Week 12 and Week 13 Participation Assignment (2 of 3)

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Contents

1 Part 2

1) $\begin{bmatrix} -1 & -1 & 0 \\ 2 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$, the complex eigenvalues are $-1+i, -1-i$

2) $\begin{bmatrix} 5 & -5 & -5 \\ -1 & 4 & 2 \\ 3 & -5 & -3 \end{bmatrix}$, the complex eigenvalues are $2+i, 2-i$

Find the complex eigenvectors for the above matrices and write it as

$$\vec{a} + i\vec{b}, \vec{a} - i\vec{b}$$

1.1 1)

$$\lambda_0 = -1 + i$$
$$\lambda_1 = -1 - i$$

$$\boldsymbol{A} = \begin{bmatrix} -1-\lambda_0 & -1 & 0 \\ 2 & -1-\lambda_0 & 1 \\ 0 & 1 & -1-\lambda_0 \end{bmatrix}$$
$$\boldsymbol{A} = \begin{bmatrix} -1-(-1+i) & -1 & 0 \\ 2 & -1-(-1+i) & 1 \\ 0 & 1 & -1-(-1+i) \end{bmatrix}$$
$$\boldsymbol{A} = \begin{bmatrix} -i & -1 & 0 \\ 2 & -i & 1 \\ 0 & 1 & -i \end{bmatrix}$$

$$\boldsymbol{A}_1 = \frac{1}{-i}\boldsymbol{A}_1$$
$$\boldsymbol{A}_2 \leftrightarrows \boldsymbol{A}_3$$
$$\boldsymbol{A} = \begin{bmatrix} 1 & -i & 0 \\ 0 & 1 & -i \\ 2 & -i & 1 \end{bmatrix}$$

$$\boldsymbol{A}_3 = \boldsymbol{A}_3 - 2\boldsymbol{A}_1$$
$$\boldsymbol{A}_3 = \boldsymbol{A}_3 - i\boldsymbol{A}_2$$
$$\boldsymbol{A} = \begin{bmatrix} 1 & -i & 0 \\ 0 & 1 & -i \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{A}_1 = \boldsymbol{A}_1 + (i)\boldsymbol{A}_2$$
$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -i \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -i \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(1)x_0 + (0)x_1 + (1)x_2 = 0$$
$$x_0 = -x_2$$

$$(0)x_0 + (1)x_1 + (-i)x_2 = 0$$
$$x_1 = (i)x_2$$

$$\mathbf{x} = \begin{bmatrix} -x_2 \\ (i)x_2 \\ x_2 \end{bmatrix}$$
$$\mathbf{x} = x_2 \begin{bmatrix} -1 \\ i \\ 1 \end{bmatrix}$$
$$\mathbf{x} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ i \\ 0 \end{bmatrix}$$
$$\mathbf{x} = \mathbf{a} + i\mathbf{b}$$
$$\mathbf{x} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + i \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\boxed{\mathbf{x} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + i \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}$$

$$\boldsymbol{A} = \begin{bmatrix} -1-\lambda_1 & -1 & 0 \\ 2 & -1-\lambda_1 & 1 \\ 0 & 1 & -1-\lambda_1 \end{bmatrix}$$
$$\boldsymbol{A} = \begin{bmatrix} -1-(-1-i) & -1 & 0 \\ 2 & -1-(-1-i) & 1 \\ 0 & 1 & -1-(-1-i) \end{bmatrix}$$
$$\boldsymbol{A} = \begin{bmatrix} i & -1 & 0 \\ 2 & i & 1 \\ 0 & 1 & i \end{bmatrix}$$

$$\boldsymbol{A}_2 \leftrightarrows \boldsymbol{A}_3$$
$$\boldsymbol{A}_1 = \frac{1}{i}\boldsymbol{A}_1$$
$$\boldsymbol{A} = \begin{bmatrix} 1 & i & 0 \\ 0 & 1 & i \\ 2 & i & 1 \end{bmatrix}$$

$$\boldsymbol{A}_3 = \boldsymbol{A}_3 - 2\boldsymbol{A}_1$$
$$\boldsymbol{A}_3 = \boldsymbol{A}_3 + i\boldsymbol{A}_2$$
$$\boldsymbol{A}_1 = \boldsymbol{A}_1 - i\boldsymbol{A}_2$$
$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & i \\ 0 & 0 & 0 \end{bmatrix}$$

$$(0)x_0 + (1)x_1 + (i)x_2 = 0$$
$$x_1 = (-i)x_2$$

$$(1)x_0 + (0)x_1 + (1)x_2 = 0$$
$$x_0 = -x_2$$

$$\mathbf{x} = \begin{bmatrix} -x_2 \\ (-i)x_2 \\ x_2 \end{bmatrix}$$
$$\mathbf{x} = x_2 \begin{bmatrix} -1 \\ -i \\ 1 \end{bmatrix}$$
$$\mathbf{x} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + i \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$\boxed{\mathbf{x} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} - i \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}$$

1.2 2)

$$\lambda_0 = 2 + i$$
$$\lambda_1 = 2 - i$$

$$\boldsymbol{A} = \begin{bmatrix} 5-\lambda_0 & -5 & -5 \\ -1 & 4-\lambda_0 & 2 \\ 3 & -5 & -3-\lambda_0 \end{bmatrix}$$
$$\boldsymbol{A} = \begin{bmatrix} 3-i & -5 & -5 \\ -1 & 2-i & 2 \\ 3 & -5 & -5-i \end{bmatrix}$$
$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & \frac{2}{5}+\frac{i}{5} \\ 0 & 0 & 0 \end{bmatrix}$$

$$(0)x_0 + (1)x_1 + \left(\frac{2}{5} + \frac{i}{5}\right) x_2 = 0$$
$$x_1 = \left(-\frac{2}{5} - \frac{i}{5}\right) x_2$$

$$(1)x_0 + (0)x_1 + (-1)x_2 = 0$$
$$x_0 = x_2$$

$$\mathbf{x} = \begin{bmatrix} x_2 \\ \left(-\frac{2}{5} - \frac{i}{5}\right) x_2 \\ x_2 \end{bmatrix}$$
$$\mathbf{x} = \begin{bmatrix} 1 \\ -\frac{2}{5} \\ 1 \end{bmatrix} + i \begin{bmatrix} 0 \\ -\frac{1}{5} \\ 0 \end{bmatrix}$$
$$\boxed{\mathbf{x} = \begin{bmatrix} 1 \\ -\frac{2}{5} \\ 1 \end{bmatrix} - i \begin{bmatrix} 0 \\ \frac{1}{5} \\ 0 \end{bmatrix}}$$

$$\boldsymbol{A} = \begin{bmatrix} 5-\lambda_1 & -5 & -5 \\ -1 & 4-\lambda_1 & 2 \\ 3 & -5 & -3-\lambda_1 \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} i+3 & -5 & -5 \\ -1 & i+2 & 2 \\ 3 & -5 & i-5 \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & \frac{2}{5}-\frac{i}{5} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} (0)x_0 + (1)x_1 + \left(\frac{2}{5} - \frac{i}{5}\right) x_2 = 0 \\ x_1 = \left(-\frac{2}{5} + \frac{i}{5}\right) x_2 \end{aligned}$$

$$\begin{aligned} (1)x_0 + (0)x_1 + (-1)x_2 = 0 \\ x_0 = x_2 \end{aligned}$$

$$\mathbf{x} = \begin{bmatrix} x_2 \\ \left(-\frac{2}{5}+\frac{i}{5}\right) x_2 \\ x_2 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} 1 \\ -\frac{2}{5} \\ 1 \end{bmatrix} + i \begin{bmatrix} 0 \\ \frac{1}{5} \\ 0 \end{bmatrix}$$

$$\boxed{\mathbf{x} = \begin{bmatrix} 1 \\ -\frac{2}{5} \\ 1 \end{bmatrix} + i \begin{bmatrix} 0 \\ \frac{1}{5} \\ 0 \end{bmatrix}}$$