Contents

1 Inertial Frames of Reference

Reference frame - framework for measurement:

- origin
- positive axes
- t = 0

1.1 Principle of Relativity

There is no preferred reference frame. All physics is equivalent in every frame.

- Inertial: Two frames related by only a velocity transformation
- Non inertial frames - You can still transform from 1 frame to another, but it requires "fictitious force"

In order to notate the relative velocity between two objects,

$$\vec{v}_{\frac{\text{object}}{\text{frame}}}$$

Velocity of object with respect to frame

1.2 Example - Utilizing Relativity/Reference Frames

$$\begin{aligned}
\left\|\vec{v}_{\frac{W}{E}}\right\| &= 0.3\,\mathrm{m\,s^{-1}} \\
\left\|\vec{v}_{\frac{S}{E}}\right\| &= 0.4\,\mathrm{m\,s^{-1}} \\
\left\|\vec{v}_{\frac{S}{W}}\right\| &= ? \\
\theta &= ?
\end{aligned}$$

Establish vector form

$$\begin{aligned}
\vec{v}_{\frac{S}{E}} &= 0\hat{x} + v_{\frac{S}{E}}\hat{y} \\
\vec{v}_{\frac{W}{E}} &= -v_{\frac{W}{E}}\hat{x} + 0\hat{y} \\
\vec{v}_{\frac{S}{W}} &= v_{\frac{S}{W}}\sin(\theta)\hat{x} + v_{\frac{S}{W}}\cos(\theta)\hat{y}
\end{aligned}$$

$$\vec{v}_{\frac{W}{E}} + \vec{v}_{\frac{S}{W}} = \vec{v}_{\frac{S}{E}}$$

Solving x components

$$\begin{aligned}
\vec{v}_{\frac{W}{E}x} + \vec{v}_{\frac{S}{W}x} &= \vec{v}_{\frac{S}{E}x} \\
-\vec{v}_{\frac{W}{E}} + \vec{v}_{\frac{S}{W}}\sin(\theta) &= 0 \\
\vec{v}_{\frac{S}{W}}\sin(\theta) &= \vec{v}_{\frac{W}{E}}
\end{aligned}$$

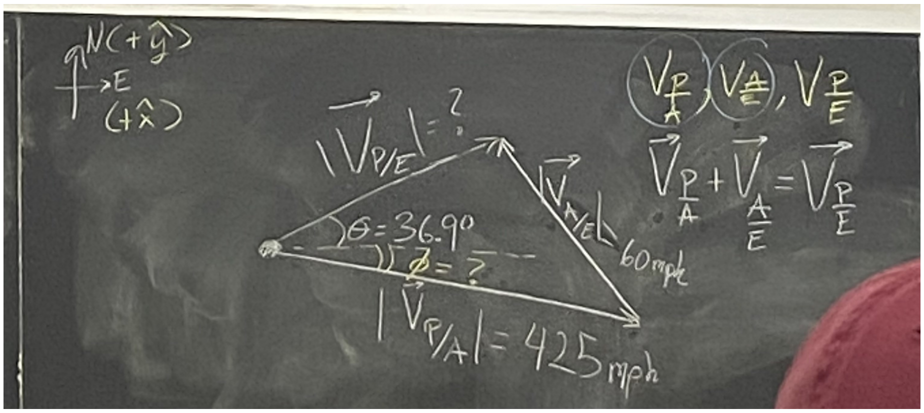
Solving y components

$$\begin{aligned}
\vec{v}_{\frac{W}{E}y} + \vec{v}_{\frac{S}{W}y} &= \vec{v}_{\frac{S}{E}y} \\
0 + \vec{v}_{\frac{S}{W}}\cos(\theta) &= \vec{v}_{\frac{S}{E}} \\
\vec{v}_{\frac{S}{W}}\cos(\theta) &= \vec{v}_{\frac{S}{E}}
\end{aligned}$$

$$\begin{aligned}
\tan(\theta) &= \frac{\vec{v}_{\frac{W}{E}}}{\vec{v}_{\frac{S}{E}}} \\
\theta &= \arctan\left(\frac{\vec{v}_{\frac{W}{E}}}{\vec{v}_{\frac{S}{E}}}\right) \\
\theta &= \arctan\left(\frac{\vec{v}_{\frac{W}{E}}}{\vec{v}_{\frac{S}{E}}}\right) \\
\theta &= \arctan\left(\frac{0.3\,\mathrm{m\,s^{-1}}}{0.4\,\mathrm{m\,s^{-1}}}\right) \\
\theta &= 37^\circ
\end{aligned}$$

$$\begin{aligned}
\vec{v}_{\frac{S}{W}}\sin(\theta) &= \vec{v}_{\frac{W}{E}} \\
\vec{v}_{\frac{S}{W}} &= \frac{v_{\frac{W}{E}}}{\sin(\theta)} \\
&= \frac{0.3\,\mathrm{m\,s^{-1}}}{\sin(37^\circ)} \\
\vec{v}_{\frac{S}{W}} &= 0.5\,\mathrm{m\,s^{-1}}
\end{aligned}$$

1.3 Example - Airplane



$$\theta = 36.9^\circ$$
$$\beta = 45.0^\circ$$

$$\vec{v}_{\frac{P}{A}} = v_{\frac{P}{A}} \cos(\phi)\hat{x} - v_{\frac{P}{A}} \sin(\phi)\hat{y}$$
$$\vec{v}_{\frac{A}{E}} = -v_{\frac{A}{E}} \sin(\beta)\hat{x} + v_{\frac{A}{E}} \cos(\beta)\hat{y}$$
$$\vec{v}_{\frac{P}{E}} = v_{\frac{P}{E}} \cos(\theta)\hat{x} + v_{\frac{P}{E}} \sin(\theta)\hat{y}$$

$$v_{\frac{P}{A}x} + v_{\frac{P}{E}x} = v_{\frac{A}{E}x}$$
$$v_{\frac{P}{A}} \cos(\phi) + v_{\frac{P}{E}} \cos(\theta) = v_{\frac{A}{E}} \sin(\beta)$$
$$(425\,\mathrm{mi\,h^{-1}}) \cos(\phi) + v_{\frac{P}{E}} \cos(36.9^\circ) = (60\,\mathrm{mi\,h^{-1}}) \sin(45.0^\circ)$$

$$v_{\frac{A}{E}y} + v_{\frac{P}{E}y} = v_{\frac{P}{A}y}$$
$$v_{\frac{A}{E}} \cos(\beta) + v_{\frac{P}{E}} \sin(\theta) = v_{\frac{P}{A}} \sin(\phi)$$
$$(60\,\mathrm{mi\,h^{-1}}) \cos(45^\circ) + v_{\frac{P}{E}} \sin(36.9^\circ) = (425\,\mathrm{mi\,h^{-1}}) \sin(\phi)$$

1.4 Example - Airplane 2

A pilot wishes to fly due North. Their plane has an airspeed of $300\,\mathrm{mi\,h^{-1}}$. There is a $50\,\mathrm{mi\,h^{-1}}$ wind blowing $\phi = 25^\circ$ W of N. What angle θ, should the pilot steer?

$$\vec{v}_{\frac{P}{E}} = 0\hat{x} + v_{\frac{P}{E}}\hat{y}$$
$$\vec{v}_{\frac{A}{E}} = (-50\,\mathrm{mi\,h^{-1}}) \sin(25^\circ)\hat{x} + (50\,\mathrm{mi\,h^{-1}}) \cos(25^\circ)$$
$$\vec{v}_{\frac{P}{A}} = (300\,\mathrm{mi\,h^{-1}}) \cos(\theta)\hat{x} + (300\,\mathrm{mi\,h^{-1}}) \sin(\theta)\hat{x}$$

$$\begin{aligned}
\vec{v}_{\frac{A}{E}x} + \vec{v}_{\frac{P}{A}x} &= \vec{v}_{\frac{P}{E}x} \\
(-50\,\mathrm{mi\,h^{-1}})\sin(25^\circ) + (300\,\mathrm{mi\,h^{-1}})\cos(\theta) &= 0 \\
\theta &= \arccos\left(\frac{(50\,\mathrm{mi\,h^{-1}})\sin(25^\circ)}{300\,\mathrm{mi\,h^{-1}}}\right) \\
\theta &= 86.0^\circ
\end{aligned}$$

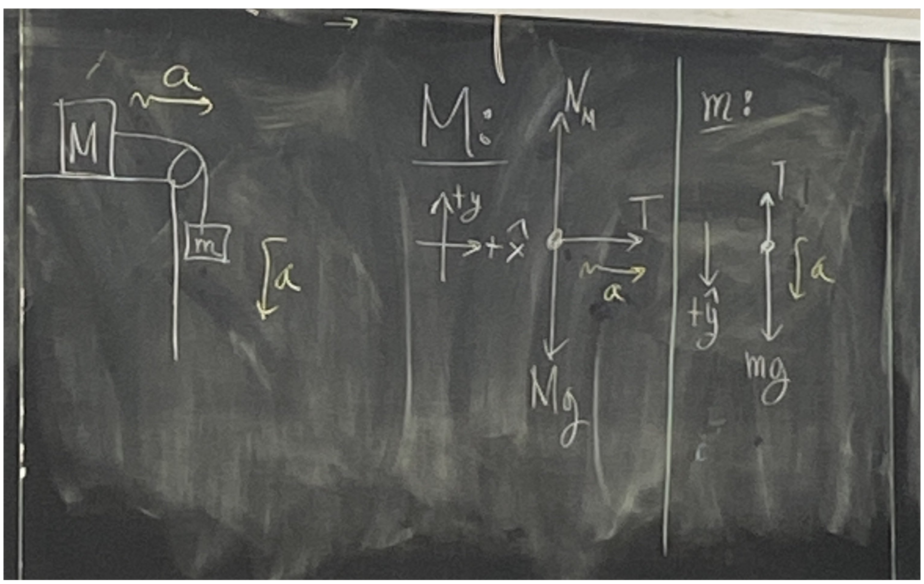
2 Newton's Second Law

$$F \equiv \frac{d\vec{p}}{dt}, \quad \frac{\text{momentum}}{\text{time}} \tag{1}$$

$$\equiv \frac{dm}{dt}v + m\frac{d\vec{v}}{dt}, \quad \frac{dm}{dt} = 0 \tag{2}$$

$$\sum \vec{F}^{(m)} = m\vec{a} \tag{3}$$

2.1 Example - Pulley



Find a and T

$$\begin{aligned}
\sum F_x^{(M)} &= Ma \\
T &= Ma
\end{aligned}$$

$$\begin{aligned}
\sum F_y^{(M)} &= 0 \\
N_M &= Mg
\end{aligned}$$

$$
\begin{aligned}
\sum F_y^{(m)} &= ma \\
mg - T &= ma \\
mg - Ma &= ma \\
a(m + M) &= mg \\
a &= \frac{mg}{m + M}
\end{aligned}
$$

$$
\boxed{T = Ma, a = \frac{mg}{m + M}}
$$