Contents

1 Section 2.4 - Homework Problems

1.1 2.4.1

Apply Euler's method twice to approximate the solution to the initial value problem on the interval $\left[0, \frac{1}{2}\right]$, first with step size $h = 0.25$, then with step size $h = 0.1$. Compare the three-decimal place values of the two approximations at $x = \frac{1}{2}$ with the value of $y\left(\frac{1}{2}\right)$ of the actual solution. (Round to three decimal places)

$$y' = -y, y(0) = 7, y(x) = 7e^{-x}$$

(a)

$$\begin{aligned} x_0 &= 0 \\ y_0 &= 7 \\ f(x,y) &= -y \\ h &= 0.25 \end{aligned}$$

$$\begin{aligned} y_1 &= (7) + (0.25)\left[-(7)\right] = 5.25 \\ y_2 &= (5.25) + (0.25)\left[-(5.25)\right] = 3.9375 \end{aligned}$$

The Euler approximation when $h = 0.25$ of $y\left(\frac{1}{2}\right)$ is

$$\boxed{3.938}$$

(b)

$$\begin{aligned} x_0 &= 0 \\ y + 0 &= 7 \\ f(x,y) &= -y \\ h &= 0.1 \end{aligned}$$

$$\begin{aligned} y_1 &= (7) + (0.1)\left[-(7)\right] = 6.3 \\ y_2 &= (6.3) + (0.1)\left[-(6.3)\right] = 5.67 \\ y_3 &= (5.67) + (0.1)\left[-(5.67)\right] = 5.103 \\ y_4 &= (5.103) + (0.1)\left[-(5.103)\right] = 4.5927 \\ y_5 &= (4.5927) + (0.1)\left[-(4.5927)\right] = 4.13343 \end{aligned}$$

The Euler approximation when $h = 0.1$ of $y\left(\frac{1}{2}\right)$ is

$$\boxed{4.133}$$

(c)

$$\begin{aligned} y &= \left(7e^{-x}\right) \\ &= \left(7e^{-\frac{1}{2}}\right) \\ y &= 4.24571 \end{aligned}$$

The value of $y\left(\frac{1}{2}\right)$ using the actual solution is

$$\boxed{4.246}$$

(d) The approximation 4.133, using the <u>lesser</u> value of h, is closer to the value of $y\left(\frac{1}{2}\right)$ found using the actual solution.

1.2 2.4.7

Apply Euler's method twice to approximate the solution to the initial value problem on the interval $\left[0, \frac{1}{2}\right]$, first with step size $h = 0.25$, then with step size $h = 0.1$. Compare the three-decimal-place values of the two approximations at $x = \frac{1}{2}$ with the value of $y\left(\frac{1}{2}\right)$ of the actual solution. (Round to three decimal places)

$$y' = -3x^2y, y(0) = 8, y(x) = 8e^{-x^3}$$

(a)

$$\begin{aligned} x_0 &= 0 \\ y_0 &= 8 \\ f(x, y) &= -3x^2y \\ h &= 0.25 \end{aligned}$$

$$\begin{aligned} y_1 &= (8) + (0.25)\left[-3(0)^2(8)\right] = 8 \\ y_2 &= (8) + (0.25)\left[-3(0.25)^2(8)\right] = 7.625 \end{aligned}$$

The Euler approximation when $h = 0.25$ of $y\left(\frac{1}{2}\right)$ is

$$\boxed{7.625}$$

(b)

$$\begin{aligned} x_0 &= 0 \\ y_0 &= 8 \\ f(x, y) &= -3x^2y \\ h &= 0.1 \end{aligned}$$

$$
\begin{aligned}
y_1 &= (8) + (0.1)\left[-3(0)^2(8)\right] = 8 \\
y_2 &= (8) + (0.1)\left[-3(0.1)^2(8)\right] = 7.976 \\
y_3 &= (7.976) + (0.1)\left[-3(0.2)^2(7.976)\right] \approx 7.88029 \\
y_4 &\approx (7.88029) + (0.1)\left[-3(0.3)^2(7.88029)\right] \approx 7.66752 \\
y_5 &\approx (7.66752) + (0.1)\left[-3(0.4)^2(7.66752)\right] \approx 7.29948
\end{aligned}
$$

The Euler approximation when h $h = 0.1$ of $y\left(\frac{1}{2}\right)$ is

$$\boxed{7.299}$$

(c)

$$
\begin{aligned}
y &= 8e^{-x^3} \\
&= 8e^{-\left(\frac{1}{2}\right)^3} \\
y &= 7.05998
\end{aligned}
$$

The value of $y\left(\frac{1}{2}\right)$ using the actual solution is

$$\boxed{7.060}$$

(d) The approximation 7.299, using the <u>lesser</u> value of h, is closer to the value of $y\left(\frac{1}{2}\right)$ found using the actual solution.