Contents

1 Section 6.2

1.1 6.2.1

Determine whether or not the given matrix $\boldsymbol{A}$ is diagonalizable. If it is, find a diagonalizing matrix $\boldsymbol{P}$ and diagonal matrix $\boldsymbol{D}$ such that $\boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P} = \boldsymbol{D}$.

$$\boldsymbol{A} = \begin{bmatrix} 6 & -4 \\ 3 & -1 \end{bmatrix}$$

$$\begin{aligned} \det(\boldsymbol{A} - \lambda\mathbf{I}) &= \begin{bmatrix} 6-\lambda & -4 \\ 3 & -1-\lambda \end{bmatrix} \\ \det(\boldsymbol{A} - \lambda\mathbf{I}) &= (6-\lambda)(-1-\lambda) - (-4)(3) \\ \det(\boldsymbol{A} - \lambda\mathbf{I}) &= \lambda^2 - 5\lambda + 6 = (\lambda-3)(\lambda-2) \\ \lambda_{1,2} &= 3, 2 \end{aligned}$$

$$\begin{aligned} [\boldsymbol{A} - \lambda_1]\,\mathbf{v} &= 0 \\ \begin{bmatrix} 3 & -4 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} &= 0 \end{aligned}$$

$$\begin{aligned} (3)\mathbf{v}_1 + (-4)\mathbf{v}_2 = 0 \\ \mathbf{v}_1 = \left(\frac{4}{3}\right)\mathbf{v}_2 \end{aligned}$$

$$\begin{aligned} (3)\mathbf{v}_1 + (-4)\mathbf{v}_2 = 0 \\ (3)\left(\frac{4}{3}\right)\mathbf{v}_2 + (-4)\mathbf{v}_2 = 0 \\ 0 = 0 \end{aligned}$$

$$\begin{aligned} \mathbf{v} &= \begin{bmatrix} \left(\frac{4}{3}\right)\mathbf{v}_2 \\ \mathbf{v}_2 \end{bmatrix} \\ \mathbf{v} &= \mathbf{v}_2 \begin{bmatrix} \frac{4}{3} \\ 1 \end{bmatrix} \end{aligned}$$

$$
\begin{aligned}
[\boldsymbol{A} - \lambda_2]\,\mathbf{v} &= 0 \\
\begin{bmatrix} 4 & -4 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} &= 0
\end{aligned}
$$

$$
\begin{aligned}
(4)\mathbf{v}_1 + (-4)\mathbf{v}_2 &= 0 \\
\mathbf{v}_1 &= \mathbf{v}_2
\end{aligned}
$$

$$
\begin{aligned}
(3)\mathbf{v}_1 + (-3)\mathbf{v}_2 &= 0 \\
\mathbf{v}_1 &= \mathbf{v}_2
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{v} &= \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_1 \end{bmatrix} \\
\mathbf{v} &= \mathbf{v}_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{P} &= \begin{bmatrix} \frac{4}{3} & 1 \\ 1 & 1 \end{bmatrix} \\
\boldsymbol{D} &= \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}
\end{aligned}
$$

1.2 6.2.5

Determine whether or not the given matrix $\boldsymbol{A}$ is diagonalizable. If it is, find a diagonalizing matrix $\boldsymbol{P}$ and diagonal matrix $\boldsymbol{D}$ such that $\boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P} = \boldsymbol{D}$.

$$
\boldsymbol{A} = \begin{bmatrix} 5 & -3 \\ 1 & 1 \end{bmatrix}
$$

$$
\begin{aligned}
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= \begin{bmatrix} 5-\lambda & -3 \\ 1 & 1-\lambda \end{bmatrix} \\
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= (5-\lambda)(1-\lambda) - (-3)(1) \\
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= \lambda^2 - 6\lambda + 8 = (\lambda-4)(\lambda-2) \\
\lambda_{1,2} &= 4, 2
\end{aligned}
$$

$$
\begin{aligned}
[\boldsymbol{A} - \lambda_1]\,\mathbf{v} &= 0 \\
\begin{bmatrix} 1 & -3 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} &= 0
\end{aligned}
$$

$$
\begin{aligned}
(1)\mathbf{v}_1 + (-3)\mathbf{v}_2 &= 0 \\
\mathbf{v}_1 &= (3)\mathbf{v}_2
\end{aligned}
$$

$$\begin{aligned}(1)\mathbf{v}_1 + (-3)\mathbf{v}_2 &= 0\\(1)(3)\mathbf{v}_2 + (-3)\mathbf{v}_2 &= 0\\0 &= 0\end{aligned}$$

$$\begin{aligned}\mathbf{v} &= \begin{bmatrix}(3)\mathbf{v}_2\\ \mathbf{v}_2\end{bmatrix}\\\mathbf{v} &= \mathbf{v}_2\begin{bmatrix}3\\1\end{bmatrix}\end{aligned}$$

$$\begin{aligned}\left[\boldsymbol{A} - \lambda_2\right]\mathbf{v} &= 0\\\begin{bmatrix}3 & -3\\1 & -1\end{bmatrix}\begin{bmatrix}\mathbf{v}_1\\ \mathbf{v}_2\end{bmatrix} &= 0\end{aligned}$$

$$\begin{aligned}(3)\mathbf{v}_1 + (-3)\mathbf{v}_2 &= 0\\\mathbf{v}_1 &= \mathbf{v}_2\end{aligned}$$

$$\begin{aligned}(1)\mathbf{v}_1 + (-1)\mathbf{v}_2 &= 0\\\mathbf{v}_2 &= \mathbf{v}_2\\0 &= 0\end{aligned}$$

$$\begin{aligned}\mathbf{v} &= \begin{bmatrix}\mathbf{v}_2\\ \mathbf{v}_2\end{bmatrix}\\\mathbf{v} &= \mathbf{v}_2\begin{bmatrix}1\\1\end{bmatrix}\end{aligned}$$

1.3 6.2.10

Determine whether or not the given matrix $\boldsymbol{A}$ is diagonalizable. If it is, find a diagonalizing matrix $\boldsymbol{P}$ and diagonal matrix $\boldsymbol{D}$ such that $\boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P} = \boldsymbol{D}$.

$$\boldsymbol{A} = \begin{bmatrix}3 & -1\\1 & 1\end{bmatrix}$$

$$\begin{aligned}\det(\boldsymbol{A} - \lambda\mathbf{I}) &= \begin{bmatrix}3-\lambda & -1\\1 & 1-\lambda\end{bmatrix}\\\det(\boldsymbol{A} - \lambda\mathbf{I}) &= (3-\lambda)(1-\lambda) - (-1)(1)\\\det(\boldsymbol{A} - \lambda\mathbf{I}) &= \lambda^2 - 4\lambda + 4 = (\lambda - 2)^2\\\lambda &= 2\end{aligned}$$

$$
\begin{aligned}
[\boldsymbol{A} - \lambda]\,\mathbf{v} &= 0 \\
\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} &= 0
\end{aligned}
$$