Let $U \sim$ be standard uniform which is underlying distribution of $S_{ij}\left(t|logp^{l},\epsilon\right)$. When $U=1$,the survival function has the value which is greater than zero.

$$
\begin{aligned}
U &= S_{ij}\left(t|logp^{l},\epsilon\right) \\
&= exp\left\{-\int_0^T \lambda_{ij}\left(z|logp^{l},\epsilon\right)dz\right\} \\
-logU &= \int_0^T \lambda_{ij}\left(z|logp^{l},\epsilon\right)dz \\
&= \underbrace{exp\left\{X_j\beta + logp^{l}_{ij}\delta + \widehat{H_{ij}}\gamma + D_{ij}\beta_D + \nu_j + \epsilon_{ij}\right\}}_{=\Theta} \times \int_0^T \kappa_0\left(z\right)dz
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{\Theta}\times\left(-logU\right) &= \int_0^T \kappa_0\left(z\right)dz \\
\frac{1}{\Theta}\times\left(-logU\right) &= \Gamma\left(T\right) \\
T &= \Gamma^{-1}\left(-\frac{1}{\Theta}\times logU\right)
\end{aligned}
$$

$$
\begin{aligned}
F_t\left(t\right) &= Pr\left(T<t\right) \\
&= Pr\left(\Gamma^{-1}\left(-\frac{1}{\Theta}\times logU\right)<t\right) \\
&= Pr\left(logU > -\Theta\cdot\Gamma\left(t\right)\right) \\
&= Pr\left(U > exp\left\{-\Theta\cdot\Gamma\left(t\right)\right\}\right) \\
&= Pr\left(U > S_{ij}\left(t|logp^{l},\epsilon\right)\right) \\
&= 1 - Pr\left(U < S_{ij}\left(t|logp^{l},\epsilon\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
S_{ij}\left(t|logp^{l},\epsilon\right) &= 1 - F_t\left(t\right) \\
f_t\left(t\right) &= \lambda_{ij}\left(t|logp^{l},\epsilon\right)\cdot S_{ij}\left(t|logp^{l},\epsilon\right).
\end{aligned}
$$

Assume $X \sim F_x$, and simulate $U \sim U\left(0,1\right)$.

$$
X = \begin{cases} 0 & \text{if } U < 1 - S_{ij}\left(t|logp^{l},\epsilon\right) \\ 1 & \text{if } U > 1 - S_{ij}\left(t|logp^{l},\epsilon\right) \end{cases}.
$$

Define $T = F_x^{-1}\left(U\right)$. Then,

$$
\begin{aligned}
F_t\left(t\right) &= Pr\left(T<t\right) \\
&= Pr\left(F_x^{-1}\left(U\right)<t\right) \\
&= Pr\left(U < F_x\left(t\right)\right)
\end{aligned}
$$