ADHOC

```
library(ggplot2)
```

1

Create a getNodes function which generates random Nodes.

```
getNodes = function(n){
# n is numeric singleton vector
# Nodes is a matrix with n by 2

  Nodes = matrix(runif(2*n,0,100), ncol = 2)
  colnames(Nodes) <- c("x","y")
  rownames(Nodes) <- 1:n

  return(Nodes)
}
```

Check the getNodes function with random values

```
seeds = vector(length = 5, mode = "list")
set.seed(12345678)

test.node = list()

for (i in 1:5){
  seeds[[i]] = .Random.seed
  node = getNodes(5)
  test.node[[i]] = node
}

test.node
```

```
## [[1]]
##           x         y
## 1 87.4493237 52.189195
## 2 28.8790082 39.870544
## 3 94.7211719 96.001960
## 4 88.4520706 81.390361
## 5  0.8678354  9.244898
##
## [[2]]
##          x        y
## 1 22.59221 82.44000
## 2 99.31582 86.42073
## 3 22.48521 70.13601
## 4 54.65986 99.46446
## 5 83.73536 56.21510
##
## [[3]]
##          x        y
## 1 29.98234 39.87809
```

```
## 2 28.59007 47.91054
## 3 41.23781 75.04288
## 4 40.49474 30.97696
## 5 38.97566 69.44591
##
## [[4]]
##           x        y
## 1 93.313683 24.26854
## 2  6.309057 62.63485
## 3 37.157258 90.39474
## 4 60.657133 69.63902
## 5 95.841602 27.49603
##
## [[5]]
##           x         y
## 1 42.268338 85.154321
## 2 59.756995 24.049858
## 3  2.071166 64.252719
## 4 21.317885  1.449243
## 5 40.160201 76.372598
```

2

Find the smallest radius Rc. First, Set up a helper function, findTranMat, to find a transition Matrix(Marcov Matrix for random walk).

```
findTranMat = function(mat, R){
  trans.mat = as.matrix(mat)
  trans.mat[trans.mat <= R] = NA
  trans.mat[trans.mat > R] = 0
  trans.mat[is.na(trans.mat)] = 1
  trans.mat = apply(trans.mat, 1, "/", rowSums(trans.mat))
  return(trans.mat)
}
```

Second, find a second largest eigen values since the largest eigen value gives us the trivial solution as well. Set up a helper function, genEigen2() which finds a second largest eigen value.

```
getEigen2 = function(mat){
  eigenvalues = eigen(mat)$values
  SecondLargest = sort(eigenvalues, decreasing = TRUE)[2]
  return(Mod(SecondLargest))
}
```

Third, set up a function findRange() to find the range of Rc

```
findRange = function(mat){
  Mini = min(mat)
  Maxi = max(mat)
  Min.Max = c(Mini, Maxi)
  names(Min.Max) = c("Lower Bound","Upper Bound")
  return(Min.Max)
}
```

Set up a function, findRc, to find the smallest radius to make the network fully connected

```
findRc = function(nodes, tol = 0.05){
  mat = dist(nodes)
  lower = findRange(mat)[1]
  upper = findRange(mat)[2]
  midpt = sum(upper, lower) / 2

  while(upper-lower > tol){
    TranMat = findTranMat(mat, midpt)
    EigenValue = round(getEigen2(TranMat), digits = 5)
    if(EigenValue < 1){
      upper = midpt
    }else{
      lower = midpt
    }
    midpt = sum(upper, lower) / 2
  }
  names(upper) = "The smallest Radius"
  return(upper)
}
```

Check the findRc function with random Node

```
nodes = getNodes(100)
findRc(nodes)
```

```
## The smallest Radius
##            14.98706
```

3

Generate 1000 networks and for each find the value for Rc. Examine the distribution of these Rc values.

sample with size n = 20, n = 50, n = 100

```
Rc1 = c()
Rc2 = c()
Rc3 = c()

seeds = vector(length = 2, mode = "list")
set.seed(12345678)

n1 = 20
n2 = 50
n3 = 100

networks1 = vector("list",1000)
networks2 = vector("list",1000)
networks3 = vector("list",1000)

for(i in 1:1000){
  seeds[[i]] = .Random.seed
  networks1[[i]] = getNodes(n1)
  networks2[[i]] = getNodes(n2)
  networks3[[i]] = getNodes(n3)
}
```

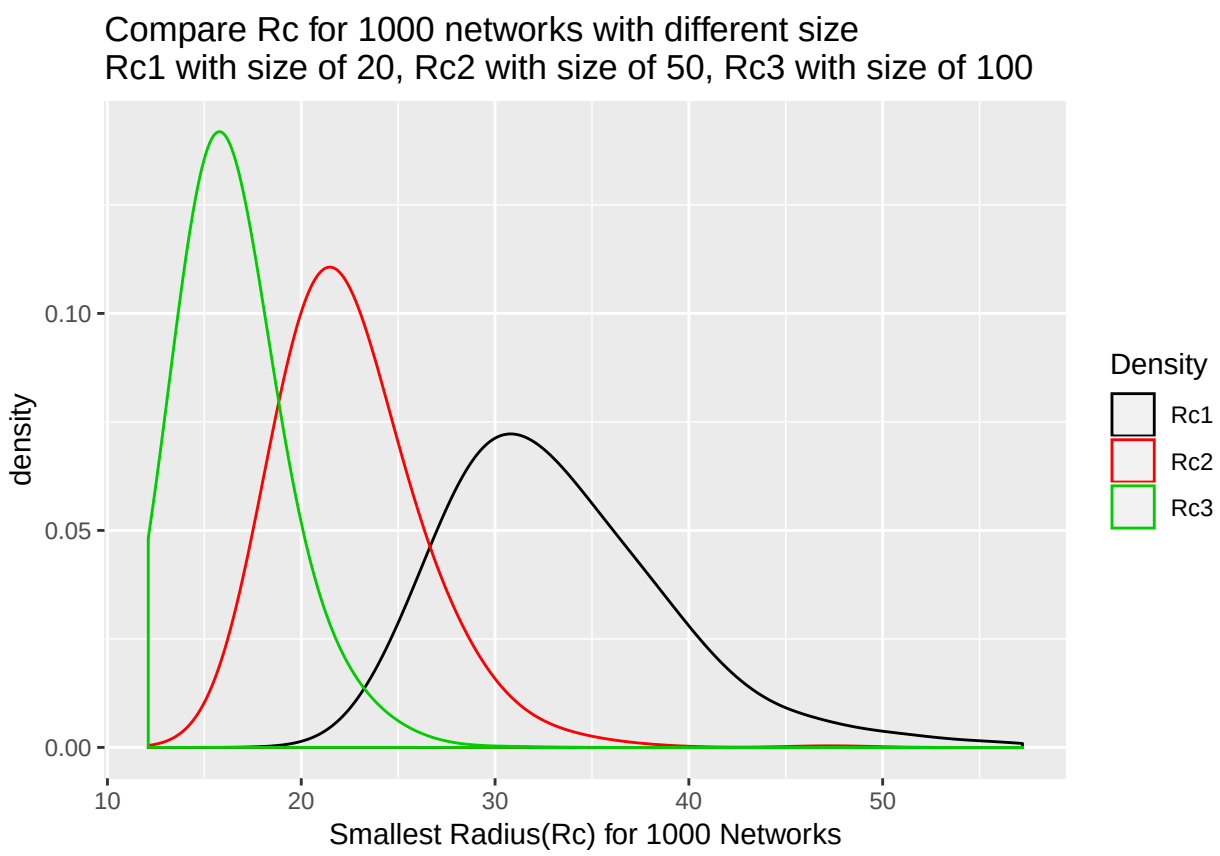
```
networks = list(networks1, networks2, networks3)

for(i in 1:1000){
  Rc1[i] = findRc(networks1[[i]])
  Rc2[i] = findRc(networks2[[i]])
  Rc3[i] = findRc(networks3[[i]])
}

Rc = list(Rc1, Rc2, Rc3)
```

As n goes up, the radius of the Rc is getter smaller. Set up a function to visualize the distribution of Rcs.

```
plotRc = function(Rc1, Rc2 = NULL, Rc3 = NULL, bw = 2, title = 'Compare Rc with different size'){
  Rcs = data.frame(Rc123 = c(Rc1, Rc2, Rc3),
                   strat = rep(c(1,2,3), times = c(length(Rc1), length(Rc2), length(Rc3))))
  ggplot(data = Rcs) +
    geom_density(aes(x = Rc123, y = ..density.., color = factor(strat)), bw = bw) +
    scale_x_continuous("Smallest Radius(Rc) for 1000 Networks") +
    ggtitle(title) +
    scale_colour_manual(values = c(1,2,3), name = 'Density', labels = c('Rc1','Rc2','Rc3'))
}
```

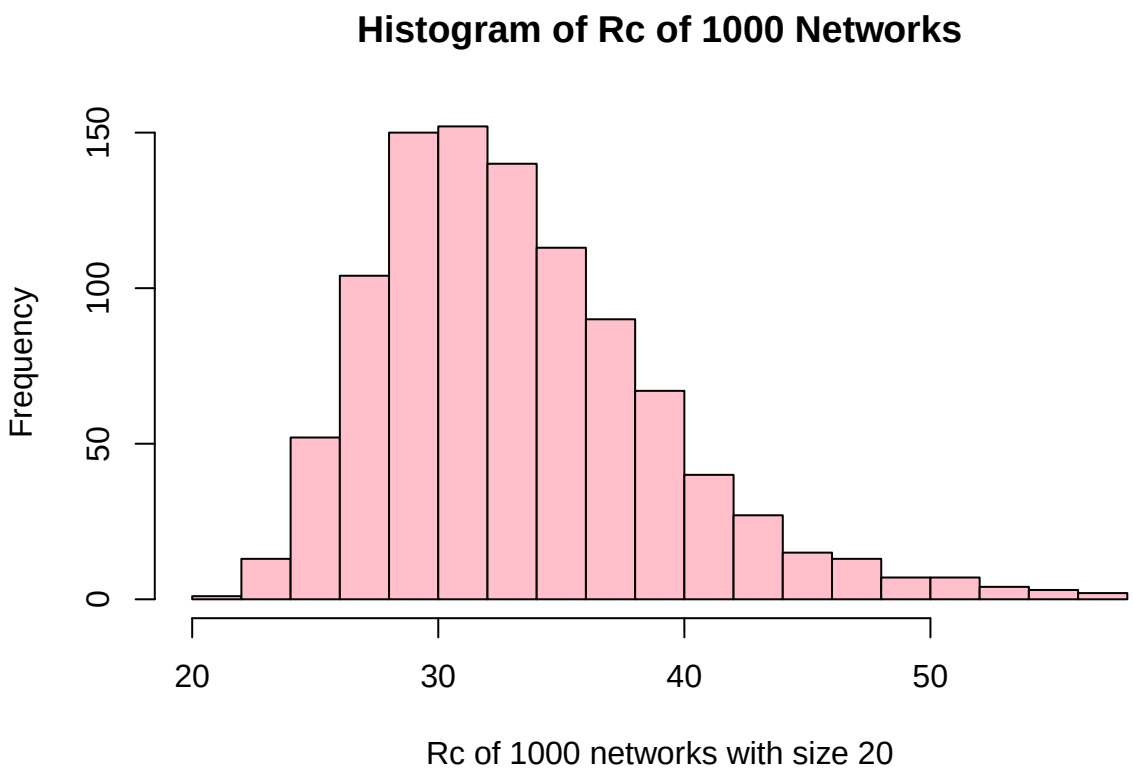
Plot distribution of Rcs.

```
plotRc(Rc1, Rc2, Rc3, title = "Compare Rc for 1000 networks with different size\nRc1 with size of 20, R
```



Plot Histogram of Rcs with summary.

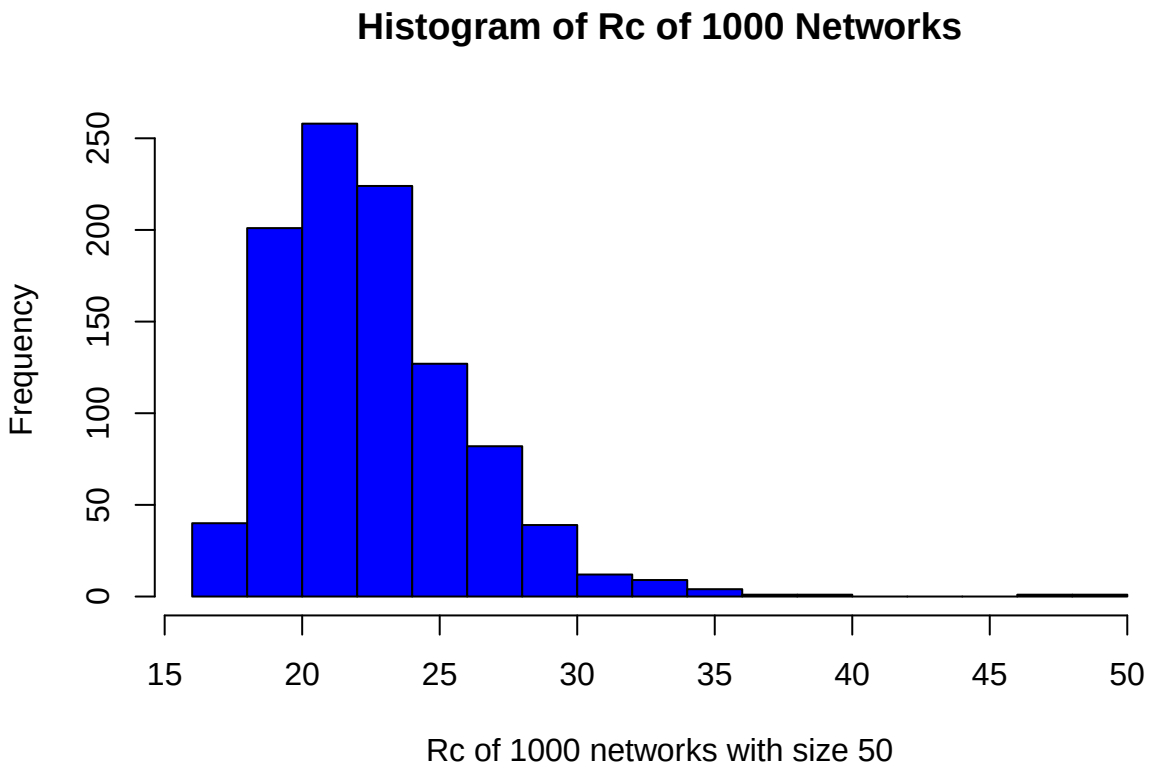
```
Rc = list(Rc1, Rc2, Rc3)
Hist1 = hist(Rc1, breaks = 20, col = "pink",
             xlab = "Rc of 1000 networks with size 20",
             main = "Histogram of Rc of 1000 Networks")
```




```
Hist1
```

```
## $breaks
##  [1] 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58
##
## $counts
##  [1]   1  13  52 104 150 152 140 113  90  67  40  27  15  13   7   7   4
## [18]   3   2
##
## $density
##  [1] 0.0005 0.0065 0.0260 0.0520 0.0750 0.0760 0.0700 0.0565 0.0450 0.0335
## [11] 0.0200 0.0135 0.0075 0.0065 0.0035 0.0035 0.0020 0.0015 0.0010
##
## $mids
##  [1] 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57
##
## $xname
## [1] "Rc1"
##
## $equidist
## [1] TRUE
##
## attr(,"class")
## [1] "histogram"
```

```
Hist2 = hist(Rc2, breaks = 20, col = "blue",
             xlab = "Rc of 1000 networks with size 50",
             main = "Histogram of Rc of 1000 Networks")
```

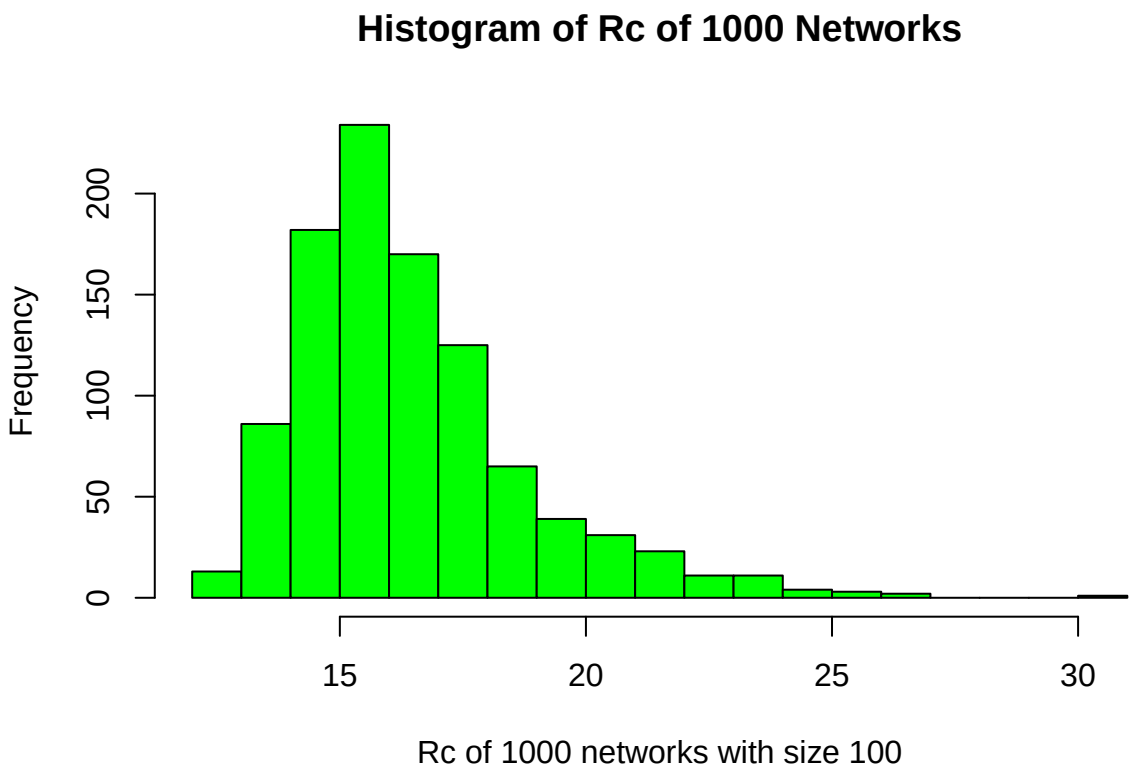



```
Hist2
```

```
## $breaks
##  [1] 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
##
## $counts
##  [1]  40 201 258 224 127  82  39  12   9   4   1   1   0   0   0   1   1
##
## $density
##  [1] 0.0200 0.1005 0.1290 0.1120 0.0635 0.0410 0.0195 0.0060 0.0045 0.0020
## [11] 0.0005 0.0005 0.0000 0.0000 0.0000 0.0005 0.0005
##
## $mids
##  [1] 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49
##
## $xname
## [1] "Rc2"
##
## $equidist
## [1] TRUE
##
## attr(,"class")
## [1] "histogram"
```

```
Hist3 = hist(Rc3, breaks = 20, col = "green",
             xlab = "Rc of 1000 networks with size 100",
```

```
            main = "Histogram of Rc of 1000 Networks")
```




```
Hist3
```

```
## $breaks
##  [1] 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
##
## $counts
##  [1]  13  86 182 234 170 125  65  39  31  23  11  11   4   3   2   0   0
## [18]   0   1
##
## $density
##  [1] 0.013 0.086 0.182 0.234 0.170 0.125 0.065 0.039 0.031 0.023 0.011
## [12] 0.011 0.004 0.003 0.002 0.000 0.000 0.000 0.001
##
## $mids
##  [1] 12.5 13.5 14.5 15.5 16.5 17.5 18.5 19.5 20.5 21.5 22.5 23.5 24.5 25.5
## [15] 26.5 27.5 28.5 29.5 30.5
##
## $xname
## [1] "Rc3"
##
## $equidist
## [1] TRUE
##
## attr(,"class")
## [1] "histogram"
```

Set up min, median, mean, and max values for Rc

```
Min = c()
Median = c()
Mean = c()
Max = c()

for (i in 1:3){
  Min[i] = min(Rc[[i]])
  Median[i] = median(Rc[[i]])
  Mean[i] = mean(Rc[[i]])
  Max[i] = max(Rc[[i]])
}

i = c()
j = c()
k = c()
l = c()

for (h in 1:3){
  i[h] = which(abs(Rc[[h]] - Min[h]) == min(abs(Rc[[h]] - Min[h])))[1]
  j[h] = which(abs(Rc[[h]] - Median[h]) == min(abs(Rc[[h]] - Median[h])))[1]
  k[h] = which(abs(Rc[[h]] - Mean[h]) == min(abs(Rc[[h]] - Mean[h])))[1]
  l[h] = which(abs(Rc[[h]] - Max[h]) == min(abs(Rc[[h]] - Max[h])))[1]
}
```

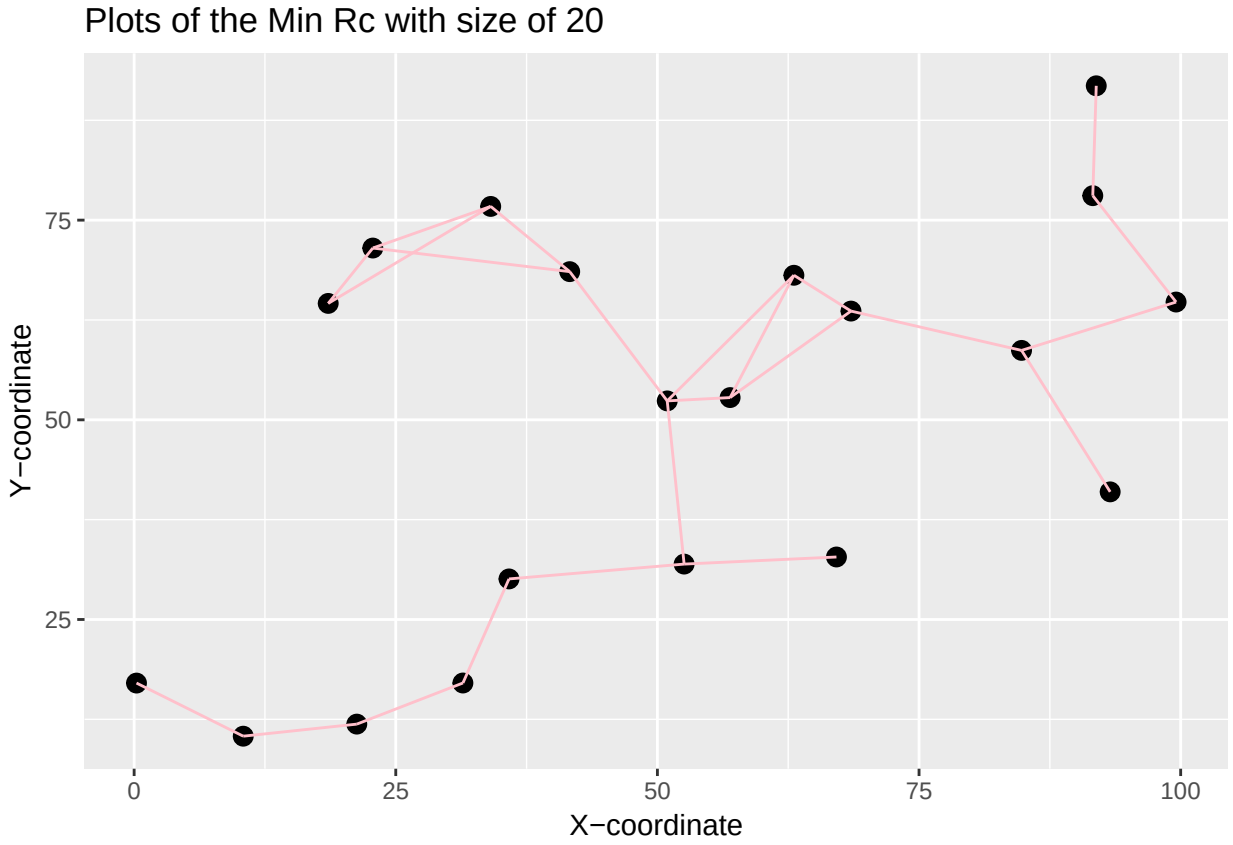
4

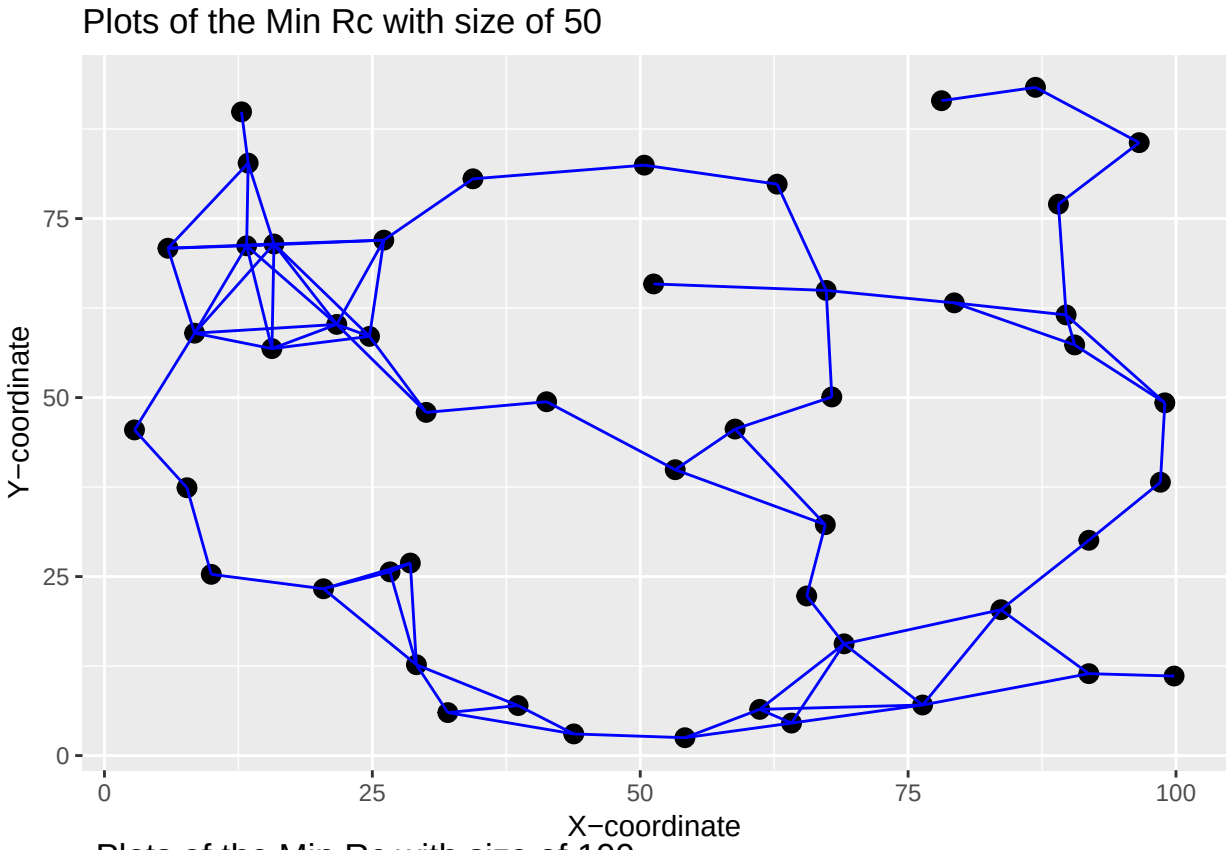
Plots of Rc with the four values(min, median, mean, max)

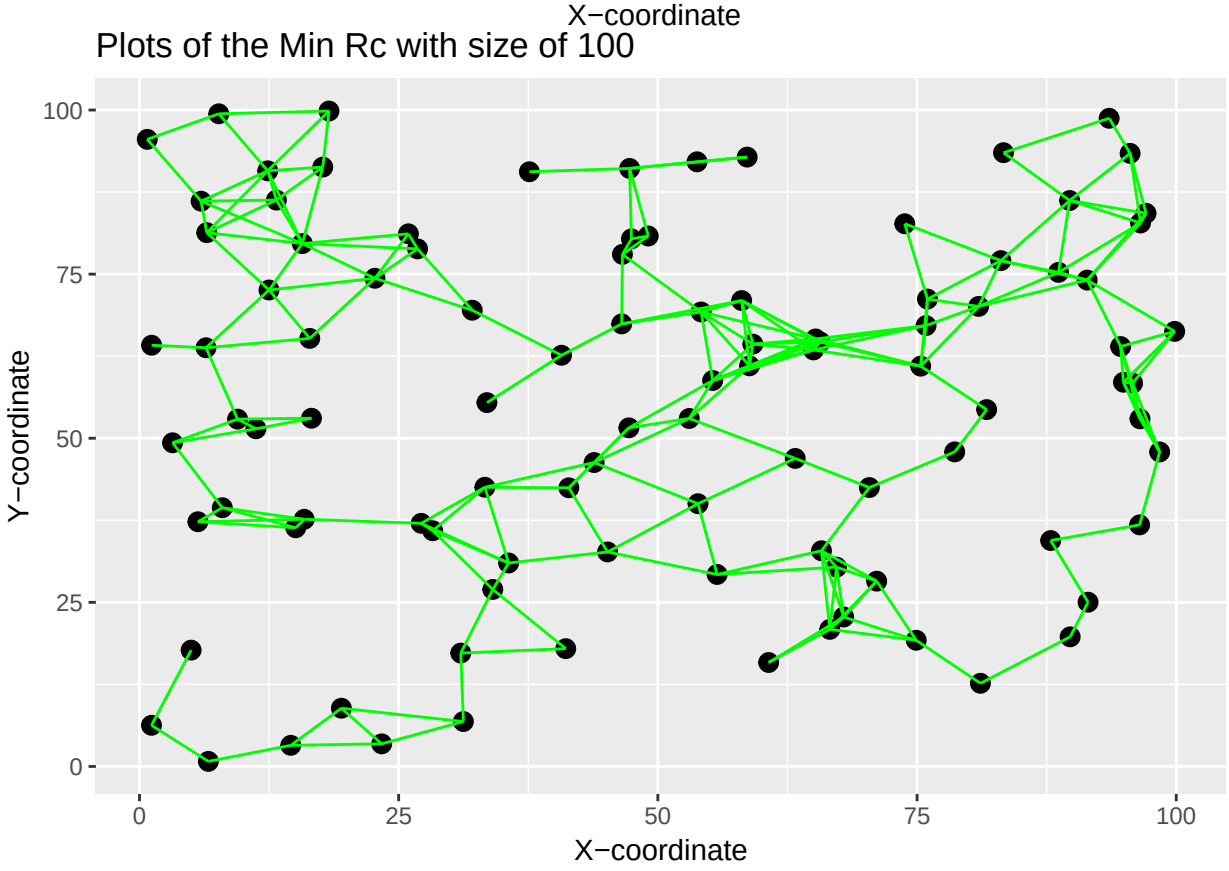
Plots of Rc with the min

```
mindf = list()
connected = list()
connections = list()

for (h in 1:3){
  mindf[[h]] = data.frame(networks[[h]][i[h]])
  comb = combn(nrow(mindf[[h]]), 2)
  connected[[h]] = comb[,which(as.numeric(dist(data.frame(networks[[h]][i[h]])))<=Rc[[h]][i[h]])]
  connections[[h]] = data.frame(
    from = mindf[[h]][connected[[h]][1, ], 1:2],
    to = mindf[[h]][connected[[h]][2, ], 1:2]
    )
  names(connections[[h]]) = c("x1", "y1", "x2", "y2")
}

color = c("pink", "blue", "green")
title = c("Plots of the Min Rc with size of 20", "Plots of the Min Rc with size of 50", "Plots of the M

for (h in 1:3){
  plot =
    ggplot(mindf[[h]], aes(x = mindf[[h]]$x, y = mindf[[h]]$y)) +
    geom_point(col = "black", size = 3) +
    geom_segment(data = connections[[h]],
```

```
                aes(x = x1, y = y1, xend = x2, yend = y2), col = color[h]) +
    labs(x = "X-coordinate", y = "Y-coordinate", title = title[h])
  print(plot)
}
```

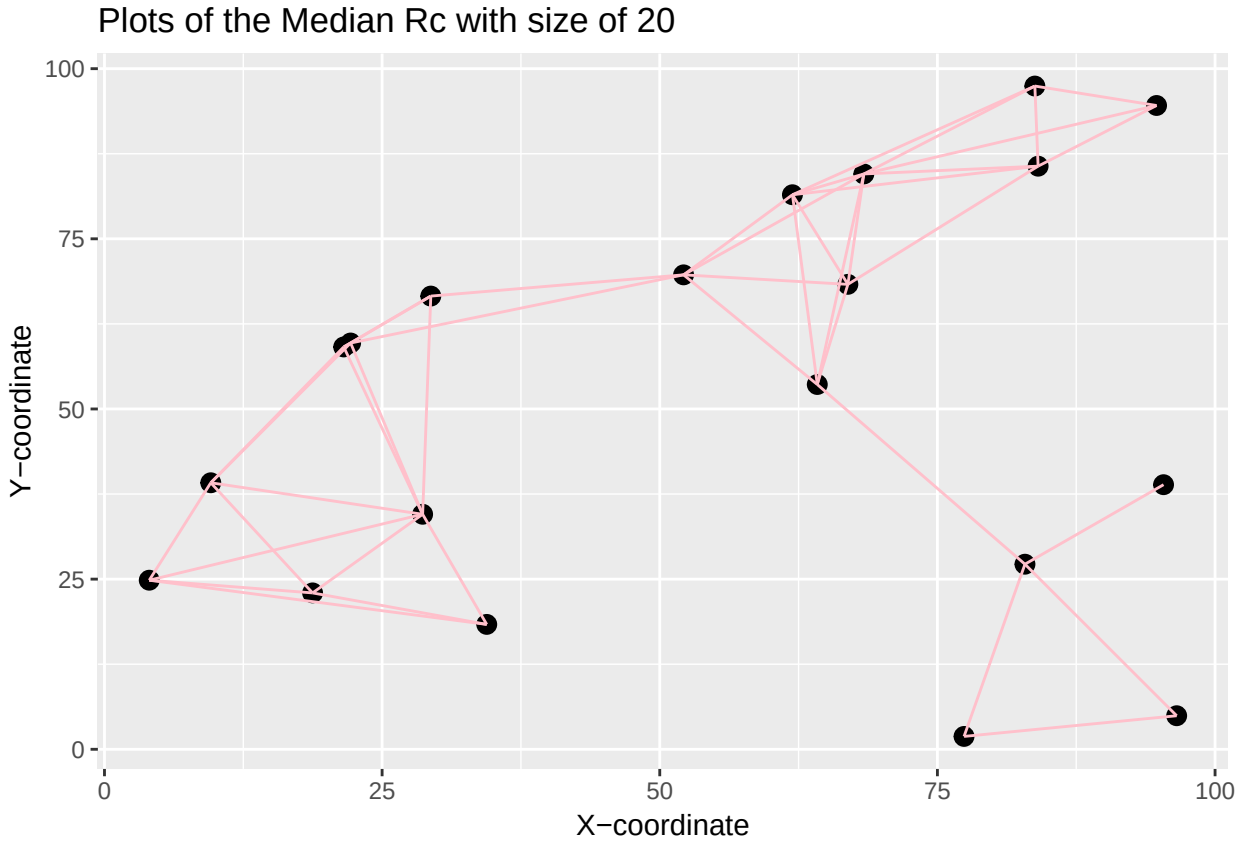


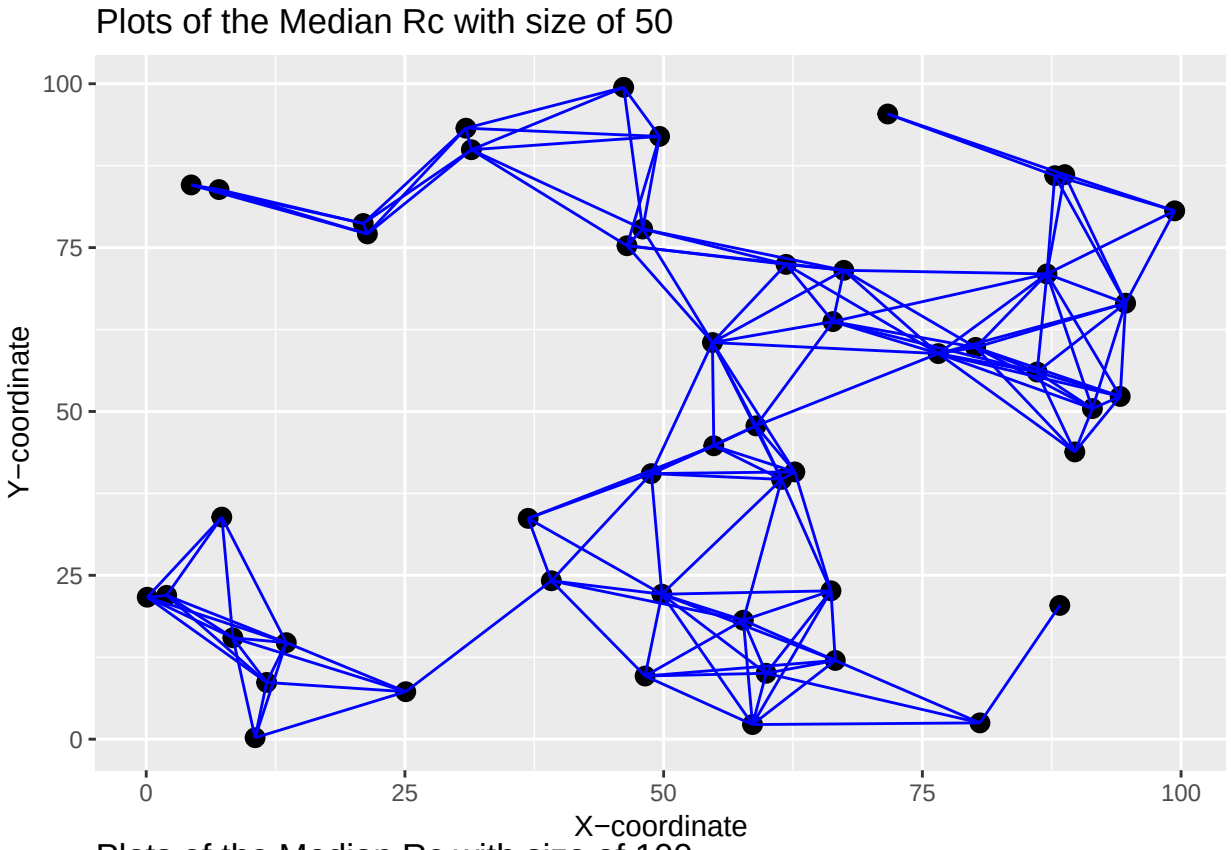


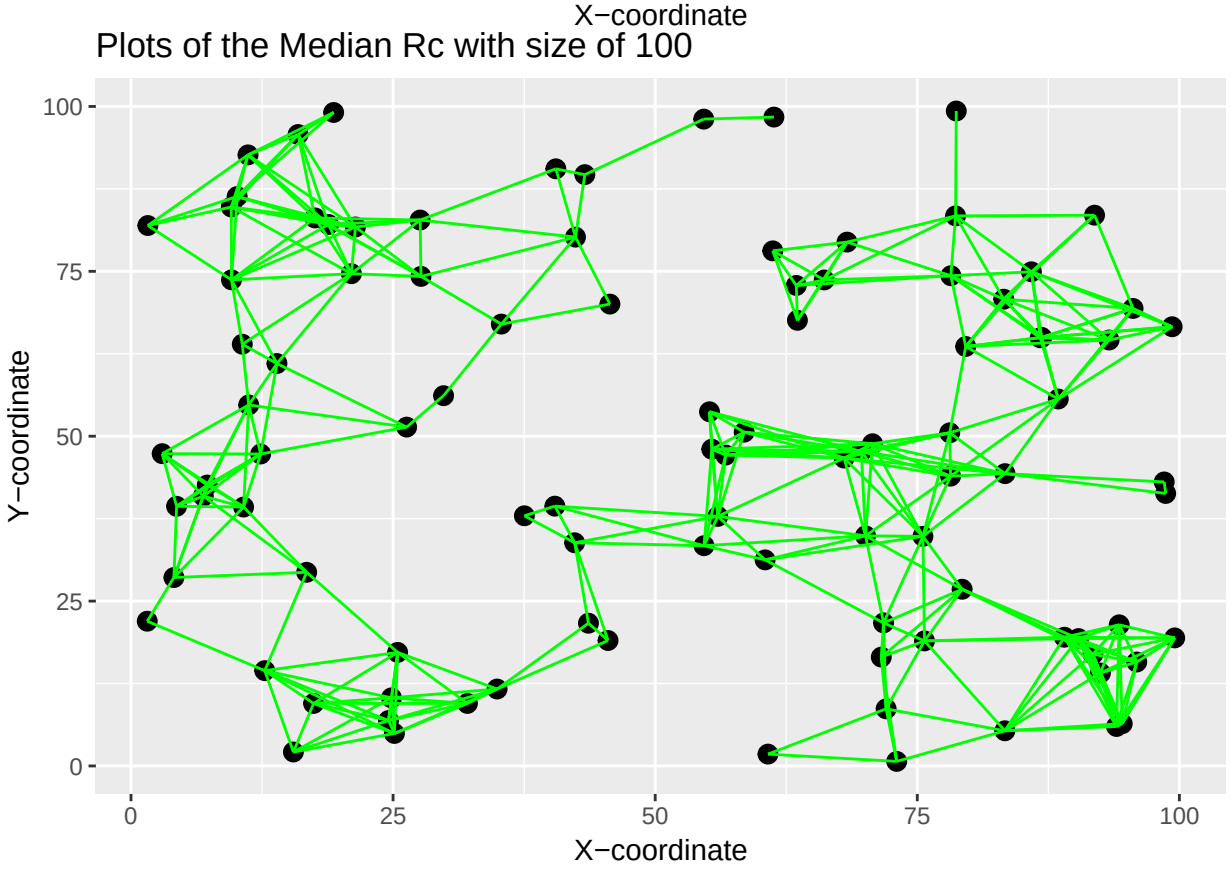


Plots of Rc with the median

```
mindf = list()
connected = list()
connections = list()

for (h in 1:3){
  mindf[[h]] = data.frame(networks[[h]][j[h]])
  comb = combn(nrow(mindf[[h]]), 2)
  connected[[h]] = comb[,which(as.numeric(dist(data.frame(networks[[h]][j[h]])))<=Rc[[h]][j[h]])]
  connections[[h]] = data.frame(
    from = mindf[[h]][connected[[h]][1, ], 1:2],
    to = mindf[[h]][connected[[h]][2, ], 1:2]
    )
  names(connections[[h]]) = c("x1", "y1", "x2", "y2")
}

color = c("pink", "blue", "green")
title = c("Plots of the Median Rc with size of 20", "Plots of the Median Rc with size of 50", "Plots of

for (h in 1:3){
  plot =
    ggplot(mindf[[h]], aes(x = mindf[[h]]$x, y = mindf[[h]]$y)) +
    geom_point(col = "black", size = 3) +
    geom_segment(data = connections[[h]],
                 aes(x = x1, y = y1, xend = x2, yend = y2), col = color[h]) +
    labs(x = "X-coordinate", y = "Y-coordinate", title = title[h])
  print(plot)
}
```

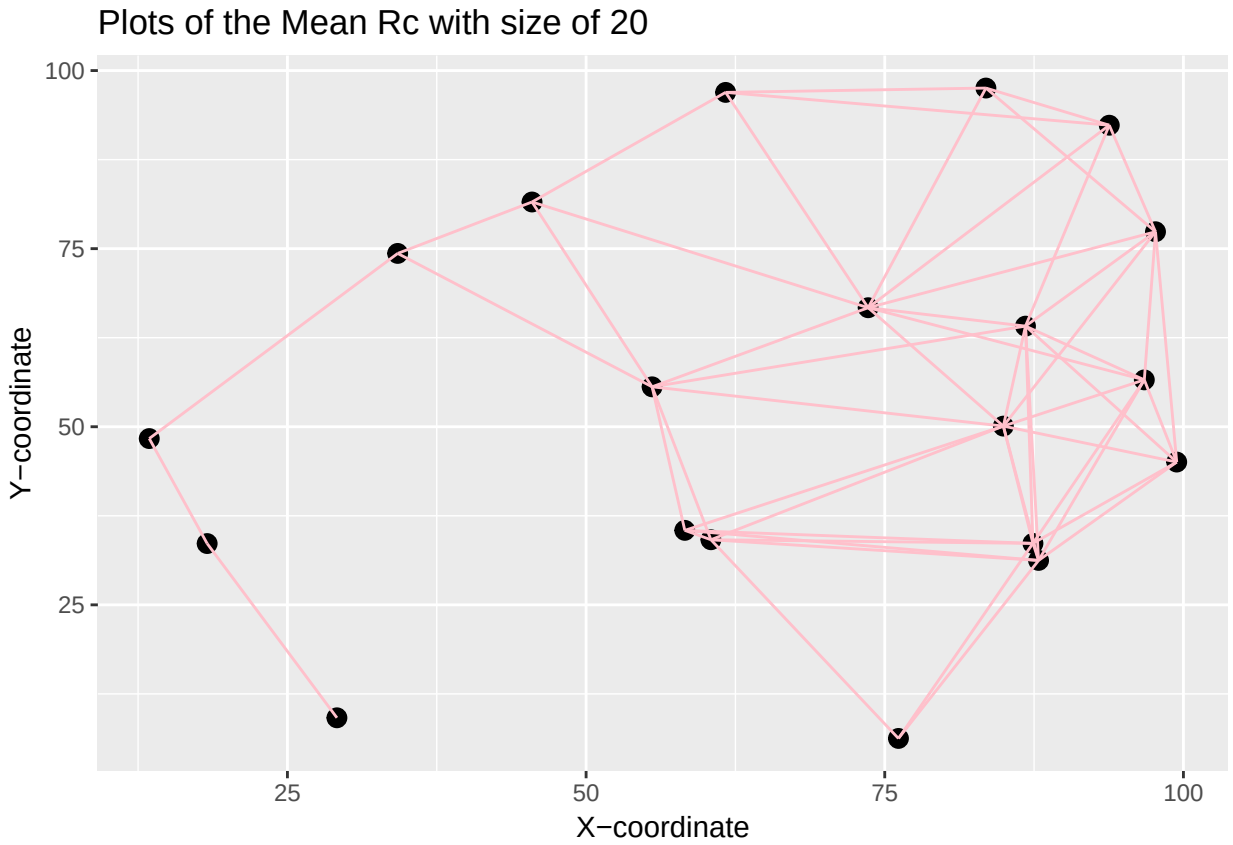


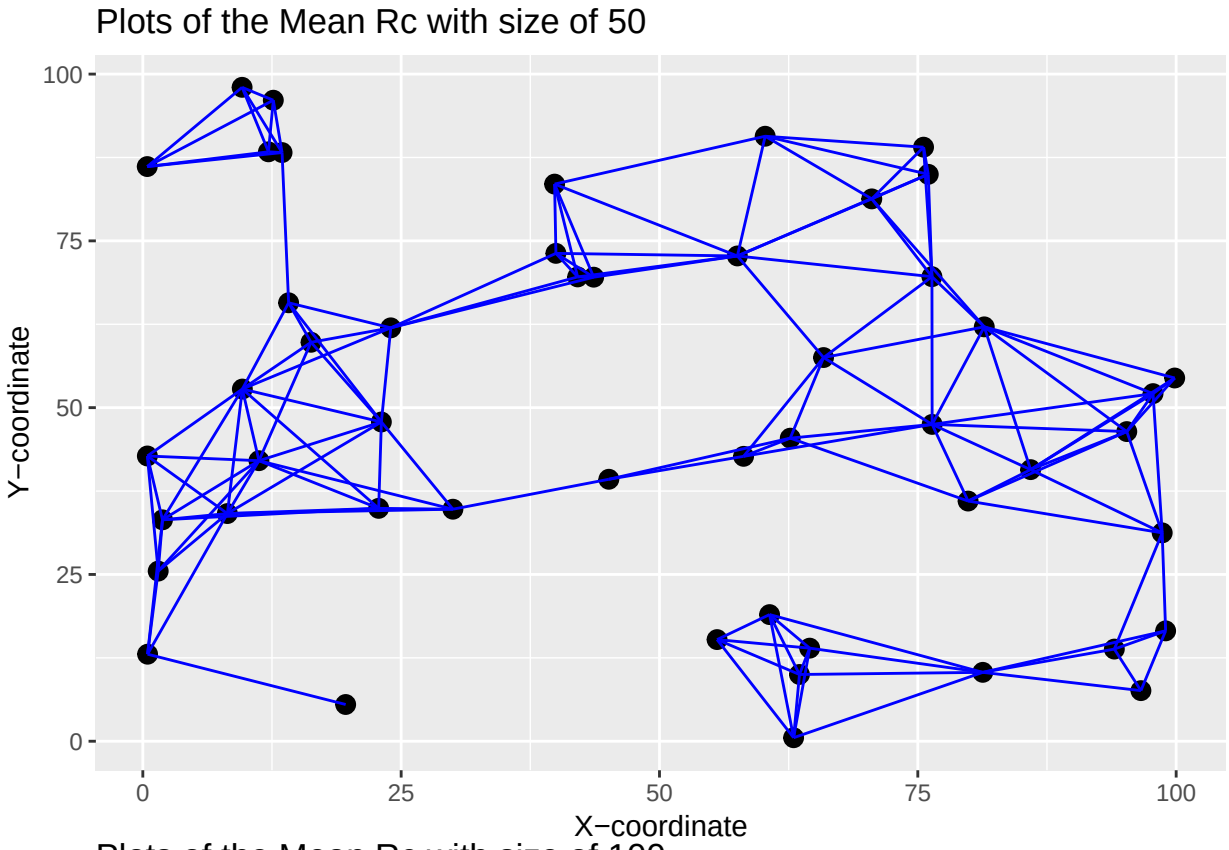


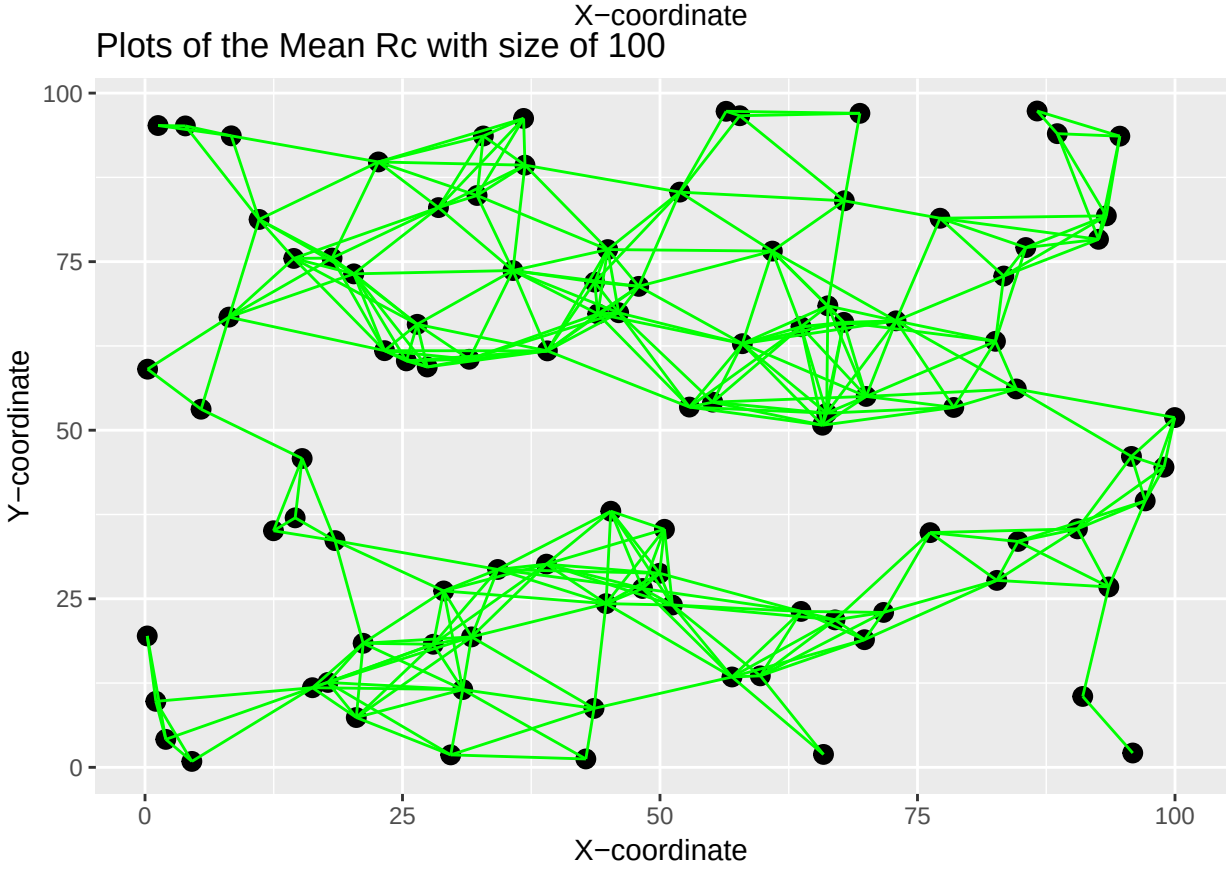


Plots of Rc with mean

```r
mindf = list()
connected = list()
connections = list()

for (h in 1:3){
  mindf[[h]] = data.frame(networks[[h]][k[h]])
  comb = combn(nrow(mindf[[h]]), 2)
  connected[[h]] = comb[,which(as.numeric(dist(data.frame(networks[[h]][k[h]])))<=Rc[[h]][k[h]])]
  connections[[h]] = data.frame(
    from = mindf[[h]][connected[[h]][1, ], 1:2],
    to = mindf[[h]][connected[[h]][2, ], 1:2]
    )
  names(connections[[h]]) = c("x1", "y1", "x2", "y2")
}

color = c("pink", "blue", "green")
title = c("Plots of the Mean Rc with size of 20", "Plots of the Mean Rc with size of 50", "Plots of the

for (h in 1:3){
  plot =
    ggplot(mindf[[h]], aes(x = mindf[[h]]$x, y = mindf[[h]]$y)) +
    geom_point(col = "black", size = 3) +
    geom_segment(data = connections[[h]],
                 aes(x = x1, y = y1, xend = x2, yend = y2), col = color[h]) +
    labs(x = "X-coordinate", y = "Y-coordinate", title = title[h])
  print(plot)
}
```



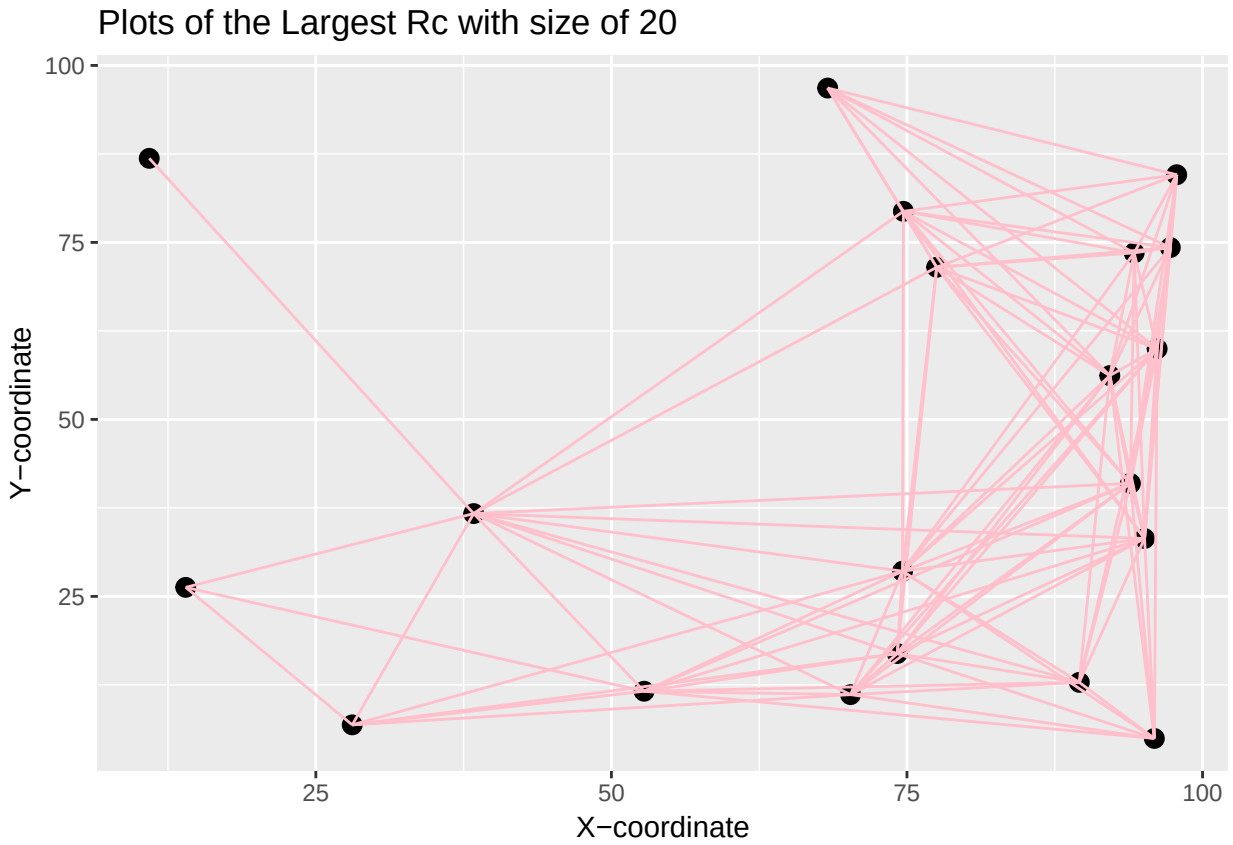


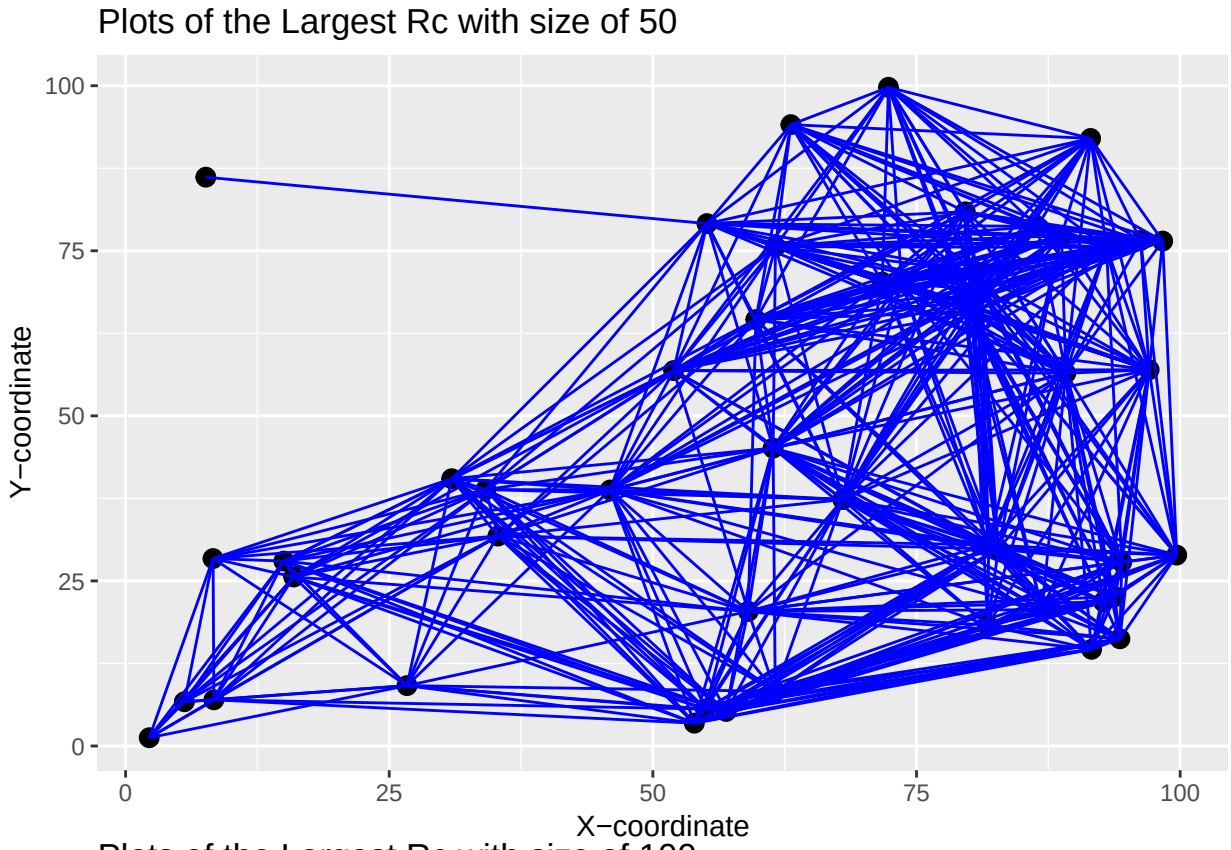


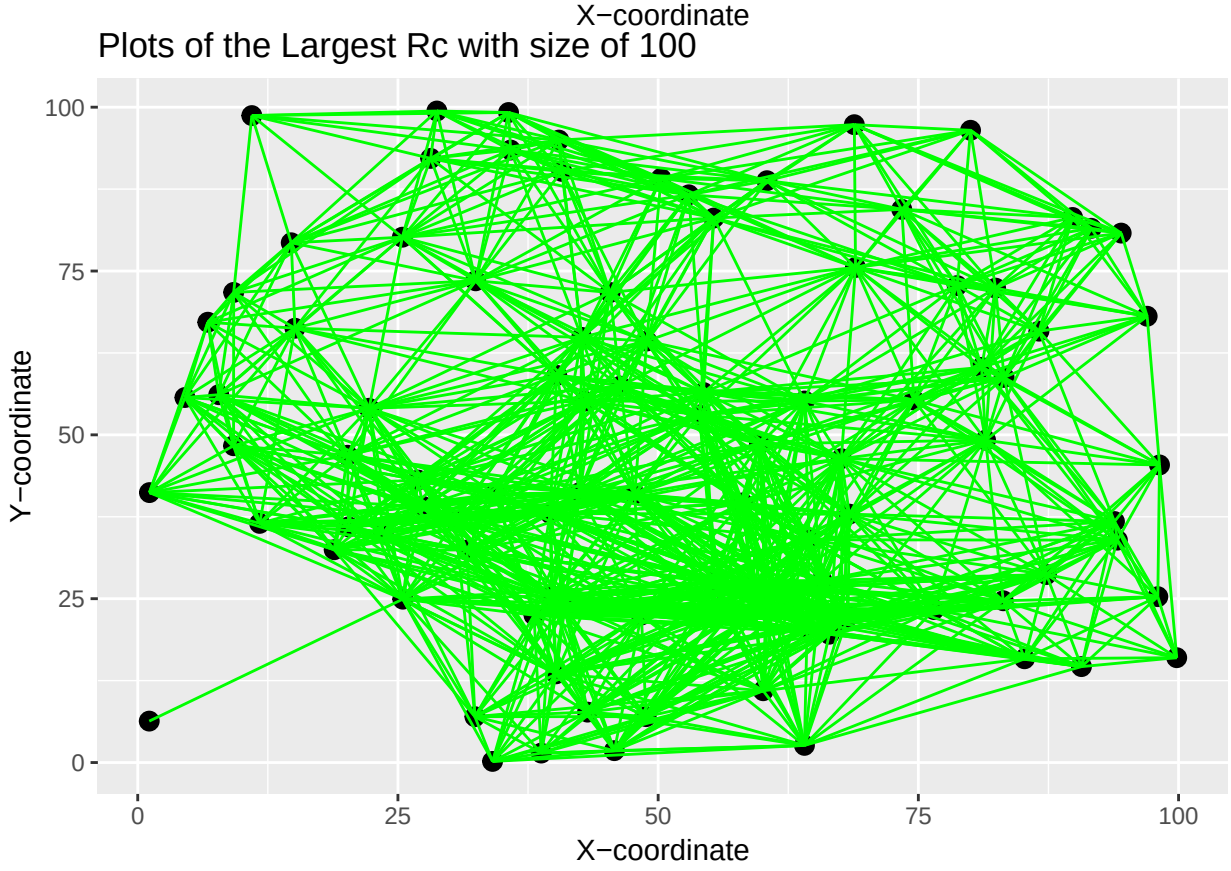
Plots of Rc with the largest

```
mindf = list()
connected = list()
connections = list()

for (h in 1:3){
  mindf[[h]] = data.frame(networks[[h]][1[h]])
  comb = combn(nrow(mindf[[h]]), 2)
  connected[[h]] = comb[,which(as.numeric(dist(data.frame(networks[[h]][1[h]])))<=Rc[[h]][1[h]])]
  connections[[h]] = data.frame(
    from = mindf[[h]][connected[[h]][1, ], 1:2],
    to = mindf[[h]][connected[[h]][2, ], 1:2]
    )
  names(connections[[h]]) = c("x1", "y1", "x2", "y2")
}

color = c("pink", "blue", "green")
title = c("Plots of the Largest Rc with size of 20", "Plots of the Largest Rc with size of 50", "Plots
```

```
for (h in 1:3){
  plot =
    ggplot(mindf[[h]], aes(x = mindf[[h]]$x, y = mindf[[h]]$y)) +
    geom_point(col = "black", size = 3) +
    geom_segment(data = connections[[h]],
                 aes(x = x1, y = y1, xend = x2, yend = y2), col = color[h]) +
    labs(x = "X-coordinate", y = "Y-coordinate", title = title[h])
  print(plot)
}
```




Plots of the Largest Rc with size of 50
Y−coordinate
X−coordinate
0
25
50
75
100


Plots of the Largest Rc with size of 100
Y−coordinate
X−coordinate
0
25
50
75
100