Contents

1 Section 4.6

1.1 4.6.1

Determine whether the following vectors are mutually orthogonal.

$$\vec{u}_1 = (1, -2, 1), \vec{u}_2 = (0, 1, 2), \vec{u}_3 = (-5, -2, 1)$$

$$\begin{aligned}
\vec{u}_1 \cdot \vec{u}_2 &= 1 \cdot 0 + -2 \cdot 1 + 1 \cdot 2 = 0 \therefore \vec{u}_1 \perp \vec{u}_2 \\
\vec{u}_1 \cdot \vec{u}_3 &= 1 \cdot -5 + -2 \cdot -2 + 1 \cdot 1 = 0 \therefore \vec{u}_1 \perp \vec{u}_3 \\
\vec{u}_2 \cdot \vec{u}_3 &= 0 \cdot -5 + 1 \cdot -2 + 2 \cdot 1 = 0 \therefore \vec{u}_2 \perp \vec{u}_3
\end{aligned}$$

The vectors are mutually orthogonal because each pair of distinct vectors is orthogonal.

1.2 4.6.5

The three vertices A, B, and C, of a triangle are given. Prove that the triangle is a right triangle by showing that its sides a, b, and c satisfy the Pythagorean relation $a^2 + b^2 = c^2$.

Find the length of each side a, b, and c.

$$\begin{aligned}
d(B, C) &= \sqrt{(3-4)^2 + (8-9)^2 + (3-6)^2 + (5-3)^2} \\
d(B, C) &= \sqrt{15}
\end{aligned}$$

$$\begin{aligned}
d(A, C) &= \sqrt{(3-4)^2 + (8-7)^2 + (3-5)^2 + (5-8)^2} \\
d(A, C) &= \sqrt{15}
\end{aligned}$$

$$\begin{aligned}
d(A, B) &= \sqrt{((4-4)^2 + (9-7)^2 + (6-5)^2 + (3-8)^2} \\
d(A, B) &= \sqrt{30}
\end{aligned}$$

As the distance $d(A, B)$ is the longest of the three sides, it is therefore the hypotenuse.

$$\begin{gathered}
a^2 + b^2 = c^2 \\
(\sqrt{15})^2 + (\sqrt{15})^2 = (\sqrt{30})^2 \\
0 = 0
\end{gathered}$$

1.3 4.6.13

The vector $\vec{v}_1 = \begin{bmatrix} 1 \\ -7 \\ 8 \end{bmatrix}$ spans a subspace $\mathbf{V}$ of the indicated Euclidean space. Find a basis for the orthogonal complement $\mathbf{V}^\perp$ of $\mathbf{V}$.

$$\mathbf{V} = \begin{bmatrix} 1 & -7 & 8 \end{bmatrix}$$

$$\begin{aligned} x_1 - 7x_2 + 8x_3 &= 0 \\ x_1 &= 7x_2 - 8x_3 \\ \mathbf{V}^\perp &= \begin{bmatrix} 7x_2 - 8x_3 \\ x_2 \\ x_3 \end{bmatrix} \\ \mathbf{V}^\perp &= x_2 \begin{bmatrix} 7 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -8 \\ 0 \\ 1 \end{bmatrix} \end{aligned}$$

$$\boxed{\text{A basis for the orthogonal complement } \mathbf{V}^\perp \text{ is } \left\{ \begin{bmatrix} 7 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -8 \\ 0 \\ 1 \end{bmatrix} \right\}}$$