Practice Exam for Midterm

Question 1

Manchester Bookseller buys books from two publishers. Rodeo House offers a package of 5 mysteries and 5 romance novels for $50, and Lovie Press offers a package of 5 mysteries and 10 romance novels for $80. Manchester wants to buy at least 2,500 mysteries and 3,500 romance novels.

(a) Formulate the problem in LP (don't worry about the integrality) so that Manchester can optimally decide how many packages to buy from each publisher; that is, minimize the total cost while satisfying given constraints.

(b) In the optimality, **how many romance novels should Manchester buy**? Answer this question by manually solving the problem. [Please draw a clean graph showing the feasible region and the optimal solution.]

(c) Manchester has promised Lovie that at least 25% of the total number of packages it purchases will come from Lovie Press. (i) Add this constraint to the formulation and (ii) derive the optimal solution. [Please draw a clean graph showing the feasible region and the optimal solution.]

Question 2

Dog Daze manufacturing produces a dog food product. The manager has developed a following LP model to minimize the total cost for using the three raw material inputs for the product while satisfying some nutritional requirements. He also obtained the corresponding sensitivity report as below. The three decision variables are defined as: x_1 = bags of raw material 1, x_2 = bags of raw material 2, x_3 = bags of raw material 3. The following is the formulation of the problem.

Minimize	$38 x_1 + 19 x_2 + 60 x_3$		[Minimize the total cost]
Subject to	$2.5 x_1 + 4 x_2 + 3 x_3 >= 794$ kilograms		[Protein]
	$3 x_1 + 2 x_2 + 4 x_3 >= 400$ kilograms		[Fiber]
	$2 x_1 + 1 x_2 + 2 x_3 >= 600$ kilograms		[Fat]
	$x_1, x_2, x_3 >= 0$		[Non-negativity]

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B1	Quantities of raw mat. 1	292	0	38	0	26.125
C1	Quantities of raw mat. 2	16	0	19	41.8	0
D1	Quantities of raw mat. 3	0	22	60	1E+30	22

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E5	Protein requirements	794	0	794	1606	44
E6	Fiber requirements	908	0	400	508	1E+30
E7	Fat requirements	600	19	600	35.2	399.1428571

(a) Which constraints are binding on the optimal solution? How do you know?

(b) If the fat requirement for the food reduces to 560 kilograms, how would this affect the optimal solution? Does the binding constraints change? Also, would the optimal total cost change? If so, by how much?

(c) The supplier providing the raw material 1 (i.e., x_1) is asking for a 10% price increase for its supplies. Can you say anything about how this would affect the usage of the three raw materials? If so, how?

(d) The supplier for the raw materials is asking for a 10% price increase for each raw material. Can you say anything about how that would affect the usage of the three raw materials? If so, how?

Question 3

Photon Technologies, Inc., a manufacturer of batteries for mobile phones, signed a contract with a large electronics manufacturer to produce three models of lithium-ion battery packs for a new line of phones. Photon Technologies can manufacture the batter packs at plants located in the Philippines and Mexico. The unit cost of the battery packs differs at the two plants because of differences in production equipment and wage rates. The contract production quantities as well as the unit costs for each batter pack at each battery pack at each manufacturing plant are given as follows:

Battery pack	Contract production quantity	Plant	
		Philippines	Mexico
PT-100	200,000	$0.95	$0.98
PT-200	100,000	$0.98	$1.06
PT-300	150,000	$1.34	$1.15

PT-100 and PT-200 battery packs are produced using similar production equipment available at both plants. However, each plant has a limited capacity for the total number of PT-100 and PT-200 battery packs produced. The combined PT-100 and PT-200 production capacities are 175,000 units at the Philippines plant and 160,000 units at the Mexico plant. The PT-300 production capacities are 75,000 units at the Philippines plant and 100,000 units at the Mexico plant. The cost of shipping from the Philippines plant is $0.18 per unit, and the cost of shipping from the Mexico plant is $0.10 per unit.

Formulate an LP that Photon Technologies can use to determine how many units of each battery pack to produce at each plant to minimize the total production and shipping cost associated with the new contract.

Question 4

NH Bank is planning to open another branch inside the SNU campus. Based on benchmark (the Jahayeon branch), NH plans to staff 10 to 18 tellers depending on the time of day. Lunchtime, from noon to 2PM, is usually the heaviest time. The table below indicates the workers needed at various hours throughout a regular day.

Time Period	Number of tellers required	Time Period	Number of tellers required
9AM-10AM	10	1PM-2PM	18
10AM-11AM	12	2PM-3PM	17
11AM-Noon	14	3PM-4PM	15
Noon-1PM	16	4PM-5PM	10

The bank is planning to hire new staffs, mixing full-time and part-time tellers, to satisfy the above requirements. A part-time employee must put in exactly 4 hours per day but can start anytime between 9AM and 1PM. Part-timers are a fairly inexpensive labor pool. Full-timers, on the other hand, work from 9AM to 5PM and are allowed one hour for lunch. Half of the full-timers eat at 11AM, the other half at noon. Full-timers thus provide 35 hours per week of productive labor time.

Part-timers earn KRW18,000/hour (i.e., KRW72,000 a day) on average, whereas the average full-timer earns KRW25,000/hour (i.e., KRW 200,000 a day). Suppose the bank has an objective to minimize its total daily cost of salary, while satisfying the given requirements. Answer the following questions.

(a) Formulate the above problem in LP. For simplicity let's ignore the integrality and focus on the scheduling aspect of the problem.

(b) By corporate policy, the bank limits part-time hours to a maximum of 50% of the day's total requirement. Add a constraint to reflect this policy, and briefly explain.

(c) In addition, the corporate policy stipulates that the number of full-timers must be at least as large as the number of part-timers at any given time throughout the day. Add a constraint to reflect this policy, and briefly explain.

(d) In the above, how many redundant constraints are there?

Question 5

Below is sample data for Daily High Temperature and Bottled Water Sales at Queensland Amusement Park.

(a) How can you obtain the covariance and correlation between Daily High Temperature and Bottled Water Sales? Use Excel formula to compute these values.

Temperature (°F)	Bottled Water Sales
78	23
79	20
80	22
82	23
84	26
85	27
86	25
87	28
91	30
93	31

(b) Compute the covariance and correlation between Daily High Temperature and Bottled Water Sales. Interpret these results.

Question 6

The International Organization of Motor Vehicle Manufacturers provides data on worldwide vehicle production by manufacturer. The following table shows vehicle production numbers for four different manufacturers for five recent years. Data are in millions of vehicles.

	Production (Millions of vehicles)				
Manufacturer	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**
Toyota	8.04	8.53	9.24	7.23	8.56
GM	8.97	9.35	8.28	6.46	8.48
Volkswagen	5.68	6.27	6.44	6.07	7.34
Hyundai	2.51	2.62	2.78	4.65	5.76

(a) Draw a line chart for the time series data for years 1 through 5 showing the number of vehicles manufactured by each automotive company. Show the time series for all four manufacturers on the same graph. NOTE: Make sure to add labels for the x-axis and y-axis and for each line.

(b) What does the line chart indicate about vehicle production amounts from years 1 through 5? Discuss.

Question 7

The Scholastic Aptitude Test (or SAT) is a standardized college entrance test that is used by colleges and universities as a means for making admission decisions. The critical reading and mathematics components of the SAT are reported on a scale from 200 to 800. Several universities believe these scores are strong predictors of an incoming student's potential success, and they use these scores as important inputs when making admission decisions on potential freshman. The Ruggles College Director of Admissions obtained freshman year GPA and the critical reading and mathematics SAT scores for a random sample of 200 students who recently completed their freshman year at Ruggles College.

The Ruggles College Director of Admissions developed the following regression model with freshman year GPA as the dependent variable and critical reading and mathematics SAT scores as independent variables using Excel.

	A	B	C	D	E	F	G	H	I
1	SUMMARY OUTPUT								
2									
3	*Regression Statistics*								
4	Multiple R	0.878764528							
5	R Square	0.772227096							
6	Adjusted R Square	0.769914681							
7	Standard Error	0.318207048							
8	Observations	200							
9									
10	ANOVA								
11		*df*	*SS*	*MS*	*F*	*Significance F*			
12	Regression	2	67.62835012	33.81417506	333.9482776	5.17551E-64			
13	Residual	197	19.94737788	0.101255725					
14	Total	199	87.575728						
15									
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 99.0%*	*Upper 99.0%*
17	Intercept	-2.671709674	0.21944785	-12.17469059	8.64934E-26	-3.104478178	-2.23894117	-3.242496964	-2.100922384
18	Reading	0.004343448	0.000239975	18.09959107	9.06411E-44	0.003870198	0.004816697	0.003719269	0.004967626
19	Math	0.004534844	0.000242036	18.73627058	1.20495E-45	0.004057531	0.005012157	0.003905306	0.005164382

(a) What is the estimated regression equation?

(b) How much variation in freshman GPA is explained by this model?

(c) Test whether each of the coefficients for critical reading and mathematics SAT scores is equal to zero at a 0.05 level of significance. What are the correct interpretations of the estimated regressions parameters? Are these interpretations reasonable?

(d) Using the model above, what is the predicted freshman GPA of Bobby Eagle, a student who has been admitted to Ruggles college with a 660 SAT score on critical reading and at a 630 SAT score on mathematics?

(e) The Ruggles College Director of Admissions believes that the relationship between a student's scores on the critical reading component of the SAT and the student's freshman GPA varies with the student's score on the mathematics component of the SAT. Hence, the director suggests extending the original model by capturing the "interaction" between the reading and math scores. Specifically, they generate a new variable by multiplying the values of reading and math scores together, and developed an estimated regression equation that includes critical reading and mathematics SAT scores and their interaction as independent variables. Below is the Excel output after including the interaction term (referred to as "ReadMath"):

	A	B	C	D	E	F	G	H	I
1	SUMMARY OUTPUT								
2									
3	*Regression Statistics*								
4	Multiple R	0.886287861							
5	R Square	0.785506173							
6	Adjusted R Square	0.782223104							
7	Standard Error	0.30957882							
8	Observations	200							
9									
10	ANOVA								
11		*df*	*SS*	*MS*	*F*	*Significance F*			
12	Regression	3	68.79127497	22.93042499	239.2597374	2.99604E-65			
13	Residual	196	18.78445303	0.095839046					
14	Total	199	87.575728						
15									
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 99.0%*	*Upper 99.0%*
17	Intercept	0.797737205	1.018616886	0.783157255	0.434480039	-1.211119146	2.806593555	-1.851833349	3.447307758
18	Reading	-0.001119552	0.001585573	-0.70608628	0.480972921	-0.004246526	0.002007423	-0.005243858	0.003004755
19	Math	-0.00096018	0.001594962	-0.602008133	0.547864481	-0.00410567	0.00218531	-0.005108908	0.003188548
20	ReadMath	8.65563E-06	2.48482E-06	3.483409928	0.000610558	3.75522E-06	1.3556E-05	2.19227E-06	1.5119E-05

The estimated regression equation is $\widehat{GPA} = 0.7977 - 0.0011 * Reading - 0.0010 * Math + 0.000009 * Reading * Math$. Test whether the coefficient for the interaction term is equal to zero at a 0.05 level of significance. Also, explain how results in (c) change.

(f) What other factors could be included in the model as independent variables?