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1 7.1

1.1 8

What is the probability that a five-card poker hand contains the ace of hearts?

$$\begin{aligned}
\mathbb{P}(A\heartsuit) &= \frac{|E|}{|S|} \\
&= \frac{C(51,4)}{C(52,5)} \\
&= \frac{51 \cdot 50 \cdot 49 \cdot 48}{1 \cdot 2 \cdot 3 \cdot 4} \cdot \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48} \\
&= \frac{5}{52}
\end{aligned}$$

1.2 Birthday Problem

Given a group of n people, what is the probability of at least two people having the same birthday?

2 7.2

$\mathbb{P}\colon S \to [0,1]$ is called probability if

1. $\forall s \in S, 0 \le \mathbb{P}(S) \le 1$
2. $\sum_{s \in S} \mathbb{P}(s) = 1$

Disjoint (mutually exclusive)

$$\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F)$$

$$\begin{aligned}
\mathbb{P}(E \cup F) &= \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F) \\
\mathbb{P}(E \cap F) &= 0
\end{aligned}$$

Independent

$$\mathbb{P}(E \cap F) = \mathbb{P}(E) \cdot \mathbb{P}(F)$$

2.1 Random Variable

$$\begin{aligned}
X &: S \to \mathbb{R} \\
\mathbb{P}(X) &\to [0, 1] \\
S &\to \mathbb{R} \to [0, 1]
\end{aligned}$$

2.2 Conditional Probability

$$\mathbb{P}(E \mid F) = \frac{\mathbb{P}(E \cap F)}{\mathbb{P}(F)}$$

Probably of E if F happened. (after F happened)

3 7.3

3.1 Bayes's Theorem

$$\mathbb{P}(E \mid F) = \frac{\mathbb{P}(F \mid E) \cdot \mathbb{P}(E)}{\mathbb{P}(F \mid E) \cdot \mathbb{P}(E) + \mathbb{P}(F \mid \overline{E}) \cdot \mathbb{P}(\overline{E})} \tag{1}$$

$$\begin{aligned}
E &\equiv E \cap U \\
&\equiv E \cap (F \cup \overline{F}) \\
&\equiv (E \cap F) \cup (E \cap \overline{F})
\end{aligned}$$

Recall

$$\mathbb{P}(E \mid F) = \frac{\mathbb{P}(E \cap F)}{\mathbb{F})}$$

3.2 Example 7.3.7

a)

$$\mathbb{P}(\overline{O} \mid -) = ?$$

$$\mathbb{P}(\overline{O} \mid -) = \frac{\mathbb{P}(- \mid \overline{O}) \cdot \mathbb{P}(\overline{O})}{\mathbb{P}(- \mid \overline{O}) \cdot \mathbb{P}(\overline{O}) + \mathbb{P}(- \mid O) \cdot \mathbb{P}(O)}$$

$$\mathbb{P}(- \mid \overline{O}) = \frac{\mathbb{P}(- \cap \overline{O})}{\mathbb{P}(\overline{O})}$$

$$\begin{aligned}
\mathbb{P}(\overline{O}) &= \mathbb{P}(+ \cap \overline{O}) + \mathbb{P}(- \cap \overline{O}) \\
\mathbb{P}(- \cap \overline{O}) &= \mathbb{P}(\overline{O}) - \mathbb{P}(+ \cap \overline{O}) \\
\mathbb{P}(- \cap \overline{O}) &= 0.99 - 0.02 \cdot 0.99 \\
\mathbb{P}(- \cap \overline{O}) &= 0.97
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}(- \mid \overline{O}) &= \frac{\mathbb{P}(- \cap \overline{O})}{\mathbb{P}(\overline{O})} \\
\mathbb{P}(- \mid \overline{O}) &= \frac{0.97}{0.99} \\
\mathbb{P}(- \mid \overline{O}) &= 0.98
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}(\overline{O} \mid -) &= \frac{\mathbb{P}(- \mid \overline{O}) \cdot \mathbb{P}(\overline{O})}{\mathbb{P}(- \mid \overline{O}) \cdot \mathbb{P}(\overline{O}) + \mathbb{P}(- \mid O) \cdot \mathbb{P}(O)} \\
\mathbb{P}(\overline{O} \mid -) &= \frac{0.98 \cdot 0.99}{0.98 \cdot 0.99 + 0.05 \cdot 0.01} \\
\mathbb{P}(\overline{O} \mid -) &= 0.9995
\end{aligned}$$

4 7.4