Homework 4 - 1D Motion

Corey Mostero

Student ID: 256652

Contents

1 Book

1.1 2.13

Turtle's position function

$$x(t) = 50.0\,\mathrm{cm} + (2.00\,\mathrm{cm\,s^{-1}})t - (0.0625\,\mathrm{cm\,s^{-2}})t^2$$

(a) Find the turtle's initial velocity (v_0), initial position (x_0), and initial acceleration (a_0).

$$x_0 = x(0) = 50.0\,\mathrm{cm} + (2.00\,\mathrm{cm\,s^{-1}})(0) - (0.0625\,\mathrm{cm\,s^{-2}})(0)^2$$
$$x_0 = 50.0\,\mathrm{cm}$$

Velocity function $x'(t)$

$$x'(t) = 2.00\,\mathrm{cm\,s^{-1}} - (0.125\,\mathrm{cm\,s^{-2}})t$$

$$v_0 = x'(0) = 2.00\,\mathrm{cm\,s^{-1}} - (0.125\,\mathrm{cm\,s^{-2}})(0)$$
$$v_0 = 2.00\,\mathrm{cm\,s^{-1}}$$

Acceleration function $x''(t)$

$$x''(t) = -0.125\,\mathrm{cm\,s^{-2}}$$

$$a_0 = x''(0) = -0.125\,\mathrm{cm\,s^{-2}}$$
$$a_0 = -0.125\,\mathrm{cm\,s^{-2}}$$

$$\boxed{x_0 = 50.0\,\mathrm{cm}, v_0 = 2.00\,\mathrm{cm\,s^{-1}}, a_0 = -0.125\,\mathrm{cm\,s^{-2}}}$$

(b) Find t when $v = 0$

$$
\begin{aligned}
x'(t) &= 2.00\,\mathrm{cm\,s^{-1}} - (0.125\,\mathrm{cm\,s^{-2}})t \\
2.00\,\mathrm{cm\,s^{-1}} - (0.125\,\mathrm{cm\,s^{-2}})t &= 0 \\
t &= 16\,\mathrm{s}
\end{aligned}
$$

$$\boxed{t = 16\,\mathrm{s}}$$

(c) Find t when $x = 50.0\,\mathrm{cm}$

$$
\begin{aligned}
x(t) &= 50.0\,\mathrm{cm} + (2.00\,\mathrm{cm\,s^{-1}})t - (0.0625\,\mathrm{cm\,s^{-2}})t^2 \\
50.0\,\mathrm{cm} &= 50.0\,\mathrm{cm} + (2.00\,\mathrm{cm\,s^{-1}})t - (0.0625\,\mathrm{cm\,s^{-2}})t^2 \\
t &= 0\,\mathrm{s}, 32\,\mathrm{s}
\end{aligned}
$$

$$\boxed{t = 32\,\mathrm{s}}$$

(d) Find t when $x = 60.0\,\mathrm{cm}$ and $x = 40.0\,\mathrm{cm}$. Find v at each time.

$$
\begin{aligned}
50.0\,\mathrm{cm} + (2.00\,\mathrm{cm\,s^{-1}})t - (0.0625\,\mathrm{cm\,s^{-2}})t^2 &= 40.0\,\mathrm{cm} \\
t &= -4.40\,\mathrm{s}, 36.4\,\mathrm{s} \\
t &= 36.4\,\mathrm{s}
\end{aligned}
$$

$$
\begin{aligned}
50.0\,\mathrm{cm} + (2.00\,\mathrm{cm\,s^{-1}})t - (0.0625\,\mathrm{cm\,s^{-2}})t^2 &= 60.0\,\mathrm{cm} \\
t &= 6.20\,\mathrm{s}, 25.8\,\mathrm{s}
\end{aligned}
$$

$$
\begin{aligned}
x'(36.4\,\mathrm{s}) &= 2.00\,\mathrm{cm\,s^{-1}} - (0.125\,\mathrm{cm\,s^{-2}})(36.4\,\mathrm{s}) \\
v_1 &= -2.55\,\mathrm{cm\,s^{-1}} \\
x'(6.20\,\mathrm{s}) &= 2.00\,\mathrm{cm\,s^{-1}} - (0.125\,\mathrm{cm\,s^{-2}})(6.20\,\mathrm{s}) \\
v_2 &= 1.225\,\mathrm{cm\,s^{-1}} \\
x'(25.8\,\mathrm{s}) &= 2.00\,\mathrm{cm\,s^{-1}} - (0.125\,\mathrm{cm\,s^{-2}})(25.8\,\mathrm{s}) \\
v_3 &= -1.225\,\mathrm{cm\,s^{-1}}
\end{aligned}
$$

$$\boxed{t = 36.4\,\mathrm{s}, 6.20\,\mathrm{s}, 25.8\,\mathrm{s}, v = -2.55\,\mathrm{cm\,s^{-1}}, 1.23\,\mathrm{cm\,s^{-1}}, -1.23\,\mathrm{cm\,s^{-1}}}$$

(e) Graphs

1.2 2.20

$$
\begin{aligned}
v_o &= 0 \\
v_f &= 73.14\,\mathrm{m\,s^{-1}} \\
t &= 30.0\,\mathrm{ms} = 0.03\,\mathrm{s}
\end{aligned}
$$

(a) Find acceleration a during serve

$$\begin{aligned}
v_f &= v_o + at \\
a &= \frac{v_f - v_o}{t} \\
a &= \frac{73.14\,\mathrm{m\,s^{-1}} - 0}{0.03\,\mathrm{s}} \\
a &= 2438\,\mathrm{m\,s^{-2}}
\end{aligned}$$

$$\boxed{a = 2438\,\mathrm{m\,s^{-2}}}$$

(b) Find distance x traveled during serve

$$\begin{aligned}
\Delta x &= v_o t + \frac{1}{2}at^2 \\
\Delta x &= (0)(0.03\,\mathrm{s}) + \frac{1}{2}(2438\,\mathrm{m\,s^{-2}})(0.03\,\mathrm{s})^2 \\
\Delta x &= 1.0971\,\mathrm{m}
\end{aligned}$$

$$\boxed{x = 1.097\,\mathrm{m}}$$

1.3 2.29

(a) Find acceleration a at $t = 3\,\mathrm{s}, 7\,\mathrm{s}, 11\,\mathrm{s}$

$$a_t = \frac{y_1 - y_0}{x_1 - x_0}$$

$$\begin{aligned}
a_3 &= \frac{20\,\mathrm{m\,s^{-1}} - 20\,\mathrm{m\,s^{-1}}}{3\,\mathrm{s} - 0} \\
a_3 &= 0 \\
a_7 &= \frac{45\,\mathrm{m\,s^{-1}} - 20\,\mathrm{m\,s^{-1}}}{9\,\mathrm{s} - 5\,\mathrm{s}} \\
a_7 &= 6.25\,\mathrm{m\,s^{-2}} \\
a_{11} &= \frac{0\,\mathrm{m\,s^{-1}} - 45\,\mathrm{m\,s^{-1}}}{13\,\mathrm{s} - 9\,\mathrm{s}} \\
a_{11} &= -11.25\,\mathrm{m\,s^{-2}}
\end{aligned}$$

$$\boxed{a_3 = 0, a_7 = 6.25\,\mathrm{m\,s^{-2}}, a_{11} = -11.25\,\mathrm{m\,s^{-2}}}$$

(b) Find distance traveled x at $t = 5\,\mathrm{s}, 9\,\mathrm{s}, 13\,\mathrm{s}$

$$\begin{aligned}
v_{5,0} &= 20\,\mathrm{m\,s^{-1}} \\
x_{5,0} &= (5\,\mathrm{s})(20\,\mathrm{m\,s^{-1}}) \\
x_{5,0} &= 100\,\mathrm{m}
\end{aligned}$$

$$
\begin{aligned}
v_5 &= 20\,\mathrm{m\,s^{-1}} \\
t &= 9\,\mathrm{s} - 5\,\mathrm{s} = 4\,\mathrm{s} \\
a_7 &= 6.25\,\mathrm{m\,s^{-2}} \\
\Delta x_{9,5} &= v_5 t + \frac{1}{2} a_7 t^2 \\
\Delta x_{9,5} &= (20\,\mathrm{m\,s^{-1}})(4\,\mathrm{s}) + \frac{1}{2}(6.25\,\mathrm{m\,s^{-2}})(4\,\mathrm{s})^2 \\
\Delta x_{9,5} &= 130\,\mathrm{m} \\
x_{9,0} &= x_{5,0} + x_{9,5} \\
x_{9,0} &= 230\,\mathrm{m}
\end{aligned}
$$

$$
\begin{aligned}
v_9 &= 45\,\mathrm{m\,s^{-1}} \\
t &= 13\,\mathrm{s} - 9\,\mathrm{s} = 4\,\mathrm{s} \\
a_{11} &= -11.25\,\mathrm{m\,s^{-2}} \\
\Delta x_{13,9} &= v_9 t + \frac{1}{2} a_{11} t^2 \\
\Delta x_{13,9} &= (45\,\mathrm{m\,s^{-1}})(4\,\mathrm{s}) + \frac{1}{2}(-11.25\,\mathrm{m\,s^{-2}})(4\,\mathrm{s})^2 \\
\Delta x_{13,9} &= 90\,\mathrm{m} \\
x_{13,0} &= x_{9,0} + x13, 9 \\
x_{13,0} &= 320\,\mathrm{m}
\end{aligned}
$$

$$
\boxed{x_{5,0} = 100\,\mathrm{m}, x_{9,0} = 230\,\mathrm{m}, x_{13,0} = 320\,\mathrm{m}}
$$

1.4 2.39

$$
\begin{aligned}
\Delta y &= y_f - y_o \\
y_f &= 0 \quad \text{(floor)} \\
y_o &= d = ? \\
v_{0_y} &= 0 \\
t &= ? \\
a_y &= g = -10\,\mathrm{m\,s^{-2}}
\end{aligned}
$$

(a)

$$
\begin{aligned}
\Delta y &= v_{o_y} t + \frac{1}{2} a_y t^2 \\
0 - y_o &= (0)t + \frac{1}{2}(-10\,\mathrm{m\,s^{-2}})t^2 \\
-y_o &= (-5\,\mathrm{m\,s^{-2}})t^2 \\
d &= (5\,\mathrm{m\,s^{-2}})t^2
\end{aligned}
$$

$$\boxed{d = (5\,\mathrm{m\,s^{-2}})t^2}$$

(b)

$$\begin{aligned} d &= 17.6\,\mathrm{cm} = 0.176\,\mathrm{m} \\ 0.176\,\mathrm{m} &= (5\,\mathrm{m\,s^{-2}})t^2 \\ t &= 0.188\,\mathrm{s} \end{aligned}$$

$$\boxed{t = 0.188\,\mathrm{s}}$$

1.5 2.51

Acceleration of motorcycle

$$a_x(t) = At - Bt^2$$

$$\begin{aligned} A &= 1.50\,\mathrm{m\,s^{-3}} \\ B &= 0.120\,\mathrm{m\,s^{-4}} \\ v_o &= 0 \end{aligned}$$

(a)

$$\begin{aligned} v_x(t) &= \int a_x(t)dt \\ &= \int \left(At - Bt^2\right) dt \\ v_x(t) &= \frac{1}{2}At^2 - \frac{1}{3}Bt^3 + C \end{aligned}$$

Assume constant is of value 0

$$\boxed{v_x(t) = (0.75\,\mathrm{m\,s^{-3}})t^2 - (0.04\,\mathrm{m\,s^{-4}})t^3}$$

$$\begin{aligned} x(t) &= \int v_x(t)dt \\ &= \int \left(\frac{1}{2}At^2 - \frac{1}{3}Bt^3 + C\right) \\ x(t) &= \frac{1}{6}At^3 - \frac{1}{12}Bt^4 + Ct + D \end{aligned}$$

Assume constants to be of value 0

$$\boxed{x(t) = (0.25\,\mathrm{m\,s^{-3}})t^3 - (0.01\,\mathrm{m\,s^{-4}})t^4}$$

(b) Find v when $a = 0$ (maximum velocity)

$$\begin{aligned} a_x(t) &= (1.50\,\mathrm{m\,s^{-3}})t - (0.120\,\mathrm{m\,s^{-4}})t^2 = 0 \\ t &= 0, 12.5\,\mathrm{s} \\ v_x(12.5\,\mathrm{s}) &= (0.75\,\mathrm{m\,s^{-3}})(12.5\,\mathrm{s})^2 - (0.04\,\mathrm{m\,s^{-4}})(12.5\,\mathrm{s})^3 \\ v &= 39.1\,\mathrm{m\,s^{-1}} \end{aligned}$$

$$\boxed{v_{\mathrm{max}} = 39.1\,\mathrm{m\,s^{-1}}}$$

1.6 2.71

Acceleration of particle

$$a_x(t) = -2.00\,\mathrm{m\,s^{-2}} + (3.00\,\mathrm{m\,s^{-3}})t$$

(a) Find v_{o_x} so that x at $t = 0, 4.00\,\mathrm{s}$ are the same

$$\begin{aligned} v_x(t) &= \int a_x(t)dt \\ &= \int \left(-2.00\,\mathrm{m\,s^{-2}} + (3.00\,\mathrm{m\,s^{-3}})t\right) dt \\ v_x(t) &= (-2.00\,\mathrm{m\,s^{-2}})t + (1.5\,\mathrm{m\,s^{-3}})t^2 + C \\ x(t) &= \int v_x(t)dt \\ &= \int \left((-2.00\,\mathrm{m\,s^{-2}})t + (1.5\,\mathrm{m\,s^{-3}})t^2 + C\right) dt \\ x(t) &= (-1.00\,\mathrm{m\,s^{-2}})t^2 + (0.5\,\mathrm{m\,s^{-3}})t^3 + Ct + D \end{aligned}$$

$$\begin{aligned} x(0) &= D \\ x(4.00\,\mathrm{s}) &= (-1.00\,\mathrm{m\,s^{-2}})(4.00\,\mathrm{s})^2 + (0.5\,\mathrm{m\,s^{-3}})(4.00\,\mathrm{s})^3 + C(4.00\,\mathrm{s}) + D \\ x(4.00\,\mathrm{s}) &= 16.0\,\mathrm{m} + (4.00\,\mathrm{s})C + D \\ x(0) &= x(4.00\,\mathrm{s}) \\ D &= 16.0\,\mathrm{m} + (4.00\,\mathrm{s})C + D \\ C &= -4.00\,\mathrm{m\,s^{-1}} \end{aligned}$$

$$\boxed{v_{0_x} = -4.00\,\mathrm{m\,s^{-1}}}$$

(b) Find velocity at $t = 4.00\,\mathrm{s}$

$$\begin{aligned} v_x(4.00\,\mathrm{s}) &= (-2.00\,\mathrm{m\,s^{-2}})(4.00\,\mathrm{s}) + (1.5\,\mathrm{m\,s^{-3}})(4.00\,\mathrm{s})^2 + (-4.00\,\mathrm{s}) \\ v &= 12.0\,\mathrm{m\,s^{-1}} \end{aligned}$$

$$\boxed{v_{4.00\,\mathrm{s}} = 12.0\,\mathrm{m\,s^{-1}}}$$

2 Lab Manual

2.1 471

(a)

$$v = at(1 + 2 + 3 + \cdots + n)$$
$$\sum_{t=1}^{n} t = \frac{n(n+1)}{2}$$
$$v = at\left(\frac{n(n+1)}{2}\right)$$
$$\boxed{v = at\left(\frac{n(n+1)}{2}\right)}$$

(b)

$$x = \frac{1}{2}at^2(1 + 4 + 9 + 16 + \cdots + n^2)$$
$$\sum_{t=1}^{n} t^2 = \frac{n(n+1)(2n+1)}{6}$$
$$x = \frac{1}{2}at^2\left(\frac{n(n+1)(2n+1)}{6}\right)$$
$$x = \frac{n(2n^2 + 3n + 1)at^2}{12}$$
$$\boxed{x = \frac{n(2n^2 + 3n + 1)at^2}{12}}$$

2.2 474

$$\begin{aligned} v_{o_A} &= 0 \\ a_A &= 8.00\,\mathrm{m\,s^{-2}} \\ x_{0_{B,A}} &= 30\,\mathrm{m} \\ v_B &= 40\,\mathrm{m\,s^{-1}} \end{aligned}$$

Find distance traveled within the first two seconds using data related to car A. (At $t = 2\,\mathrm{s}$ car A and B are at the same position, so using either car's data is valid)

$$\begin{aligned} \Delta x &= v_{o_A}t + \frac{1}{2}a_A t^2 \\ &= (0)(2\,\mathrm{s}) + \frac{1}{2}(8.00\,\mathrm{m\,s^{-2}})(2\,\mathrm{s})^2 \\ \Delta x &= 16.0\,\mathrm{m} \end{aligned}$$

Now find the when car A and B meet

$$
\begin{aligned}
v_{f_A} &= v_{o_A} + a_A t \\
&= 0 + (8.00\,\mathrm{m\,s^{-2}})(2\,\mathrm{s}) \\
v_{f_A} &= 16.0\,\mathrm{m\,s^{-1}} \\
\Delta x_A &= v_{o_A} t + \frac{1}{2} a_A t^2 \\
\Delta x_A &= (16.0\,\mathrm{m\,s^{-1}})t + (4.0\,\mathrm{m\,s^{-2}})t^2
\end{aligned}
$$

$$
\begin{aligned}
\Delta x_B &= v_{o_B} t + \frac{1}{2} b_A t^2 \\
\Delta x_B &= (40\,\mathrm{m\,s^{-1}})t + (3.0\,\mathrm{m\,s^{-2}})t^2
\end{aligned}
$$

Set the distance equations equal to each other to find t (the time the cars meet)

$$
\begin{gathered}
\Delta x_A = \Delta x_B \\
(16.0\,\mathrm{m\,s^{-1}})t + (4.0\,\mathrm{m\,s^{-2}})t^2 = (40\,\mathrm{m\,s^{-1}})t + (3.0\,\mathrm{m\,s^{-2}})t^2 \\
t = 0, 24\,\mathrm{s}
\end{gathered}
$$

It is found that car A and B meet at $24\,\mathrm{s}$ after car A initially overtakes car B. Use $t = 24\,\mathrm{s}$ to find the distance that they meet at.

$$
\begin{aligned}
\Delta x_A(24\,\mathrm{s}) &= (16.0\,\mathrm{m\,s^{-1}})(24\,\mathrm{s}) + (4.0\,\mathrm{m\,s^{-2}})(24\,\mathrm{s})^2 \\
x &= 2688\,\mathrm{m}
\end{aligned}
$$

Combine these values with the initial distance / time to find the total distance / time.

$$
\begin{aligned}
x &= 16.0\,\mathrm{m} + 2688\,\mathrm{m} = 2704\,\mathrm{m} \\
t &= 2\,\mathrm{s} + 24\,\mathrm{s} = 26\,\mathrm{s}
\end{aligned}
$$

$$\boxed{x = 2704\,\mathrm{m}, t = 26\,\mathrm{s}}$$