Contents

1 Section 6.2

1.1 6.2.1

Determine whether or not the given matrix $\boldsymbol{A}$ is diagonalizable. If it is, find a diagonalizing matrix $\boldsymbol{P}$ and diagonal matrix $\boldsymbol{D}$ such that $\boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P} = \boldsymbol{D}$.

$$\boldsymbol{A} = \begin{bmatrix} 6 & -4 \\ 3 & -1 \end{bmatrix}$$

$$\begin{aligned} \det(\boldsymbol{A} - \lambda\mathbf{I}) &= \begin{bmatrix} 6-\lambda & -4 \\ 3 & -1-\lambda \end{bmatrix} \\ \det(\boldsymbol{A} - \lambda\mathbf{I}) &= (6-\lambda)(-1-\lambda) - (-4)(3) \\ \det(\boldsymbol{A} - \lambda\mathbf{I}) &= \lambda^2 - 5\lambda + 6 = (\lambda-3)(\lambda-2) \\ \lambda_{1,2} &= 3, 2 \end{aligned}$$

$$\begin{gathered} [\boldsymbol{A} - \lambda_1]\,\mathbf{v} = 0 \\ \begin{bmatrix} 3 & -4 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} = 0 \end{gathered}$$

$$\begin{aligned} (3)\mathbf{v}_1 + (-4)\mathbf{v}_2 = 0 \\ \mathbf{v}_1 = \left(\frac{4}{3}\right)\mathbf{v}_2 \end{aligned}$$

$$\begin{aligned} (3)\mathbf{v}_1 + (-4)\mathbf{v}_2 &= 0 \\ (3)\left(\frac{4}{3}\right)\mathbf{v}_2 + (-4)\mathbf{v}_2 &= 0 \\ 0 &= 0 \end{aligned}$$

$$\mathbf{v} = \begin{bmatrix} \left(\frac{4}{3}\right)\mathbf{v}_2 \\ \mathbf{v}_2 \end{bmatrix}$$
$$\mathbf{v} = \mathbf{v}_2 \begin{bmatrix} \frac{4}{3} \\ 1 \end{bmatrix}$$

$$[\boldsymbol{A} - \lambda_2]\,\mathbf{v} = 0$$
$$\begin{bmatrix} 4 & -4 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} = 0$$

$$(4)\mathbf{v}_1 + (-4)\mathbf{v}_2 = 0$$
$$\mathbf{v}_1 = \mathbf{v}_2$$

$$(3)\mathbf{v}_1 + (-3)\mathbf{v}_2 = 0$$
$$\mathbf{v}_1 = \mathbf{v}_2$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_1 \end{bmatrix}$$
$$\mathbf{v} = \mathbf{v}_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\boldsymbol{P} = \begin{bmatrix} \frac{4}{3} & 1 \\ 1 & 1 \end{bmatrix}$$
$$\boldsymbol{D} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

1.2 6.2.5

Determine whether or not the given matrix $\boldsymbol{A}$ is diagonalizable. If it is, find a diagonalizing matrix $\boldsymbol{P}$ and diagonal matrix $\boldsymbol{D}$ such that $\boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P} = \boldsymbol{D}$.

$$\boldsymbol{A} = \begin{bmatrix} 5 & -3 \\ 1 & 1 \end{bmatrix}$$

$$\det(\boldsymbol{A} - \lambda\mathbf{I}) = \begin{bmatrix} 5-\lambda & -3 \\ 1 & 1-\lambda \end{bmatrix}$$
$$\det(\boldsymbol{A} - \lambda\mathbf{I}) = (5-\lambda)(1-\lambda) - (-3)(1)$$
$$\det(\boldsymbol{A} - \lambda\mathbf{I}) = \lambda^2 - 6\lambda + 8 = (\lambda-4)(\lambda-2)$$
$$\lambda_{1,2} = 4, 2$$

$$
\begin{aligned}
[\boldsymbol{A} - \lambda_1]\,\mathbf{v} &= 0 \\
\begin{bmatrix} 1 & -3 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} &= 0
\end{aligned}
$$

$$
\begin{aligned}
(1)\mathbf{v}_1 + (-3)\mathbf{v}_2 &= 0 \\
\mathbf{v}_1 &= (3)\mathbf{v}_2
\end{aligned}
$$

$$
\begin{aligned}
(1)\mathbf{v}_1 + (-3)\mathbf{v}_2 &= 0 \\
(1)(3)\mathbf{v}_2 + (-3)\mathbf{v}_2 &= 0 \\
0 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{v} &= \begin{bmatrix} (3)\mathbf{v}_2 \\ \mathbf{v}_2 \end{bmatrix} \\
\mathbf{v} &= \mathbf{v}_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
[\boldsymbol{A} - \lambda_2]\,\mathbf{v} &= 0 \\
\begin{bmatrix} 3 & -3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} &= 0
\end{aligned}
$$

$$
\begin{aligned}
(3)\mathbf{v}_1 + (-3)\mathbf{v}_2 &= 0 \\
\mathbf{v}_1 &= \mathbf{v}_2
\end{aligned}
$$

$$
\begin{aligned}
(1)\mathbf{v}_1 + (-1)\mathbf{v}_2 &= 0 \\
\mathbf{v}_2 &= \mathbf{v}_2 \\
0 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{v} &= \begin{bmatrix} \mathbf{v}_2 \\ \mathbf{v}_2 \end{bmatrix} \\
\mathbf{v} &= \mathbf{v}_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\end{aligned}
$$

1.3 6.2.10

Determine whether or not the given matrix $\boldsymbol{A}$ is diagonalizable. If it is, find a diagonalizing matrix $\boldsymbol{P}$ and diagonal matrix $\boldsymbol{D}$ such that $\boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P} = \boldsymbol{D}$.

$$
\boldsymbol{A} = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}
$$

$$\det(\boldsymbol{A}-\lambda\mathbf{I}) = \begin{bmatrix} 3-\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix}$$
$$\det(\boldsymbol{A}-\lambda\mathbf{I}) = (3-\lambda)(1-\lambda)-(-1)(1)$$
$$\det(\boldsymbol{A}-\lambda\mathbf{I}) = \lambda^2-4\lambda+4 = (\lambda-2)^2$$
$$\lambda = 2$$

$$[\boldsymbol{A}-\lambda]\,\mathbf{v} = 0$$
$$\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} = 0$$

$$(1)\mathbf{v}_1 + (-1)\mathbf{v}_2 = 0$$
$$\mathbf{v}_1 = \mathbf{v}_2$$

$$(1)\mathbf{v}_1 + (-1)\mathbf{v}_2 = 0$$
$$\mathbf{v}_1 = \mathbf{v}_2$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_1 \end{bmatrix}$$
$$\mathbf{v} = \mathbf{v}_1\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The matrix is not diagonalizable.

1.4 6.2.12

Determine whether or not the given matrix $\boldsymbol{A}$ is diagonalizable. If it is, find a diagonalizing matrix $\boldsymbol{P}$ and diagonal matrix $\boldsymbol{D}$ such that $\boldsymbol{P}^{-1}\boldsymbol{AP} = \boldsymbol{D}$.

$$\boldsymbol{A} = \begin{bmatrix} 10 & 8 \\ -18 & -14 \end{bmatrix}$$

$$\det(\boldsymbol{A}-\lambda\mathbf{I}) = \begin{bmatrix} 10-\lambda & 8 \\ -18 & -14-\lambda \end{bmatrix}$$
$$\det(\boldsymbol{A}-\lambda\mathbf{I}) = (10-\lambda)(-14-\lambda)-(8)(-18)$$
$$\det(\boldsymbol{A}-\lambda\mathbf{I}) = \lambda^2+4\lambda+4 = (\lambda+2)^2$$
$$\lambda = -2$$

$$[\boldsymbol{A}-\lambda]\,\mathbf{v} = 0$$
$$\begin{bmatrix} 12 & 8 \\ -18 & -12 \end{bmatrix}\begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} = 0$$

$$\begin{aligned}(12)\mathbf{v}_1 + (8)\mathbf{v}_2 &= 0 \\ \mathbf{v}_2 &= \left(-\frac{3}{2}\right)\mathbf{v}_1\end{aligned}$$

$$\begin{aligned}(-18)\mathbf{v}_1 + (-12)\mathbf{v}_2 &= 0 \\ (-18)\mathbf{v}_1 + (-12)\left(-\frac{3}{2}\right)\mathbf{v}_1 &= 0 \\ 0 &= 0\end{aligned}$$

$$\begin{aligned}\mathbf{v} &= \begin{bmatrix}\mathbf{v}_1 \\ \left(-\frac{3}{2}\right)\mathbf{v}_1\end{bmatrix} \\ \mathbf{v} &= \mathbf{v}_1\begin{bmatrix}1 \\ -\frac{3}{2}\end{bmatrix}\end{aligned}$$

The matrix is not diagonalizable.

1.5 6.2.13

Determine whether or not the given matrix $\boldsymbol{A}$ is diagonalizable. If it is, find a diagonalizing matrix $\boldsymbol{P}$ and diagonal matrix $\boldsymbol{D}$ such that $\boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P} = \boldsymbol{D}$.

$$\boldsymbol{A} = \begin{bmatrix}2 & 3 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3\end{bmatrix}$$

$$\begin{aligned}\det(\boldsymbol{A} - \lambda\mathbf{I}) &= \begin{bmatrix}2-\lambda & 3 & 0 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & 3-\lambda\end{bmatrix} \\ \det(\boldsymbol{A} - \lambda\mathbf{I}) &= (2-\lambda)((3-\lambda)(3-\lambda) - 0) - 0 - 0 \\ \det(\boldsymbol{A} - \lambda\mathbf{I}) &= -(\lambda-3)^2(\lambda-2) \\ \lambda_{1,2} &= 3, 2\end{aligned}$$

$$\begin{aligned}[\boldsymbol{A} - \lambda_1]\,\mathbf{v} &= 0 \\ \begin{bmatrix}-1 & 3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3\end{bmatrix} &= 0\end{aligned}$$

$$\begin{aligned}(-1)\mathbf{v}_1 + (3)\mathbf{v}_2 + (0)\mathbf{v}_3 &= 0 \\ \mathbf{v}_1 &= (3)\mathbf{v}_2\end{aligned}$$

$$
\begin{aligned}
(0)\mathbf{v}_1 + (0)\mathbf{v}_2 + (0)\mathbf{v}_3 &= 0 \\
0 &= 0
\end{aligned}
$$

$$
\begin{aligned}
(0)\mathbf{v}_1 + (0)\mathbf{v}_2 + (0)\mathbf{v}_3 &= 0 \\
0 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{v} &= \begin{bmatrix} (3)\mathbf{v}_2 \\ \mathbf{v}_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \mathbf{v}_3 \end{bmatrix} \\
\mathbf{v} &= \mathbf{v}_2 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + \mathbf{v}_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\left[\boldsymbol{A} - \lambda_2\right] \mathbf{v} &= 0 \\
\begin{bmatrix} 0 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \end{bmatrix} &= 0
\end{aligned}
$$

$$
\begin{aligned}
(0)\mathbf{v}_1 + (3)\mathbf{v}_2 + (0)\mathbf{v}_3 &= 0 \\
\mathbf{v}_2 &= 0
\end{aligned}
$$

$$
\begin{aligned}
(0)\mathbf{v}_1 + (1)\mathbf{v}_2 + (0)\mathbf{v}_3 &= 0 \\
\mathbf{v}_2 &= 0
\end{aligned}
$$

$$
\begin{aligned}
(0)\mathbf{v}_1 + (0)\mathbf{v}_2 + (1)\mathbf{v}_3 &= 0 \\
\mathbf{v}_3 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{v} &= \begin{bmatrix} \mathbf{v}_1 \\ 0 \\ 0 \end{bmatrix} \\
\mathbf{v} &= \mathbf{v}_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{v}_{1,2,3} &= \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\
\boldsymbol{D} &= \begin{bmatrix} 3 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \\
\boldsymbol{P} &= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}
\end{aligned}
$$

1.6 6.2.16

Determine whether or not the given matrix $\boldsymbol{A}$ is diagonalizable. If it is, find a diagonalizing matrix $\boldsymbol{P}$ and diagonal matrix $\boldsymbol{D}$ such that $\boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P} = \boldsymbol{D}$.

$$\begin{bmatrix} 1 & -3 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= \begin{bmatrix} 1-\lambda & -3 & 3 \\ 0 & 2-\lambda & -1 \\ 0 & 0 & 1-\lambda \end{bmatrix} \\
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= (1-\lambda)((2-\lambda)(1-\lambda) - 0) - (-3)(0) - (3)(0) \\
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= -(\lambda-2)(\lambda-1)^2 \\
\lambda_{1,2} &= 2, 1
\end{aligned}$$

$$\begin{aligned}
[\boldsymbol{A} - \lambda_1]\,\mathbf{v} &= 0 \\
\begin{bmatrix} -1 & -3 & 3 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \end{bmatrix} &= 0
\end{aligned}$$

$$\begin{aligned}
(0)\mathbf{v}_1 + (0)\mathbf{v}_2 + (-1)\mathbf{v}_3 &= 0 \\
\mathbf{v}_3 &= 0
\end{aligned}$$

$$\begin{aligned}
(-1)\mathbf{v}_1 + (-3)\mathbf{v}_2 + (3)\mathbf{v}_3 &= 0 \\
(-1)\mathbf{v}_1 + (-3)\mathbf{v}_2 + (3)(0) &= 0 \\
\mathbf{v}_1 &= (-3)\mathbf{v}_2
\end{aligned}$$

$$\mathbf{v} = \mathbf{v}_2 \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{aligned}
[\boldsymbol{A} - \lambda_2]\,\mathbf{v} &= 0 \\
\begin{bmatrix} 0 & -3 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \end{bmatrix} &= 0
\end{aligned}$$

$$\begin{aligned}
(0)\mathbf{v}_1 + (0)\mathbf{v}_2 + (0)\mathbf{v}_3 &= 0 \\
0 &= 0
\end{aligned}$$

$$
\begin{aligned}
(0)\mathbf{v}_1 + (1)\mathbf{v}_2 + (-1)\mathbf{v}_3 &= 0 \\
\mathbf{v}_2 &= \mathbf{v}_3
\end{aligned}
$$

$$
\begin{aligned}
(0)\mathbf{v}_1 + (-3)\mathbf{v}_2 + (3)\mathbf{v}_3 &= 0 \\
(0)\mathbf{v}_1 + (-3)(\mathbf{v}_3) + (3)\mathbf{v}_3 &= 0 \\
0 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{v} &= \begin{bmatrix} \mathbf{v}_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{v}_3 \\ \mathbf{v}_3 \end{bmatrix} \\
\mathbf{v} &= \mathbf{v}_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \mathbf{v}_3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}
\end{aligned}
$$

1.7 6.2.19

Determine whether or not the given matrix $\boldsymbol{A}$ is diagonalizable. If it is, find a diagonalizing matrix $\boldsymbol{P}$ and diagonal matrix $\boldsymbol{D}$ such that $\boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P} = \boldsymbol{D}$.

$$
\boldsymbol{A} = \begin{bmatrix} 2 & -1 & 1 \\ -4 & 2 & 4 \\ -2 & -1 & 5 \end{bmatrix}
$$

$$
\begin{aligned}
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= \begin{bmatrix} 2-\lambda & -1 & 1 \\ -4 & 2-\lambda & 4 \\ -2 & -1 & 5-\lambda \end{bmatrix} \\
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= (2-\lambda)((2-\lambda)(5-\lambda) - (4)(-1) \\
&\quad + (-1)((-4)(5-\lambda) - (4)(-2)) \\
&\quad + (1)((-4)(-1) - (2-\lambda)(-2)) \\
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= -\lambda^3 + 9\lambda^2 - 26\lambda + 24 = -(\lambda-4)(\lambda-3)(\lambda-2) \\
\lambda_{1,2,3} &= 4, 3, 2
\end{aligned}
$$