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1 Section 4.1

1.1 4.1.1

Find $|a-b|, 2a+b, 3a-4b$

$$a = \begin{bmatrix} 5 \\ 5 \\ -6 \end{bmatrix}, b = \begin{bmatrix} 2 \\ -2 \\ -5 \end{bmatrix}$$

$$\begin{aligned} \|a-b\| &= \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix} \\ &= \sqrt{(3)^2 + (7)^1 + (-1)^2} \\ \|a-b\| &= \sqrt{59} \end{aligned}$$

$$\boxed{\|a-b\| = \sqrt{59}}$$

$$\begin{aligned} 2a+b &= \begin{bmatrix} 12 \\ 8 \\ -17 \end{bmatrix} \\ 2a+b &=< 12, 8, -17 > \end{aligned}$$

$$\boxed{2a+b =< 12, 8, -17 >}$$

$$\begin{aligned} 3a-4b &= \begin{bmatrix} 7 \\ 23 \\ 2 \end{bmatrix} \\ 3a-4b &=< 7, 23, 2 > \end{aligned}$$

$$\boxed{3a-4b =< 7, 23, 2 >}$$

1.2 4.1.3

Find $\|a-b\|$, $2a+b$, and $4a-5b$.

$$a = 5\hat{\imath} + 2\hat{\jmath} + 7\hat{k}, b = 7\hat{\imath} + 8\hat{\jmath} - 5\hat{k}$$

$$\|a-b\| = \sqrt{(5-7)^2 + (2-8)^2 + (7-(-5))^2}$$
$$\|a-b\| = 2\sqrt{46}$$

$$\boxed{2\sqrt{46}}$$

$$2a+b = \begin{bmatrix} 2(5)+7 \\ 2(2)+8 \\ 2(7)+(-5) \end{bmatrix}$$
$$2a+b = \begin{bmatrix} 17 \\ 12 \\ 9 \end{bmatrix}$$

$$\boxed{17\hat{\imath} + 12\hat{\jmath} + 9\hat{k}}$$

$$4a-5b = \begin{bmatrix} 4(5)-5(7) \\ 4(2)-5(8) \\ 4(7)-5(-5) \end{bmatrix}$$
$$4a-5b = \begin{bmatrix} -15 \\ -32 \\ 53 \end{bmatrix}$$

$$\boxed{-15\hat{\imath} - 32\hat{\jmath} + 53\hat{k}}$$

1.3 4.1.5

Determine whether the given vectors u and v are linearly dependent or linearly independent.

$$u = \begin{bmatrix} 4 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\boxed{\text{Linearly Dependent, } u = \frac{3}{4}v}$$

1.4 4.1.7

Determine whether the given vectors u and v are linearly dependent or linearly independent.

$$u = \begin{bmatrix} -9 \\ 9 \end{bmatrix}, v = \begin{bmatrix} 9 \\ 9 \end{bmatrix}$$

$$a \begin{bmatrix} -9 \\ 9 \end{bmatrix} + b \begin{bmatrix} 9 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} -9a + 9b &= 0 \\ 9b &= 9a \\ b &= a \end{aligned}$$

$$\begin{aligned} 9a + 9b &= 0 \\ 9a + 9a &= 0 \\ 18a &= 0 \\ a &= 0 \end{aligned}$$

$$\begin{aligned} -9a + 9b &= 0 \\ -9(0) + 9b &= 0 \\ b &= 0 \end{aligned}$$

$$\boxed{\text{Linearly Independent}}$$

1.5 4.1.9

Express w as a linear combination of u and v.

$$u = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, v = \begin{bmatrix} -1 \\ 8 \end{bmatrix}, w = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 5 \\ -3 & 8 & 0 \end{bmatrix}$$

$$\begin{aligned} A_2 &= A_2 + 3A_1 \\ A &= \begin{bmatrix} 1 & -1 & 5 \\ 0 & 5 & 15 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} 5b &= 15 \\ b &= 3 \end{aligned}$$

$$\begin{aligned} a-b&=5\\ a-(3)&=5\\ a&=8 \end{aligned}$$

$$\boxed{w=8u+3v}$$

1.6 4.1.13

Express w as a linear combination of u and v.

$$u=\begin{bmatrix}7\\8\end{bmatrix}, v=\begin{bmatrix}5\\7\end{bmatrix}, w=\begin{bmatrix}-4\\-2\end{bmatrix}$$

$$A=\begin{bmatrix}7&5&-4\\8&7&-2\end{bmatrix}$$

$$\begin{aligned} A_1&=\frac{1}{7}A_1\\ A&=\begin{bmatrix}1&\frac{5}{7}&-\frac{4}{7}\\8&7&-2\end{bmatrix} \end{aligned}$$

$$\begin{aligned} A_2&=A_2-8A_1\\ A&=\begin{bmatrix}1&\frac{5}{7}&-\frac{4}{7}\\0&\frac{9}{7}&\frac{18}{7}\end{bmatrix} \end{aligned}$$

$$\begin{aligned} \frac{9}{7}b&=\frac{18}{7}\\ b&=2 \end{aligned}$$

$$\begin{aligned} a+\frac{5}{7}b&=-\frac{4}{7}\\ a+\frac{5}{7}(2)&=-\frac{4}{7}\\ a&=-2 \end{aligned}$$

$$\boxed{w=-2u+2v}$$

1.7 4.1.15

Use the theorem for three linearly independent vectors (that is, calculate a determinant) to determine whether the given vectors u, v, and w are linearly dependent or independent.

$$u=\begin{bmatrix}3\\-1\\4\end{bmatrix}, v=\begin{bmatrix}4\\6\\-7\end{bmatrix}, w=\begin{bmatrix}7\\5\\-3\end{bmatrix}$$

$$\det(A) = \begin{bmatrix} 3 & 4 & 7 \\ -1 & 6 & 5 \\ 4 & -7 & -3 \end{bmatrix}$$
$$\det(A) = 3\begin{bmatrix} 6 & 5 \\ -7 & -3 \end{bmatrix} - 4\begin{bmatrix} -1 & 5 \\ 4 & -3 \end{bmatrix} + 7\begin{bmatrix} -1 & 6 \\ 4 & -7 \end{bmatrix}$$
$$\det(A) = 3(6 \cdot -3 - 5 \cdot -7) - 4(-1 \cdot -3 - 5 \cdot 4) + 7(-1 \cdot -7 - 6 \cdot 4)$$
$$\det(A) = 0$$

$$\boxed{\det(A) = 0}$$

1.8 4.1.19

Solve a linear system to determine whether the given vectors u, v, and w are linearly independent or dependent. If they are linearly dependent, find scalars a, b, and c not all zero such that $au + bv + cw = 0$.

$$u = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, v = \begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix}, w = \begin{bmatrix} -6 \\ -2 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -3 & -6 \\ 0 & 1 & -2 \\ 1 & -1 & -4 \end{bmatrix}$$

$$A_1 = \frac{1}{2}A_1$$
$$A_3 = A_3 - A_1$$
$$A = \begin{bmatrix} 1 & -\frac{3}{2} & -3 \\ 0 & 1 & -2 \\ 0 & \frac{1}{2} & -1 \end{bmatrix}$$

$$A_3 = A_3 - \frac{1}{2}A_2$$
$$A = \begin{bmatrix} 1 & -\frac{3}{2} & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$b - 2c = 0$$
$$b = 2c$$

$$a - \frac{3}{2}b - 3c = 0$$
$$a - \frac{3}{2}(2c) - 3c = 0$$
$$a = 6c$$

If $c = 1$, then

$$a = 6(1) = 6$$
$$b = 2(1) = 2$$

$$\boxed{6u + 2v + w = 0}$$

1.9 4.1.27

Express the vector t as a linear combination of the vectors u, v, and w.

$$t = \begin{bmatrix} 0 \\ 0 \\ 14 \end{bmatrix}, u = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}, v = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}, w = \begin{bmatrix} 4 \\ -1 \\ -13 \end{bmatrix}$$

$$A = \left[\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 3 & -1 & -3 & 0 \\ 4 & 2 & -10 & 14 \end{array}\right]$$

$$A_3 = A_3 - 4A_1$$
$$A = \left[\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 0 & 2 & -12 & 0 \\ 0 & 6 & -22 & 14 \end{array}\right]$$

$$A_3 = A_3 - 3A_2$$
$$A = \left[\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 0 & 2 & -12 & 0 \\ 0 & 0 & 14 & 14 \end{array}\right]$$

$$14c = 14$$
$$c = 1$$

$$2b - 12c = 0$$
$$2b - 12(1) = 0$$
$$b = 6$$

$$a - b + 3c = 0$$
$$a - (6) + 3(1) = 0$$
$$a = 3$$

$$\boxed{t = 3u + 6v + w}$$

1.10 4.1.29

Show that the given set V is closed under addition and multiplication by scalars and is therefore a subspace of $\mathbb{R}^3$. V is the set of all $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that $x = 0$. Let $a = \begin{bmatrix} 0 \\ y \\ z \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ v \\ w \end{bmatrix}$ be two vectors in V. Find their sum $a + b$.

$$a + b = \begin{bmatrix} 0 + 0 \\ y + v \\ z + w \end{bmatrix}$$

$$a + b = \begin{bmatrix} 0 \\ y + v \\ z + w \end{bmatrix}$$

$$\boxed{a + b = \begin{bmatrix} 0 \\ y + v \\ z + w \end{bmatrix}}$$