

# Homework 12

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### 0.1 Question 1

Density: What is the radius of a sphere that has a density of  $5000 \text{ kg m}^{-3}$  and a mass of  $6.00 \text{ kg}$ ?

$$\rho = 5000 \text{ kg m}^{-3}$$

$$m = 6.00 \text{ kg}$$

$$r = ?$$

$$\rho = \frac{m}{V}$$

$$V = \frac{m}{\rho}$$

$$\frac{4}{3}\pi r^3 = \frac{m}{\rho}$$

$$r = \sqrt[3]{\frac{3m}{4\pi\rho}}$$

$$r = \sqrt[3]{\frac{3(6.00 \text{ kg})}{4\pi(5000 \text{ kg m}^{-3})}}$$

$$r = 0.065922 \text{ m} = 6.59 \text{ cm}$$

$r = 6.59 \text{ cm}$

### 0.2 Question 2

Pressure in a fluid: A cubical box,  $5.00 \text{ cm}$  on each side, is immersed in a fluid. The gauge pressure at the top surface of the box is  $594 \text{ Pa}$  and the gauge pressure on the bottom surface is  $1133 \text{ Pa}$ . What is the density of the fluid?

$$h = 5.00 \text{ cm} = 0.05 \text{ m}$$

$$p_0 = 594 \text{ Pa}$$

$$p_1 = 1133 \text{ Pa}$$

$$\rho = ?$$

$$\begin{aligned}
p_1 &= p_0 + \rho gh \\
\rho &= \frac{p_1 - p_0}{gh} \\
\rho &= \frac{1133 \text{ Pa} - 594 \text{ Pa}}{(9.80 \text{ m s}^{-2})(0.05 \text{ m})} \\
\rho &= 1100 \text{ kg m}^{-3} \\
\boxed{\rho &= 1100 \text{ kg m}^{-3}}
\end{aligned}$$

### 0.3 Question 3

Pressure in a fluid: As shown in the figure, a container has a vertical tube, whose inner radius is 32.00 mm, connected to it at its side. An unknown liquid reaches level *A* in the container and level *B* in this tube - level *A* being 5.0 cm higher than level *B*. The liquid supports a 20.0 cm high column of oil, between levels *B* and *C*, whose density is  $460 \text{ kg m}^{-3}$ . What is the density of the unknown liquid?

$$\begin{aligned}
y_{A,B} &= 5.0 \text{ cm} = 0.05 \text{ m} \\
y_{B,C} &= 20.0 \text{ cm} = 0.20 \text{ m} \\
\rho_{oil} &= 460 \text{ kg m}^{-3} \\
\rho_{unknown} &=?
\end{aligned}$$

$$\begin{aligned}
\rho_{unk} g y_{A,B} &= \rho_{oil} g y_{B,C} \\
\rho_{unk} &= \frac{\rho_{oil} y_{B,C}}{y_{A,B}} \\
\rho_{unk} &= \frac{(460 \text{ kg m}^{-3})(0.20 \text{ m})}{0.05 \text{ m}} \\
\rho_{unk} &= 1840 \text{ kg m}^{-3}
\end{aligned}$$

$$\boxed{\rho_{unknown} = 1840 \text{ kg m}^{-3} \approx 1800 \text{ kg m}^{-3}}$$

### 0.4 Question 4

Pressure in a fluid: In the figure, an open tank contains a layer of oil floating on top of a layer of water (of density  $1000 \text{ kg m}^{-3}$ ) that is 3.0 m thick, as shown. What must be the thickness of the oil layer if the gauge pressure at the bottom of the tank is to be  $5.0 \times 10^4 \text{ Pa}$ ? The density of the oil is  $510 \text{ kg m}^{-3}$ .

$$\begin{aligned}
\rho_{water} &= 1000 \text{ kg m}^{-3} \\
y_{water} &= 3.0 \text{ m} \\
p_0 &= 5.0 \times 10^4 \text{ Pa} \\
\rho_{oil} &= 510 \text{ kg m}^{-3} \\
y_{oil} &=?
\end{aligned}$$

$$\begin{aligned}
\rho_{oil}gy_{oil} &= p_0 + \rho_{water}gy_{water} \\
y_{oil} &= \frac{p_0 + \rho_{water}gy_{water}}{\rho_{oil}g} \\
y_{oil} &= \frac{5.0 \times 10^4 \text{ Pa} + (1000 \text{ kg m}^{-3})(9.80 \text{ m s}^{-2})(3.0 \text{ m})}{(510 \text{ kg m}^{-3})(9.80 \text{ m s}^{-2})} \\
y_{oil} &= 15.8864 \text{ m} = 15.9 \text{ m}
\end{aligned}$$

$$y_{oil} = 15.9 \text{ m} \approx 16.0 \text{ m}$$

### 0.5 Question 5

Pascal's principle: A 12 000 N car is raised using a hydraulic lift, which consists of a U-tube with arms of unequal areas, filled with incompressible oil and capped at both ends with tight-fitting pistons. The wider arm of the U-tube has a radius of 18.0 cm and the narrower arm has a radius of 5.00 cm. The car rests on the piston on the wider arm of the U-tube. The pistons are initially at the same level. What is the initial force that must be applied to the smaller piston in order to start lifting the car?

$$\begin{aligned}
w_{car} &= 12\,000 \text{ N} \\
r_{wide} &= 18.0 \text{ cm} = 0.18 \text{ m} \\
r_{narrow} &= 5.00 \text{ cm} = 0.05 \text{ m} \\
F &=?
\end{aligned}$$

$$\begin{aligned}
\frac{F}{A_{narrow}} &= \frac{w_{car}}{A_{wide}} \\
F &= \frac{w_{car}\pi r_{narrow}^2}{\pi r_{wide}^2} \\
F &= \frac{(12\,000 \text{ N})(0.05 \text{ m})^2}{(0.18 \text{ m})^2} \\
F &= 925.926 \text{ N} = 926.0 \text{ N}
\end{aligned}$$

$$F = 926.0 \text{ N}$$

### 0.6 Question 6

Buoyancy: A board that is 20.0 cm wide, 5.00 cm thick, and 3.00 m long has a density  $350 \text{ kg m}^{-3}$ . The board is floating partially submerged in water of density  $1000 \text{ kg m}^{-3}$ . What fraction of the volume is above the surface of the water?