Week 01 and Week 02 Participation Assignment (4 of 4)

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1 Part 4

First order linear differential equation has the form of $\frac{dy}{dx} + p(x) \cdot y = q(x)$. Determine whether the given equation is linear or not (You may review on separable if you would like to identify as separable). If it is linear, identify p(x) and q(x). Pay attention that the coefficient of $\frac{dy}{dx}$ in the form must be 1.

1.1 Problem 1

$$x^2 \frac{dy}{dx} + \sin(x) - y = 0$$

$$\begin{aligned} x^2 \frac{dy}{dx} + \sin(x) - y &= 0 \\ x^2 \frac{dy}{dx} - y &= -\sin(x) \\ \frac{dy}{dx} - \frac{1}{x^2}(y) &= \frac{-\sin(x)}{x^2} \end{aligned}$$

$$\boxed{\text{Linear: } p(x) = \frac{-1}{x^2},\ q(x) = \frac{-\sin(x)}{x^2}}$$

1.2 Problem 2

$$\frac{dx}{dt} + xt = e^x$$

Nonlinear: Can't arrange into first-order linear differential equation form.

1.3 Problem 3

$$\left(t^2 + 1\right) \frac{dy}{dt} = yt - y$$

$$\left(t^2+1\right)\frac{dy}{dt} = yt - y$$
$$\left(t^2+1\right)\frac{dy}{dt} = y\left(t-1\right)$$
$$\frac{dy}{dt}\cdot\frac{1}{y} = \frac{t-1}{t^2+1}$$

Nonlinear: Can't arrange into first-order linear differential equation form.

1.4 Problem 4

$$3t = e^t\frac{dy}{dt} + y\ln(t)$$

$$3t = e^t\frac{dy}{dt} + y\ln(t)$$
$$\frac{dy}{dt} + \frac{\ln(t)}{e^t}\cdot y = \frac{3t}{e^t}$$

Linear: $p(t) = \dfrac{\ln(t)}{e^t}$, $q(t) = \dfrac{3t}{e^t}$

1.5 Problem 5

$$x\frac{dx}{dt} + t^2x = \sin(t)$$

$$x\frac{dx}{dt} + t^2x = \sin(t)$$
$$x\left(\frac{dx}{dt} + t^2\right) = \sin(t)$$

Nonlinear: Can't arrange into first-order linear differential equation form.

1.6 Problem 6

$$3r = \frac{dr}{d\theta} - \theta^3$$

$$3r = \frac{dr}{d\theta} - \theta^3$$
$$\frac{dr}{d\theta} - 3r = \theta^3$$

Linear: $p(\theta) = -3$, $q(\theta) = \theta^3$