Homework 9 - Circular Motion

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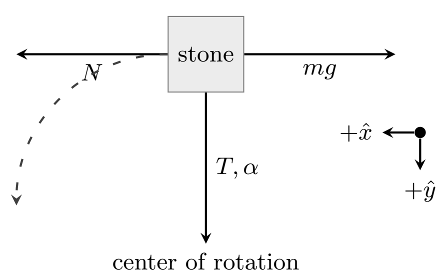
Contents

1 Book

1.1 5.43

$$m = 0.80\,\mathrm{kg}$$
$$L = 0.90\,\mathrm{m}$$
$$T = 60.0\,\mathrm{N}$$

(a) Draw a free-body diagram of the stone.



(b) Find the maximum speed the stone can attain without the string breaking.

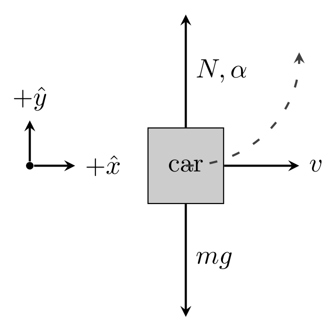
$$\begin{aligned}
\sum F_c &= m\alpha \\
T &= m\left(\frac{v_{max}^2}{L}\right) \\
v_{max} &= \sqrt{\frac{TL}{m}} \\
v_{max} &= \sqrt{\frac{(60.0\,\mathrm{N})(0.90\,\mathrm{m})}{0.80\,\mathrm{kg}}} \\
v_{max} &= 8.22\,\mathrm{m\,s^{-1}}
\end{aligned}$$

$$\boxed{v_{max} = 8.22\,\mathrm{m\,s^{-1}}}$$

1.2 5.45

$$\begin{aligned}
m &= 1.60\,\mathrm{kg} \\
v &= 12.0\,\mathrm{m\,s^{-1}} \\
r &= 5.00\,\mathrm{m}
\end{aligned}$$

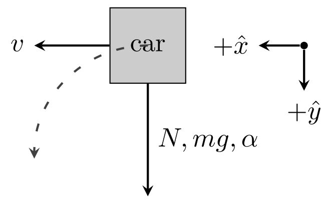
(a) What is the normal force at point A?



$$\begin{aligned}
\sum F_c &= m\alpha \\
N &= m\left(\frac{v^2}{r}\right) + mg \\
N &= (1.60\,\mathrm{kg})\left(\frac{(12.0\,\mathrm{m\,s^{-1}})^2}{5.00\,\mathrm{m}}\right) + (1.60\,\mathrm{kg})(10.0\,\mathrm{m\,s^{-2}}) \\
N &= 62.1\,\mathrm{m\,s^{-2}}
\end{aligned}$$

$$\boxed{N = 62.1\,\mathrm{m\,s^{-2}}}$$

(b) What is the normal force at point B?



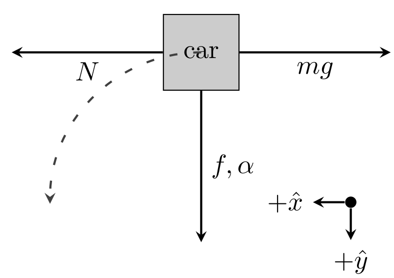
$$\sum F_c = m\alpha$$
$$N + mg = m\left(\frac{v^2}{r}\right)$$
$$N = m\left(\frac{v^2}{r}\right) - mg$$
$$N = 1.60\,\mathrm{kg}\left(\frac{(12.0\,\mathrm{m\,s^{-1}})^2}{5.00\,\mathrm{m}} - 10.0\,\mathrm{m\,s^{-2}}\right)$$
$$N = 30.1\,\mathrm{m\,s^{-2}}$$

$$\boxed{N = 30.1\,\mathrm{m\,s^{-2}}}$$

1.3 5.48

$$r = 230.0\,\mathrm{m}$$
$$v = 28.0\,\mathrm{m\,s^{-1}}$$

(a) What is the minimum coefficient of static friction that will prevent sliding?



$$\sum F_x = 0$$
$$N = mg$$

$$
\begin{aligned}
\sum F_y &= m\alpha \\
f &= m\left(\frac{v^2}{r}\right) \\
\mu N &= m\left(\frac{v^2}{r}\right), \quad N = mg \\
\mu &= \frac{v^2}{rg} \\
\mu &= \frac{(28.0\,\mathrm{m\,s^{-1}})^2}{(230.0\,\mathrm{m})(10.0\,\mathrm{m\,s^{-2}})} \\
\mu &= 0.341
\end{aligned}
$$

$$\boxed{\mu = 0.341}$$

(b) Suppose that the highway is icy and the coefficient of static friction between the tires and pavement is only one-third of what you found in part **(a)**. What should be the maximum speed of the car so that it can round the curve safely?

$$
\begin{aligned}
\sum F_y &= m\alpha \\
\frac{\mu}{3}mg &= m\left(\frac{v^2}{r}\right) \\
v &= \sqrt{\frac{\mu g r}{3}} \\
v &= \sqrt{\frac{(0.341)(10.0\,\mathrm{m\,s^{-2}})(230.0\,\mathrm{m})}{3}} \\
v &= 16.2\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$\boxed{v = 16.2\,\mathrm{m\,s^{-1}}}$$

1.4 5.54

$$
\begin{aligned}
D &= 100\,\mathrm{m} \\
r &= \frac{D}{2} = 50.0\,\mathrm{m} \\
\mathrm{rpm} &= 1\,\mathrm{rev\,min^{-1}}
\end{aligned}
$$

(a) Find the speed of the passengers when the Ferris wheel is rotating at this rate.

$$
\begin{aligned}
v &= \frac{2\pi r}{T} \\
v &= \frac{2\pi(50.0\,\mathrm{m})}{60.0\,\mathrm{s}} \\
v &= 5.24\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$\boxed{v = 5.24\,\mathrm{m\,s^{-1}}}$$

(b) A passenger weighs 902 N at the weight-guessing booth on the ground. What is his apparent weight at the highest and at the lowest point on the Ferris wheel?

$$m = \frac{w}{g} = \frac{902\,\mathrm{N}}{10.0\,\mathrm{m\,s^{-2}}} = 90.2\,\mathrm{kg}$$

$$
\begin{aligned}
\sum F_y^{top} &= m\alpha \\
mg &= m\left(\frac{v^2}{r}\right) + N_{top} \\
N_{top} &= m\left(-\frac{v^2}{r} + g\right) \\
N_{top} &= (90.2\,\mathrm{kg})\left(-\frac{(5.24\,\mathrm{m\,s^{-1}})^2}{50.0\,\mathrm{m}} + 10.0\,\mathrm{m\,s^{-2}}\right) \\
N_{top} &= 852.5\,\mathrm{N}
\end{aligned}
$$

$$
\begin{aligned}
\sum F_y^{bottom} &= m\alpha \\
N_{bottom} &= m\left(\frac{v^2}{r}\right) + mg \\
N_{bottom} &= m\left(\frac{v^2}{r} + g\right) \\
N_{bottom} &= (90.2\,\mathrm{kg})\left(\frac{(5.24\,\mathrm{m\,s^{-1}})^2}{50.0\,\mathrm{m}} + 10.0\,\mathrm{m\,s^{-2}}\right) \\
N_{bottom} &= 951.5\,\mathrm{N}
\end{aligned}
$$

$$\boxed{N_{top} = 852.5\,\mathrm{N}, N_{bottom} = 951.5\,\mathrm{N}}$$

(c) What would be the time for one revolution if the passenger's apparent weight at the highest point were zero?

$$
\begin{aligned}
v &= \frac{2\pi r}{T} \\
T &= \frac{2\pi r}{v}
\end{aligned}
$$

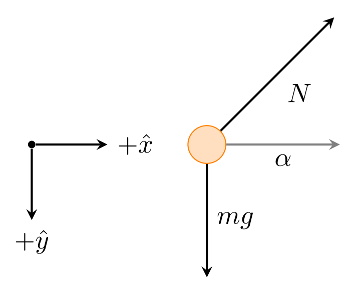
$$\sum F_y^{top} = m\alpha$$
$$mg = m\left(\frac{v^2}{r}\right) + N$$
$$v = \sqrt{gr - \frac{N}{m}}, \quad N = 0$$
$$v = \sqrt{gr}$$
$$v = \sqrt{(10.0\,\mathrm{m\,s^{-2}})(50.0\,\mathrm{m})}$$
$$v = 22.4\,\mathrm{m\,s^{-1}}$$

$$T = \frac{2\pi(50.0\,\mathrm{m})}{22.4\,\mathrm{m\,s^{-1}}}$$
$$T = 14.0\,\mathrm{s}$$

$$\boxed{T = 14.0\,\mathrm{s}}$$

1.5 5.107

$$r = 0.100\,\mathrm{m}$$
$$\mathrm{rpm} = 4.80\,\mathrm{rev\,s^{-1}}$$



(a) Find the angle β at which the bead is in vertical equilibrium.

$$\sum F_x^{bead} = m\alpha$$
$$N_x \sin(\beta) = m\alpha$$
$$N_x = \frac{m\frac{v^2}{r}}{\sin(\beta)} = \frac{mv^2}{r\sin(\beta)}$$

$$\sum F_y^{bead} = 0$$
$$N_y \cos(\beta) = mg$$
$$N_y = \frac{mg}{\cos(\beta)}$$

$$
\begin{aligned}
N_x &= N_y \\
\frac{mv^2}{r\sin(\beta)} &= \frac{mg}{\cos(\beta)} \\
v^2\cos(\beta) &= gr\sin(\beta) \\
\tan(\beta) &= \frac{v^2}{gr} \\
\beta &= \arctan\left(\frac{v^2}{gr}\right) \\
\beta &= \arctan\left(\frac{\left(\frac{2\pi r}{T}\right)^2}{gr}\right) \\
\beta &= \arctan\left(\frac{4\pi^2 r}{gT^2}\right) \\
\beta &= \arctan\left(\frac{4\pi^2(0.100\,\mathrm{m})}{(10.0\,\mathrm{m\,s^{-2}})\left(\frac{1}{4.80}\mathrm{s}\right)^2}\right) \\
\beta &= 83.7^\circ
\end{aligned}
$$

$$\boxed{\beta = 83.7^\circ}$$

(b) Is it possible for the bead to "ride" at the same elevation as the center of the hoop?

$$\beta = 90.0^\circ$$

$$
\begin{aligned}
\frac{v^2}{r\sin(\beta)} &= \frac{g}{\cos(\beta)} \\
v &= \sqrt{\frac{gr\sin(\beta)}{\cos(\beta)}} \\
v &= \sqrt{\frac{(10.0\,\mathrm{m\,s^{-2}})(0.100\,\mathrm{m})\sin(90.0^\circ)}{\cos(90.0^\circ)}} \\
v &= 0
\end{aligned}
$$

No it is not possible as the velocity would have to be zero, but would instead mean that the bead isn't moving.

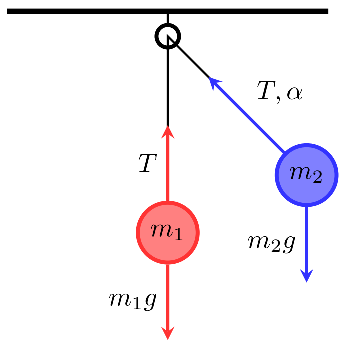
(c) What will happen if the hoop rotates at $1.00\,\mathrm{rev\,s^{-1}}$?

$$
\begin{aligned}
\beta &= \arctan\left(\frac{4\pi^2(0.100\,\mathrm{m})}{(10.0\,\mathrm{m\,s^{-2}})(1.00\,\mathrm{s})^2}\right) \\
\beta &= 21.5^\circ
\end{aligned}
$$

The bead ends up swinging at an angle lower and closer to the vertical axis.

2 Lab Manual

2.1 1072



$$\sum F_y^{m_1} = 0$$
$$T = m_1 g$$

$$\sum F_y^{m_2} = m_2 g$$
$$T\cos(\theta) = m_2 g$$

$$\sum F_x^{m_2} = m_2 \alpha$$
$$T\sin(\theta) = m_2 \left(\frac{\left(\frac{2\pi(c-a)}{t}\right)^2}{c-a} \right)$$
$$m_1 g \sin(\theta) = m_2 \left(\frac{4\pi^2(c-a)}{t^2} \right)$$
$$t = 2\pi \sqrt{\frac{m_2(c-a)}{m_1 g}}$$

$$\boxed{t = 2\pi \sqrt{\frac{m_2(c-a)}{m_1 g}}}$$

2.2 1073

$$\sum F_c = dm\alpha$$
$$\sum F_c = dm\omega^2 r$$

$$\begin{aligned}\sum F_x &= 0\\ T\cos\left(\frac{d\theta}{2}\right) - T\cos\left(\frac{d\theta}{2}\right) &= 0\\ 0 &= 0\end{aligned}$$

$$\begin{aligned}\sum F_y &= 0\\ 2T\sin\left(\frac{d\theta}{2}\right) &= 0\\ Td\theta &= 0\end{aligned}$$

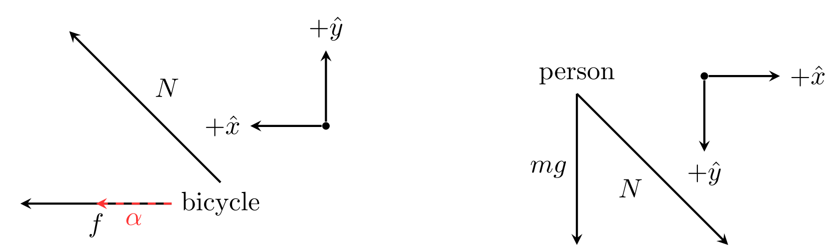
$$\begin{aligned}\sum F_c &= dm\omega^2 r\\ 0 + Td\theta &= dm\omega^2 r\\ T &= \frac{dm}{d\theta}\omega^2 r\\ T &= (\rho A r)\omega^2 r\\ T &= \rho A\omega^2 r^2\end{aligned}$$

$$\boxed{T = \rho A\omega^2 R^2}$$

2.3 1082

$$\begin{aligned}v &= 12\,\text{ft s}^{-1}\\ \theta &= 53^\circ\\ m &= 180\,\text{lb}\end{aligned}$$

(a) Draw a diagram showing all the forces on the bicycle and rider. What direction does he tend to tip (clockwise or counter-clockwise) if he inclines at angle greater than 53°? What causes him to tip?



If he continues to inline at an angle greater than 53°, he will begin to tip clockwise. His centripetal force becomes stronger than his weight pulling him down, causing him to tip. A more general answer as to why he tips is if the boy is unable to balance his centripetal force with his (the boy and bicycle's) weight, he will not be able to keep his balance.

(b) Find the radius of the curve.

$$\begin{aligned}
\sum F_y^{(person)} &= 0 \\
N\sin(\theta) + mg &= 0 \\
N &= -\frac{mg}{\sin(\theta)}
\end{aligned}$$

$$\begin{aligned}
\sum F_x^{(bicycle)} &= m\alpha \\
N\cos(\theta) &= m\alpha \\
\left(-\frac{mg}{\sin(\theta)}\right)\cos(\theta) &= m\left(\frac{v^2}{r}\right) \\
-\frac{gm}{\tan(\theta)} &= \frac{mv^2}{r} \\
r &= -\frac{\tan(\theta)v^2}{g} \\
r &= -\frac{\tan(53°)(12\,\mathrm{ft\,s^{-1}})^2}{32.17\,\mathrm{ft\,s^{-2}}} \\
r &= -5.94\,\mathrm{m}
\end{aligned}$$

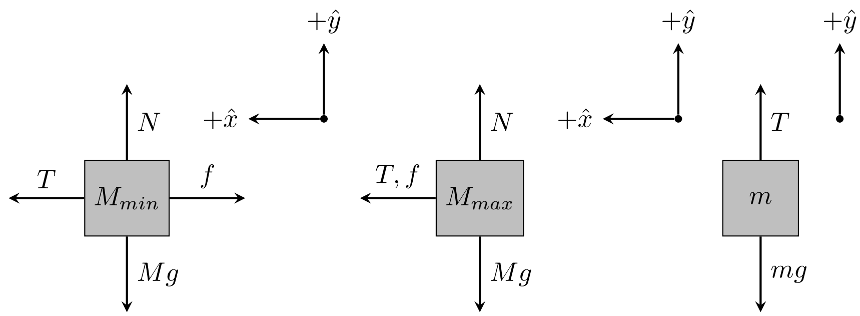
The radius is 5.94 m to the left.

(c) Find the friction force between the road and wheel.

$$\begin{aligned}
\sum F &= 0 \\
N\cos(\theta) + f &= 0 \\
f &= -N\cos(\theta) \\
f &= \frac{gw}{\tan(\theta)} \\
f &= \frac{180\,\mathrm{lb}}{\tan(53°)} \\
f &= 136.0\,\mathrm{lb}
\end{aligned}$$

$$\boxed{f = 136.0\,\mathrm{lb}}$$

2.4 1087



$$\sum F_y^{(m)} = 0$$
$$T = mg$$

$$\sum F_y^{(M)} = 0$$
$$N = Mg$$

$$\sum F_x^{(M)} = M\omega^2\mu_{min}$$
$$T = M\omega^2\mu_{min} + f$$
$$\mu_{min} = \frac{T - f}{M\omega^2}$$
$$\mu_{min} = \frac{mg - \mu Mg}{M\omega^2}$$

$$\sum F_x^{(M)} = M\omega^2\mu_{max}$$
$$T + f = M\omega^2\mu_{max}$$
$$\mu_{max} = \frac{T + f}{M\omega^2}$$
$$\mu_{max} = \frac{mg + \mu Mg}{M\omega^2}$$

$$\boxed{\mu_{min} = \frac{mg - \mu Mg}{M\omega^2}, \mu_{max} = \frac{mg + \mu Mg}{M\omega^2}}$$

2.5 1088

$$\sum F_x = m\alpha$$
$$N\sin(\theta) = m\left(\frac{v^2}{L\cos(\theta)}\right)$$

$$\sum F_y = 0$$
$$N\cos(\theta) = mg$$

$$\frac{N\sin(\theta)}{N\cos(\theta)} = \frac{m\left(\frac{v^2}{L\cos(\theta)}\right)}{mg}$$
$$\tan(\theta) = \frac{v^2}{gL\cos(\theta)}$$
$$v = \sqrt{gL\sin(\theta)}$$

$$\boxed{v = \sqrt{gL\sin(\theta)}}$$

$$m\mathcal{L}\{\ddot{x}\} + b\mathcal{L}\{\dot{x}\} + k\mathcal{L}\{x\} = \mathcal{L}\{0\}$$
$$m\left[s^2\mathcal{L}\{x\} - sx(0) - \dot{x}(0)\right] + b\left[s\mathcal{L}\{x\} - x(0)\right] + k\mathcal{L}\{x\} = 0$$
$$\mathcal{L}\{x\}(ms^2 + bs + k) - smx(0) - m\dot{x}(0) - bx(0) = 0$$
$$\mathcal{L}\{x\} = \frac{smx(0) + m\dot{x}(0) + bx(0)}{ms^2 + bs + k}$$
$$x = \mathcal{L}^{-1}\left\{\frac{smx(0) + m\dot{x}(0) + bx(0)}{ms^2 + bs + k}\right\}$$