Financial Markets and Fluctuations in Uncertainty

Cristina Arellano, Yan Bai, and Patrick Kehoe

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Motivation

Recent recession

- Output and labor drop, accounted for
  - Mainly by a worsening of labor wedge
  - Less by a fall in TFP

$\Rightarrow$ "Labor-wedge driven recession"

Popular story

- Increase in "uncertainty" at firm level
- Interacts with financial frictions

$\Rightarrow$ Firms shrink level of employment

This paper

Our goal

- Build a model to formalize gist of popular story
- Generate a labor-wedge driven recession quantitatively

Our formalization

- "Uncertainty shock"
  - Model as increase in volatility of firm idiosyncratic shocks
  - Quantify increase using dispersion of firms' growth rate
- Financial frictions
  - Model as uncontingent debt; allow costly default

Question

Can an increase in volatility of firms' idiosyncratic shocks that generates observed increase in firms' dispersion deliver

- Large contraction in output?
  - Yes: 67% of output drop
- Worsening of labor wedge?
  - Yes: 41% of labor wedge worsening

Key Elements in Model

- Firms produce before knowing current idiosyncratic demand shock
  - In high states 'too small' and in low states 'too big'
- Firms have limited ability to insure idiosyncratic shock
  - If scale too big, can't pay wage bill and might default
- Costly default
  - Liquidated, so lose future profits that are covering entry cost

$\Longrightarrow$ Labor wedge

- Risk of default create a wedge between MPL and wage

Volatility shock generates labor-wedge driven recession

- Increase in volatility
  - Increases risk of default for a given scale
  - Induces firms to choose smaller scale
  - So increases wedge between MPL and wage

Literature

- Uncertainty shocks important for aggregates
  - Bloom (2008), Bloom, Floetotto, and Jaimovich (2010), Bachmann and Bayer (2009), Christiano, Motto, and Rostagno (2009)
- Firm heterogeneity and financial frictions
  - Cooley and Quadrini (2001), Gilchrist, Sim, and Zakrajsek (2010), Thomas and Khan (2011)
- Financial shocks
  - Jermann and Quadrini (2012), Guerrieri and Lorenzoni (2011)

Simple Example

Simple Example

Two points

- Complete financial markets
  - Constant labor wedge
  - Increased volatility $\longrightarrow$ no effect on output or labor wedge
- Incomplete financial markets
  - Varying labor wedge
  - Increased volatility $\longrightarrow$ output declines, labor wedge worsens

Simple Example

- Period 1:
  - Firms hire labor and produce before the demand shock z
  - Demand shock is realized; firms choose price p given demand function

$$y^d(p; z) = (z/p)^\gamma Y$$

  - Firms are liquidated if dividend is negative
- Period 2:
  - Firms get future value V only if not liquidated

Complete financial markets

- End of period 1, given ℓ, firms choose price p

$$\pi(z;\ell) = \max_p \left\{ p y^d(p;z) - w\ell \right\}$$

$$y^d(p;z) \leq \ell^\theta$$

- Optimal to set $y^d(p;z) = \ell^\theta$ when $\gamma \geq 1$
- Price $p(z) = zY^{1/\gamma}\ell^{-\theta/\gamma}$

Complete financial markets

- Firms choose ℓ to maximize the expected value

$$\max_{\ell} \int_0^\infty \left[p(z)\ell^\theta - w\ell + V \right] f(z)dz$$

- Optimal scale chosen to maximize short term profits

$$\underbrace{Ep(z)\theta\ell^{\theta-1}}_{\text{value } MPL} = \underbrace{\frac{\gamma}{(\gamma-1)}}_{\text{constant labor wedge}} w$$

- Use state-contingent debt to pay dividends and avoid liquidation

- Increased volatility $\longrightarrow$ no effect on output or labor wedge

Incomplete financial markets

- Firms are liquidated when demand shocks are low $(z < \widehat{z})$
  - For each ℓ, $\widehat{z}$ is lowest z s.t. $p(z)\ell^{\theta} \geq w\ell$
- Firms choose $(\ell, \widehat{z})$ to maximize the expected value

$$\max_{\ell, \widehat{z}} \int_{\widehat{z}}^{\infty} \left[p(z)\ell^{\theta} - w\ell \right] f(z)dz + \int_{\widehat{z}}^{\infty} Vf(z)dz$$

s.t

$$p(\widehat{z})\ell^{\theta} - w\ell = 0$$

- Optimal scale chosen to maximize short term profits *and* future value

$$\underbrace{E[p(z)|z \geq \widehat{z}]\theta\ell^{\theta-1}}_{MPL} = \frac{\gamma}{(\gamma - 1)} \left[w + \underbrace{V \frac{f(\hat{z})}{1 - F(\hat{z})} \frac{d\hat{z}}{d\ell}}_{\text{Wedge}} \right]$$

- Increased volatility reduces labor and output and worsens labor wedge

Model

Our model

Dynamic general equilibrium model with

- Households (standard)
  - Provide labor
  - Sell uncontingent debt to firms
  - Own firms
- Final goods firms
  - Aggregate intermediate goods with CES aggregator
- Firms

Final Goods Firms

- CES aggregator across goods x from measure of firms Υ

$$Y = \left(\int z(x) y(x)^{\frac{\gamma-1}{\gamma}} d\Upsilon(x) \right)^{\frac{\gamma}{\gamma-1}}$$

- Yields a demand function

$$y(x) = \left(\frac{z(x)}{p(x)} \right)^{\gamma} Y$$

- Demand shocks
    - Idiosyncratic shocks z with common stochastic volatility σ
    - Markov processes: $\pi_z(z_t | z_{t-1}, \sigma_{t-1})$ and $\pi_\sigma(\sigma_t | \sigma_{t-1})$

Firms

- Hire labor and produce $y = \ell^\theta$ before demand shock z
- Issue uncontingent debt b and can default on it
- Costly default
  - Pay a fixed cost ξ to start a business –> profits after entry are positive
  - If default, liquidated so lose positive PV of profits
- Dividends non-negative
- Aggregate state: $S = (\sigma, \Upsilon)$, Υ is measure of firms over (ℓ, b, z)

Firms

- Maximize discounted value of dividends

$$d = p(z)\ell^{\theta} - w\ell - b + q(\ell', b'|z, S)b' \geq 0$$

- Firms with high debt must default and set $\phi = 0$
- Generates bond price schedule $q(\ell', b'|z, S)$
  - Compensates for default risk
  - Different for each choice of ℓ' and b'
  - Implies borrowing limits $B(z, S) = \max_{\ell', b'} q(\ell', b'|z, S)b'$

Firms' problem

$$V(\ell, b, z, S) = \max_{\{d,p,b',\ell'\}} d + \delta \sum_{z',\sigma'} Q\left(\sigma'|S\right) \pi_z(z'|z,\sigma) V(\ell', b', z', S')$$

$$d = p\ell^\theta - w\ell - b + q(\ell', b'|z, S)b' \geq 0$$

$$(z/p)^\gamma Y = \ell^\theta$$

$$\mathrm{Y}' = G(S)$$

- Firms discount future more than consumer ($\delta < 1$)
  - Lower incentive for firms to self-insure
  - Reduced form: tax benefit of debt, other reasons why firms hold debt
  - Discipline quantitatively with average debt/sales

Firm Entry

- New entrants

$$V^e(S) = \max_{\ell'_e} \; -\xi + \delta \sum_{z',\sigma'} Q(\sigma'|S)\pi_z^e(z'|\sigma)V'(\ell'_e, 0, z', S')$$

Enter if and only if $V^e(S) \geq 0$

- Free entry condition implies positive expected value after entry

$$\delta \sum_{z',\sigma'} Q(\sigma'|S)\pi_z^e(z'|\sigma)V'(\ell'_e, 0, z', S') = \xi > 0$$

  - Cost of default: Firm exits so loses expected value of future profits

- The measure of firms is time-varying

Bond Price

- Compensates intermediaries for the loss in default

$$q(\ell', b'|z, S)b' = \sum_{z',\sigma'} Q\left(\sigma\prime|S\right)\pi_z(z'|z,\sigma)\phi(\ell', b', z', S')b\prime$$

- Firms maintain a *buffer stock* of potential funds

$$B(z, S) - q(\ell', b'|z, S)b'$$

Households

- Provide employment at the beginning of period and consumption and assets after shocks

$$V_t^H = \max_{L_t} \left\{ \sum_{\sigma_t} \pi_\sigma(\sigma|\sigma_{-1}) \max_{C,\{A'(\sigma\prime)\}} \left[U(C,L) + \beta V_{t+1}^H \right] \right\}$$

subject to their budget constraint

$$C + \sum_{\sigma'} Q(\sigma'|S)A'(\sigma') = wL + A(\sigma) + D - T$$

Experiments and Results

Quantifying volatility shocks

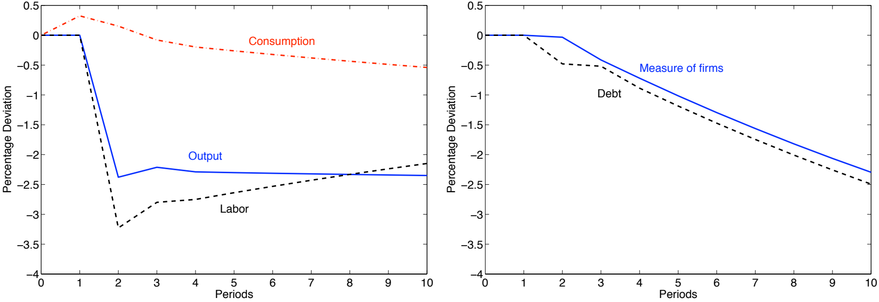
- Use cross-section firm dispersion to parameterize volatility shocks
- Firm dispersion:
  - Interquartile range of sales growth (differences between 75% and 25%)
- Parameter values: $\rho_z = 0.70$, $\mu_\sigma = 0.18$, $\rho_\sigma = 0.85$

Other parameters

$$u(c,h) = \frac{C^{1-\rho}}{1-\rho} - \chi \frac{L^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

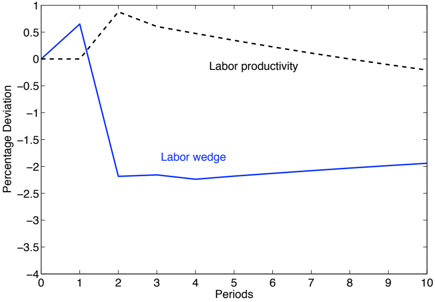
Labor elasticity	$\nu = 2$	Rogerson and Wallenius (2009)
Labor share	$\theta = 0.70$	U.S. National Accounts
Risk aversion	$\rho = 2$	Common value
Markup	$\gamma/(\gamma - 1) = 1.15$	Basu and Fernald (1997)
Discount for HH	$\beta = 0.99$	Interest rate 1%
Entry costs	$\xi/\overline{y} = 0.32$	BLS, entrants labor/total labor=1.7%
Death Shock	$\pi(z = z_0) = 2.5\%$	U.S. failure rates
Discount for firms	$\delta = 0.7$	liability/sales ratio (Compustat)

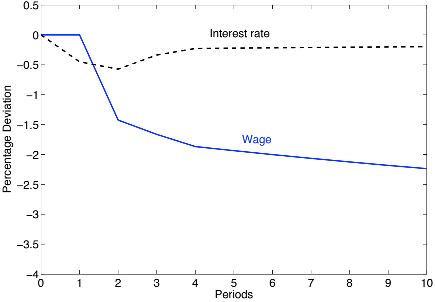
Aggregate impulse response to high volatility



- Labor falls more than output, measure of firms and debt fall

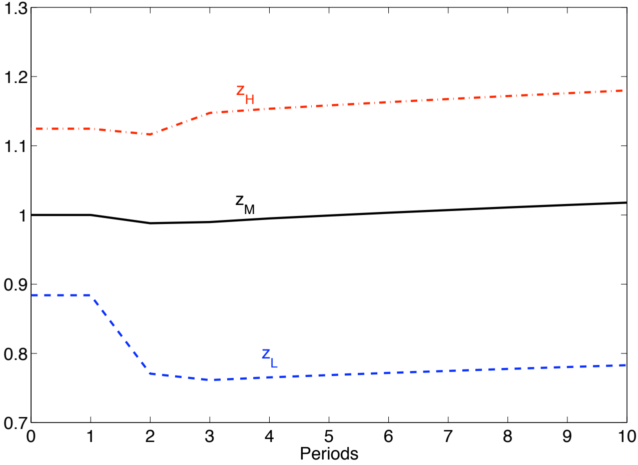
Aggregate impulse response to high volatility





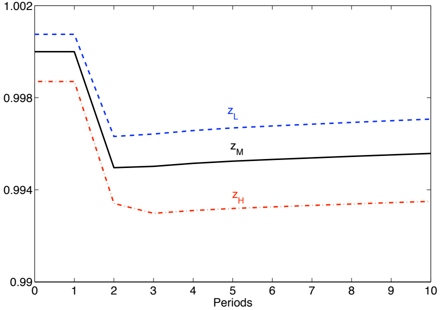
- Labor wedge falls; productivity unchanged
- Wage falls; interest rate falls

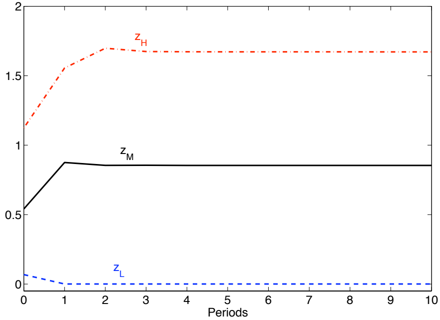
Firm employment response to high volatility



- z_M firms decreases employment
- z_L decreases a lot; z_H increases

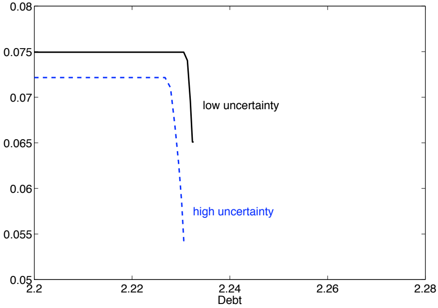
Firm debt response to high volatility

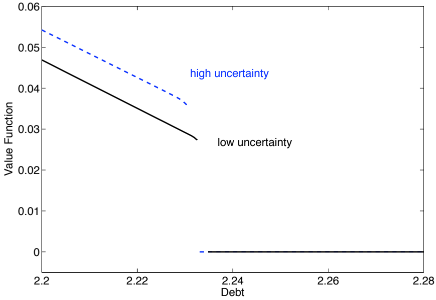




- Firm debt falls, buffer stock rises for most firms

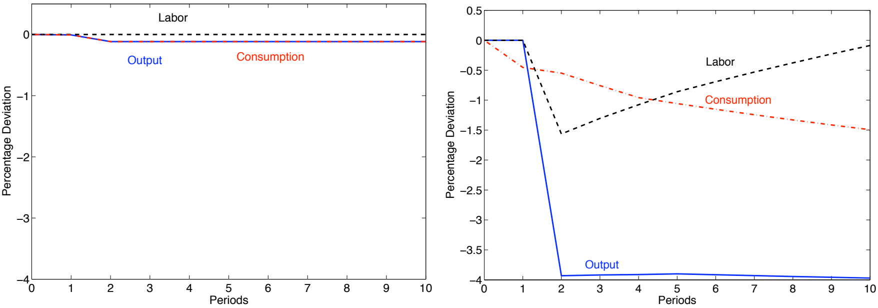
Firm employment and value functions





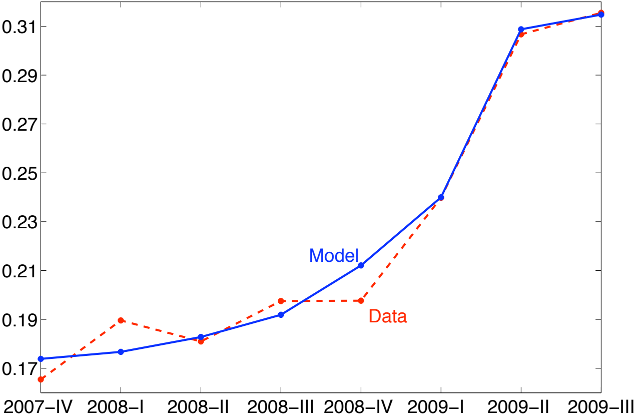
- Firms with high debt choose lower employment
- Default is due to liquidity problems

Aggregate impulse: Two reference models



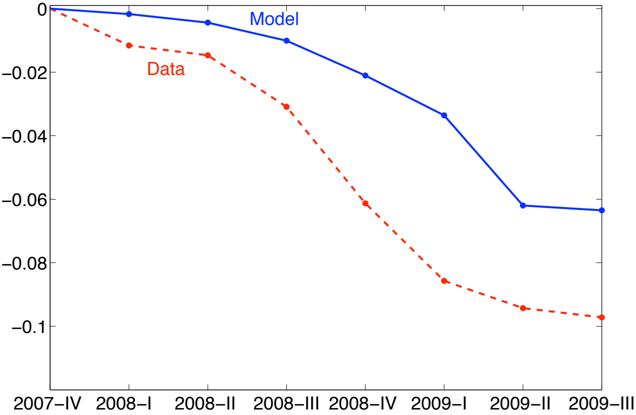
- Financial frictions are essential
- Labor wedge results from financial frictions & volatility shocks

Experiment



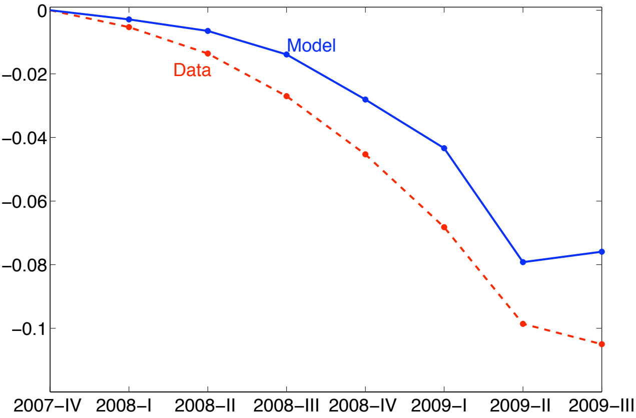
- Choose sequence of σ_t to match observed IQR sales growth

Output



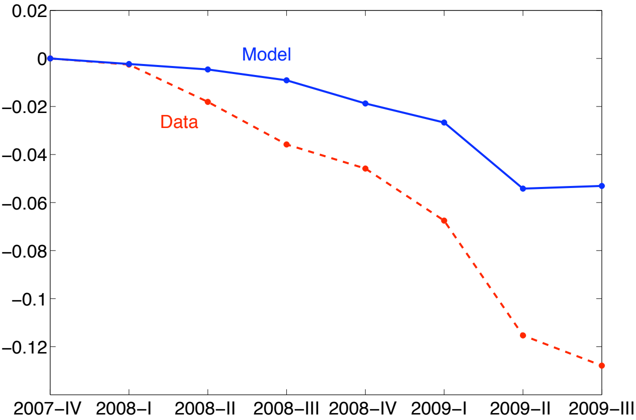
- Model output matches 67% of the output decline

Labor



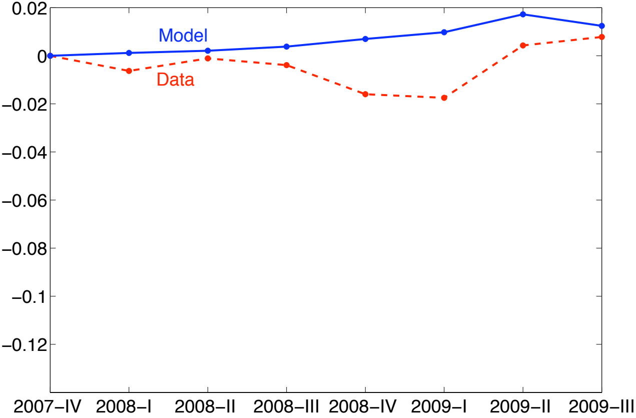
- Model labor matches 73% of the labor decline

Labor wedge



- Model can account for 41% of the worsening in the labor wedge

Productivity



- Abstract from TFP variation

Business Cycles

	Data		Model	
	std(x)	$\frac{std(x)}{std(GDP)}$	Std(x)	$\frac{std(x)}{std(GDP)}$
GDP	3.2		2.4	
Labor	4.1	1.27	3.1	1.26
Consumption	2.7	0.83	1.2	0.48
Labor Wedge	5.4	1.69	2.3	0.95

- Volatility shocks can account for:
  - 75% of the variability of labor relative to output
  - 60% of the variability in the labor wedge relative to output

Conclusion

- Framework that combines volatility shocks with financial markets imperfections
- Generates movements in output, labor, and the labor wedge linked to financial frictions