Homework #3 Solution

1. a.

Decision variables: $x_j = 1$ *if invest in startup i; 0 otherwise for all companies i = 1, … , 20.*
$y_j = 1$ *if additional industry risk considered for j; 0 otherwise for all industry j = {1, 2, 3, 4, 5} = {Health, Tech, Food, Nonprofit, SocialMedia}.*

We also define I_i, E_i, R_i *to be investment level, expected return, and risk assessment value for each startup i; specific values are given in the table.*

Formulation:

Minimize $\sum_{i=1}^{20} R_i x_i + \sum_{j=1}^{5} 4 y_j$

Subject to

$\sum_{i=1}^{20} I_i x_i \geq 65{,}000{,}000$ *(bounds on budget)*

$\sum_{i=1}^{20} I_i x_i \leq 85{,}000{,}000$

$\sum_{i=1}^{20} (E_i - I_i)\, x_i \geq 0.33 \sum_{i=1}^{20} I_i\, x_i$ *(minimum investment gain)*

$\sum_{i=1}^{4} I_i x_i \leq 0.4 \sum_{i=1}^{20} I_i\, x_i$ *(40% cap on single industry)*

$\sum_{i=5}^{8} I_i x_i \leq 0.4 \sum_{i=1}^{20} I_i\, x_i$

$\sum_{i=9}^{12} I_i x_i \leq 0.4 \sum_{i=1}^{20} I_i\, x_i$

$\sum_{i=13}^{16} I_i x_i \leq 0.4 \sum_{i=1}^{20} I_i\, x_i$

$\sum_{i=17}^{20} I_i x_i \leq 0.4 \sum_{i=1}^{20} I_i\, x_i$

$\sum_{i=1}^{4} x_i \leq 1 + 3\, y_j$ *(added risk factor)*

$\sum_{i=5}^{8} x_i \leq 1 + 3\, y_j$

$\sum_{i=9}^{12} x_i \leq 1 + 3\, y_j$

$\sum_{i=13}^{16} x_i \leq 1 + 3\, y_j$

$\sum_{i=17}^{20} x_i \leq 1 + 3\, y_j$

$x_i, y_j \in \{0, 1\}$ for all *i* and *j*

b.

The optimal solution invests in startups 1−3, 5−6, 8−10, 12, 14, and 17−20. This means we incur the additional health, tech, food, and social media industry risks. The total risk is 67.

Input (Data)

Startup ID	1	2	3	4	5	6	7	8	9
Industry	Health	Health	Health	Health	Tech	Tech	Tech	Tech	Fooc
Investment	$3,330,105.00	$6,117,680.23	$4,050,126.00	$5,280,131.00	$4,650,072.00	$4,526,987.16	$3,420,068.00	$3,900,129.00	$5,147,7
Expected Return	$6,560,498.80	$6,780,075.00	$6,319,693.93	$9,865,219.86	$5,886,612.77	$4,650,092.00	$10,641,062.95	$6,748,642.23	$5,550,0
Risk	1	6	6	10	4	5	7	5	3
Expected gain	$3,230,393.80	$662,394.77	$2,269,567.93	$4,585,088.86	$1,236,540.77	$123,104.84	$7,220,994.95	$2,848,513.23	$402,2

Decision Variables

	1	2	3	4	5	6	7	8	9
invest	1	1	1	0	1	1	0	1	1

	1	2	3	4	5
additional risk indicator	1	1	1	0	1

Objective Function

total risk	67

Constraints

investment bounds	65413778.15	>=	65000000
	65413778.15	<=	85000000
investment gains	30394076.57	>=	21586546.79
40% cap	6162356.5	<=	26165511.26
	4208158.84	<=	26165511.26
	10326658.48	<=	26165511.26
	4502007.9	<=	26165511.26
	5194894.85	<=	26165511.26
added risk	3	<=	4
	3	<=	4
	3	<=	4
	1	<=	1
	4	<=	4

2. *a.*

distance	A	B	C	D	E	F	G
A	0	7	4	5	12	7	14
B	7	0	11	12	5	14	12
C	4	11	0	7	16	3	11
D	5	12	7	0	16	5	9
E	12	5	16	16	0	14	7
F	7	14	3	5	14	0	8
G	14	12	11	9	7	8	0

b. Decision variables:

X_j =1 if a store is opened at node $j \in \{A, B, \dots, G\}$; 0 otherwise. Thus, we defined 7 binary decision variables of X_A, X_B, X_C, X_D, X_E, X_F, X_G.

Formulation:

Define a matrix a_{ij} which indicates whether a demand at node i can be covered by a facility located at node j or not. That is, a_{ij} is set to 1 if the distance between the nodes i and j, d_{ij}, is less than or equal to 8. Otherwise, it is set to 0.

$$\begin{array}{lll} \textit{Minimize} & \sum_{j \in \{A,\dots,G\}} X_j & \\ \textit{Subject to} & \sum_j a_{ij} X_j \geq 1 \quad \text{for all } i, j \in \{A, B, \dots, G\} & \text{(coverage)} \\ & X_j = \{0,1\} \quad \text{for all } j \in \{A, B, \dots, G\}. & \text{(binary)} \end{array}$$

Equivalently, one may express the formulation as follows:

$$\begin{array}{ll} \textit{Minimize} & X_A + X_B + X_C + X_D + X_E + X_F + X_G \\ \textit{Subject to} & X_A + X_B + X_C + X_D + X_F >= 1 \\ & X_A + X_B + X_E >= 1 \end{array}$$

$$X_A + X_C + X_D + X_F \geq 1$$
$$X_A + X_C + X_D + X_F \geq 1$$
$$X_B + X_E + X_G \geq 1$$
$$X_A + X_C + X_D + X_F + X_G \geq 1$$
$$X_E + X_F + X_G \geq 1$$
$$X_A,\ X_B,\ X_C,\ X_D,\ X_E,\ X_F,\ X_G = \{1, 0\}.$$

c. The following is the coverage matrix based on the distance matrix:

coverage	A	B	C	D	E	F	G
A	1	1	1	1	0	1	0
B	1	1	0	0	1	0	0
C	1	0	1	1	0	1	0
D	1	0	1	1	0	1	0
E	0	1	0	0	1	0	1
F	1	0	1	1	0	1	1
G	0	0	0	0	1	1	1

Decision Variables							
	A	**B**	**C**	**D**	**E**	**F**	**G**
Locations	1	0	0	0	1	0	0

The minimum number of facility to open is 2.

**Note that the above solution is not the only optimal solution. There are some alternative optimal solutions for this problem; for example, one can open two facilities at B and F and still cover all the markets.*

d. We can modify the formulation accordingly as follows. First, we define another decision variable which indicates whether a demand node is covered or not. Specifically, we have:

Y_i =1 if a demand at node $i \in \{A, B, \dots, G\}$ is covered; 0 otherwise. Thus, we define 7 additional binary decision variables of Y_A, Y_B, Y_C, Y_D, Y_E, Y_F, Y_G.

In addition, we use h_i to indicate the corresponding demand at node i. Using these, the formulation for the problem can be expressed as follows:

Maximize $\sum_{i \in \{A,\dots,G\}} h_i Y_i$

Subject to $\sum_{j \in \{A,\dots,G\}} a_{ij} X_j \geq Y_i$ *for all* $i, j \in \{A, B, \dots, G\}$ *(coverage)*

$\sum_j X_j \leq 2$ *for all* $i, j \in \{A, B, \dots, G\}$ *(budget)*

$X_j = \{0,1\}$ *for all* $j \in \{A, B, \dots, G\}$ *(binary)*

$Y_i = \{0,1\}$ *for all* $j \in \{A, B, \dots, G\}$. *(binary)*

e.

Input (Data)

distance	A	B	C	D	E	F	G
A	0	7	4	5	12	7	14
B	7	0	11	12	5	14	12
C	4	11	0	7	16	3	11
D	5	12	7	0	16	5	9
E	12	5	16	16	0	14	7
F	7	14	3	5	14	0	8
G	14	12	11	9	7	8	0

demand	A	B	C	D	E	F	G
	100	200	120	45	250	80	75

Cover distance	6		Budget	2

coverage	A	B	C	D	E	F	G
A	1	0	1	1	0	0	0
B	0	1	0	0	1	0	0
C	1	0	1	0	0	1	0
D	1	0	0	1	0	1	0
E	0	1	0	0	1	0	0
F	0	0	1	1	0	1	0
G	0	0	0	0	0	0	1

Decision Variables

	A	B	C	D	E	F	G
X	0	1	1	0	0	0	0
Y	1	1	1	0	1	1	0

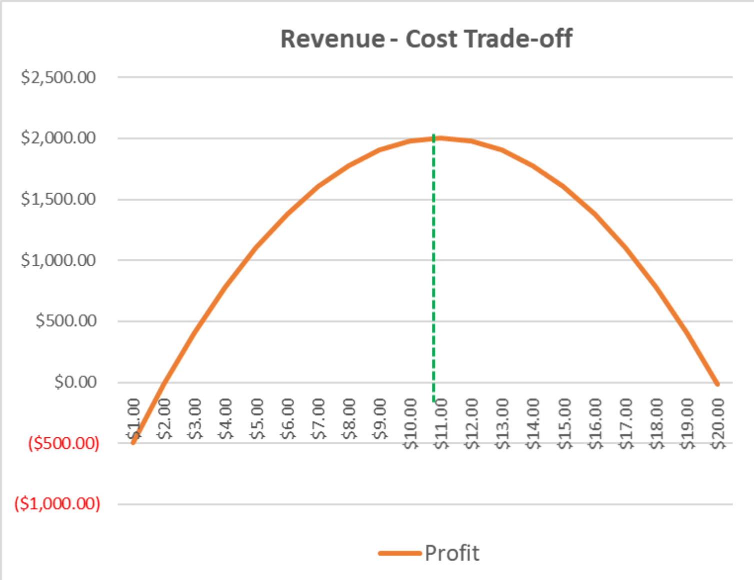
Objective Function

Total Demand	750

Constraints

total coverage		Cover or not?
1	>=	1
1	>=	1
1	>=	1
0	>=	0
1	>=	1
1	>=	1
0	>=	0
# of facility		budget
2	<=	2

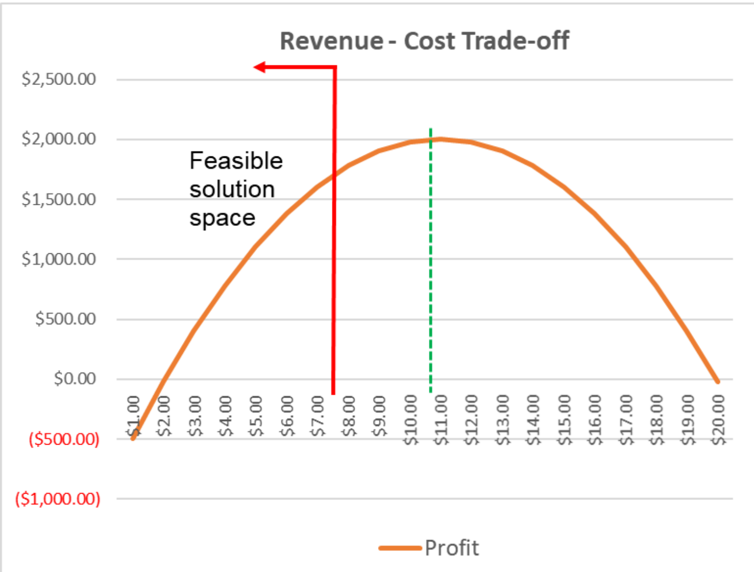
3. a. This is an unconstrained optimization problem. The objective is to maximize the profit which is given by: p(500 – 25p) – 2*(500 – 25p) – 20.*
One can use the solver to obtain the optimal pricing for the problem. The optimal solution is p=$11 with a resulting profit of $2005. The resulting optimal profit is illustrated in the figure below.*



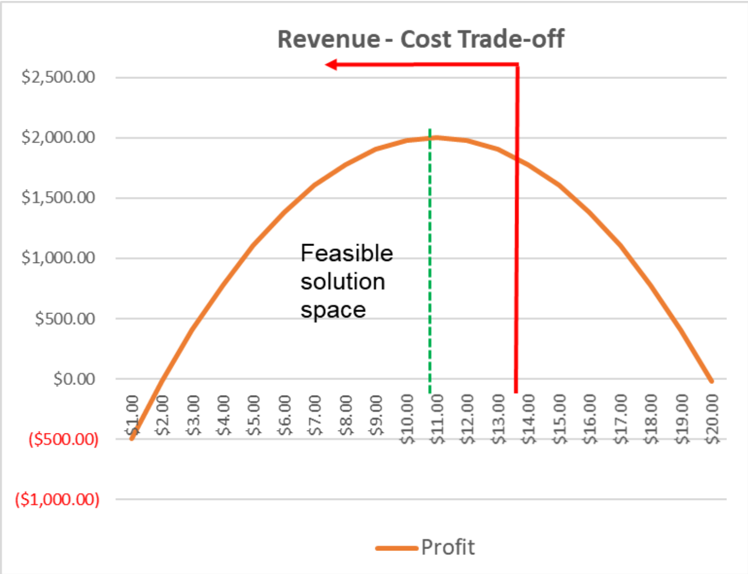
b. We can simply add a constraint stipulating that p >= $8. Hence, the formulation is given as follows:

maximize	*p*(500 – 25p) – 2*(500 – 25p) – 20*
subject to:	*p <= 8, p >= 0.*

The optimal solution is p=$8 with a resulting profit of $1780. The resulting optimal profit is illustrated in the figure below.*



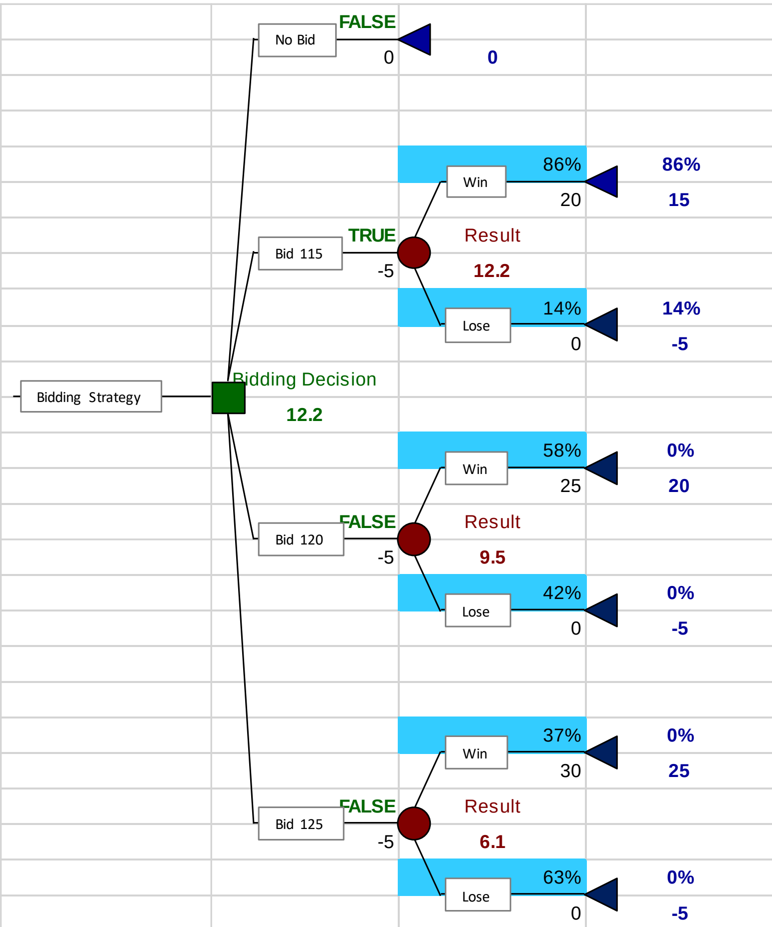
c. We note that the objective function is concave and it peaks at $11 (as shown in part a). Hence, the new constraint p <= $14 does not bind, and therefore the optimal solution is identical to part a; p=$11 with a resulting profit of $2005.*



4.

PGT has four decision strategies: no bid, bid 115, bid 120, or bid 125.

a. Decision Tree for Pangyo Tech's bidding strategy:



b. Optimal strategy is to bid KRW 115 million, and the corresponding expected payoff is KRW 12 million.

		Decision					
		Competitors' Lowest Bid Scenarios					
Payoff Table		No Bid	<115	115 <= x <120	120 <= x < 125	>= 125	Expected Result
	No Bid	₩0	₩0	₩0	₩0	₩0	₩0
Decision	Bid 115	₩15	(₩5)	₩15	₩15	₩15	₩12
	Bid 120	₩20	(₩5)	(₩5)	₩20	₩20	₩10
	Bid 125	₩25	(₩5)	(₩5)	(₩5)	₩25	₩6
	Probability	0.3	0.14	0.28	0.21	0.07	
				Maximum Expected Payoff			**₩12**
				Optimal decision			**Bid 115**