Week 03 Participation Assignment (1 of 3)

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Contents

1 Part 1

To test for a first order differential equation to be exact or not, we must write it in the form of $M(x,y)dx + N(x,y)dy = 0$. Determine whether the given equation is exact or not. (For your own practice, you may also identify the equation as separable or linear as well as exact equation).

1.1 a)

$$(2x + yx^{-1})dx + (xy - 1)dy = 0$$

$$\begin{aligned}\frac{\partial M}{\partial y} &= \frac{\partial}{\partial y}\left(2x + yx^{-1}\right)\\ &= (0) + x^{-1}\\ &= x^{-1}\\ \frac{\partial N}{\partial x} &= \frac{\partial}{\partial x}(xy - 1)\\ &= y - (0)\\ &= y\end{aligned}$$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \therefore \text{not exact}}$$

1.2 b)

$$(2y^3 + 2y^2)dx + (3xy^2 + 2xy)dy = 0$$

$$\begin{aligned}\frac{\partial M}{\partial y} &= \frac{\partial}{\partial y}\left(2y^3 + 2y^2\right)\\ &= 6y^2 + 4y\\ \frac{\partial N}{\partial x} &= \frac{\partial}{\partial x}\left(3xy^2 + 2xy\right)\\ &= 3y^2 + 2y\end{aligned}$$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \therefore \text{not exact}}$$

1.3 c)

$$(2x + y)dx + (x - 2y)dy = 0$$

$$\begin{aligned}
\frac{\partial M}{\partial y} &= \frac{\partial}{\partial y}(2x + y) \\
&= (0) + 1 \\
\frac{\partial N}{\partial x} &= \frac{\partial}{\partial x}(x - 2y) \\
&= 1 - (0)
\end{aligned}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \therefore \text{exact}}$$

1.4 d)

$$(y^2 + 2xy)dx - x^2 dy = 0$$

$$\begin{aligned}
\frac{\partial M}{\partial y} &= \frac{\partial}{\partial y}\left(y^2 + 2xy\right) \\
&= 2y + 2x \\
\frac{\partial N}{\partial x} &= \frac{\partial}{\partial x}\left(-x^2\right) \\
&= -2x
\end{aligned}$$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \therefore \text{not exact}}$$

1.5 e)

$$(x^2 \sin(x) + 4y)dx + xdy = 0$$

$$\begin{aligned}
\frac{\partial M}{\partial y} &= \frac{\partial}{\partial y}\left(x^2 \sin(x) + 4y\right) \\
&= (0) + 4 \\
&= 4 \\
\frac{\partial N}{\partial x} &= \frac{\partial}{\partial x}(x) \\
&= 1
\end{aligned}$$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \therefore \text{not exact}}$$

1.6 f)

$$[\sin(xy) + xy\cos(xy)]dx + [1 + x^2\cos(xy)]dy = 0$$

$$\begin{aligned}
\frac{\partial M}{\partial x} &= \frac{\partial}{\partial x}\left(\sin(xy) + xy\cos(xy)\right) \\
&= x\cos(xy) + (x \cdot \cos(xy)) + (-x\sin(xy) \cdot xy) \\
&= 2x\cos(xy) - x^2 y\sin(xy) \\
\frac{\partial N}{\partial x} &= \frac{\partial}{\partial x}\big(1 + x^2\cos(xy)\big) \\
&= (0) + (2x \cdot \cos(xy)) + (-y\sin(xy) \cdot x^2) \\
&= 2x\cos(xy) - x^2 y\sin(xy)
\end{aligned}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \therefore \text{exact}}$$