Contents

1 Section 4.4

1.1 4.4.1

Determine whether or not the given vectors in $\mathbb{R}^2$ form a basis for $\mathbb{R}^2$.

$$\vec{v}_1 = \begin{bmatrix} 8 \\ 9 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$$

Do the given vectors form a basis for $\mathbb{R}^2$?

$$\boxed{\text{Yes, because } \vec{v}_1 \text{ and } \vec{v}_2 \text{ are linearly independent.}}$$

1.2 4.4.5

Determine whether or not the given vectors in $\mathbb{R}^3$ form a basis for $\mathbb{R}^3$.

$$\vec{v}_1 = \begin{bmatrix} 0 \\ -7 \\ 9 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ 7 \\ -4 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

Do the given vectors form a basis for $\mathbb{R}^3$?

Begin by finding the determinant:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 \\ -7 & 7 & -5 \\ 9 & -4 & -3 \end{bmatrix}$$

$$\mathbf{A}_2 = 7\mathbf{A}_2 + 9\mathbf{A}_1$$
$$\mathbf{A} = \begin{bmatrix} -7 & 7 & -5 \\ 0 & 35 & -66 \\ 0 & 0 & 0 \end{bmatrix}$$

$\det(\mathbf{A}) = -7 \cdot 35 \cdot 0 = 0$

1.3 4.4.7

Determine whether or not the given vectors in $\mathbb{R}^3$ form a basis for $\mathbb{R}^3$.

$$v_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 8 \\ 3 \\ 13 \end{bmatrix}, v_3 = \begin{bmatrix} 7 \\ 2 \\ 9 \end{bmatrix}$$

Do the given vectors form a basis for $\mathbb{R}^3$?

$$\mathbf{A} = \begin{bmatrix} 0 & 8 & 7 \\ 0 & 3 & 2 \\ 1 & 13 & 9 \end{bmatrix}$$

$$\begin{aligned} &\mathbf{A}_1 \leftrightarrows \mathbf{A}_3 \\ &\mathbf{A}_3 = 3\mathbf{A}_3 - 8\mathbf{A}_2 \\ &\mathbf{A} = \begin{bmatrix} 1 & 13 & 9 \\ 0 & 3 & 2 \\ 0 & 0 & 5 \end{bmatrix} \end{aligned}$$

$$\boxed{\det(\mathbf{A}) = 1 \cdot 3 \cdot 5 = 15 \therefore \text{ Linearly Independent}}$$

1.4 4.4.9

Find a basis for the indicated subspace of $\mathbb{R}^3$.

The plane with equation $x - 8y + 9z = 0$.

$$\mathbf{A} = \begin{bmatrix} 1 \\ -8 \\ 9 \end{bmatrix}$$

$$x = 8y - 9z$$

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} 8y - 9z \\ y \\ z \end{bmatrix} \\ \mathbf{A} &= \begin{bmatrix} 8y \\ y \\ 0 \end{bmatrix} + \begin{bmatrix} -9z \\ 0 \\ z \end{bmatrix} \\ \mathbf{A} &= y \begin{bmatrix} 8 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -9 \\ 0 \\ 1 \end{bmatrix} \end{aligned}$$

$$\boxed{\text{A basis for the indicated subspace of } \mathbb{R}^3 \text{ is } \left\{ \begin{bmatrix} 8 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -9 \\ 0 \\ 1 \end{bmatrix} \right\}.}$$

1.5 4.4.13

Find a basis for the indicated subspace of $\mathbb{R}^4$.

The set of all vectors of the form (a, b, c, d) such that $a = 6c$ and $b = 2d$.

$$\mathbf{A} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 6c \\ 2d \\ c \\ d \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 6c \\ 0 \\ c \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2d \\ 0 \\ d \end{bmatrix}$$

$$\mathbf{A} = c \begin{bmatrix} 6 \\ 0 \\ 1 \\ 0 \end{bmatrix} + d \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{\text{A basis for the indicated subspace of } \mathbb{R}^4 \text{ is } \left\{ \begin{bmatrix} 6 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}.}$$

1.6 4.4.15

Find a basis for the solution space of the given homogeneous linear system.

$$\begin{cases} x_1 - 2x_2 + 11x_3 = 0 \\ 2x_1 - 3x_2 + 14x_3 = 0 \end{cases}$$

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 11 \\ 2 & -3 & 14 \end{bmatrix}$$

$$\mathbf{A}_2 = \mathbf{A}_2 - 2\mathbf{A}_1$$
$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 11 \\ 0 & 1 & -8 \end{bmatrix}$$

$$x_1 = 2x_2 - 11x_3$$
$$x_2 = 8x_3$$
$$x_1 = 2(8x_3) - 11x_3$$
$$x_1 = 5x_3$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$
$$x = \begin{bmatrix} 5x_3 \\ 8x_3 \\ x_3 \end{bmatrix}$$
$$x = x_3 \begin{bmatrix} 5 \\ 8 \\ 1 \end{bmatrix}$$

A basis for the solution space of the given homogeneous linear system is $\left\{ \begin{bmatrix} 5 \\ 8 \\ 1 \end{bmatrix} \right\}$.

1.7 4.4.17

Find a basis for the solution space of the given homogeneous linear system.

$$\begin{cases} x_1 - 3x_2 + \ \ 3x_3 - 3x_4 = 0 \\ 2x_1 - 5x_2 + 11x_3 - 4x_4 = 0 \end{cases}$$

$$\mathbf{A} = \begin{bmatrix} 1 & -3 & 3 & -3 \\ 2 & -5 & 11 & -4 \end{bmatrix}$$

$$\mathbf{A}_2 = \mathbf{A}_2 - 2\mathbf{A}_1$$
$$\mathbf{A} = \begin{bmatrix} 1 & -3 & 3 & -3 \\ 0 & 1 & 5 & 2 \end{bmatrix}$$

$$
\begin{aligned}
x_1 - 3x_2 + 3x_3 - 3x_4 &= 0 \\
x_1 &= 3x_2 - 3x_3 + 3x_4 \\
x_2 + 5x_3 + 2x_4 &= 0 \\
x_2 &= -5x_3 - 2x_4 \\
x_1 &= 3(-5x_3 - 2x_4) - 3x_3 + 3x_4 \\
x_1 &= -18x_3 - 3x_4
\end{aligned}
$$

$$
\begin{aligned}
x &= \begin{bmatrix} -18x_3 - 3x_4 \\ -5x_3 - 2x_4 \\ x_3 \\ x_4 \end{bmatrix} \\
x &= \begin{bmatrix} -18x_3 \\ -5x_3 \\ x_3 \\ 0 \end{bmatrix} + \begin{bmatrix} -3x_4 \\ -2x_4 \\ 0 \\ x_4 \end{bmatrix} \\
x &= x_3 \begin{bmatrix} -18 \\ -5 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \end{bmatrix}
\end{aligned}
$$

A basis for the solution space of the given homogeneous linear system is $\left\{ \begin{bmatrix} -18 \\ -5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$.

1.8 4.4.21

Find a basis for the solution space of the given homogeneous linear system.

$$
\begin{cases}
x_1 - 3x_2 - 9x_3 - 5x_4 = 0 \\
2x_1 + x_2 - 4x_3 + 11x_4 = 0 \\
x_1 - x_2 - 5x_3 + x_4 = 0
\end{cases}
$$

$$
\mathbf{A} = \begin{bmatrix} 1 & -3 & -9 & -5 \\ 2 & 1 & -4 & 11 \\ 1 & -1 & -5 & 1 \end{bmatrix}
$$

$$
\begin{aligned}
\mathbf{A}_2 &= \mathbf{A}_2 - 2\mathbf{A}_1 \\
\mathbf{A}_3 &= \mathbf{A}_3 - \mathbf{A}_1 \\
\mathbf{A} &= \begin{bmatrix} 1 & -3 & -9 & -5 \\ 0 & 7 & 14 & 21 \\ 0 & 2 & 4 & 6 \end{bmatrix}
\end{aligned}
$$

$$\mathbf{A}_3 = 7\mathbf{A}_3 - 2\mathbf{A}_2$$
$$\mathbf{A}_2 = \frac{1}{7}\mathbf{A}_2$$
$$\mathbf{A} = \begin{bmatrix} 1 & -3 & -9 & -5 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x_2 + 2x_3 + 3x_4 &= 0 \\ x_2 &= -2x_3 - 3x_4 \\ x_1 - 3x_2 - 9x_3 - 5x_4 &= 0 \\ x_1 &= 3x_2 + 9x_3 + 5x_4 \\ x_1 &= 3(-2x_3 - 3x_4) + 9x_3 + 5x_4 \\ x_1 &= 15x_3 - 4x_4 \end{aligned}$$

$$x = \begin{bmatrix} 15x_3 - 4x_4 \\ -2x_3 - 3x_4 \\ x_3 \\ x_4 \end{bmatrix}$$
$$x = \begin{bmatrix} 15x_3 \\ 2x_3 \\ x_3 \\ 0 \end{bmatrix} + \begin{bmatrix} -4x_4 \\ -3x_4 \\ 0 \\ x_4 \end{bmatrix}$$
$$x = x_3 \begin{bmatrix} 15 \\ 2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -4 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{\text{A basis for the solution space of the given homogeneous linear system is } \left\{ \begin{bmatrix} 15 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}.}$$