

# Zero Divisors of $\mathbb{Z}_m$

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*Proof.* Suppose  $k \in \mathbb{Z}_m$ . If  $\gcd(k, m) = 1$ , then there is  $x, y \in \mathbb{Z}$  such that  $kx + my = 1$ . Thus,  $x$  is the inverse of  $k \in \mathbb{Z}_m$ . If  $\gcd(k, m) = d > 1$ . Then  $\frac{m}{d} \in \mathbb{Z}_m$  is nonzero such that  $k(\frac{m}{d}) = 0 \in \mathbb{Z}_m$  so that  $k$  is a zero divisor. Therefore every nonzero element of  $\mathbb{Z}_m$  is a zero divisor.  $\square$