

Question 3: Exercise 4.1.3, sections b, c

b)  $f$  is not a function when  $x$  is 2 and  $-2$ ;  $\frac{1}{0}$

c)  $f$  is a function, its range is all positive real numbers

Exercise 4.1.5, section, b, d, h, i, l

b)  $\{4, 9, 16, 25\}$

d)  $\{0, 1, 2, 3, 4, 5\}$

h)  $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

i)  $\{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$

l)  $\emptyset, \{2\}, \{2, 3\}, \{3\}$

subtracting from the power set  $\emptyset, \{1\}, \{1, 2\}, \{1, 3\}, \{2\}, \{2, 1\}, \{2, 3\}, \{3\}$

Question 4:

Exercise 4.2.2, sections, c,g,k

- c) It is one to one, but not onto ( $x \neq -2$ )
- g) It is one to one, but not onto because  $y$  can never be odd because  $y$  can never be odd
- k) It is not one to one because when  $x = 2$ ,  $y = 2$  and  $x = 1$ ,  $y = 4$  map to the same value, it is not onto because  $y$  can never be one.

Exercise 4.2.4, sections b, c, d, g

b) It is neither one to one nor onto. It is not one to one because  $f(100)$  and  $f(000)$  map to the same target. It is not onto because  $y = 000$  is never mapped to.

c) It is one to one and onto.

d) It is one to one, but not onto because  $y = 0001$  is not mapped to anything.

g) It is not one to one because  $f(\{1,2\})$  and  $f(\{2\})$  maps to the same  $y$ ; it is not onto because nothing maps to  $\{1,2\}$ .

II. Give an example of a function from the set of integers to the set of positive integers that is:

a) one to one, but not onto:  $f(x) = x^2 + 2$

b) onto, not one to one:  $f(x) = |x| + 1$

c) one to one and onto:

$$f(x) = -2x, x < 0$$

$$f(x) = 2x + 1, x \geq 0$$

d) neither one to one nor onto:  $f(x) = x^2$

Question 5:

Exercise 4.3.2, sections c,d,g,i

- c) It has a well defined inverse, the inverse is  $f^{-1}(x) = x - \frac{3}{2}$
- d) It is not a well defined inverse because both  $\{1,2\}$  and  $\{3,4\}$  maps to 2
- g) It has a well defined inverse, the inverse is just  $f^{-1} = f$  output is obtained by taking the input string and reversing the bits
- i) It has a well defined inverse, the inverse is  $f^{-1}(x, y) = (x - 5, y + 2)$

Exercise 4.4.8, sections c,d

- c)  $2(x^2 + 1) + 3$
- d)  $(2x + 3)^2 + 1$

Exercise 4.4.2, sections b-d

- b)  $\left(\left\lceil \frac{x}{5} \right\rceil\right)^2 \rightarrow \left(\left\lceil \frac{52}{5} \right\rceil\right)^2 \rightarrow (11)^2 \rightarrow 121$
- c)  $4^2 = 16 \rightarrow \left\lceil \frac{16}{5} \right\rceil = 4 \rightarrow 2^4 = 16$
- d)  $\left(\left\lceil \frac{x^2}{5} \right\rceil\right)^2$

Exercise 4.4.6, sections c-e

- c)  $h(f(010)) \rightarrow h(110) \rightarrow 111$
- d)  $\{000, 001, 010, 011, 100, 101, 110, 111\} \rightarrow \{100, 101, 110, 111\} \rightarrow \{101, 111\}$
- e)  $\{000, 001, 010, 011, 100, 101, 110, 111\} \rightarrow \{100, 101, 110, 111\} \rightarrow \{001, 101, 011, 111\}$