1 A Review of 2A Error Theory

If

$$x = x_{\text{best}} \pm \delta x$$

– Where x_{best} is th ebest guess at the measurement and δx is the uncertainty – on a measurement device uncertainty is the the resolution

Rule 1: δx has same units as x_{best}

Rule 2: δx always is rounded to one sig. fig.

Rule 3: x_{best} <u>must</u> have the same precision as δx

For example 6.874 15 cm ± 0.5 mm, turn this into properly reported value:

Step 1: Same units 68.7415 mm ± 0.5 mm

Step 2: Round x_{best} to match δx

$$68.7(5)\,\text{mm}$$

$$x_{\text{best}} \pm \delta x$$

$$y_{\text{best}} \pm \delta y$$

for $x_{\text{best}} \pm y_{\text{best}}$

$$\delta_{x+y} = \delta_x + delta_y$$

$$\delta x - y = \delta_x + delta_y$$

for $x_{\text{best}} \cdot y_{\text{best}}$ or $\frac{x_{\text{best}}}{y_{\text{best}}}$

$$\frac{\delta_{xy}}{xy} = \frac{\delta_x}{x} + \frac{\delta_y}{y}$$

where $\frac{\delta_y}{y}$ is the fractional error, relative error, or percent error

$$\delta f = \left.\frac{\partial f}{\partial x}\right|_{x_{\text{best}}} \delta x$$

$$\frac{df}{dx} = \frac{df}{dx}$$

$$df = \frac{df}{dx} dx$$

For determinate errors:

$$\delta f = \sum \frac{\partial f}{\partial x_i} \delta x_i$$

in 2D:

$$\delta f = \left|\frac{\partial f}{\partial x}\right| \delta x + \left|\frac{\partial f}{\partial y}\right| \delta y$$

Given

$$f = x + y$$

Taking the partial derivative in respect to x

$$\frac{\partial f}{\partial x} = 1$$

Taking the partial derivative in respect to y

$$\frac{\partial f}{\partial y} = 1$$

$$\delta f = (1)(\delta x) + (1)(\delta y) = \delta x + \delta y$$

Given

$$f = xy$$

$$\frac{\partial f}{\partial x} = y$$

$$\frac{\partial f}{\partial y} = x$$

$$\delta f = y\delta x + x\delta y$$

Now dividing by f (which is equal to xy)

$$\frac{\delta f}{f} = \frac{y\delta x}{xy} + \frac{x\delta y}{xy} = \frac{\delta x}{x} + \frac{\delta y}{y}$$

Given

$$f = ax^n$$

$$\frac{\partial f}{\partial x} = \left|anx^{n-1}\right|$$

$$\frac{\partial f}{f} = \frac{\frac{\partial f}{\partial x}\delta x}{f}$$

$$\frac{\partial f}{f} = \left|\frac{anx^{n-1}}{ax^n}\delta x\right|$$

$$\frac{\partial f}{f} = \left|n\frac{\delta x}{x}\right|$$

deviation = $\bar{x} - x_i$

Root Mean Square

$$\sigma_x - \sqrt{\frac{\sum(\bar{x} - x_i)^2}{N}}$$

Where σ_x is standard deviation

The best value for statistical data is given by the average

$$x_{\text{best}} = \bar{x} = \frac{\sum x_i}{N}$$

δ_x for a statistical spread is:

$$\delta_x = \frac{\sigma_x}{\sqrt{N}}$$

$$\begin{aligned} x_{\text{best}} &= \bar{x} \pm \frac{\sigma_x}{\sqrt{N}} \\ &= 1.960\,462\,\text{s} \pm 0.069\,127\,\text{s} \\ &= 1.96(7)\,\text{s} \end{aligned}$$

$$\boxed{x_{\text{best}} = 1.96(7)\,\text{s}}$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

$$T = 2\pi l^{1/2} g^{-1/2}$$

$$g^{1/2} = 2\pi l^{1/2} T^{-1}$$

$$g = 4pi^2 l T^{-2}$$

For <u>statistical</u> uncertainties

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2}$$

$$\frac{\partial g}{\partial l} = 4\pi^2 T^{-2}$$

$$\frac{\partial g}{\partial T} = (-2)4\pi^2 l T^{-3}$$

$$\begin{aligned}
\frac{\delta g}{g} &= \frac{\sqrt{\left(\frac{\partial g}{\partial l}\delta l\right)^2 + \left(\frac{\partial g}{\partial T}\delta T\right)^2}}{g} \\
\frac{\delta g}{g} &= \sqrt{\left(\frac{\partial g}{\partial l}\frac{\delta l}{g}\right)^2 + \left(\frac{\partial g}{\partial l}\frac{\delta T}{g}\right)^2} \\
\frac{\partial g}{g} &= \sqrt{\left(\frac{\partial l}{l}\right)^2 + 4\left(\frac{\partial T}{T}\right)^2} \\
&= \sqrt{\left(\frac{2}{100}\right)^2 + 4\left(\frac{0.07}{1.96}\right)^2} \\
&= 7
\end{aligned}$$

$$\begin{aligned}
g &= 4\pi^2 l T^{-2} \\
&= 4\pi^2 (1.00\,\mathrm{m})(1.96\,\mathrm{s}^{-2} \\
g &= 10.276\,556\,02\,\mathrm{m/s^2}
\end{aligned}$$

$$\begin{aligned}
\delta g &= (0.07)(10.276\,55\,\mathrm{m/s^2} \\
&= 0.7\,\mathrm{m/s^2} \\
g &= 10.3(7)\,\mathrm{m/s^2}
\end{aligned}$$