Outliers in Time Series

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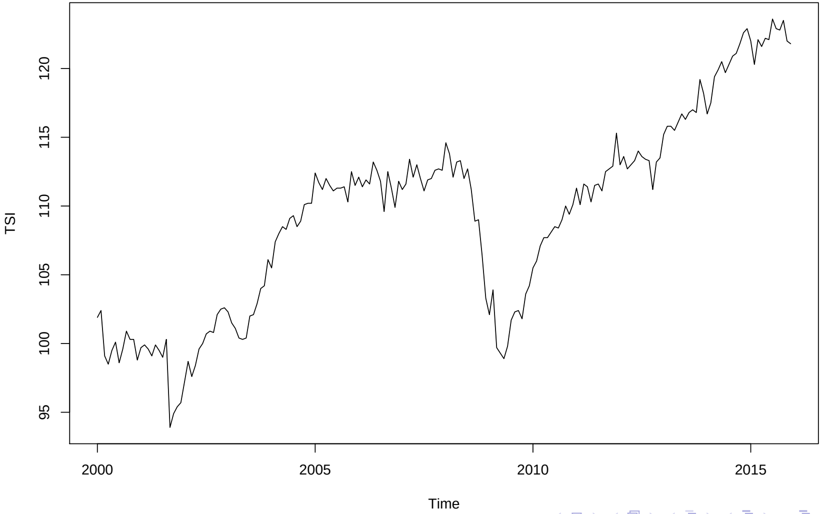
Outline

- Motivating example
- Models to describe four types of outliers
- Estimating outlier effects using linear regression
- Using estimated effects to detect outliers
- R function `tsoutliers::tso`

Motivating Example

- Transportation Services Index (TSI): monthly measure of volume of services provided by for-hire transportation sector



Motivating Example

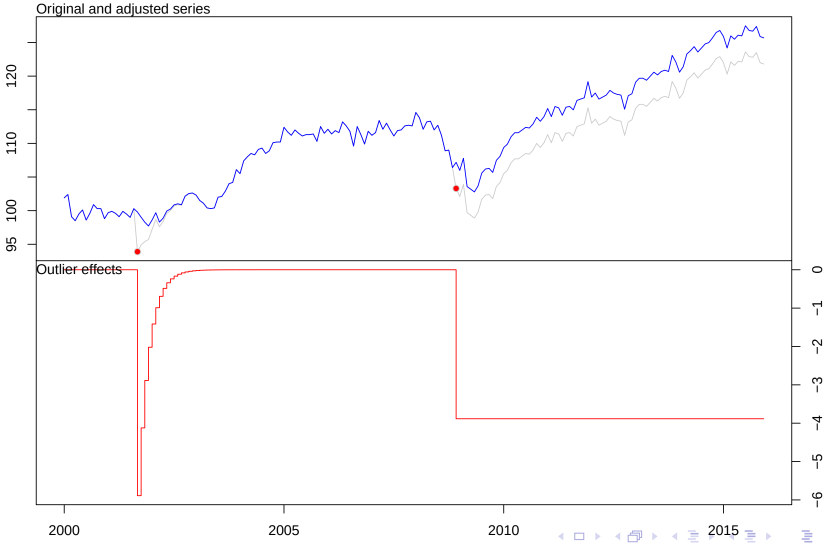
- The `tso` function in the `tsoutliers` package automatically detects and fits a model to the series with outlier effects removed
  - We'll describe how this works
- Two outliers detected:
  - Temporary Change outlier in Sept. 2001
  - Level Shift outlier in Dec. 2008

```
tso_output$outliers
```

```
##   type ind    time   coefhat     tstat
## 1   TC  21 2001:09 -5.889364 -5.928143
## 2   LS 108 2008:12 -3.884195 -3.633127
```

Motivating Example

```
plot(tso_output)
```



Motivating Example

- Fitting ARIMA(1, 1, 0) model...
- Model fit without adjusting for outliers:
  - $\hat{\alpha}_1 = -0.165952$
  - $\hat{\sigma}_2 = 1.425164$
- Model fit after adjusting for outliers:
  - $\hat{\alpha}_1 = -0.2159957$
  - $\hat{\sigma}_2 = 1.1362642$
- Failing to adjust for outliers can result in
  - Wrong model or biased parameter estimates
  - Increased forecasting error

Outlier Models

- ARIMA(p, d, q) process

$$X_t = \frac{\theta(B)}{\alpha(B)\phi(B)} Z_t$$

- Roots of $\theta(B), \phi(B)$ outside unit circle
- $\alpha(B) = (1 - B)^d$
- $Z_t \sim_{iid}$ Normal$(0, \sigma^2)$

Outlier Models

- Observed series

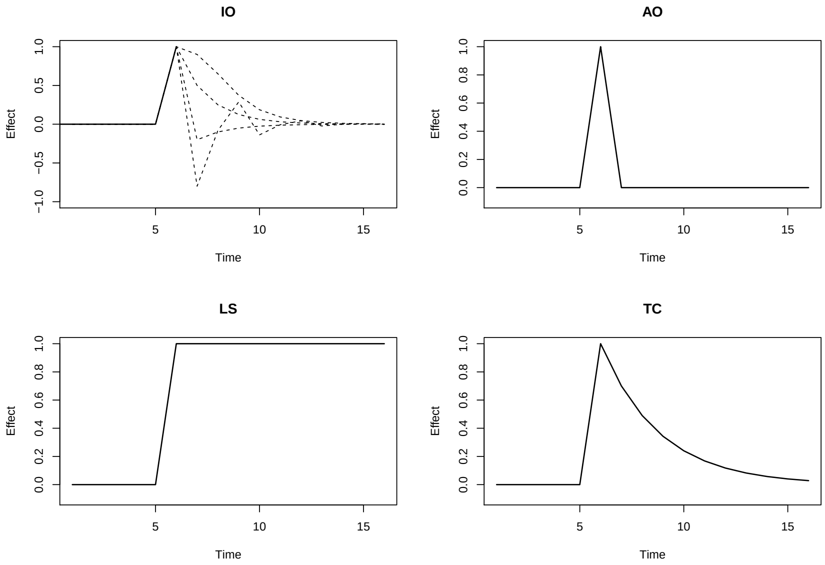
$$X_t^* = X_t + \text{ outlier effect}$$

- Four models for outlier effect:
  - Additive outlier (AO)
  - Level shift (LS)
  - Temporary change (TC)
  - Innovational outlier (IO)

Outlier Models

$$
\begin{aligned}
\text{AO:} \qquad & X_t^* = X_t + \omega I_t(t_1) \\
\text{LS:} \qquad & X_t^* = X_t + \frac{1}{1-B}\omega I_t(t_1) \\
\text{TC:} \qquad & X_t^* = X_t + \frac{1}{(1-\delta B)}\omega I_t(t_1) \\
\text{IO:} \qquad & X_t^* = X_t + \frac{\theta(B)}{\alpha(B)\phi(B)}\omega I_t(t_1) \\
& \quad = \frac{\theta(B)}{\alpha(B)\phi(B)}\left[Z_t + \omega I_t(t_1)\right]
\end{aligned}
$$

Outlier Models



Outlier Estimation

- Obtain residuals $\hat{e}_t$ from the observed series X_t^* by applying
$$\pi(B) = \frac{\alpha(B)\phi(B)}{\theta(B)} = 1 - \pi_1 B - \pi_2 B^2 - \pi_3 B^3 - \ldots$$
  - (Remember $X_t = \frac{\theta(B)}{\alpha(B)\phi(B)} Z_t$)
- If there were no outliers, result is white noise: $\pi(B) X_t = Z_t$
- When outlier present at $t = t_1$, residuals $\hat{e}_t = \pi(B) X_t^*$ for $t = t_1, \ldots, n$ reveal outlier effect

Outlier Estimation

- Residuals for each type of outlier:

$$
\begin{aligned}
\text{IO:} \qquad & \hat{e}_t = \omega I_t(t_1) + Z_t \\
\text{AO:} \qquad & \hat{e}_t = \omega \pi(B) I_t(t_1) + Z_t \\
\text{LS:} \qquad & \hat{e}_t = \omega \frac{\pi(B)}{1 - B} I_t(t_1) + Z_t \\
\text{TC:} \qquad & \hat{e}_t = \omega \frac{\pi(B)}{1 - \delta B} I_t(t_1) + Z_t
\end{aligned}
$$

- All have the form of simple linear regression (w/o intercept):

$$\hat{e}_t = \omega x_t + Z_t$$

Outlier Estimation

- Least-squares estimate:

$$\hat{\omega} = \frac{\sum_{t=t_1}^{n} \hat{e}_t x_t}{\sum_{t=t_1}^{n} x_t^2}$$

- Divide by standard error:

$$\hat{\tau} = \frac{\hat{\omega}}{\hat{\sigma} / \sqrt{\sum_{t=t_1}^{n} x_t^2}}$$

  - Approximately $\sim$ Normal(0, 1)

Outlier Detection

- At each $t = 1, \dots, n$, for each outlier type (AO, LS, TC, IO),
  - Estimate outlier effect $\hat{\omega}$ and calculate $\hat{\tau}$
  - Large $|\hat{\tau}|$ $(> C)$ indicates an outlier
- Once outlier is detected, estimated effect can be subtracted to obtain adjusted series

Outlier Detection

- Iterative procedure for detecting outliers, adjusting series, and fitting (seasonal) ARIMA model:
  - Chen, C. and Liu, Lon-Mu (1993), "Joint Estimation of Model Parameters and Outlier Effects in Time Series," *Journal of the American Statistical Association*, 88, 284–297.

1. Estimate model parameters, then locate outliers one-by-one, biggest $|\hat{\tau}|$ first, adjusting series each time
2. Re-estimate model parameters, drop any outliers where $|\hat{\tau}|$ no longer big enough after accounting for other outlier effects
3. Estimate final model parameters, repeat (1) and (2) using fixed parameters to get final set of outliers and effects

Outlier Detection

- Implemented in `tso` function in `tsoutliers` R package

```
tso(y, cval = NULL, delta = 0.7,
    types = c("AO", "LS", "TC"),
    maxit = 1, maxit.iloop = 4,
    tsmethod = c("auto.arima", "arima", "stsm"),
    args.tsmethod = NULL)
```

The End

- Thank you
- “Please clap” — Jeb Bush