CS101 Homework Assignment 2

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1. **Academic Honesty Declaration**

   In the process of finishing this homework:

   (a) I had conversations about the contents and solutions of this assignment with the following people: Melat Feseha

   (b) I consulted the following resources, such as books, articles, webpages:
   - https://wch.github.io/latexsheet/
   - https://en.wikibooks.org/wiki/LaTeX/Basics

   (c) I did not look at the answers of any other students.

   (d) I did not provide my answers to other students.

2. **Writing Component**

(a) Give a careful proof of the following proposition using proof by induction : For all $i \leq 0$, and all strings $w \epsilon \sum$, $(w^R)^i = (w^i)^R$, where w^R is the reverse of string w.

Solution: Let a integers i. Let a string w be given. We go by induction on i.

- $(i = 0)$
  - (RHS) $(W^R)^i = (W^R)^0 = \epsilon$
  - (LHS) $(W^i)^R = (W^0)^R = \epsilon$

  Since $\epsilon \ = \ \epsilon$, this is immediate and we are finished.
- $(i = n+1)$ We must prove that $(w^R)^{n+1} = (w^{n+1})^R$. Our IH shows that $(w^R)^n = (w^n)^R$.

$$\begin{aligned}
(w^R)^{n+1} &= (w^R)^n \parallel (w^R)^1 \\
&= (w^n)^R \parallel (w^R)^1 \text{ by IH} \\
&= (w^n)^R \parallel (w^R) \text{ by 2.1 section theorem} \\
&= (w \times w^n)^R \\
&= (w^{n+1})^R
\end{aligned}$$

Since $(w^{n+1}) \ = \ (w^{n+1})$, this is immediate and we are finished.

(b) Are the following sets closed under the following operations? If not, what are their respective closures?

i. The even length strings over the alphabet $\{a, b\}$ under Kleene star.
Solution: The even length strings over alphabet $\{a, b\}$ are closed under the operation Kleene Star because the result would be strings of even lengths which is in the set of even length strings.

ii. The odd length strings over the alphabet $\{a, b\}$ under concatenation.
Solution: The odd length strings over alphabet $\{a, b\}$ are not closed under the concatenation operation because some of the results would be strings of even lengths which are not present in the set of odd length strings.