CS101 Homework Assignment 10

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1. **Academic Honesty Declaration**

   In the process of finishing this homework:

   (a) I had conversations about the contents and solutions of this assignment with the following people:

   (b) I consulted the following resources, such as books, articles, webpages:

   - https://wch.github.io/latexsheet/
   - https://en.wikibooks.org/wiki/LaTeX/Basics

   (c) I did not look at the answers of any other students.

   (d) I did not provide my answers to other students.

2. **Writing Component**

   (a) **Variants of TM's**

   Another useful variant of a Turing machine has a single tape, but two reading heads. That is, the two reading heads can be looking at two cells of the tape at any time. Thus one of these Turing machines makes a transition based on the current state and the contents under each of the two read heads. It then writes something under each read head and moves each of the heads.Describe informally a two-head Turing machine that decides the language $L = \{ww \mid w\epsilon\ \{a, b\}^*\}$.

   Solution:

   First, we move one of the two reading heads all the way to right until we hit a blank space. Then, we can move one to the right with the reading head all the way to left and move the reading head to the right by one. With each move by one, we change an a to a 1 and a b to 0, we do this to keep track of the comparison later on to make sure the string is in the form ww. Then, when we encounter our first numbers 0 and 1 we move the leftmost reading head until we hit a blank space, then move by one. Now, we move the leftmost reading head by one until we hit a blank space and the rightmost reading head by one until we hit a blank space, but we compare the contents each time to make sure it's the form ww. If they don't match at any point, we reject. When the rightmost reading head hits a blank space, we can halt and accept since the string since it is the form ww.

(b) **Closure Properties of TM's**
Show that the decidable languages are closed under intersection and set difference. I.e., if L1 and L2 are decidable, then so are L1 ∩ L2 and L1 - L2.

Solution:

A Turing Machine that decides L1 ∩ L2: Given L_1 and L_2 are Turing Recognizable and Decidable languages by Turing Machines M1 and M2, respectively. On input x:

1) We run M1 on x. If M1 rejects, we reject. If M1 accepts, we run M2 on x.
2) If M2 accepts the accept, else reject.

A Turing Machine that decides L1 - L2: Given L_1 and L_2 are Turing Recognizable and Decidable languages by Turing Machines M1 and M2, respectively. On input x:

1) We run M1 and M2 on x. If M1 accepts and M2 rejects, we accept.

(c) **Closure Properties of TM's**

Prove that the following question is undecidable. Given a Turing machine M, is L(M) finite? (Prove without using Rice's Theorem)

Solution:

1) E(M) is true iff L(M) is finite.

2) $M_{<p,w>}$ discards own input, simulates p on w; accepts when halted.

3) Then,
If p halts on w, $L(M_{<p,w>})$ = everything = $N(M_{<p,w>})$ says no
If p does not halt on $w = \emptyset = N(M_{<p,w>})$ says yes

4) $H(p, w) = !N(M_{<p,w>})$

5) Contradiction!
Since we know the halting problem is undecidable, our assumption that that $L(M)$ is finite which is supposed to be decidable is incorrect.

(d) **Reduction**

Recall the domain of a function is the set of elements that it can be applied to, while the range is the set of elements obtained from the function. More carefully, if f is a function, Domain(f) = $\{x|f(x)$ is defined $\}$ and Range(f) = {y— there is an x such that f(x) = y}.

(a) Show that a set L is semi-decidable iff it is the domain of a Turing-computable (partial) function.

Solution:

Let us that there's a set E that's semi-decidable. Since we know that E is semi-decidable, that means there exists a Turing machine, M, that when given an element in set E it will halt or accept. As a result, any x in E is part of the domain of the Turing machine M which means that it is also in the domain of of a Turing computable function which represents our machine M. If it is the case that an element is not in the set E, we can reject it and halt, or run forever. Allowing our machine M to run forever allows us to build another machine $M^{'}$ that takes this machine M and a string r as input. If the machine M rejects on r, the outer machine $M^{'}$ runs forever which means its domain is only the elements in set E because for all other inputs it not defined.

(b) Show that a set L is semi-decidable iff it is the range of a Turing-computable (partial) function.

Solution:

Extra