Collatz Conjecture

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Math 361B

March 18, 2019

1 Conjecture

When you take some number x and double it, so $2x$, you will get one more step than x did. For example, when you input 100 terms, you get 26 steps to reach 1, but when you input 200 terms, you receive an output of 27 steps to reach 1. Another example, when you input 500 terms, your output is 111 steps to reach 1, but with 1000 terms your output is 112 steps to reach 1.

Proof. Let $x \in \mathbb{N}$. Suppose that $2x$ terms gives you $n+1$ steps to reach 1 in the Collatz Conjecture. Then,

$$\begin{aligned} 2(\frac{x}{2}) &= \frac{2x}{2} \\ &= x \\ &\vdots \end{aligned}$$

On the other hand,

$$\begin{aligned} 2(3x+1) &= 6x+3 \\ &= 3(2x+1) \\ &\vdots \end{aligned}$$

Therefore, when x is doubled, it takes $n+1$ steps to reach 1. □

2 Conjecture

When you take 2^n terms like 1, 2, 8 or higher, it takes $n+1$ terms to reach 1. For example, the input of 1 term gives you an output of 1 step to reach 1, and for the input of 2 terms, the output is 2 steps to reach 1.The pattern goes on so for the input of 2^2 or 4, the output is 3 steps to reach 1, and so on.

3 Conjecture

When you have some odd number $(2n + 1)$ for your input, you will receive an even output $(2m)$. For example, the input of 5 gives you an output of 6 steps to reach 1, and the input of 109 gives you an output of 114 steps to reach 1.