Week 04 Participation Assignment

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Contents

1 Part 01

Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two functions where f is strictly increasing and g is strictly decreasing.

Prove that both $f \circ g : \mathbb{R} \to \mathbb{R}$ and $g \circ f : \mathbb{R} \to \mathbb{R}$ are strictly decreasing.

As f is strictly increasing, $\forall x \forall y\,(x < y \implies f(x) < f(y))$, and as g is strictly decreasing, $\forall x \forall y\,(x < y \implies g(x) > g(y))$.

Case by Case Proof

- $f \circ g : \mathbb{R} \to \mathbb{R}$

$$\begin{aligned}(f \circ g) &= f(g)\\ (f \circ g) &= f(g(y), g(x)), \quad g(y) < g(x)\\ (f \circ g) &= f(g(y)) < f(g(x))\\ (f \circ g)\,(y) &< (f \circ g)\,(x)\end{aligned}$$

  The composition $f \circ g$ displays that for when $x < y$, $f(g(y)) < f(g(x))$. In other words, the output $g(y)$ of a larger input y will be less than the output $g(x)$ of a smaller input x.

- $g \circ f : \mathbb{R} \to \mathbb{R}$

$$\begin{aligned}(g \circ f) &= g(f)\\ (g \circ f) &= g(f(x), g(f(y)), \quad f(x) < f(y)\\ (g \circ f) &= g(f(x)) < g(f(y))\\ (g \circ f)\,(x) &< (g \circ f)\,(y)\end{aligned}$$

  The composition $g \circ f$ displays that for when $x < y$, $g(f(x)) < g(f(y))$. In other words, the output $f(x)$ of a smaller input x will be less than the output $f(y)$ of a larger input y.