Discrete Mathematics Hw1 Name: Taehyun Yang 2021-14284

Problem 1

1) $p \to (\neg q \wedge r)$

2) $(\neg q \wedge r) \to p$

3) To have atleast 2GB of Ram and 20GB of free space, you must install the program.

4) $\neg(\neg q \wedge r) \to \neg p$

$q \vee \neg r \to \neg p$

5) If your computer has less than 2GB of Ram or don't have 20GB of free space, then you can't install the program

Problem 2 $(P \to Q) \to R \equiv P \to (Q \to R)$?

P	Q	R	$P \to Q$	$(P \to Q) \to R$	$Q \to R$	$P \to (Q \to R)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	F	T	T
				1		2

$1 \neq 2$

Truth table does not match

Problem 3) For $P(i,j,n)$ when $i=$ row $j=$ column $n=$ Value in current Square.

$$\bigwedge_{i=1}^{9} \bigwedge_{j=1}^{9} \bigvee_{n=1}^{9} P(i,j,n)$$

Problem 49

L: House is next to a lake
K: Treasure is in the kitchen
E: The tree in the front yard is an elm
P: Treasure is buried under the flagpole
G: Treasure is in the garage

1. $L \to \neg K$
2. $E \to K$
3. L
4. $E \vee \neg P$
5. $P \oplus G$

b)

	L	from statement 3
	$L \to \neg K$	from 1
x)	$\neg K \to \neg E$	Contrapositive of 2
	$E \vee \neg P$	from 4
	$\neg E$	from x
$\therefore$	$\neg P$	Disjunctive syllogism from
	$P \oplus G$	from 5
	$\neg P$	exclusive or
$\therefore$	G	Treasure is under the garage.

Problem 5

1) True

2) True

3) For all x, there exists a y $(y > x)$

False when x is 36 there is no y that $y > x$

4) True

5) False

when $x = 36$ x is odd $=$ false $\wedge \exists y\ y > x$ $=$ false

Problem 6 Contra positive

For any integer, if n is even, then (n^2+n+1) is odd

for any integer k, $n = 2k$

$$(2k)^2 + 2k + 1 = 4k^2 + 2k + 1$$
$$2k(2k+1) + 1$$
$$\text{even\#} \cdot \text{odd\#} + 1$$
$$= \text{even\#} + 1$$
$$= \text{odd}$$

Contrapositive is true, therefore the original statement:

For any integer n, if (n^2+n+1) is even, then n is odd is True

Problem 7

Contradiction irrational number x satisfies $ax^2+bx+c=0$
when $a, b, c,$ are odd integers,

$$ax^2+bx+c = (Ax+B)(Cx+D)$$

$$a = AC$$
$$b = AD+BC$$
$$c = BD$$

if a, b, c are odd

$a = AC$ $\quad$ A, C must be odd
$c = B \cdot D$ $\quad$ B D must be odd
$b = AD+BC$
odd·odd + odd·odd
odd + odd = even
odd $\neq$ even

Thus x is irrational

$\therefore$ original statement: real number x satisfies $ax^2+bx+c=0$
when a, b, c are odd is true

Problem 8

A	B	C	D	A	B	C	D	A	B
B	C	D	A	B	C	D	A	B	C
C	D	A	B	C	D	A	B	C	D
D	A	B	C	D	A	B	C	D	A
A	B	C	D	A	B	C	D	A	B
B	C	D	A	B	C	D	A	B	C
C	D	A	B	C	D	A	B	C	D
D	A	B	C	D	A	B	C	D	A
A	B	C	D	A	B	C	D	A	B
B	C	D	A	B	C	D	A	B	C

To fill a $10 \times 10 = 100$ board completely, you need $25 \times 4 = 100$, 25 straight tetrominos

However, counting the board there are

A: 25
B: 26
C: 25
D: 24

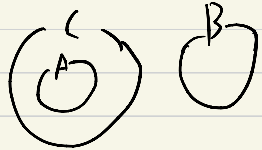
In this board, all straight tetromino  will cover only one A, one B, one C, and one D.
but you would need atleast 26 tetrominoes

but $25 \neq 26$ therefore Not possible.

Problem 1

a) if $A \vee C \subseteq B \vee C$ then $A \subseteq B$

counter example:

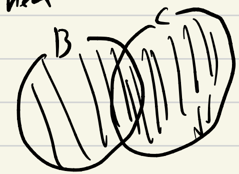


$A \not\subseteq B$

false

b) if $A \vee C \subseteq B \vee C$

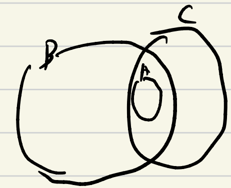
then



Then $A \vee C$ is contained in the black are

but since $A \wedge C \subseteq B \wedge C$ A must be in

$\therefore$



which means $A \subseteq B$

Problem 10

a) Yes it is subjective

$\forall y \exists x \; (f(x) = y)$

Assuming x only has 1 element, $f(\{x\}) = x$

when $x \in \mathbb{Z}$, $f(\{x\})$ can be any integer. Hence, for all y, there exists an x that $f(x) = y$. Therefore, it is subjective.

b) $\forall x \forall y \; (f(x) = f(y) \rightarrow x = y)$ counter example

$f(\{0\}) = 0$ $\quad f(x) = f(y) \rightarrow x = y$ $\quad$ No it is not injective

$f(\{-5, 5\}) = 0$ $\quad$ T $\quad$ F

problem 11

a) bijection iff $\overbrace{\forall y \exists x (f(x)=y)}^{\text{Surjection}} \wedge \overbrace{\forall x \forall y (f(x)=f(y) \rightarrow x=y)}^{\text{injection}}$

Counter example

$f(2,2) = 0$

$f(1,1) = 0$

$f(2,2) = f(1,1) \rightarrow (2,2) \neq (1,1)$

T

Not an injection, hence not a bijection

b) Subset $S \equiv \{(1,1),(1,2),(1,3)\ldots\} \vee \{(2,1),(3,1),(4,1)\ldots\}$

Surjective?

Range covers all of $\mathbb{Z}$, $\{\ldots -2,-1,0,1,2\ldots\}$ so surjective.

injective?

every $x-y$ of subset S is unique, non repeating therefore injective

Surjective $\wedge$ injective = bijective

Problem 12 Assuming function f existed,

and $T = \{s \in S \mid s \notin f(s)\}$

$\therefore T \in P(S)$ by definition of power set.

if $f(s) = T$ for $s \in S$ then $s \notin f(s) \rightarrow s \notin T$

but if $s \notin f(s)$ then $s \in T$

$$s \notin T \neq s \in T \quad \text{Contradiction}$$

Therefore, by contradiction, onto function f does not exist.

Since onto function does not exist A to B is not one to one. So $|S| \neq |P(S)|$

However, according to Cantor's Theorem

If S is a set, then $|S| < |P(S)|$.

https://www.whitman.edu/mathematics/higher_math_online/section04.10.html