CE 369: Applied Geostatistics

Homework 3

Cross-Statistics

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Contents

1 Objective

This assignment uses the same Berea sandstone data as Homework 2 to understand how to calculate cross-statistics for a given lag distance. All the code written to execute the desired calculations for this assignment can be found in the following GitHub repository: `https://github.com/tadler2014/AppliedGeostatistics`

2 Horizontal Lag

To understand the relationship between the resistivity and permeability of the Berea Sandstone across space, cross-statistical tools are employed such as cross h-scatterplots, cross correlation and cross variance/semivariance. The underlying principles and statistics of this analysis are the same as those utilized Homework 2. However, instead of pairing the value of one variable with the value of the same variable at another location, the values of different variables at different locations are paired. In this assignment, we are looking at the relationship between permeability at point (a) and water resistivity at point (a+h). This cross variable relationship is important to understand because identifying lags of a secondary variable can be useful predictors of primary variables in places where the primary variable is unknown.

Just as with Homework 2, the spatial extent of this analysis is subdivided into two planes of the Berea sandstone, the horizontal ($\theta = 0^o$) and the vertical ($\theta = 90^o$). With lag distances of 3, 6, 9, 12, 15, and 18 mm in the horizontal direction, the spatial continuity and corresponding cross-statistics are assessed (Figure 1). As it can noted in the cross h-scatterplots of Figure 1, the permeability and resistivity of the Berea Sandstone lack any clear form of spatial continuity in the horizontal direction. There is a weak inverse relationship between the two parameters which appears to randomly fluctuate in strength with increased lag distance as noted in the summary statistics (Figure 1). These results contrast with what was observed in Homework 2 where variables that were closer together in space are more similar. Clearly, this doesn't hold true for the permeability and resistivity cross-variable relationship in space in the same way.

3 Vertical Lag

Similarly to the horizontal lag assessment, the spatial continuity of permeability at point (a) and resistivity at point (a+h) is examined in the vertical direction for lag distances of 3, 6, 9, 12, 15, and 18 mm. In the vertical direction, the spatial continuity appear to lack a strong relationship between the two variables as seen in Figure 2. This relationship is weaker than then the one observed in the horizontal direction which is not too surprising considering that in the Homework 2 analysis of single variable continuity, there was a stronger spatial relationship in the horizontal direction. As noted in Figure 2, differences in the lag distance do not appear to considerably change the correlation or covariance of the relationship. This lack of spatial continuity undermines the ability to use resistivity as a secondary variable for permeability over space.

4 Spatial Correlation

To visualize the results of spatial correlation differently, the experimental data points (i.e. the statistics calculated above for the cross-correlogram, cross-variance, and cross-semivariance) are plotted over lag distance (Figures 3 & 4). In the horizontal direction (Figure 3), the pearson correlation coefficient appears to fluctuate around 0.42 as the lag increases. The covaraince and semivariance appear to show no trend, as noted earlier, which suggests that there is a lack of significant spatial correlation between resistivity and permeability. The same can be said for the semivariance in the vertical direction (Figure 4). The correlation coefficient and covariance in the vertical direction both appear to increase as the lag distance increases from 6 and 15mm, but with only 6 lag distances assessed no confident interpretations of this trend can be made yet.

For further analyses, it would likely be beneficial to increase the number of lag distances assessed so as to get a clearer image of any spatial correlation trends. Using the three statistics in tandem with each other is useful in that they provide a quantitative summary of the information contained in the h-scatter plots. However the coarse scale this analysis limits the visual interpretation of the data as shown in Figure 3 & 4. Furthermore, it might be interesting to significantly increase the lag distance. It is quite possible that the scale of this assessment in millimeters obscures relationships in permeability and resistivity across larger scales. In 1986, Katz and Thompson put forward an equation to describe the constraint of electrical conductivity (σ) on permeability (k) as shown in Equation 1 (Katz et al, 1986).

$$k = cl_c^2(\frac{\sigma}{\sigma_f}) \tag{1}$$

The validity of this model has been affirmed, but only on scale of meter to kilometers. Perhaps our spatial lens is a little too zoomed in to notice the underlying the relationship between our two variables of interest.

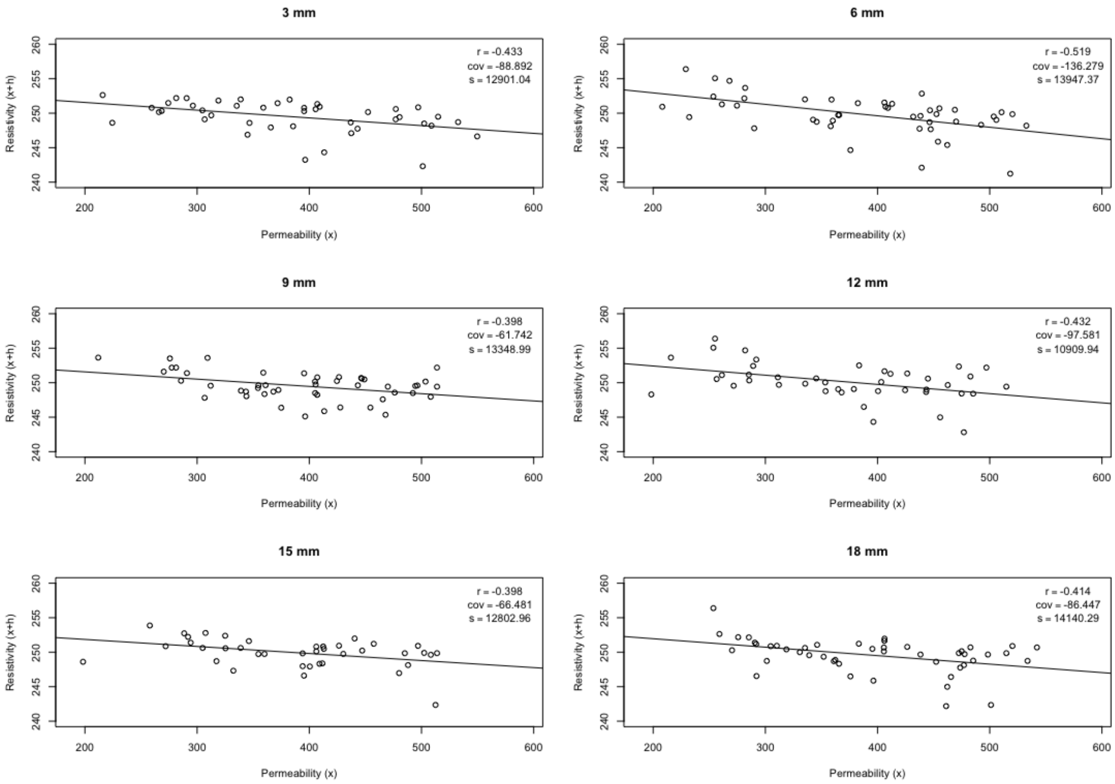

Figure 1: Cross h-scatterplots of Resistivity and Permeability in the Horizontal Direction.

Cross-Statistics:
Correlation Coefficient (r), Covariance (cov), Semivariance (s)

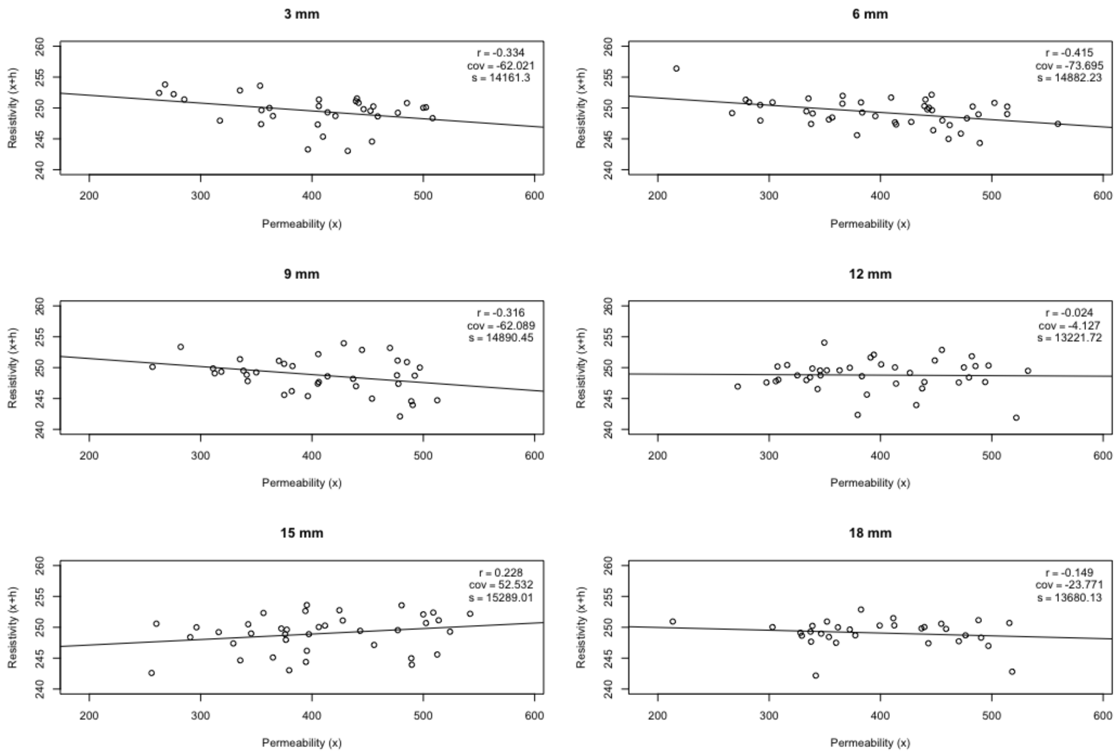

Figure 2: Cross h-scatterplots of Resistivity and Permeability in the Vertical Direction.

Cross-Statistics:
Correlation Coefficient (r), Covariance (cov), Semivariance (s)

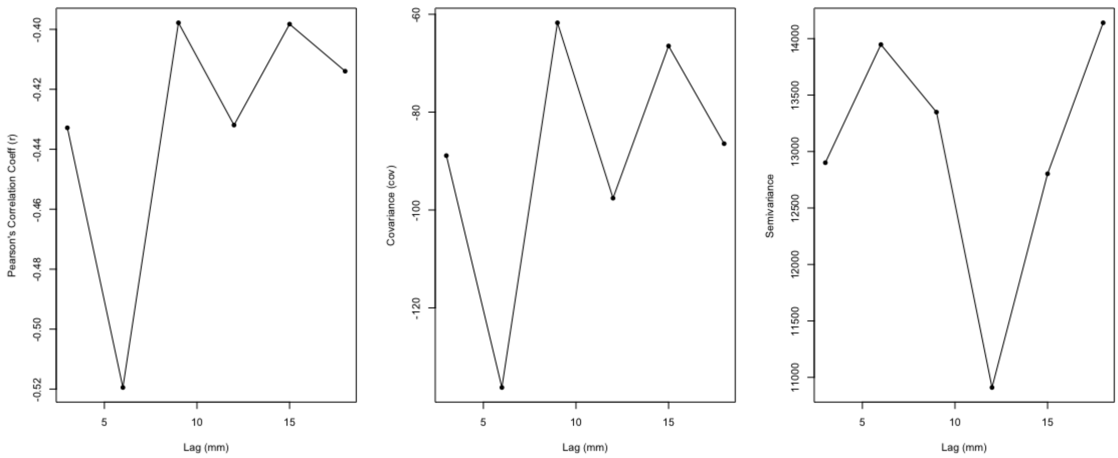

Figure 3: Cross-Statistics in the Horizontal Direction

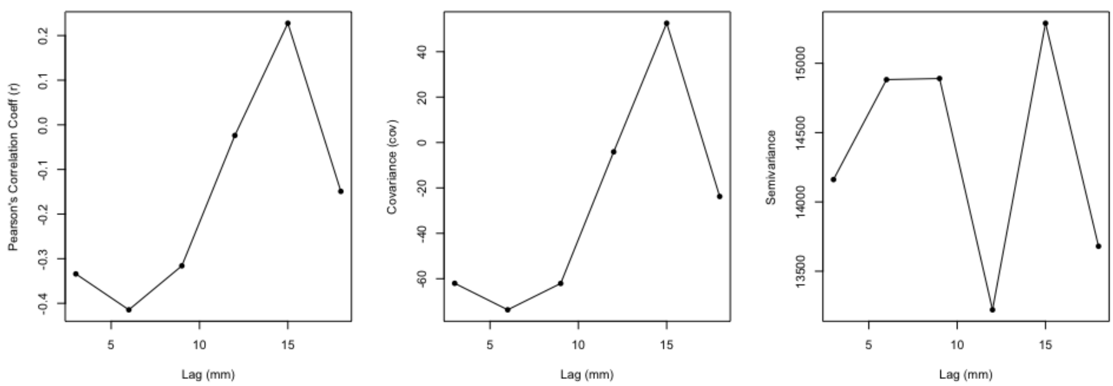

Figure 4: Cross-Statistics in the Vertical Direction

5 References

Katz, A.J. and Thompson, A.H., 1986. Quantitative prediction of permeability in porous rock. Physical Review B, 34, 8179 - 8181