Homework 6 - Force Dynamics

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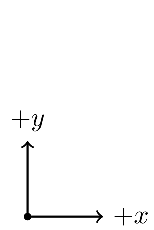
Contents

1 Book

1.1 5.12

$$m = 125\,\mathrm{kg}$$
$$F_{\text{thrust}} = 1720\,\mathrm{N}$$
$$F_{\text{ps}} = 15.5\,\mathrm{N}$$

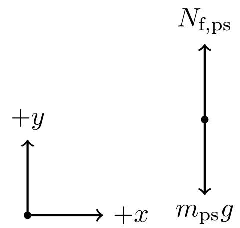
(a)



$$\sum F_y = ma$$
$$F_{\text{thrust}} = ma + mg$$
$$a = \frac{F_{\text{thrust}} - mg}{m}$$
$$a = \frac{1720\,\mathrm{N} - (125\,\mathrm{kg})(10\,\mathrm{m\,s^{-2}})}{125\,\mathrm{kg}}$$
$$a = 3.76\,\mathrm{m\,s^{-2}}$$

$$\boxed{a = 3.76\,\mathrm{m\,s^{-2}}}$$

(b)



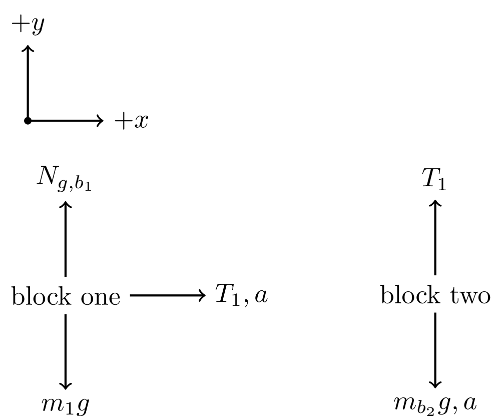
$$
\begin{aligned}
\sum F_y &= m_{\mathrm{ps}}a \\
N_{\mathrm{f,ps}} - m_{\mathrm{ps}}g &= m_{\mathrm{ps}}a \\
N_{\mathrm{f,ps}} &= m_{\mathrm{ps}}(g+a) \\
N_{\mathrm{f,ps}} &= \left(\frac{F_{\mathrm{ps}}}{g}\right)(g+a) \\
N_{\mathrm{f,ps}} &= \left(\frac{15.5\,\mathrm{N}}{10\,\mathrm{m\,s^{-2}}}\right)(10\,\mathrm{m\,s^{-2}} + 3.76\,\mathrm{m\,s^{-2}}) \\
N_{\mathrm{f,ps}} &= 21.33\,\mathrm{N}
\end{aligned}
$$

$$\boxed{N_{\mathrm{f,ps}} = 21.33\,\mathrm{N}}$$

1.2 5.17

$$
\begin{aligned}
m_1 &= 4.70\,\mathrm{kg} \\
\mu &= 0 \\
m_2 &= ? \\
T &= 13.6\,\mathrm{N}
\end{aligned}
$$

(a)



(b)

$$
\begin{aligned}
\sum F_x^{(b_1)} &= m_{b_1} a \\
T_1 &= m_{b_1} a \\
a &= \frac{T_1}{m_{b_1}} \\
a &= \frac{13.6\,\mathrm{N}}{4.70\,\mathrm{kg}} \\
a &= 2.89\,\mathrm{m\,s^{-2}}
\end{aligned}
$$

$$\boxed{a = 2.89\,\mathrm{m\,s^{-2}}}$$

(c)

$$
\begin{aligned}
\sum F_y^{(b_2)} &= -m_{b_2} a \\
T_1 - m_{b_2} g &= -m_{b_2} a \\
m_{b_2}(-a+g) &= T_1 \\
m_{b_2} &= \frac{T_1}{-a+g} \\
m_{b_2} &= \frac{13.6\,\mathrm{N}}{-(2.89\,\mathrm{m\,s^{-2}}) + 10\,\mathrm{m\,s^{-2}}} \\
m_{b_2} &= 1.91\,\mathrm{kg}
\end{aligned}
$$

$$\boxed{m_{b_2} = 1.91\,\mathrm{kg}}$$

(d) The weight of the hanging block (w_{b_2}) can be calculated using

$$w_{b_2} = m_{b_2} g,$$

solved as so:

$$
\begin{aligned}
w_{b_2} &= m_{b_2} g \\
&= (1.91\,\mathrm{kg})(10\,\mathrm{m\,s^{-2}}) \\
w_{b_2} &= 19.1\,\mathrm{N}
\end{aligned}
$$

$$\boxed{\therefore \text{ it can be shown that } w_{b_2} > T_1}$$

1.3 5.21

$$
\begin{aligned}
m &= 2.10\,\mathrm{kg} \\
v &= 8.50\,\mathrm{m\,s^{-1}} \\
t &= 0 \\
F(t) &= (6.00\,\mathrm{N\,s^{-2}})t^2
\end{aligned}
$$

(a) Using the force function and NSL, solve for the acceleration function and integrate to get the velocity function.

$$F(t) = ma$$
$$a(t) = \frac{F}{m}$$
$$a(t) = \frac{-6.00\,\mathrm{N\,s^{-2}}}{2.10\,\mathrm{kg}}t^2$$
$$a(t) = (-2.86\,\mathrm{m\,s^{-4}})t^2$$

$$v(t) = \int a(t)dt = \int (-2.86\,\mathrm{m\,s^{-4}})t^2 dt$$
$$v(t) = (-0.953\,\mathrm{m\,s^{-4}})t^3 + v$$
$$v(t) = (-0.953\,\mathrm{m\,s^{-4}})t^3 + 8.50\,\mathrm{m\,s^{-1}}$$

Find t when the velocity is 0.

$$v(t) = (-0.953\,\mathrm{m\,s^{-4}})t^3 + 8.50\,\mathrm{m\,s^{-1}} = 0$$
$$(0.953\,\mathrm{m\,s^{-4}})t^3 = 8.50\,\mathrm{m\,s^{-1}}$$
$$t = 2.07\,\mathrm{s}$$

Integrate and find the distance at time, $t = 2.07\,\mathrm{s}$.

$$x(t) = \int v(t)dt = \int (-0.953\,\mathrm{m\,s^{-4}})t^3 + 8.50\,\mathrm{m\,s^{-1}} dt$$
$$x(t) = (-0.238\,\mathrm{m\,s^{-2}})t^4 + (8.50\,\mathrm{m\,s^{-1}})t + 0$$

$$x(2.07\,\mathrm{s}) = (-0.238\,\mathrm{m\,s^{-2}})(2.07\,\mathrm{s})^4 + (8.50\,\mathrm{m\,s^{-1}})(2.07\,\mathrm{s})$$
$$x(2.07\,\mathrm{s}) = 13.2\,\mathrm{m}$$

$$\boxed{x = 13.2\,\mathrm{m}}$$

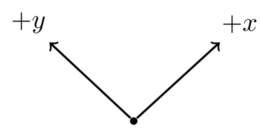
(b) Find v at time $t = 3.00\,\mathrm{s}$.

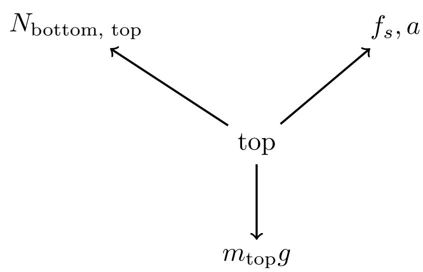
$$v(3.00\,\mathrm{s}) = (-0.953\,\mathrm{m\,s^{-4}})(3.00\,\mathrm{s})^3 + 8.50\,\mathrm{m\,s^{-1}}$$
$$v(3.00\,\mathrm{s}) = -17.2\,\mathrm{m\,s^{-1}}$$

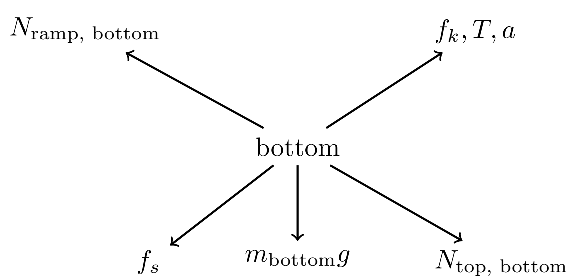
$$\boxed{v = -17.2\,\mathrm{m\,s^{-1}}}$$

1.4 5.33

$$
\begin{aligned}
m_{\text{top}} &= 32.0\,\text{kg} \\
m_{\text{bottom}} &= 48.0\,\text{kg} \\
\Delta y &= 2.50\,\text{m} \\
\Delta x &= 4.75\,\text{m} \\
v &= 15.0\,\text{cm}\,\text{s}^{-1} \\
\mu_k &= 0.444 \\
\mu_s &= 0.800 \\
a_x &= 0 \text{ (constant)}
\end{aligned}
$$







(a)

$$
\begin{aligned}
\tan(\theta) &= \frac{y}{x} \\
\theta &= \arctan\left(\frac{y}{x}\right) \\
\theta &= \arctan\left(\frac{2.50\,\text{m}}{4.75\,\text{m}}\right) \\
\theta &= 27.8^\circ
\end{aligned}
$$

$$
\begin{aligned}
\sum F_y^{(\text{t, b})} &= m_{\text{t, b}} g \cos(27.8^\circ) \\
N_{\text{b, t}} - N_{\text{t, b}} + N_{\text{r, b}} &= m_{\text{t, b}} g \cos(27.8^\circ) \\
N_{\text{r, b}} &= m_{\text{t, b}} g \cos(27.8^\circ) \\
N_{\text{r, b}} &= (32.0\,\text{kg} + 48.0\,\text{kg})(10\,\text{m}\,\text{s}^{-2}) \cos(27.8^\circ) \\
N_{\text{r, b}} &= 707.7\,\text{N}
\end{aligned}
$$

$$
\begin{aligned}
\sum F_x^{(\text{t, b})} &= m_{\text{t, b}} a \\
f_s - f_s + f_k + T - m_{\text{t, b}} g \sin(27.8^\circ) &= (m_{\text{t, b}})(0) \\
\mu_k N_{\text{r, b}} + T - m_{\text{t, b}} g \sin(27.8^\circ) &= 0 \\
T &= m_{\text{t, b}} g \sin(27.8^\circ) - \mu_k N_{\text{r, b}} \\
T &= (32.0\,\text{kg} + 48.0\,\text{kg})(10\,\text{m}\,\text{s}^{-2}) \sin(27.8^\circ) - (0.444)(707.7\,\text{N}) \\
T &= 58.9\,\text{N}
\end{aligned}
$$

$$
\boxed{T = 58.9\,\text{N}}
$$

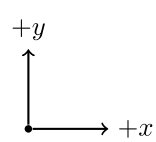
(b)

$$
\begin{aligned}
\sum F_x^{(\text{top})} &= 0 \\
f_s &= m_{\text{top}} g \sin(27.8^\circ) \\
f_s &= (32.0\,\text{kg})(10.0\,\text{m}\,\text{s}^{-2}) \sin(27.8^\circ) \\
f_s &= 149.2\,\text{N}
\end{aligned}
$$

$$
\boxed{f_s = 149.2\,\text{N at } \theta = 27.8^\circ}
$$

1.5 5.34

$$
\begin{aligned}
w_A &= 45.0\,\text{N} \\
w_B &= 25.0\,\text{N} \\
a_B &= 0
\end{aligned}
$$



$$
\begin{tikzcd}
 & N_{\text{tabletop, A}} & & & T \\
f_{\text{tabletop, A}} & A \arrow[l] \arrow[r] \arrow[u] \arrow[d] & T, a & & B \arrow[u] \arrow[d] \\
 & m_A g & & & m_B g, a
\end{tikzcd}
$$

(a)

$$
\begin{aligned}
w_A &= m_A g \\
m_A &= \frac{w_A}{g} \\
m_A &= \frac{45.0\,\mathrm{N}}{10\,\mathrm{m\,s^{-2}}} \\
m_A &= 4.5\,\mathrm{kg}
\end{aligned}
$$

$$
\begin{aligned}
w_B &= m_B g \\
m_B &= \frac{w_B}{g} \\
m_B &= \frac{25.0\,\mathrm{N}}{10\,\mathrm{m\,s^{-2}}} \\
m_B &= 2.5\,\mathrm{kg}
\end{aligned}
$$

$$
\begin{aligned}
\sum F_y^{(B)} &= -m_B a \\
T - m_B g &= (-m_B)(0) \\
T &= m_B g \\
T &= 25.0\,\mathrm{N}
\end{aligned}
$$

$$
\begin{aligned}
\sum F_y^{(A)} &= 0 \\
N_{\text{t, A}} &= m_A g \\
N_{\text{t, A}} &= 45.0\,\mathrm{N}
\end{aligned}
$$

$$
\begin{aligned}
\sum F_x^{(A)} &= m_A a \\
T - \mu N_{\text{t, A}} &= (m_A)(0) \\
\mu &= \frac{T}{N_{\text{t, A}}} \\
\mu &= \frac{25.0\,\mathrm{N}}{45.0\,\mathrm{N}} \\
\mu &= 0.556
\end{aligned}
$$

$$\boxed{\mu = 0.556}$$

(b)

$$\begin{aligned}
\sum F_y^{(A)} &= 0 \\
N_{\text{t, A}} - m_A g &= 0 \\
N_{\text{t, A}} &= m_A g \\
N_{\text{t, A}} &= 2(45.0\,\mathrm{N}) \\
N_{\text{t, A}} &= 90.0\,\mathrm{N}
\end{aligned}$$

$$\begin{aligned}
\sum F_x^{(A)} &= m_A a \\
T - f_{\text{t, A}} &= m_A a \\
T &= \mu N_{\text{t, A}} + m_A a
\end{aligned}$$

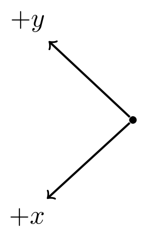
$$\begin{aligned}
w_A &= m_A g \\
m_A &= \frac{90.0\,\mathrm{N}}{10.0\,\mathrm{m\,s^{-2}}} \\
m_A &= 9.00\,\mathrm{kg}
\end{aligned}$$

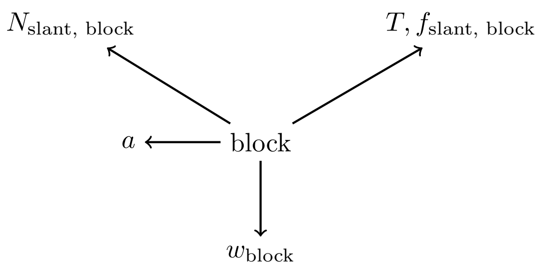
$$\begin{aligned}
\sum F_y^{(B)} &= -m_B a \\
T - m_B g &= -m_B a \\
(\mu N_{\text{t, A}} + m_A a) - m_B g &= -m_B a \\
-m_A a - m_B a &= \mu N_{\text{t, A}} - m_B g \\
a &= \frac{\mu N_{\text{t, A}} - m_B g}{-m_A - m_B} \\
a &= \frac{(0.556)(90.0\,\mathrm{N}) - 25.0\,\mathrm{N}}{-(9.00\,\mathrm{kg}) - 2.5\,\mathrm{kg}} \\
a &= -2.18\,\mathrm{m\,s^{-2}}
\end{aligned}$$

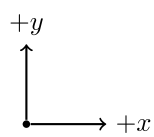
$$\boxed{a = -2.18\,\mathrm{m\,s^{-2}}}$$

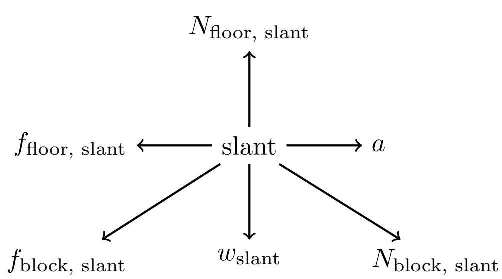
2 Lab Manual

2.1 571









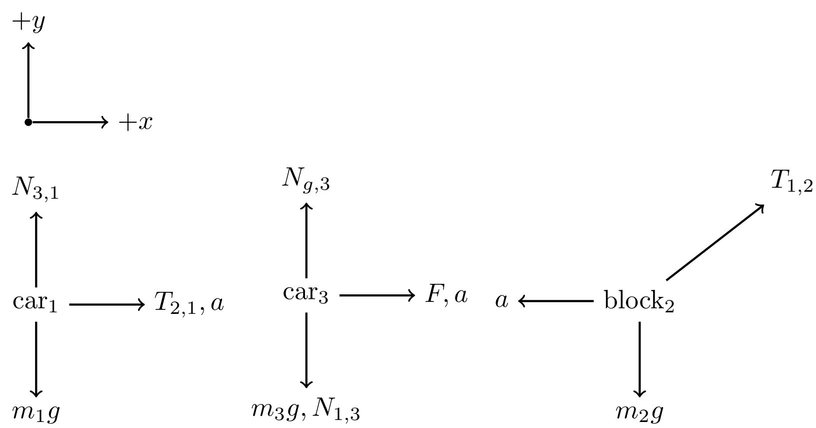
$$\begin{aligned}
\sum F_x^{(\text{slant})} &= m_s a \\
-f_{f,s} - f_{b,s}\cos(\theta) + N_{b,s}\sin(\theta) &= m_s a \\
a &= \frac{-f_{f,s} - f_{b,s}\cos(\theta) + N_{b,s}\sin(\theta)}{m_s} \\
a &= \frac{-\mu N_{f,s} - \mu N_{b,s}\cos(\theta) + N_{b,s}\sin(\theta)}{m_s}
\end{aligned}$$

$$
\begin{aligned}
\sum F_y^{(\text{slant})} &= 0 \\
N_{f,s} - f_{b,s}\sin(\theta) - N_{b,s}\cos(\theta) - w_s &= 0 \\
N_{f,s} &= \mu N_{b,s}\sin(\theta) + N_{b,s}\cos(\theta) + m_s g
\end{aligned}
$$

$$
\begin{aligned}
\sum F_y^{(\text{block})} &= 0 \\
N_{s,b} - w_b\cos(\theta) + m_b a\sin(\theta) &= 0
\end{aligned}
$$

$$
\begin{aligned}
\sum F_x^{(\text{block})} &= m_b a \\
-T - f_{s,b} + w_b\sin(\theta) + m_b a\cos(\theta) &= m_b a
\end{aligned}
$$

2.2 575



$$
\begin{aligned}
m_1 &= 5\,\mathrm{kg} \\
m_2 &= 4\,\mathrm{kg} \\
m_3 &= 21\,\mathrm{kg} \\
F &= ?
\end{aligned}
$$

$$
\begin{aligned}
\sum F_y^{(\text{car}_1)} &= 0 \\
N_{3,1} &= m_1 g \\
N_{3,1} &= (5\,\mathrm{kg})(10\,\mathrm{m\,s^{-2}}) \\
N_{3,1} &= 50\,\mathrm{N}
\end{aligned}
$$

$$
\begin{aligned}
\sum F_y^{(\text{car}_3)} &= 0 \\
N_{g,3} &= m_3 g + N_{1,3} \\
N_{g,3} &= (21\,\mathrm{kg})(10\,\mathrm{m\,s^{-2}}) + 50\,\mathrm{N} \\
N_{g,3} &= 260\,\mathrm{N}
\end{aligned}
$$

$$\sum F_y^{(\text{block}_2)} = 0$$
$$T_{1,2}\cos(\theta) = m_2 g$$

$$\sum F_x^{(\text{block}_2)} = -m_2 a$$
$$T_{1,2}\sin(\theta) = -m_2 a$$
$$a = -\frac{T_{1,2}\sin(\theta)}{m_2}$$

$$\sum F_x^{(\text{car}_1)} = m_1 a$$
$$T_{2,1} = m_1 a$$
$$a = \frac{T_{2,1}}{m_1}$$

$$-\frac{T_{1,2}\sin(\theta)}{m_2} = \frac{T_{2,1}}{m_1}$$
$$-\frac{\sin(\theta)}{m_2} = \frac{1}{m_1}$$
$$\theta = \arcsin\left(-\frac{m_2}{m_1}\right)$$
$$\theta = \arcsin\left(-\frac{4\,\text{kg}}{5\,\text{kg}}\right)$$
$$\theta = -53.13^\circ$$

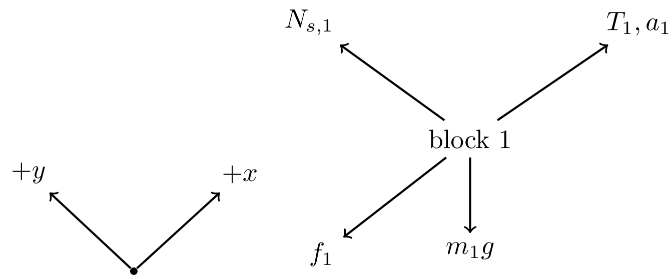
$$T_{1,2}\cos(\theta) = m_2 g$$
$$T_{1,2} = \frac{m_2 g}{\cos(\theta)}$$
$$T_{1,2} = \frac{(4\,\text{kg})(10\,\text{m}\,\text{s}^{-2})}{\cos(-53.13^\circ)}$$
$$T_{1,2} = 66.67\,\text{N}$$

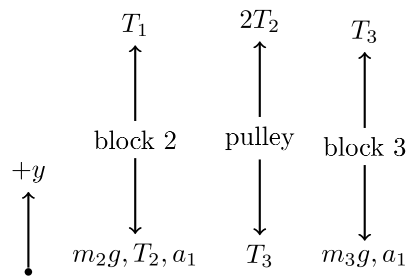
$$a = \frac{T_{2,1}}{m_1}$$
$$a = \frac{66.67\,\text{N}}{5\,\text{kg}}$$
$$a = 13.33\,\text{m}\,\text{s}^{-2}$$

$$F = (m_1 + m_2 + m_3)a$$
$$F = (30\,\text{kg})(13.33\,\text{m}\,\text{s}^{-2})$$
$$F = 399.9\,\text{N}$$

$$\boxed{F = 399.9\,\text{N}}$$

2.3 577





(a)

$$\begin{aligned}
\sum F_y^{(b_1)} &= 0 \\
N_{s,1} &= m_1 g \cos(\theta)
\end{aligned}$$

$$\begin{aligned}
\sum F_x^{(b_1)} &= m_1 a_1 \\
T_1 - f_1 - \frac{m_1 g}{\sin(\theta)} &= m_1 a_1 \\
T_1 - \mu N_{s,1} &= m_1 a_1 \\
a_1 &= \frac{T_1 - \mu m_1 g \cos(\theta)}{m_1}
\end{aligned}$$

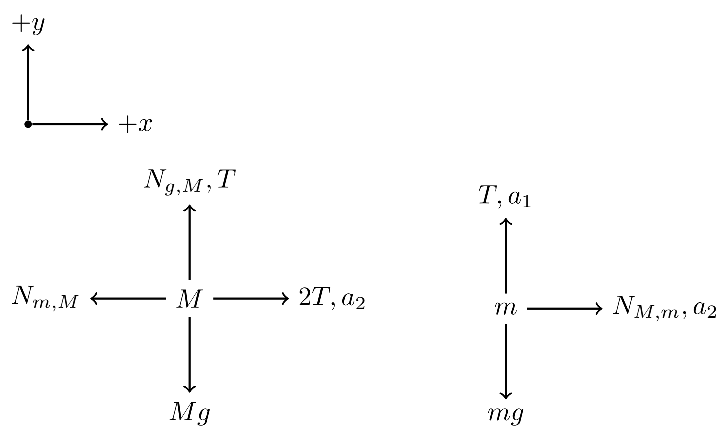
$$\begin{aligned}
\sum F_y^{(b_2)} &= -m_2 a_1 \\
T_1 - m_2 g - T_2 &= -m_2 a_1 \\
a_1 &= \frac{-T_1 + m_2 g + T_2}{m_2}
\end{aligned}$$

$$\begin{aligned}
\sum F_y^{(p)} &= 0 \\
2T_2 &= T_3 \\
T_2 &= \frac{1}{2} T_3
\end{aligned}$$

$$
\begin{aligned}
\sum F_y^{(b_3)} &= -m_3 a_1 \\
T_3 - m_3 g &= -m_3 a_1 \\
T_3 &= -m_3 a_1 + m_3 g \\
a_1 &= \frac{-T_3 + m_3 g}{m_3}
\end{aligned}
$$

$$
\begin{aligned}
T_2 &= \frac{1}{2} T_3 \\
T_2 &= \frac{-m_3 a_1 + m_3 g}{2}
\end{aligned}
$$

2.4 578



$$
\begin{aligned}
\sum F_y^{(M)} &= 0 \\
N_{g,M} + T &= Mg
\end{aligned}
$$

$$
\begin{aligned}
\sum F_x^{(M)} &= Ma_2 \\
2T &= N_{m,M} + Ma_2 \\
T &= \frac{N_{m,M} + Ma_2}{2}
\end{aligned}
$$

$$
\begin{aligned}
\sum F_y^{(m)} &= -ma_1 \\
T &= -ma_1 + mg
\end{aligned}
$$

$$
\begin{aligned}
\sum F_x^{(m)} &= ma_2 \\
N_{M,m} &= ma_2
\end{aligned}
$$

$$T = \frac{N_{m,M} + Ma_2}{2}$$
$$T = \frac{ma_2 + Ma_2}{2}$$

$$T = -ma_1 + mg$$
$$ma_1 = mg - T$$
$$a_1 = \frac{mg - T}{m}$$
$$a_1 = \frac{mg - \frac{ma_2 + Ma_2}{2}}{m}$$
$$a_1 = \frac{4mg}{5m + M}$$
$$a_1 = \frac{4(0.5\,\mathrm{kg})(10\,\mathrm{m\,s^{-2}})}{5(0.5\,\mathrm{kg}) + 3.0\,\mathrm{kg})}$$
$$a_1 = 3.636\,\mathrm{m\,s^{-2}}$$

$$\Delta x = v_o t + \frac{1}{2} a_1 t^2$$
$$1.6\,\mathrm{m} = (0)t + \frac{1}{2}(3.636\,\mathrm{m\,s^{-2}})t^2$$
$$t = \pm 0.9381\,\mathrm{s}$$

$$\boxed{t = 0.9381\,\mathrm{s}}$$