Contents

1 Section 10.2

Laplace transform of the second derivative of a function

$$\mathcal{L}(f'') = s^2\mathcal{L}(f) - sf(0) - f'(0) \tag{1}$$

1.1 10.2.1

Solve the following differential equation by Laplace transforms. The function is subject to the given conditions.

$$y'' + 64y = 0, y(0) = 0, y'(0) = 1$$

$$\begin{aligned}
\mathcal{L}(y'') + 64\mathcal{L}(y) &= \mathcal{L}(0)\\
\left[s^2\mathcal{L}(y) - sy(0) - y'(0)\right] + 64\mathcal{L}(y) &= \mathcal{L}(0)\\
s^2\mathcal{L}(y) - 0 - 1 + 64\mathcal{L}(y) &= \mathcal{L}(0)\\
\mathcal{L}(y)(s^2+64) &= 1\\
\mathcal{L}(y) &= \frac{1}{s^2+64}\\
y &= \mathcal{L}^{-1}\left(\frac{1}{s^2+64}\right)\\
y &= \frac{1}{8}\sin(8t)
\end{aligned}$$

1.2 10.2.3

Solve the following differential equation by Laplace transforms. The function is subject to the given conditions.

$$y'' + 4y = 4, y(0) = 1, y'(0) = 2$$

$$\begin{aligned}
\mathcal{L}(y'') + 4\mathcal{L}(y) &= \mathcal{L}(4)\\
\left[s^2\mathcal{L}(y) - sy(0) - y'(0)\right] + 4\mathcal{L}(y) &= \frac{4}{s}\\
\mathcal{L}(y)(s^2+4) - s - 2 &= \frac{4}{s}\\
\mathcal{L}(y) &= \frac{s^2+2s+4}{s(s^2+4)}
\end{aligned}$$

$$
\begin{aligned}
y &= \mathcal{L}^{-1}\left(\frac{2}{s^2+4}\right) + \mathcal{L}^{-1}\left(\frac{1}{s}\right) \\
y(t) &= \sin(2t) + 1
\end{aligned}
$$

1.3 10.2.19

Find the inverse Laplace transform of the following function using the theorem of transforms of integrals.

$$F(s) = \frac{1}{s(s^2+9)}$$

$$
\begin{aligned}
F(s) &= \frac{1}{s(s^2+9)} \\
F(s) &= \frac{1}{9}\left(\frac{1}{s} - \frac{s}{s^2+9}\right) \\
f(t) &= \frac{1}{9}\left(\mathcal{L}^{-1}\left(\frac{1}{s}\right) - \mathcal{L}^{-1}\left(\frac{s}{s^2+9}\right)\right) \\
f(t) &= \frac{1}{9}\left(1 - \cos(3t)\right)
\end{aligned}
$$

1.4 10.2.21

Find the inverse Laplace transform of the following function using the theorem of transforms of integrals.

$$F(s) = \frac{1}{s^2(s^2+36)}$$

$$
\begin{aligned}
F(s) &= \frac{1}{s^2(s^2+36)} \\
F(s) &= \frac{1}{36}\left(\frac{1}{s^2} - \frac{1}{s^2+36}\right) \\
f(t) &= \frac{1}{36}\left(\mathcal{L}^{-1}\left(\frac{1}{t^2}\right) - \mathcal{L}^{-1}\left(\frac{1}{t^2+36}\right)\right) \\
f(t) &= \frac{t}{36} - \frac{\sin(6t)}{216}
\end{aligned}
$$