Homework 8 - Momentum

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Contents

1 Book

1.1 8.16

$$\begin{aligned}
m_{(a)stronaut} &= 65.5\,\mathrm{kg} \\
m_{(t)ool} &= 2.50\,\mathrm{kg} \\
v_{t_1} &= 3.10\,\mathrm{m\,s^{-1}} \\
v_{a_1} &= ?
\end{aligned}$$

$$\begin{aligned}
P_0 &= P_1 \\
m_a v_{a_0} + m_t v_{t_0} &= m_a v_{a_1} + m_t v_{t_1} \\
0 + 0 &= m_a v_{a_1} + m_t v_{t_1} \\
v_{a_1} &= -\frac{m_t v_{t_1}}{m_a} \\
v_{a_1} &= -\frac{(2.50\,\mathrm{kg})(3.10\,\mathrm{m\,s^{-1}})}{65.5\,\mathrm{kg}} \\
v_{a_1} &= -0.118\,\mathrm{m\,s^{-1}}
\end{aligned}$$

The astronaut will move at a speed of $0.118\,\mathrm{m\,s^{-1}}$ opposite of the tool's direction.

1.2 8.21

$$
\begin{aligned}
m_A &= 0.245\,\mathrm{kg} \\
m_B &= 0.360\,\mathrm{kg} \\
v_{B_0} &= 0 \\
v_{A_1} &= -0.118\,\mathrm{m\,s^{-1}} \\
v_{B_1} &= 0.660\,\mathrm{m\,s^{-1}} \\
v_{A_0} &= ?
\end{aligned}
$$

$$-\hat{x} \longleftarrow\!\!\bullet\!\!\longrightarrow +\hat{x}$$

(a) What was the speed of puck A before the collision?

$$
\begin{aligned}
P_0 &= P_1 \\
m_A v_{A_0} + m_B v_{B_0} &= m_A v_{A_1} + m_B v_{B_1} \\
m_A v_{A_0} + 0 &= m_A v_{A_1} + m_B v_{B_1} \\
v_{A_0} &= \frac{m_A v_{A_1} + m_B v_{B_1}}{m_A} \\
v_{A_0} &= \frac{(0.245\,\mathrm{kg})(-0.118\,\mathrm{m\,s^{-1}}) + (0.360\,\mathrm{kg})(0.660\,\mathrm{m\,s^{-1}})}{0.245\,\mathrm{kg}} \\
v_{A_0} &= 0.852\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$\boxed{v_{A_0} = 0.852\,\mathrm{m\,s^{-1}}}$$

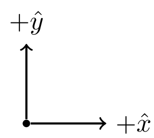
(b) Calculate the change in the total kinetic energy of the system that occurs during the collision.

$$
\begin{aligned}
\Delta PE &= E_{A_1} + E_{B_1} - E_{A_0} + E_{B_0} \\
\Delta PE &= \frac{1}{2} m_A v_{A_1}^2 + \frac{1}{2} m_B v_{B_1}^2 - \frac{1}{2} m_A v_{A_0}^2 + 0 \\
\Delta PE &= \frac{1}{2}(0.245\,\mathrm{kg})(-0.118\,\mathrm{m\,s^{-1}})^2 + \frac{1}{2}(0.360\,\mathrm{kg})(0.660\,\mathrm{m\,s^{-1}})^2 \\
&\quad - \frac{1}{2}(0.245\,\mathrm{kg})(0.852\,\mathrm{m\,s^{-1}})^2 \\
\Delta PE &= -0.008\,81\,\mathrm{J} = 8.81 \times 10^{-3}\,\mathrm{J}
\end{aligned}
$$

$$\boxed{\Delta PE = -0.008\,81\,\mathrm{J} = 8.81 \times 10^{-3}\,\mathrm{J}}$$

1.3 8.30

$$\begin{aligned}
m_A = m_B &= ? \\
v_{A_0} &= 40.0\,\mathrm{m\,s^{-1}} \\
\theta_A &= 30.0^\circ \\
v_{B_0} &= 0 \\
\theta_B &= -45.0^\circ \\
v_{A_1} &= ? \\
v_{B_1} &= ?
\end{aligned}$$



(a) Find the speed of each asteroid after the collision.
Speed of asteroid in $\hat{x}$ direction:

$$\begin{aligned}
v_{A_0} &= 40.0\,\mathrm{m\,s^{-1}}\cos(0^\circ) = 40.0\,\mathrm{m\,s^{-1}} \\
v_{B_0} &= 0 \\
v_{A_1} &= v_{A_1}\cos(30.0^\circ) \\
v_{B_1} &= v_{B_1}\cos(-45.0^\circ)
\end{aligned}$$

$$\begin{aligned}
P_{0_x} &= P_{1_x} \\
m_A v_{A_0} + m_B v_{B_0} &= m_A v_{A_1} + m_B v_{B_1} \\
v_{A_0} &= v_{A_1} + v_{B_1} \\
40.0\,\mathrm{m\,s^{-1}} &= v_{A_1}\cos(30.0^\circ) + v_{B_1}\cos(-45.0^\circ)
\end{aligned}$$

Speed of asteroid in $\hat{y}$ direction:

$$\begin{aligned}
v_{A_0} &= 40.0\,\mathrm{m\,s^{-1}}\cos(90^\circ) = 0 \\
v_{B_0} &= 0 \\
v_{A_1} &= v_{A_1}\sin(30.0^\circ) \\
v_{B_1} &= v_{B_1}\sin(-45.0^\circ)
\end{aligned}$$

$$\begin{aligned}
P_{0_y} &= P_{1_y} \\
m_A v_{A_0} + m_B v_{B_0} &= m_A v_{A_1} + m_B v_{B_1} \\
0 &= v_{A_1} + v_{B_1} \\
v_{A_1}\sin(30.0^\circ) + v_{B_1}\sin(-45.0^\circ) &= 0
\end{aligned}$$

$$[\boldsymbol{A}|\mathbf{v}] = \left[\begin{array}{cc|c} \cos(30.0°) & \cos(-45.0°) & 40.0\,\mathrm{m\,s^{-1}} \\ \sin(30.0°) & \sin(-45.0°) & 0 \end{array}\right]$$

$$\boldsymbol{A}_2 = \boldsymbol{A}_2 - \boldsymbol{A}_1\frac{\sqrt{3}}{3}$$
$$[\boldsymbol{A}|\mathbf{v}] = \left[\begin{array}{cc|c} \cos(30.0°) & \cos(-45.0°) & 40.0\,\mathrm{m\,s^{-1}} \\ 0 & -1.12 & -23.1\,\mathrm{m\,s^{-1}} \end{array}\right]$$

$$\begin{aligned} -1.12\mathbf{v}_B &= -23.1\,\mathrm{m\,s^{-1}} \\ \mathbf{v}_B &= 20.6\,\mathrm{m\,s^{-1}} \end{aligned}$$

$$\begin{aligned} (\cos(30.0°))\mathbf{v}_A + (\cos(-45.0°))\mathbf{v}_B &= 40.0\,\mathrm{m\,s^{-1}} \\ (\cos(30.0°))\mathbf{v}_A + (\cos(-45.0°))(20.6\,\mathrm{m\,s^{-1}}) &= 40.0\,\mathrm{m\,s^{-1}} \\ \mathbf{v}_A &= 29.4\,\mathrm{m\,s^{-1}} \end{aligned}$$

Asteroid A moves $29.4\,\mathrm{m\,s^{-1}}$ at $30.0°$ above the horizontal while asteroid B moves $20.6\,\mathrm{m\,s^{-1}}$ at $-45.0°$ below the horizontal.

(b) What fraction of the original kinetic energy of asteroid A dissipates during this collision.

$$\begin{aligned} E_1 : E_0 &= \frac{E_1}{E_0} \\ E_1 : E_0 &= \frac{\frac{1}{2}m_A v_{A_1}^2 + \frac{1}{2}m_B v_{B_1}^2}{\frac{1}{2}m_A v_{A_0}^2 + \frac{1}{2}m_B v_{B_0}^2} \\ E_1 : E_0 &= \frac{v_{A_1}^2 + v_{B_1}^2}{v_{A_0}^2} \\ E_1 : E_0 &= \frac{(29.4\,\mathrm{m\,s^{-1}}) + (20.6\,\mathrm{m\,s^{-1}})}{40.0\,\mathrm{m\,s^{-1}}} \\ E_1 : E_0 &= 0.805 = 80.5\,\% \end{aligned}$$

80.5 % of asteroid A's kinetic energy is conserved; therefore also meaning that 19.5 % is dissipated during collision.

1.4 8.34

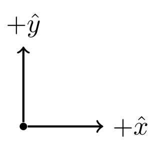
$$\begin{aligned} m_{(a)pple} &= M \\ v_{a_0} &= 0 \\ m_{(d)art} &= \frac{M}{4} \\ v_{d_0} &= v_0 \end{aligned}$$

$$\begin{aligned}
P_0 &= P_1 \\
m_a v_{a_0} + m_d v_{d_0} &= v_1(m_a + m_d) \\
0 + m_d v_{d_0} &= v_1(m_a + m_d) \\
v_1 &= \frac{m_d v_{d_0}}{m_a + m_d}
\end{aligned}$$

$$\begin{aligned}
E_0 &= E_1 \\
\frac{1}{2}mv_0^2 + mgh_0 &= \frac{1}{2}mv_1^2 + mgh_1 \\
\frac{1}{2}(m_a + m_d)\left(\frac{m_d v_{d_0}}{m_a + m_d}\right)^2 + 0 &= 0 + (m_a + m_d)gh_1 \\
h_1 &= \frac{m_d^2 v_0^2}{2(m_a + m_d)^2 g} \\
h_1 &= \frac{\left(\frac{M}{4}\right)^2 v_0^2}{2\left(M + \frac{M}{4}\right)^2 g} \\
h_1 &= \frac{v_0^2}{50g}
\end{aligned}$$

$$\boxed{\text{The height of the collided apple and dart reach } h_1 = \tfrac{v_0^2}{50g}.}$$

1.5 8.41



$$\begin{aligned}
m_{(c)ar} &= 950\,\mathrm{kg} \\
m_{(t)ruck} &= 1900\,\mathrm{kg} \\
v_1 &= 16.0\,\mathrm{m\,s^{-1}} \\
\theta &= 24.0\,^\circ\,\text{East of North} \\
v_{c_0} &=? \\
v_{t_0} &=?
\end{aligned}$$

Find conservation of momentum in $\hat{x}$ direction:

$$\begin{aligned}
v_{c_0} &= v_{c_0}\cos(0^\circ) = v_{c_0} \\
v_{t_0} &= v_{t_0}\cos(90.0^\circ) = 0 \\
v_{c_1} &= 16.0\,\mathrm{m\,s^{-1}}\sin(24.0^\circ) = 6.51\,\mathrm{m\,s^{-1}} \\
v_{t_1} &= 16.0\,\mathrm{m\,s^{-1}}\sin(24.0^\circ) = 6.51\,\mathrm{m\,s^{-1}}
\end{aligned}$$

$$
\begin{aligned}
P_{0_x} &= P_{1_x} \\
m_c v_{c_0} + m_t v_{t_0} &= m_c v_{c_1} + m_t v_{t_1} \\
m_c v_{c_0} + 0 &= v_1\left(m_c + m_t\right) \\
v_{c_0} &= \frac{v_1\left(m_c + m_t\right)}{m_c} \\
v_{c_0} &= \frac{(6.51\,\mathrm{m\,s^{-1}})(950\,\mathrm{kg} + 1900\,\mathrm{kg})}{950\,\mathrm{kg}} \\
v_{c_0} &= 19.53\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

Find conservation of momentum in $\hat{y}$ direction:

$$
\begin{aligned}
v_{c_0} &= v_{c_0}\cos(90.0^\circ) = 0 \\
v_{t_0} &= v_{t_0}\cos(0^\circ) = v_{t_0} \\
v_{c_1} &= 16.0\,\mathrm{m\,s^{-1}}\cos(24.0^\circ) = 14.6\,\mathrm{m\,s^{-1}} \\
v_{t_1} &= 16.0\,\mathrm{m\,s^{-1}}\cos(24.0^\circ) = 14.6\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
m_c v_{c_0} + m_t v_{t_0} &= m_c v_{c_1} + m_t v_{t_1} \\
0 + m_t v_{t_0} &= v_1(m_c + m_t) \\
v_{t_0} &= \frac{v_1(m_c + m_t)}{m_t} \\
v_{t_0} &= \frac{(14.6\,\mathrm{m\,s^{-1}})(950\,\mathrm{kg} + 1900\,\mathrm{kg})}{1900\,\mathrm{kg}} \\
v_{t_0} &= 21.9\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

The speed of the car before collision was $v_{c_0} = 19.53\,\mathrm{m\,s^{-1}}$, and the speed of the truck before collision was $v_{t_0} = 21.9\,\mathrm{m\,s^{-1}}$.

1.6 8.44

$-\hat{x} \longleftarrow\!\bullet\!\longrightarrow +\hat{x}$

$$
\begin{aligned}
m_{(b)lock} &= 15.0\,\mathrm{kg} \\
k &= 575.0\,\mathrm{N\,m^{-1}} \\
v_{b_0} &= 0 \\
m_{(s)tone} &= 3.00\,\mathrm{kg} \\
v_{s_0} &= 8.00\,\mathrm{m\,s^{-1}} \\
v_{s_1} &= -2.00\,\mathrm{m\,s^{-1}} \\
\Delta x_{b,max} &=?
\end{aligned}
$$

Find the velocity of the upon collision v_1:

$$\begin{aligned}
P_0 &= P_1 \\
m_s v_{s_0} + m_b v_{b_0} &= m_s v_{s_1} + m_b v_{b_1} \\
m_s v_{s_0} + 0 &= m_s v_{s_1} + m_b v_{b_1} \\
v_{b_1} &= \frac{m_s v_{s_0} - m_s v_{s_1}}{m_b} \\
v_{b_1} &= \frac{(3.00\,\mathrm{kg})(8.00\,\mathrm{m\,s^{-1}}) - (3.00\,\mathrm{kg})(-2.00\,\mathrm{m\,s^{-1}})}{15.0\,\mathrm{kg}} \\
v_{b_1} &= 2.00\,\mathrm{m\,s^{-1}}
\end{aligned}$$

Find the max distance using conservation of energy x_{max}:

$$\begin{aligned}
E_0 &= E_1 \\
\frac{1}{2} m_b v_0^2 &= \frac{1}{2} k x_{max}^2 \\
x_{max} &= \sqrt{\frac{m_b v_0^2}{k}} \\
x_{max} &= \sqrt{\frac{(15.0\,\mathrm{kg})(2.00\,\mathrm{m\,s^{-1}})^2}{575.0\,\mathrm{N\,m^{-1}}}} \\
x_{max} &= 0.323\,\mathrm{m}
\end{aligned}$$

$$\boxed{\text{The steel ball moves the block to a maximum of } x_{max} = 0.323\,\mathrm{m}.}$$

1.7 8.48

$-\hat{x} \longleftarrow\!\bullet\!\longrightarrow +\hat{x}$

$$\begin{aligned}
m_{(s)mall} &= 10.0\,\mathrm{g} \\
v_{s_0} &= -0.400\,\mathrm{m\,s^{-1}} \\
m_{(l)arge} &= 30.0\,\mathrm{g} \\
v_{l_0} &= 0.200\,\mathrm{m\,s^{-1}}
\end{aligned}$$

(a) Find the velocity of each marble after the collision.

$$\begin{aligned}
P_0 &= P_1 \\
m_s v_{s_0} + m_l v_{l_0} &= m_s v_{s_1} + m_l v_{l_1} \\
v_{s_1} &= \frac{m_s v_{s_0} + m_l v_{l_0} - m_l v_{l_1}}{m_s} \\
v_{l_1} &= \frac{m_s v_{s_0} + m_l v_{l_0} - m_s v_{s_1}}{m_l}
\end{aligned}$$

(b) Calculate the *change in momentum* for each marble.

(c) Calculate the *change in kinetic energy* for each marble.

1.8 8.62

1.9 8.87

2 Lab Manual

2.1 972

2.2 975

2.3 986

3 Problem B

Consider a Tsiolkovsky Rocket in a gravitational field, g. At time $t = 0$, the velocity of the rocket is $v = v_0$, and the mass is $m = m_0$. Let the mass loss rate of the rocket be constant in time: $\dot{m} = -km_0$ [recall that a variable with a dot on top is the time derivative: $\dot{m} = \frac{dm}{dt}$, $\dot{v} = \frac{dv}{dt}$, etc.]

1. Show that the acceleration of the rocket is

$$a = \dot{v} = -\frac{u_{rel}}{m}\dot{m} - g$$

2. Show that the mass as a function of time is

$$m = m_0(1 - kt)$$

3. Show that acceleration can also be written as

$$a = \dot{v} = \frac{ku_{rel}}{1 - kt} - g$$

4. Show that the ΔV for a constant mass loss rate rocket is given by:

$$\Delta V = u_{rel} \ln\left[\frac{1}{1 - kt}\right] - gt$$