Homework: Section 1.5

1 Practice Problems

1.1 Problem 1

Find the general solution of the differential equation. Then, use the initial condition to find the corresponding particular solution.

$$\frac{dy}{dx} - 4y = 6e^{4x},\ y(0) = 0$$

$$\begin{aligned}
\frac{dy}{dx} - 4y &= 6e^{4x} \text{ already separated} \\
&\therefore P(x) = -4, \rho(x) = e^{\int(-4)dx} \\
y(x) &= e^{\int(4)dx}\left[\int\left(6e^{4x}e^{\int(-4)dx}\right)dx + C\right] \\
&= e^{4x}\int\left(6e^{4x}e^{-4x}\right)dx + Ce^{4x} \\
&= e^{4x}\int(6)dx + Ce^{4x} \\
y(0) &= e^{4(0)}(6(0)) + Ce^{4(0)} = 0 \\
C &= 0
\end{aligned}$$

$$\boxed{y(x) = \frac{6x}{e^{-4x}} + Ce^{4x}, y(0) \to y(x) = \frac{6x}{e^{-4x}}}$$

1.2 Problem 2

Find the general solution of the differential equation. Then, use the initial condition to find the corresponding particular solution.

$$xy' + 5y = 6x$$

First multiply each side by the integrating factor $\frac{1}{x}$ to get the form $y' + P(x)y = Q(x), y(3) = 6$

$$
\begin{aligned}
y' + \frac{5}{x}y &= 6 \\
&\therefore P(x) = \frac{5}{x}, \rho(x) = e^{\int (\frac{5}{x})dx} \\
y(x) &= e^{-\int (\frac{5}{x})dx} \left[\int \left(6e^{\int (\frac{5}{x})dx} \right) dx + C \right] \qquad = x^{-5} \left[\int (6x^5)dx + C \right] \\
&= x^{-5} \left[x^6 + C \right] \\
&= x + Cx^{-5} \\
y(3) &= (3) + C(3)^{-5} = 6 \\
\frac{1}{243}C &= 3 \\
C &= 729
\end{aligned}
$$

$$\boxed{y(x) = x + Cx^{-5}, y(3) \rightarrow y(x) = x + 729x^{-5}}$$