Homework 9 - Circular Motion

Corey Mostero - 2566652

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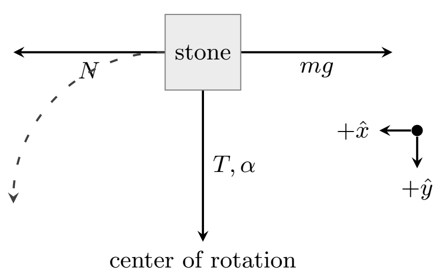
Contents

1 Book

1.1 5.43

$$m = 0.80\,\mathrm{kg}$$
$$L = 0.90\,\mathrm{m}$$
$$T = 60.0\,\mathrm{N}$$

(a) Draw a free-body diagram of the stone.



(b) Find the maximum speed the stone can attain without the string breaking.

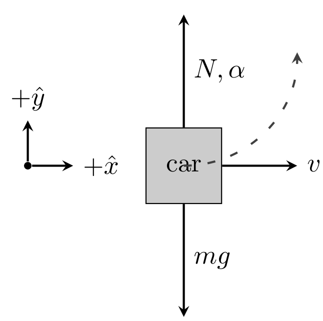
$$\sum F_c = m\alpha$$
$$T = m\left(\frac{v_{max}^2}{L}\right)$$
$$v_{max} = \sqrt{\frac{TL}{m}}$$
$$v_{max} = \sqrt{\frac{(60.0\,\mathrm{N})(0.90\,\mathrm{m})}{0.80\,\mathrm{kg}}}$$
$$v_{max} = 8.22\,\mathrm{m\,s^{-1}}$$

$$\boxed{v_{max} = 8.22\,\mathrm{m\,s^{-1}}}$$

1.2 5.45

$$m = 1.60\,\mathrm{kg}$$
$$v = 12.0\,\mathrm{m\,s^{-1}}$$
$$r = 5.00\,\mathrm{m}$$

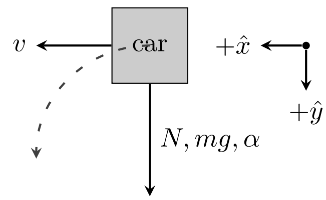
(a) What is the normal force at point A?



$$\sum F_c = m\alpha$$
$$N = m\left(\frac{v^2}{r}\right) + mg$$
$$N = (1.60\,\mathrm{kg})\left(\frac{(12.0\,\mathrm{m\,s^{-1}})^2}{5.00\,\mathrm{m}}\right) + (1.60\,\mathrm{kg})(10.0\,\mathrm{m\,s^{-2}})$$
$$N = 62.1\,\mathrm{m\,s^{-2}}$$

$$\boxed{N = 62.1\,\mathrm{m\,s^{-2}}}$$

(b) What is the normal force at point B?



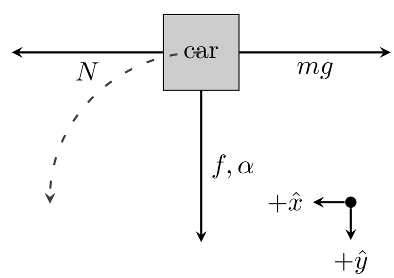
$$
\begin{aligned}
\sum F_c &= m\alpha \\
N + mg &= m\left(\frac{v^2}{r}\right) \\
N &= m\left(\frac{v^2}{r}\right) - mg \\
N &= 1.60\,\mathrm{kg}\left(\frac{(12.0\,\mathrm{m\,s^{-1}})^2}{5.00\,\mathrm{m}} - 10.0\,\mathrm{m\,s^{-2}}\right) \\
N &= 30.1\,\mathrm{m\,s^{-2}}
\end{aligned}
$$

$$\boxed{N = 30.1\,\mathrm{m\,s^{-2}}}$$

1.3 5.48

$$
\begin{aligned}
r &= 230.0\,\mathrm{m} \\
v &= 28.0\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

(a) What is the minimum coefficient of static friction that will prevent sliding?



$$
\begin{aligned}
\sum F_x &= 0 \\
N &= mg
\end{aligned}
$$

$$\begin{aligned}
\sum F_y &= m\alpha \\
f &= m\left(\frac{v^2}{r}\right) \\
\mu N &= m\left(\frac{v^2}{r}\right), \quad N = mg \\
\mu &= \frac{v^2}{rg} \\
\mu &= \frac{(28.0\,\mathrm{m\,s^{-1}})^2}{(230.0\,\mathrm{m})(10.0\,\mathrm{m\,s^{-2}})} \\
\mu &= 0.341
\end{aligned}$$

$$\boxed{\mu = 0.341}$$

(b) Suppose that the highway is icy and the coefficient of static friction between the tires and pavement is only one-third of what you found in part **(a)**. What should be the maximum speed of the car so that it can round the curve safely?

$$\begin{aligned}
\sum F_y &= m\alpha \\
\frac{\mu}{3}mg &= m\left(\frac{v^2}{r}\right) \\
v &= \sqrt{\frac{\mu g r}{3}} \\
v &= \sqrt{\frac{(0.341)(10.0\,\mathrm{m\,s^{-2}})(230.0\,\mathrm{m})}{3}} \\
v &= 16.2\,\mathrm{m\,s^{-1}}
\end{aligned}$$

$$\boxed{v = 16.2\,\mathrm{m\,s^{-1}}}$$

1.4 5.54

$$\begin{aligned}
D &= 100\,\mathrm{m} \\
r &= \frac{D}{2} = 50.0\,\mathrm{m} \\
\mathrm{rpm} &= 1\,\mathrm{rev\,min^{-1}}
\end{aligned}$$

(a) Find the speed of the passengers when the Ferris wheel is rotating at this rate.

$$v = \frac{2\pi r}{T}$$
$$v = \frac{2\pi(50.0\,\mathrm{m})}{60.0\,\mathrm{s}}$$
$$v = 5.24\,\mathrm{m\,s^{-1}}$$

$$\boxed{v = 5.24\,\mathrm{m\,s^{-1}}}$$

(b) A passenger weighs 902 N at the weight-guessing booth on the ground. What is his apparent weight at the highest and at the lowest point on the Ferris wheel?

$$m = \frac{w}{g} = \frac{902\,\mathrm{N}}{10.0\,\mathrm{m\,s^{-2}}} = 90.2\,\mathrm{kg}$$

$$\begin{aligned}
\sum F_y^{top} &= m\alpha \\
mg &= m\left(\frac{v^2}{r}\right) + N_{top} \\
N_{top} &= m\left(-\frac{v^2}{r} + g\right) \\
N_{top} &= (90.2\,\mathrm{kg})\left(-\frac{(5.24\,\mathrm{m\,s^{-1}})^2}{50.0\,\mathrm{m}} + 10.0\,\mathrm{m\,s^{-2}}\right) \\
N_{top} &= 852.5\,\mathrm{N}
\end{aligned}$$

$$\begin{aligned}
\sum F_y^{bottom} &= m\alpha \\
N_{bottom} &= m\left(\frac{v^2}{r}\right) + mg \\
N_{bottom} &= m\left(\frac{v^2}{r} + g\right) \\
N_{bottom} &= (90.2\,\mathrm{kg})\left(\frac{(5.24\,\mathrm{m\,s^{-1}})^2}{50.0\,\mathrm{m}} + 10.0\,\mathrm{m\,s^{-2}}\right) \\
N_{bottom} &= 951.5\,\mathrm{N}
\end{aligned}$$

$$\boxed{N_{top} = 852.5\,\mathrm{N}, N_{bottom} = 951.5\,\mathrm{N}}$$

(c) What would be the time for one revolution if the passenger's apparent weight at the highest point were zero?

$$v = \frac{2\pi r}{T}$$
$$T = \frac{2\pi r}{v}$$

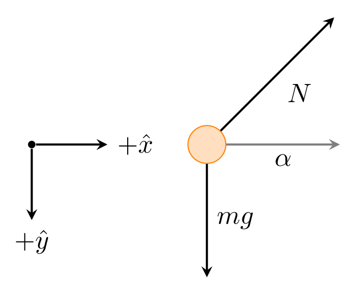
$$
\begin{aligned}
\sum F_y^{top} &= m\alpha \\
mg &= m\left(\frac{v^2}{r}\right) + N \\
v &= \sqrt{gr - \frac{N}{m}}, \quad N = 0 \\
v &= \sqrt{gr} \\
v &= \sqrt{(10.0\,\mathrm{m\,s^{-2}})(50.0\,\mathrm{m})} \\
v &= 22.4\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
T &= \frac{2\pi(50.0\,\mathrm{m})}{22.4\,\mathrm{m\,s^{-1}}} \\
T &= 14.0\,\mathrm{s}
\end{aligned}
$$

$$
\boxed{T = 14.0\,\mathrm{s}}
$$

1.5 5.107

$$
\begin{aligned}
r &= 0.100\,\mathrm{m} \\
\mathrm{rpm} &= 4.80\,\mathrm{rev\,s^{-1}}
\end{aligned}
$$



(a) Find the angle β at which the bead is in vertical equilibrium.

$$
\begin{aligned}
\sum F_x^{bead} &= m\alpha \\
N_x \sin(\beta) &= m\alpha \\
N_x &= \frac{m\frac{v^2}{r}}{\sin(\beta)} = \frac{mv^2}{r\sin(\beta)}
\end{aligned}
$$

$$
\begin{aligned}
\sum F_y^{bead} &= 0 \\
N_y \cos(\beta) &= mg \\
N_y &= \frac{mg}{\cos(\beta)}
\end{aligned}
$$

$$
\begin{aligned}
N_x &= N_y \\
\frac{mv^2}{r\sin(\beta)} &= \frac{mg}{\cos(\beta)} \\
v^2\cos(\beta) &= gr\sin(\beta) \\
\tan(\beta) &= \frac{v^2}{gr} \\
\beta &= \arctan\left(\frac{v^2}{gr}\right) \\
\beta &= \arctan\left(\frac{\left(\frac{2\pi r}{T}\right)^2}{gr}\right) \\
\beta &= \arctan\left(\frac{4\pi^2 r}{gT^2}\right) \\
\beta &= \arctan\left(\frac{4\pi^2(0.100\,\mathrm{m})}{(10.0\,\mathrm{m\,s^{-2}})\left(\frac{1}{4.80}\mathrm{s}\right)^2}\right) \\
\beta &= 83.7^\circ
\end{aligned}
$$

$$\boxed{\beta = 83.7^\circ}$$

(b) Is it possible for the bead to "ride" at the same elevation as the center of the hoop?

$$\beta = 90.0^\circ$$

$$
\begin{aligned}
\frac{v^2}{r\sin(\beta)} &= \frac{g}{\cos(\beta)} \\
v &= \sqrt{\frac{gr\sin(\beta)}{\cos(\beta)}} \\
v &= \sqrt{\frac{(10.0\,\mathrm{m\,s^{-2}})(0.100\,\mathrm{m})\sin(90.0^\circ)}{\cos(90.0^\circ)}} \\
v &= 0
\end{aligned}
$$

No it is not possible as the velocity would have to be zero, but would instead mean that the bead isn't moving.

(c) What will happen if the hoop rotates at $1.00\,\mathrm{rev\,s^{-1}}$?

$$
\begin{aligned}
\beta &= \arctan\left(\frac{4\pi^2(0.100\,\mathrm{m})}{(10.0\,\mathrm{m\,s^{-2}})(1.00\,\mathrm{s})^2}\right) \\
\beta &= 21.5^\circ
\end{aligned}
$$

The bead ends up swinging at an angle lower and closer to the vertical axis.

2 Lab Manual

2.1 1072

2.2 1073

2.3 1082

2.4 1087

2.5 1088