Contents

1 Section 10.3

1.1 10.3.1

Apply the translation theorem to find the Laplace transform of the following function.

$$f(t) = t^5 e^{\pi t}$$

$$\mathcal{L}\left\{t^5\right\} = \frac{5!}{s^6}$$
$$\mathcal{L}\left\{t^5 e^{\pi t}\right\} = \frac{5!}{(s-\pi)^6}$$

1.2 10.3.3

Apply the translation theorem to find the Laplace transform of the following function.

$$f(t) = e^{-9t}\sin(7\pi t)$$

$$\mathcal{L}\left\{7\pi t\right\} = \frac{7\pi}{s^2 + 49\pi^2}$$
$$\mathcal{L}\left\{e^{-9t}\sin(7\pi t)\right\} = \frac{7\pi}{(s+9)^2 + 49\pi^2}$$

1.3 10.3.5

Apply the translation theorem to find the inverse Laplace transform of the following function.

$$F(s) = \frac{5}{2s - 18}$$

$$\begin{aligned}
F(s) &= \frac{5}{2}\left(\frac{1}{s-9}\right) \\
\mathcal{L}^{-1}\left\{\frac{1}{s-9}\right\} &= e^{9t} \\
\mathcal{L}^{-1}\left\{\frac{5}{2s-18}\right\} &= \frac{5e^{9t}}{2}
\end{aligned}$$

1.4 10.3.9

Apply the translation theorem to find the inverse Laplace transform of the following function.

$$F(s) = \frac{7s+13}{s^2-6s+73}$$

$$\begin{aligned}
F(s) &= \frac{7s+13}{s^2-6s+73} \\
F(s) &= \frac{7(s+3-3)+13}{(s-3)^2+64} \\
F(s) &= \frac{7(s-3)+34}{(s-3)^2+64} \\
F(s) &= \frac{7(s-3)}{(s-3)^2+64} + \frac{34}{(s-3)^2+64} \\
\mathcal{L}^{-1}\left\{\frac{7(s-3)}{(s-3)^2+64}\right\} &= e^{3t}\cos(8t) \\
\mathcal{L}^{-1}\left\{\frac{34}{(s-3)^2+64}\right\} &= e^{3t}\frac{34}{8}\sin(8t) \\
\mathcal{L}^{-1}\left\{F(s)\right\} &= e^{3t}\left(\cos(8t) + \frac{17\sin(t)}{4}\right)
\end{aligned}$$

1.5 10.3.13

Use partial fractions to find the inverse Laplace transform of the following function.

$$F(s) = \frac{-50-15s}{s^2+13s+36}$$

$$\begin{aligned}
F(s) &= \frac{2}{s+4} - \frac{17}{x+9} \\
\mathcal{L}^{-1}\left\{F(s)\right\} &= 2e^{-4t} - 17e^{-9t}
\end{aligned}$$

1.6 10.3.15

Use partial fractions to find the inverse Laplace transform of the following function.

$$F(s) = \frac{5}{s^3-4s^2}$$

$$
\begin{aligned}
F(s) &= \frac{5}{16(s-4)} - \frac{5}{16s} - \frac{5}{4s^2} \\
F(s) &= \frac{5e^{4t}}{16} - \frac{5}{16} - \frac{5t}{4}
\end{aligned}
$$

1.7 10.3.37

Use Laplace transforms to solve the following initial value problem.

$$x'' + 10x' + 29x = te^{-t}; x(0) = 0, x'(0) = 3$$

$$
\begin{aligned}
\mathcal{L}\left\{x''\right\} + 10\mathcal{L}\left\{x'\right\} + 29\mathcal{L}\left\{x\right\} &= \mathcal{L}\left\{te^{-t}\right\} \\
\left[s^2\mathcal{L}\left\{x\right\} - sx(0) - x'(0)\right] + 10\left[s\mathcal{L}\left\{x\right\} - x(0)\right] + 29\mathcal{L}\left\{x\right\} &= \frac{1}{(s+1)^2} \\
\mathcal{L}\left\{x\right\}(s^2 + 10s + 29) - 0 - 3 - 0 &= \frac{1}{(s+1)^2} \\
\mathcal{L}\left\{x\right\} &= \frac{3s^2 + 6s + 4}{(s+1)^2(s^2 + 10s + 29)}
\end{aligned}
$$

$$
\begin{aligned}
x &= \mathcal{L}^{-1}\left\{\frac{3s^2 + 6s + 4}{(s+1)^2(s^2 + 10s + 29)}\right\} \\
x &= \frac{1}{50}\mathcal{L}^{-1}\left\{\frac{s}{s^2 + 10s + 29}\right\} + \frac{313}{100}\mathcal{L}^{-1}\left\{\frac{1}{s^2 + 10s + 29}\right\} - \frac{1}{50}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{20}\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}^{-1}\left\{\frac{s}{s^2 + 10s + 29}\right\} &= \mathcal{L}^{-1}\left\{\frac{s + 5 - 5}{(s+5)^2 + 4}\right\} \\
&= \mathcal{L}^{-1}\left\{\frac{s+5}{(s+5)^2 + 4}\right\} - 5\mathcal{L}^{-1}\left\{\frac{1}{(s+5)^2 + 4}\right\} \\
&= e^{-5t}\cos(2t) - 5e^{-5t}\frac{1}{2}\sin(2t) \\
&= e^{-5t}\left(\cos(2t) - \frac{5}{2}\sin(2t)\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}^{-1}\left\{\frac{1}{s^2 + 10s + 29}\right\} &= \mathcal{L}^{-1}\left\{\frac{1}{(s+5)^2 + 4}\right\} \\
&= \frac{e^{-5t}\sin(2t)}{2}
\end{aligned}
$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} = te^{-t}$$

$$x(t) = \frac{1}{50}e^{-5t}\left(\cos(2t) - \frac{5}{2}\sin(2t)\right) + \frac{313}{100}\frac{e^{-5t}\sin(2t)}{2} - \frac{1}{50}e^{-t} + \frac{1}{20}te^{-t}$$