Week 10 Participation Assignment (1 of 2)

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Contents

1 Part 1

Let's consider the following matrix

$$A = \begin{bmatrix} 3 & -1 & 3 & 7 & 2 & 2 & 15 \\ -4 & 3 & 11 & 4 & 2 & 3 & -17 \\ -3 & 2 & 6 & 1 & 1 & 1 & -16 \\ 1 & 4 & 40 & 37 & 12 & 17 & 24 \\ -5 & 3 & 7 & -1 & 0 & 1 & -22 \end{bmatrix}$$

We can define the following subspaces:

$$W_1 = \text{colspace}(A), W_2 = \text{rowspace}(A), W_3 = \text{nullspace}(A)$$

Questions:

1) Write W_1, W_2 as spaces of vectors. Make sure you write all the vectors as $\langle x_1, x_2, x_3, \cdots, x_k \rangle$.

2) Identify the ambient spaces of colspace(A), rowspace(A), and nullspace(A).

3) Next, we can define $W_1^\perp$, $W_2^\perp$, $W_3^\perp$. Then what are the ambient spaces of the orthogonal complements?

1.1 1)

1.1.1 Find rref(A)

$$A_2 = 3A_2 + 4A_1$$

$$A = \begin{bmatrix} 3 & -1 & 3 & 7 & 2 & 2 & 15 \\ 0 & 5 & 45 & 40 & 14 & 17 & 9 \\ -3 & 2 & 6 & 1 & 1 & 1 & -16 \\ 1 & 4 & 40 & 37 & 12 & 17 & 24 \\ -5 & 3 & 7 & -1 & 0 & 1 & -22 \end{bmatrix}$$

$A_3 = A_3 + A_1$

$$A = \begin{bmatrix} 3 & -1 & 3 & 7 & 2 & 2 & 15 \\ 0 & 5 & 45 & 40 & 14 & 17 & 9 \\ 0 & 1 & 9 & 8 & 3 & 3 & -1 \\ 1 & 4 & 40 & 37 & 12 & 17 & 24 \\ -5 & 3 & 7 & -1 & 0 & 1 & -22 \end{bmatrix}$$

$A_3 = 5A_3 - A_2$

$$A = \begin{bmatrix} 3 & -1 & 3 & 7 & 2 & 2 & 15 \\ 0 & 5 & 45 & 40 & 14 & 17 & 9 \\ 0 & 0 & 0 & 0 & 1 & -2 & -14 \\ 1 & 4 & 40 & 37 & 12 & 17 & 24 \\ -5 & 3 & 7 & -1 & 0 & 1 & -22 \end{bmatrix}$$

$A_4 = 3A_4 - A_1$

$$A = \begin{bmatrix} 3 & -1 & 3 & 7 & 2 & 2 & 15 \\ 0 & 5 & 45 & 40 & 14 & 17 & 9 \\ 0 & 0 & 0 & 0 & 1 & -2 & -14 \\ 0 & 13 & 117 & 104 & 34 & 49 & 57 \\ -5 & 3 & 7 & -1 & 0 & 1 & -22 \end{bmatrix}$$

$A_4 = 5A_4 - 13A_2$

$$A = \begin{bmatrix} 3 & -1 & 3 & 7 & 2 & 2 & 15 \\ 0 & 5 & 45 & 40 & 14 & 17 & 9 \\ 0 & 0 & 0 & 0 & 1 & -2 & -14 \\ 0 & 0 & 0 & 0 & -12 & 24 & 168 \\ -5 & 3 & 7 & -1 & 0 & 1 & -22 \end{bmatrix}$$

$A_4 = A_4 + 12A_3$

$$A = \begin{bmatrix} 3 & -1 & 3 & 7 & 2 & 2 & 15 \\ 0 & 5 & 45 & 40 & 14 & 17 & 9 \\ 0 & 0 & 0 & 0 & 1 & -2 & -14 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -5 & 3 & 7 & -1 & 0 & 1 & -22 \end{bmatrix}$$

Swap A_4, A_5

$A_4 = 3A_4 + 5A_1$

$$A = \begin{bmatrix} 3 & -1 & 3 & 7 & 2 & 2 & 15 \\ 0 & 5 & 45 & 40 & 14 & 17 & 9 \\ 0 & 0 & 0 & 0 & 1 & -2 & -14 \\ 0 & 4 & 36 & 32 & 10 & 13 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_4 = 5A_4 - 4A_2$$
$$A = \begin{bmatrix} 3 & -1 & 3 & 7 & 2 & 2 & 15 \\ 0 & 5 & 45 & 40 & 14 & 17 & 9 \\ 0 & 0 & 0 & 0 & 1 & -2 & -14 \\ 0 & 0 & 0 & 0 & -6 & -3 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_4 = A_4 + 6A_3$$
$$A = \begin{bmatrix} 3 & -1 & 3 & 7 & 2 & 2 & 15 \\ 0 & 5 & 45 & 40 & 14 & 17 & 9 \\ 0 & 0 & 0 & 0 & 1 & -2 & -14 \\ 0 & 0 & 0 & 0 & 0 & -15 & -75 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_1 = \frac{1}{3}A_1$$
$$A_2 = \frac{1}{5}A_2$$
$$A_4 = -\frac{1}{15}A_4$$
$$A = \begin{bmatrix} 1 & -\frac{1}{3} & 1 & \frac{7}{3} & \frac{2}{3} & \frac{2}{3} & 5 \\ 0 & 1 & 9 & 8 & \frac{14}{5} & \frac{17}{5} & \frac{9}{5} \\ 0 & 0 & 0 & 0 & 1 & -2 & -14 \\ 0 & 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Using calculator to find reduced form:

$$\boxed{\text{rref}(A) = \begin{bmatrix} 1 & 0 & 4 & 5 & 0 & 0 & 3 \\ 0 & 1 & 9 & 8 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}$$

$W_1 = \text{colspace}(A) =$

$$\text{span}\left(c_1 \begin{bmatrix} 3 \\ -4 \\ -3 \\ 1 \\ -5 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 3 \\ 2 \\ 4 \\ 3 \end{bmatrix} + c_3 \begin{bmatrix} 3 \\ 11 \\ 6 \\ 40 \\ 7 \end{bmatrix} + c_4 \begin{bmatrix} 7 \\ 4 \\ 1 \\ 37 \\ -1 \end{bmatrix} + c_5 \begin{bmatrix} 2 \\ 2 \\ 1 \\ 12 \\ 0 \end{bmatrix} + c_6 \begin{bmatrix} 2 \\ 3 \\ 1 \\ 17 \\ 1 \end{bmatrix} + c_7 \begin{bmatrix} 15 \\ -17 \\ -16 \\ 24 \\ -22 \end{bmatrix} \right)$$

$$W_2 = \text{rowspace}(A) =$$
$$\text{span}\left(c_1\begin{bmatrix}3\\-1\\3\\7\\2\\2\\15\end{bmatrix} + c_2\begin{bmatrix}-4\\3\\11\\4\\2\\3\\-17\end{bmatrix} + c_3\begin{bmatrix}-3\\2\\6\\1\\1\\1\\-16\end{bmatrix} + c_4\begin{bmatrix}1\\4\\40\\37\\12\\17\\24\end{bmatrix} + c_5\begin{bmatrix}-5\\3\\7\\-1\\0\\1\\-22\end{bmatrix}\right)$$

1.2 2)

W_1 ambient space: $\mathbb{R}^7$

W_2 ambient space: $\mathbb{R}^5$

W_3 ambient space: $\mathbb{R}^7$

1.3 3)

$W_1^\perp$ ambient space: $\mathbb{R}^7$

$W_2^\perp$ ambient space: $\mathbb{R}^5$

$W_3^\perp$ ambient space: $\mathbb{R}^7$