Chapter 1 - Vectors & Scalars

1 Introduction to Vectors & Scalars

1.1 Vectors & Scalars

Scalar = Magnitude
Vector = Magnitude + Direction

Vectors are represented by arrows – tail and tip

$\longrightarrow$

Triangle addition:

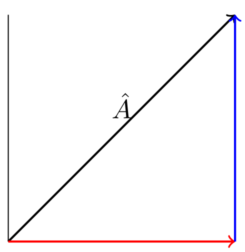
$$\hat{A} = 20 \text{ @ } 40° \text{ above } + \hat{x}$$

$\hat{x} \rightarrow$ Unit vector gives direction with Magnitude 1 "the x direction"

Vector B is 45 @ 60 ° Right of $+\hat{y}$

1.2 Algebra as an alternative to Euclidean Geometry

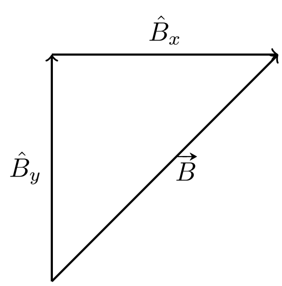
In order to add vectors, first break vectors into components



Any vector is the sum of its components

$$\boxed{\hat{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}} \tag{1}$$

$$\hat{A} = A\cos(\theta)\hat{x} + A\sin(\theta)\hat{y}\ (+0\hat{z})$$



Write $\vec{B}$ in component form:

$$\vec{B} = B\sin(\phi)\hat{x} + B\cos(\phi)\hat{y}$$

$$= (45)\sin(60°)\hat{x} + (45)\cos(60°)\hat{y}$$

$$\boxed{\vec{B} = 39\hat{x} + 23\hat{y}}$$

To add two vectors, add each direction separately:

$$15\hat{x} + 13\hat{y}$$

$$39\hat{x} + 23\hat{y}$$

$$\vec{A} + \vec{B} = 54\hat{x} + 36\hat{y}$$

To substract two vectors, subtract each direction seperately:

$$15\hat{x} + 13\hat{y}$$

$$39\hat{x} + 23\hat{y}$$

$$\vec{A} - \vec{B} = -24\hat{x} - 10\hat{y}$$

To compose (get magnitude and direction) of a vector, use:

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\theta_{\vec{A}} = \arctan\left(\frac{A_i}{A_j}\right)$$

$$|\vec{A} + \vec{B}| = \sqrt{(A+B)_x^2 + (A+B)_y^2}$$

$$= \sqrt{54^2 + 36^2} = 65$$

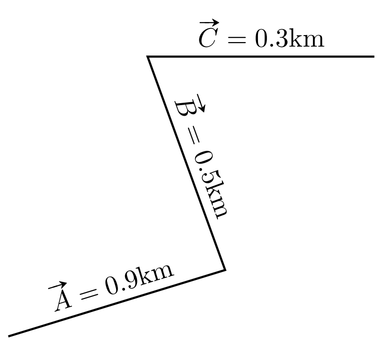
$$\theta = \arctan\left(\frac{(A+B)_y}{(A+B)_x}\right) = \arctan\left(\frac{36}{54}\right) = 34°$$

1.3 Scalar Multiplication

$$\gamma\vec{A} = \gamma A_x\hat{x} + \gamma A_y\hat{y} + \gamma A_z\hat{z}$$

$$-\vec{A} = (-A_x)\,\hat{x} + (-A_y)\,\hat{y} + (-A_z)\,\hat{z}$$

Given three vectors connected from head to tail, find the distance from the tail of $\vec{A}$ to the head of $\vec{C}$:



$$\vec{C} = 0.3\text{km}\hat{x}$$
$$\vec{A} = +A\cos(\theta)\hat{x} + A\sin(\theta)\hat{y}$$
$$= (0.9\text{km})\cos(17°)\hat{x} + (0.9\text{km})\sin(17°)\hat{y}$$
$$\vec{A} = 0.86\text{km}\hat{x} + 0.26\text{km}\hat{y}$$
$$\vec{B} = -B\sin(\phi)\hat{x} + B\cos(\phi)\hat{y}$$
$$\vec{B} = -0.17\text{km}\hat{x} + 0.47\text{km}\hat{y}$$

$$\vec{A} = 0.86\text{km}\hat{x} + 0.26\text{km}\hat{y}$$
$$\vec{B} = -0.17\text{km}\hat{x} + 0.47\text{km}\hat{y}$$
$$\vec{C} = 0.3\text{km}\hat{x} + 0\hat{y}$$

$$\boxed{\vec{A} + \vec{B} + \vec{C} = 0.99\text{km}\hat{x} + 0.73\text{km}\hat{y}}$$

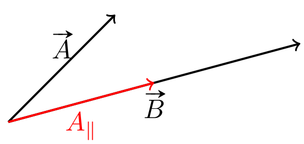
1.4 Multiplying Vectors

A dot product multiplies two vectors and returns a scalar.

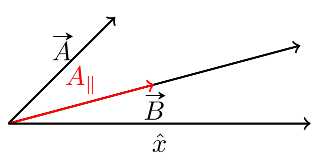
Notation:

$$\vec{A} \cdot \vec{B}$$

Essentially means: "What part of $\vec{A}$ lies in the direction of $\vec{B}$?"



Another example to show importance of angle between vectors:



$$A_{\parallel} = A\cos(\theta)_{AB}$$
$$\vec{A} \cdot \vec{B} = (A\cos(\theta)_{AB})\,(B)$$
$$\vec{A} \cdot \vec{B} = (20\cos(10°))\,(45)$$
$$\boxed{\vec{A} \cdot \vec{B} = 886}$$