Homework 5 - 2D Motion

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Contents

1 Problems

1.1 A

$$\begin{aligned} (x_0, y_0) &= (0, 0) \\ \text{Initial speed} &= v_0 \\ \text{Initial angle} &= \theta \\ d &= R \\ \Delta x &= R \\ \Delta y &= 0 \end{aligned}$$

a)

$$\begin{aligned} \Delta x &= v_{0_x} t + \frac{1}{2} a_x t^2 \\ R &= v_0 \cos(\theta) t + \frac{1}{2}(0) t^2 \\ R &= v_0 \cos(\theta) t \end{aligned}$$

$$\begin{aligned} \Delta y &= v_{0_y} t + \frac{1}{2} g t^2 \\ 0 &= v_0 \sin(\theta) t + \frac{1}{2} g t^2 \\ t &= 0, \frac{2 v_0 \sin(\theta)}{g} \end{aligned}$$

$$R = v_0 \cos(\theta) \left(\frac{2v_0 \sin(\theta)}{g} \right)$$
$$R = \frac{\sin(2\theta) v_0^2}{g}$$

$$\boxed{R = \frac{\sin(2\theta) v_0^2}{g}}$$

b)

$$\begin{aligned}
R(\theta) &= \frac{\sin(2\theta) v_0^2}{g} \\
R'(\theta) &= \frac{2v^2 \cos(2\theta)}{g} \\
R'(\theta) &= 0 \\
\frac{2v^2 \cos(2\theta)}{g} &= 0 \\
2v^2 \cos(2\theta) &= 0 \\
\theta &= \frac{\pi}{4}(+\pi n)
\end{aligned}$$

$$\boxed{\theta = \frac{\pi}{4} = 45^\circ}$$

2 Book

2.1 3.38

$$\begin{aligned}
\Delta x_{\mathrm{B,A}} &= 1500\,\mathrm{m} = 1.5\,\mathrm{km} \\
v_{\frac{b}{w}} &= 4.00\,\mathrm{km\,h^{-1}} \\
v_{\frac{p}{w}} &= 4.00\,\mathrm{km\,h^{-1}} \\
a &= 0 \\
v_w &= 2.80\,\mathrm{km\,h^{-1}}
\end{aligned}$$

$$\begin{aligned}
\Delta x &= v_f t_{b_0} - \frac{1}{2} a t_{b_0}^2 \\
1.5\,\mathrm{km} &= (4.00\,\mathrm{km\,h^{-1}} + 2.80\,\mathrm{km\,h^{-1}}) t - \frac{1}{2}(0) t^2 \\
t_{b_0} &= 0.221\,\mathrm{h}
\end{aligned}$$

$$
\begin{aligned}
\Delta x &= v_f t_{b_1} - \frac{1}{2} a t_{b_1}^2 \\
1.5\,\mathrm{km} &= (4.00\,\mathrm{km\,h^{-1}} - 2.80\,\mathrm{km\,h^{-1}})t - \frac{1}{2}(0)t^2 \\
t_{b_1} &= 1.25\,\mathrm{h}
\end{aligned}
$$

$$
\begin{aligned}
t_b &= 0.221\,\mathrm{h} + 1.25\,\mathrm{h} \\
t_b &= 1.471\,\mathrm{h}
\end{aligned}
$$

$$
\begin{aligned}
2\Delta x &= v_f t_{p_1} - \frac{1}{2} a t_{p_1}^2 \\
2(1.5\,\mathrm{km}) &= (4.00\,\mathrm{km\,h^{-1}})t_{p_1} - \frac{1}{2}(0)t_{p_1}^2 \\
t_p &= 0.75\,\mathrm{h}
\end{aligned}
$$

$$\boxed{t_b = 1.471\,\mathrm{h}, t_p = 0.75\,\mathrm{h}}$$

2.2 3.41

$$
\begin{aligned}
\Delta x_r &= 500\,\mathrm{m} \\
v_{b/e} &= 0\hat{x} + 2.0\,\mathrm{m\,s^{-1}}\hat{y} \\
v_{b/r} &= 4.2\,\mathrm{m\,s^{-1}}\hat{x} + 0\hat{y} \\
v_{b/e} &= 4.2\,\mathrm{m\,s^{-1}}\hat{x} + 2.0\,\mathrm{m\,s^{-1}}\hat{y}
\end{aligned}
$$

(a)

$$
\begin{aligned}
v_{b/e} &= \sqrt{(v_{r_x} + v_{b_x})^2 + (v_{r_y} + v_{b_y})^2} \\
&= \sqrt{(0 + 4.2\,\mathrm{m\,s^{-1}})^2 + (2.0\,\mathrm{m\,s^{-1}} + 0)^2} \\
v_{b/e} &= 4.652\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
\tan(\theta) &= \frac{v_{b/e_y}}{v_{b/e_x}} \\
\theta &= \arctan\left(\frac{v_{b/e_y}}{v_{b/e_x}}\right) \\
\theta &= \arctan\left(\frac{2.0\,\mathrm{m\,s^{-1}}}{4.2\,\mathrm{m\,s^{-1}}}\right) \\
\theta &= 25.46^\circ
\end{aligned}
$$

$$\boxed{v_{b/e} = 4.652\,\mathrm{m\,s^{-1}}, \theta = 25.46\,^\circ\,\mathrm{S\,of\,E}}$$

(b)

$$\begin{aligned}
\Delta x_r &= v_{b/e_x} t - \frac{1}{2}at^2 \\
500\,\mathrm{m} &= (4.2\,\mathrm{m\,s^{-1}})t - \frac{1}{2}(0)t^2 \\
t &= 119.0\,\mathrm{s}
\end{aligned}$$

$$\boxed{t = 119.0\,\mathrm{s}}$$

(c)

$$y_0 = 0$$

$$\begin{aligned}
\Delta y &= v_{b/e_y} t - \frac{1}{2}at^2 \\
y_1 - 0 &= (2.0\,\mathrm{m\,s^{-1}})(119.0\,\mathrm{s}) - \frac{1}{2}(0)(119.0\,\mathrm{s}) \\
y_1 &= 238.0\,\mathrm{m}
\end{aligned}$$

$$\boxed{\Delta y = 238.0\,\mathrm{m}}$$

2.3 3.42

(a)

$$\begin{aligned}
\sin(\theta) &= \frac{v_{w/e}}{v_{b/w}} \\
\theta &= \arcsin\left(\frac{v_{w/e}}{v_{b/w}}\right) \\
\theta &= \arcsin\left(\frac{2.0\,\mathrm{m\,s^{-1}}}{4.2\,\mathrm{m\,s^{-1}}}\right) \\
\theta &= 28.44\,^\circ\,\mathrm{N\,of\,E}
\end{aligned}$$

$$\boxed{\theta = 28.44\,^\circ\,\mathrm{N\,of\,E}}$$

(b)

$$\begin{aligned}
v_{b/e} &= \sqrt{(v_{b/w_x} + v_{w/e_x})^2 + (v_{b/w_y} + v_{w/e_y})^2} \\
v_{b/e} &= \sqrt{((4.2\,\mathrm{m\,s^{-1}})\cos(28.44^\circ) + 0)^2 + ((4.2\,\mathrm{m\,s^{-1}})\sin(28.44^\circ) + 2.0\,\mathrm{m\,s^{-1}})^2} \\
v_{b/e} &= 3.693\,\mathrm{m\,s^{-1}}
\end{aligned}$$

$$\boxed{v_{b/e} = 3.693\,\mathrm{m\,s^{-1}}}$$

(c)

$$\Delta x_r = v_{b/e} t - \frac{1}{2} a t^2$$
$$500\,\text{m} = (3.693\,\text{m}\,\text{s}^{-1}) t - \frac{1}{2}(0) t^2$$
$$t = 135.4\,\text{s}$$

$$\boxed{t = 135.4\,\text{s}}$$

2.4 3.43

$$v_{w/e} = (0)\hat{x} + (80.0\,\text{km}\,\text{h}^{-1})\hat{y}$$

(a)

$$v_{a/w} = 320.0\,\text{km}\,\text{h}^{-1}$$

$$\sin(\theta) = \frac{v_{w/e_y}}{v_{a/w}}$$
$$\theta = \arcsin\left(\frac{80.0\,\text{km}\,\text{h}^{-1}}{320.0\,\text{km}\,\text{h}^{-1}}\right)$$
$$\theta = 14.48\,^\circ\,\text{N of W}$$

$$\boxed{\theta = 14.48\,^\circ\,\text{N of W}}$$

(b)

$$\cos(\theta) = \frac{v_{a/e}}{v_{a/w}}$$
$$v_{a/e} = (v_{a/w})\cos(\theta)$$
$$v_{a/e} = (320.0\,\text{km}\,\text{h}^{-1})\cos(14.48^\circ)$$
$$v_{a/e} = 309.8\,\text{km}\,\text{h}^{-1}$$

$$\boxed{v_{a/e} = 309.8\,\text{km}\,\text{h}^{-1}}$$

3 Lab Manual

3.1 473

$$\begin{aligned}
v_{r/b} &= v \\
d_A &= d \\
v_{A/r} &= c \\
a_A &= 0 \\
d_B &= d \\
v_{B/r} &= c \\
a_B &= 0
\end{aligned}$$

(a) First find t_A. Subscripts $0, 1$ denote the first and second trip (0-indexed).

$$\begin{aligned}
\Delta x_0 &= v_{A/r} t_0 + \frac{1}{2} a_A t_0^2 \\
d_A &= (c - v) t_0 + \frac{1}{2}(0) t_0^2 \\
t_0 &= \frac{d_A}{c - v}
\end{aligned}$$

$$\begin{aligned}
\Delta x_0 &= v_{A/r} t_0 + \frac{1}{2} a_A t_0^2 \\
d_A &= (c + v) t_1 + \frac{1}{2}(0) t_1^2 \\
t_1 &= \frac{d_A}{c + v}
\end{aligned}$$

$$\begin{aligned}
t_A &= t_0 + t_1 \\
t_A &= \frac{d_A}{c - v} + \frac{d_A}{c + v} \\
t_A &= d_A \left(\frac{1}{c\left(1 - \frac{v}{c}\right)} + \frac{1}{c\left(1 + \frac{v}{c}\right)} \right) \\
t_A &= \frac{d_A}{c} \left(\frac{1}{1 - \frac{v}{c}} + \frac{1}{1 + \frac{v}{c}} \right) \\
t_A &= \frac{d_A}{c} \left(\frac{1 + \frac{v}{c}}{(1 - \frac{v}{c})(1 + \frac{v}{c})} + \frac{1 - \frac{v}{c}}{(1 + \frac{v}{c})(1 - \frac{v}{c})} \right) \\
t_A &= \frac{2\frac{d_A}{c}}{-(\frac{v}{c})(\frac{v}{c}) + \frac{v}{c} - \frac{v}{c} + 1} \\
t_A &= \frac{2\frac{d_A}{c}}{1 - (\frac{v}{c})^2}
\end{aligned}$$

$$\boxed{t_A = \frac{2\frac{d}{c}}{1-(\frac{v}{c})^2}}$$

Find t_B. Subscripts $0, 1$ denote the first and second trip (0-indexed).

$$v_{B/r}^2 - v_{r/b}^2 = v_{B/b}^2$$
$$v_{B/b} = \sqrt{c^2 - v^2}$$

$$2\Delta x = v_{B/b} t_B + \frac{1}{2} a_B t_B^2$$
$$2d_B = \left(\sqrt{c^2 - v^2}\right) t_B + \frac{1}{2}(0) t_B^2$$
$$t_B = \frac{2d_B}{\sqrt{c^2\left(1 - \frac{v^2}{c^2}\right)}}$$
$$t_B = \frac{2d_B}{c\sqrt{1 - \frac{v^2}{c^2}}}$$
$$t_B = \frac{2\frac{d_B}{c}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$\boxed{t_B = \frac{2\frac{d}{c}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}$$

(b) Utilize the negative binomial series $(x+1)^{-n} = 1 - nx + \frac{1}{2}n(n+1)x^2 - \cdots$ as the prompt states that $\frac{v}{c} << 1$.

$$t_A - t_B \approx \frac{v^2 d}{c^3}$$
$$2\frac{d}{c}\left(1 - \left(\frac{v}{c}\right)^2\right)^{-1} - 2\frac{d}{c}\left(1 - \left(\frac{v}{c}\right)^2\right)^{-1/2} \approx \frac{v^2 d}{c^3}$$
$$2\frac{d}{c}\left(\left(1 + \left(\frac{v}{c}\right)^2\right) - \left(1 + \frac{1}{2}\left(\frac{v}{c}\right)^2\right)\right) \approx \frac{v^2 d}{c^3}$$
$$\frac{v^2 d}{c^3} \approx \frac{v^2 d}{c^3}$$

$$\boxed{t_A - t_B \approx \frac{v^2 d}{c^3}}$$

3.2 475

(a)

$$v_{p/e} = (8\,\mathrm{mi\,h^{-1}}\sin(\theta))\hat{x} + (8\,\mathrm{mi\,h^{-1}}\cos(\theta))\hat{y}$$
$$v_{w/e} = (0)\hat{x} + (20\,\mathrm{mi\,h^{-1}})\hat{y}$$

$$\sin(\theta) = \frac{v_{p/e}}{v_{w/e}}$$
$$\theta = \arcsin\left(\frac{v_{p/e}}{v_{w/e}}\right)$$
$$\theta = \arcsin\left(\frac{8\,\mathrm{mi\,h^{-1}}}{20\,\mathrm{mi\,h^{-1}}}\right)$$
$$\theta = 23.58^\circ$$

$$\boxed{\theta = 23.58^\circ}$$

(a)

$$\text{distance} = D$$
$$\frac{dV}{dx} = v_{r/g} \cdot A$$
$$\text{wetness} = \frac{dV}{dx} t$$

$$\text{wetness} = (v_{r/g} \cdot A)t$$
$$t = \frac{\Delta D}{v_{r/g}}$$
$$\text{wetness} = \left(v_{r/g} \cdot A\right)\left(\frac{D}{v_{r/g}}\right)$$
$$\text{wetness} = AD$$

$$\boxed{\text{wetness} = AD}$$

In the end, the angle that the rain hits the engineer ends up having no effect whether they are running or walking. More specifically, when the engineer walks, the value of theta is closer to 90° meaning the rainfall per distance $\frac{dV}{dx}$ is of smaller volume. As the engineer runs faster, more rainfall will hit the engineer, though the speed at which they're traveling makes the total volume of rain hit the same (given adequate/perfectly consistent variables).

3.3 477

$$\begin{aligned}
v_{A/e} &= 50\,\mathrm{mi\,h^{-1}} \\
v_{B/e} &= 50\,\mathrm{mi\,h^{-1}} \\
d &= 100\,\mathrm{mi} \\
t &= 0.5\,\mathrm{h}
\end{aligned}$$

(a) Reference

$$\begin{aligned}
(v_{A/e} + v_{B/e})t_n &= d - 2v_{A/e}\sum_{i=1}^{n-1} t_i \\
(50\,\mathrm{mi\,h^{-1}} + 50\,\mathrm{mi\,h^{-1}})t_n &= 100\,\mathrm{mi} - 2(50\,\mathrm{mi\,h^{-1}})\sum_{i=1}^{n-1} t_i \\
(100\,\mathrm{mi\,h^{-1}})t_n &= 100\,\mathrm{mi} - 100\,\mathrm{mi\,h^{-1}}\sum_{i=1}^{n-1} t_i \\
t_n &= 1 - \sum_{i=1}^{n-1} t_i \\
t_n - t_{n-1} &= -t_{n-1} \\
t_n &= \frac{1}{1}t_{n-1}
\end{aligned}$$

3.4 670

$$\begin{aligned}
\cos(\theta) &= \frac{\text{adj.}}{v_o} \\
\text{adj.} &= (v_o)(\cos(\theta)) \\
\cos(\beta) &= \frac{x}{\text{adj.}} \\
\cos(\beta) &= \frac{x}{(v_o)\cos(\theta)} \\
x &= (v_o)\cos(\theta)\cos(\beta)
\end{aligned}$$

$$\boxed{x = (v_o)\cos(\theta)\cos(\beta)}$$

3.5 672

$$\begin{aligned}
\theta &= 60^\circ \\
\phi &= 40^\circ \\
v_o &= 200\,\mathrm{ft\,s^{-1}} \\
a_x &= 0
\end{aligned}$$

(a)

$$
\begin{aligned}
v_{f_x} &= v_{o_x} + a_x t \\
v_{f_x} &= (200\,\mathrm{ft\,s^{-1}})\cos(60°) + (0)t \\
v_{f_x} &= (200\,\mathrm{ft\,s^{-1}})\cos(60°)
\end{aligned}
$$

$$
\begin{aligned}
v_{f_y} &= v_{o_y} + gt \\
v_{f_y} &= (200\,\mathrm{ft\,s^{-1}})\sin(60°) + (-32.17\,\mathrm{ft\,s^{-2}})t
\end{aligned}
$$

$$
\begin{aligned}
\tan(\phi) &= \frac{v_{f_y}}{v_{f_x}} \\
\tan(40°) &= \frac{(200\,\mathrm{ft\,s^{-1}})\sin(60°) + (-32.17\,\mathrm{ft\,s^{-2}})t}{(200\,\mathrm{ft\,s^{-1}})\cos(60°)} \\
t &= 2.78\,\mathrm{s}
\end{aligned}
$$

$$\boxed{t = 2.78\,\mathrm{s}}$$

(b)

$$
\begin{aligned}
\Delta y &= v_o t + \frac{1}{2}gt^2 \\
\Delta y &= (200\,\mathrm{ft\,s^{-1}})(2.78\,\mathrm{s}) + \frac{1}{2}(-32.17\,)(2.78\,\mathrm{s})^2 \\
\Delta y &= 431.7\,\mathrm{ft}
\end{aligned}
$$

$$\boxed{\Delta y = 431.7\,\mathrm{ft}}$$

3.6 676

$$
\begin{aligned}
v_{o_x} &= v_o\cos(\theta) = v_o\cos(37°) \\
v_{o_y} &= v_o\sin(\theta) = v_o\sin(37°) \\
a_x &= 0 \\
g &= -32.17\,\mathrm{ft\,s^{-2}}
\end{aligned}
$$

$$
\begin{aligned}
\Delta y &= v_{o_y} t + \frac{1}{2}gt^2 \\
0 &= (v_o\sin(37°))t + \frac{1}{2}(-32.17\,\mathrm{ft\,s^{-2}})t^2 \\
t &= \frac{2\sin(37°)}{-32.17\,\mathrm{ft\,s^{-2}}}v_o
\end{aligned}
$$

$$
\begin{aligned}
\Delta x &= v_{o_x} t - \frac{1}{2} a_x t^2 \\
192\,\mathrm{ft} &= (v_o \cos(37^\circ))(\frac{2\sin(37^\circ)}{-32.17\,\mathrm{ft\,s^{-2}}} v_o) - 0 \\
v_o &= 80.16\,\mathrm{ft\,s^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
\Delta y &= v_{o_y} t - \frac{1}{2} g t^2 \\
-160\,\mathrm{ft} &= (80.16\,\mathrm{ft\,s^{-1}})\sin(37^\circ)t - \frac{1}{2}(32.17\,\mathrm{ft\,s^{-1}})t^2 \\
t &= 4.992\,\mathrm{s}
\end{aligned}
$$

$$
\begin{aligned}
\Delta x &= v_o \cos(\theta)t - \frac{1}{2} a_x t^2 \\
\Delta x &= (80.16\,\mathrm{ft\,s^{-1}})\cos(37^\circ)(4.992\,\mathrm{s}) - \frac{1}{2}(0)(4.992\,)^2 \\
\Delta x &= 319.6\,\mathrm{ft}
\end{aligned}
$$

$$
\boxed{\Delta x = 319.6\,\mathrm{ft}}
$$

3.7 678

$$
\begin{aligned}
v_0 &= 160\,\mathrm{ft\,s^{-1}} \\
\theta &= 53^\circ \\
\phi &= 45^\circ
\end{aligned}
$$

(a)

$$
\begin{aligned}
v_{o_x} &= v_{f_x} \\
v_o \cos(53^\circ) &= v_f \cos(45^\circ) \\
v_f &= \frac{(160\,\mathrm{ft\,s^{-1}})\cos(53^\circ)}{\cos(45^\circ)} \\
v_f &= 136.2\,\mathrm{ft\,s^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
v_{f_y} &= v_{o_y} + gt \\
t &= \frac{v_{f_y} - v_{o_y}}{g} \\
t &= \frac{-136.2\,\mathrm{ft\,s^{-1}}\sin(45^\circ) - (160\,\mathrm{ft\,s^{-1}})\sin(53^\circ)}{-32.17\,\mathrm{ft\,s^{-2}}} \\
t &= 6.967\,\mathrm{s}
\end{aligned}
$$

$$
\boxed{t = 6.967\,\mathrm{s}}
$$

(b)

$$\Delta x = v_{o_x} t + \frac{1}{2} a_x t^2$$
$$\Delta x = (160\,\mathrm{ft\,s^{-1}} \cos(53°))(6.967\,\mathrm{s}) + \frac{1}{2}(0)(6.967\,\mathrm{s})$$
$$\Delta x = 670.9\,\mathrm{ft}$$

$$\Delta y = v_{o_y} t + \frac{1}{2} g t^2$$
$$\Delta y = (160\,\mathrm{ft\,s^{-1}} \sin(53°))(6.967\,\mathrm{s}) + \frac{1}{2}(-32.17\,\mathrm{ft\,s^{-2}})(6.967\,\mathrm{s})^2$$
$$\Delta y = 109.5\,\mathrm{ft}$$

$$\boxed{(670.9\,\mathrm{ft}, 109.5\,\mathrm{ft})}$$