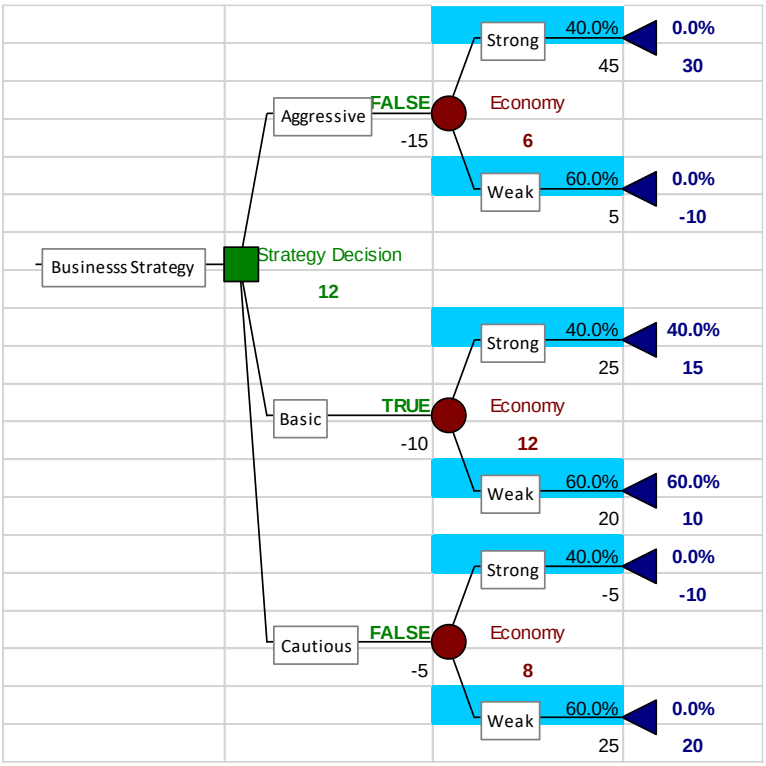
Homework #4 Solution

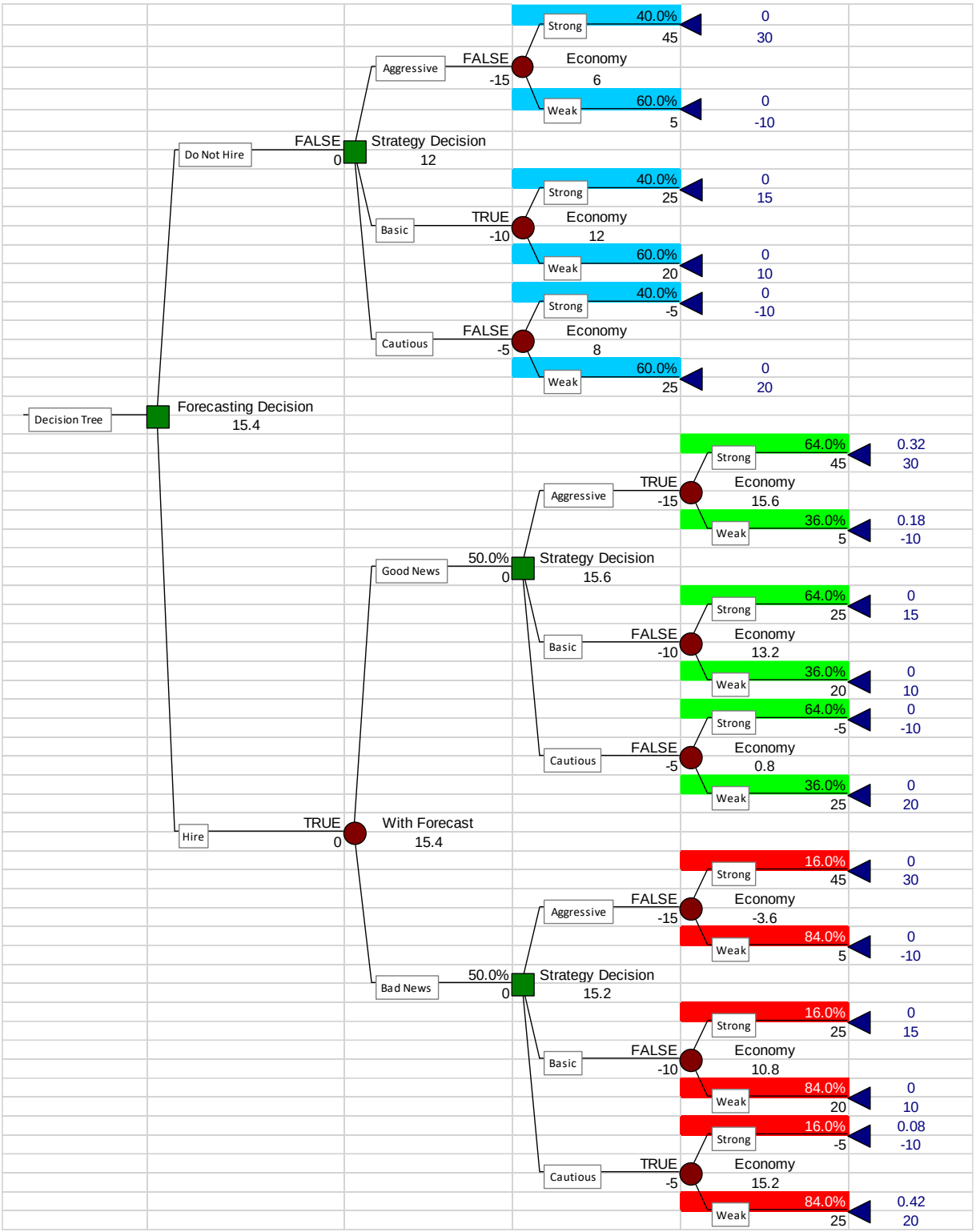
1. *a. Decision Tree for Business strategy decision:*



Optimal strategy is to choose the basic investment strategy, and the corresponding expected payoff is $12 million.

		Decision			
		Environment e =			
Payoff Table	R_d (e)	s	w	Investment	E[R_d (e)]
	A	$45.00	$5.00	$15.00	$6.00
Decision d =	B	$25.00	$20.00	$10.00	$12.00
	C	($5.00)	$25.00	$5.00	$8.00
	Maximum Expected Payoff				**$12.00**
	Optimal decision				**B**

b. Decision tree with the option to consult Seefar.



Optimal strategy is to hire Seefar for consultation. Consequently, choose the aggressive strategy if the news is good; otherwise, choose the cautious strategy if the news is bad. The corresponding expected payoff is $15.4 million.

	Information					Decision					
	Environment e =					Environment e =				Expected Payoff	
Prior probability	Strong (s)	Weak (w)		**Payoff Table**	R_d (e)	e = s	e = w	Investment	Without	With Good	With Bad
P(e) =	0.4	0.6			A	$45.00	$5.00	$15.00	$6.00	$15.60	
				Decision d =	B	$25.00	$20.00	$10.00	$12.00	$13.20	$10.80
Information i =	"Good" (g)	"Bad" (b)			C	($5.00)	$25.00	$5.00	$8.00	$0.80	$15.20
					Maximum Expected Payoff				$12.00	$15.60	$15.20
Reliability					Optimal Decision						
P(i | e)	e = s	e=w									
i = g	0.8	0.3			Expected Maximum E(Rd | i)				$ 15.40		
i = b	0.2	0.7									
					Expected Value of Information				$ 3.40		
Joint Probability			**Marginal Probability**								
P(i and e) = P(i | e)P(e)	e = s	e=w	P(i)					**Formulas:**			
i = g	0.32	0.18	0.5					J5. =Sumproduct(G5:H5,$B$5:$C$5)-I5			
i = b	0.08	0.42	0.5					K5. =Sumproduct(G5:H5,$B$21:$C$21)-I5			
								L5. =Sumproduct(G5:H5,$B$22:$C$22)-I5			
Posterior Probability									copied to J6:J7, K6:K7, L6:L7		
P(e | i) = P(e and i)/P(i)								J8. =Max(J5:J7) copied to K8:L8			
i = g	0.64	0.36						J11. =K8*D16+L8*D17			
i = b	0.16	0.84						J13. =J11-J8			

2. a. Below is the snapshot of the simulation process. Specifically, left part represents the simulation of one iteration for finding out a birthday match from a 30 student class. The right part represents the main simulation which repeats the left part for 1000 times.

Note that, even if more than two students have the same birthday it still counts as one match. One way to implement this is to use "=IF(COUNTIF(B5:$B$33,B4)=1,1,0)". The total #Matches(G4) simply adds up the matches.

	A	B	C	D	E	F	G	H
1		**Birthday Matching Problem**						Not sure how t
2		One Simulation				Main (repeat the left 1000 times)		
3	Person	Birthday	Match?	Match?		Class #	#Matches	Value
4	1	5-2	0	0		0	2	Probability
5	2	5-27	0	0		1	1	Cumulative
6	3	4-23	0	0		2	1	
7	4	2-8	0	0		3	3	
8	5	11-30	0	0		4	3	
9	6	2-10	0	0		5	0	
10	7	2-21	0	0		6	1	
11	8	3-27	0	0		7	2	
12	9	5-28	0	0		8	4	
13	10	6-20	1	1		9	0	
14	11	6-26	0	0		10	2	
15	12	11-17	0	0		11	2	
16	13	7-8	0	0		12	2	
17	14	5-25	0	0		13	0	
18	15	1-22	0	0		14	0	
19	16	2-16	0	0		15	1	
20	17	8-18	0	0		16	0	
21	18	6-20	0	0		17	2	
22	19	12-11	0	0		18	1	
23	20	7-12	1	0		19	2	
24	21	1-15	0	0		20	3	
25	22	8-14	0	0		21	1	
26	23	10-3	0	0		22	2	
27	24	2-1	0	0		23	0	
28	25	7-12	0	1		24	1	
29	26	7-18	0	0		25	3	

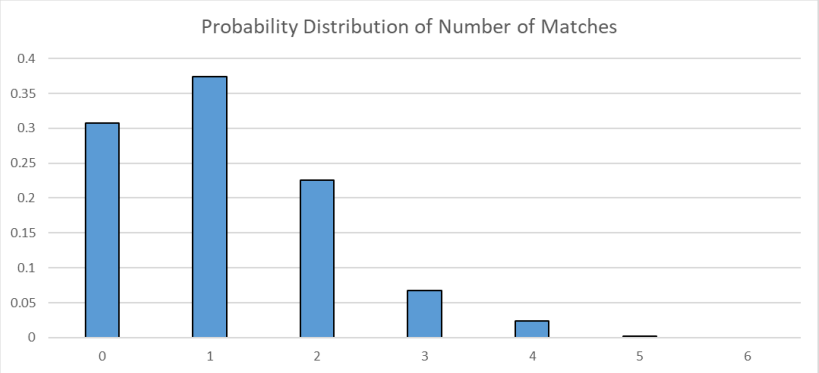
The following is the resulting probability distribution for the number of birthday matches.

Value	0	1	2	3	4	5	6
Probability	0.307	0.374	0.226	0.067	0.024	0.002	0
Cumulative	0.307	0.681	0.907	0.974	0.998	1	1

b. The following is the resulting statistics for the simulation:

Statistics:	Average	1.133
	Std Dev	1.016042983

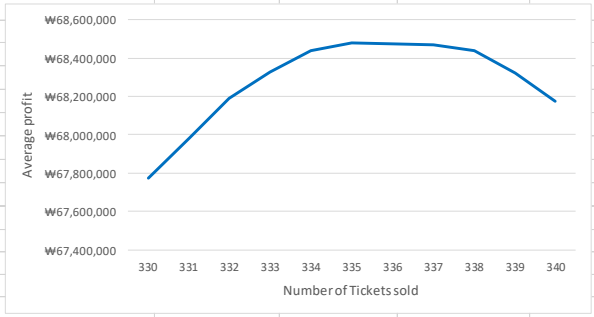
c. The associated graph of the probability distribution is as follows:



3. a. *The normal approximation for B(330,0.9) will be N(297, 29.7). Note that 29.7 here is the variance and thus the standard deviation is 5.45.*

*b. For 5,000 runs of Normal approximation, the optimal number of tickets to sell was 337, and the corresponding profit was KRW68,511,439. Note, to feed the excel, you need to use =NORMINV(rand(), 330*0.9, sqrt(330*0.9*0.1)).*

Outcome:	Avg Profit	₩67,706,676
	330	₩67,770,968
	331	₩68,052,902
	332	₩68,262,142
Decision	333	₩68,290,714
No. of Tickets Sold	334	₩68,393,306
	335	₩68,423,622
Optimal Decision	336	₩68,455,338
	337	₩68,511,439
	338	₩68,406,921
	339	₩68,383,538
	340	₩68,176,732



c. For the 5,000 runs of binomial distribution, the optimal number of tickets to sell was 336, and the corresponding profit was KRW68,497,750.

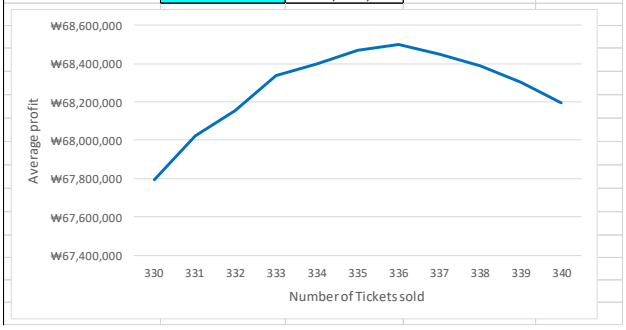
Optimal Stopping: Job Search

Offers (in thousands)	Mean	Std Dev
	₩50,000	₩5,000
Search Cost	₩100	
Reservation Level	₩65,000	

Offer Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Offer Size	₩58,230	₩58,927	₩50,776	₩61,268	₩38,963	₩57,459	₩48,601	₩49,653	₩47,076	₩52,775	₩40,354	₩44,731	₩57,737	₩49,016	₩47,086	₩49,608	₩50,387
Payoff per Period	-₩100	-₩100	-₩100	-₩100	-₩100	-₩100	-₩100	-₩100	-₩100	-₩100	-₩100	-₩100	-₩100	-₩100	-₩100	-₩100	-₩100

		Reservation Levels															
Toatl Payoff	(₩10,000)	₩50,000	₩51,000	₩52,000	₩53,000	₩54,000	₩55,000	₩56,000	₩57,000	₩58,000	₩59,000	₩60,000	₩61,000	₩62,000	₩63,000	₩64,000	₩65,000
Simulation No.																	
1		₩56,461	₩54,526	₩53,031	₩55,838	₩54,624	₩55,506	₩55,574	₩57,731	₩54,965	₩60,660	₩56,883	₩56,934	₩63,141	(₩10,000)	(₩10,000)	(₩10,000)
2		₩56,119	₩51,046	₩56,016	₩57,290	₩55,804	₩57,394	₩57,828	₩56,272	₩58,687	₩56,843	₩60,338	₩61,565	₩61,115	(₩10,000)	(₩10,000)	₩58,346
3		₩50,714	₩60,612	₩55,137	₩53,790	₩60,798	₩57,160	₩58,101	₩57,474	₩57,779	₩57,182	₩59,807	₩57,455	₩56,971	(₩10,000)	₩63,811	(₩10,000)
4		₩55,922	₩55,613	₩52,226	₩54,395	₩53,763	₩65,688	₩59,845	₩58,653	₩58,574	₩60,875	₩58,875	(₩10,000)	(₩10,000)	(₩10,000)	₩62,304	(₩10,000)
497		₩53,357	₩53,550	₩58,333	₩57,292	₩53,642	₩59,380	₩53,405	₩56,928	₩55,562	₩57,182	₩55,784	₩57,415	(₩10,000)	(₩10,000)	₩62,269	(₩10,000)
498		₩50,061	₩56,232	₩56,593	₩53,716	₩58,037	₩57,838	₩56,327	₩53,791	₩57,029	₩58,861	₩56,891	(₩10,000)	₩52,463	(₩10,000)	(₩10,000)	(₩10,000)
499		₩52,702	₩51,249	₩56,990	₩58,527	₩54,994	₩58,312	₩60,313	₩61,771	₩60,106	₩55,871	₩60,099	₩57,304	(₩10,000)	(₩10,000)	₩63,148	(₩10,000)
500		₩55,033	₩58,007	₩56,145	₩56,485	₩56,473	₩54,961	₩54,945	₩57,214	₩58,206	₩58,697	₩54,981	(₩10,000)	₩55,192	₩63,677	(₩10,000)	(₩10,000)
Average		₩53,773	₩54,271	₩55,151	₩55,633	₩56,054	₩56,890	₩57,506	₩57,914	₩58,015	₩57,328	₩52,002	₩42,228	₩28,341	₩18,765	₩4,118	₩586
										Optimal							

Outcome:	Avg Profit	₩67,750,150
	330	₩67,795,200
	331	₩68,021,750
	332	₩68,155,100
Decision	333	₩68,335,200
No. of Tickets Sold	334	₩68,397,850
	335	₩68,468,200
Optimal Decision	336	₩68,497,750
	337	₩68,446,700
	338	₩68,389,800
	339	₩68,301,100
	340	₩68,197,000



Given that the probability p = 0.9 (0.1) is not an extreme value (such as 0.9999 or 0.0001) and that the number of trials n=300 is large enough (>20), normal approximation would work fine. However, we find that their optimal solutions are not identical (although "close enough"). This is the nature of the "approximation". Also, this indicates that a simulation of 5,000 runs may not be sufficiently large enough to provide sufficient robustness to our results.

4. a. The following is the simulation result for the job search process. The optimal reservation salary level, based on 500 simulation run, is KRW 58,000,000.

b. The following is the simulation result for the job search process with a weekly discount rate of 0.5%. The optimal reservation salary level, based on 500 simulation run, is KRW 56,000,000.

Optimal Stopping: Job Search

Offers (in thousands)	Mean	Std Dev															
	₩50,000	₩5,000															
discount rate	0.5%																
Reservation Level	₩55,000																
Offer Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Offer Size	₩46,569	₩49,395	₩52,954	₩51,148	₩47,870	₩54,334	₩44,561	₩51,089	₩48,508	₩46,018	₩59,615	₩57,302	₩51,680	₩49,858	₩38,870	₩36,478	₩43,897
Payoff per Period	₩0	₩0	₩0	₩0	₩0	₩0	₩0	₩0	₩0	₩0	₩59,615	₩0	₩0	₩0	₩0	₩0	₩0
Present value	₩0	₩0	₩0	₩0	₩0	₩0	₩0	₩0	₩0	₩0	₩56,701	₩0	₩0	₩0	₩0	₩0	₩0
						Reservation Levels											
Toatl Payoff	**₩56,701**	**₩50,000**	**₩51,000**	**₩52,000**	**₩53,000**	**₩54,000**	**₩55,000**	**₩56,000**	**₩57,000**	**₩58,000**	**₩59,000**	**₩60,000**	**₩61,000**	**₩62,000**	**₩63,000**	**₩64,000**	**₩65,000**
Simulation No.	1	₩54,772	₩57,851	₩56,811	₩52,916	₩54,749	₩56,035	₩54,271	₩41,072	₩61,315	₩42,521	₩57,414	₩57,481	₩0	₩0	₩0	₩0
	2	₩54,615	₩52,420	₩52,731	₩53,481	₩59,774	₩56,229	₩54,694	₩55,282	₩55,184	₩58,804	₩58,368	₩48,614	₩62,331	₩0	₩0	₩0
	3	₩53,370	₩52,065	₩49,308	₩60,305	₩51,038	₩58,253	₩58,512	₩55,008	₩58,808	₩40,028	₩48,205	₩43,549	₩0	₩0	₩61,179	₩0
	4	₩56,450	₩54,525	₩52,890	₩53,530	₩53,052	₩53,444	₩56,420	₩54,551	₩53,914	₩0	₩45,320	₩54,822	₩0	₩42,004	₩0	₩0
	497	₩48,761	₩53,564	₩55,275	₩55,035	₩54,108	₩58,762	₩57,195	₩55,354	₩55,130	₩57,000	₩56,338	₩49,400	₩39,823	₩0	₩0	₩0
	498	₩53,351	₩54,873	₩53,180	₩53,512	₩53,704	₩56,425	₩56,455	₩60,815	₩54,172	₩61,625	₩62,979	₩0	₩0	₩47,734	₩0	₩0
	499	₩52,278	₩53,304	₩54,269	₩54,402	₩60,619	₩58,508	₩57,294	₩57,938	₩51,488	₩55,092	₩48,939	₩44,548	₩62,513	₩59,334	₩0	₩54,673
	500	₩51,128	₩54,287	₩55,221	₩53,362	₩52,565	₩56,580	₩58,509	₩56,895	₩54,211	₩58,542	₩42,765	₩57,568	₩0	₩62,587	₩0	₩56,797
	Average	**₩53,762**	**₩54,260**	**₩54,586**	**₩55,261**	**₩55,762**	**₩56,104**	**₩56,321**	**₩56,035**	**₩55,353**	**₩53,223**	**₩48,331**	**₩39,602**	**₩27,346**	**₩19,530**	**₩11,214**	**₩6,565**
								Optimal									