Homework 12

Corey Mostero - 2566652

27 June 2023

Contents

0.1 Question 1

Density: What is the radius of a sphere that has a density of $5000\,\mathrm{kg\,m^{-3}}$ and a mass of $6.00\,\mathrm{kg}$?

$$\begin{aligned} \rho &= 5000\,\mathrm{kg\,m^{-3}} \\ m &= 6.00\,\mathrm{kg} \\ r &= ? \end{aligned}$$

$$\begin{aligned} \rho &= \frac{m}{V} \\ V &= \frac{m}{\rho} \\ \frac{4}{3}\pi r^3 &= \frac{m}{\rho} \\ r &= \sqrt[3]{\frac{3m}{4\pi\rho}} \\ r &= \sqrt[3]{\frac{3(6.00\,\mathrm{kg})}{4\pi(5000\,\mathrm{kg\,m^{-3}})}} \\ r &= 0.065\,922\,\mathrm{m} = 6.59\,\mathrm{cm} \end{aligned}$$

$$\boxed{r = 6.59\,\mathrm{cm}}$$

0.2 Question 2

Pressure in a fluid: A cubical box, $5.00\,\mathrm{cm}$ on each side, is immersed in a fluid. The gauge pressure at the top surface of the box is $594\,\mathrm{Pa}$ and the gauge pressure on the bottom surface is 1133 Pa. What is the density of the fluid?

$$\begin{aligned} h &= 5.00\,\mathrm{cm} = 0.05\,\mathrm{m} \\ p_0 &= 594\,\mathrm{Pa} \\ p_1 &= 1133\,\mathrm{Pa} \\ \rho &= ? \end{aligned}$$

$$\begin{aligned} p_1 &= p_0 + \rho g h \\ \rho &= \frac{p_1 - p_0}{gh} \\ \rho &= \frac{1133\,\mathrm{Pa} - 594\,\mathrm{Pa}}{(9.80\,\mathrm{m\,s^{-2}})(0.05\,\mathrm{m})} \\ \rho &= 1100\,\mathrm{kg\,m^{-3}} \end{aligned}$$

$$\boxed{\rho = 1100\,\mathrm{kg\,m^{-3}}}$$

0.3 Question 3

Pressure in a fluid: As shown in the figure, a container has a vertical tube, whose inner radius is 32.00 mm, connected to it at its side. An unknown liquid reaches level A in the container and level B in this tube - level A being 5.0 cm higher than level B. The liquid supports a 20.0 cm high column of oil, between levels B and C, whose density is $460\,\mathrm{kg\,m^{-3}}$. What is the density of the unknown liquid?

$$\begin{aligned} y_{A,B} &= 5.0\,\mathrm{cm} = 0.05\,\mathrm{m} \\ y_{B,C} &= 20.0\,\mathrm{cm} = 0.20\,\mathrm{m} \\ \rho_{oil} &= 460\,\mathrm{kg\,m^{-3}} \\ \rho_{unknown} &= ? \end{aligned}$$

$$\begin{aligned} \rho_{unk} g y_{A,B} &= \rho_{oil} g y_{B,C} \\ \rho_{unk} &= \frac{\rho_{oil} y_{B,C}}{y_{A,B}} \\ \rho_{unk} &= \frac{(460\,\mathrm{kg\,m^{-3}})(0.20\,\mathrm{m})}{0.05\,\mathrm{m}} \\ \rho_{unk} &= 1840\,\mathrm{kg\,m^{-3}} \end{aligned}$$

$$\boxed{\rho_{unknown} = 1840\,\mathrm{kg\,m^{-3}} \approx 1800\,\mathrm{kg\,m^{-3}}}$$

0.4 Question 4

Pressure in a fluid: In the figure, an open tank contains a layer of oil floating on top of a layer of water (of density $1000\,\mathrm{kg\,m^{-3}}$) that is 3.0 m thick, as shown.

What must be the thickness of the oil layer if the gauge pressure at the bottom of the tank is to be 5.0×10^4 Pa? The density of the oil is $510\,\mathrm{kg\,m^{-3}}$.

$$\begin{aligned}
\rho_{water} &= 1000\,\mathrm{kg\,m^{-3}} \\
y_{water} &= 3.0\,\mathrm{m} \\
p_0 &= 5.0 \times 10^4\,\mathrm{Pa} \\
\rho_{oil} &= 510\,\mathrm{kg\,m^{-3}} \\
y_{oil} &=?
\end{aligned}$$

$$\begin{aligned}
\Delta p &= \rho_{water} g y_{water} + \rho_{oil} g y_{oil} \\
y_{oil} &= \frac{\Delta p - \rho_{water} g y_{water}}{\rho_{oil} g} \\
y_{oil} &= \frac{5.0 \times 10^4\,\mathrm{Pa} - (1000\,\mathrm{kg\,m^{-3}})(9.80\,\mathrm{m\,s^{-2}})(3.0\,\mathrm{m})}{(510\,\mathrm{kg\,m^{-3}})(9.80\,\mathrm{m\,s^{-2}})} \\
y_{oil} &= 4.121\,65\,\mathrm{m} = 4.12\,\mathrm{m}
\end{aligned}$$

$$\boxed{y_{oil} = 4.12\,\mathrm{m} \approx 4.1\,\mathrm{m}}$$

0.5 Question 5

Pascal's principle: A 12 000 N car is raised using a hydraulic lift, which consists of a U-tube with arms of unequal areas, filled with incompressible oil and capped at both ends with tight-fitting pistons. The wider arm of the U-tube has a radius of 18.0 cm and the narrower arm has a radius of 5.00 cm. The car rests on the piston on the wider arm of the U-tube. The pistons are initially at the same level. What is the initial force that must be applied to the smaller piston in order to start lifting the car?

$$\begin{aligned}
w_{car} &= 12\,000\,\mathrm{N} \\
r_{wide} &= 18.0\,\mathrm{cm} = 0.18\,\mathrm{m} \\
r_{narrow} &= 5.00\,\mathrm{cm} = 0.05\,\mathrm{m} \\
F &=?
\end{aligned}$$

$$\begin{aligned}
\frac{F}{A_{narrow}} &= \frac{w_{car}}{A_{wide}} \\
F &= \frac{w_{car} \pi r_{narrow}^2}{\pi r_{wide}^2} \\
F &= \frac{(12\,000\,\mathrm{N})(0.05\,\mathrm{m})^2}{(0.18\,\mathrm{m})^2} \\
F &= 925.926\,\mathrm{N} = 926.0\,\mathrm{N}
\end{aligned}$$

$$\boxed{F = 926.0\,\mathrm{N}}$$

0.6 Question 6

Buoyancy: A board that is 20.0 cm wide, 5.00 cm thick, and 3.00 m long has a density 350 kg m^{-3}. The board is floating partially submerged in water of density 1000 kg m^{-3}. What fraction of the volume is above the surface of the water?

$$\begin{aligned}
w_{plank} &= 20.0\,\text{cm} = 0.20\,\text{m} \\
h_{plank} &= 5.00\,\text{cm} = 0.05\,\text{m} \\
l_{plank} &= 3.00\,\text{m} \\
\rho_{plank} &= 350\,\text{kg}\,\text{m}^{-3} \\
\rho_{water} &= 1000\,\text{kg}\,\text{m}^{-3}
\end{aligned}$$

$$\begin{aligned}
\rho_{plank} &= \frac{m_{plank}}{V_{plank}} \\
m_{plank} &= \rho_{plank} V_{plank} \\
m_{plank} &= (350\,\text{kg}\,\text{m}^{-3})(0.20\,\text{m})(0.05\,\text{m})(3.00\,\text{m}) \\
m_{plank} &= 10.5\,\text{kg}
\end{aligned}$$

$$\begin{aligned}
\sum F_y &= 0 \\
B &= w_{plank} \\
\rho_{water} V_{water} g &= m_{plank} g \\
V_{water} &= \frac{m_{plank}}{\rho_{water}} \\
V_{water} &= \frac{10.5\,\text{kg}}{1000\,\text{kg}\,\text{m}^{-3}} \\
V_{water} &= 0.0105\,\text{m}^3
\end{aligned}$$

Find the volume of the plank, then find its ratio compared with V_{water}.

$$\begin{aligned}
V_{plank} &: V_{water} \\
(0.20\,\text{m})(0.05\,\text{m})(3.00\,\text{m}) &: 0.0105\,\text{m}^3 \\
0.03\,\text{m}^3 &: 0.0105\,\text{m}^3 \\
\frac{0.03\,\text{m}^3}{0.03\,\text{m}^3} &: \frac{0.0105\,\text{m}^3}{0.03\,\text{m}^3} \\
1.00 &: 0.35
\end{aligned}$$

In other words, for every 1.00 (the entirety of the block) block, 0.35 of it is submerged in the water. The fraction of volume that is above the surface of the water would be the difference:

$$\boxed{1.00 - 0.35 = 0.650}$$

0.7 Question 7

Buoyancy: A rock is suspended from a scale reads 20.0 N. A beaker of water (having a density of $1000\,\mathrm{kg\,m^{-3}}$) is raised up so the rock is totally submerged in the water. The scale now reads 12.5 N. What is the density of the rock?

$$
\begin{aligned}
w_{rock_0} &= 20.0\,\mathrm{N} \\
w_{rock_1} &= 12.5\,\mathrm{N} \\
\rho_{rock} &= ?
\end{aligned}
$$

$$
\begin{aligned}
B &= w_{air} - w_{water} \\
B &= w_{rock_0} - w_{rock_1} \\
B &= 20.0\,\mathrm{N} - 12.5\,\mathrm{N} \\
B &= 7.50\,\mathrm{N}
\end{aligned}
$$

$$
\begin{aligned}
\sum F_y &= 0 \\
B &= w_{rock_1} \\
B &= \rho_{water} V_{rock} g \\
V_{rock} &= \frac{B}{\rho_{water} g} \\
V_{rock} &= \frac{7.50\,\mathrm{N}}{(1000\,\mathrm{kg\,m^{-3}})(9.80\,\mathrm{m\,s^{-2}})} \\
V_{rock} &= 0.000\,765\,\mathrm{m^3} = 7.65 \times 10^{-4}\,\mathrm{m^3}
\end{aligned}
$$

$$
\begin{aligned}
\rho_{rock} &= \frac{m_{rock}}{V_{rock}} \\
\rho_{rock} &= \frac{\frac{20.0\,\mathrm{N}}{9.80\,\mathrm{m\,s^{-2}}}}{7.65 \times 10^{-4}\,\mathrm{m^3}} \\
\rho_{rock} &= 2667.73\,\mathrm{kg\,m^{-3}} = 2.67 \times 10^3\,\mathrm{kg\,m^{-3}}
\end{aligned}
$$

$$\boxed{\rho_{rock} = 2.67 \times 10^3\,\mathrm{kg\,m^{-3}}}$$

0.8 Question 8

Buoyancy: A 7.8 kg solid sphere, made of metal whose density is $2500\,\mathrm{kg\,m^{-3}}$, is suspended by a card. When the sphere is immersed in water (of density $1000\,\mathrm{kg\,m^{-3}}$), what is the tension in the cord?

$$
\begin{aligned}
m_{sphere} &= 7.8\,\mathrm{kg} \\
\rho_{metal} &= 2500\,\mathrm{kg\,m^{-3}} \\
\rho_{water} &= 1000\,\mathrm{kg\,m^{-3}} \\
T &= ?
\end{aligned}
$$

$$\rho_{sphere} = \frac{m_{sphere}}{V_{sphere}}$$
$$V_{sphere} = \frac{m_{sphere}}{\rho_{sphere}}$$

$$\sum F_y = 0$$
$$T + B = w_{sphere}$$
$$T + \rho_{water} V_{sphere} g = m_{sphere} g$$
$$T = m_{sphere} g \left(1 - \frac{\rho_{water}}{\rho_{sphere}}\right)$$
$$T = (7.8\,\mathrm{kg})(9.80\,\mathrm{m\,s^{-2}}) \left(1 - \frac{1000\,\mathrm{kg\,m^{-3}}}{2500\,\mathrm{kg\,m^{-3}}}\right)$$
$$T = 45.9\,\mathrm{N}$$

$$\boxed{T = 45.9\,\mathrm{N} \approx 46.0\,\mathrm{N}}$$

0.9 Question 9

Buoyancy: A circular cylinder of height 1.20 m having faces of diameter 0.620 m is immersed in water of density $1.00 \times 10^3\,\mathrm{kg\,m^{-3}}$ with its axis vertical so that its faces are parallel to the surface of the water. The upper face is 2.50 m below the surface of the water. The net force on this cylinder is observed to be 1120 N downward. Atmospheric pressure is 1.01×10^5 Pa. What is the weight of the cylinder?

$$h = 1.20\,\mathrm{m}$$
$$d = 0.620\,\mathrm{m}$$
$$\rho_{water} = 1.00 \times 10^3\,\mathrm{kg\,m^{-3}}$$
$$\Delta y = 2.50\,\mathrm{m}$$
$$F_{total} = 1120\,\mathrm{N}$$
$$p_{atm} = 1.01 \times 10^5\,\mathrm{Pa}$$

$$\sum F_y = 0$$
$$F_{total} + B = w$$
$$w = F_{total} + \rho_{water} V_{cylidner} g$$
$$w = 1120\,\mathrm{N} + (1.00 \times 10^3\,\mathrm{kg\,m^{-3}}) \left(\pi \left(\frac{0.620\,\mathrm{m}}{2}\right)^2 (1.20\,\mathrm{m})\right) (9.80\,\mathrm{m\,s^{-2}})$$
$$w = 4670.43\,\mathrm{N}$$

$$\boxed{w = 4670.43\,\mathrm{N} \approx 4670\,\mathrm{N}}$$

0.10 Question 10

Flow rate: Water flowing through a pipe suddenly comes to a section of pipe where the pipe diameter decreases to 86 % of its previous value. If the speed of the water in the larger section of the pipe was $36\,\mathrm{m\,s^{-1}}$, what is its speed in this smaller section?

$$\begin{aligned} d_1 &= (0.86)d_0 \\ v_0 &= 36\,\mathrm{m\,s^{-1}} \\ v_1 &= ? \end{aligned}$$

$$\begin{aligned} A_0 v_0 &= A_1 v_1 \\ \pi\left(\frac{d_0}{2}\right)^2 v_0 &= \pi\left(\frac{(0.86)d_0}{2}\right)^2 v_1 \\ v_1 &= \frac{v_0}{0.86^2} \\ v_1 &= \frac{36\,\mathrm{m\,s^{-1}}}{0.86^2} \\ v_1 &= 48.675\,\mathrm{m\,s^{-1}} \end{aligned}$$

$$\boxed{v_1 = 48.675\,\mathrm{m\,s^{-1}} \approx 49\,\mathrm{m\,s^{-1}}}$$

0.11 Question 11

Bernoulli's principle: Incompressible water flows out of a large reservoir through a pipe that opens to the atmosphere 5.70 m below the level of the water in the reservoir. What is the speed of the water as it comes out of the pipe?

$$\begin{aligned} y_0 &= 5.70\,\mathrm{m} \\ y_1 &= 0 \\ v_0 &= 0 \\ \rho_{water} &= 1000\,\mathrm{kg\,m^{-3}} \\ v_1 &= ? \end{aligned}$$

$$\begin{aligned} \rho_{water} g y_0 &= \frac{1}{2}\rho_{water} v_1^2 \\ v_1 &= \sqrt{2 g y_0} \\ v_1 &= \sqrt{2(9.80\,\mathrm{m\,s^{-2}})(5.70\,\mathrm{m})} \\ v_1 &= 10.5698\,\mathrm{m\,s^{-1}} = 10.6\,\mathrm{m\,s^{-1}} \end{aligned}$$

$$\boxed{v_1 = 10.6\,\mathrm{m\,s^{-1}}}$$

0.12 Question 12

Bernoulli's principle: A horizontal tube consists of a 7.0 cm diameter pipe that narrows to a 2.0 cm diameter throat. In the pipe, the water pressure is twice atmospheric pressure and the water flows with a speed of $0.40\,\mathrm{m\,s^{-1}}$. What is the pressure in the throat, assuming that the water behaves like an ideal fluid? The density of water is $1000\,\mathrm{kg\,m^{-3}}$, and atmospheric pressure is 1.01×10^5 Pa.

$$\begin{aligned}
d_0 &= 7.0\,\mathrm{cm} = 0.07\,\mathrm{m}\\
d_1 &= 2.0\,\mathrm{cm} = 0.02\,\mathrm{m}\\
p_0 &= 2(1.01 \times 10^5\,\mathrm{Pa}) = 2.02 \times 10^5\,\mathrm{Pa}\\
v_0 &= 0.40\,\mathrm{m\,s^{-1}}\\
p_1 &= ?
\end{aligned}$$

$$\begin{aligned}
A_0 v_0 &= A_1 v_1\\
v_1 &= \frac{\pi r_0^2 v_0}{\pi r_1^2}\\
v_1 &= \frac{\left(\frac{0.07\,\mathrm{m}}{2}\right)^2 (0.40\,\mathrm{m\,s^{-1}})}{\left(\frac{0.02\,\mathrm{m}}{2}\right)^2}\\
v_1 &= 4.9\,\mathrm{m\,s^{-1}}
\end{aligned}$$

$$\begin{aligned}
p_0 + \frac{1}{2}\rho_{water} v_0^2 &= p_1 + \frac{1}{2}\rho_{water} v_1^2\\
p_1 &= p_0 + \frac{1}{2}\rho_{water} v_0^2 - \frac{1}{2}\rho_{water} v_1^2\\
p_1 &= 2.02 \times 10^5\,\mathrm{Pa} + \frac{1}{2}(1000\,\mathrm{kg\,m^{-3}})\left[(0.40\,\mathrm{m\,s^{-1}})^2 - (4.9\,\mathrm{m\,s^{-1}})^2\right]\\
p_1 &= 190\,075\,\mathrm{Pa}
\end{aligned}$$

$$\boxed{p_1 = 190\,075\,\mathrm{Pa} \approx 1.9\,\mathrm{atm}}$$

0.13 Question 13

Bernoulli's principle: Consider a very small hole in the bottom of a tank 20.0 cm in diameter filled with water to a height of 50.0 cm. Find the speed at which the water exits the tank through the hole.

$$\begin{aligned}
d &= 20.0\,\mathrm{cm} = 0.20\,\mathrm{m}\\
r &= {}^{d}\!/_{2} = 0.10\,\mathrm{m}\\
h &= 50.0\,\mathrm{cm} = 0.50\,\mathrm{m}\\
v_1 &= ?
\end{aligned}$$

$$
\begin{aligned}
\rho_{water} g h &= \frac{1}{2}\rho_{water} v_1^2 \\
v_1 &= \sqrt{2gh} \\
v_1 &= \sqrt{2(9.80\,\mathrm{m\,s^{-2}})(0.50\,\mathrm{m})} \\
v_1 &= 3.1305\,\mathrm{m\,s^{-1}} = 3.13\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$\boxed{v_1 = 3.13\,\mathrm{m\,s^{-1}}}$$

0.14 Question 14

Bernoulli's principle: Water flows in the horizontal pipe shown in the figure. At point A the area is $25.0\,\mathrm{cm^2}$ and the speed of the water is $2.00\,\mathrm{m\,s^{-1}}$. At B the area is $16.0\,\mathrm{cm^2}$. The fluid in the manometer is mercury, which has a density of $13\,600\,\mathrm{kg\,m^{-3}}$. We can treat water as an ideal fluid having a density of $1000\,\mathrm{kg\,m^{-3}}$. What is the manometer reading h?

$$
\begin{aligned}
A_0 &= 25.0\,\mathrm{cm^2} = 0.25\,\mathrm{m^2} \\
v_0 &= 2.00\,\mathrm{m\,s^{-1}} \\
A_1 &= 16.0\,\mathrm{cm^2} = 0.16\,\mathrm{m^2} \\
v_1 &= ? \\
\rho_{mercury} &= 13\,600\,\mathrm{kg\,m^{-3}} \\
\rho_{water} &= 1000\,\mathrm{kg\,m^{-3}} \\
h &= ?
\end{aligned}
$$

$$
\begin{aligned}
A_0 v_0 &= A_1 v_1 \\
v_1 &= \frac{A_0 v_0}{A_1} \\
v_1 &= \frac{(0.25\,\mathrm{m^2})(2.00\,\mathrm{m\,s^{-1}})}{0.16\,\mathrm{m^2}} \\
v_1 &= 3.125\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
p_0 + \frac{1}{2}\rho_{water} v_0^2 &= p_1 + \frac{1}{2}\rho_{water} v_1^2 \\
\frac{1}{2}\rho_{water}(v_0^2 - v_1^2) &= p_1 - p_0 \\
\frac{1}{2}\rho_{water}(v_0^2 - v_1^2) &= \rho_{mercury} g h \\
h &= \frac{\rho_{water}(v_0^2 - v_1^2)}{2\rho_{mercury} g} \\
h &= \frac{(1000\,\mathrm{kg\,m^{-3}})((2.00\,\mathrm{m\,s^{-1}})^2 - (3.125\,\mathrm{m\,s^{-1}})^2)}{2(13\,600\,\mathrm{kg\,m^{-3}})(9.80\,\mathrm{m\,s^{-2}})} \\
h &= -0.021\,63\,\mathrm{m} = 0.021\,63\,\mathrm{m}
\end{aligned}
$$

$$\boxed{h = 0.021\,63\,\mathrm{m} = 2.16\,\mathrm{cm}}$$

0.15 Question 15

Water stands at a depth H in a large, open tank whose side walls are vertical, as shown in the figure. A hole is made in one of the walls at a depth h below the water surface. At what distance R from the foot of the wall does the emerging stream strike the floor? Approach: Use conservation of energy to determine the final velocity, then use kinematics to find distance R.

$$\rho_{water} g h_{hole} = \frac{1}{2}\rho_{water} v_1^2$$
$$v_1 = \sqrt{2gh_{hole}}$$

$$\Delta y = v_{0_y} t + \frac{1}{2}gt^2$$
$$h = 0 + \frac{1}{2}gt^2$$
$$t = \sqrt{\frac{2h}{g}}$$

$$\Delta x = v_1 t - \frac{1}{2}a_x t^2$$
$$R = v_1 t$$
$$R = (\sqrt{2gh_{hole}})\left(\sqrt{\frac{2h}{g}}\right)$$
$$R = \sqrt{\frac{4g(H-h)h}{g}}$$
$$R = 2\sqrt{h(H-h)}$$

$$\boxed{R = 2\sqrt{h(H-h)}}$$