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# 1 Chapter 1

### 1.1 Boolean Algebra

A	$\mathbf{B}$	$\mathbf{A} \wedge \mathbf{B}$	$\mathbf{A} \lor \mathbf{B}$	$\mathbf{A}\oplus\mathbf{B}$
Т	Т	Т	Т	F
Т	F	F	Т	Т
F	Т	F	Т	Т
F	F	F	F	F

## 1.2 Quantifiers

- $\bullet$  For all:  $\forall$
- There exist(s):  $\exists$

#### 1.3 Sets

$$\begin{split} \mathbb{N} &: \{0,1,2,3,\cdots\} \\ \mathbb{Z} &: \{0,\pm 1,\pm 2,\cdots\} \\ \mathbb{Q} &: \frac{a}{b} \mid b \neq 0 \mid a,b \in \mathbb{Z} \\ \mathbb{R} &: x^2 = 2, x = ? \\ \mathbb{C} &: x^2 = -1, x = ? \end{split}$$

#### 1.4 Truth Tables

For 
$$p \implies q, P \implies Q$$

Prove by truth table that

$$(p \implies q) \iff (\neg q \implies \neg p)$$

p	q	$p \implies q$
Τ	Τ	${ m T}$
T	F	$\mathbf{F}$
F	Τ	${ m T}$
F	F	${ m T}$

p	q	$\neg p$	$\neg q$	$  \neg p \implies \neg q  $
Τ	Τ	F	F	T
Τ	F	F	Τ	F
F	Τ	Τ	$\mathbf{F}$	T
F	F	Τ	Τ	$ $

#### 1.4.1 1.30

Show that

$$(p \lor q) \land (\neg p \lor r) \implies (q \lor r)$$

is a tautology.

p	q	r	$\neg p$	$p \lor q$	$\neg p \lor r$	$(p \vee q) \wedge (\neg p \vee r)$	$q \vee r$	$(p \lor q) \land (\neg p \lor r) \implies (q \lor r)$
Т	Т	Т	F	Т	Т	Τ	Т	T
T	Т	F	F	T	F	$\mathbf{F}$	${ m T}$	T
T	F	Τ	F	T	Т	${ m T}$	${ m T}$	${ m T}$
T	F	F	F	T	F	$\mathbf{F}$	F	${ m T}$
F	Т	Τ	Τ	T	Т	${ m T}$	${ m T}$	${ m T}$
F	Т	$\mathbf{F}$	Τ	T	Т	${ m T}$	${ m T}$	${ m T}$
F	F	T	Τ	F	Т	$\mathbf{F}$	${ m T}$	${ m T}$
F	F	$\mathbf{F}$	Τ	F	Т	$\mathbf{F}$	F	T

### 1.4.2 Logical Equivalence Proof

Prove

$$p \implies q \equiv \neg q \implies \neg p$$

by case analysis.

Case 1 p is true

Subcase 1 q is true

- Then  $p \implies q$  is true
- $\neg q$  is false  $\implies \neg p \implies \neg p$  is true