1.1
only 2

Inductive proof let $P(t)$ be If s is not a root of the characteristic equation of the associated linear homogeneous recurrence relation, then there is a particular solution of the form

and If s is a root of this characteristic equation and its multiplicity is m, then there is a particular solution of the form

$t=0$

If s is not a root, of a_n,

then $C_1 p_t s^{n-1} + C_2 p_{t1}^{n-2} \ldots C_k p_0 s^{n-k}) + b_0 s^n$

$= p_0 a_n^T + b_0 s^n$

$p_0 a_n^T + F(n)$

$= a_n$

If S is a root

then $C_1 p_0 (n-1)^t s^{n-1} + C_2 p_0 (n-2)^t s^{n-2} \ldots C_k p_0 (n-k)^t s^{n-k}$

$+ b_0 s^n$

$= p_0 (n-1)(n-2) \ldots (n-k) a_n^T + F(n)$

$= a_n$

$P(0)$ is true.

$P(1), P(k) \ldots$

2.1 Algorithm will take 2 unique people so 2n people and see whether the votes get more than n/2 each. There can be up to 3 winners because sum of 4 numbers all $> \frac{n}{2}$ is larger than 2n.

When n=1, Both people on the slip are picked.

Recursive step:
Divide the list to as equally if it is odd. Equally if even. Then continue until we get sublists with at most 3 people. Apply to both lists for total of six people. Then repeat the entire list to compare the freq. of the winners. This requires at most 12 comparisons for a list of 2n

$$c(n) = 2c(\frac{n}{2}) + 12n$$

2.2) master theorem: $a = 2 = b^2$

$a = 2$ $b = 2$

$= O(n^{\log_2 2}) =$ worst case time complexity $= O(n \log n)$

3.1) If initial set is divided to $\frac{n}{4}$, $\{A, B, C, D\}$

1. Ask if x is in $A \cup D$
2. Ask if x is in $A \cup C$

If yes is answered in both then x is not in D

we can also eliminate another subset similarly for answers {Yes, Yes}, {Yes, No}, {No, Yes}, {No, No}

Set can be divided by 4 by asking 2 questions.

3-2 $f(n)$ is divisible by 4, then

$f(n) = f(3n/4) + 2$

we can eliminate $\frac{1}{4}$ subsets by asking 2 questions and $\frac{3}{4}$ elements remain

then, $f(n) = f(\frac{3}{4}(n)) + 2 = f(\frac{3n}{4}) + 2$

$f((\frac{3}{4})^2 n) + 2 + 2$

$= 2(\log_{\frac{4}{3}} n) + 2 + 2 \ldots$

$\frac{2\log n}{\log \frac{4}{3}} \sim 4.8 \log n$

4 Total permutation is $26!$

fish containing permutation $= F$ $\quad |F| = 23!$

rat $= R$ $\quad |R| = 24!$

bird $= B$ $\quad |B| = 23!$

inclusion exclusion

$|F \cup R \cup B| = |F| + |R| + |B| - |F \cap R| - |F \cap B| - |R \cap B| + |F \cap R \cap B|$

$F \cap B$ and $R \cap B$ is 0 because I and r can't appear twice, making $|F \cap R \cap B|$ also 0.

$|F \cap R| = 21!$

$= 24! + 23! + 23! - 21!$

$26! - (24! + 2 \cdot 23! - 21!)$

5.1 If $w_j > w$

$M(j, w)$

However, the item that exceeds w cannot be included.
So total weight of first items in j but doesn't $> w$
can be shown as

$M(j-1, w)$

Otherwise,
The max weight for subset of $j-1$ items can be shown as
$M(j-1, w-w_j)$

by adding item j,

$M(j,w) = M(j-1, w-w_j) + w_j$
Max weight not exceeding w can be shown as
$\max\{M(j-1,w), w_j + M(j-1, w-w_j)\}$

2. If $W > w_i$

Find $1 \leq j \leq n$, $0 \leq w \leq W$ $M(j,w)$

dynamic algorithm (All $w \leq W$)

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for (all w ≤ W)
M(0,w) = 0
    for (all j ≤ n)
        for (all w ≤ W)
        if w ≥ w_j
          then  M(j,w) = max{M(j-1,w), w_j + M(j-1,w-w_j)}
          else  M(j,w) = M(j-1,w)
          return M
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5. 3 We can use the values M(j,w) to find a subset of items with max total weight no exceed W by altering the algorithm in 2. By changing the if conditionals to check whether the weight doesnt exceed the first j item that does not exceed w, and if it doesnt exceed, redefining m(j,w) to wj+ M(j-1, w-wj) will return the accurate subset. However in the opposite case, redefining m(j,w) to M(j-1,w) and returning the respective item will find the maximum total weight. By giving a dynamic conditional, we can give the accurate answer for a special case.

6 If R is anti symmetric and $R \cap R^{-1}$ is no $\emptyset$.

Assume $(a,b) \in R \cap R^{-1}$

$(a,b) \in R^{-1}$

$(b,a) \in R$ and $(a,b) \in R$

Since R is anti symmetric, $a=b$

So $R \cap R^{-1}$ only contains (x,x) but $(x,x) \in \Delta$

$R \cap R^{-1} \subseteq \Delta$

if $R \cap R^{-1} \subseteq \Delta$, $(a,b), (b,a) \in R$

$(a,b) \in R^{-1}$, (b,a) also is $\in R^{-1}$

therefore $(a,b), (b,a) \in R \cap R^{-1}$

then

$(a,b) \in \Delta$

$(b,a) \in \Delta$

and since $\Delta = \{(a,a) \mid a \in A\}$ $a=b$

If R is anti symmetric then $R \cap R^{-1} \subseteq \Delta$ and also the case for vice versa. Therefore R is anti symmetric if and only if $R \cap R^{-1} \subseteq \Delta$

7 No, not necessarily. For example for $\{(0,1),(2,1)\}$ and $\{0,1,2\}$

Symmetric closure is $\{(0,1),(1,1),(2,2),(0,2),(2,0),(2,1),(1,2)\}$

reflexive closure is $\{(0,0),(1,1),(2,2),(0,2),(1,2)\}$

Transitive closure is $\{(0,1),(2,1)\}$
but as you can see $(0,2)$ and $(2,1)$ are in relation
but $(0,1)$ is not. Therefore not necessarily the case.

8 https://math.stackexchange.com/questions/3384971/is-a-graph-being-bipartite-an-isomorphic-variant
used this forum to see if I am going on the right track

Assume two graphs G_1, G_2 are isomorphic and G_1 is bipartite. Prove G_2 is also bipartite

$G_1 := (V_1, G_1)$ $G_2 := (V_2, G_2)$ but they are isomorphic, therefore V_1 and V_2 are one to one and onto, so $f: V_1 \to V_2$

G_1 is bipartite, so $V_1 = V_x \cup V_y$ so V_x and V_y are dissociated.

if f is a bijection,
$V_2 = f(V_x) \cup f(V_y)$, $f(V_x) \cap f(V_y) = \emptyset$
No elements are adjacent for both $f(V_x)$ and $f(V_y)$

Therefore G_2 is also a bipartite graph