Question 1:

A. Convert the following numbers to their decimal representation. Show your work.

1. $10011011_2 = 155_{10}$
   $(2^0 \times 1) + (2^1 \times 1) + (2^2 \times 0) + (2^3 \times 1) + (2^4 \times 1) + (2^5 \times 0) + (2^6 \times 0) + (2^7 \times 1)$
   $1 + 2 + 8 + 16 + 128 = 155_{10}$

2. $456_7 = 237_{10}$
   $(7^0 \times 6) + (7^1 \times 5) + (7^2 \times 4)$
   $6 + 35 + 196 = 237_{10}$

3. $38A_{16} = 906_{10}$
   $\left(16^0 \times A\right) + \left(16^1 \times 8\right) + \left(16^2 \times 3\right)$
   $10 + 128 + 768 = 906_{10}$

4. $2214_5 = 309_{10}$
   $(5^0 \times 4) + (5^1 \times 1) + (5^2 \times 2) + (5^3 \times 2)$
   $4 + 5 + 50 + 250 = 309_{10}$

B. Convert the following numbers to their binary representation:

1. $69_{10} = 1000101_2$
   $\frac{69}{2} = 34r1, \frac{34}{2} = 17r0, \frac{17}{2} = 8r1, \frac{8}{2} = 4r0, \frac{4}{2} = 2r0, \frac{2}{2} = 1r0, \frac{1}{2} = 0r1 = 1000101_2$

2. $485_{10} = 111100101_2$
   $\frac{485}{2} = 242r1, \frac{242}{2} = 121r0, \frac{121}{2} = 60r1, \frac{60}{2} = 30r0, \frac{30}{2} = 15r0, \frac{15}{2}7r1, \frac{7}{2} = 3r1, \frac{3}{2} = 1r1, \frac{1}{2} = 0r1$
   $= 111100101_2$

3. $6D1A_{10} = 0110110100011010_2$
   Used the hexadecimal binary conversion list mentioned in the webinar

C. Convert the following numbers to their hexadecimal representation:

1. $1101011_2 = 6B_6$
   $(2^0 \times 1) + (2^1 \times 1) + (2^2 \times 0) + (2^3 \times 1) + (2^4 \times 0) + (2^5 \times 1) + (2^6 \times 1) = 107$
   $107/16 = 6$ r 11, $6/16 = 0$ r $6 = 6B_6$

2. $895_{10} = 37F$
   $\frac{895}{16} = 55 = r = 15, \frac{55}{16} = 3 = r = 7, \frac{3}{16} = 0 = r = 3 = 37F$

Question 2:

Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8 = 14303_8$
   $\left(11\%83, \frac{11}{8} = 1, 8\%8 = 0, \frac{8}{8} = 1, 11\%8 = 3, \frac{11}{8} = 1, 12\%8 = 4, \frac{12}{8} = 1\right) = 14303_8$
2. $10110011_2 + 1101_2 = 11000000_2$
   $\left(2\%2 = 0, \frac{2}{2} = 1, 2\%2 = 0, \frac{2}{2} = 1, 2\%2 = 0, \frac{2}{2} = 1, 2\%2 = 0, \frac{2}{2} = 1, 2\%2 = 0, \frac{2}{2} = 1\right) = 11000000_2$
3. $7A66_{16} + 45C5_{16} = C02B_{16}$
   $\left(6 + 5 = B, 18\%16 = 2, \frac{18}{16} = 1, 16\%16 = 0, \frac{16}{16} = 1, 1 + 7 + 4 = 12 = C\right) = \text{C02B}$
4. $3022_5 - 2433_5 = 34_5$
   $((2 - 3 = 7 - 2 = 4) \rightarrow (1 - 3 = 6 - 3 = 3) \rightarrow (0 - 4 = 4 - 4 = 0) \rightarrow (2 - 2 = 0)) = 0034_5$

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. $124_{10} = 01111100_{10}$
   $\left(\frac{124}{2} = r = 0, \frac{62}{2} = r = 0, \frac{31}{2} = r = 1, \frac{15}{2} = r = 1, \frac{7}{2} = r = 1, \frac{3}{2} = r = 1, \frac{1}{2} = r = 1\right)$
   add extra zero because it is 8-bit $= 01111100_{10}$
2. $-124_{10} = 10000100_{10}$
   using last answer and flipping it and adding 1 byte $(01111100_{10} = 10000011_{10}) \rightarrow (10000011_{10} + 00000001_{10} = 10000100_{10})$
3. $109_{10} = 01101101_{10}$
   $\left(\frac{109}{2} = r = 1, \frac{54}{2} = r = 0, \frac{27}{2} = r = 1, \frac{13}{2} = r = 1, \frac{6}{2} = r = 0, \frac{3}{2} = r = 1, \frac{1}{2} = r = 1\right) = 01101101_{10}$
4. $-79_{10} = 10111001_{10}$
   $\left(\frac{79}{2} = r = 1, \frac{39}{2} = r = 1, \frac{19}{2} = r = 1, \frac{8}{2} = r = 0, \frac{4}{2} = r = 0, \frac{2}{2} = r = 0, \frac{1}{2} = r = 1\right) = 01000111_{10}$
   flip it because it is negative and add one byte
   $10111000_{10} + 1 = 10111001_{10}$

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. 000111108 bit 2's comp $= 30_{10}$
   $(2^0 \times 0) + (2^1 \times 1) + (2^2 \times 1) + (2^3 \times 1) + (2^4 \times 1) + (2^5 \times 0) + (2^6 \times 0) + (2^7 \times 0)$
   $2 + 4 + 8 + 16 = 30_{10}$
2. 111001108 bit 2's comp $= -26_{10}$
   $(2^0 \times 0) + (2^1 \times 1) + (2^2 \times 1) + (2^3 \times 0) + (2^4 \times 0) + (2^5 \times 1) + (2^6 \times 1) + (2^7 \times -1)$
   $2 + 4 + 32 + 64 - 128 = -26_{10}$
3. 001011018 bit 2's comp $= 45_{10}$
   $(2^0 \times 1) + (2^1 \times 0) + (2^2 \times 1) + (2^3 \times 1) + (2^4 \times 0) + (2^5 \times 1) + (2^6 \times 0) + (2^7 \times 0)$
   $1 + 4 + 8 + 32 = 45$
4. 100111108 bit 2's comp $= -98_{10}$
   $(2^0 \times 0) + (2^1 \times 1) + (2^2 \times 1) + (2^3 \times 1) + (2^4 \times 1) + (2^5 \times 0) + (2^6 \times 0) + (2^7 \times -1)$
   $2 + 4 + 8 + 16 - 128 = -98_{10}$

Question 4:
Solve the following questions from the Discrete Math zyBook:

Exercise 1.2.4, sections b, c

Write a truth table for each expression.

(b) $\neg(p \vee q)$

p	**q**	$p \vee q$	$\neg p \vee q$
T	T	T	F
T	F	T	T
F	T	F	T
F	F	T	T

(c) $r \vee (p \wedge \neg q)$

p	**q**	**r**	$\neg q$	$p \wedge \neg q$	$r \vee (p \wedge \neg q)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

Exercise 1.3.4, sections b, d

Give a truth table for each expression.

(b)$(p\to q)\to(q\to p)$

p	**q**	$p\to q$	$q\to p$	$(p\to q)\to(q\to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(d)$(p\leftrightarrow q)\oplus(p\leftrightarrow\neg q)$

p	**q**	$\neg q$	$p\leftrightarrow q$	$p\leftrightarrow\neg q$	$(p\leftrightarrow q)\oplus(p\leftrightarrow\neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Question 5:
Solve the following questions from the Discrete Math zyBook:
1. Exercise 1.2.7, sections b, c

Consider the following pieces of identification a person might have in order to apply for a credit card:

B: Applicant presents a birth certificate.
D: Applicant presents a driver's license.
M: Applicant presents a marriage license.

Write a logical expression for the requirements under the following conditions:

(b) The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

$(B \wedge D) \vee (B \wedge M) \vee (M \wedge D)$

(c) Applicant must present either a birth certificate or both a driver's license and a marriage license.

$B \vee (D \wedge M)$

2. Exercise 1.3.7, sections b – e

Define the following propositions:

s: a person is a senior
y: a person is at least 17 years of age
p: a person is allowed to park in the school parking lot

Express each of the following English sentences with a logical expression:

(b) A person can park in the school parking lot if they are a senior or at least seventeen years of age.

$(s \vee y) \rightarrow p$

(c) Being 17 years of age is a necessary condition for being able to park in the school parking lot.

$y \leftrightarrow p$

(d) A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

$p \leftrightarrow (s \wedge y)$

(e) Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

$p \rightarrow (s \vee y)$

Question 6:
Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.3.6, sections b – d

Give an English sentence in the form "If…then…" that is equivalent to each sentence.

(b) Maintaining a B average is necessary for Joe to be eligible for the honors program.

If Joe wants to be eligble for honors then he needs to maintain a B average.

(c) Rajiv can go on the roller coaster only if he is at least four feet tall.

If Rajiv can ride the rollercoaster, then he is atleast four feet tall.

(d) Rajiv can go on the roller coaster if he is at least four feet tall.

If Rajiv is atleast 4 feet tall then he can ride the rollercoaster.

2. Exercise 1.3.10, sections c – f

The variable p is true, q is false, and the truth value for variable r is unknown. Indicate whether the truth value of each logical expression is true, false, or unknown.

(c) $(p \lor r) \leftrightarrow (q \land r)$ - False

(d) $(p \land r) \leftrightarrow (q \land r)$ - Unknown

(e) $p \rightarrow (r \lor q)$ - Unknown

(f) $(p \land q) \rightarrow r$ - True

Question 7:
Solve Exercise 1.4.5, sections b – d, from the Discrete Math zyBook:

Define the following propositions:
j : Sally got the job.
l : Sally was late for her interview
r : Sally updated her resume.
Express each pair of sentences using logical expressions. Then prove whether the two expressions are logically equivalent.

(b)
If Sally did not get the job, then she was late for her interview or did not update her resume.
If Sally updated her resume and was not late for her interview, then she got the job.
$\neg j \rightarrow (l \lor \neg r) \equiv (r \land \neg l) \rightarrow j$
$\neg j \rightarrow (l \lor \neg r)$ - premise
$\neg(\neg j) \lor (l \lor \neg r) \equiv \neg(r \land \neg l) \rightarrow j$ - conditional identities
$\neg\neg j \lor l \lor \neg r \equiv \neg r \lor \neg\neg l) \lor j$ - demograns law
$j \lor l \lor \neg r \equiv \neg r \lor l \lor j$ - double negation rule
$j \lor l \lor \neg r \equiv j \lor l \lor \neg r$ - commutative rule
$j \lor l \lor \neg r \equiv j \lor l \lor \neg r$

(c)
If Sally got the job then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.
$(j \rightarrow \neg l) \equiv (\neg j \rightarrow l)$ - premise
$(\neg j \lor \neg l) \equiv (\neg\neg j \lor l)$ - conditional identities
$(\neg j \lor \neg l) \equiv (j \lor l)$ - double negation rule
$(\neg j \lor \neg l)$ not equal $(j \lor l)$

(d)
If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.
$(r \lor \neg l) \rightarrow j \equiv j \rightarrow (r \land \neg l)$
$(r \lor \neg l) \rightarrow j$ - premise
$\neg(r \lor \neg l) \rightarrow j$ - negation rule
$(\neg r \land \neg\neg l) \rightarrow j$ - demorgans law
$(\neg r \land l) \rightarrow j \equiv j \rightarrow (r \land \neg l)$ - premise
$\neg(\neg r \land l) \lor j \equiv \neg j \lor (r \land \neg l)$ - conditional identities
$(r \lor \neg l) \lor j \equiv \neg j \lor (r \land \neg l)$ - double negation rule
$(r \lor \neg l) \lor j$ not equal $\neg j \lor (r \land \neg l)$

Question 8:
Solve the following questions from the Discrete Math zyBook: Exercise 1.5.2, sections c, f, i

Exercise 1.5.2: Using the laws of logic to prove logical equivalence.
Use the laws of propositional logic to prove the following:

(c) $(\boldsymbol{p} \to \boldsymbol{q}) \wedge (\boldsymbol{p} \to \boldsymbol{r}) \equiv \boldsymbol{p} \to (\boldsymbol{q} \wedge \boldsymbol{r})$

$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$ - premise
$(\neg p \vee q) \wedge (\neg p \vee r)$ - conditional identities
$p \to (q \wedge r)$ - premise
$\neg p \vee (q \wedge r)$ - conditional identities
$(\neg p \vee q) \wedge (\neg p \vee r)$ - distributive laws
$(\boldsymbol{\neg p} \vee \boldsymbol{q}) \wedge (\boldsymbol{\neg p} \vee \boldsymbol{r}) \equiv (\boldsymbol{\neg p} \vee \boldsymbol{q}) \wedge (\boldsymbol{\neg p} \vee \boldsymbol{r})$

(f) $\neg(\boldsymbol{p} \vee (\boldsymbol{\neg p} \wedge \boldsymbol{q})) \equiv -\boldsymbol{p} \wedge \neg \boldsymbol{q}$
$\neg(p \vee (\neg p \wedge q))$ - premise
$\neg p \wedge \neg(\neg p \wedge q)$ - demorgans law
$\neg p \wedge (\neg\neg p \vee \neg q)$ - demorgans law
$\neg p \wedge (p \vee \neg q)$ - double negation rule
$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$ - distributive law
$F \vee (\neg p \wedge \neg q) \equiv \neg p \wedge \neg q$ - complement law
$\boldsymbol{\neg p} \wedge \boldsymbol{\neg q} \equiv \boldsymbol{\neg p} \wedge \boldsymbol{\neg q}$

(i) $(\boldsymbol{p} \wedge \boldsymbol{q}) \to \boldsymbol{r} \equiv (\boldsymbol{p} \wedge \neg \boldsymbol{r}) \to -\boldsymbol{q}$
$\neg(p \wedge q) \vee r \equiv \neg(p \wedge \neg r) \to \neg q$ - negation rule
$\neg p \vee \neg q \vee r \equiv \neg p \vee \neg\neg r \vee \neg q$ - demorgans law
$\neg p \vee \neg q \vee r \equiv \neg p \vee r \vee \neg q$ - double negation rule
$\neg p \vee \neg q \vee r \equiv \neg p \vee \neg q \vee r$ - commutative rule
$\boldsymbol{\neg p} \vee \boldsymbol{\neg q} \vee \boldsymbol{r} \equiv \boldsymbol{\neg p} \vee \boldsymbol{\neg q} \vee \boldsymbol{r}$

Exercise 1.5.3, sections c, d

Use the laws of propositional logic to prove that each statement is a tautology.

(c) $\boldsymbol{r} \vee (\neg \boldsymbol{r} \to \boldsymbol{p})$
$\neg r \vee (\neg r \to p)$ - premise
$\neg r \vee (\neg\neg r \vee p)$ - conditional identities
$\neg r \vee (r \vee p)$ - double negation rule
$r \vee \neg r \vee p$ - commutative law
$T \vee p$ - complement law
True - domination law

(d) $\neg(\boldsymbol{p} \to \boldsymbol{q}) \to -\boldsymbol{q}$
$\neg(p \to q) \to -q$ - premise
$\neg(\neg p \vee q) \to \neg q$ - condition identities
$(\neg\neg p \wedge \neg q) \to \neg q$ - demorgans law
$\neg(\neg\neg p \wedge \neg q) \vee \neg q$ - conditional identities
$\neg(p \wedge \neg q) \vee \neg q$ - double negation rule
$\neg p \vee \neg\neg q \vee \neg q$ - demorgans law
$\neg p \vee q \vee \neg q$ - double negation rule
$\neg p \vee T$ - complement
True - domination law

Question 9:
Solve the following questions from the Discrete Math zyBook: Exercise 1.6.3, sections c, d

Consider the following statements in English. Write a logical expression with the same meaning. The domain is the set of all real numbers.

(c) There is a number that is equal to its square.

$\exists x(x = x^2)$

(d) Every number is less than or equal to its square plus 1.

$\forall x(x \leq x^2 + 1)$

Exercise 1.7.4, sections b - d

In the following question, the domain is a set of employees who work at a company. Ingrid is one of the employees at the company. Define the following predicates:

S(x):x was sick yesterday
W(x):x went to work yesterday
V(x):x was on vacation yesterday

Translate the following English statements into a logical expression with the same meaning.

(b) Everyone was well and went to work yesterday.

$\forall x(\neg S(x) \wedge W(x))$

(c) Everyone who was sick yesterday did not go to work.

$\forall x(S(x) \to \neg W(x))$

(d) Yesterday someone was sick and went to work.

$\exists x(S(x) \wedge W(x))$

Question 10:
Solve the following questions from the Discrete Math zyBook: Exercise 1.7.9, sections c – i

The domain for this question is the set {a,b,c,d,e}. The following table gives the value of predicates P,Q, and R for each element in the domain. For example, Q(c)=T because the truth value in the row labeled c and the column Q is T. Using these values, determine whether each quantified expression evaluates to true or false.

	P(x)	Q(x)	R(x)
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

(c) $\exists \boldsymbol{x}((\boldsymbol{x} = \boldsymbol{c}) \rightarrow \boldsymbol{P}(\boldsymbol{x}))$ **- True**

P(a) = True

(d) $\exists \boldsymbol{x}(\boldsymbol{Q}(\boldsymbol{x}) \wedge \boldsymbol{R}(\boldsymbol{x}))$ **- True**

Q(a) and R(e) = True

(e) $\boldsymbol{Q}(\boldsymbol{a}) \wedge \boldsymbol{P}(\boldsymbol{d})$ **- True**

Q(a) = True and P(d) = True

(f) $\forall \boldsymbol{x}((\boldsymbol{x} \neq \boldsymbol{b}) \rightarrow \boldsymbol{Q}(\boldsymbol{x}))$ **- True**

Q(a,c,d,e) = True

(g) $\forall \boldsymbol{x}(\boldsymbol{P}(\boldsymbol{x}) \vee \boldsymbol{R}(\boldsymbol{x}))$ **- False**

P(c) or R(c) = False

(h) $\forall \boldsymbol{x}(\boldsymbol{R}(\boldsymbol{x}) \rightarrow \boldsymbol{P}(\boldsymbol{x}))$ **- True**

R(a,b,c,d) = false so the conditional is always true
R(e) = true and P(e) = true, so the conditional is true

(i) $\exists \boldsymbol{x}(\boldsymbol{Q}(\boldsymbol{x}) \vee \boldsymbol{R}(\boldsymbol{x}))$ **- True**

Q(e) and R(e) = True

Question 10 (cont.):
Exercise 1.9.2, sections b - i

The tables below show the values of predicates P(x,y),Q(x,y), and S(x,y) for every possible combination of values of the variables x and y. The row number indicates the value for x and the column number indicates the value for y. The domain for x and y is {1,2,3}.

P	1	2	3	Q	1	2	3	S	1	2	3
1	T	F	T	1	F	F	F	1	F	F	F
2	T	F	T	2	T	T	T	2	F	F	F
3	T	T	F	3	T	F	F	3	F	F	F

Indicate whether each of the quantified statements is true or false.
(b) $\exists x \forall y Q(x, y)$ - True

Q(2,y)

(c) $\exists y \forall x P(x, y)$ - True

P(x,1)

(d) $\exists x \exists y S(x, y)$ - False

(e) $\forall x \exists y Q(x, y)$ - False

(f) $\forall x \exists y P(x, y)$ - True

P(1,y), P(2,y), P(3,y)

(g) $\forall x \forall y P(x, y)$ - False

(h) $\exists x \exists y Q(x, y)$ - True

Q(2,2)

(i) $\forall x \forall y \neg S(x, y)$ - True

$S(x, y) =$ True then $\neg S(x, y) =$ False - So the statement is True

Question 11:
Solve the following questions from the Discrete Math zyBook:
1. Exercise 1.10.4, sections c − g

Translate each of the following English statements into logical expressions. The domain is the set of all real numbers.

(c) There are two numbers whose sum is equal to their product.

$\exists x \exists y(x+y=xy)$

(d) The ratio of every two positive numbers is also positive.

$\forall(x,y)(x>0 \land y>0) \rightarrow \frac{x}{y}>0$

(e) The reciprocal of every positive number less than one is greater than one.

$\forall(x)(x>0 \land x<1) \rightarrow \frac{1}{x}>1$

(f) There is no smallest number.

$\forall x \exists y(x>y)$

(g) Every number other than 0 has a multiplicative inverse.

$\forall x\big((x \neq 0) \rightarrow \big(x \times \frac{1}{x}=1\big)\big)$

2. Exercise 1.10.7, sections c − f

The domain is a group working on a project at a company. One of the members of the group is named Sam Define the following predicates.

$P(x{,}y)$:x knows y ‘s phone number. (A person may or may not know their own phone number.)
$D(x)$:x missed the deadline.
$N(x)$:x is a new employee.

Give a logical expression for each of the following sentences.

(c) There is at least one new employee who missed the deadline.

$\exists x(N(x) \land D(x))$

(d) Sam knows the phone number of everyone who missed the deadline.

$\forall x(D(x) \land \text{Sam} \neq y) \rightarrow \forall y\ P(\text{Sam},y)$

(e) There is a new employee who knows everyone’s phone number.

$\exists x(x \neq y \land N(x) \land \forall y P(x,y))$

(f) Exactly one new employee missed the deadline.

$\exists x(N(x) \land D(x)) \land \forall y(y \neq x \rightarrow \neg(D(y)))$

Question 11 (cont.):

3. Exercise 1.10.10, sections c − f

The domain for the first input variable to predicate T is a set of students at a university. The domain for the second input variable to predicate T is the set of Math classes offered at that university. The predicate $T(x,y)$ indicates that student x has taken class y. Sam is a student at the university and Math 101 is one of the courses offered at the university. Give a logical expression for each sentence.

(c) Every student has taken at least one class other than Math 101.

$\forall x \exists y (T(x, y) \land y \neq \text{Math } 101)$

(d) There is a student who has taken every math class other than Math 101.

$\exists x, \forall y (y \neq \text{Math } 101 \rightarrow T(x, y))$

(e) Everyone other than Sam has taken at least two different math classes.

$\forall x (x \neq \text{Sam} \rightarrow \exists y, z (T(x, y) \land T(x, z)))$

(f) Sam has taken exactly two math classes.

$\exists x (x = \text{Sam} \land \exists y, z (T(\text{Sam}, y) \land T(\text{Sam}, z)) \land y \neq z \land \forall c ((c \neq y) \land (c \neq z) \rightarrow \neg T(\text{Sam}, c))$

Question 12:
Solve the following questions from the Discrete Math zyBook:

Exercise 1.8.2, sections b – e

In the following question, the domain is a set of male patients in a clinical study. Define the following predicates:

P(x):x was given the placebo
D(x):x was given the medication
M(x):x had migraines

Translate each statement into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English. Sample question: Some patient was given the placebo and the medication.

- ∃x(P(x)∧D(x))
- Negation: -∃x(P(x)∧D(x))
- Applying De Morgan's law: ∀x(-P(x)∨¬D(x))
- English: Every patient was either not given the placebo or not given the medication (or both).

(b) Every patient was given the medication or the placebo or both.

$\forall x(M(x) \lor P(x) \lor (M(x) \land P(x)))$ - premise
$\neg\forall x(M(x) \lor P(x) \lor (M(x) \land P(x)))$ - negation
$\exists x\neg(M(x) \lor P(x) \lor (M(x) \land P(x)))$ - demorgans law
$\exists x(\neg M(x) \land \neg P(x) \land \neg(M(x) \land P(x)))$ - demorgans law

(c) There is a patient who took the medication and had migraines.

$\exists x(D(x) \land M(x))$ - premise
$\neg\exists x(D(x) \land M(x))$ - negation
$\forall x\neg(D(x) \land M(x))$ - demorgans law
$\forall x(\neg D(x) \lor \neg M(x))$ - demorgans law

(d) Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, p→q≡-p∨q.)

$\forall x(P(x) \to M(x))$ - premise
$\neg\forall x(P(x) \to M(x))$ - negation
$\exists x\neg(P(x) \to M(x))$ - demorgans law
$\exists x\neg(\neg P(x) \lor M(x))$ conditional identities
$\exists x(\neg\neg P(x) \land \neg M(x))$ demorgans law
$\exists x(P(x) \land \neg M(x))$ double negation

(e) There is a patient who had migraines and was given the placebo.

$\exists x(M(x) \land P(x))$ - premise
$\neg\exists x(M(x) \land P(x))$ - negation
$\forall x\neg(M(x) \land P(x))$ - demorgans law
$\forall x(\neg M(x) \lor \neg P(x))$ - demorgans law

Question 12 (cont.):

Exercise 1.9.4, sections c – e

Write the negation of each of the following logical expressions so that all negations immediately precede predicates. In some cases, it may be necessary to apply one or more laws of propositional logic.

(c) ∃x∀y(P(x,y)→Q(x,y))

$\exists x \forall y (P(x,y) \rightarrow Q(x,y))$ - premise
$\neg \exists x \forall y (P(x,y) \rightarrow Q(x,y))$ - negation
$\forall x \exists y \neg (P(x,y) \rightarrow Q(x,y))$ - demorgans law
$\forall x \exists y \neg (\neg P(x,y) \lor Q(x,y))$ - conditional identities
$\forall x \exists y (\neg\neg P(x,y) \land \neg Q(x,y))$ - demorgans law
$\forall x \exists y (P(x,y) \land \neg Q(x,y))$ double negation

(d) ∃x∀y(P(x,y)↔P(y,x))

$\exists x \forall y (P(x,y) \leftrightarrow P(y,x))$ - premise
$\neg \exists x \forall y (P(x,y) \leftrightarrow P(y,x))$ - negation
$\forall x \exists y \neg (P(x,y) \leftrightarrow P(y,x))$ - demorgans law
$\forall x \exists y \neg ((P(x,y) \rightarrow P(y,x)) \land (P(y,x) \rightarrow P(x,y)))$ - conditional identities
$\forall x \exists y \neg ((\neg P(x,y) \lor P(y,x)) \land (\neg P(y,x) \lor P(x,y)))$ - conditional identities
$\forall x \exists y (\neg(\neg P(x,y) \lor P(y,x)) \lor \neg(\neg P(y,x) \lor P(x,y)))$ - demorgans law
$\forall x \exists y ((\neg\neg P(x,y) \land \neg P(y,x)) \lor (\neg\neg P(y,x) \land \neg P(x,y)))$ - demorgans law
$\forall x \exists y ((P(x,y) \land \neg P(y,x)) \lor (P(y,x) \land \neg P(x,y)))$ - double negation

(e) ∃x∃yP(x,y)∧∀x∀yQ(x,y)

$\exists x \exists y P(x,y) \land \forall x \forall y Q(x,y)$ - premise
$\neg(\exists x \exists y P(x,y) \land \forall x \forall y Q(x,y))$ - negation
$(\neg \exists x \exists y P(x,y) \lor \neg \forall x \forall y Q(x,y))$ - demorgans law
$\forall x \forall y \neg P(x,y) \lor \exists x \exists y \neg Q(x,y)$ - demorgans law