Contents

1 Chapter 18 - Thermal Properties of Matter

Avogadro's number

$$N_A = 6.02 \times 10^{23}\,\mathrm{mol} \tag{1}$$

1.1 The Ideal Gas Law

Ideal gas: a collection of atoms or molecules that move randomly and exert no long-range forces on each other.

Number of moles

$$n = \frac{N}{N_A} = \frac{m_{particle}N}{m_{particle}N_A} = \frac{m}{M} \tag{2}$$

The **molar mass** M **(molecular weight)** is the mass per mole. The total mass of n moles is $m_{total} = nM$.

Ideal-gas equation

$$pV = nRT \tag{3}$$

Universal gas constant

$$R = 8.31\,\mathrm{J\,mol^{-1}\,K^{-1}} = 0.0821\,\mathrm{L\,atm\,mol^{-1}\,K^{-1}} \tag{4}$$

The volume occupied by 1 mol of any ideal gas at atmospheric pressure and at 0 °C is 22.4 L.

1.1.1 Question

$$
\begin{aligned}
V &= 22.4 \times 10^{-3}\,\mathrm{L} \\
T &= 273.15\,\mathrm{K} \\
p &= 1.013 \times 10^{5}\,\mathrm{Pa} = 1.0\,\mathrm{atm} \\
n &= ?
\end{aligned}
$$

$$
\begin{aligned}
pV &= nRT \\
n &= \frac{pV}{RT} \\
n &= \frac{(1.0\ \mathrm{atm})(22.4\,\mathrm{L})}{(0.0821\,\mathrm{L\,atm\,mol^{-1}\,K^{-1}})(273.15\,\mathrm{K})} \\
n &= 1.000\,\mathrm{mol}
\end{aligned}
$$

1.1.2 18.3

$$
\begin{aligned}
V_0 &= 0.110\,\mathrm{m^3} \\
p_0 &= 0.355\,\mathrm{atm} \\
V_1 &= 0.390\,\mathrm{m^3} \\
T &= \text{constant} \\
p_1 &= ?
\end{aligned}
$$

$$
\begin{aligned}
p_0 V_0 &= p_1 V_1 \\
p_1 &= \frac{p_0 V_0}{V_1} \\
p_1 &= \frac{(0.355\,\mathrm{atm})(0.110\,\mathrm{m^3})}{0.390\,\mathrm{m^3}} \\
p_1 &= 0.1001\,\mathrm{atm}
\end{aligned}
$$

1.1.3 18.4

$$
\begin{aligned}
V_0 &= 3.00\,\mathrm{L} \\
p_0 &= 3.00\,\mathrm{atm} \\
T_0 &= 20.0\,^{\circ}\mathrm{C} = 293\,\mathrm{K} \\
p_1 &= 1.00\,\mathrm{atm}
\end{aligned}
$$

(a)

$$
\begin{aligned}
pV &= nRT \\
\frac{p}{T} &= \frac{nR}{V} \\
\frac{p_0}{T_0} &= \frac{p_1}{T_1} \\
T_1 &= \frac{p_1 T_0}{p_0} \\
T_1 &= \frac{(1.00\,\text{atm})(293\,\text{K})}{3.00\,\text{atm}} \\
T_1 &= 97.7\,\text{K} = -175.3\,^\circ\text{C}
\end{aligned}
$$

1.1.4 18.7

$$
\begin{aligned}
V_0 &= 499\,\text{cm}^3 = 499 \times 10^{-6}\,\text{m}^3 \\
p_0 &= 1.01 \times 10^5\,\text{Pa} \\
T_0 &= 27.0\,^\circ\text{C} = 300\,\text{K} \\
V_1 &= 46.2\,\text{cm}^3 = 46.2 \times 10^{-6}\,\text{m}^3 \\
p_1 &= 2.72 \times 10^6\,\text{Pa} + 1\,\text{atm} = 2.821 \times 10^6\,\text{Pa} \\
T_1 &= ?
\end{aligned}
$$

$$
\begin{aligned}
pV &= nR\Delta T \\
\frac{p_0 V_0}{T_0} &= \frac{p_1 V_1}{T_1} \\
T_1 &= \frac{T_0 p_1 V_1}{p_0 V_0} \\
T_1 &= \frac{(300\,\text{K})(2.821 \times 10^6\,\text{Pa})(46.2 \times 10^{-6}\,\text{m}^3)}{(1.01 \times 10^5\,\text{Pa})(499 \times 10^{-6}\,\text{m}^3)} \\
T_1 &= 755.79\,\text{K}
\end{aligned}
$$

1.1.5 18.13

$$
\begin{aligned}
p_0 &= 1\,\text{atm} V_0 & &= V_{earth} \\
V_1 &= V_{venus} \\
T_1 &= 1003\,^\circ\text{C} = 1276\,\text{K} \\
p_1 &= 92\,\text{atm} \\
T_0 &= 273\,\text{K}
\end{aligned}
$$

$$
\begin{aligned}
pV &= nR\Delta T \\
\frac{p_0 V_0}{T_0} &= \frac{p_1 V_1}{T_1} \\
V_1 &= \frac{T_1 p_0}{T_0 p_1} V_0 \\
V_1 &= \frac{(1276\,\mathrm{K})(1\,\mathrm{atm})}{(273\,\mathrm{K})(92\,\mathrm{atm}} \\
V_1 &= (0.051) V_0
\end{aligned}
$$

1.1.6 18.16

$$
\begin{aligned}
n &= 3\,\mathrm{mol} \\
l &= 0.300\,\mathrm{m}
\end{aligned}
$$

(a)

$$
T = 20.0\,^\circ\mathrm{C} = 293\,\mathrm{K}
$$

$$
\begin{aligned}
F &= pA \\
F &= \frac{nRTA}{V} \\
F &= \frac{(3\,\mathrm{mol})(8.31\,\mathrm{J\,mol^{-1}\,K^{-1}})(293\,\mathrm{K})(0.300\,\mathrm{m})^2}{(0.300\,\mathrm{m})^3} \\
F &= 24\,348.3\,\mathrm{N} = 2.43 \times 10^4\,\mathrm{N}
\end{aligned}
$$

(b)

$$
T = 100.0\,^\circ\mathrm{C} = 373\,\mathrm{K}
$$

$$
\begin{aligned}
F &= \frac{nRTA}{V} \\
F &= \frac{(3\,\mathrm{mol})(8.31\,\mathrm{J\,mol^{-1}\,K^{-1}})(373\,\mathrm{K})(0.300\,\mathrm{m})^2}{(0.300\,\mathrm{m})^3} \\
F &= 30\,996.3\,\mathrm{N} = 3.10 \times 10^4\,\mathrm{N}
\end{aligned}
$$

1.1.7 18.18

$$
\begin{aligned}
\Delta y &= 11\,000\,\mathrm{m} \\
T &= -56.5\,^\circ\mathrm{C} = 216.5\,\mathrm{K} \\
\rho &= 0.364\,\mathrm{kg\,m^{-3}} \\
p &= ?
\end{aligned}
$$

$$\begin{aligned}
\rho &= \frac{m}{V} \\
m &= \rho V \\
n &= \frac{m}{M} \\
n &= \frac{\rho V}{M}
\end{aligned}$$

$$\begin{aligned}
pV &= nRT \\
pV &= \left(\frac{\rho V}{M}\right) RT \\
p &= \frac{\rho RT}{M} \\
p &= \frac{(0.364\,\mathrm{kg\,m^{-3}})(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})(216.5\,\mathrm{K})}{28.8 \times 10^{-3}\,\mathrm{kg\,mol^{-1}}} \\
p &= 22\,749.8\,\mathrm{Pa} = 2.27 \times 10^{4}\,\mathrm{Pa}
\end{aligned}$$

1.1.8 Question

$$\begin{aligned}
T &= 0.00\,^{\circ}\mathrm{C} = 273\,\mathrm{K} \\
g &= 9.80\,\mathrm{m\,s^{-2}}
\end{aligned}$$

$$\begin{aligned}
\frac{dp}{dy} &= -\rho g \\
p &= \rho RT \\
\rho &= \frac{p}{RT} \\
\frac{dp}{dy} &= -\left(\frac{p}{RT}\right) g \\
\frac{dp}{dy} &= -\frac{pg}{RT} \\
p' &= -p \cdot \frac{g}{RT} \\
\mathcal{L}\left\{p'\right\} + \mathcal{L}\left\{p\right\} &= \frac{g}{RT} \\
sF(s) - f(0) + F(s) &= \frac{g}{RT} \\
F(s)(s-1) &= \frac{g}{RT} \\
F(s) &= \frac{g}{RT} \cdot \frac{1}{s-1} \\
s &= \frac{ge^{t}}{RT}
\end{aligned}$$

2 Molecules and Intermolecular Forces

2.1 The Van Der Waals Equation

The model used for the ideal-gas equation ignores the volumes of molecules and the attractive forces between them.

$$\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT \tag{5}$$

2.2 Kinetic-Molecular Model of an Ideal Gas

1. Molecules are in constant motion and undergo perfectly elastic collisions.

2.3 Collisions and Gas Pressure

$$\begin{aligned} \Delta\mathbf{P_y} &= m\Delta\mathbf{v_y} = 0 \\ \Delta\mathbf{P_x} &= m\Delta\mathbf{v_x} = 2m\mathbf{v_x} \end{aligned}$$

The amount of molecules per volume that collide with a given wall area A in a time interval dt:

$$\begin{aligned} V &= Ah \\ \Delta\mathbf{x} &= v_{0_y}t + \frac{1}{2}a_y\Delta t^2 \\ \Delta\mathbf{x} &= v_{0_y}t \\ V &= A|v_x|dt \end{aligned}$$

$$\frac{1}{2}\left(\frac{N}{V}\right)(A|v_x|dt)$$

For all molecules in the gas, the total momentum change dP_x during dt is the number of collisions multiplied by the momentum change.

$$\begin{aligned} dP_x &= \frac{1}{2}\left(\frac{N}{V}\right)(A|v_x|dt)(2m|v_x|) \\ &= \frac{NAmv_x^2dt}{V} \end{aligned}$$

The rate of change of momentum component P_x:

$$\frac{dP_x}{dt} = \frac{NAmv_x^2}{V}$$

Pressure:

$$p = \frac{F}{A} = \frac{m\frac{\Delta\mathbf{p}}{\Delta t}}{A} = \frac{\frac{NAmv_x^2}{V}}{A} = \frac{Nmv_x^2}{V} \tag{6}$$

2.3.1 Pressure and Molecular Kinetic Energies

The speed v of a molecule is related to the velocity components by

$$v^2 = v_x^2 + v_y^2 + v_z^2 \tag{7}$$
$$(v^2)_{av} = (v_x^2)_{av} + (v_y^2)_{av} + (v_z^2)_{av} \tag{8}$$

As there is no real difference in our model between directions:

$$(v_x^2)_{av} = \frac{1}{3}(v^2)_{av} \tag{9}$$

$$\begin{aligned}
p &= \frac{Nmv_x^2}{V} \\
pV &= nRT \\
\left[\frac{Nmv_x^2}{V}\right] V &= nRT \\
\frac{N}{3}(2K) &= nRT, \quad KE = \frac{1}{2}mv^2, n = \frac{m}{M} = \frac{N}{N_A} \\
K &= \frac{3}{2}nRT
\end{aligned}$$

$$\begin{aligned}
pV &= nRT \\
nV &= \frac{N}{N_A}RT \\
pV &= Nk_BT
\end{aligned}$$

2.3.2 18.24

$$\begin{aligned}
v_{rmt} &= 2\sqrt{\frac{3RT}{m}} \\
v_{rmt} &= \sqrt{\frac{3R}{m} \cdot 4T}
\end{aligned}$$

$$\begin{aligned}
p_1V &= nRT_1 \\
p_1 &= 2p_2 \\
2p_2V &= n_2R(4T_1) \\
n_2 &= \frac{2}{4} \cdot \frac{p_1V}{RT_1} \\
n_2 &= \frac{1}{2}n_1
\end{aligned}$$

2.3.3 18.28

$$
\begin{aligned}
V &= 1.64\,\mathrm{L} \\
m &= 0.226\,\mathrm{kg} \\
v_{rms} &= 182\,\mathrm{m\,s^{-1}} \\
p &= ?
\end{aligned}
$$

$$
\begin{aligned}
v_{rms} &= \sqrt{\frac{3RT}{M}} \\
v_{rms}^2 &= \frac{3RT}{M} \\
RT &= \frac{v_{rms}^2 M}{3}
\end{aligned}
$$

$$
\begin{aligned}
pV &= nRT \\
pV &= n\left(\frac{v_{rms}^2 M}{3}\right) \\
p &= \frac{\frac{m}{M}\left(\frac{v_{rms}^2 M}{3}\right)}{V} \\
p &= \frac{m v_{rms}^2}{3V} \\
p &= \frac{(0.226\,\mathrm{kg})(182\,\mathrm{m\,s^{-1}})^2}{3(1.64\,\mathrm{L})} \\
p &= 1521.55\,\mathrm{Pa}
\end{aligned}
$$

2.3.4 18.30

$$
\begin{aligned}
M_{mars} &= 44.0\,\mathrm{g\,mol^{-1}} = 0.044\,\mathrm{kg\,mol^{-1}} \\
P_{mars} &= 650\,\mathrm{Pa} \\
T_0 &= 0.0\,^{\circ}\mathrm{C} = 273\,\mathrm{K} \\
T_1 &= -100.0\,^{\circ}\mathrm{C} = 173\,\mathrm{K}
\end{aligned}
$$

(a)

$$
\begin{aligned}
v_{rms} &= \sqrt{\frac{3RT}{M}} \\
v_{rms} &= \sqrt{\frac{3(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})(273\,\mathrm{K})}{0.044\,\mathrm{kg\,mol^{-1}}}} \\
v_{rms} &= 393.4\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$
$$v_{rms} = \sqrt{\frac{3(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})(173\,\mathrm{K})}{0.044\,\mathrm{kg\,mol^{-1}}}}$$
$$v_{rms} = 313.2\,\mathrm{m\,s^{-1}}$$

(b)

$$pV = nRT$$
$$pV = \left(\frac{m}{M}\right) RT$$
$$m = \frac{pVM}{RT}$$
$$\rho = \frac{m}{V}$$
$$\rho = \frac{\frac{pVM}{RT}}{V}$$
$$\rho = \frac{pM}{RT}$$

$$\rho_0 = \frac{(650\,\mathrm{Pa})(0.044\,\mathrm{kg\,mol^{-1}})}{(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})(273\,\mathrm{K})}$$
$$\rho_0 = 0.0126\,\mathrm{kg\,m^{-3}} = 12.6\,\mathrm{g\,m^{-3}} \cdot \frac{1}{44.0\,\mathrm{g\,mol^{-1}}} = 0.286\,\mathrm{mol\,m^{-3}}$$

$$\rho_1 = \frac{(650\,\mathrm{Pa})(0.044\,\mathrm{kg\,mol^{-1}})}{(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})(173\,\mathrm{K})}$$
$$\rho_1 = 0.0199\,\mathrm{kg\,m^{-3}} = 19.9\,\mathrm{g\,m^{-3}} \cdot \frac{1}{44.0\,\mathrm{g\,mol^{-1}}} = 0.452\,\mathrm{mol\,m^{-3}}$$

3 Collisions Between Molecules