Week 05 Participation Assignment (1 of 2)

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Contents

1 Part 1

When we use the method of Undetermined Coefficients to find a participation solution for a given differential equation (non-homogeneous), it is limited to the case that $f(x)$ is in the form of sine function, cosine function, exponential function, polynomial and product/sum of the functions mentioned. This method won't work if $f(x) = \sec(3x)$, for example, $y'' + 9y = 2\sec(3x)$. Then we will use another method called Variation of Parameter.

Let's use this method to solve the differential equation $y'' + 9y = 2\sec(3x)$

1.1 1

$$\begin{aligned}
y'' + 9y &= 2\sec(3x) \\
r^2 + 9 &= 0 \\
r &= 0 \pm 3i \\
y(x) &= c_1\cos(3x) + c_2\sin(3x) \\
y_1(x) &= \cos(3x) \\
y_1'(x) &= -3\sin(3x) \\
y_2(x) &= \sin(3x) \\
y_2'(x) &= 3\cos(3x)
\end{aligned}$$

$$\begin{aligned}
W &= \begin{vmatrix} \cos(3x) & \sin(3x) \\ -3\sin(3x) & 3\cos(3x) \end{vmatrix} \\
&= \cos(3x)\cdot 3\cos(3x) - \sin(3x)\cdot -3\sin(3x) \\
W &= 3\cos^2(3x) + 3\sin^2(3x) \\
W &= 3\left(\cos^2(3x) + \sin^2(3x)\right) \\
W &= 3
\end{aligned}$$

$$\begin{aligned}
W_1 &= \begin{vmatrix} 0 & \sin(3x) \\ 2\sec(3x) & 3\cos(3x) \end{vmatrix} \\
W_1 &= -2\tan(3x)
\end{aligned}$$

$$\begin{aligned}
W_2 &= \begin{vmatrix} \cos(3x) & 0 \\ -3\sin(3x) & 2\sec(3x) \end{vmatrix} \\
W_2 &= 2
\end{aligned}$$

$$\begin{aligned}
\int \frac{W_1}{W} dx &= \int \frac{-2\tan(3x)}{3} dx \\
&= -\frac{2}{3} \int \frac{\sin(3x)}{\cos(3x)} dx \\
&= \frac{2}{3 \cdot 3} \int \frac{1}{u} du \\
\int \frac{W_1}{W} dx &= \frac{2}{9} \ln(\cos(3x)) + C_3
\end{aligned}$$

$$\begin{aligned}
\int \frac{W_2}{W} dx &= \int \frac{2}{3} dx \\
\int \frac{W_2}{W} dx &= \frac{2x}{3} + C_4
\end{aligned}$$

$$\begin{aligned}
y(x) &= \left(\frac{2}{9} \ln(\cos(3x)) + C_3\right) \cos(3x) + \left(\frac{2x}{3} + C_4\right) \sin(3x) \\
y(x) &= \frac{2}{9} \ln(\cos(3x)) \cos(3x) + C_3 \cos(3x) + \frac{2}{3} x \sin(3x) + C_4 \sin(3x)
\end{aligned}$$

$$\boxed{y(x) = \frac{2}{9} \ln(\cos(3x)) \cos(3x) + C_3 \cos(3x) + \frac{2}{3} x \sin(3x) + C_4 \sin(3x)}$$