Question 5:

Exercise 1.12.2 sections b,e

Use the rules of inference and the laws of propositional logic to prove that each argument is valid. Number each line of your argument and label each line of your proof "Hypothesis" or with the name of the rule of inference used at that line. If a rule of inference is used, then include the numbers of the previous lines to which the rule is applied.

b.
$p \to (q \land r)$
$\neg q$

$\therefore \neg p$

1. $p \to (q \land r)$ - hypothesis
2. $\neg q$ - hypothesis
3. $p \to q$ - simplification on 1
4. $\neg p$ - Modus tollens on 2 and 3

e.
$p \lor q$
$\neg p \lor r$
$\neg q$

$\therefore r$

1. $p \lor q$ - hypothesis
2. $\neg q$ - premise
3. $q \lor p$ - commutative law on 1
4. p - disjunctive syllogism on 2 and 3
5. $\neg p \lor r$ - hypothesis
6. $\neg\neg p$ - double negation on 4
7. r - disjunctive syllogism on 5 and 6

Exercise 1.12.3, section c

Some of the rules of inference can be proven using the other rules of inference and the laws of propositional logic.

c.
$p \lor q$
$\neg p$

$\therefore q$

1. $p \lor q$ - hypothesis
2. $\neg\neg p \lor q$ - double negation on 1
3. $\neg p \to q$ - conditional identities on 2
4. $\neg p$ - hypothesis
5. q - Modus ponens 3 and 4

Question 5(cont.):

Exercise 1.12.5, sections c,d

c.
I will buy a new car and a new house only if I get a job.
I am not going to get a job.

∴ I will not buy a new car.

c = I will buy a new car
h = I will buy a new house
j = I get a job

$(h \land c) \to j$
$\neg j$

$\therefore \neg c$

1. $(h \land c) \to j$ - hypothesis
2. $c \to j$ - simplification on 1
3. $\neg j$ - hypothesis
4. $\neg c$ - Modus tollens on 2 and 3

VALID ARGUMENT

d.
I will buy a new car and a new house only if i get a job.
I am not going to get a job.
I will buy a new house.

∴ I will not buy a new car.

c = I will buy a new car
h = I will buy a new house
j = I get a job

$c \lor h \to j$
$\neg j$
h

$\therefore \neg c$

$c \lor h \to j$ - hypothesis
$\neg j$ - hypothesis
$\neg(c \lor h)$ - Modus tollens on 1 and 2
$\neg c \land \neg h$ - demorgans law on 3
$\neg c$ - simplification on 4
h - hypothesis
$\neg c \land h$ - conjunction on 5 and 6
$\neg c$ - simplification on 7

VALID ARGUMENT

Question 5(cont.):

Exercise 1.13.3, section b

b:
$\exists x(P(x) \vee Q(x))$
$\exists x \neg Q(x)$

$\therefore \exists x P(x)$

	P	Q
a	F	T
b	F	F

The argument is invalid because when it is P(a) and Q(b) the first and second hypothesis is true, but the conclusion is false.

Exercise 1.13.5, section d, e

d.
Every student who missed class got a detention.
Penelope is a student in the class.
Penelope did not miss class.

$\therefore$ Penelope did not get a detention.

$\forall x(m(x) \rightarrow d(x))$
Penelope is a student in the class
$\neg m$(Penelope)

$\therefore \neg d$(Penelope)

This is an invalid argument because when m(x) = False and d(x) = True, both hypothesis are true, but the conclusion is false.

e.
Every student who missed class or got a detention did not get an A
Penelop is a student in the class.
Penelope got an A.

$\therefore$ Penelope did not get a detention.

$\forall x[(m(x) \wedge d(x)) \rightarrow A(x)]$
Penelope is a student
A(Penelope)

$\therefore \neg d$(Penelope)

1. $\forall x[(m(x) \wedge d(x)) \rightarrow A(x)]$ - hypothesis
2. Penelope is a student - hypothesis

3. $[(m(\text{Penelope}) \wedge d(\text{Penelope})) \rightarrow A(\text{Penelope})]$ - Universal Instantiation on 1 and 2
4. $A(\text{Penelope})$ - hypothesis
5. $\neg\neg A(\text{Penelope})$ - double negation law on 4
6. $\neg(m(\text{Penelope}) \vee d(\text{Penelope}))$ - Modus tollens on 3 and 5
7. $\neg m(\text{Penelope}) \wedge d(\text{Penelope})$ - demorgans on 6
8. $\neg d(\text{Penelope})$ - simplification on 7

Question 6:

Solve Exercise 2.4.1, section d; Exercise 2.4.3, section b, from the Discrete Math zyBook

d. The product of two integers is an odd integer.

If the integer is odd then the product of the two integers are odd. Let a and b be integers and assume the integer is odd
ab= $2k + 1$ - ab = 2k+1 because the product of both integer are odd
$(2k + 1)(2k + 1) = 4k^2 + 4k + 1$ - plugging in the 2k+1 for ab to see if the product is odd
$2(2k^2 + 2k) + 1$ =ab - you can take out a 2 to put it in 2k+1 format
$2(m) + 1 =$ ab where m is $(2k^2 + 2k)$
Because ab are both integers and is in the format 2m + 1, the product of two odd integers will always be odd. ■

b. If x is a real number and $x \leq 3$, then $12 - 7x + x^2 \geq 0$.
Let x be a real number and assume that if x is ≤ 3 then $12 - 7x + x^2 \geq 0$
$x^2 - 7x + x^2 \geq 0$ - multiply each side by -1
$-x^2 + 7x - x^2 \leq 0$ - now that the inequalities are the same side plug in $x \leq 3$
$-(3)^2 + 7(3) - 12 = 0$
$0 \leq 3$
Because 0 is less than 3 the statement is true. ■

Question 7:

Solve Exercise 2.5.1, section d; Exercise 2.5.4, sections a, b; Exercise 2.5.5, section c, from the Discrete Math zyBook:
d.

For every integer n, if $n^2 - 2n + 7$ is even, then n is odd.

If n is even then $n^2 - 2n + 7$ is odd. - using proof by contraposition
$n = 2k$ - we are assuming n is even
(2k)^2 + 2(2k) + 7 - we plug in 2k for n
$4k^2 + 4k + 7$ - we multiply it out
$4k^2 + 4k + 6 + 1$ - separate the 7 to 6 and 1 to factor our smoothly
$2(2k^2 + 2k + 3) + 1$ - now it is in the format 2k+1
$n^2 - 2n + 7 = 2m + 1$ where $m = 2k^2 + 2k + 3$
With the use of proof by contraposition we proved that the original theory is true. ∎

a.
For every pair of real numbers x and y, if $x^3 + xy^2 \leq x^2y + y^3$, then $x \leq y$.
If $x > y$ then $x^3 + xy^2 > x^2y + y^3$ - using proof by contraposition
$x^3 + xy^2 > x^2y + y^3$
$x(x^2 + y^2) > y(x^2 + y^2)$ - distribute the x out
$x(z) > y(m)$ where z is $x^2 + y^2$ and y is $x^2 + y^2$
$x > y$
Because z and m are both the same exact value and x is greater than y, with the proof by contraposition we are able to prove our original theory true. ∎

b.
For every pair of real numbers x amd y, if x+y > 20, then x > 10 or y > 10.
$(x \leq 10 \land x \leq 10) \rightarrow x + y \leq 20$ - proof by contrapostion
$x + y \leq 20$ - add both inequalities
Since when you add both inequalities it is equal to the conclusion, with the proof by contraposition the original theory true. ∎

c.
For every non-zero real number x, if x is irrational, then 1/x is also irrational.
If $\frac{1}{x}$ is rational, then x is rational. - using proof by contraposition
$\frac{1}{x} = \frac{a}{b}$ - 1/x = ab where and b does not equal 0
$1 = a\frac{x}{b}$ - you multiply both sides by x
$b = ax$ - then divide both sides by a
$\frac{b}{a} = x$ - you reach x = b/a
Since a and b are non-zero real numbers, that means $\frac{b}{a}$ has to be rational, thus proving x is rational.
∎

Question 8:

Solve Exercise 2.6.6, sections c, d, from the Discrete Math zyBook:

c. The average of three real numbers is greater than or equal to at least
$\frac{a+b+c}{3} \geq a, b, c$ - premise
$\frac{a+b+c}{3} < a, b, c$ - proof by contradiction
$\frac{a+b+c}{3} + \frac{a+b+c}{3} + \frac{a+b+c}{3} < a + b + c$ - the sum of the average should be less than the sum of abc
$\frac{3a+3b+3c}{3} < a + b + c$ - simplify the left side
$a + b + c < a + b + c$ - they are equal to each other

Since these are equal rather than less than to each other, this is a contradiction, proving the theory correct. ■

d. There is no smallest integer
There is a smallest integer
Let x be the smallest integer and y is a integer
$y = x - 1$
Since y is always less than x, there is a contradiction which proves the theory correct. ■

Question 9:

Solve Exercise 2.7.2, section b, from the Discrete Math zyBook:
b. If x and y have the same parity, then $x + y$ is even.

Case 1
x and y is odd = x + y
Let x and y be odd (2k + 1) we can plug the numbers into $x + y$ because $x = 2k + 1, y = 2k + 1$
(2m + 1) + (2n + 1) = 4m+2
2(2m + 1) = x + y
When x and y is odd, $x + y$ is in the format $2k$, thus it is even and the theory is correct. ■

Case 2
x and y is even = x + y
Let x and y be even (2k) - we can plug the numbers into $x + y$ because $x = 2k, y = 2k$
(2m) + (2n) = 2(m + n)
2(m + n) = x + y
When x and y is even, $x + y$ is in the format $2k$, thus it is even and the theory is correct. ■