

Assignment

Ans 1) Asymptotic means towards infinity (the size of the input is very large). These notations are used to tell the complexity of an algorithm.

Notations :-

1) Big O - Let $f(n) = O(g(n))$ $f(n) \leq c \cdot g(n)$
means $g(n)$ is tight upper bound of $f(n)$, $f(n)$ can never go beyond $g(n)$.

2) Big Omega (Ω) :
 $f(n) = \Omega(g(n))$ $f(n) \geq c \cdot g(n)$
 $g(n)$ is tight lower bound of $f(n)$, $f(n)$ can never perform better than $g(n)$.

3) Theta (Θ) :
Theta gives both 'tight' lower and upper bound.

4) small-O (o) :-
 o gives upper bound (not tight). $f(n) < c \cdot g(n)$

5) small-omega (ω) -
 ω gives lower bound (not tight) $f(n) > c \cdot g(n)$

Ans 2) for ($i=1$ to n)?
 $i = i * 2$
}

Time complexity = $O(\log n)$.

Ans 3) $T(n) = 3T(n-1)$
 $T(1) = 1$

$$T(2) = 3T(n-1) = 3$$

$$T(3) = 3T(2) = 9$$

$$T(4) = 27$$

$$\begin{aligned}\text{Time complexity} &= 3 + 9 + 27 + \dots + (n-1)^3 \\ &= 3^n \\ &= O(3^n).\end{aligned}$$

Ans 4) $T(n) = 2(T(n-1) - 1)$
 $T(n-1) = 2(T(n-2) - 1)$
 $T(n) = 4T(n-2) - 2 - 1$

$$T(n-2) = 2T(n-3) - 1$$

$$T(n) = 8T(n-3) - 4 - 2 - 1$$

$$T(n) = 2^k \dots 2^2 - 2^1 - 2^0$$

$$\text{Time complexity} = O(1)$$

Ans 5)

S	i
1	1
3	2
6	3
10	4

$$\text{Time complexity} = O(\sqrt{n})$$

Ans 6) $i * i = n$
 $i = \sqrt{n}$
 $T(n) = O(\sqrt{n})$

Ans 10) n^k is $O(n^c)$ as -

If $n = 2$ $k = 2$, $c = 2$

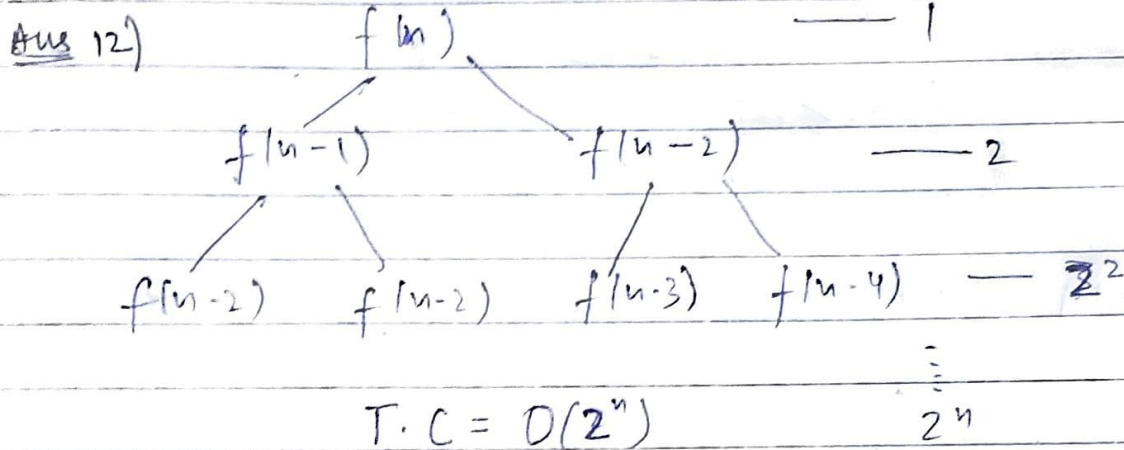
Then $2^2 < 2^2$

So, e^n is the upper limit of n^k .

Ans 11)

$j=1$	$i=0$
1	1
2	3
3	6
4	10

Time Comp = $O(2^n)$



Ans 13) $n \log n$
 for ($i=0$ to n)
 for ($j=0$ to $n : j=j*2$)
 {++;

n^3

for ($i=0$ to n)
 for ($j=0$ to n)
 for ($k=0$ to n)
 {++;

• $\log(\log n)$


```

int func (int n)
{
    if (n == 1)
        return n;
    else
        return func(5n) + func(5n);
}

```

Ans 14) $T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + Cn^2$

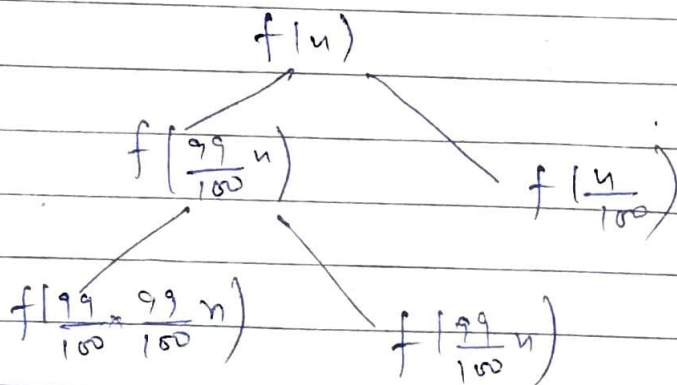
Using Master theorem.

$$a = 2 \quad b = 2 \quad c = 1$$

$$f(n) \geq n^2 \quad f(n^2) \neq 1$$

$$T.C. = O(n^2)$$

Ans 17) $T(n) = T\left(\frac{99}{100}n\right) + T\left(\frac{n}{100}\right)$



$$T.C. = O(\log n)$$

Ans 18.) a) $100 < \log(\log n) < \log n < \sqrt{n} < n < \log(n!)$
 $< n \log n < n^2 < 2^n < 2^{2n} < 4^n < n!$

b) $1 < \log(\log n) < \sqrt{\log n} < \log 2^n < \log n < 2 \log(n) < n \log(n) < n < 2n < 4n < n^2 < \log(n!) < 2 \cdot (2^n) < n!$

$$e) 96 < \log_2 n < \log_8 n < \log_5 n < \log n! < n \log_6 (n) < n \log_2 (n) \\ < 10n^2 < 7n^3 < 8^{2n} < n!$$

Ans 19) linear (arr, key)

```
{
    for (int i=0 ; i<n ; i++)
        if (arr[i] == key)
            return i;
    return -1;
}
```

<u>Ans 21)</u>	Algo	Best	Avg	Worst	Space. Com
	Bubble	$O(n^2)$	$O(n^2)$	$O(n^2)$	1
	Selection	$O(n^2)$	$O(n^2)$	$O(n^2)$	1
	Insertion	$O(n^2)$	$O(n^2)$	$O(n^2)$	1

Ans 24) $T(n) = T\left(\frac{n}{2}\right) + 1$