Contents

1 Energy

1.1 Example

$$
\begin{aligned}
h_{0_b} &= 1\,\mathrm{m} \\
h_{1_b} &= 0.7\,\mathrm{m} \\
h_{0_t} &= 0.8\,\mathrm{m} \\
h_{1_t} &= 0.4\,\mathrm{m}
\end{aligned}
$$

$$
\begin{aligned}
\epsilon &= \left|\frac{v_{ref_f}}{v_{ref_i}}\right| \\
mgh &= \frac{1}{2}mv^2 \\
v_i &= \sqrt{2gh_0} \\
v_f &= \sqrt{2gh_1}
\end{aligned}
$$

$$
\begin{aligned}
\epsilon &= \sqrt{\frac{2gh_1}{2gh_0}} \\
\epsilon &= \sqrt{\frac{h_1}{h_0}} \\
\epsilon_b &= 0.84 \\
\epsilon_t &= 0.71
\end{aligned}
$$

A basketball and tennis ball are dropped so that the tennis ball sits on top of the basket ball. $m_{tb} = 0.0568\,\mathrm{kg}$, $m_{bb} = 0.4848\,\mathrm{kg}$, $h_{o_b} = 1.5\,\mathrm{m}$, $D_b = 0.20\,\mathrm{m}$.

1) How fast does the basketball hit the ground?

$$
\begin{aligned}
E_{1.5_m} &= E_{0_m} \\
m_{bb}gh_0 &= \frac{1}{2}m_{bb}u_A^2 \\
u_A &= \sqrt{2gh_0} \\
u_A &= 5.5\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

2) How fast does the basketball rebound?

$$
\begin{aligned}
u_B &= \epsilon u_A \\
u_B &= (0.84)(5.5\,\mathrm{m\,s^{-1}}) \\
u_B &= 4.6\,\mathrm{m\,s^{-1}} = u_i \\
v_i &= 5.5\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

3) How fast does the tennis ball hit the basketball?

$$
\begin{aligned}
v_{CM} &= \frac{m_{TB}v_i + m_{BB}u_i}{m_{TB} + m_{BB}} \\
v_{CM} &= \frac{(0.0568\,\mathrm{kg})(5.5\,\mathrm{m\,s^{-1}}) - (0.4848\,\mathrm{kg})(4.6\,\mathrm{m\,s^{-1}})}{0.0568\,\mathrm{kg} + 0.4848\,\mathrm{kg}} \\
v_{CM} &= -3.54\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

initial	LAB	$-v_{CM}$	ZMF
$\vec{v}$	$5.5\,\mathrm{m\,s^{-1}}$	$3.54\,\mathrm{m\,s^{-1}}$	$9.04\,\mathrm{m\,s^{-1}}$
$\vec{u}$	$-4.6\,\mathrm{m\,s^{-1}}$	$3.54\,\mathrm{m\,s^{-1}}$	$-1.06\,\mathrm{m\,s^{-1}}$

$$
\begin{aligned}
v_f^{ZMF} &= -\epsilon v_i^{ZMF} \\
v_f^{ZMF} &= -(0.71)(9.04\,\mathrm{m\,s^{-1}}) \\
v_f^{ZMF} &= -6.42\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
v_f^{LAB} &= v_{CM} + v_f^{ZMF} \\
v_f^{LAB} &= -6.42\,\mathrm{m\,s^{-1}} - 3.54\,\mathrm{m\,s^{-1}} \\
v_f^{LAB} &= -9.96\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

4) How high does the tennis ball go?

$$
\begin{aligned}
E_c &= E_2 \\
\frac{1}{2}mv_c^2 + mgh_c &= \frac{1}{2}mv_2^2 + mgh_2 \\
\frac{1}{2}v_c^2 + gh_c &= 0 + gh_2 \\
h_2 &= \frac{v_c^2 + gh_c}{2g} \\
h_2 &= \frac{(9.9\,\mathrm{m\,s^{-1}})^2 + (10\,\mathrm{m\,s^{-2}})(0.2\,\mathrm{m})}{2(10\,\mathrm{m\,s^{-2}})} \\
h_2 &= 5.0005\,\mathrm{m}
\end{aligned}
$$

2 Momentum Conservation - Rockets

2.1 Tsiolkovsky Rocket

$$\sum \vec{F}_{ext} = \frac{d\vec{p}}{dt} \tag{1}$$

Momentum of rocket:

$$P_i = mv + vdm$$

If the rocket speeds up:

$$P_f = m(v + dv) + udm$$

What is the limit definition of the derivative?

$$\begin{aligned}
\frac{dp}{dt} \lim_{\Delta t \to 0} \frac{\Delta p}{\Delta t} &= \frac{P_f - P_i}{dt} \\
\frac{dp}{dt} &= \frac{mv + mdv + udm - mv - vdm}{dt} \\
\frac{dp}{dt} &= m\frac{dv}{dt} + (u - v)\frac{dm}{dt} \\
\frac{dp}{dt} &= m\frac{dv}{dt} + u_{rel}\frac{dm}{dt}
\end{aligned}$$

$$\sum F = \frac{dp}{dt} = m\frac{dv}{dt} + u_{rel}\frac{dm}{dt} \tag{2}$$

2.2 Free Space Rocket

$$\begin{aligned}
\sum F &= \frac{dp}{dt} \\
0 &= \frac{dp}{dt} \\
0 &= m\frac{dv}{dt} + u_{rel}\frac{dm}{dt}
\end{aligned}$$

$$\begin{aligned}
\text{initial mass} &= m_0 + \Delta m \\
\text{final mass} &= m_0 \\
v_f - v_i &= \Delta v \quad \text{(Overall change in speed)}
\end{aligned}$$

$$
\begin{aligned}
m\frac{dv}{dt} + u_{rel}\frac{dm}{dt} &= 0 \\
mdv + u_{rel}dm &= 0 \\
mdv &= -u_{rel}dm \\
dv &= -\frac{u_{rel}}{m}dm \\
\int_{v_i}^{v_f} dv &= -u_{rel}\int_{m_0+\Delta m}^{m_0} \frac{1}{m}dm \\
\Delta v &= -u_{rel}\ln\left(\frac{m_0}{m_0+\Delta m}\right) \\
\Delta v &= u_{rel}\ln\left(\frac{m_0+\Delta m}{m_0}\right)
\end{aligned}
$$

$$
\Delta v = u_{rel}\ln\left(\frac{m_0+\Delta m}{m_0}\right) \tag{3}
$$

2.3 Rocket Against Gravity

$$
\begin{aligned}
\sum F &= \frac{dp}{dt} \\
m\frac{dv}{dt} + u_{rel}\frac{dm}{dt} + mg + gdm &= 0 \\
mdv + u_{rel}dm + mgdt + gdmdt &= 0 \quad \text{(The } dmdt \text{ can be assumed to equal 0)} \\
dv + \frac{u_{rel}}{m}dm + gdt &= 0 \\
dv &= -\frac{u_{rel}}{m}dm - gdt \\
\int_{v_i}^{v_f} dv &= -u_{re}\int_{m+\Delta m}^{m_0} \frac{1}{m}dm - g\int_{t_i}^{t_f} dt \\
\Delta v &= u_{rel}\ln\left(\frac{m_0+\Delta m}{m_0}\right) - g\Delta t
\end{aligned}
$$

$$
\Delta v = u_{rel}\ln\left(\frac{m_0+\Delta m}{m_0}\right) - g\Delta t \tag{4}
$$