$

WE CONSIDER THE BANDS: VISIBLE, NEAR INFRARED/THERMAL INFRARED, MICROWAVE

WE HAVE A FAST DECAY AT HIGH FREQUENCIES AND A SLOW DECAY AT LOW FREQUENCIES: USUAL OBJECTS AT ROOM TEMPERATURE DON'T EMIT RADIATION IN THE VISIBLE BUT INSTEAD THEY EMIT SMALL DETECTABLE QUANTITIES OF RADIATION IN THE MW AVE BAND (USED FOR PASSIVE SYSTEMS).

ELECTROMAGNETIC RADIATION PRINCIPLES

ENERGY TRANSFER WAYS: CONDUCTION, CONVECTION AND RADIATION

TO UNDERSTAND HOW ELECTROMAGNETIC RADIATION IS CREATED, IT PROPAGATES AND INTERACTS WITH MATTER, WE CAN USE TWO DIFFERENT MODELS: WAVE MODEL AND PARTICLE MODEL.

WAVE MODEL

THE ELECTROMAGNETIC RADIATION (EMR) IS AN ELECTROMAGNETIC WAVE THAT TRAVELS THROUGH SPACE AT THE SPEED OF LIGHT (MAXWELL). THE ELECTROMAGNETIC WAVE CONSISTS OF TWO FLUCTUATING FIELDS, ELECTRIC AND MAGNETIC, ORTHOGONAL TO EACH OTHER AND PERPENDICULAR TO THE DIRECTION OF TRAVEL.

ELECTROMAGNETIC RADIATION IS GENERATED WHEN AN ELECTRICAL CHARGE IS ACCELERATED. $c = \lambda \cdot \nu$

THE SUN PRODUCES A CONTINUOUS SPECTRUM OF ELECTROMAGNETIC RADIATION FROM γ RAYS TO RADIO WAVES. (CAN BE SEEN AS A 6000 K BLACKBODY) DUE TO NUCLEAR FUSION ON THE SURFACE

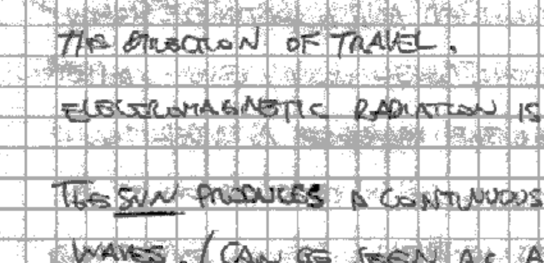
$$\boxed{M_\lambda = \sigma T^4}$$ = TOTAL EMITTED RADIATION FROM A BLACKBODY (STEFAN-BOLTZMANN LAW) NB.: EMITTED RADIATION IS A FUNCTION OF TEMPERATURE

σ = STEFAN BOLTZMANN CONSTANT

$$\boxed{\lambda_{max} = \frac{k}{T}}$$ = DOMINANT WAVELENGTH OF A BLACKBODY (WEIN'S DISPLACEMENT LAW) NB.: 0.48 μm FOR THE SUN (GREEN LIGHT)

k = CONSTANT

THE EARTH CAN BE SEEN AS A 300 K BLACKBODY, $\lambda_{max} = 9.7\ \mu m$



THE AREA UNDER EACH CURVE IS THE TOTAL RADIANT ENERGY M_λ EMITTING EACH OBJECT

$T \uparrow \Rightarrow \lambda_{max} \downarrow, M_\lambda \uparrow$

PARTICLE MODEL

LIGHT IS A PARTICULAR KIND OF MATTER (EINSTEIN). WHEN LIGHT INTERACTS WITH MATTER IT BEHAVES THOUGHT IT IS COMPOSED OF MANY INDIVIDUAL BODIES CALLED PHOTONS THAT HAVE AN ENERGY AND A MOMENTUM.

QUANTUM THEORY: ENERGY IS TRANSFERRED IN DISCRETE PACKETS CALLED QUANTA OR PHOTONS

$$\boxed{Q = h \times \nu}$$ = ENERGY OF A QUANTUM

ν = FREQUENCY; h = PLANCK CONSTANT

$$Q = \frac{h \times c}{\lambda}$$ THE ENERGY OF THE QUANTUM IS INVERSELY PROPORTIONAL TO THE WAVELENGTH

ELECTRONS ROTATE AROUND THE NUCLEUS OF AN ATOM. DIFFERENT SUBSTANCES ARE MADE BY ATOMS WITH DIFFERENT N° OF ELECTRONS AND DISPLACEMENT.

ELECTRON'S MOTION IS RESTRICTED TO A DEFINITE RANGE FROM THE NUCLEUS. ORBITAL PATHS OF ELECTRONS CAN BE THOUGHT OF AS ENERGY CLASSES OR LEVELS. TO CLIMB TO A HIGHER CLASS WORK MUST BE PERFORMED. THE ENERGY TO JUMP MUST BE AT LEAST SUFFICIENT TO EXCEED THE STATES. THE ELECTRON IN HIGHER STATE HAS POTENTIAL ENERGY. AFTER $\sim 10^{-8}$ s IT FALLS BACK TO THE LOWEST EMPTY ORBIT, EMITTING RADIATION. THE WAVELENGTH OF RADIATION IS A FUNCTION OF THE AMOUNT OF WORK DONE ON THE ATOM (THE QUANTUM OF ENERGY ABSORBED TO CAUSE THE ELECTRON TO JUMP TO A HIGHER ORBIT).

RADIATION IS PRODUCED BY CHANGES IN THE ENERGY LEVELS OF THE OUTER VALENCE ELECTRONS.

THE WAVELENGTHS OF ENERGY PRODUCED ARE FUNCTION OF THE PARTICULAR ORBITAL LEVELS OF THE ELECTRONS INVOLVED. IF THE ATOM ABSORBS ENOUGH ENERGY TO BECOME IONIZED AND IF A FREE ELECTRON DROPS IN TO FILL THE VACANT ENERGY LEVEL, THEN THE RADIATION GIVEN OFF IS UNQUANTIZED AND A CONTINUOUS SPECTRUM IS PRODUCED. EVERY TIME AN ELECTRON JUMPS FROM A HIGHER TO A LOWER ENERGY LEVEL, A PHOTON MOVES AWAY AT THE SPEED OF LIGHT. ELECTRON ORBITS ARE LIKE THE RUNGS OF A LADDER: AN ELECTRON DOESN'T ALWAYS USE CONSECUTIVE RUNGS BUT HAS SOME SELECTION RULES.

SUBSTANCES HAVE DIFFERENT COLORS BECAUSE OF DIFFERENCES IN THEIR ENERGY LEVELS AND THE SELECTION RULES.

QUANTUM LEAP: WHEN AN ELECTRON DISAPPEARS FROM ITS ORIGINAL ORBIT AND REAPPEARS IN ITS DESTINATION ORBIT WITHOUT EVER HAVING TO TRAVERSE ANY OF THE POSITIONS IN BETWEEN. IT IS POSSIBLE FOR THE ELECTRON TO LEAP FROM AN EXCITED ORBIT TO THE GROUND STATE IN A SERIES OF JUMPS. THE ENERGIES EMITTED IN TWO DIFFERENT JUMPS MUST SUM THE TOTAL OF THE SINGLE LARGE JUMP.

N.B.: WHEN ENERGY INTERACTS WITH MATTER IT IS USEFUL TO DESCRIBE IT AS DISCRETE PACKETS OF ENERGY, OR QUANTA.

SCATTERING

SCATTERING DIFFERS FROM REFLECTION IN THAT THE DIRECTION ASSOCIATED WITH SCATTERING IS UNPREDICTABLE. THREE TYPES OF SCATTERING: RAYLEIGH, MIE AND NON-SELECTIVE

RAYLEIGH: OCCURS WHEN THE DIAMETER OF THE MATTER IS MANY TIMES SMALLER THAN THE WAVELENGTH OF THE INCIDENT ELECTROMAGNETIC RADIATION.

THERE IS ABSORPTION AND RE-EMISSION OF RADIATION, IT IS IMPOSSIBLE TO PREDICT THE DIRECTION.

AMOUNT OF SCATTERING $\propto 1/\lambda^4$ OF THE RADIATION WAVELENGTH

RAYLEIGH SCATTERING IS RESPONSIBLE FOR THE BLUE SKY; SHORT VIOLET AND BLUE WAVELENGTHS ARE MORE EFFICIENTLY SCATTERED THAN THE OTHERS. AT SUNSET/SUNRISE THEY ARE SCATTERED MORE (LONGER PATH) WE SEE THEN THE RESIDUE THAT IS NOT SCATTERED (ORANGE AND RED).

MIE: OCCURS WHEN THERE ARE SPHERICAL PARTICLES IN THE ATMOSPHERE WITH DIAMETER APPROXIMATELY EQUAL TO THE WAVELENGTH OF RADIATION CONSIDERED (FOR VISIBLE LIGHT: WATER VAPOR, DUST, POLLUTION).

LONGER WAVELENGTHS RESPECT TO RAYLEIGH AND GREATER AMOUNT

N.B.: ALSO POLLUTION CONTRIBUTES TO SUNSET/SUNRISE COLOR: MORE PARTICLES WILL BE PRESENT

AND MORE BLUE/VIOLET LIGHT WILL BE SCATTERED AWAY.

<u>NON SELECTIVE</u>: PRODUCED WHEN THERE ARE PARTICLES IN THE ATMOSPHERE SEVERAL TIMES THE DIAMETER OF THE RADIATION BEING TRANSMITTED. NON SELECTIVE BECAUSE ALL WAVELENGTHS OF LIGHT ARE SCATTERED. FOR EXAMPLE WATER DROPLETS, FOG AND CLOUDS CAUSE THEM TO APPEAR **WHITE**.

N.B.: SCATTERING IS VERY IMPORTANT IN REMOTE SENSING AS IT CAN SEVERELY **REDUCE** THE INFORMATION CONTENT OF DATA.

ABSORPTION

IS THE PROCESS BY WHICH RADIANT ENERGY IS ABSORBED AND CONVERTED INTO OTHER FORMS OF ENERGY. WE HAVE ATMOSPHERIC WINDOWS IN THE ATMOSPHERE WHERE THE ENERGY IS EFFECTIVELY TRANSMITTED. OCCURS WHEN ENERGY OF THE SAME FREQUENCY AS THE RESONANT FREQUENCY OF AN ATOM OR MOLECULE IS ABSORBED, PRODUCING AN EXCITED STATE. IF INSTEAD OF RE-RADIATING A PHOTON THE ENERGY IS TRANSFORMED INTO HEAT AND IS RADIATED AT LONGER WAVELENGTHS, ABSORPTION OCCURS. TRANSMISSION IS INVERSELY RELATED TO THE EXTINCTION COEFFICIENT TIMES THE THICKNESS OF THE LAYER.

REFLECTANCE

IS THE PROCESS WHEREBY RADIATION BOUNCES OFF AN OBJECT LIKE A CLOUD OR THE TERRAIN. THIS PROCESS IS MORE COMPLICATED AND INVOLVES RE-RADIATION OF PHOTONS IN UNISON BY ATOMS OR MOLECULES IN A LAYER ONE-HALF WAVELENGTH DEEP. THREE KINDS OF REFLECTANCE:

<u>SPECULAR REFLECTION</u>: WHEN THE SURFACE IS SMOOTH

<u>DIFFUSE REFLECTION</u>: WHEN THE SURFACE IS ROUGH (E.G.: WHITE PAPER OR POWDER)

<u>PERFECTLY DIFFUSE REFLECTION</u>: VERY ROUGH **SURFACE** / LAMBERTIAN SURFACE THE RADIANT FLUX LEAVING THE SURFACE IS CONSTANT FOR ANY ANGLE OF REFLECTANCE

RADIATION BUDGET EQUATION

$$\boxed{\Phi_{i_\lambda} = \Phi_{reflected_\lambda} + \Phi_{absorbed_\lambda} + \Phi_{transmitted_\lambda}} \rightarrow [W]$$

TOTAL AMOUNT OF RADIANT FLUX INCIDENT TO TERRAIN

RADIANT FLUXES REFLECTED, ABSORBED AND TRANSMITTED

<u>HEMISPHERICAL REFLECTANCE</u> $\rho_\lambda = \Phi_{reflected} / \Phi_{i\lambda}$

<u>HEMISPHERICAL TRANSMITTANCE</u> $\tau_\lambda = \Phi_{transmitted} / \Phi_{i\lambda}$

<u>HEMISPHERICAL ABSORBANCE</u> $\alpha_\lambda = \Phi_{absorbed} / \Phi_{i\lambda}$

N.B.: BY MULTIPLYING x100 THE ABOVE QUANTITIES WE HAVE THE PERCENTAGES.

IRRADIANCE: AMOUNT OF RADIANT FLUX INCIDENT UPON A SURFACE PER UNIT AREA $\boxed{E_\lambda = \frac{\Phi_\lambda}{A}}$ $\left[\frac{W}{m^2}\right]$

EXITANCE: AMOUNT OF RADIANT FLUX LEAVING PER UNIT AREA OF THE PLANE SURFACE $\boxed{M_\lambda = \frac{\Phi_\lambda}{A}}$ $\left[W/m^2\right]$

RADIANCE: IS THE RADIANT FLUX PER UNIT SOLID ANGLE LEAVING AN EXTENDED SOURCE IN A GIVEN DIRECTION PER UNIT PROJECTED SOURCE AREA IN THAT DIRECTION $\boxed{L_\lambda = \frac{\Phi/\Omega}{A\cos\theta}}$ $\left[\frac{W}{m^2 \cdot sr}\right]$

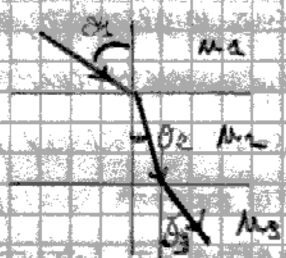
REFRACTION INDEX

IS A MEASURE OF THE OPTICAL DENSITY OF A SUBSTANCE $\boxed{n = \frac{c}{c_n}}$ ← SPEED OF LIGHT IN VACUUM

N.B.: THE SPEED OF LIGHT IN A SUBSTANCE CAN NEVER REACH THE SPEED OF LIGHT IN VACUUM SO ITS REFRACTIVE INDEX IS ALWAYS > 1. LIGHTS IN PRACTICE TRAVELS MORE SLOWLY BECAUSE OF HIGHER DENSITY.

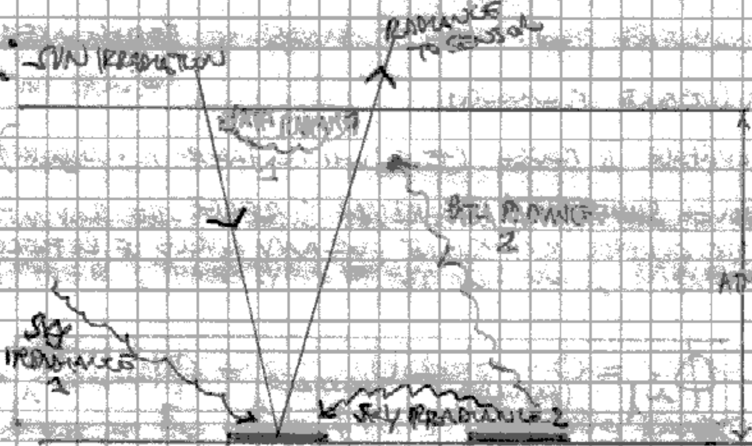
SNELL'S LAW

DESCRIBES REFRACTION FOR THE TRANSMITTED WAVE $\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$



THE INCIDENT ENERGY IS BENT FROM ITS NORMAL TRAJECTORY

NOTE:



PLANCK RADIATION

EVERY PHYSICAL BODY SPONTANEOUSLY AND CONTINUOUSLY EMITS ELECTROMAGNETIC RADIATION

PLANCK'S LAW DESCRIBES (NEARLY) THE EMITTED RADIATION IN THERMODYNAMIC EQUILIBRIUM CONDITION

PLANCK RADIATION IS SAID TO BE THERMAL, AS IT DEPENDS ON TEMPERATURE (MORE TEMPERATURE, MORE RADIATION). THE ENERGY IS TRANSFERED AS HEAT.

PLANCK RADIATION IS THE GREATEST AMOUNT OF RADIATION THAT ANY BODY AT THERMAL EQUILIBRIUM CAN EMIT FROM ITS SURFACE

THE EMISSIVITY OF AN INTERFACE IS KNOWN AS ITS TRANSMITTANCE OR AS ITS **ABSORBANCE**

$0 < \text{EMISSIVITY} < 1$, EMISSIVITY + REFLECTIVITY = 1

N.B.: AN IDEALLY PERFECTLY REFLECTING INTERFACE HAS EMISSIVITY $\emptyset$, REFLECTIVITY 1
AN IDEALLY PERFECTLY TRANSMITTING INTERFACE HAS EMISSIVITY 1, REFLECTIVITY 0

BLACK BODY

A BLACK BODY IS A BODY THAT ABSORBES ALL THE RADIATION INCIDENT ON IT AND THAT IT IS INTERFACED WITH ANOTHER MEDIUM WITH EMISSIVITY ONE (PERFECTLY TRANSMITTING).

A BLACK BODY IS REPRESENTED BY A CAVITY IN THERMAL EQUILIBRIUM. AT THE EQUILIBRIUM THE RADIATION INSIDE THIS ENCLOSURE FOLLOWS PLANCK'S LAW.

THE SURFACE OF THE BLACK BODY CAN BE MODELED BY A SMALL HOLE IN THE WALL OF A LARGE BOX, WHICH IS AT UNIFORM TEMPERATURE, WITH RIGID OPAQUE WALLS THAT ARE NOT PERFECTLY REFLECTING AT ANY WAVELENGTH.

PLANCK DISTRIBUTION

PLANCK DISTRIBUTION FOR PHOTONS, WHICH ARE NON INTERACTING AND HAVE ZERO MASS, IS THE UNIQUE MAXIMUM ENTROPY ENERGY DISTRIBUTION. IF THE PHOTON GAS IS NOT INITIALLY PLANCKIAN, THE 2^{ND} LAW OF THERMODYNAMICS GUARANTEES THAT THE INTERACTIONS, BETWEEN PHOTONS AND OTHER PARTICLES OR BETWEEN THE PHOTONS THEMSELVES, WILL CAUSE THE PHOTON ENERGY DISTRIBUTION TO BE PLANCKIAN.

$B_\nu(T)$ = SPECTRAL RADIANCE $\left[\frac{W}{sr \cdot m^2 \cdot Hz}\right]$

$B_\nu(T)\cos\theta \cdot dA \cdot d\Omega \cdot d\nu$ = INFINITESIMAL AMOUNT OF POWER RADIATED IN DIRECTION θ FROM THE SURFACE NORMAL TO dA WITH INFINITESIMAL ANGLE $d\Omega$ CENTERED ON FREQUENCY ν → INTEGRATING THIS QUANTITY I GET THE TOTAL RADIATED POWER (STEFAN-BOLTZMANN LAW)

N.B.: THE SPECTRAL RADIANCE FROM A BLACK BODY HAS THE SAME VALUE FOR EVERY DIRECTION AND ANGLE OF POLARIZATION, SO IT IS CALLED A LAMBERTIAN RADIATOR

PLANCK'S LAW

PLANCK'S LAW DESCRIBES THE ELECTROMAGNETIC RADIATION EMITTED BY A BLACK BODY IN THERMAL EQUILIBRIUM AT A DEFINITE TEMPERATURE

$$\boxed{B_\nu(T) = \frac{2h\nu^3}{c^2}\cdot\frac{1}{e^{\frac{h\nu}{k_BT}}-1}} \qquad \left[\frac{W}{sr\cdot m^2\cdot Hz}\right]$$

$$\boxed{B_\lambda(T) = \frac{2hc^2}{\lambda^5}\cdot\frac{1}{e^{\frac{hc}{\lambda k_BT}}-1}} \qquad \left[\frac{W}{sr\cdot m^3}\right]$$

PLANCKIAN RADIATION

B = SPECTRAL RADIANCE

k_B = BOLTZMANN CONSTANT

$$\boxed{u_i(T) = \frac{4\pi}{c} B_i(T)}$$

IN FORM OF SPECTRAL ENERGY, ENERGY PER VOLUME PER SPECTRAL UNIT.

N.B.: AT LOW FREQUENCIES TENDS TO RAYLEIGH JEANS LAW

AT HIGH FREQUENCIES TENDS TO WIEN APPROXIMATION

N.B.: PLANCK DISTRIBUTION IS THE ONLY ONE STABLE FOR RADIATION IN THERMODYNAMIC EQUILIBRIUM

[redacted]

CONVERSION BETWEEN THE VARIOUS FORMS OF PLANCK'S LAW:

$$B_\lambda(\lambda,T)\,d\lambda = -B_\nu(\nu(\lambda),T)\,d\nu$$

ENERGY INCREMENT "−" BECAUSE AN INCREMENT OF FREQUENCY CORRESPONDS WITH DECREMENT OF WAVELENGTH

$$B_\lambda(\lambda,T) = -\frac{d\nu}{d\lambda}B_\nu(\nu(\lambda),T)$$

$$\nu(\lambda) = c/\lambda \qquad d\nu/d\lambda = -c/\lambda^2$$

PLANCK'S LAW DERIVATION

CONSIDER A BLACK BODY; A CUBE OF SIDE L WITH CONDUCTING WALLS FILLED WITH ELECTROMAGNETIC RADIATION IN THERMAL EQUILIBRIUM AT TEMPERATURE T, WITH A SMALL HOLE ON ONE WALL. THE RADIATION EMITTED IS SO THE CHARACTERISTIC OF A PERFECT BLACK BODY

WE CALCULATE THE SPECTRAL ENERGY DENSITY WITHIN THE CAVITY AND THEN DETERMINE THE SPECTRAL RADIANCE OF THE EMITTED RADIATION.

AT THE WALLS OF THE CUBE THE PARALLEL COMPONENTS OF THE ELECTRIC FIELD AND THE ORTHOGONAL COMPONENT OF THE MAGNETIC FIELD MUST VANISH.

ANALOGOUS TO THE WAVE FUNCTION OF A PARTICLE IN A BOX, THE FIELDS ARE SUPERPOSITIONS OF PERIODIC FUNCTIONS. THE THREE WAVELENGTHS ARE $\lambda_i = 2L/n_i$ (n_i = INTEGERS)

FOR EACH n_i WE HAVE TWO MODES (LINEARLY INDEPENDENT SOLUTIONS) = POLARIZATION STATES

THE ENERGY LEVELS OF A MODE ARE $\boxed{E_{n_1,n_2,n_3}(r) = \left(r+\frac{1}{2}\right)\frac{hc}{2L}\sqrt{n_1^2+n_2^2+n_3^2}}$

r CAN BE INTERPRETED AS THE NUMBER OF PHOTONS IN THE MODE.

FOR $r=0$ THE ENERGY OF THE MODE IS NOT ZERO! THIS VACUUM ENERGY OF THE E.M. FIELD IS RESPONSIBLE FOR THE CASIMIR EFFECT

NOW WE CALCULATE THE INTERNAL ENERGY OF THE BOX AT ABSOLUTE TEMPERATURE T:

$$P_n = \frac{\exp(-\beta \cdot E(n))}{Z(\beta)}$$ = PROBABILITY DISTRIBUTION OVER THE ENERGY LEVELS OF A PARTICULAR MODE (STATISTICAL MECHANICS)

$$\beta = 1/(k_B T)$$

$$Z(\beta) = \sum_{n=0}^{+\infty} e^{-\beta E(n)} = \frac{e^{-\beta \varepsilon/2}}{1-e^{-\beta\varepsilon}}$$ = PARTITION FUNCTION OF A SINGLE MODE (TO NORMALIZE P_n)

$$\varepsilon = \frac{hc}{2L}\sqrt{n_1^2+n_2^2+n_3^2}$$ = ENERGY OF A SINGLE PHOTON

$$\langle E \rangle = -\frac{d \log(Z)}{d\beta} = \frac{\varepsilon}{2} + \frac{\varepsilon}{e^{\beta\varepsilon}-1}$$ = AVERAGE ENERGY IN A MODE, SPECIAL CASE OF THE GENERAL FORMULA FOR PARTICLES OBEYING BOSE-EINSTEIN STATISTICS. CHEMICAL POTENTIAL IS ZERO SINCE THERE IS NO RESTRICTION ON THE TOTAL N° OF PHOTONS

THE TOTAL ENERGY IN THE BOX FOLLOWS BY SUMMING $\langle E\rangle - \varepsilon/2$ OVER ALL ALLOWED SINGLE PHOTON STATES. AS $L \to \infty$ ε BECOMES CONTINUOUS AND WE CAN INTEGRATE $\langle E\rangle - \varepsilon/2$ OVER ε.

TO COMPUTE THE ENERGY WE NEED TO EVALUATE HOW MANY PHOTON STATES THERE ARE IN A GIVEN ENERGY RANGE:

$$U = \int_0^{+\infty} \frac{\varepsilon}{e^{\beta\varepsilon}-1} g(\varepsilon)\, d\varepsilon$$

$g(\varepsilon)$ = DENSITY OF STATES

$g(\varepsilon)d\varepsilon$ = N° OF PHOTON STATES WITH ENERGIES BETWEEN ε AND $\varepsilon + d\varepsilon$

$$\varepsilon = \frac{hc}{2L} n \quad , \quad n = \sqrt{n_1^2+n_2^2+n_3^2}$$ (SIMPLE RE-WRITING)

FOR EVERY n WITH COMPONENTS ≥ 0 THERE ARE TWO PHOTON STATES, SO THE N° OF PHOTON STATES IN A CERTAIN REGION OF n-SPACE IS TWICE THE VOLUME OF THAT REGION.

$$dn = \left(\frac{2L}{hc}\right) d\varepsilon$$ SHELL OF THICKNESS dn DUE TO AN ENERGY RANGE $d\varepsilon$ IN n-SPACE. THE SHELL SPANS $1/8$ OF SPHERE

SO THE N° OF PHOTON STATES IS:

$$g(\varepsilon)\, d\varepsilon = 2 \cdot \frac{1}{8} \cdot 4\pi \cdot n^2\, dn = \frac{8\pi L^3}{h^3 c^3} \varepsilon^2 d\varepsilon$$

SUBSTITUTING:

$$U = L^3 \frac{8\pi}{h^3 c^3} \int_0^{+\infty} \frac{\varepsilon^3}{e^{\beta\varepsilon}-1} d\varepsilon$$

$$\left(\frac{U}{L^3} = \int_0^{+\infty} u_\nu(T)\, d\nu \quad , \quad u_\nu(T) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/k_BT} - 1} \quad , \quad \frac{U}{L^3} = \int_0^{+\infty} u_\lambda(T)\, d\lambda \right.$$

$$\left. u_\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda k_BT} - 1} \right)$$

$$\varepsilon = k_B T x \rightarrow u(T) = \frac{8\pi (k_BT)^4}{(hc)^3} J \quad , \quad J = \int_0^{+\infty} \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$$

$\boxed{\frac{U}{V} = \frac{8\pi^5 (k_BT)^4}{15(hc)^3}}$ = TOTAL ELECTROMAGNETIC ENERGY INSIDE THE BOX ($V = L^3$ = BOX VOLUME)

$\boxed{\frac{U}{V} = \frac{4\sigma T^4}{c}}$ $\qquad 4\sigma/c$ = RADIATION CONSTANT

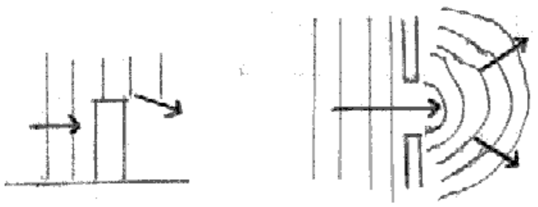
THE RADIATION IS THE SAME IN ALL DIRECTIONS AND PROPAGATES AT SPEED c. THE SPECTRAL RADIANCE OF RADIATION EXITING THE HOLE IS:

$$\boxed{B_\nu(T) = \frac{u_\nu(T)\, c}{4\pi} = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_BT} - 1}}$$

IT CAN BE CONVERTED IN WAVELENGTHS ($B_\lambda(T)$) BY SUBSTITUTING ν BY c/λ AND EVALUATING $B_\lambda(T) = B_\nu(T) \left| \frac{d\nu}{d\lambda} \right|$.

DIFFRACTION

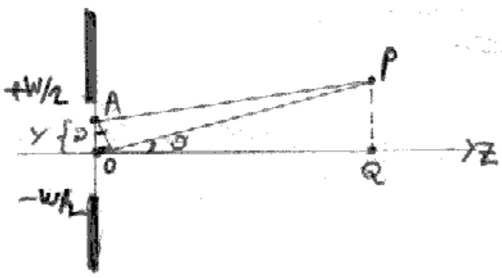
PHENOMENON THAT INVOLVES THE VARIATION OF THE PROPAGATION DIRECTION OF AN E.M. WAVE CAUSED BY AN OBSTACLE ON ITS PATH



WHEN THE WAVE ENCOUNTERS AN OBSTACLE, ITS WAVEFRONT HAS TO CHANGE: THE WAVE TENDS TO "BEND AROUND AN OBSTACLE" OR "SPREAD OUT" AS IT GOES THROUGH A GAP.
THE EFFECT DOES NOT DEPEND ON THE TYPE OF MATTER

HUYGEN'S PRINCIPLE: THE PROPAGATION OF A WAVE CAN BE STUDIED BY ASSUMING THAT EACH POINT OF THE WAVEFRONT IS THE SOURCE OF A SECONDARY SPHERICAL WAVE.
THE WAVEFRONT PROPAGATES AS THE ENVELOPE OF ALL SECONDARY WAVEFRONTS.
THE SECONDARY SPHERICAL WAVES INTERACT, THE PHASE DIFFERENCE DEPENDS ON THE DIFFERENCE OF THE DISTANCES FROM THE TWO SOURCE POINTS.

INTERFERENCE



SLIT OF WIDTH w, POINT P VERY FAR FROM THE SLIT
THE WAVES GENERATED BY "O" AND "A" HAVE DIFFERENT PHASES
IF P IS VERY FAR, $\overline{OP}$ AND $\overline{AP}$ ARE ALMOST PARALLEL AND THE PATH DIFFERENCE IS ABOUT $y\sin\theta$

THE PHASE OF THE WAVE GENERATED BY A IN P IS $k y \sin\theta$
(k = WAVE NUMBER)

THE COMPLEX CONTRIBUTION OF A IN P IS $\boxed{e^{jky\sin\theta}}$

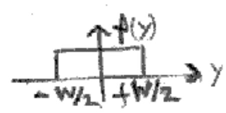
IF A MOVES ON THE WHOLE SLIT $\boxed{A(\theta) = \int_{-W/2}^{W/2} e^{jky\sin\theta}\, dy}$

IF THE SLIT IS A SEGMENT WITH TRANSPARENCY $f(y)$ WE CONSIDER THE SECONDARY WAVES TO HAVE DIFFERENT AMPLITUDES

$$\boxed{A(\theta) = \int_{-\infty}^{+\infty} f(y)\, e^{jky\sin\theta}\, dy}$$ FRAUNHOFER'S DIFFRACTION INTEGRAL

IT IS LIKE A FOURIER TRANSFORM COMPUTED IN THE VARIABLE $-k\sin\theta$, $A(\theta) = \int_{-\infty}^{+\infty} f(y)\, e^{-j\frac{yk\sin\theta}{2\pi}}\, dy$

THE IDEAL SLIT IS; $f(y) = 1$ IF $|y| \leq W/2$



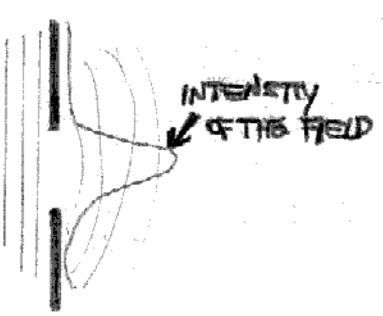
$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$

$$A(\theta) \propto \mathrm{sinc}\left(\frac{Wk\sin\theta}{2\pi}\right)$$

$$I(\theta) \propto \mathrm{sinc}^2\left(\frac{Wk\sin\theta}{2\pi}\right)$$ (INTENSITY OF THE WAVE)

FIRST ZERO $\sin\theta = \pm \frac{2\pi}{Wk} = \pm \frac{\lambda}{W}$ (BEAM WIDTH)

IF $W \gg \lambda$ THEN $\sin\theta \simeq \theta \rightarrow \theta \simeq \frac{\lambda}{W}$

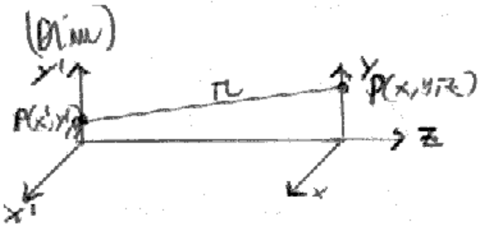


FAR FIELD

WE CONSIDER THE APPROXIMATION $\overline{AP} \parallel \overline{OP}$ VALID IF THE DIFFERENCE BETWEEN $\overline{AQ}$ AND $\overline{OQ}$ IS LESS THAN $\lambda/4$.

TO BE TRUE FOR THE WHOLE SLIT IT HAS TO BE $\boxed{\frac{w^2}{8z} < \frac{\lambda}{4}}$

THAT IS, z IS MUCH LARGER THAN THE FRESNEL DISTANCE $\boxed{z_F = \frac{w^2}{2\lambda}}$



$|r| = \sqrt{(x-x')^2+(y-y')^2+z^2}$

$\rho^2 \triangleq (x-x')^2+(y-y')^2$ $\rightarrow |r| = \sqrt{\rho^2+z^2} = z\left(1+\frac{\rho^2}{z^2}\right)^{1/2}$

TAYLOR $(1+u)^{1/2} \simeq 1+u/2-u^2/8$

$$|r| \cong z\left(1+\frac{1}{2}\cdot\frac{\rho^2}{z^2}-\frac{1}{8}\cdot\frac{\rho^4}{z^4}\right)$$

(1) $z(1+0) \cong z$ IF z IS VERY LARGE THAT IS WHEN THE 2^{nd} TERM IS SMALL IN PHASE SHIFT RESPECT TO 2π $\quad \frac{k\rho^2}{2z} \ll 2\pi \rightarrow \frac{\rho^2}{2z} \ll \lambda \rightarrow \left(\frac{w}{2}\right)^2\cdot\frac{1}{2z} \ll \lambda \rightarrow \frac{w^2}{8z} \ll \lambda$ (FRAUNHOFER APPROX. FAR FIELD)

(2) $z\left(1+\frac{\rho^2}{2z^2}+0\right) \cong z+\frac{z\rho^2}{2z^2}$ (FRESNEL APPROX. NEAR FIELD) $\quad \frac{k\rho^4}{8z^3} \ll 2\pi \rightarrow \frac{\rho^4}{8z^3\lambda} \ll 1$

TWO DIMENSIONAL APERTURE

$$A(\theta_x,\theta_y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\, e^{j(kx\sin\theta_x + ky\sin\theta_y)}\, dx\, dy$$

IT IS THE 2D-TRANSFORM OF $f(x,y)$

RECTANGULAR APERTURE

SOLVED IN SEPARABLE WAY BY MULTIPLYING THE RESULTS OF THE TWO 1-D TRANSFORMS IN THE TWO ORTHOGONAL DIRECTIONS

CIRCULAR APERTURE

WE HAVE TO COMPUTE THE FOURIER TRANSFORM OF A CIRCLE, EASY IN POLAR COORDINATES, EXPRESS THE INTEGRAL IN TERMS OF BESSEL'S FUNCTIONS $\quad A(\theta_x,\theta_y) \propto \frac{J_1\left(\frac{kD\sin\theta_r}{2}\right)}{\frac{kD\sin\theta_r}{2}}$

SPATIAL RESOLUTION

DIFFRACTION HAS IMPORTANT CONSEQUENCES ON THE RESOLUTION OF ACQUISITION SYSTEMS. IT HAPPENS WITH CAMERAS.
THINK TO THE CONVERSE PROCESS; RAYS ARRIVE AT THE SENSOR ALONG DIFFERENT PATHS WITH CONSTRUCTIVE/DESTRUCTIVE INTERFERENCE

THE ANGULAR RESOLUTION IS SUCH THAT: $\sin\theta_r \simeq 1.22\frac{\lambda}{D}$

AND FOR SMALL θ_r: $\boxed{\theta_r \simeq 1.22\frac{\lambda}{D} \simeq \frac{\lambda}{D}}$

EXAMPLE:

$\lambda = 0.5\,\mu m$, $D = 5\,cm$ (LENS), $\frac{\lambda}{D} = 10^{-5}$ rad THE SPATIAL RESOLUTION IS 10 m AT 1000 km DISTANCE

$\lambda = 3\,cm$, $D = 1\,m$ (ANTENNA), $\frac{\lambda}{D} = 0.03$ rad THE SPATIAL RESOLUTION IS 30 km AT 1000 km DISTANCE

PROPAGATION IN MATTER

MAXWELL'S EQUATIONS IN MATTER

$$\boxed{\begin{aligned} \nabla \cdot D &= \rho \\ \nabla \cdot B &= 0 \\ \nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla \times H &= J + \frac{\partial D}{\partial t} \end{aligned}}$$

$D = \varepsilon E$ — $\varepsilon = \varepsilon_r \varepsilon_0$ ELECTRICAL PERMITTIVITY

$B = \mu H$ — $\mu = \mu_r \mu_0$ MAGNETIC PERMEABILITY

$J = \gamma E$ — γ = CONDUCTANCE

HOMOGENEOUS MEDIA μ, ε DON'T DEPEND ON THE POSITION

ISOTROPIC MEDIA μ, ε ARE SCALAR

DISPERSIVE MEDIA IN SINUSOIDAL REGIME ε, μ DEPEND ON THE E.M. WAVE FREQUENCY

NON-DISPERSIVE MEDIA μ, ε ARE CONSTANT IN f

IF THE MEDIA ARE LINEAR WE CAN APPLY FOURIER DECOMPOSITION AND ~~[illegible]~~ SOLVE THE PROPAGATION OF EACH COMPONENT WITH ITS OWN PARAMETERS.

THE MEDIA OF INTEREST ARE: LINEAR, HOMOGENEOUS, ISOTROPIC, DISPERSIVE.

PROPAGATION VELOCITY

$$\boxed{\frac{k}{\omega} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}}$$ = PHASE VELOCITY (VELOCITY OF A MONOCHROMATIC WAVE)

IF THE MEDIUM IS DISPERSIVE ε_r AND μ_r ARE NOT CONSTANT FOR VARYING $k \rightarrow \underline{\omega = \omega(k)}$

REFRACTIVE INDEX

$$\boxed{n = \frac{c}{v} = \sqrt{\varepsilon_r \mu_r}}$$

$F = \frac{S_0}{2Z}$ (IRRADIANCE)

$$\boxed{Z = Z_0 \sqrt{\frac{\mu_r}{\varepsilon_r}}}$$ (MEDIUM IMPEDANCE)

ATTENUATION

THE MATERIAL USUALLY ABSORBS ENERGY AND IT IS RELATED TO THE CONDUCTANCE OF THE MATERIAL.

THIS CAN VARY FOR VARYING WAVE NUMBER.

IN SINUSOIDAL REGIME ATTENUATION CAN BE STUDIED BY THE ε_r.

WE CAN ALMOST ALLWAYS ASSUME THAT $\mu_r = 1$.

IF $\gamma \neq 0$, IN PRESENCE OF AN ELECTRIC FIELD, AN ELECTRIC CURRENT WITH DENSITY J IS GENERATED:

$$\boxed{J = \gamma E}$$

IN SINUSOIDAL REGIME WE CAN USE FOURIER TRANSFORM:

$$\boxed{\nabla \times H = \gamma E + j\omega\varepsilon E = j\omega\varepsilon_c E}$$

$$\boxed{\varepsilon_c = \varepsilon - j\frac{\gamma}{\omega}}$$

LINEAR ATTENUATION CAN BE DESCRIBED BY SIMPLY ADDING AN IMAGINARY PART TO THE DIELECTRIC CONSTANT: $\boxed{\varepsilon_c = (\varepsilon' - j\varepsilon'')\varepsilon_0} = \varepsilon_r\varepsilon_0$

IF THERE IS NO CONDUCTANCE (γ) THEN THE DIELECTRIC CONSTANT IS REAL ???

$$\boxed{n^2 = \varepsilon_r = \varepsilon' - j\varepsilon'' = (n - jk)^2 = \underbrace{n^2 - k^2}_{\varepsilon'} - \underbrace{2jnk}_{\varepsilon''}}$$

$$E_x = E_0 e^{j(\omega t - kz)} \quad , \quad k = \frac{\omega n}{c} = \frac{\omega(n - jk)}{c}$$

$$E_x = E_0 e^{j\left(\omega t - \frac{k\omega n z}{c}\right)} = E_0 e^{-\frac{\omega k z}{c}} \cdot e^{j\left(\omega t - \frac{\omega n z}{c}\right)}$$

AMPLITUDE OF ELECTRIC FIELD EXPONENTIALLY DECREASES IN Z

$$F = F_0 e^{-\frac{2\omega k z}{c}} \quad \text{(IRRADIANCE)}$$

$$\boxed{l_A = \frac{c}{2\omega k}} \quad \text{(ABSORPTION DISTANCE)}$$

N.B.: THE IMAGINARY PART OF THE REFRACTIVE INDEX DETERMINES THE ATTENUATION

THE REAL PART DETERMINES THE WAVELENGTH $\boxed{\lambda = \frac{2\pi c}{\omega n}}$

IN VACUUM: $\lambda_0 = \frac{2\pi c}{\omega} \rightarrow \lambda = \frac{\lambda_0}{n}$

INTERACTION WITH MATTER

TRANSMISSION: THE WAVE OR PART OF IT PROCEEDS ON ITS PATH THROUGH THE MEDIUM
ABSORPTION: PART OF THE ENERGY OF THE E.M. WAVE IS TRANSFERRED TO THE MEDIUM AS A CONSEQUENCE OF THE INTERACTION. THIS ENERGY CAN BE GIVEN BACK AS A RADIATION BUT USUALLY AT DIFFERENT WAVELENGTHS.
SCATTERING: THE WAVE IS DEVIATED IN MANY DIRECTIONS BY SMALL PARTICLES OR BY SMALL SURFACE ELEMENTS
REFLECTION: THE WAVE IS DEVIATED IN ONE DIRECTION PREDOMINANT, AFTER HITTING A SURFACE

DIELECTRIC CONSTANT OF GASES
ASSUMPTION: NO STRONG ABSORPTION

$$\boxed{\varepsilon_r \approx 1 + \frac{N\alpha}{\varepsilon_0}}$$

N = gas molecules density
α = molecular polarizability
α/ε_0 IS A VOLUME

$$\boxed{n = \sqrt{\varepsilon_r} = \sqrt{1 + \frac{N\alpha}{\varepsilon_0}} \approx 1 + \frac{N\alpha}{2\varepsilon_0}}$$ IF $\frac{N\alpha}{\varepsilon_0} \ll 1$

DIELECTRIC CONSTANT OF SOLIDS AND INSULATING LIQUIDS
NON POLAR MATERIALS: USUALLY HAVE A CONSTANT ε_r (POSSIBLY COMPLEX)
POLAR MATERIALS: POSSIBLE TO MODEL THE RESONANCE AND WRITE ε_r WITH DEBYE'S EQUATIONS

$$\boxed{\varepsilon' = \varepsilon_\infty + \frac{\varepsilon_p}{1+\omega^2\tau^2}} \qquad \boxed{\varepsilon'' = \frac{\omega\tau\varepsilon_p}{1+\omega^2\tau^2}}$$

τ = RELAXATION TIME
ε_∞ = ASYMPTOTIC CONSTANT AT HIGH FREQUENCIES
ε_p = POLAR CONTRIBUTION TO DIELECTRIC CONSTANT

DIELECTRIC CONSTANT OF METALS
ELECTRICAL PROPERTIES DETERMINED BY THE VERY HIGH DENSITY OF DELOCALIZED ELECTRONS

$$\boxed{\varepsilon' = 1 - \frac{\sigma\tau}{\varepsilon_0(1+\omega^2\tau^2)}} \qquad \boxed{\varepsilon'' = \frac{\sigma}{\varepsilon_0\omega(1+\omega^2\tau^2)}} \qquad \tau = \frac{m_e\sigma}{Ne^2}$$

m_e = ELECTRON MASS
e = ELECTRON CHARGE

$\tau \approx 10^{-15} \div 10^{-14}$ [s]

WE STUDY THE BEHAVIOR IN TWO REGIONS: $\omega \gg 1/\tau$ AND $\omega \ll 1/\tau$

LOW FREQUENCIES (RADIO)

$$\varepsilon_r \approx -j\frac{\sigma}{\varepsilon_0\omega}, \quad n = k = \sqrt{\frac{\sigma}{2\varepsilon_0\omega}}, \quad l_a = c\sqrt{\frac{\sigma}{2\varepsilon_0\omega}} = \frac{\lambda}{4\pi}$$

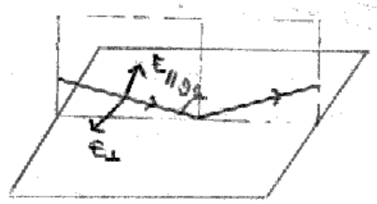
HIGH FREQUENCIES (OPTICAL)

$$\varepsilon_r \approx 1 - \frac{Ne^2}{\varepsilon_0 m_e \omega^2} = 1 - \frac{\sigma}{\varepsilon_0\omega^2\tau}$$

AT SUFFICIENTLY HIGH FREQUENCIES THE ELECTRIC CONSTANT IS REAL AND ALMOST EQUAL TO ONE, SO AT HIGH FREQUENCIES (U.V., X-RAYS) METALS ARE ALMOST TRANSPARENT

THE PHASE VELOCITY CAN BE LARGER THAN c BUT THE GROUP VELOCITY IS SMALLER

PLANE BOUNDARIES



WE CAN DECOMPOSE THE INCIDENT RAY IN TWO COMPONENTS:

$E_{\parallel}$ PARALLEL TO THE PLANE THAT CONTAINS THE RAYS (TM)

$E_{\perp}$ ORTHOGONAL TO THE PLANE THAT CONTAINS THE RAYS (TE)

N.B.: IF THE MEDIA ARE HOMOGENEOUS AND ISOTROPIC EACH COMPONENT ONLY GENERATES A COMPONENT OF THE SAME KIND

REFLECTION AND TRANSMISSION COEFFICIENTS OBTAINED BY SOLVING THE MAXWELL'S EQUATIONS AT THE SURFACE:

$$r_{\perp} = \frac{Z_2 \cos\theta_1 - Z_1 \cos\theta_2}{Z_2 \cos\theta_1 + Z_1 \cos\theta_2} \qquad t_{\perp} = \frac{2 Z_2 \cos\theta_1}{Z_2 \cos\theta_1 + Z_1 \cos\theta_2}$$

$$r_{\parallel} = \frac{Z_2 \cos\theta_2 - Z_1 \cos\theta_1}{Z_2 \cos\theta_2 + Z_1 \cos\theta_1} \qquad t_{\parallel} = \frac{2 Z_2 \cos\theta_1}{Z_2 \cos\theta_2 + Z_1 \cos\theta_1}$$

IN MANY CASES MEDIUM 1 IS VACUUM OR AIR AND WE CAN USE $n_1 = 1$ WITH GOOD APPROXIMATION

FRESNEL'S EXPRESSIONS:

$$r_{\perp} = \frac{\cos\theta_1 - \sqrt{\varepsilon_{r2} - \sin^2\theta_1}}{\cos\theta_1 + \sqrt{\varepsilon_{r2} - \sin^2\theta_1}} \qquad r_{\parallel} = \frac{\sqrt{\varepsilon_{r2} - \sin^2\theta_1} - \varepsilon_{r2}\cos\theta_1}{\sqrt{\varepsilon_{r2} - \sin^2\theta_1} + \varepsilon_{r2}\cos\theta_1}$$

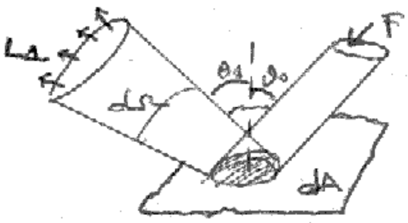
$$t_{\perp} = \frac{2\cos\theta_1}{\cos\theta_1 + \sqrt{\varepsilon_{r2} - \sin^2\theta_1}} \qquad t_{\parallel} = \frac{2\sqrt{\varepsilon_{r2}}\cos\theta_1}{\sqrt{\varepsilon_{r2} - \sin^2\theta_1} + \varepsilon_{r2}\cos\theta_1}$$

IF MEDIUM 2 IS NON-ABSORBING:

$$r_{\perp} = \frac{\cos\theta_1 - \sqrt{n_2^2 - \sin^2\theta_1}}{\cos\theta_1 + \sqrt{n_2^2 - \sin^2\theta_1}} \qquad r_{\parallel} = \frac{\sqrt{n_2^2 - \sin^2\theta_1} - n_2^2\cos\theta_1}{\sqrt{n_2^2 - \sin^2\theta_1} + n_2^2\cos\theta_1}$$

THE PARALLEL COMPONENT IS NOT REFLECTED WHEN $\theta_1 = \theta_B$ WHERE θ_B IS THE BREWSTER ANGLE WHICH SATISFIES $\tan\theta_B = n_2$

SURFACE SCATTERING



F = IRRADIANT FLUX

E = IRRADIANCE = $F \cdot \cos\theta_0$

R = BIDIRECTIONAL REFLECTANCE DISTRIBUTION FUNCTION $= \frac{L_1}{E}$

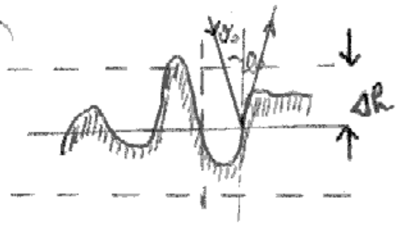
BISTATIC SCATTERING COEFFICIENT; $\gamma = 4\pi R \cos\theta_1$

WHEN $\theta_0 = \theta_1$ WE DEFINE THE BACKSCATTER COEFFICIENT $\sigma^0 = \gamma \cos\theta_0 = 4\pi R \cos^2\theta_0$

TO FIND THE TOTAL RADIANT FLUX M IN ALL DIRECTIONS WE INTEGRATE R OVER ALL ANGLES (ALBEDO).

IF THE SURFACE IS NOT PERFECTLY SMOOTH THE INCIDENT WAVE IS REFLECTED IN MANY DIRECTIONS WITH DIFFERENT INTENSITIES.

WE CONSIDER THE SCATTERING FROM A SUFFICIENTLY SMOOTH SURFACE USING THE RAYLEIGH MODEL:



ASPERITIES ARE WITHIN Δh FROM THE PLANE

THE PATH DIFFERENCE BETWEEN A RAY REFLECTED ON THE REFERENCE PLANE AND ONE REFLECTED BY THE ASPERITY IS NEARLY $2\Delta h \cos\theta_0$, THE PHASE DIFFERENCE IS:

$$\boxed{\Delta\phi = \frac{4\pi \Delta h \cos\theta_0}{\lambda}}$$

IF WE WANT TWO RAYS IN PHASE WE NEED A PHASE DIFFERENCE $< \pi/2$ THUS $\boxed{\Delta h = \frac{\lambda}{2\cos\theta_0}}$

THE SURFACE IS MORE OR LESS REFLECTIVE ACCORDING TO THE INCIDENT ANGLE

FOR RAYS INCIDENT IN THE ORTHOGONAL DIRECTION WE HAVE REFLECTIONS IF ASPERITIES ARE IN THE ORDER OF $\lambda/8$ (INDICATIVELY)

EXAMPLE:

OPTICAL FREQUENCIES: A SURFACE REFLECTS IF $\Delta h \approx 60\,nm$ (ARTIFICIALLY TREATED / LIQUID)

RADIO FREQUENCIES (UHF): $\Delta h = 40\,cm$ SO MANY SURFACES ARE PERFECTLY REFLECTIVE.

IT IS POSSIBLE TO STUDY THE SCATTERING WITH THE MODEL OF SMALL PERTURBATIONS (THE MODEL USED FOR FRAUNHOFER DIFFRACTION IN THE FAR FIELD)

PATH DIFFERENCE: $\Delta l = x\overbrace{(\sin\theta_0 - \sin\theta_1)}^{\alpha} - z(x)\overbrace{(\cos\theta_0 + \cos\theta_1)}^{\beta}$

PHASE DIFFERENCE: $\Delta\phi = k\alpha x - k\beta z(x)$

TOTAL COMPLEX AMPLITUDE:

$$E = \int_{-\infty}^{+\infty} e^{-j\phi(x)}\, dx = \int_{-\infty}^{+\infty} e^{-jk\alpha x} \cdot e^{jk\beta z(x)}\, dx \quad \text{(FOURIER TRANSFORM)}$$

FOR SMALL $z(x)$ (TAYLOR): $e^{jk\beta z(x)} \approx 1 + jk\beta z(x) + \dots$

CONTRIBUTIONS:

FIRST TERM: $\int_{-\infty}^{+\infty} e^{-jk\alpha x}\,dx \propto \delta(k\alpha)$ = COMPONENT ASSOCIATED TO THE IDEAL REFLECTION

SECOND TERM: $\int_{-\infty}^{+\infty} (jk\beta\zeta)\, e^{-jk\alpha x}\,dx$ $\propto$ TO THE FOURIER TX OF THE ~~[illegible]~~ $\zeta(x)$ = THE AMPLITUDE IN DIRECTION α IS GIVEN BY THE COMPONENT OF $\zeta(x)$ WITH SPATIAL FREQ. $k\alpha$

IF THE HIGHER ORDER TERMS CAN BE NEGLECTED, WE SPEAK OF BRAGG SCATTERING

APPROXIMATION OK IF: $k\beta\Delta h \ll 1$ = $\Delta h \ll \dfrac{\lambda}{2\pi(\cos\theta_0+\cos\theta_1)}$ = RAYLEIGH CRITERION SATISFIED

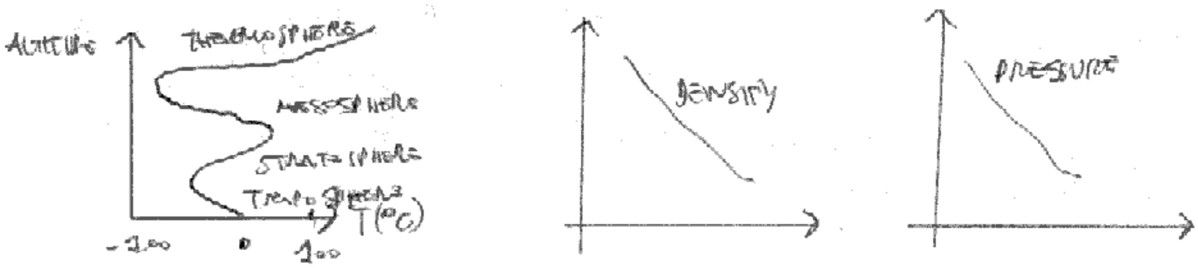
MODEL EXTREMELY ACCURATE TO DESCRIBE THE EFFECT OF WATER WAVELETS (1cm) OF THE μWAVES SCATTERING

IT IS NEEDED A STATISTICAL DESCRIPTION OF RAYS AND THEN WE CAN COMPUTE AN AVERAGE

THE CORRELATION FUNCTION OF $\zeta(x)$ BECOMES IMPORTANT AND IT IS TYPICALLY ASSUMED GAUSSIAN

(CLEAR WATER VERY SMALL REFLECTANCE)

ATMOSPHERE LAYERS: TROPOSPHERE (0-11 km)
STRATOSPHERE (11-50 km)
MESOSPHERE (50-80 km)
THERMOSPHERE (>80 km)
IONOSPHERE (60-450 km) → T CAN REACH 1500°C BUT LOW THERMAL EXCHANGE



ABSORPTION DETERMINES THE WINDOWS THAT CAN BE USED FOR REMOTE SENSING

WINDOWS: VISIBLE-NEAR INFRARED (0.4-2μm)
3 WINDOWS IN THERMAL INFRARED (3μm, 5μm, 8-14 μm)
μWAVE WINDOW (>1 mm)

ATMOSPHERIC SCATTERING: DUE TO GAS MOLECULES, AEROSOL, WATER PARTICLES IN THE CLOUDS, SUSPENDED ICE CRYSTALS, GRADIME

RATIO BETWEEN PARTICLE DIMENSION AND WAVELENGTH $\boxed{\alpha = \dfrac{2\pi r}{\lambda}}$ (CRITICAL PARAMETER)

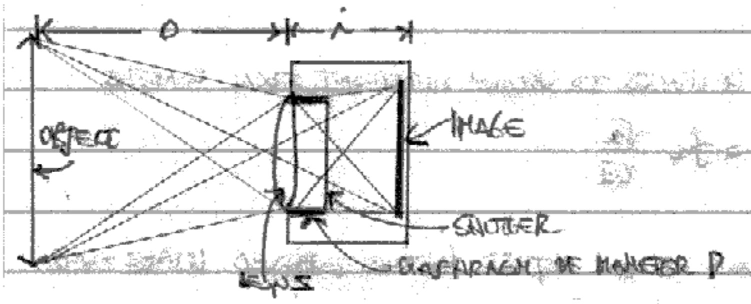
$\alpha < 0.001$	NEGLIGIBLE SCATTERING
$0.001 < \alpha < 0.1$	RAYLEIGH SCATTERING
$0.1 < \alpha < 50$	MIE SCATTERING
$\alpha > 50$	OPTICAL GEOMETRY

PHOTOGRAPHIC SYSTEMS

ANALOG CAMERAS (+) GEOMETRIC RESOLUTION
SIMPLE TO BUILD
RELIABLE
GOOD WHERE SPATIAL RESOLUTION AND GEOMETRIC PRECISION ARE MORE IMPORTANT THAN RADIOMETRIC/SPECTRAL RESOLUTION
(−) DATA HANDLING NEEDS DIGITIZATION
CAN BE USED ONLY UP TO NEAR INFRARED
ARCHIVING

DIGITAL CAMERAS (+) ALLOW TRANSMISSION OF IMAGES
CAN BE USED IN THERMAL INFRARED
NUMERICAL DATA DIRECTLY AVAILABLE
(−) LESS RELIABLE
NEED CAREFUL CALIBRATION OF THE SENSORS
ARCHIVING

ACQUISITION SCHEME



LENS: FOCUSES LIGHT RAYS ON THE IMAGE PLANE

DIAPHRAGM: DETERMINES LENS OPENING DURING EXPOSURE

SHUTTER: CONTROLS DURATION OF EXPOSURE

o: DISTANCE FROM LENS TO OBJECT

i: DISTANCE FROM LENS TO IMAGE PLANE

f: FOCAL LENGTH OF THE LENS $\boxed{\frac{1}{f} = \frac{1}{o} + \frac{1}{i}}$

f IS CONSTANT: OBJECTS AT DIFFERENT DISTANCES GENERATE IMAGES AT DIFFERENT DISTANCES IN THE CAMERA (USUALLY MOVES THE LENS IN ORDER TO KEEP THE IMAGE FOCUSED ON A FIXED POSITION)

IF OBJECT IS VERY FAR $o \to \infty$ $\boxed{f \simeq i}$ (IN REMOTE SENSING APPROXIMATION PRECISE!!)

THE FOCAL LENGTH IS THE DISTANCE FROM THE LENS OF THE IMAGE GENERATED BY AN INFINITELY FAR OBJECT

CHROMATIC ABERRATION: DIFFERENT COLORS ARE FOCUSED ON DIFFERENT PLANES (REFRACTIVE INDEX DEPENDS ON λ). TO CONTRAST, THIN LENSES OF DIFFERENT REFRACTIVE PROPERTIES ARE COMBINED

TOTAL IRRADIANT FLUX ON THE LENS $\boxed{\pi\left(\frac{D}{2}\right)^2 \Omega L}$

L = RADIANCE
D = LENS DIAMETER
Ω = SOLID ANGLE
ΩL = TOTAL IRRADIANCE AT THE LENS
Ωf^2 = SURFACE WHERE THE FLUX IS DISTRIBUTED

$\boxed{\pi\left(\frac{D}{2}\right)^2 \frac{L}{f^2}}$ IF THERE IS NO LOSS IN THE LENS THE FLUX IS DISTRIBUTED ON AN IMAGE PORTION WITH SURFACE Ωf^2

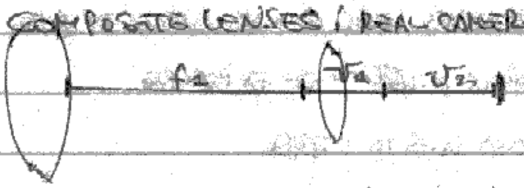
EXPOSURE $\boxed{E = \pi\left(\frac{D}{2}\right)^2 \frac{L}{f^2} \cdot t = \frac{\pi L}{4}\left(\frac{D}{f}\right)^2 t}$

D = LENS OPENING REGULATED BY THE DIAPHRAGM
t = EXPOSURE TIME REGULATED BY THE SHUTTER

F-STOP $\boxed{F = f\text{-STOP} = f/D}$

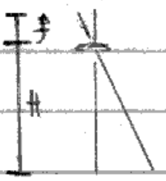
THE EXPOSURE IS USUALLY REGULATED BY SPECIFYING THE EXPOSURE TIME t AND f-STOP. F-STOP VARIES BY POWERS OF $\sqrt{2}$ WHILE THE EXPOSURE TIME BY POWERS OF 2. THIS ALLOW TO KEEP A CONSTANT EXPOSURE WITH DIFFERENT EXPOSURE TIMES.

COMPOSITE LENSES: REAL CAMERA USE COMBINATION OF LENSES TO OBTAIN DIFFERENT FOCAL LENGTHS



FOCAL LENGTH $= f_1 \frac{v_2}{v_1}$

SCALE: IS THE RATIO BETWEEN THE LINEAR EXTENSION OF AN OBJECT IN THE FORMED IMAGE AND ITS REAL SIZE



$\boxed{f/H = \text{SCALE}}$

EXAMPLE: 1:20000

BLACK AND WHITE: BLACK AND WHITE PHOTOS MADE WITH PANCHROMATIC OR INFRARED SENSITIVE FILMS
VERY DIFFICULT TO BUILD FILMS SENSITIVE ABOVE 0.9 μm (ONLY SCIENTIFIC EXPERIMENTS)
IR IMAGE GIVES VERY USEFUL INFORMATION ABOUT WATER AND WET SOIL (MUCH DARKER IN IR REGION)
IT IS POSSIBLE TO APPLY FILTERS TO ACQUIRE IN SOME GIVEN REGIONS OF THE SPECTRUM.

COLOR FILM: COLOR FILMS CONTAIN DIFFERENT LAYERS WITH DIFFERENT SENSITIVITIES
(THE SAME FOR COLOR INFRARED)

SOME NATURAL PHENOMENA AT HIGH TEMPERATURES CAN BE STUDIED IN THE NEAR-IR BY MEANS OF THE EMITTED RADIATION.

MAPPING CAMERAS

FILM SIZE = 240 x 240 mm

IMAGE SIZE = 230 x 230 mm

ADDITIONAL DATA WRITTEN ON THE FILM (CLOCK, ALTITUDE, ...)

FOCAL LENGTH ≈ 150 mm (FOR MAPPING PURPOSES) ALSO 90/210 mm ARE USED

300 mm USED FOR VERY HIGH ALTITUDE APPLICATIONS

CAMERA LENSES: NORMAL ANGLE = ANGULAR FIELD OF VIEW UP TO 75°

WIDE ANGLE = FIELD OF VIEW FROM 75° TO 100°

SUPERWIDE ANGLE = TOTAL FIELD OF VIEW > 100°

DURATION EXPOSURE FROM 1/1000 TO 1/100 s

TO COMPENSATE THE IMAGE SHIFT WHEN THE SHUTTER IS OPEN, MANY CAMERAS HAVE A BUILT-IN IMAGE MOTION COMPENSATION APPARATUS WHICH MOVES THE FILM TO AVOID BLUR.

PANORAMIC CAMERAS

HAVE LENS WITH A SMALL ANGULAR VIEW WHICH ACQUIRES THROUGH A NARROW SLIT. THE LENS IS PHYSICALLY MADE TO ROTATE SO AS TO ACQUIRE A VERY LARGE AREA IN THE ACROSS FLIGHT DIRECTION.

(+) VERY DETAILED IMAGES (DUE TO NARROW SLIT) OF VERY LARGE AREAS (DUE TO LENS SWINGING)

USED AT VERY HIGH ALTITUDES TO COVER A HUGE AREA

(−) LACK OF GEOMETRY FIDELITY, DIFFERENT ILLUMINATION AND ATMOSPHERIC DISTORTIONS IN DIFFERENT PORTIONS OF THE IMAGE

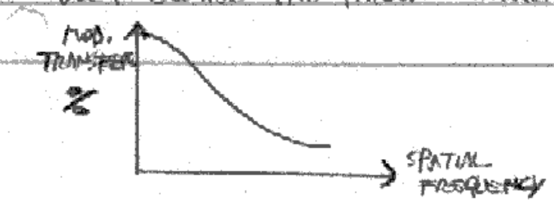
FILM RESOLUTION: (OF A FILM) NUMBER OF LINES PER mm THAT CAN BE DISTINGUISHED

DEPENDS ON THE SIZE OF THE CRYSTALS. LARGER CRYSTALS MORE SENSITIVE (REQUIRE LESS ILLUMINATION) BUT LOWER RESOLUTION.

FOR AERIAL PHOTOGRAPHY RESOLUTION FROM 20 TO 200 LINES PER mm

ANOTHER CLASSIFICATION IS THROUGH MODULATION: WE MEASURE THE MODULATION THAT IS THE RATIO BETWEEN THE CONTRAST OBTAINED ON THE FILM AND THE CONTRAST OF THE ORIGINAL PATTERN

WE INCREASE THE PATTERN SPATIAL FREQUENCY AND SEE WHAT HAPPENS TO THE MODULATION.



GROUND RESOLUTION DISTANCE : THE MINIMAL DISTANCE ON THE GROUND BETWEEN TWO DISTINGUISHABLE OBJECTS

$$\boxed{GRD\,[mm] = \frac{SCALE^{-1}}{\text{FILM RESOLUTION}\,[mm^{-1}]}}$$

A 23x23 cm FILM WITH RESOLUTION OF 400 LINES (PAIRS) PER mm CONTAINS SOMETHING LIKE 2.1 GIGAPIXELS. 120 m OF FILM CONTAIN 500 PHOTOGRAPHS = 1 TERABYTE

DIFFRACTION

RESOLUTION DEPENDS NOT ONLY ON THE FILM RESOLUTION BUT ALSO ON DIFFRACTION

ANGULAR RESOLUTION LIMITED BY λ/D DUE TO FRAUNHOFER DIFFRACTION

RESOLUTION $\approx \boxed{\frac{\lambda}{D} f}$ ON THE FILM PLANE

IF THE VALUE $\frac{\lambda f}{D}$ IS LARGER THAN THE SPACING BETWEEN DISTINGUISHABLE [illegible] THEN THE RESOLUTION OF THE SYSTEM IS LIMITED BY THE DIFFRACTION.

EXAMPLE: $D = 4\,cm$, $f = 15\,cm$, $\lambda = 0.5\,\mu m$ → RESOLUTION $7.5\,\mu m$ → >70 lines/mm IS USELESS!

DIGITAL CAMERAS

IMAGE ACQUIRED BY A CCD RATHER THAN BY A FILM. DIFFERENT COLORS ARE SAMPLED IN DIFFERENT POSITIONS ON A GRID. EACH LIGHT-PIXEL IS IDENTICAL, IMAGE SENSORS RECORD ONLY THE GRAYSCALE, SO THEY ONLY CAPTURE BRIGHTNESS, TO GET COLOUR, COLOUR FILTERS ARE ADDED IN FRONT OF THE SENSORS.

DIGITAL CAMERAS RESOLUTION : DISTANCE BETWEEN TWO SENSORS. GROUND PIXEL SIZE IS THE SIDE OF A SQUARE OF LAND SURFACE WHICH IS MAPPED TO ONE SINGLE PIXEL IN THE DIGITAL IMAGES. IT IS THE SAME AS THE INSTANTANEOUS FIELD OF VIEW (IFOV). IF THE SENSORS ARE SPACED BY d [mm] THEN THE IFOV IS $\boxed{IFOV_{LINEAR}\,[mm] = \frac{d}{SCALE}}$

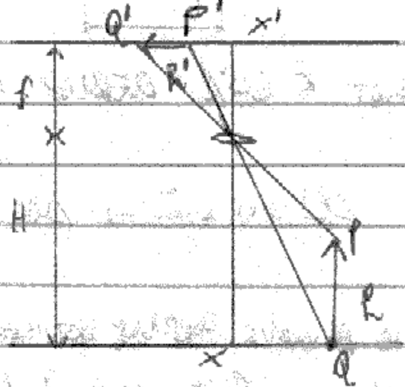
RELIEF DISPLACEMENT : IMPORTANT PHENOMENON IN AERIAL PHOTOGRAPHY

A POINT $P=(X,Y,Z)$ WITH $Z=h$ IS MAPPED BY THE PHOTOGRAPH TO A POINT $P'=(x',y',z')$ WITH

$$x' = -\frac{fx}{H-h}$$

$$y' = -\frac{fy}{H-h}$$

$$z' = H+f$$



WE NEGLECT THE y COMPONENT

$$\boxed{h' =} -\frac{fx}{H-h} - \frac{fx}{H} = -\frac{hfx}{H(H-h)} = \frac{hx'}{H-h} \boxed{\approx h\frac{x'}{H}}$$

PROPORTIONAL FACTOR $\frac{x'}{H}$ BETWEEN h AND h' : $\boxed{\Delta h' \approx \Delta h \cdot \frac{x'}{H}}$

DISPLACEMENT IS LARGER FOR POINTS FAR FROM THE FOCAL AXIS

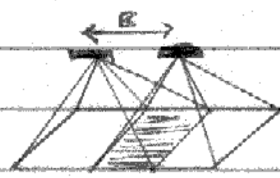
RESOLUTION IN THE DETECTABLE HEIGHTS ; WE MAKE x' TO ITS LARGEST VALUE ($W/2$ FOR A $W \times W$ IMAGE) $\boxed{\Delta h \approx \frac{2H\Delta h'}{W}}$

EXAMPLE: $W=23$ cm , $\Delta h' = 0,1$ mm $\Delta h \approx H/4000$

STEREOGRAMMETRY

THE DISPLACEMENT PHENOMENON CAN BE USED EFFICIENTLY BY COMBINING TWO DIFFERENT PHOTOGRAPHS EXAMPLE FROM POSITION $(0,0,H)$ AND $(B,0,H)$

$$x_2' = -\frac{f(x-B)}{H-h}$$



WE SOLVE:

$$X = -\frac{x'B}{x'-x_2'} , \quad Y = \frac{y'B}{x'-x_2'} , \quad h = H + \frac{fB}{x'-x_2'}$$

TO IMPROVE THE ALTITUDE RESOLUTION WE HAVE TO INCREASE B, BUT THIS REDUCES THE OVERLAP REGION TO A LENGTH $\boxed{\frac{WH}{f} - B}$

TYPICALLY $B \approx 0.4 \frac{WH}{f}$ THAT IS 60% OVERLAP AND $\boxed{\Delta h \approx \frac{H \Delta l'}{0.4 W}}$

EXAMPLE: $W = 23cm$, $f = 15cm$, $\Delta l' \cong 0.1mm \rightarrow \Delta h \simeq H/900$ WITH THE WHOLE OVERLAP REGION

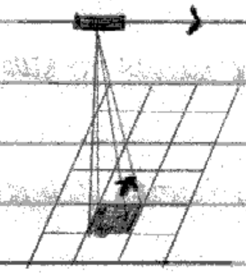
DURING A FLIGHT ACTUALLY THERE IS ALWAYS OVERLAP BETWEEN IMAGES FOR A ROBUST ACQUISITION USUALLY 60% OVERLAP ~~ACROSS~~ ALONG TRACK AND 20% ACROSS TRACK

MULTISPECTRAL SCANNERS

WE HAVE ELECTRO-OPTICAL SYSTEMS AND THE SENSOR IS THUS COMPOSED OF ONE OR MORE ELEMENTS SENSITIVE TO THE E.M. RADIATION. RADIATION IS MEASURED BY ONE OR MORE SENSORS COMBINED WITH OPTICAL AND MECHANICAL DEVICES (MOVABLE MIRRORS, PRISMS, FILTERS) THAT ALLOW TO SCAN THE DIFFERENT LAND REGIONS AND SPLIT THE DIFFERENT COMPONENTS FOR DIFFERENT SENSORS.

(+) LARGER USABLE BAND (E.G. THERMAL INFRARED), DON'T NEED DEVELOPING PROCESSES.

WHISK BROOM (ACROSS TRACK SCANNING)



- ONE SINGLE SENSOR FOR EACH BAND
- MECHANICAL ROTATION OF THE MIRROR ALLOWS SEQUENTIAL ACQUISITION OF GROUND PIXELS (IN THE DIRECTION TRANSVERSAL TO THE MOTION DIRECTION)
- A SYSTEM OF LENSES AND FILTERS SPLITS THE DIFFERENT BANDS

(+) SMALLER N° OF SENSORS ; HIGH HOMOGENEITY OF MEASURES ; LESS CALIBRATION PROBLEMS

(−) MECHANICAL SCANNING CAN MAKE THE SYSTEM LESS RELIABLE AND INDUCE VIBRATIONS / LESS TIME AVAILABLE FOR EACH PIXEL AND SO, LESS RADIOMETRIC RESOLUTION.

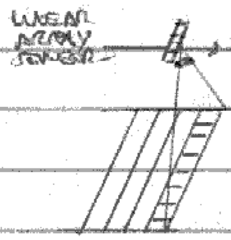
TO SPLIT I CAN USE PRISMS (REFRACTION) OR GRIDS (DIFFRACTION)

DICHROIC GRATING SEPARATES THERMAL AND OPTICAL BANDS

TEMPERATURE OF SENSORS FOR THERMAL COMPONENTS MUST BE KEPT LOW AND UNDER PROCESS CONTROL.

IF THE MIRROR HAS A CONSTANT ANGULAR VELOCITY THE IMAGES EXHIBIT A TANGENTIAL-SCALE DISTORTION THAT MUST BE CORRECTED.

PUSHBROOM (ALONG TRACK SCANNING)

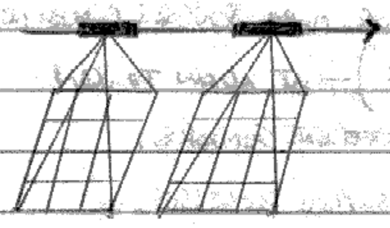


- FOR EACH SUBBAND AN ENTIRE ROW OF PIXELS IS ACQUIRED BY AN ARRAY OF SENSORS
- IT IS NOT NEEDED A MOVING MECHANICAL APPARATUS

(+) MORE RELIABLE SYSTEM, HIGHER EXPOSITION ALLOWS LARGER RADIOMETRIC RESOLUTION

(−) DIFFICULT TO CALIBRATE ALL SENSORS, ORIGINALLY THEY COULD NOT BE USED IN THE THERMAL INFRARED (NOW YES... LDCM).

STEP STARE (SENSOR MATRICES)



GROUND RESOLUTION: AT LOW ALTITUDES RESOLUTION IS NOT CONSTANT ACROSS THE TRACK
(NEGLIGIBLE FOR SATELLITES)

HYPERSPECTRAL SCANNERS: EACH PIXEL HAS ASSOCIATED A CONTINUOUS SPECTRUM. THIS ALLOWS TO STUDY COMPOSITIONS OF SOIL, ROCKS AND SO ON.

THERMAL SCANNERS: ACQUIRE IN THE THERMAL INFRARED REGION. THEY CATCH EMITTED RADIATION RATHER THAN THE REFLECTED RADIATION.

WITH ELECTRO-OPTICAL SENSORS I CAN ACQUIRE IN THE TERMAL INFRARED

MAIN WINDOWS ARE 3-5 μm AND 8-14 μm DUE TO ABSORPTION.

IFOV CANNOT BE TOO SMALL. THE RESOLUTION IS SMALLER THAN IN THE VISIBLE BAND DUE TO LOW ENERGY OF PHOTONS.

ACROSS ~~SCAN~~ TRACK SCANNING DEVICES USUALLY USED

IMAGES PRODUCED ARE THERMOGRAMS AND ASSOCIATE LIGHTER TEMPERATURES TO HOTTER REGIONS (RESOLUTION OF 0.1 °C).

FALSE COLORS USED TO RENDER INFORMATION (RED→HOT / BLUE→COLD)

TO OBTAIN ABSOLUTE VALUES A COMPLEX CALIBRATION STAGE IS REQUIRED.

LIDAR (LIGHT DETECTION AND RANGING)

AN ACTIVE SENSOR EMITS A LASER PULSE OR WAVEFORM IN THE TARGET DIRECTION, THE RETURN REACHES THE SENSOR WHICH MEASURES THE INTENSITY AND/OR TIME OF FLIGHT/ WHOLE DELAY.

INFORMATIONS
- TARGET DISTANCE (TOPOGRAPHIC LIDAR): MEASURES TIME OF FLIGHT
- CHEMICAL COMPOSITION (DIAL): MEASURES DIFFERENT ABSORPTION OF PULSES AT DIFFERENT WAVELENGTHS
- TARGET SPEED (DOPPLER LIDAR): DOPPLER EFFECT DUE TO RELATIVE MOTION

LASER (LIGHT AMPLIFICATION BY STIMULATED EMISSION OF RADIATION): BEAM OF LIGHT WITH VERY HIGH COHERENCE, ALMOST PERFECTLY MONOCHROMATIC. STIMULATED COHERENT EMISSION IN A CAVITY, THE BEAM IS VERY HIGH COLLIMATED.

MONOSTATIC: TX AND RX ARE IN THE SAME POSITION

- COAXIAL: THE LASER BEAM AXIS IS COINCIDENT WITH THE RECEIVER OPTICS AXIS
  THE RECEIVER CAN SEE THE LASER BEAM, THIS CAUSES NEAR-FIELD BACKSCATTERING THAT CAN SATURATE THE PHOTODETECTOR. IT IS POSSIBLE TO GATE THE PHOTODETECTOR OR USE A FAST SHUTTER.
- BIAXIAL: THE LASER BEAM AND THE RECEIVER AXIS ARE SEPARATED. THE LASER BEAM ONLY ENTERS THE RECEIVER FIELD OF VIEW BEYOND SOME PREDETERMINED RANGE.
  THIS HELPS AVOIDING NEAR FIELD BACKSCATTERED RADIATION.

BISTATIC: THE TRANSMITTER AND THE RECEIVER ARE NOT AT THE SAME LOCATION

PULSED SYSTEMS (USUALLY MONOSTATIC): THE DEVICE EMITS A SEQUENCE OF LASER PULSES (200000 pulses/sec) AND MEASURES TIME OF FLIGHT, PRECISION UP TO GPS ≈ 1 cm

CONTINUOUS WAVEFORM (USUALLY BISTATIC): CONTINUOUS WAVEFORM WITH SINUSOIDAL ENVELOPE, THE PHASE DIFFERENCE IS MEASURED. TYPICALLY USED IN LIDAR AS DOPPLER, FOR WIND PROFILING

IMU (INERTIAL MEASUREMENT UNIT) AND GPS ARE FUNDAMENTAL. ALLOW TO ESTIMATE WITH HIGH PRECISION THE ABSOLUTE POSITION AND THE SPEED OF THE AIRPLANE IN ANY MOMENT (PRECISION ≈ 15 cm)

RAW LIDAR CONSISTS OF A LIST OF (x,y,z) COORDINATES, WE MEASURE THE RETURN INTENSITY

SCANNING MECHANISMS → OSCILLATING MIRROR ; SAWTOOTH/Z-SHAPED/SINUSOIDAL
→ ROTATING POLYGON ; PARALLEL LINES
→ NUTATING MIRROR (PALMER SCAN) ; ELLIPTICAL.

LIDAR	PHOTOGRAMMETRIC
DAY/NIGHT ACQUISITION	ONLY DAY ACQUISITION
DIRECT 3D ACQUISITION	COMPLICATED
VERTICAL ACCURACY BETTER THAN PLANIMETRIC	PLANIMETRIC ACCURACY BETTER THAN VERTICAL
DIFFICULT TO DERIVE SEMANTIC INFORMATION	RICH OF SEMANTIC INFORMATION
ENRICHED WITH INTENSITY VALUES	

MULTIPLE RETURNS: POSSIBLE TO EXTRACT INFORMATION ON PROFILE OF COMPLEX STRUCTURES (TREES)

- DISCRETE RETURN: LOGS WHEN THRESHOLD IS EXCEEDED
- WAVEFORM RECORDING: INTENSITY PROFILE
- PHOTON COUNTING: LOW POWER, GOOD FOR CLOUD PENETRATION

ATMOSPHERIC LIDAR (USUALLY BISTATIC, WAVEFORM, DOPPLER)

DETECTION OF ATMOSPHERIC AEROSOLS AND DENSITIES. CANNOT TELL THE COMPOSITION OF THE ATMOSPHERE IN FACT ONLY THE SCATTERING INTENSITY IS DETECTED BUT NOT THE SPECTRA! CONTAINS SPECIFIC INFORMATIONS RELATED TO SPECIES WHICH COULD BE USED TO DETERMINE THE COMPOSITION OF THE OBJECT REGION

DOPPLER EFFECT + BOLTZMANN DISTRIBUTION LEAD TO MORE SOPHISTICATED SPECTRAL ANALYSIS FOR WIND/TEMPERATURE.

DOPPLER LIDAR: EXPLOITS DOPPLER EFFECT TO ESTIMATE THE TARGET SPEED (E.G.: WIND SPEED) SPEED OF PARTICLES THANKS TO DOPPLER EFFECT, COMBINED WITH TIME OF FLIGHT, BEAM INTENSITY, CAN GIVE EXTREMELY PRECISE INFO ON THE PROFILE OF WIND SPEED.

DIAL LIDAR (DIFFERENTIAL ABSORPTION LIDAR): TO DETECT CONCENTRATION LEVEL OF SUBSTANCES, BASED ON ABSORPTION PROPERTIES OF THE SUBSTANCES TO STUDY. TWO PULSES WITH DIFFERENT FREQUENCIES ARE USED (ONE AT ABSORPTION FREQUENCY, THE OTHER AT A DIFFERENT ONE). BY MEASURING DIFFERENT INTENSITIES WE DEDUCE THE ABSORPTION AND SO THE CONCENTRATION

FLUORESCENCE LIDAR: PULSE INDUCE A FLUORESCENCE REACTION. ENERGY OF PULSE ABSORBED BY THE TARGET. THEN THE TARGET EMITS RADIATION AT A DIFFERENT λ. (DETECTION OF OIL SPILL)

MICROWAVES REMOTE SENSING

PASSIVE: 1-200 GHz, SAME PRINCIPLE OF THERMAL INFRARED, VERY LOW EMISSIVITY ⇒ VERY LARGE IFOV
STUDY OF SNOWFIELDS, GLACIERS, WIDE AREA PRECIPITATIONS.

ACTIVE: NON IMAGING = 1-D MEASUREMENTS OF A GIVEN TARGET LIKE DISTANCE OR SPEED (ALTIMETER, WIND SCATTEROMETER)
IMAGING = 2-D RECONSTRUCTION OF THE CHARACTERISTICS OF AN ENTIRE SCENE

(+) ACQUIRES DATA ALSO AT NIGHT
FULL CONTROL OF SIGNAL CHARACTERISTICS (WAVELENGTH, POLARIZATION, INCIDENCE ANGLE)
USE FREQUENCIES THAT ALLOW OBSERVATION WHEN NOTHING CAN BE OBSERVED (VISIBLE)
POSSIBILITY OF MEASURING THE TIME OF FLIGHT

1-10 GHz TRASMISSIVITY IS 100% INDEPENDENTLY FROM PRECIPITATIONS / CLOUDS, REMOTE SENSING IS SO POSSIBLE WHEN VISIBILITY IS LIMITED (TROPICS ALSO)
22 GHz TRASMISSIVITY ≤ 85% DUE TO WATER VAPOR ABSORPTION
60 GHz TOTAL ABSORPTION BAND FOR OXIGEN

INTERACTION WAVES-MATERIALS IS DIFFERENT RESPECT TO VISIBLE, IN FACT DIFFERENT PENETRATIONS
SHORT WAVELENGTHS HAVE SMALL PENETRATION

RADAR (RADIO DETECTING AND RANGING): AN IMPULSE IS EMITTED BY A SOURCE AND REFLECTED BY THE TARGET
THE CHARACTERISTICS OF THE RETURN SIGNAL ALLOW ONE TO STUDY THE PROPERTIES OF THE TARGET. INTERPRETATION OF 2D IMAGES IN IR REGION IS MORE DIFFICULT.

PHYSICAL PRINCIPLE SIMILAR TO LIDAR, A SERIES OF PULSES USED. THE TIME OF FLIGHT ALLOWS TO MEASURE THE DISTANCE. THE BEAM HAS LARGER SPOT SO DIFFERENT OBJECTS REFLECT EACH PULSE!! THE INTENSITY OF REFLECTION IS USED TO CREATE AN IMAGE.

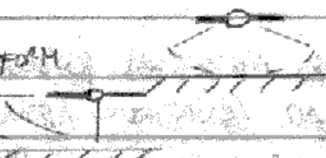
REAL APERTURE RADAR: PLATFORM IS IN FIXED POSITION / THE FUNCTION IS TO ACQUIRE DATA OVER DIFFERENT REGIONS

SYNTHETIC APERTURE RADAR: EXPLOITS THE PLATFORM MOTION TO EMULATE A LARGER ANTENNA.

SURFACE SCATTERING: SMALL INCIDENT ANGLES (ALMOST ⊥) THE SMOOTH SURFACE GIVES BACK MORE POWER, AT > ANGLES A ROUGH SURFACE GIVES MORE BACKSCATTERED POWER.

POLARIZATION: VERY IMPORTANT. CONSIDER POLARIZATION OF EMITTED AND RECEIVED WAVE. FOUR COMBINATIONS HH, HV, VH, VV.
SO IT IS POSSIBLE TO UNDERSTAND ALIGNMENT OF OBJECT RESPECT TO RADAR, SCATTERING, ROUGHNESS, CORNER REFLECTORS...

NADIR LOOKING: ACQUISITION IN THE VERTICAL DIRECTION BELOW THE PLATFORM



SIDE LOOKING (SAR, AIRBORNE): IT ACQUIRES IMAGES ON ONLY ONE LATERAL SIDE
BECAUSE IT CAN ONLY MEASURE DISTANCES AND CANNOT DISTINGUISH THE LEFT HAND SIDE FROM THE RIGHT HAND ONE.

OPTICAL IMAGING ⇒ NADIR LOOKING TYPE

IMAGING → SIDE-LOOKING MODE

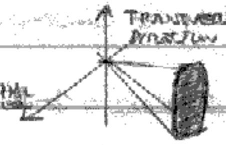
(41)

ANTENNA: HIGHLY NON ISOTROPIC (VERY DIRECTIVE)

$$\boxed{\underset{\text{POWER}}{\Phi_T}(R,\theta,\varphi) = \frac{\overset{\text{TOTAL POWER}}{P_T}}{4\pi R^2}\,\overset{\text{GAIN}}{G(\theta,\varphi)}}$$

POWER FLUX FOR THE ISOTROPIC ONE

FOR SIDE-LOOKING RADARS THE ANGULAR WIDTH OF THE LOBE IN THE AZIMUTHAL DIRECTION IS MUCH LOWER THAN IN THE TRANSVERSE DIRECTION



$\boxed{P_{\text{BACKSCATTERED}} = \Phi_T \sigma}$ THE TARGET WILL REFLECT DEPENDING ON ITS SHAPE AND SIZE EXPRESSED BY σ COEFFICIENT! KNOWN FOR SIMPLE SHAPES, OTHERWISE WE HAVE TO DETERMINE EXPERIMENTALLY

$\boxed{\Phi_R = \frac{\Phi_T \sigma}{4\pi R^2} = \frac{P_T G \sigma}{(4\pi)^2 R^4}}$ POWER FLUX AT THE RECEIVER

$\boxed{P_R = \frac{P_T \cdot G \cdot \sigma \cdot A_{eff}}{(4\pi)^2 R^4}}$ INTERCEPTED POWER

$G = \frac{4\pi A_{eff}}{\lambda^2}$ ANTENNA GAIN

$\boxed{P_R = \frac{P_T G^2 \sigma \lambda^2}{(4\pi)^3 R^4}}$ RADAR EQUATION (IN THE MONOSTATIC CASE)

DISTORTIONS: DISTORTION NEAR THE NADIR REGION (FORESHORTENING, LAYOVER, SHADOWING).

RESOLUTION

SLANT RANGE RESOLUTION: RESOLUTION IN RADIAL DIRECTION: MINIMAL DIFFERENCE BETWEEN THE DISTANCES OF TWO TARGETS THAT THE ANTENNA CAN DISTINGUISH.

$\boxed{\rho_{SR} = \frac{\tau c}{2}}$ τ = PULSE DURATION

GROUND RANGE RESOLUTION: RESOLUTION IN THE DIRECTION OF THE MAIN EXTENSION OF THE WAVE BEAM; DEPENDS ON THE ANGLE AND THUS ON THE TARGET DISTANCE FROM THE VERTICAL BELOW THE PLATFORM. RESOLUTION IS LOWER IN FAR RANGE (BETTER!)

$\boxed{\rho_{GR} = \frac{\rho_{SR}}{\sin\theta}}$

AZIMUTHAL RESOLUTION: MINIMAL DETECTABLE DISTANCE IN THE FLIGHT DIRECTION

$\boxed{\rho_{AR} = \frac{h\beta}{\cos\theta}}$

FUNDAMENTAL PARAMETER: DOESN'T DEPEND ON SLANT RANGE RESOLUTION AND SO DOESN'T DEPEND ON DURATION OF PULSE SHAPE. IT DEPENDS ONLY ON THE WIDTH OF THE LOBE IN THE FLIGHT DIRECTION AND ON THE ALTITUDE

$\beta \approx \frac{\lambda}{L}$ LOBEWIDTH AT $-3\,dB$, $\rho_{AR} \approx \frac{h\lambda}{L\cos\theta}$

TO IMPROVE GROUND RANGE RESOLUTION IT IS NECESSARY TO SHORTEN THE PULSES/INCREASE BANDWIDTH BUT THE ENERGY DECREASES. I CANNOT USE TOO MUCH POWER → SOLUTION: CHIRPED PULSES: LONG IN DURATION WITH LOW POWER, INSTANTANEOUS FREQUENCY INCREASES IN TIME.

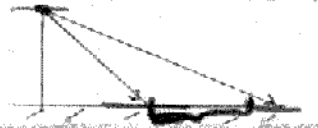
(+) SAME RESOLUTION WITH ALMOST THE SAME BANDWIDTH OF SHORT PULSES, LOWER POWER. RECEIVER CAN DISTINGUISH CLOSE TARGETS

(-) HIGHER RECEIVER COMPLEXITY

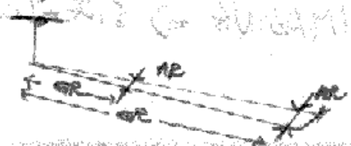
TO IMPROVE AZIMUTHAL RESOLUTION WE COULD USE LONGER ANTENNAS/SHORTER WAVELENGTHS BUT IT'S IMPRACTICAL. SOLUTION: SAR (SYNTHETIC APERTURE RADARS): EXPLOITS PLATFORM MOVEMENT TO SIMULATE A LONGER ANTENNA IN THE AZIMUTHAL DIRECTION. RADAR EMITS HIGHLY COHERENT WAVE AND ANALYZES ALL THE ECHOES FROM A GIVEN POINT FLYING OVER SO AS TO SIMULATE WHAT WOULD HAVE BEEN RECEIVED BY A LONGER ANTENNA THAN THE PHYSICAL ONE. IT IS ACTUALLY A SMART USAGE OF THE DOPPLER EFFECT.

SLANT RANGE

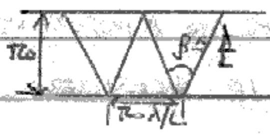
GROUND RANGE



AZIMUTHAL

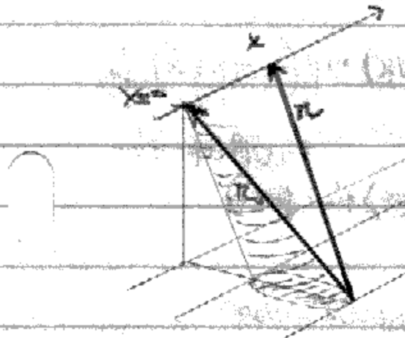


SYNTHETIC APERTURE: LENGTH OF THE PATH DURING WHICH THE TARGET IS VISIBLE
IT EQUALS THE WIDTH OF THE LOBE AT GROUND IN THE AZIMUTHAL DIRECTION



ANGULAR RESOLUTION: $\boxed{\alpha_{SA} = \frac{\lambda}{2L_{SA}} = \frac{L}{2R_0}}$

$\boxed{\rho_{SA} = \alpha_{SA}\cdot R_0 = \frac{L}{2}}$ GROUND RESOLUTION IN AZIMUTHAL DIRECTION IS HALF THE REAL APERTURE OF THE ANTENNA



$$R = \sqrt{R_0^2+x^2} = R_0\sqrt{1+\frac{x^2}{R_0^2}} \approx R_0\left(1+\frac{x^2}{2R_0^2}\right) = R_0 + \frac{x^2}{2R_0} \quad (\text{IF } R_0 \gg x)$$

$$\varphi(x) = 2\cdot\frac{2\pi}{\lambda}\cdot\left(R_0+\frac{x^2}{2R_0}\right) = \frac{4\pi R_0}{\lambda} + \frac{2\pi x^2}{\lambda R_0} = \underbrace{\frac{4\pi R_0}{\lambda}}_{\text{CONSTANT}} + \frac{2\pi v^2 t^2}{\lambda R_0} \quad (\text{PHASE})$$

$$f(t) = \frac{1}{2\pi}\cdot\frac{\partial\varphi(t)}{\partial t} = \underbrace{\frac{2v^2}{\lambda R_0}}_{\text{CHIRP RATE}} t = K t \quad (\text{INSTANTANEOUS FREQUENCY})$$

$t_{min} = -\frac{T}{2} = -\frac{L_{SA}}{2v} = -\frac{\beta_{RA} R_0}{2v}$ $\qquad t_{max} = \frac{T}{2} = \frac{\beta_{RA} R_0}{2v}$

$B_a = f(t_{max}) - f(t_{min}) = \frac{2v^2}{\lambda R_0}\cdot\frac{\beta_{RA} R_0}{v} = \frac{2\beta_{RA} v}{\lambda}$

SAR BASED ON PULSES: WE NEED TO USE SHANNON THEOREM AND USE PULSES SUFFICIENTLY CLOSE TO ALLOW THE RECONSTRUCTION OF SIGNAL WITH SUCH A BAND.

$\boxed{PRF = 2B_a = \frac{4\beta_{RA} v}{\lambda}}$ PULSE REPETITION FREQUENCY: LOWER BOUND FOR THE FREQUENCY OF PULSES

$\boxed{T = \frac{\lambda}{4\beta_{RA} v}}$ SAMPLING PERIOD ⇒ AZIMUTHAL DISTANCE BETWEEN TWO SAMPLES $\frac{\lambda}{4\beta_{RA}} \approx \frac{L}{4}$

FOCUSING: EACH POINT TARGET IS OBSERVED ON A SEQUENCE OF PULSES AND NOT ONLY JUST PULSE. THE RETURN PULSES OF ONE TARGET IN GENERAL OVERLAP WITH THE RETURN PULSES OF OTHER TARGETS. USUALLY WE EXPRESS THIS BY SAYING THAT THE SAR IS NOT FOCUSED. WE NEED TO PROCESS THE SIGNAL (FOCUSING) SO AS TO CONCENTRATE ALL THE ENERGY OF A RETURN PULSE FROM A TARGET IN ONE SINGLE POINT.

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SOLUTION: PROCESSING IN AZIMUTH

PROCESSING IN AZIMUTH: WE PROCESS THE SIGNAL USING THE PHASE HISTORY.

WE USE A REFERENCE PHASE THAT IS Ø AT $t=0$, THE PHASE CAN BE $\boxed{\varphi(t) = \pi \cdot K \cdot t^2}$

IF BACKSCATTERING IS INDEPENDENT FROM THE ANGLE AND CONSTANT IN TIME THE AMPLITUDE WILL BE CONSTANT.

FOR EXAMPLE WE RECEIVE $\boxed{S(t) = A_0 \cdot \exp(j\pi k t^2)}$ in $[t_{min}, t_{max}]$

WE COMPUTE A CORRELATION WITH A REFERENCE PULSE SO AS TO COMPRESS THE SIGNAL ON A SHORTER DOMAIN. WE USE AN IDEAL SIGNAL $\boxed{R(t) = \exp(-j\pi k t^2)}$

SO WE HAVE CORRELATION BETWEEN $S(t)$ AND $R(t)$ $\Rightarrow \delta(t)$ COMPRESSION IN THE ORIGIN

$S(t)$ IS ALWAYS WINDOWED $S(t) \cdot W(t)$

CORRELATION $V(t) = \int_{-\infty}^{+\infty} S(\alpha)\, W(\alpha)\, R(t+\alpha)\, d\alpha = A_0 \exp(-jk\pi t^2) \underbrace{\int W(\alpha) \exp(-j2\pi k t \alpha)\, d\alpha}_{\text{FOURIER !!!}}$

$= A_0 \exp(-j\pi k t^2) \cdot \hat{W}(kt)$

$\mathrm{RECT}(\frac{t}{T}) \Leftrightarrow \mathrm{sinc}(kTt)$

THE PULSE RESOLUTION DEPENDS ON THE ANALYSIS WINDOW.

WIDTH OF THE PULSE IS HALF OF THE WIDTH OF THE MAIN LOBE IN CASE OF A RECTANGULAR WINDOW

ZEROS $t = \pm 1/kT$ AZIMUTH RESOLUTION $\rho_{SA} = \frac{V}{KT} = \frac{V}{B_a} = \frac{L}{2}$

PROCESSING IN RANGE: TO COMPRESS PULSES $S(t) = A_0 \exp(j\pi k t^2)\, W(t)$ (CHIRP)

USED BAND DEPENDS ON $W(t)$ AND ON CHIRP RATE k $B = kT$, $R(t) = A_0 \exp(-j\pi k t^2)$

$V(t) = A_0 \exp(-j\pi k t^2) \cdot \hat{W}(kt)$ $\boxed{\rho_{SR} = \frac{c\tau}{2} = \frac{c}{2B}}$

SPECKLE NOISE: TYPICAL SAR DISTURBANCE. IT APPEARS AS A NOISE TO OUR EYES. DUE TO THE INTERFERENCE BETWEEN THE RETURN WAVES FROM DIFFERENT TARGETS WHICH COMPOSE EACH IMAGED PIXEL. EACH PIXEL IS OBTAINED BY CONSIDERING THE CONTRIBUTIONS OF DIFFERENT TARGETS WHICH BACKSCATTER THE PULSES WITH DIFFERENT AMPLITUDES AND PHASES, WHICH EXACT VALUES CANNOT BE PREDICTED. RECEIVED WAVE IS THE VECTORIAL SUM OF THE CONTRIBUTIONS THAT INTERFERE IN A RANDOM WAY.

IT IS POSSIBLE TO REDUCE NOISE BY SACRIFICING RESOLUTION AND AVERAGING THE VALUES ON A GIVEN REGION.

SAR INTERFEROMETRY: FOR AERIAL PHOTOGRAPHY WE USE STEREOGRAMMETRY TO INFER ALTITUDE. WITH SAR WE MEASURE TWO DIFFERENT POSITIONS TO INFER THE ANGLES FROM THE DISTANCES. WE CALL INTERFEROMETRY BECAUSE THE INFORMATION IS BROUGHT ON THE PHASE DIFFERENCE.

$\phi = 2\frac{2\pi}{\lambda}(R_1 - R_2) = \frac{4\pi}{\lambda}\Delta R$ (PHASE DIFFERENCE) $\Delta R \simeq B \sin(\theta - \alpha)$

(!) PHASE DIFFERENCES ARE ONLY KNOWN AT MULTIPLES OF 2π → NECESSARY TO PERFORM THE PHASE UNWRAPPING

POLARIZATION: USEFUL TO INFER IMPORTANT INFORMATION ON THE OBSERVED TARGET

- HH: PREFERRED FOR STUDY OF SOILS (VERTICAL STRUCTURES MORE TRANSPARENT), WATER/ICE, TARGETS ON WATER
- VV: WATER SURFACES ROUGHNESS / WIND VELOCITY
- HV/VH: USEFUL WITH MULTIPLE REFLECTIONS.