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The Beauty of Bresenham's Algorithm

A simple implementation to plot lines, circles, ellipses and Bézier curves.

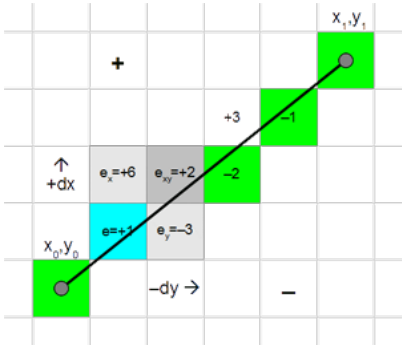
The Algorithm

This page introduces a compact and efficient implementation of Bresenham's algorithm to plot lines, circles, ellipses and Bézier curves.
A detailed documentation of the algorithm and more program examples are availble in PDF: Bresenham.pdf.
Some C-program examples of the document are listed below.
You can try the Bresenham algorithm online on this scratchpad.
The source code of the samples is also available in C: bresenham.c

Line

A simple example of Bresenham's line algorithm.

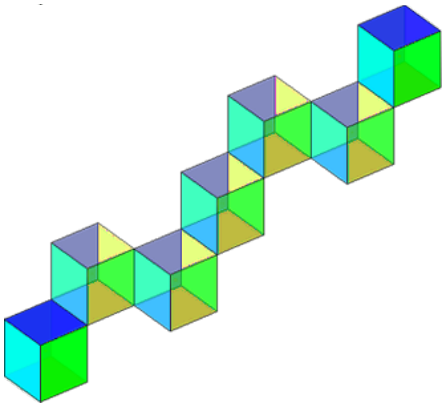
```
void plotLine(int x0, int y0, int x1, int y1)
{
   int dx =  abs(x1-x0), sx = x0<x1 ? 1 : -1;
   int dy = -abs(y1-y0), sy = y0<y1 ? 1 : -1;
   int err = dx+dy, e2; /* error value e_xy */

   for(;;){  /* loop */
      setPixel(x0,y0);
      if (x0==x1 && y0==y1) break;
      e2 = 2*err;
      if (e2 >= dy) { err += dy; x0 += sx; } /* e_xy+e_x > 0 */
      if (e2 <= dx) { err += dx; y0 += sy; } /* e_xy+e_y < 0 */
   }
}
```



Bresenham in 3D

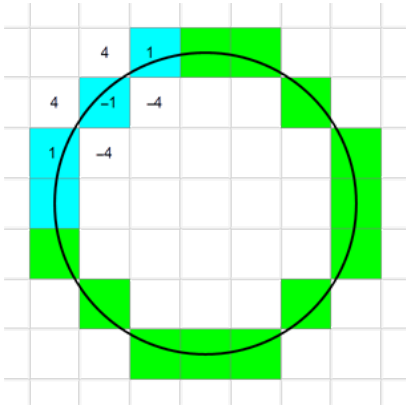
The algorithm could be extended to three (or more) dimensions.

```
void plotLine3d(int x0, int y0, int z0, int x1, int y1, int z1)
{
   int dx = abs(x1-x0), sx = x0<x1 ? 1 : -1;
   int dy = abs(y1-y0), sy = y0<y1 ? 1 : -1;
   int dz = abs(z1-z0), sz = z0<z1 ? 1 : -1;
   int dm = max(dx,dy,dz), i = dm; /* maximum difference */
   x1 = y1 = z1 = dm/2; /* error offset */

   for(;;) {  /* loop */
      setPixel(x0,y0,z0);
      if (i-- == 0) break;
      x1 -= dx; if (x1 < 0) { x1 += dm; x0 += sx; }
      y1 -= dy; if (y1 < 0) { y1 += dm; y0 += sy; }
      z1 -= dz; if (z1 < 0) { z1 += dm; z0 += sz; }
   }
}
```



Circle

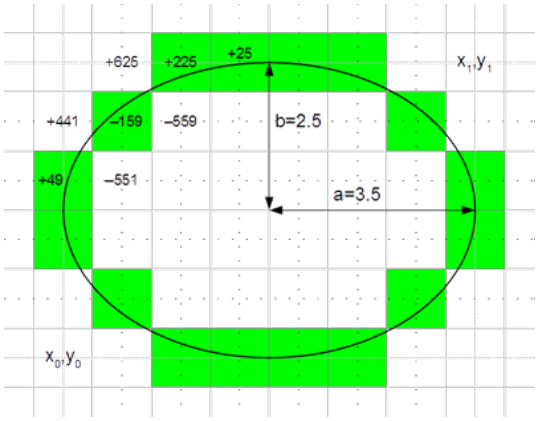
This is an implementation of the circle algorithm.




```
void plotCircle(int xm, int ym, int r)
{
   int x = -r, y = 0, err = 2-2*r; /* II. Quadrant */
   do {
      setPixel(xm-x, ym+y); /*   I. Quadrant */
      setPixel(xm-y, ym-x); /*  II. Quadrant */
      setPixel(xm+x, ym-y); /* III. Quadrant */
      setPixel(xm+y, ym+x); /*  IV. Quadrant */
      r = err;
      if (r <= y) err += ++y*2+1;           /* e_xy+e_y < 0 */
      if (r > x || err > y) err += ++x*2+1; /* e_xy+e_x > 0 or no 2nd y-step */
   } while (x < 0);
}
```

Ellipse

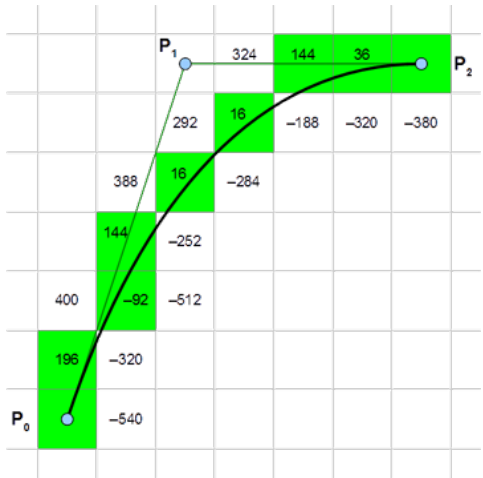
This program example plots an ellipse inside a specified rectangle.




```
void plotEllipseRect(int x0, int y0, int x1, int y1)
{
   int a = abs(x1-x0), b = abs(y1-y0), b1 = b&1; /* values of diameter */
   long dx = 4*(1-a)*b*b, dy = 4*(b1+1)*a*a; /* error increment */
   long err = dx+dy+b1*a*a, e2; /* error of 1.step */

   if (x0 > x1) { x0 = x1; x1 += a; } /* if called with swapped points */
   if (y0 > y1) y0 = y1; /* .. exchange them */
   y0 += (b+1)/2; y1 = y0-b1;   /* starting pixel */
   a *= 8*a; b1 = 8*b*b;

   do {
       setPixel(x1, y0); /*   I. Quadrant */
       setPixel(x0, y0); /*  II. Quadrant */
       setPixel(x0, y1); /* III. Quadrant */
       setPixel(x1, y1); /*  IV. Quadrant */
       e2 = 2*err;
       if (e2 <= dy) { y0++; y1--; err += dy += a; }  /* y step */
       if (e2 >= dx || 2*err > dy) { x0++; x1--; err += dx += b1; } /* x step */
   } while (x0 <= x1);

   while (y0-y1 < b) {  /* too early stop of flat ellipses a=1 */
       setPixel(x0-1, y0); /* -> finish tip of ellipse */
       setPixel(x1+1, y0++);
       setPixel(x0-1, y1);
       setPixel(x1+1, y1--);
   }
}
```

Bézier curve

This program example plots a quadratic Bézier curve limited to gradients without sign change.



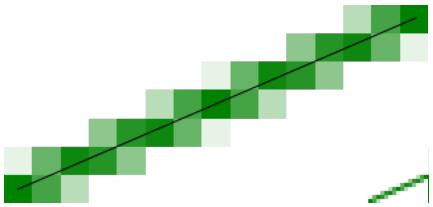

```
void plotQuadBezierSeg(int x0, int y0, int x1, int y1, int x2, int y2)
{
  int sx = x2-x1, sy = y2-y1;
  long xx = x0-x1, yy = y0-y1, xy;         /* relative values for checks */
  double dx, dy, err, cur = xx*sy-yy*sx;                    /* curvature */

  assert(xx*sx <= 0 && yy*sy <= 0);  /* sign of gradient must not change */

  if (sx*(long)sx+sy*(long)sy > xx*xx+yy*yy) { /* begin with longer part */
    x2 = x0; x0 = sx+x1; y2 = y0; y0 = sy+y1; cur = -cur;  /* swap P0 P2 */
  }
  if (cur != 0) {                                    /* no straight line */
    xx += sx; xx *= sx = x0 < x2 ? 1 : -1;           /* x step direction */
    yy += sy; yy *= sy = y0 < y2 ? 1 : -1;           /* y step direction */
    xy = 2*xx*yy; xx *= xx; yy *= yy;          /* differences 2nd degree */
    if (cur*sx*sy < 0) {                           /* negated curvature? */
      xx = -xx; yy = -yy; xy = -xy; cur = -cur;
    }
    dx = 4.0*sy*cur*(x1-x0)+xx-xy;             /* differences 1st degree */
    dy = 4.0*sx*cur*(y0-y1)+yy-xy;
    xx += xx; yy += yy; err = dx+dy+xy;                /* error 1st step */
    do {
      setPixel(x0,y0);                                     /* plot curve */
      if (x0 == x2 && y0 == y2) return;  /* last pixel -> curve finished */
```

```
      y1 = 2*err < dx;                  /* save value for test of y step */
      if (2*err > dy) { x0 += sx; dx -= xy; err += dy += yy; } /* x step */
      if (    y1    ) { y0 += sy; dy -= xy; err += dx += xx; } /* y step */
    } while (dy < dx );           /* gradient negates -> algorithm fails */
  }
  plotLine(x0,y0, x2,y2);                  /* plot remaining part to end */
}
```

Anti-aliased line

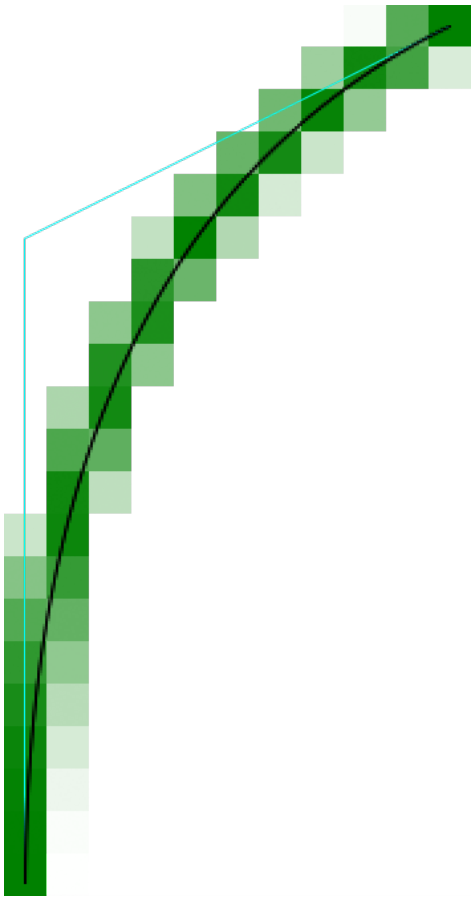
The algorithm could be modified to draw an anti-aliased line.

```
void plotLineAA(int x0, int y0, int x1, int y1)
{
   int dx = abs(x1-x0), sx = x0<x1 ? 1 : -1;
   int dy = abs(y1-y0), sy = y0<y1 ? 1 : -1;
   int err = dx-dy, e2, x2;                           /* error value e_xy */
   int ed = dx+dy == 0 ? 1 : sqrt((float)dx*dx+(float)dy*dy);

   for ( ; ; ){                                         /* pixel loop */
      setPixelAA(x0,y0, 255*abs(err-dx+dy)/ed);
      e2 = err; x2 = x0;
      if (2*e2 >= -dx) {                                    /* x step */
         if (x0 == x1) break;
         if (e2+dy < ed) setPixelAA(x0,y0+sy, 255*(e2+dy)/ed);
         err -= dy; x0 += sx;
      }
      if (2*e2 <= dy) {                                     /* y step */
         if (y0 == y1) break;
         if (dx-e2 < ed) setPixelAA(x2+sx,y0, 255*(dx-e2)/ed);
         err += dx; y0 += sy;
    }
  }
}
```



Anit-aliased quadratic Bézier curve

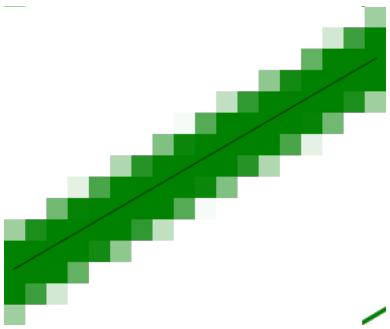
This program example draws a segment of an anti-alised quadratic Bézier curve.



```
void plotQuadBezierSegAA(int x0, int y0, int x1, int y1, int x2, int y2)
{
   int sx = x2-x1, sy = y2-y1;
   long xx = x0-x1, yy = y0-y1, xy;         /* relative values for checks */
   double dx, dy, err, ed, cur = xx*sy-yy*sx;                /* curvature */

   assert(xx*sx >= 0 && yy*sy >= 0);  /* sign of gradient must not change */

   if (sx*(long)sx+sy*(long)sy > xx*xx+yy*yy) { /* begin with longer part */
      x2 = x0; x0 = sx+x1; y2 = y0; y0 = sy+y1; cur = -cur; /* swap P0 P2 */
   }
   if (cur != 0)
   {                                                  /* no straight line */
      xx += sx; xx *= sx = x0 < x2 ? 1 : -1;          /* x step direction */
      yy += sy; yy *= sy = y0 < y2 ? 1 : -1;          /* y step direction */
      xy = 2*xx*yy; xx *= xx; yy *= yy;         /* differences 2nd degree */
      if (cur*sx*sy < 0) {                          /* negated curvature? */
         xx = -xx; yy = -yy; xy = -xy; cur = -cur;
      }
      dx = 4.0*sy*(x1-x0)*cur+xx-xy;            /* differences 1st degree */
      dy = 4.0*sx*(y0-y1)*cur+yy-xy;
      xx += xx; yy += yy; err = dx+dy+xy;               /* error 1st step */
      do {
         cur = fmin(dx+xy,-xy-dy);
         ed = fmax(dx+xy,-xy-dy);           /* approximate error distance */
         ed = 255/(ed+2*ed*cur*cur/(4.*ed*ed+cur*cur));
         setPixelAA(x0,y0, ed*fabs(err-dx-dy-xy));          /* plot curve */
         if (x0 == x2 && y0 == y2) return;/* last pixel -> curve finished */
         x1 = x0; cur = dx-err; y1 = 2*err+dy < 0;
         if (2*err+dx > 0) {                                  /* x step */
            if (err-dy < ed) setPixelAA(x0,y0+sy, ed*fabs(err-dy));
            x0 += sx; dx -= xy; err += dy += yy;
         }
         if (y1) {                                            /* y step */
            if (cur < ed) setPixelAA(x1+sx,y0, ed*fabs(cur));
            y0 += sy; dy -= xy; err += dx += xx;
         }
      } while (dy < dx);              /* gradient negates -> close curves */
   }
   plotLineAA(x0,y0, x2,y2);              /* plot remaining needle to end */
}
```

Anti-aliased thick line

This algorithm draws an anti-aliased line of `wd` pixel width.



```
void plotLineWidth((int x0, int y0, int x1, int y1, float wd)
{
   int dx = abs(x1-x0), sx = x0 < x1 ? 1 : -1;
   int dy = abs(y1-y0), sy = y0 < y1 ? 1 : -1;
   int err = dx-dy, e2, x2, y2;                          /* error value e_xy */
   float ed = dx+dy == 0 ? 1 : sqrt((float)dx*dx+(float)dy*dy);

   for (wd = (wd+1)/2; ; ) {                                    /* pixel loop */
      setPixelColor(x0,y0,max(0,255*(abs(err-dx+dy)/ed-wd+1)));
      e2 = err; x2 = x0;
      if (2*e2 >= -dx) {                                            /* x step */
         for (e2 += dy, y2 = y0; e2 < ed*wd && (y1 != y2 || dx > dy); e2 += dx)
            setPixelColor(x0, y2 += sy, max(0,255*(abs(e2)/ed-wd+1)));
         if (x0 == x1) break;
         e2 = err; err -= dy; x0 += sx;
      }
      if (2*e2 <= dy) {                                             /* y step */
         for (e2 = dx-e2; e2 < ed*wd && (x1 != x2 || dx < dy); e2 += dy)
            setPixelColor(x2 += sx, y0, max(0,255*(abs(e2)/ed-wd+1)));
         if (y0 == y1) break;
         err += dx; y0 += sy;
      }
   }
}
```

Features of the rasterising algorithm:

- Universal: This algorithm plots lines, circles, ellipses, Bézier curves and more

- Fast: Draws complex curves nearly as fast as lines.
- Simple: Short and compact implementation.
- Exact: No approximation of the curve.
- Smooth: Apply anti-aliasing to any curve.
- Flexible: Adjustable line thickness.

The principle of the algorithm could be used to rasterize any curve.
