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1 7.1

1.1 8

What is the probability that a five-card poker hand contains the ace of hearts?

$$\begin{aligned}
 \mathbb{P}(A\heartsuit) &= \frac{|E|}{|S|} \\
 &= \frac{C(51, 4)}{C(52, 5)} \\
 &= \frac{51 \cdot 50 \cdot 49 \cdot 48}{1 \cdot 2 \cdot 3 \cdot 4} \cdot \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48} \\
 &= \frac{5}{52}
 \end{aligned}$$

1.2 Birthday Problem

Given a group of n people, what is the probability of at least two people having the same birthday?

2 7.2

$\mathbb{P}: S \rightarrow [0, 1]$ is called probability if

1. $\forall s \in S, 0 \leq \mathbb{P}(s) \leq 1$
2. $\sum_{s \in S} \mathbb{P}(s) = 1$

Disjoint (mutually exclusive)

$$\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F)$$

$$\begin{aligned}
 \mathbb{P}(E \cup F) &= \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F) \\
 \mathbb{P}(E \cap F) &= 0
 \end{aligned}$$

Independent

$$\mathbb{P}(E \cap F) = \mathbb{P}(E) \cdot \mathbb{P}(F)$$

2.1 Random Variable

$$\begin{aligned}X &: S \rightarrow \mathbb{R} \\ \mathbb{P}(X) &\rightarrow [0, 1] \\ S &\rightarrow \mathbb{R} \rightarrow [0, 1]\end{aligned}$$

2.2 Conditional Probability

$$\mathbb{P}(E \mid F) = \frac{\mathbb{P}(E \cap F)}{\mathbb{P}(F)}$$

Probably of E if F happened. (after F happened)