CS101 Homework Assignment 7

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1. **Academic Honesty Declaration**

   In the process of finishing this homework:

   (a) I had conversations about the contents and solutions of this assignment with the following people: Melat Feseha

   (b) I consulted the following resources, such as books, articles, webpages:
   - https://wch.github.io/latexsheet/
   - https://en.wikibooks.org/wiki/LaTeX/Basics

   (c) I did not look at the answers of any other students.

   (d) I did not provide my answers to other students.

2. **Writing Component**

   (a) **Context-free Grammars**

   A book I have claims that $L = \{w\epsilon\{0, ..., 9\}^*|$the number of odd digits in w is equal to the number of even digits in $w\}$ is generated by the following grammar:

   - $S \longrightarrow ESO$
   - $S \longrightarrow OSE$
   - $S \longrightarrow \epsilon$
   - $E \longrightarrow 0|2|4|6|8$
   - $O \longrightarrow 1|3|5|7|9$

   Find a string in L not generated by this grammar. Argue why it is not. What language does it generate? Describe it as carefully as you can (though you need not prove it).

   Solution:

   A string w that is in L but not generated by this grammar would be the 53200235 because it is comprised of digits from the finite alphabet $\{0, ..., 9\}^*$ and the amount of odd and even digits are equal. However, w cannot be generated by this grammar because the present grammar has no pathway to process strings with symmetrical structure; in other words, there are no recursive cases ESE and OSO. This grammar generates a language in which strings in that language must be comprised of digits from the finite alphabet $\{0, ..., 9\}^*$, the amount of odd and even digits in that string must be equal, and no symmetrical structure such as the first character being mirrored as the last character is allowed.

(b) **Pumping Lemma**

Let $L_2 = \{a^i b^j c^k : i, j, k \geq 0 \; and \; j > max(i, k)\}$. Use the pumping lemma to show L_2 is not context-free.

Solution:

i. Opponent picks a pumping lenght p.

$p =$ any natural number (that does not generate an empty string)

ii. I pick a string w

$w = a^p b^{p+1} c^p$

iii. They pick a decomposition.

$w = uvxyz$ such that $|vxy| \leq p$ and $vy \neq \epsilon$ such that $uv^i xy^i z$ is in L_2 (CFL) forall $i \geq 0$.

iv. Case I: vxy does not straddle boundaries.

A. In the case where vxy is within region a^p, if $i = 2$ the string produced by $uv^i xy^i z$ will not be in L because the number of a's will be greater than the number of b's which violates the requirements for a string in Language L.

B. In the case where vxy is within region b^{p+1}, if $i = 0$ the string produced by $uv^i xy^i z$ will not be in L because the number of b's will be less than the number of a's and c's which violates the requirements for a string in Language L.

C. In the case where vxy is within region c^p, if $i = 2$ the string produced by $uv^i xy^i z$ will not be in L because the number of c's will be greater than the number of b's which violates the requirements for a string in Language L.

v. Case II: vxy straddles regions a^p and b^{p+1}.

if $i = 2$, the string produced by $uv^i xy^i z$ will not be in L the number of a's will be greater than the number of b's which violates the requirements for a string in Language L.

vi. Case III: vxy straddles regions b^{p+1} and c^p.

if $i = 2$, the string produced by $uv^i xy^i z$ will not be in L the number of c's will be greater than the number of b's which violates the requirements for a string in Language L.

vii. Conclude

Because one of these cases above must happen, there will always be a contradiction where a certain string is pumped out of the Language L. Thus, making the assumption that L is a context free language false, and L must not be a context free language.

(c) **Pumping Lemma and Closure Properties**

Let $L = \{xay : x, y \; \epsilon \; \{0,1\}^*$ and x occurs somewhere as a sub string of $y\}$.

Hint: Show $L_1 = L \cap 0^*1^*a0^*1^*$ is not context-free and use closure properties to show L is not context-free as a result.

Solution:

To generate L_1, we need to acknowledge that L has the requirements where x must be a subset of y, and the regular expression $0^*1^*0^*1^*$ has the requirement where the order of 0's and 1's must be $01a01$. As a result, $L_1 = \{0^n1^ma0^p1^k : n \leq p$ and $m \leq k\ \}$. The requirements that $n \leq p$ and $m \leq k$ insure that first part of the string (part before a) is a subset of the second part (part after a), and the order of 0's and 1's is represented in the general structure of the string by $0^n1^ma0^p1^k$. Now, we will use the Pumping Lemma to show L_1 is not context free.

i. Opponent picks a pumping length p.

   $p =$ any natural number (that does not generate an empty string)

ii. I pick a string w

   $w = 0^p1^pa0^p1^p$

iii. They pick a decomposition.

   $w = uvxyz$ such that $|vxy| \leq p$ and $vy \neq \epsilon$ such that uv^ixy^iz is in L_2 (CFL) forall $i \geq 0$.

iv. Case I: vxy is within region 0^p (left side of a).

   if $i = 2$, the string produced by uv^ixy^iz will not be in L_1 because the number of 0's on the left side of a will not be less than or equal to the number of 0's on the right side of a which violates the requirements for a string in Language L_1.

v. Case II: vxy straddles regions 0^p and 1^p (left side of a).

   if $i = 2$, the string produced by uv^ixy^iz will not be in L_1 because the number of 0's on the left side of a will not be less than or equal to the number of 0's on the right side a. In addition, the number of 1's on the left side of a will not be less than or equal to the number of 1's on the right side a. These violate the requirement for a string in Language L_1.

vi. Case III: vxy is within region 1^p (left side of a).

   if $i = 2$, the string produced by uv^ixy^iz will not be in L_1 because the number of 1's on the left side of a will not be less than or equal to the number of 1's on the right side a. This violates the requirements for a string in Language L_1.

vii. Case IV: vxy straddles regions a and 1^p (left side of a).

   if $i = 2$, the string produced by uv^ixy^iz will not be in L_1 because the number of a's will be more than one which violates the requirement for a string in Language L_1 to only have one a in it's structure.

viii. Case V: vxy is within region a.

if $i = 2$, the string produced by uv^ixy^iz will not be in L_1 because the number of a's will be more than one which violates the requirement for a string in Language L_1 to only have one a in it's structure.

ix. Case VI: vxy straddles regions a and 0^p (right side of a).

if $i = 2$, the string produced by uv^ixy^iz will not be in L_1 because the number of a's will be more than one which violates the requirement for a string in Language L_1 to only have one a in it's structure.

x. Case VII: vxy is within region 0^p (right side of a).

if $i = 0$, the string produced by uv^ixy^iz will not be in L_1 because the number of 0's on the right side of a will be less than the number of 0's on the left side of a. This violates the requirement for a string in language L_1.

xi. Case VII: vxy straddles regions 0^p and 1^p (right side of a).

if $i = 0$, the string produced by uv^ixy^iz will not be in L_1 because the number of 0's on the right side of a will be less than the number of 0's on the left side of a or the number of 1's on the right side of a will be less than the number of 1's on the left side of a.These violate the requirement for a string in Language L_1.

xii. Case IX: vxy is within region 1^p (right side of a).

if $i = 0$, the string produced by uv^ixy^iz will not be in L_1 because the number of 1's on the right side of a will be less than the number of 1's on the left side of a. This violates the requirement for a string in language L_1.

xiii. Conclude

Because one of these cases above must happen, there will always be a contradiction where a certain string is pumped out of the Language L_1. Thus, making the assumption that L_1 is a context free language false, and L_1 must not be a context free language.

We know that the given regular expression $0^*1^*a0^*1^*$ is regular and we also know the intersection of a regular language and a context free language would yield a context free language. However, our Pumping Lemma proof demonstrated that L_1 is not context free; as a result, it must be the case that the language L_1 is not context free.

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