Question 7:

Exercise 3.1.1, sections a-g

(a) $27 \in A$ - True
(b) $27 \in B$ - False
(c) $100 \in B$ - True
(d) $E \subseteq C$ or $C \subseteq E$. - False
(e) $E \subseteq A$ - True
(f) $A \subseteq E$ - False
(g) $E \in A$ - False

Exercise 3.1.2 sections a-e

(a) $15 \subset A$ - False
(b) $\{15\} \subset A$ - True
(c) $\emptyset \subset C$ - True
(d) $D \subseteq D$ - True
(e) $\emptyset \in B$ - False

Exercise 3.1.5, sections b, d

(b) $s = \{x \in \mathbb{Z}^+ \mid x \text{ is a multiple of } 3\}$
(d) $s = \{x \in \mathbb{N} \mid x \text{ is multiple of } 10 \text{ and is } \leq 1000\}$, $|101|$

Exercise 3.2.1, sections a-k

Let $X = \{1, \{1\}, \{1, 2\}, 2, \{3\}, 4\}$. Which statements are true?

(a) $2 \in X$ - True
(b) $\{2\} \subseteq X$ - True
(c) $\{2\} \in X$ - False
(d) $3 \in X$ - False
(e) $\{1, 2\} \in X$ - True
(f) $\{1, 2\} \subseteq X$ - True
(g) $\{2, 4\} \subseteq X$ - True
(h) $\{2, 4\} \in X$ - False
(i) $\{2, 3\} \subseteq X$ - False
(j) $\{2, 3\} \in X$ - False
(k) $|X| = 7$ - False

Question 8

Exercise 3.2.4 sections b

Let $\{A = \{1, 2, 3\}$, What is $X \in P(A) : 2 \in X\}$

$P(A) = \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$

All the sets that contain the element $2 = \{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}$

Question 9

Exercise 3.3.1 sections c-e

(c) $A \cap C$
$S = \{-3, 1, 17\}$
(d) $A \cup (B \cap C)$
$S = \{-5, -3, 0, 1, 4, 17\}$
(e) $A \cap B \cap C$
$S = \{1\}$

Exercise 3.3.3 sections a, b, e, f

(a) = $\{1\}$
$A^2 \cap A^3 \cap A^4 \cap A^5$
$\{1, 2, 4\} \cap \{1, 3, 9\} \cap \{1, 4, 16\} \cap \{1, 5, 25\} = \{1\}$
(b) = $\{1, 2, 3, 4, 5, 9, 16, 25\}$
$A^2 \cup A^3 \cup A^4 \cup A^5$
$\{1, 2, 4\} \cup \{1, 3, 9\} \cup \{1, 4, 16\} \cup \{1, 5, 25\} = \{1, 2, 3, 4, 5, 9, 16, 25\}$
(e) = $\left\{x \in \mathbb{R} : -\frac{1}{100} \leq x \leq \frac{1}{100}\right\}$ - Because it is an intersection and in order for it to be true, the sets in comparison must contain each elements, the result would be the smallest number in the range.
(f) = $\{x \in \mathbb{R} : -1 < x < 1\}$ Because it is a union it would include all the numbers of the set, so naturally the largest number of the set range would include every number.

Exercise 3.3.4 sections b,d

(b) $P(A \cup B)$ - $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$

(d) $P(A) \cup P(B)$

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

$P(A) \cup P(B) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{c\}, \{b, c\}\}$

Question 10

Exercise 3.5.1, sections b, c

(b) Write an element from the set $B \times A \times C$

{foam, tall, non-fat}

(c) Write set $B \times C$ using roster notation.

$S =$ {foam,non-fat}, {foam, whole}, {no foam, non-fat}, {no foam, whole}

Exercise 3.5.3, sections b, c, e

(b) True - all elements in an integer is also included in real numbers
(c) False - 64 belongs to both a perfect square and a perfect cube
(e) True - If every element in A belongs to B, then every element in $A^2$ should belong to $B^2$

Exercise 3.5.6, sections d, e

(d) $\{xy$ :where $x \in \{0\} \cup \{0\}^2$and $y \in \{1\} \cup \{1\}^2\}$
$x \in \{0, 00\}, y \in \{1, 11\}$
$S = \{01, 011, 001, 0011\}$
(e) $\{xy : x \in \{aa, ab\}$ and $y \in \{a\} \cup \{a\}^2\}$
$x \in \{aa, ab\}, y \in \{aa, a\}$
$S = \{aaa, aaaa, aba, abaa\}$

Exercise 3.5.7, sections c, f, g

(c) $(A \times B) \cup (A \times C)$
$(a, b), (a, c) \cup (a, a)(a, b)(a, d)$
$\{ab, ac, aa, ad\}$

(f) $P(A \times B)$
$P((a, b), (a, c))$
$\{\emptyset, \{ab\}, \{ac\}\{abac\}\}$

(g) $P(A) \times P(B)$. Use ordered pair notation for elements of the Cartesian product.
$P(A) = \emptyset, \{a\}$
$P(B) = \emptyset, \{b\}, \{c\}, \{b, c\}$
$\{(\emptyset, \emptyset), (\emptyset, \{b\}), (\emptyset, \{c\})(\emptyset, \{b, c\}), (\{a\}, \emptyset), (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{b, c\})\}$

Question 11

Exercise 3.6.2, sections b, c
(b) $(B \cup A) \cap (\overline{B} \cup A) = A$
$(A \cup B) \cap$ (A $\cup \overline{B})$ - commutative law
$A \cup (\overline{B} \cap B)$ - distributive law
$(\overline{B} \cap B)$ = $\emptyset$ - complement law
$A \cup \emptyset = A$ - identity law

(c)
c = complement
$\left(A \cap B^{C}\right)^{C} = A^{C} \cup B$
$A^{C} \cup \left(B^{C}\right)^{C}$ - demorgans law
$A^{C} \cup B$ - double complement law
$A^{C} \cup B$ = $A^{C} \cup B$

Exercise 3.6.3, sections b,d
(b) $A - (B \cap A) = A$
$B = \{3, 2, 1\}$
$A = \{1\}$
$B \cap A = 1$
$A - 1 = \emptyset$
$\emptyset \neq A$
(d) $(B - A) \cup A = A$
$B = \{1, 2\}$
$A = \{1\}$
$B - A = \{2\}$
$\{2\} \cup A = \{1, 2\}$
$\{1, 2\} \neq A$

Exercise 3.6.4, sections b,c
(b) $A \cap (B - A) = \emptyset$
$B - A = B \cap \overline{A}$ - subtraction law
$A \cap \left(B \cap \overline{A}\right) = B \cap \left(A \cap \overline{A}\right)$ - associative law
$A \cap \overline{A} = \emptyset$ - complement law
$B \cap \emptyset = \emptyset$ - domination law
$\emptyset = \emptyset$

(c) $A \cup (B - A) = A \cup B$
$A \cup (B - A) = B \cap \overline{A}$ - subtraction law
$A \cup B \cap \overline{A} = (A \cup B) \cap \left(A \cup \overline{A}\right)$ - distributive law
$A \cup \overline{A} = U$ - complement law
$(A \cup B) \cap U = A \cup B$ - identity law
$A \cup B = A \cup B$