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# 1 5.1 Weak Induction

# 2 5.2 Strong Induction

## 2.1 Example

Prove that any positive integer can be written as a sum of $2^k$, where $k$'s are distinct.

**Proof**:

1. Basis step:
$$1 = 2^0, 2 = 2^1, 3 = 2^0 + 2^1$$

2. Inductive step:

   We assume that $k$ can be written as a sum of distinct powers of 2.

   Where $1 \leq k \leq n$, we want to show that $n+1$ is a sum of distinct powers of 1.

   Consider the following cases

   (a) $n = \sum 2^r$, where $r$ is some distinct positive integers. Then $n+1 = 1 + \sum 2^r = 2^0 + \sum 2^r$ is a sum of distinct powers of 2.

   (b) $n = 1 + \sum 2^s$, where $s$ is some distinct positive integers. Then $n+1 = 1 + \sum 2^s + 1 = 2 + \sum 2^s = 2\left(1 + \sum 2^{s-1}\right)$. Since $n \geq 1$, $\frac{n+1}{2} = 1 + \sum 2^{s-1}$, then $\frac{n+1}{2}$ is a sum of distinct powers of 2.