Week 12 and Week 13 Participation Assignment (1 of 3)

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Contents

1 Part 1

a) Write the given system in the matrix form $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$. Then identify whether it is a homogeneous or non-homogeneous equation.

1) $\begin{cases} x'(t) = \ 3x(t) - y(t) + t^2 \\ y'(t) = -x(t) + 2y(t) + e^t \end{cases}$

2) $\begin{cases} \frac{dx}{dt} = \ x + y + z \\ \frac{dy}{dt} = 2x - y + 3z \\ \frac{dz}{dt} = \ \ x + 5z \end{cases}$

b) Rewrite the given scalar equation as a first order system and then express the system in the matrix form $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}$.

$$y''(t) - 3y'(t) - 10y(t) = \sin(t)$$

1.1 a)

1)

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} t^2 \\ e^t \end{bmatrix}$$

The system can also be expressed similarly if we let $\mathbf{x} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ and $\mathbf{x}' = \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix}$.

$$\mathbf{x}' = \boldsymbol{A}\mathbf{x} + \mathbf{f}$$
$$\mathbf{x}' = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} t^2 \\ e^t \end{bmatrix}$$

2)

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 0 & 5 \end{bmatrix}\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} + 0$$

If we let $\mathbf{x} = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$ and $\mathbf{x}' = \begin{bmatrix} x'(t) \\ y'(t) \\ z'(t) \end{bmatrix}$:

$$\mathbf{x} = \boldsymbol{A}\mathbf{x} + \mathbf{f}$$
$$\mathbf{x}' = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 0 & 5 \end{bmatrix} \mathbf{x}$$

1.2 b)

$$y''(t) - 3y'(t) - 10y(t) = \sin(t)$$

Let $x_0 = y'(t), x_1 = y''(t)$; $\therefore x_1' = y''(t)$. A linear system of ODE can now be constructed.

$$\begin{aligned} y''(t) &= 3y'(t) + 10y(t) + \sin(t) \\ x_1 &= 3x_0 + 10y + \sin(t) \\ x_0 &= y'(t) \end{aligned}$$

$$\frac{d}{dt} \begin{bmatrix} y \\ x_0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 10 & 3 \end{bmatrix} \begin{bmatrix} y \\ x_0 \end{bmatrix} + \begin{bmatrix} \sin(t) \\ \sin(t) \end{bmatrix}$$