

# Outliers in Time Series

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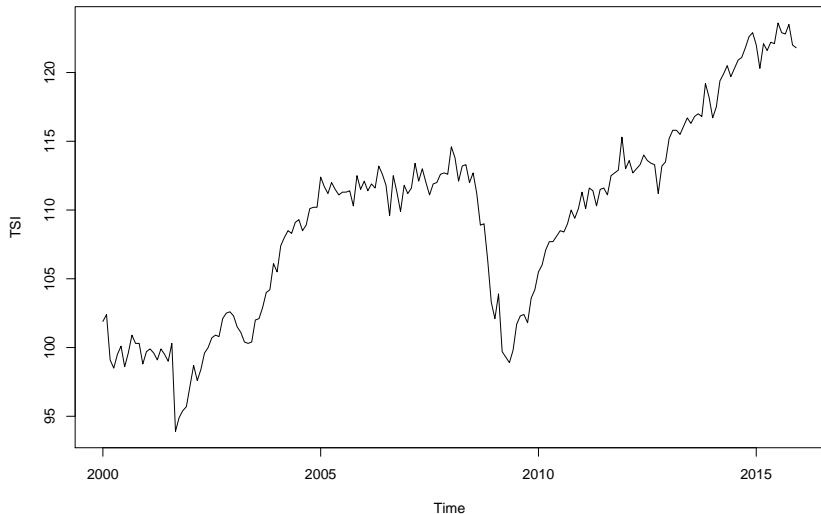
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# Outline

- ▶ Motivating example
- ▶ Models to describe four types of outliers
- ▶ Estimating outlier effects using linear regression
- ▶ Using estimated effects to detect outliers
- ▶ R function `tsoutliers::tso`

# Motivating Example

- ▶ Transportation Services Index (TSI): monthly measure of volume of services provided by for-hire transportation sector



# Motivating Example

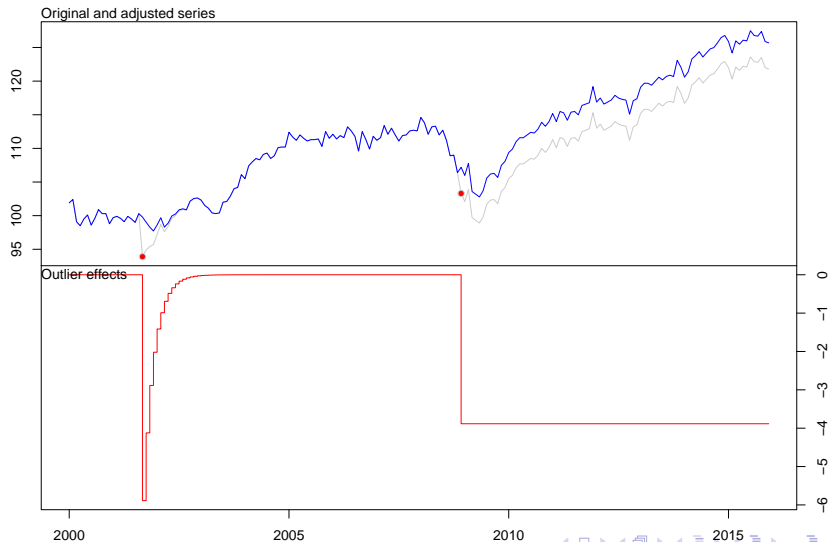
- ▶ The `tso` function in the `tsoutliers` package automatically detects and fits a model to the series with outlier effects removed
  - ▶ We'll describe how this works
- ▶ Two outliers detected:
  - ▶ Temporary Change outlier in Sept. 2001
  - ▶ Level Shift outlier in Dec. 2008

```
tso_output$outliers
```

##	type	ind	time	coefhat	tstat
## 1	TC	21	2001:09	-5.889364	-5.928143
## 2	LS	108	2008:12	-3.884195	-3.633127

# Motivating Example

```
plot(tso_output)
```



# Motivating Example

- ▶ Fitting ARIMA(1, 1, 0) model...
- ▶ Model fit without adjusting for outliers:
  - ▶  $\hat{\alpha}_1 = -0.165952$
  - ▶  $\hat{\sigma}_2 = 1.425164$
- ▶ Model fit after adjusting for outliers:
  - ▶  $\hat{\alpha}_1 = -0.2159957$
  - ▶  $\hat{\sigma}_2 = 1.1362642$
- ▶ Failing to adjust for outliers can result in
  - ▶ Wrong model or biased parameter estimates
  - ▶ Increased forecasting error

# Outlier Models

- ▶ ARIMA( $p, d, q$ ) process

$$X_t = \frac{\theta(B)}{\alpha(B)\phi(B)} Z_t$$

- ▶ Roots of  $\theta(B), \phi(B)$  outside unit circle
- ▶  $\alpha(B) = (1 - B)^d$
- ▶  $Z_t \sim_{iid} \text{Normal}(0, \sigma^2)$

# Outlier Models

- ▶ Observed series

$$X_t^* = X_t + \text{outlier effect}$$

- ▶ Four models for outlier effect:
  - ▶ Additive outlier (AO)
  - ▶ Level shift (LS)
  - ▶ Temporary change (TC)
  - ▶ Innovational outlier (IO)



# Outlier Models

$$\text{AO:} \quad X_t^* = X_t + \omega l_t(t_1)$$

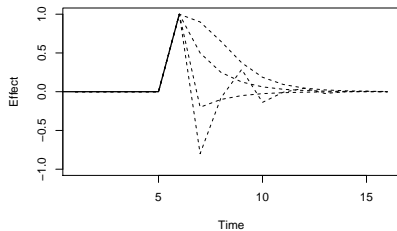
$$\text{LS:} \quad X_t^* = X_t + \frac{1}{1-B} \omega l_t(t_1)$$

$$\text{TC:} \quad X_t^* = X_t + \frac{1}{(1-\delta B)} \omega l_t(t_1)$$

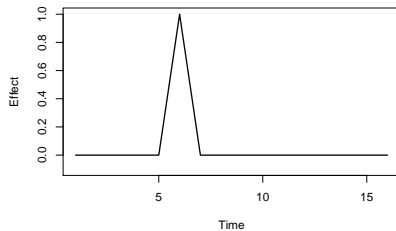
$$\text{IO:} \quad X_t^* = \frac{\theta(B)}{\alpha(B)\phi(B)} [Z_t + \omega l_{t_1}(t)]$$

# Outlier Models

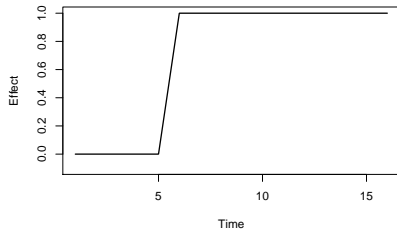
**IO**



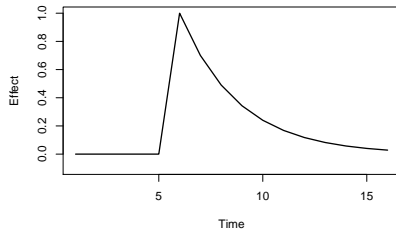
**AO**



**LS**



**TC**



# Outlier Estimation

- ▶ Obtain residuals  $\hat{e}_t$  from the observed series  $X_t^*$  by applying

$$\pi(B) = \frac{\alpha(B)\phi(B)}{\theta(B)} = 1 - \pi_1 B - \pi_2 B^2 - \pi_3 B^3 - \dots$$

- ▶ (Remember  $X_t = \frac{\theta(B)}{\alpha(B)\phi(B)}$ )
- ▶ If there were no outliers, result is white noise:  $\pi(B)X_t = Z_t$
- ▶ When outlier present, residuals  $\hat{e}_t = \pi(B)X_t^*$  reveal outlier effect

# Outlier Estimation

- Residuals for each type of outlier:

$$\text{IO:} \quad \hat{e}_t = \omega I_t(t_1) + Z_t$$

$$\text{AO:} \quad \hat{e}_t = \omega \pi(B) I_t(t_1) + Z_t$$

$$\text{LS:} \quad \hat{e}_t = \omega \frac{\pi(B)}{1-B} I_t(t_1) + Z_t$$

$$\text{TC:} \quad \hat{e}_t = \omega \frac{\pi(B)}{1-\delta B} I_t(t_1) + Z_t$$

- All have the form of simple linear regression:

$$\hat{e}_t = \omega x_t + Z_t$$

# Outlier Estimation

- ▶ Least-squares estimate:

$$\hat{\omega} = \frac{\sum_{t=t_1}^n \hat{e}_t x_t}{\sum_{t=t_1}^n x_t^2}$$

- ▶ Divide by standard error:

$$\hat{\tau} = \frac{\hat{\omega}}{\hat{\sigma} / \sqrt{\sum_{t=t_1}^n x_t^2}}$$

- ▶ Approximately  $\sim \text{Normal}(0, 1)$

# Outlier Detection

- ▶ At each  $t = 1, \dots, n$ , for each outlier type (AO, LS, TC, IO),
  - ▶ Estimate outlier effect  $\hat{\omega}$  and calculate  $\hat{\tau}$
  - ▶ Large  $|\hat{\tau}|$  indicates an outlier
- ▶ Once outlier is detected, effect can be subtracted to obtain adjusted series

# Outlier Detection

- ▶ Iterative procedure for detecting outliers, adjusting series, and fitting (seasonal) ARIMA model:
  - ▶ Chen, C. and Liu, Lon-Mu (1993), “Joint Estimation of Model Parameters and Outlier Effects in Time Series,” *Journal of the American Statistical Association*, 88, 284–297.
- ▶ Three stages with many iterations of outlier detection, series adjustment, and re-fitting the model
  - ▶ Necessary to deal with masking, other issues with estimating outlier effects one-at-a-time when multiple outliers are present
- ▶ Implemented in `tso` function in `tsoutliers` R package

# Outlier Detection

```
tso(y, cval = NULL, delta = 0.7,  
     types = c("AO", "LS", "TC"),  
     maxit = 1, maxit.iloop = 4,  
     tsmethod = c("auto.arima", "arima", "stsm"),  
     args.tsmethod = NULL)
```



# The End

- ▶ Thank you
- ▶ “Please clap” — Jeb Bush