Week 11 Participation Assignment (2 of 2)

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Contents

1 Part 2

Given a square matrix $\boldsymbol{A}$, we can find the eigenvalues and the corresponding eigenvectors. Then with the eigenvalues and the eigenvectors, we can construct the matrices $\boldsymbol{P}$ and $\boldsymbol{D}$ such that $\boldsymbol{A} = \boldsymbol{PDP}^{-1}$.
For the following given information, construct at least three ways for the matrix $\boldsymbol{D}$.

1) For $\lambda = 1 : \{\langle -1, 0, 1\rangle\}$, for $\lambda = 3 : \{\langle 1, 0, 1\rangle\}$, for $\lambda = 5 : \{\langle 0, 1, 0\rangle\}$

$$\boldsymbol{P} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$
$$\boldsymbol{P}^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$
$$\boldsymbol{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$
$$\boldsymbol{A} = \boldsymbol{PDP}^{-1}$$
$$\boldsymbol{A} = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

2) For $\lambda = 7 : \{\langle 1, 1, 1\rangle\}$, for $\lambda = 4 : \{\langle -1, 1, 0\rangle, \langle -1, 0, 1\rangle\}$

$$\boldsymbol{P} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$
$$\boldsymbol{P}^{-1} = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$$
$$\boldsymbol{D} = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$
$$\boldsymbol{A} = \begin{bmatrix} 5 & -1 & -1 \\ -1 & 5 & 1 \\ -1 & 1 & 5 \end{bmatrix}$$

3) For $\lambda = 4 : \{\langle 4, 0, 5\rangle, \langle 2, -5, 0\rangle\}$, for $\lambda = 7 : \{\langle 1, -1, 1\rangle\}$

$$\boldsymbol{P} = \begin{bmatrix} 4 & 0 & 5 \\ 2 & -5 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\boldsymbol{P}^{-1} = \begin{bmatrix} 1 & 1 & -5 \\ \frac{2}{5} & \frac{1}{5} & -2 \\ -\frac{3}{5} & -\frac{4}{5} & 4 \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} -5 & -12 & 60 \\ 0 & 4 & 0 \\ -\frac{9}{5} & -\frac{12}{5} & 16 \end{bmatrix}$$