Homework #2

1. The Fuel Economy Guide provided by the U.S. Department of Energy has fuel efficiency data for cars and trucks. A portion of the data for 311 cars is contained in the provided worksheet. The Class column shows the size of the car: Compact, Midsize, or Large. The Displacement column shows the engine's displacement in liters. The FuelType column shows whether the car uses premium (P) or regular (R) fuel, and the HwyMPG column shows the fuel efficiency rating for highway driving in terms of miles per gallon.

a. Develop an estimated regression equation that can be used to predict the fuel efficiency for highway driving given the engine's displacement. Test for significance using the 0.05 level of significance. How much of the variation in the sample values of HwyMPG does this estimated regression equation explain?

b. Create a scatter plot with HwyMPG as the dependent variable and displacement as the independent variable for which the points representing compact, midsize, and large automobiles are shown in different shapes and/or colors. What does this chart suggest about the relationship between the class of automobile (compact, midsize, and large) and HwyMPG?

c. Now consider the addition of the variables Class and FuelType to the linear regression model in part (a). Develop the estimated regression equation that can be used to predict the fuel efficiency for highway driving, given the engine's displacement, class type, and fuel type. How much of the variation in the sample values of HwyMPG is explained by this estimated regression equation?

d. An automobile manufacturer is designing a new compact model with a displacement of 2.9 liters with the objective of creating a model that will achieve at least 25 estimated highway MPG. The manufacturer must now decide if the car can be designed to use premium fuel and still achieve the objective of 25 MPG on the highway. Use the model developed in part (c) to recommend a decision to this manufacturer.

2. Solve the following problem (this is the minimization problem from the class slide):

$$\begin{aligned} &\textit{Minimize} && 3x_1 + 4x_2 \\ &\textit{Subject to} && 6x_1 + x_2 \geq 120 \\ & && 4x_1 + 4x_2 \geq 320 \\ & && 3x_1 + 5x_2 \geq 300 \\ & && x_1, x_2 \geq 0 \end{aligned}$$

a. Describe the feasible region (of the constraints) graphically.

b. Find the optimal solution on the graph (using the objective function).

c. Find the optimal solution using the Excel Solver.

d. Compute the amount of surplus for each constraint. What are the binding constraints?

3. SNU Bank issues five types of loans for various consumer needs (such as home, commercial, and auto). In addition, to diversify its portfolio, and to minimize risk, the bank invests in risk-free securities. The loans and the risk-free securities with their annual rate of return are given below:

Type of Loan or Security	Annual Rate of Return (%)
Home mortgage loan (type 1)	6
Home mortgage loan (type 2)	8
Commercial loan	11
Automobile loan	9
Home improvement loan	10
Risk-free securities	4

The bank's objective is to maximize the annual rate of return on investments of its available funds while satisfying the following policies, restrictions, and regulations:

- The bank has a total of KRW9,000,000,000 in available funds.
- Risk-free securities must contain at least 10% of the total funds available for investments.
- Home improvement loans cannot exceed KRW800,000,000.
- The investment in mortgage loans total must be at least 60% of all the funds invested in loans.
- The investment in type1 mortgage must be at least twice as much as the type2 mortgage loans.
- Home improvement loans cannot exceed 40% of the funds invested in type1 mortgage loans.
- Automobile loans and home improvement loans together may not exceed the commercial loans.
- Commercial loans cannot exceed 50% of the total funds invested in mortgage loans total.

a. Formulate the problem. Clearly define the decision variables, and write down the objective function and constraints.

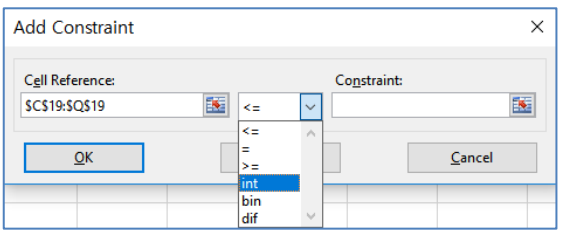
b. Solve the problem using Excel Solver.

4. Consider "The Bic Mac Attack" Problem from the class slide. You may download the excel input from the course website. Suppose you wish to select a nutritionally complete diet consisting of foods taken from this menu at the least cost. Specifically, suppose you would like to design a diet that provides at least 100% of the RDA (recommended daily allowance) of vitamins A, C, B1, B2, niacin, calcium, and iron, supplies at least 55 grams of protein, contains at most 3 grams of sodium, and provides at most 30% of its calories from fat. (Fact: 1 gram of fat contains 9 calories.)

a. Formulate the problem. Clearly define the decision variables, and write down the objective function and constraints.

b. Solve the problem using Excel Solver. That is, determine the least cost daily diet that satisfies all of the nutritional requirements. Ignore the integrality of decision variables; i.e., assume McDonalds sells 0.28 units of hamburger and 1.37 units of French fries!

c. Re-solve the above part (b), but now let's consider the integrality. That is, all items must be sold in integer units. In order to apply this, we need an extra constraint added to the solver that stipulates that all decision variables be integer. Such constraint can be added as shown in the right.



5. Serious Toys, Unlimited, produces 3 types of building blocks preschoolers. Recently, the company underwent a major reorganization of its manufacturing facility and a restructuring of its prices. As a result, a new linear programming model (ignoring the integrality) has been formulated to describe the situation where A = number of advanced sets, B = number of beginner sets, C = number of intermediate sets:

Maximize	$Z = 1.2\,A + 1.6\,B + 1.4\,C$	*(hourly profit)*
Subject to	$6\,A + 5\,B + 3\,C \leq 300$ *pounds*	*(plastic)*
	$9\,A + 4\,B + 5\,C \leq 280$ *minutes*	*(labor)*
	$2\,A + 8\,B + 4\,C \leq 320$ *minutes*	*(machine)*
	$B \geq 18$ *sets*	
	$A, B, C \geq 0$	

a. The objective function can be interpreted as follows: The firm's objective is to maximize the hourly profit where each unit sales of product A (advanced), B (beginner), and C (intermediate) contributes \$1.2, \$1.6, and \$1.4, respectively. How would you interpret the four constraints (the associated subject is given within the parenthesis)?

b. Solve the problem using Excel. Report the "sensitivity analysis" from Excel and use it to answer the following questions.

c. How would you interpret the shadow price of 0.2 for labor constraint?

d. What is the range of feasibility for the RHS of the labor constraint?

e. If the amount of labor available were to increase by 60 minutes/hour, would the optimal quantities of the decision variables change? Would the optimal profit change?

f. If machine time increased by 50 minutes/hour, by how much would profit increase?

g. If machine time increased by 100 minutes/hour, by how much would profit increase?

h. How much would another 10 pounds of plastic/hour be worth in terms of increase profit? Explain.

i. If the profit/unit on advanced sets increased by 70%, would that affect the optimal solution? Explain.