Week 03 Participation Assignment - Part 01

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Contents

1 Part 01

The purpose of this exercise is to prove that for any real number: $a : \sqrt{a^2} = |a|$.

First, we recall that the absolute value of any real number is defined by

$$|a| = \begin{cases} a \text{ if } a \geq 0, \text{ and} \\ -a \text{ if } a < 0. \end{cases}$$

a) Use the definition above to explain why for any real number $a : |a| \geq 0$.
Case by Case Proof:

- Case 1: $a \geq 0$.

$$\begin{aligned} &|a| = a, \quad a \geq 0 \\ &|a| = a \geq 0 \\ &|a| \geq a \end{aligned}$$

- Case 2: $a < 0$.

$$\begin{aligned} &|a| = -a, \quad a < 0 \implies -a > 0 \\ &|a| = -a > 0, \quad -a > 0 \implies -a \geq 0 \\ &|a| \geq 0 \end{aligned}$$

b) Again, using the definition, show that $|a|^2 = a^2$.
Case by Case Proof:

- Case 1: $a \geq 0$.

$$\begin{aligned} |a|^2 &= a^2 \\ |a| \cdot |a| &= a \cdot a, \quad |a| = a \geq 0 \\ a \cdot a &= a \cdot a \end{aligned}$$

- Case 2: $a < 0$.

$$\begin{aligned} |a|^2 &= a^2 \\ |a| \cdot |a| &= a \cdot a, \quad |a| = -a > 0 \implies |a| = -a \geq 0 \\ -a \cdot -a &= a \cdot a \\ a \cdot a &= a \cdot a \end{aligned}$$

c) Our next goal is to show that $\sqrt{b}$ is unique. In other words, prove that if c and d are two real numbers such that $c \geq 0$, and $d \geq 0$, and $b = c^2 = d^2$, then $c = d$.

$$\begin{aligned}
c^2 &= d^2 \\
c^2 - d^2 &= 0 \\
(c+d)(c-d) &= 0 \\
c &= \pm d \\
|c| &= |d|, \quad |c| = c \geq 0, |d| = d \geq 0 \\
c &= d
\end{aligned}$$

d) Rewrite the definition for $\sqrt{b}$ to define $\sqrt{a^2}$

e) Put together all the steps above to write a complete proof that $\sqrt{a^2} = |a|$.

$$\sqrt{a^2} = |a|$$

Case by Case Proof:

- Case 1: $a \geq 0$

$$\begin{aligned}
\sqrt{a^2} &= |a| \\
a^2 &= |a|^2 \\
a \cdot a &= |a| \cdot |a|, \quad |a| = a \geq 0 \\
a \cdot a &= a \cdot a
\end{aligned}$$

- Case 2: $a < 0$

$$\begin{aligned}
\sqrt{a^2} &= |a| \\
a^2 &= |a|^2 \\
a \cdot a &= |a| \cdot |a|, \quad |a| = a < 0 \implies -a > 0 \implies -a \geq 0 \\
a \cdot a &= -a \cdot -a \\
a \cdot a &= a \cdot a
\end{aligned}$$