Week 01 Participation Assignment

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Contents

1 Problem 1

Use a truth table to verify the first De Morgan law: $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

The truth values for the compound propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are the same, and are therefore logically equivalent thereby verifying the first De Morgan law.

2 Problem 2

Verify by showing either both sides are true, or both sides are false, for exactly the same combinations of truth values of the propositional variables in these expressions (except for 18).

2.1 16

Show that $p \iff q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

2.1.1 Truth Table

p	q	$p \iff q$	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
T	T	T	T	F	F	F	T
T	F	F	F	F	T	F	F
F	T	F	F	T	F	F	F
F	F	T	F	T	T	T	T

2.1.2 Proof

Start by observing that the first compound proposition $p \iff q$ is true only if $(p = T \wedge q = T) \vee (p = F \wedge q = F)$.

Looking at the second compound proposition, the first proposition $(p \wedge q)$ is only true when both $p = T \wedge q = T$.

As for the second proposition $(\neg p \wedge \neg q)$, this can only be true when both $p = F \wedge q = F$.

The second compound proposition $(p \wedge q) \vee (\neg p \wedge \neg q)$ can now be seen as true only when $(p = T \wedge q = T) \vee (p = F \wedge q = F)$.

$\therefore$ it can be concluded that each compound proposition is true if and only if: $(p = T \wedge q = T) \vee (p = F \wedge q = F)$, proving the expression.

2.2 17

Show that $\neg(p \iff q)$ and $p \iff \neg q$ are logically equivalent.

2.2.1 Truth Table

p	q	$p \iff q$	$\neg(p \iff q)$	$\neg q$	$p \iff \neg q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	T
F	F	T	F	T	F

2.2.2 Proof

Begin by observing the first compound proposition $\neg(p \iff q)$. The expression $p \iff q$ is **true** *if and only if* $((p = T) \wedge (q = T)) \vee ((p = F) \wedge (q = F))$. Including the negation connective, the LHS compound proposition is **false** under the same aforementioned truth values.

Now looking at the RHS compound proposition, we can see that it is **false** *if and only if* $p \equiv q$. In other words, the RHS compound proposition is **false** *if and only if* $((p = T) \wedge (q = T)) \vee ((p = F) \wedge (q = F))$.

$\therefore$ both sides are verified to be **false** under the same combination of truth values: $p \equiv q$, or more specifically $((p = T) \wedge (q = T)) \vee ((p = F) \wedge (q = F))$.

2.3 18

Show that $p \implies q$ and $\neg q \implies \neg p$ are logically equivalent.

p	q	$p \implies q$	$\neg q$	$\neg p$	$\neg q \implies \neg p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

2.4 19

Show that $\neg p \iff q$ and $p \iff \neg q$ are logically equivalent.

p	$\neg p$	q	$\neg p \iff q$	$\neg q$	$p \iff \neg q$
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	T	F	T
F	T	F	F	T	F

2.5 20

Show that $\neg(p \oplus q)$ and $p \iff q$ are logically equivalent.

p	q	$p \oplus q$	$\neg(p \oplus q)$	$p \iff q$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

2.6 21

Show that $\neg(p \iff q)$ and $\neg p \iff q$ are logically equivalent.

p	q	$p \iff q$	$\neg(p \iff q)$	$\neg p$	$\neg p \iff q$
T	T	T	F	F	F
T	F	F	T	F	T
F	T	F	T	T	T
F	F	T	F	T	F

2.7 22

Show that $(p \implies q) \wedge (p \implies r)$ and $p \implies (q \wedge r)$ are logically equivalent.

p	q	r	$p \implies q$	$p \implies r$	$(p \implies q) \wedge (p \implies r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

p	q	r	$q \wedge r$	$p \implies (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

2.8 23

Show that $(p \implies r) \wedge (q \implies r)$ and $(p \vee q) \implies r$ are logically equivalent.

p	q	r	$p \implies r$	$q \implies r$	$(p \implies r) \wedge (q \implies r)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

p	q	r	$p \vee q$	$(p \vee q) \implies r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

2.9 24

Show that $(p \implies q) \vee (p \implies r)$ and $p \implies (q \vee r)$ are logically equivalent.

p	q	r	$p \implies q$	$p \implies r$	$(p \implies q) \vee (p \implies r)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

p	q	r	$q \vee r$	$p \implies (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

2.10 25

Show that $(p \implies r) \vee (q \implies r)$ and $(p \wedge q) \implies r$ are logically equivalent.

p	q	r	$p \implies r$	$q \implies r$	$(p \implies r) \vee (q \implies r)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

p	q	r	$p \wedge q$	$(p \wedge q) \implies r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

2.11 26

Show that $\neg p \implies (q \implies r)$ and $q \implies (p \vee r)$ are logically equivalent.

p	q	r	$\neg p$	$q \implies r$	$\neg p \implies (q \implies r)$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

p	q	r	$p \vee r$	$q \implies (p \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	F	T

2.12 27

Show that $p \iff q$ and $(p \implies q) \wedge (q \implies p)$ are logically equivalent.

p	q	$p \iff q$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	$p \implies q$	$q \implies p$	$(p \implies q) \wedge (q \implies p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

2.13 28

Show that $p \iff q$ and $\neg p \iff \neg q$ are logically equivalent.

p	q	$p \iff q$	$\neg p$	$\neg q$	$\neg p \iff \neg q$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T