Contents

1 Section 7.3

1.1 7.3.1-T

Apply the eigenvalue method to find a general solution of the given system. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the given system.

$$x_1' = 2x_1 + 6x_2$$
$$x_2' = 6x_1 + 2x_2$$

$$\begin{bmatrix} \mathbf{x}_1' \\ \mathbf{x}_2' \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix}$$
$$\det(\boldsymbol{A} - \lambda\mathbf{I}) = \begin{bmatrix} 2-\lambda & 6 \\ 6 & 2-\lambda \end{bmatrix}$$
$$\det(\boldsymbol{A} - \lambda\mathbf{I}) = (2-\lambda)(2-\lambda) - (6)(6) = \lambda^2 - 4\lambda - 32 = (\lambda - 8)(\lambda + 4)$$
$$\lambda_{1,2} = 8, -4$$

$$[\boldsymbol{A} - \lambda_1]\,\mathbf{x} = 0$$
$$\begin{bmatrix} -6 & 6 \\ 6 & -6 \end{bmatrix} \mathbf{x} = 0$$
$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \mathbf{x} = 0$$

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_2 \\ \mathbf{x}_2 \end{bmatrix}$$
$$\mathbf{x} = \mathbf{x}_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$[\boldsymbol{A} - \lambda_2] = 0$$
$$\begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix} \mathbf{x} = 0$$
$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x} = 0$$

$$\mathbf{x} = \begin{bmatrix} -\mathbf{x}_2 \\ \mathbf{x}_2 \end{bmatrix}$$
$$\mathbf{x} = \mathbf{x}_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\mathbf{x} = C_1 \mathbf{x}_1 e^{8t} + C_2 \mathbf{x}_2 e^{-4t}$$
$$\mathbf{x} = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{8t} + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-4t}$$
$$\mathbf{x}_1 = C_1 e^{8t} - C_2 e^{-4t}$$
$$\mathbf{x}_2 = C_1 e^{8t} + C_2 e^{-4t}$$

1.2 7.3.3-T

Apply the eigenvalue method to find a general solution of the given system. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the given system.

$$\mathbf{x}_1' = 3\mathbf{x}_1 + 4\mathbf{x}_2, \mathbf{x}_2' = 3\mathbf{x}_1 + 2\mathbf{x}_2, \mathbf{x}_1(0) = \mathbf{x}_2(0) = 1$$

$$\mathbf{x}' = \begin{bmatrix} 3 & 4 \\ 3 & 2 \end{bmatrix} \mathbf{x}$$

$$\det(\boldsymbol{A} - \lambda \mathbf{I}) = \begin{bmatrix} 3-\lambda & 4 \\ 3 & 2-\lambda \end{bmatrix}$$
$$\det(\boldsymbol{A} - \lambda \mathbf{I}) = (3-\lambda)(2-\lambda) - (4)(3) = (\lambda - 6)(\lambda + 1)$$
$$\lambda_{1,2} = 6, -1$$

$$[\boldsymbol{A} - \lambda_1]\,\mathbf{x} = 0$$
$$\begin{bmatrix} -3 & 4 \\ 3 & -4 \end{bmatrix} \mathbf{x} = 0$$
$$\begin{bmatrix} 1 & \frac{4}{3} \\ 0 & 0 \end{bmatrix} \mathbf{x} = 0$$
$$\mathbf{x} = \mathbf{x}_2 \begin{bmatrix} \frac{4}{3} \\ 1 \end{bmatrix}$$

$$[\boldsymbol{A} - \lambda_2]\,\mathbf{x} = 0$$
$$\begin{bmatrix} 4 & 4 \\ 3 & 3 \end{bmatrix} \mathbf{x} = 0$$
$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x} = 0$$
$$\mathbf{x} = \mathbf{x}_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\mathbf{x}(t) = C_1\mathbf{x}_1e^{6t} + C_2\mathbf{x}_2e^{-1t}$$
$$\mathbf{x}(t) = C_1\begin{bmatrix}\frac{4}{3}\\1\end{bmatrix}e^{6t} + C_2\begin{bmatrix}-1\\1\end{bmatrix}e^{-t}$$

The general solution in matrix form is $\mathbf{x}(t) = C_1\begin{bmatrix}\frac{4}{3}\\1\end{bmatrix}e^{6t} + C_2\begin{bmatrix}-1\\1\end{bmatrix}e^{-t}$.

Now finding the particular solution.

$$\mathbf{x}_1(t) = C_1\left(\frac{4}{3}\right)e^{6t} + C_2(-1)e^{-t}$$
$$\mathbf{x}_1(0) = C_1\left(\frac{4}{3}\right)e^{6(0)} + C_2(-1)e^{-(0)} = 1$$
$$\mathbf{x}_1(0) = C_1\left(\frac{4}{3}\right) - C_2 = 1$$
$$\mathbf{x}_2(t) = C_1(1)e^{6t} + C_2(1)e^{-t}$$
$$\mathbf{x}_2(0) = C_1(1)e^{6(0)} + C_2(1)e^{-(0)} = 1$$
$$\mathbf{x}_2(0) = C_1 + C_2 = 1$$

$$[\mathbf{x}|1] = \left[\begin{array}{cc|c}\frac{4}{3} & -1 & 1\\ 1 & 1 & 1\end{array}\right]$$
$$[\mathbf{x}|1] = \left[\begin{array}{cc|c}1 & 0 & \frac{6}{7}\\ 0 & 1 & \frac{1}{7}\end{array}\right]$$

$$\mathbf{x}(t) = \left(\frac{6}{7}\right)\begin{bmatrix}\frac{4}{3}\\1\end{bmatrix}e^{6t} + \left(\frac{1}{7}\right)\begin{bmatrix}-1\\1\end{bmatrix}e^{-t}$$

1.3 7.3.7-T

Apply the eigenvalue method to find a general solution of the given system. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the given system.

$$\mathbf{x}_1' = -2\mathbf{x}_1 + 6\mathbf{x}_2, \mathbf{x}_2' = 9\mathbf{x}_1 - 5\mathbf{x}_2$$

$$\det(\boldsymbol{A} - \lambda\mathbf{I}) = \begin{bmatrix}-2-\lambda & 6\\ 9 & -5-\lambda\end{bmatrix}$$
$$\det(\boldsymbol{A} - \lambda\mathbf{I}) = (-2-\lambda)(-5-\lambda) - (6)(9) = (\lambda-4)(\lambda+11)$$
$$\lambda_{1,2} = 4, -11$$

$$
\begin{aligned}
[\boldsymbol{A} - \lambda_1]\,\mathbf{x} &= 0 \\
\begin{bmatrix} -6 & 6 \\ 9 & -9 \end{bmatrix} \mathbf{x} &= 0 \\
\mathbf{x} &= \mathbf{x}_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
[\boldsymbol{A} - \lambda_2]\,\mathbf{x} &= 0 \\
\begin{bmatrix} 9 & 6 \\ 9 & 6 \end{bmatrix} \mathbf{x} &= 0 \\
\mathbf{x} &= \mathbf{x}_2 \begin{bmatrix} -\frac{2}{3} \\ 1 \end{bmatrix}
\end{aligned}
$$

$$
\mathbf{x} = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4t} + C_2 \begin{bmatrix} -\frac{2}{3} \\ 1 \end{bmatrix} e^{-11t}
$$

1.4 7.3.15

Apply the eigenvalue method to find a general solution of the given system. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the given system.

$$
\mathbf{x}_1' = 5\mathbf{x}_1 - 10\mathbf{x}_2, \mathbf{x}_2' = 8\mathbf{x}_1 - 3\mathbf{x}_2
$$

$$
\begin{aligned}
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= \begin{bmatrix} 5-\lambda & -10 \\ 8 & -3-\lambda \end{bmatrix} \\
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= (5-\lambda)(-3-\lambda) - (-10)(8) = 1 \pm 8i \\
\lambda_{1,2} &= 1 \pm 8i
\end{aligned}
$$

$$
\begin{aligned}
[\boldsymbol{A} - \lambda_1]\,\mathbf{x} &= 0 \\
\begin{bmatrix} 4-8i & -10 \\ 8 & -8i-4 \end{bmatrix} \mathbf{x} &= 0
\end{aligned}
$$