Homework 7 - Energy

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Contents

1 Book

1.1 6.19

$$m_{\text{asteroid}} = 2.4 \times 10^{15}\,\text{kg}$$
$$v_{\text{asteroid}} = 20\,\text{km}\,\text{s}^{-1} = 2 \times 10^{4}\,\text{m}\,\text{s}^{-1}$$

(a) How much kinetic energy did this meteor deliver to the ground?

$$E = \frac{1}{2}mv^2$$
$$E = \frac{1}{2}(2.4 \times 10^{15}\,\text{kg})(2 \times 10^{4}\,\text{m}\,\text{s}^{-1})^2$$
$$E = 4.8 \times 10^{23}\,\text{kg}\,\text{m}\,\text{s}^{-1}$$

$$\boxed{E = 4.8 \times 10^{23}\,\text{J}}$$

(b) How does this energy compare to the energy released by a 1.0 Mt nuclear bomb?

$$E_{\text{asteroid}} = 4.8 \times 10^{23}\,\text{J}$$
$$E_{\text{bomb}} = 4.184 \times 10^{15}\,\text{J}$$

$$\frac{E_{\text{asteroid}}}{E_{\text{bomb}}}$$
$$\frac{4.8 \times 10^{23}\,\text{J}}{4.184 \times 10^{15}\,\text{J}} = 1.147 \times 10^{8}\,\text{J}$$

> The kinetic energy of the asteroid is 1.147×10^8 J as much kinetic energy from a 1.0 Mt nuclear bomb.

1.2 6.29

$$E_A = 27\,\text{J}$$
$$m_B = \frac{1}{4} E_A$$

(a) If object B also has 27 J of kinetic energy, is it moving faster or slower than object A? By what factor?

$$E_A = 27\,\text{J}$$

$$\begin{aligned}
E_A &= E_B \\
\frac{1}{2} m_A v_A^2 &= \frac{1}{2} m_B v_B^2 \\
m_A v_A^2 &= \left(\frac{1}{4} m_A\right) v_B^2 \\
4 v_A^2 &= v_B^2 \\
v_B &= \sqrt{4 v_A^2} \\
v_B &= 2 v_A
\end{aligned}$$

> The velocity of v_B is two times v_A, implying that object B is moving twice as fast as object A. (The factor would be 2)

(b) By what factor does the speed of each object change if total work -18 J is done on each?

$$W_{\text{total}} = -18\,\text{J}$$

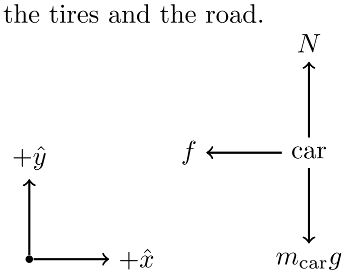
$$\begin{aligned}
W_{\text{total}} &= E_A - E_B \\
-18\,\text{J} &= E_A - 27\,\text{J} \\
E_A &= 9\,\text{J}
\end{aligned}$$

$$\frac{\frac{1}{2}m_{A_f}v_{A_f}^2}{\frac{1}{2}m_{A_i}v_{A_i}^2} = \frac{E_{A_f}}{E_{B_i}}$$
$$\frac{v_{A_f}^2}{v_{A_i}^2} = \frac{9\,\text{J}}{27\,\text{J}}$$
$$v_{A_f}^2 = \frac{1}{3}v_{A_i}^2$$
$$v_{A_f} = \frac{1}{\sqrt{3}}v_{A_i}$$

> As negative work is done upon the object A calculated above, it makes sense that the resulting (final) velocity would be less than the initial velocity (in this case specifically by the factor of $\frac{1}{\sqrt{3}}$).
> It can also be concluded that due to object A and B having both the same kinetic energy ($E_A = E_B$) and work done upon them, the factor calculated will be the same.

1.3 6.31

(a) Use the work-energy theorem to calculate the minimum stopping distance of the car in terms of v_0, g, and the coefficient of kinetic friction μ_k between the tires and the road.



$$\sum F_y = 0$$
$$N = m_{\text{car}}g$$

$$W = -fd$$
$$W = -\mu N d$$
$$W = -\mu m_{\text{car}} g d$$

$$W = E_f - E_i$$
$$-\mu m_{\text{car}} g d = 0 - \frac{1}{2}m_{\text{car}}v_i^2$$
$$d = \frac{v_i^2}{2\mu g}$$

$$\boxed{d = \frac{v_i^2}{2\mu g}}$$

(b) By what factor would the minimum stopping distance change if

(i) the coefficient of kinetic friction were doubled?

$$\mu = 2\mu$$

$$\begin{aligned} W &= E_f - E_i \\ -2\mu m_{\text{car}} g d_1 &= -\frac{1}{2} m_{\text{car}} v_i^2 \\ d_1 &= \frac{v_i^2}{4\mu g} \end{aligned}$$

$$\begin{gathered} d : d_1 \\ \frac{v_i^2}{2\mu g} : \frac{v_i^2}{4\mu g} \\ 1 : \frac{1}{2} \end{gathered}$$

$$\boxed{\frac{1}{2}}$$

(ii) the initial speed were doubled?

$$v_i = 2v_i$$

$$\begin{aligned} W &= E_f - E_i \\ -\mu m_{\text{car}} g d_1 &= -\frac{1}{2} m_{\text{car}} (2v_i)^2 \\ d_1 &= \frac{2v_i^2}{\mu g} \end{aligned}$$

$$\begin{gathered} d : d_1 \\ \frac{v_i^2}{2\mu g} : \frac{2v_i^2}{\mu g} \\ 1 : 4 \end{gathered}$$

$$\boxed{4}$$

(iii) both the coefficient of kinetic friction and the initial speed were doubled?
We can use parts **(i)** and **(ii)** to find the ratio as the W and E_f would simply expand to include the calculated values above and simplify to the expression below:

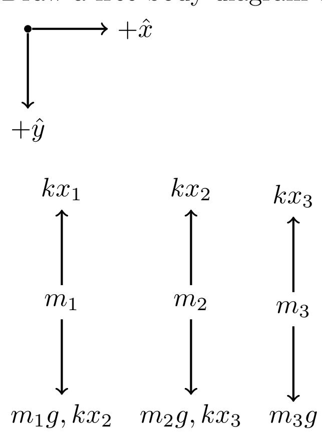
$$4 \cdot \frac{1}{2} = 2$$

$$\boxed{2}$$

1.4 6.33

$$\begin{aligned} m_1 = m_2 = m_3 &= 8.50\,\mathrm{kg} \\ k &= 7.80\,\mathrm{kN\,m^{-1}} = 7.80 \times 10^3\,\mathrm{N\,m^{-1}} \\ x &= 12.0\,\mathrm{cm} \end{aligned}$$

(a) Draw a free-body diagram of each mass.



(b) How long is each spring when hanging as shown?

$$\begin{aligned} \sum F_y^{(m_3)} &= 0 \\ kx_3 &= m_3 g \\ x_3 &= \frac{m_3 g}{k} \\ x_3 &= \frac{(8.50\,\mathrm{kg})(10\,\mathrm{m\,s^{-2}})}{7.80 \times 10^3\,\mathrm{N\,m^{-1}}} \\ x_3 &= 0.011\,\mathrm{m} = 1.1\,\mathrm{cm} \end{aligned}$$

$$
\begin{aligned}
\sum F_y^{(m_2)} &= 0 \\
kx_2 &= m_2 g + kx_3 \\
x_2 &= \frac{m_2 g + kx_3}{k} \\
x_2 &= \frac{(8.50\,\mathrm{kg})(10\,\mathrm{m\,s^{-2}}) + (7.80 \times 10^3\,\mathrm{N\,m^{-1}})(0.011\,\mathrm{m})}{7.80 \times 10^3\,\mathrm{N\,m^{-1}}} \\
x_2 &= 0.022\,\mathrm{m} = 2.2\,\mathrm{cm}
\end{aligned}
$$

$$
\begin{aligned}
\sum F_y^{(m_1)} &= 0 \\
kx_1 &= m_1 g + kx_2 \\
x_1 &= \frac{m_1 g + kx_2}{k} \\
x_1 &= \frac{(8.50\,\mathrm{kg})(10\,\mathrm{m\,s^{-2}}) + (7.80 \times 10^3\,\mathrm{N\,m^{-1}})(0.022\,\mathrm{m})}{7.80 \times 10^3\,\mathrm{N\,m^{-1}}} \\
x_1 &= 0.033\,\mathrm{m} = 3.3\,\mathrm{cm}
\end{aligned}
$$

Spring 1: $12.0\,\mathrm{cm} + x_1 = 15.3\,\mathrm{cm}$
Spring 2: $12.0\,\mathrm{cm} + x_2 = 14.2\,\mathrm{cm}$
Spring 3: $12.0\,\mathrm{cm} + x_3 = 13.1\,\mathrm{cm}$

1.5 6.45

$$
\begin{aligned}
F(x) &= 18.0\,\mathrm{N} - (0.530\,\mathrm{N\,m^{-1}})x \\
m &= 5.00\,\mathrm{kg} \\
v_0 &= 0 \\
x_0 &= 0 \\
x_1 &= 11.0\,\mathrm{m} \\
v_1 &= ? \\
E_i &= 0
\end{aligned}
$$

What is its speed after it has traveled 11.0 m?

$$
\begin{aligned}
W &= \int_{x_0}^{x_1} (F(x))\,dx \\
W &= \int_0^{11.0\,\mathrm{m}} \big(18.0\,\mathrm{N} - (0.530\,\mathrm{N\,m^{-1}})x\big)\,dx \\
W &= \big[(18.0\,\mathrm{N})x - (0.265\,\mathrm{N\,m^{-1}})x^2\big]\Big|_0^{11.0\,\mathrm{m}} \\
W &= \big[(18.0\,\mathrm{N})(11.0\,\mathrm{m}) - (0.265\,\mathrm{N\,m^{-1}})(11.0\,\mathrm{m})^2\big] - 0 \\
W &= 165.935\,\mathrm{J}
\end{aligned}
$$

$$
\begin{aligned}
W &= E_f - E_i, && E_i = 0 \text{ as } v_0 = 0 \\
W &= E_f \\
W &= \frac{1}{2} m v_f^2 \\
v_f &= \sqrt{\frac{2W}{m}} \\
v_f &= \sqrt{\frac{2(165.935\,\mathrm{J})}{5.00\,\mathrm{kg}}} \\
v_f &= 8.15\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$
\boxed{v_f = 8.15\,\mathrm{m\,s^{-1}}}
$$

1.6 6.48

$$
\begin{aligned}
d &= 5.0\,\mathrm{km} \\
v_{\mathrm{run}} &= 10\,\mathrm{km\,h^{-1}} \\
P_{\mathrm{run}} &= 700\,\mathrm{W} \\
v_{\mathrm{walk}} &= 3.0\,\mathrm{km\,h^{-1}} \\
P_{\mathrm{walk}} &= 290\,\mathrm{W}
\end{aligned}
$$

1) Which choice would burn up more energy, and how much energy (in joules) would it burn?

$$
\begin{aligned}
d &= v_{\mathrm{run}} t_{\mathrm{run}} \\
t_{\mathrm{run}} &= \frac{d}{v_{\mathrm{run}}} \\
t_{\mathrm{run}} &= \frac{5.0\,\mathrm{km}}{10\,\mathrm{km\,h^{-1}}} \\
t_{\mathrm{run}} &= 0.5\,\mathrm{h}
\end{aligned}
$$

$$
\begin{aligned}
P &= \frac{W}{t} \\
W_{\mathrm{run}} &= P_{\mathrm{run}} t_{\mathrm{run}} \\
&= (700\,\mathrm{W})(0.5\,\mathrm{h}) \\
&= 350\,\mathrm{W\,h} \\
W_{\mathrm{run}} &= (350\,\mathrm{W\,h}) \left(\frac{3600\,\mathrm{s}}{1\,\mathrm{h}} \right) = 1\,260\,000\,\mathrm{J} = 1.26 \times 10^6\,\mathrm{J}
\end{aligned}
$$

$$t_{\text{walk}} = \frac{d}{v_{\text{walk}}}$$
$$t_{\text{walk}} = \frac{5.0\,\text{km}}{3.0\,\text{km}\,\text{h}^{-1}}$$
$$t_{\text{walk}} = 1.667\,\text{h}$$

$$\begin{aligned} W_{\text{walk}} &= P_{\text{walk}} t_{\text{walk}} \\ &= (290\,\text{W})(1.667\,\text{h}) \\ &= 483.43\,\text{W}\,\text{h} \\ W_{\text{walk}} &= (483.43\,\text{W}\,\text{h})\left(\frac{3600\,\text{s}}{1\,\text{h}}\right) = 1.74 \times 10^{6}\,\text{J} \end{aligned}$$

Walking will take more energy than running.

2) Why does the more intense exercise burn up less energy than the less intense exercise?

Because I will have to take a longer time to arrive at the physics lab by walking, it takes up more energy.

1.7 6.51

$$\begin{aligned} h &= 10\,\text{m} \\ m_{\text{student}} &= 71\,\text{kg} \\ P &= 500\,\text{W} \\ t &= ? \end{aligned}$$

How long would it take the student to travel up the three flights of stairs?

$$\begin{aligned} P &= \frac{W}{t} \\ t &= \frac{W}{P} \\ t &= \frac{wh}{P} \\ t &= \frac{(71\,\text{kg})(10\,\text{m}\,\text{s}^{-2})(10\,\text{m})}{500\,\text{W}} \\ t &= 14.2\,\text{s} \end{aligned}$$

$$\boxed{t = 14.2\,\text{s}}$$

1.8 7.5

$$y_0 = 20.8\,\mathrm{m}$$
$$v_0 = 11.5\,\mathrm{m\,s^{-1}}$$
$$\theta = 50.1^\circ$$

(a) What is the speed of the ball just before it strikes the ground?

$$\begin{aligned} E_0 &= E_1 \\ \frac{1}{2}mv_0^2 + mgh_0 &= \frac{1}{2}mv_1^2 + mgh_1 \\ v_1 &= \sqrt{v_0^2 + 2gh_0 - gh_1} \\ v_1 &= \sqrt{(11.5\,\mathrm{m\,s^{-1}}))^2 + 2(10\,\mathrm{m\,s^{-2}})(20.8\,\mathrm{m}) - (10\,\mathrm{m\,s^{-2}})(0)} \\ v_1 &= 23.41\,\mathrm{m\,s^{-1}} \end{aligned}$$

$$\boxed{v_1 = 23.41\,\mathrm{m\,s^{-1}}}$$

(b) What is the answer for part **(a)** if the initial velocity is at an angle of 50.1° *below* the horizontal?

$$\theta = -50.1^\circ$$

$$\begin{aligned} v_1 &= \sqrt{v_0^2 + 2gh_0 - gh_1} \\ v_1 &= \sqrt{(11.5\,\mathrm{m\,s^{-1}}))^2 + 2(10\,\mathrm{m\,s^{-2}})(20.8\,\mathrm{m}) - (10\,\mathrm{m\,s^{-2}})(0)} \\ v_1 &= 23.41\,\mathrm{m\,s^{-1}} \end{aligned}$$

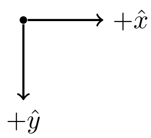
$$\boxed{v_1 = 23.41\,\mathrm{m\,s^{-1}}}$$

(c) If the affects of air resistance are included, will part **(a)** or **(b)** give the higher speed?

In part **(b)**, the tennis ball is thrown directly at the ground resulting in a shorter distance and time traveled. As the tennis ball is in the air for less time and distance, air resistance will likely have less of an overall effect when compared with throwing it above the horizontal.

1.9 7.9

$$
\begin{aligned}
m &= 0.20\,\mathrm{kg} \\
v_0 &= 0 \\
R &= 0.50\,\mathrm{m} \\
W_f &= -0.22\,\mathrm{J} \\
y_A &= 0
\end{aligned}
$$



(a) Between points A and B, how much work is done on the rock by

(i) the normal force?

$$W_N = 0$$

Forces perpendicular to the displacement have 0 work done.

(ii) gravity?

$$
\begin{aligned}
W_g &= E_B - E_A \\
W_g &= mgy_B - mgy_A \\
W_g &= (0.20\,\mathrm{kg})(10\,\mathrm{m\,s^{-2}})(0.50\,\mathrm{m}) - (0.20\,\mathrm{kg})(10\,\mathrm{m\,s^{-2}})(0\,\mathrm{m}) \\
W_g &= 1.00\,\mathrm{J}
\end{aligned}
$$

$$\boxed{W_g = 1.00\,\mathrm{J}}$$

(b) What is the speed of the rock as it reaches point B?

$$
\begin{aligned}
W &= E_B - E_A \\
W_g - W_f &= \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 \\
v_1 &= \sqrt{2\left(\frac{W_g - W_f}{m}\right)} \\
v_1 &= \sqrt{2\left(\frac{1.00\,\mathrm{J} - 0.22\,\mathrm{J}}{0.20\,\mathrm{kg}}\right)} \\
v_1 &= 2.79\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$\boxed{v_1 = 2.79\,\mathrm{m\,s^{-1}}}$$

(c) Of the three forces acting on the rock as it slides down the bowl, which (if any) are constant and which are not? Explain.

- W_N - The work done by normal force is only zero when it is entirely orthogonal to the direction of motion, but as the rock rolls on the *curve*, the angle between the direction of motion and the normal force is no longer perpendicular resulting in a **non-constant** work done by the normal force.
- W_f - Since the work done by W_N varies, the work done by friction varies resulting in it also being **non-constant**.

$$W_f = F_f d\cos(\theta) = \mu N d\cos(\theta)$$

- W_g - As long as the mass of the rock is constant, and the rock is continuously in contact with the bowl (not falling), the work down by gravity is **constant**.

(d) Just as the rock reaches point B, what is the normal force on it due to the bottom of the bowl?

$$\begin{tikzcd} & N, a \\ f & \text{rock} \arrow[l] \arrow[u] \arrow[d] \\ & mg \end{tikzcd}$$

$$\begin{aligned}
\sum F &= -ma_{cen} \\
mg &= N - ma_{cen} \\
N &= mg + m\left(\frac{v_1^2}{R}\right) \\
N &= (0.20\,\mathrm{kg})(10\,\mathrm{m\,s^{-2}}) + (0.20\,\mathrm{kg})\left(\frac{(2.79\,\mathrm{m\,s^{-1}})^2}{0.50\,\mathrm{m}}\right) \\
N &= 5.11\,\mathrm{N}
\end{aligned}$$

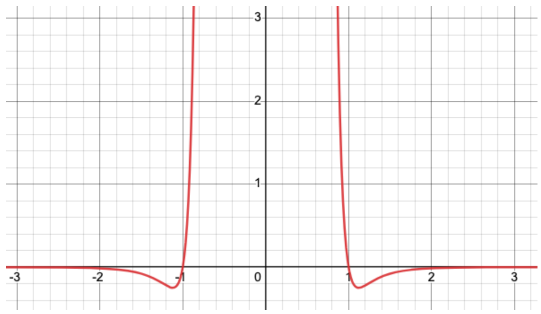
$$\boxed{N = 5.11\,\mathrm{N}}$$

1.10 7.35

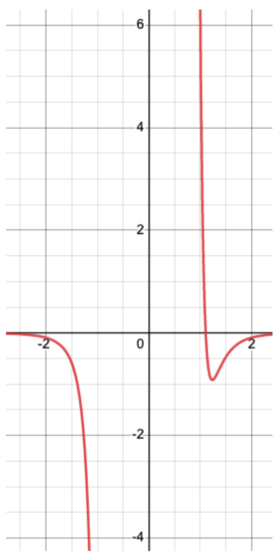
(a)

$$\begin{aligned}
F(r) &= \frac{dU}{dr} = \frac{d}{dr}\left(\left(\frac{a}{r^{12}}\right) - \left(\frac{b}{r^6}\right)\right) \\
F(r) &= \frac{12a}{r^{13}} - \frac{6b}{r^7}
\end{aligned}$$

- $U(r)$



- $F(r)$



(b) Find the equilibrium distance between the two atoms. Is this equilibrium stable?

$$\frac{12a}{r^{13}} - \frac{6b}{r^7} = 0$$
$$\frac{12a}{r^{13}} = \frac{6b}{r^7}$$
$$12a = 6br^6$$
$$r = \sqrt[6]{\frac{2a}{b}}$$

The equilibrium is stable.

(c) What minimum energy must be added to the molecule to *dissociate* it – that is, to separate the two atoms to an infinite distance apart?

$$-U = -\frac{a}{r^{12}} + \frac{b}{r^6}$$
$$-U = -\frac{a}{(\sqrt[6]{\frac{a}{b}})^{12}} + \frac{b}{(\sqrt[6]{\frac{a}{b}})^{6}}$$
$$-U = \frac{b^2}{4a}$$

$$\boxed{-U = \frac{b^2}{4a}}$$

(d) Find the values of the constants a and b.

$$d = 1.13 \times 10^{-10}\,\mathrm{m}$$
$$E = 1.54 \times 10^{-18}\,\mathrm{J}$$
$$a = ?$$
$$b = ?$$

$$r = d$$
$$\sqrt[6]{\frac{2a}{b}} = 1.13 \times 10^{-10}\,\mathrm{m}$$
$$\frac{2a}{b} = 2.08 \times 10^{-60}\,\mathrm{m}^6$$
$$a = (1.04 \times 10^{-60}\,\mathrm{m}^6)b$$

$$U = E$$
$$\frac{b^2}{4a} = 1.54 \times 10^{-18}\,\mathrm{J}$$
$$\frac{b^2}{4(1.04 \times 10^{-60}\,\mathrm{m}^6)b} = 1.54 \times 10^{-18}\,\mathrm{J}$$
$$b = 6.41 \times 10^{-78}\,\mathrm{J\,m}^6$$

$$a = (1.04 \times 10^{-60}\,\mathrm{m}^6)b$$
$$a = (1.04 \times 10^{-60}\,\mathrm{m}^6)(6.41 \times 10^{-78}\,\mathrm{J\,m}^6)$$
$$a = 6.67 \times 10^{-138}\,\mathrm{J\,m}^{12}$$

$$\boxed{a = 6.67 \times 10^{-138}\,\mathrm{J\,m}^{12}, b = 6.41 \times 10^{-78}\,\mathrm{J\,m}^6}$$

1.11 7.40

$$
\begin{aligned}
m &= 2.00\,\mathrm{kg} \\
k &= 400\,\mathrm{N\,m^{-1}} \\
x &= 0.220\,\mathrm{m} \\
\theta &= 37.0^\circ \\
v_1 &=? \\
x_1 &=?
\end{aligned}
$$

(a) What is the speed of the block as it slides along the horizontal surface after having left the spring?

$$
\begin{aligned}
E_{spring} &= E_p \\
\frac{1}{2}kx^2 &= \frac{1}{2}mv_1^2 \\
v_1 &= x\sqrt{\frac{k}{m}} \\
v_1 &= (0.220\,\mathrm{m})\sqrt{\frac{400\,\mathrm{N\,m^{-1}}}{2.00\,\mathrm{kg}}} \\
v_1 &= 3.11\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$\boxed{v_1 = 3.11\,\mathrm{m\,s^{-1}}}$$

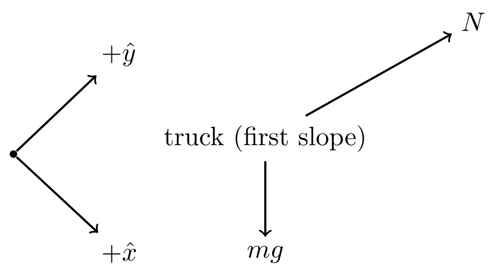
(b) How far does the block travel up the incline before starting to slide back down?

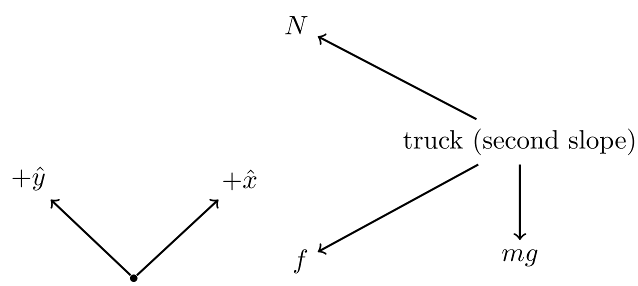
$$
\begin{aligned}
E_{spring} &= E_p \\
\frac{1}{2}kx^2 &= mgd\sin(\theta) \\
d &= \frac{kx^2}{2mg\sin(\theta)} \\
d &= \frac{(400\,\mathrm{N\,m^{-1}})(0.220\,\mathrm{m})}{2(2.00\,\mathrm{kg})(10\,\mathrm{m\,s^{-2}})\sin(37.0^\circ)} \\
d &= 0.804\,\mathrm{m}
\end{aligned}
$$

$$\boxed{d = 0.804\,\mathrm{m}}$$

1.12 7.58

$$\begin{aligned}
\text{truck mass} &= m \\
\text{first slope angle} &= \alpha \\
\text{initial speed} &= v_0 \\
\text{distance} &= L_0 \\
\text{second slope angle} &= \beta \\
\text{second slope coefficient of rolling friction} &= \mu_r \\
\text{max distance up second ramp} &= L_1 = ?
\end{aligned}$$





Since the distance the truck halts at is being found, it can be assumed that $v_1 = 0$.

$$\begin{aligned}
E_k + E_p &= E_k + E_p - W_f \\
\frac{1}{2}mv_0^2 + mgL_0\sin(\alpha) &= \frac{1}{2}mv_1^2 + mgL_1\sin(\beta) - \mu N \\
\frac{1}{2}mv_0^2 + mgL_0\sin(\alpha) &= 0 + mgL_1\sin(\beta) - (-\mu_r mgL_1\cos(\beta)) \\
L_1 &= \frac{v_0^2 + 2\sin(\alpha)gL_0}{2g(\mu\cos(\beta) + \sin(\beta))}
\end{aligned}$$

$$\boxed{L_1 = \frac{v_0^2 + 2\sin(\alpha)gL_0}{2g(\mu\cos(\beta) + \sin(\beta))}}$$

2 Lab Manual

2.1 871

(a)

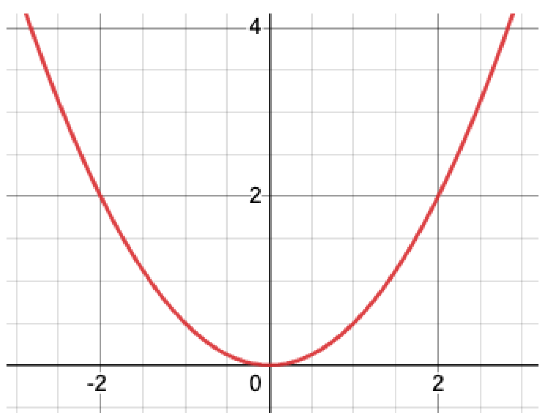
$$F_y = -\frac{dU(r)}{dy}$$
$$F_y = -\frac{d}{dy}(mgy)$$
$$F_y = -mg$$

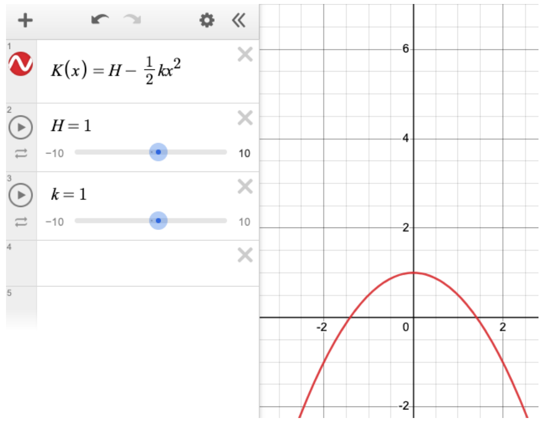
The slope of the force is the weight of the object at any point y.

(b)

$$U = \int (kx)\,dx$$
$$U = \frac{1}{2}kx^2 + C$$

"K must always be positive at all positions. Also, $K = H - U$. Therefore, $H - U$ must always be positive, or, in other words, U cannot ever be greater than H."

- U(x) 



- K(x)

(c)

$$F = \frac{dU}{dr} = \frac{d}{dr}\left(\frac{-k}{r}\right)$$
$$F = -\frac{k}{r^2}$$

(d)

(e) Found in part **(c)**:

$$F = -\frac{k}{r^2}$$

(f) Yes because as **Figure 3** extends to ∞ it gradually attracts towards 0 ergs

(g) When $H = {}^1\!/\!_2 \times 10^{-12}$, it can move between $r_0 < r <= r_6$.
When $H = -{}^1\!/\!_2 \times 10^{-12}$, it can move between $r_2 <= r <= r_4$.

(h) A curve that approaches 0 as $y \to \infty$.

2.2 876

(a)

$$F_1 = k_1 x_1$$
$$x_1 = \frac{F_1}{k_1}$$
$$F_2 = k_2 x_1$$
$$x_2 = \frac{F_2}{k_2}$$

$$
\begin{aligned}
\sum x &= x_1 + x_2 \\
\sum F &= \sum kx \\
x &= \frac{\sum F}{\sum k}
\end{aligned}
$$

$$
\begin{aligned}
\frac{F}{k_{eq}} &= \frac{F}{k_1} + \frac{F}{k_2} \\
\frac{1}{k_{eq}} &= \frac{1}{k_1} + \frac{1}{k_2} \\
k_{eq} &= \frac{k_1 k_2}{k_1 + k_2}
\end{aligned}
$$

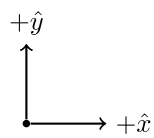
(b)

$$
\begin{aligned}
W &= \int_0^x k_{eq} x dx \\
W &= \left[\frac{1}{2} k_{eq} x^2\right]_0^x \\
W &= \frac{1}{2} k_{eq} x^2
\end{aligned}
$$

$$
\begin{aligned}
F &= kx \\
x &= \frac{F}{k} \\
W &= \frac{1}{2} k_{eq} \left(\frac{F}{k}\right)^2 \\
W &= \frac{1}{2}\frac{F^2}{k_1} + \frac{1}{2}\frac{F^2}{k_2} \\
W &= \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2
\end{aligned}
$$

2.3 884

$$
\begin{aligned}
r &= 4\,\mathrm{ft} \\
\theta &= 37^\circ \\
v_0 &= 0 \\
\mu &= 0.3
\end{aligned}
$$



$$W_{grav} + W_f = E_1 - E_0$$
$$-mgh - \mu mg\cos(\theta)\left(\frac{h}{\sin(\theta)}\right) = 0 - mgr$$
$$h = \frac{r}{\mu\cot(\theta) + 1}$$
$$h = \frac{4\,\text{ft}}{(0.3)\cot(37°) + 1}$$
$$h = 2.86\,\text{ft}$$

$$W_f = (\mu mg\cos(\theta))\left(\frac{h}{\sin(\theta)}\right)$$
$$W_f = ((0.3)\cos(37°)mg)\left(\frac{2.86\,\text{ft}}{\sin(37°)}\right)$$
$$W_f = (1.14\,\text{ft})mg$$
$$\boxed{W_f = (1.14\,\text{ft})mg}$$

2.4 885

$$L = 2\,\text{ft}$$
$$\theta = 60°$$
$$y = 0 \text{ at lowest point}$$

(a)

$$E_{low} = E_k + E_p$$
$$E_{low} = \frac{1}{2}mv_{max}^2 + mgy$$
$$E_{low} = \frac{1}{2}mv_{max}^2 + 0$$
$$E_{low} = \frac{1}{2}mv_{max}^2$$

$$E_{high} = E_k + E_p$$
$$E_{high} = \frac{1}{2}mv^2 + mgy$$
$$E_{high} = \frac{1}{2}m(0)^2 + mgy$$
$$E_{high} = mgy$$

$$
\begin{aligned}
E_{low} &= E_{high} \\
\frac{1}{2}mv_{max}^2 &= mgy \\
v_{max} &= \sqrt{2gy}, \quad \text{The height of the bob is non-constant, and can be found as } L(1-\cos(\theta)) \\
v_{max} &= \sqrt{2gL(1-\cos(\theta))}
\end{aligned}
$$

$$
\begin{aligned}
v_{max} &= \sqrt{2gL(1-\cos(\theta))} \\
v_{max} &= \sqrt{2(32.17\,\mathrm{ft/s^2})(2\,\mathrm{ft})(1-\cos(60^\circ))} \\
v_{max} &= 8.02\,\mathrm{ft/s}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\frac{1}{2}mv_{max}^2 + mgh_0 &= \frac{1}{2}m\left(\frac{1}{2}v_{max}\right)^2 + mgL(1-\cos(\theta)) \\
1-\cos(\theta) &= \frac{\frac{1}{2}v_{max}^2 - \frac{1}{2}\left(\frac{1}{2}v_{max}\right)^2}{gL} \\
\theta &= \arccos\left(-\frac{\frac{1}{2}v_{max}^2 - \frac{1}{2}\left(\frac{1}{2}v_{max}\right)^2}{gL} + 1\right) \\
\theta &= \arccos\left(-\frac{\frac{1}{2}(8.02\,\mathrm{ft/s})^2 - \frac{1}{2}\left(\frac{1}{2}(8.02\,\mathrm{ft/s})\right)^2}{(32.17\,\mathrm{ft/s^2})(2\,\mathrm{ft})} + 1\right) \\
\theta &= 51.31^\circ
\end{aligned}
$$

(c)

$$
\begin{aligned}
a &= \frac{v^2}{R} \\
a &= \frac{(8.02\,\mathrm{ft/s})^2}{2\,\mathrm{ft}} \\
a &= 32.16\,\mathrm{ft/s^2}
\end{aligned}
$$