CS101 Homework Assignment 12

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December 9, 2021

1. **Academic Honesty Declaration**

   In the process of finishing this homework:

   (a) I had conversations about the contents and solutions of this assignment with the following people:

   (b) I consulted the following resources, such as books, articles, webpages:

   - https://wch.github.io/latexsheet/
   - https://en.wikibooks.org/wiki/LaTeX/Basics

   (c) I did not look at the answers of any other students.

   (d) I did not provide my answers to other students.

2. **Writing Component**

   (a) **Recursion and Fixed Points**

      i. Define in PCF the recursive fibonacci function fib such that fib 0 = 1, fib 1 = 1, and fib n = fib (n-1) + fib (n-2) for n > 1. To simplify your answer you may assume that "+" is a built-in function of PCF (i.e., you do not have to define it).
      Solution:

      rec (fib: Int → Int) ⇒ fn(n:Int) if (isZero n) then 1 else if
      (isZero(pred n)) then 1 else
      fib(pred n) + fib(pred(pred n))

      ii. Repeat the previous part but this time write fib in the lambda calculus. This time you will use the Y-combinator to define the recursion. You may use the functions pred and plus we defined in class without repeating their definitions.
      Solution:

      F = λf.λn cond(isZero n) I
      (cond(isZero(pred n))) I
      (plus (f(pred n) f(pred(pred n))))

      let fib = YF

      iii. Use your definition of fib from above to calculate fib 2. Do it step by step, showing all of yourwork. You may assume that fib 1 = 1,fib 0 = 1. To make your life simpler you can also assume that pred, plus, and isZero all give the expected results without showing all of the reduction steps.. E.g., pred 2 = 1, pred 1 = 0, isZero 2 = false, isZero 1 = false, etc. All other β-reduction steps in the computation should be shown.
      Solution:

      Attached below

fib 2 = (F(fib)) 2

λn. cond (iszero n) I
(cond (isZero (pred n))) I
(plus (fib (pred n) fib (pred (pred n)))) 2

cond (isZero 2) I
(cond (isZero (pred 2))) I
(plus (fib (pred 2) fib (pred (pred 2))))

(cond (isZero (pred 2))) I
(plus (fib (pred 2) fib (pred (pred 2))))

plus (fib (pred 2) fib (pred (pred 2)))

plus (1 1)

= 2

(b) **Programming Component**

Begin by writing the computation rule using environments for let expressions.
Solution:

$$\frac{(\text{term, env}) \longrightarrow \text{s} \quad (\text{body, env}[\text{s/vble}]) \longrightarrow \text{s}}{\text{let vble} = \text{term in body} \longrightarrow \text{s}}$$