Contents

1 Part 1

Take the Laplace Transformation of the following initial valued problem and solve for $Y(s)$:

1)

$$y'' + 5y' + 6y = 12e^t, y(0) = -1, y'(0) = 7$$

$$\begin{aligned}
\mathcal{L}(y'') + 5\mathcal{L}(y') + 6\mathcal{L}(y) &= 12\mathcal{L}(e^t) \\
\left[s^2\mathcal{L}(y) - sy(0) - y'(0)\right] + 5\left[s\mathcal{L}(y) - y(0)\right] + 6\mathcal{L}(y) &= \frac{12}{s-1} \\
\mathcal{L}(y)(s^2 + 5s + 6) + s - 7 + 5 &= \frac{12}{s-1} \\
\mathcal{L}(y) &= \frac{-s^2 + 3s + 10}{(s-1)(s^2 + 5s + 6)} \\
\mathcal{L}(y) &= -\frac{s-5}{(s-1)(s+3)} \\
\mathcal{L}(y) &= -\frac{1}{s-1} + \frac{2}{s+3}
\end{aligned}$$

$$\begin{aligned}
y &= -\mathcal{L}^{-1}\left(\frac{1}{s-1}\right) + 2\mathcal{L}^{-1}\left(\frac{1}{s+3}\right) \\
y &= e^t - 2e^{-3t}
\end{aligned}$$

$$\boxed{y = e^t - 2e^{-3t}}$$

2)

$$y'' - 7y' + 10y = 9\cos(t) + 7\sin(t), y(0) = 5, y'(0) = -4$$

$$\begin{aligned}
\mathcal{L}(y'') - 7\mathcal{L}(y') + 10\mathcal{L}(y) &= 9\mathcal{L}(\cos(t)) + 7\mathcal{L}(\sin(t)) \\
\left[s^2\mathcal{L}(y) - sy(0) - y'(0)\right] - 7\left[s\mathcal{L}(y) - y(0)\right] + 10\mathcal{L}(y) &= \frac{9s}{s^2+1} + \frac{7}{s^2+1} \\
\mathcal{L}(y)(s^2 - 7s + 10) - 5s + 4 + 35 &= \frac{9s+7}{s^2+1} \\
\mathcal{L}(y) &= \frac{5s^3 - 39s^2 + 14s - 32}{(s^2+1)(s^2 - 7s + 10)} \\
\mathcal{L}(y) &= \frac{s}{s^2+1} + \frac{8}{s-2} - \frac{4}{s-5}
\end{aligned}$$

$$y = \mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) + 8\mathcal{L}^{-1}\left(\frac{1}{s-2}\right) - 4\mathcal{L}^{-1}\left(\frac{1}{s-5}\right)$$
$$y = \cos(t) + 8e^{2t} - 4e^{5t}$$

$$\boxed{y = \cos(t) + 8e^{2t} - 4e^{5t}}$$

3)

$$y'' - 4y = 4t - 8e^{-2t}, y(0) = 0, y'(0) = 5$$

$$\mathcal{L}(y'') - 4\mathcal{L}(y) = 4\mathcal{L}(t) - 8\mathcal{L}(e^{-2t})$$
$$\left[s^2\mathcal{L}(y) - sy(0) - y'(0)\right] - 4\mathcal{L}(y) = \frac{4}{s^2} - \frac{8}{s+2}$$
$$\mathcal{L}(y)(s^2-4) - 0 - 5 = \frac{4}{s^2} - \frac{8}{s+2}$$
$$\mathcal{L}(y) = \frac{5s^3 + 2s^2 + 4s + 8}{s^2(s+2)(s^2-4)}$$
$$\mathcal{L}(y) = -\frac{1}{s^2} - \frac{1}{s+2} + 2\frac{1}{(s+2)^2} + \frac{1}{s-2}$$

$$y = -\mathcal{L}^{-1}\left(\frac{1}{s^2}\right) - \mathcal{L}^{-1}\left(\frac{1}{s+2}\right) + 2\mathcal{L}^{-1}\left(\frac{1}{(s+2)^2}\right) + \mathcal{L}^{-1}\left(\frac{1}{s-2}\right)$$
$$y = -t - e^{-2t} + 2te^{-2t} + e^{2t}$$

$$\boxed{y = -t - e^{-2t} + 2te^{-2t} + e^{2t}}$$