Contents

1 1D Motion

$$\vec{v} \equiv \frac{d\vec{x}}{dt} \tag{1}$$

$$\vec{a} \equiv \frac{d\vec{v}}{dt} \tag{2}$$

1.1 Kinematics - Constant Acceleration

When doing calculus you must establish Boundary Conditions - A point in time where we know the values of the function.

1.2 $v = v_0 + at$ (No Δx)

During the equation of motion:

$$a = const \tag{3}$$

$$t = 0; x = x_0, v = v_0 \tag{4}$$

We want $x(t)$. Start by finding velocity:

$$a = \frac{dv}{dt} \tag{5}$$

$$dv = adt \tag{6}$$

$$\int dv = \int (a)\, dt \tag{7}$$

$$v = at + C \quad \text{at } t = 0, v = v_0 \tag{8}$$

$$v_0 = (0) + C \tag{9}$$

$$C = v_0 \tag{10}$$

$$v = v_0 + at \tag{11}$$

1.3 Equation of Motion - $x - x_0 + v_0t + \frac{1}{2}at^2$ (No v)

Integrate v to get $x(t)$:

$$v = \frac{dx}{dt} \tag{12}$$

$$v_0 + at = \frac{dx}{dt} \tag{13}$$

$$\int_{x_0}^{x} dx = \int_{0}^{t} (v_0 + at)\, dt \tag{14}$$

$$x - x_0 = v_0t + \frac{1}{2}at^2 - 0 - 0 \tag{15}$$

$$x(t) = x_0 + v_0t + \frac{1}{2}at^2 \tag{16}$$

x as a function of t is called the "equation of motion"

1.4 $x = x_0 + \bar{v}t$ or $x = x_0 + \left(\frac{v_0+v}{2}\right) t$ (No a)

(1)

$$v = v_0 + at$$

(2)

$$a = \frac{v - v_0}{t}$$

(3) Plug (1) into (2)

$$x = x_0 + v_0t + \frac{1}{2}at^2 \tag{17}$$

$$x = x_0 + v_0t + \frac{1}{2}\left(\frac{v - v_0}{t}\right)t^2 \tag{18}$$

$$x = x_0 + \left(\frac{1}{2}v_0 + \frac{1}{2}v\right)t \tag{19}$$

$$x = x_0 + \left(\frac{v_0 + v}{2}\right)t \tag{20}$$

1.5 $v^2 = v_0^2 + 2a\Delta x$ (No t)

(1) $v = v_0 + at$

(2) $t = \frac{v - v_0}{a}$

(3) Plug (1) into (3)

$$x = x_0 + \left(\frac{v_0 + v}{2}\right)\left(\frac{v - v_0}{a}\right) \tag{21}$$

$$2a\Delta x = (v + v_0)(v - v_0) \tag{22}$$

1.6 $x = x_0 - vt + \frac{1}{2}at^2$ (No v_0)

No derivation given.

1.7 Relation

- Slope of velocity $\rightarrow$ acceleration
- Slope of position $\rightarrow$ velocity
- Concavity of position $\rightarrow$ acceleration
- Derivative $\rightarrow$ slope
- Integral $\rightarrow$ area