Week 04 Participation Assignment

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1 Part 01

Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two functions where f is strictly increasing and g is strictly decreasing.

Prove that both $f \circ g : \mathbb{R} \to \mathbb{R}$ and $g \circ f : \mathbb{R} \to \mathbb{R}$ are strictly decreasing.

As f is strictly increasing, $\forall x \forall y \, (x < y \implies f(x) < f(y))$, and as g is strictly decreasing, $\forall x \forall y \, (x < y \implies g(x) > g(y))$.

We can define the functions f and g as the following strictly increasing and strictly decreasing functions:

Let $x \in \mathbb{R}$.

$$\begin{aligned} f(x) &= x \\ g(x) &= -x \end{aligned}$$

Case by Case Proof

- $f \circ g : \mathbb{R} \to \mathbb{R}$

$$\begin{aligned} (f \circ g)\,(x) &= f(g(x)) \\ (f \circ g)\,(x) &= f(-x) \\ (f \circ g)\,(x) &= -x \end{aligned}$$

- $g \circ f : \mathbb{R} \to \mathbb{R}$

$$\begin{aligned} (g \circ f)\,(x) &= g(f(x)) \\ (g \circ f)\,(x) &= g(x) \\ (g \circ f)\,(x) &= -x \end{aligned}$$

In each case, it can be seen that the composition of functions f and g is strictly decreasing as it produces a linearly strictly decreasing function $y(x) = -x$ where the domain and codomain $\in \mathbb{R}$.