Week 03 Participation Assignment - Part 01

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1 Part 01

The purpose of this exercise is to prove that for any real number: $a : \sqrt{a^2} = |a|$.

First, we recall that the absolute value of any real number is defined by

$$|a| = \begin{cases} a \text{ if } a \geq 0, \text{ and} \\ -a \text{ if } a < 0. \end{cases}$$

a) Use the definition above to explain why for any real number $a : |a| \geq 0$.
Case by Case Proof: Let $x : x \in \mathbb{R}, [0, \infty)$.

- Case 1: $a \geq 0$ and $x = a$.

$$\begin{aligned} |a| &= a \\ |x| &= a \\ x &= a \end{aligned}$$

- Case 2: $a < 0$ and $x = -a$.

$$\begin{aligned} |-a| &= -a \\ |-(-x)| &= -a \\ |x| &= -a \\ x &= -a \end{aligned}$$

b) Again, using the definition, show that $|a|^2 = a^2$.
Case by Case Proof:

- Case 1: $a \geq 0$.

$$\begin{aligned} |a|^2 &= a^2 \\ |a| \cdot |a| &= a \cdot a \\ |x| \cdot |x| &= a \cdot a \\ x \cdot x &= a \cdot a \end{aligned}$$

- Case 2: $a < 0$.

$$\begin{aligned} |-a|^2 &= a^2 \\ |-a| \cdot |-a| &= a \cdot a \\ |-(-x)| \cdot |-(-x)| &= a \cdot a \\ |x| \cdot |x| &= a \cdot a \\ x \cdot x &= a \cdot a \end{aligned}$$

c) Our next goal is to show that $\sqrt{b}$ is unique. In other words, prove that if c and d are two real numbers such that $c \geq 0$, and $d \geq 0$, and $b = c^2 = d^2$, then $c = d$.

$$\begin{aligned} c^2 &= d^2 \\ c^2 - d^2 &= 0 \\ (c+d)(c-d) &= 0 \\ c &= \pm d \\ |c| &= |d| \end{aligned}$$

d) Rewrite the definition for $\sqrt{b}$ to define $\sqrt{a^2}$

e) Put together all the steps above to write a complete proof that $\sqrt{a^2} = |a|$.

$$\begin{aligned} \sqrt{b} &= c^2 \\ \sqrt{b} &= (\pm d)^2 \\ c^2 &= |d|^2 \\ \sqrt{c^2} &= \sqrt{|d|^2} \\ \sqrt{c^2} &= \sqrt{d^2} \\ \sqrt{c^2} &= |d| \end{aligned}$$