Contents

1 Torque

Torque ($\vec{\tau}$)

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Torque is a measure of a force's ability to rotate an object around a reference point

$\vec{r} \rightarrow$ "lever arm"
$\vec{F} \rightarrow$ "applied force"

line of action: the direction that a vector points along
With torque, you are allowed to move vectors along the line of action. Any force whose line of action goes through it contributes no torque.

1.1 Two Equations of Static Equilibrium

$$\sum F = 0, \left(\frac{d\vec{v}}{dt} = 0\right) \tag{1}$$

$$\sum \tau = 0, \left(\frac{d\vec{\omega}}{dt} = 0\right) \tag{2}$$

1.2 Example 1.2

Two children $m_1 = 20\,\mathrm{kg}$ and $m_2 = 35\,\mathrm{kg}$ wish to play on an $M = 200\,\mathrm{kg}$ $L = 4\,\mathrm{m}$ seesaw which is balanced in the middle. Child m_1 sites on the left end.

Where must m_2 sit to balance the seesaw?

$$\begin{aligned}
m_1 g &= 20\,\mathrm{kg} \cdot 10\,\mathrm{m\,s^{-2}} \\
&= 200\,\mathrm{N} \\
r_{m_1} &= 2\,\mathrm{m} \\
m_2 g &= 35\,\mathrm{kg} \cdot 10\,\mathrm{m\,s^{-2}} \\
&= 350\,\mathrm{N} \\
r_{m_2} &=? \\
M &= 200\,\mathrm{kg} \cdot 10\,\mathrm{m\,s^{-2}} \\
&= 2000\,\mathrm{N} \\
N_M &=?
\end{aligned}$$

$$\begin{aligned}
\sum \tau &= 0 \\
(r_{m_1})(m_1 g) - (r_{m_2})(m_2 g) &= 0 \\
r_{m_2} m_2 g &= r_{m_1} m_1 g \\
r_{m_2} &= \frac{m_1}{m_2} r_{m_2} \\
&= \left(\frac{20\,\mathrm{kg}}{35\,\mathrm{kg}}\right)(2\,\mathrm{m}) \\
r_{m_2} &= 1.14\,\mathrm{m}
\end{aligned}$$

$$\boxed{r_{m_2} = 1.14\,\mathrm{m}}$$

1.3 Example 1.3

$$\begin{aligned}
r_F &= 12\,\mathrm{in} \\
r_M &= 6\,\mathrm{in} \\
F &= 220\,\mathrm{N} \\
mg &= 1.5\,\mathrm{N} \\
N &=? \\
r_N &= 1\,\mathrm{in} \\
F_H &=?
\end{aligned}$$

$$
\begin{aligned}
\sum \tau &= 0 \\
(r_{F_{\text{app}}})(F_{\text{app}}) + (r_m)(mg) - (r_N)(N) &= 0 \\
(r_N)(N) &= (r_{F_{\text{app}}})(F_{\text{app}}) + (r_m)(mg) \\
N &= \frac{(r_{F_{\text{app}}})(F_{\text{app}}) + (r_m)(mg)}{(r_N)} \\
&= \frac{(12\,\text{in})(220\,\text{N}) + (6\,\text{in})(1.5\,\text{N})}{1\,\text{in}} \\
N &= 2650\,\text{N}
\end{aligned}
$$

$$\boxed{N = 2650\,\text{N}}$$

$$
\begin{aligned}
\sum F_y &= 0 \\
F + mg - N - F_H &= 0 \\
F_H &= F + mg - N \\
&= 220\,\text{N} + 1.5\,\text{N} - 2650\,\text{N} \\
F_H &= -2430\,\text{N}
\end{aligned}
$$

$$\boxed{F_H = -2430\,\text{N}}$$

1.4 Example 1.4

$$
\begin{aligned}
\sum \tau &= 0 \\
\frac{L}{3} T \sin(\theta) - \frac{L}{2} w - Lw &= 0 \\
\frac{L}{3} T \sin(\theta) &= \frac{L}{2} w + Lw \\
\frac{T \sin(\theta)}{3} &= \frac{w}{2} + w \\
T \sin(\theta) &= \frac{3}{2} w + 3w \\
&= \frac{9}{2} w
\end{aligned}
$$

$$
\begin{aligned}
\sum F_x &= 0 \\
-T \cos(\theta) + N &= 0 \\
T \cos(\theta) &= N
\end{aligned}
$$

$$f = \mu N = \mu T \cos(\theta)$$

$$
\begin{aligned}
\sum \tau &= 0 \\
-\frac{fL}{3} - \frac{Lw}{6} - \frac{2Lw}{3} &= 0 \\
-\frac{f}{3} &= \frac{w}{6} + \frac{2w}{3} \\
f &= -3\left(\frac{w}{6} + \frac{2w}{3}\right) \\
&= -\frac{w}{2} - 2w \\
f &= \left|-\frac{5w}{2}\right| = \frac{5w}{2}
\end{aligned}
$$

$$
\begin{aligned}
T\cos(\theta) &= \frac{f}{\mu} \\
&= \frac{5w}{2\mu}
\end{aligned}
$$

$$
\begin{aligned}
\frac{T\sin(\theta)}{T\cos(\theta)} &= \frac{\frac{9w}{2}}{\frac{5w}{2\mu}} \\
\tan(\theta) &= \frac{18\mu w}{10w} \\
\tan(\theta) &= \frac{9\mu}{2}
\end{aligned}
$$

$$
\boxed{\tan(\theta) = \frac{9\mu}{2}}
$$