Homework 12

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0.1 Question 1

Density: What is the radius of a sphere that has a density of $5000\,\mathrm{kg\,m^{-3}}$ and a mass of $6.00\,\mathrm{kg}$?

$$\begin{aligned}\rho &= 5000\,\mathrm{kg\,m^{-3}}\\ m &= 6.00\,\mathrm{kg}\\ r &= ?\end{aligned}$$

$$\begin{aligned}\rho &= \frac{m}{V}\\ V &= \frac{m}{\rho}\\ \frac{4}{3}\pi r^3 &= \frac{m}{\rho}\\ r &= \sqrt[3]{\frac{3m}{4\pi\rho}}\\ r &= \sqrt[3]{\frac{3(6.00\,\mathrm{kg})}{4\pi(5000\,\mathrm{kg\,m^{-3}})}}\\ r &= 0.065\,922\,\mathrm{m} = 6.59\,\mathrm{cm}\end{aligned}$$

$$\boxed{r = 6.59\,\mathrm{cm}}$$

0.2 Question 2

Pressure in a fluid: A cubical box, $5.00\,\mathrm{cm}$ on each side, is immersed in a fluid. The gauge pressure at the top surface of the box is $594\,\mathrm{Pa}$ and the gauge pressure on the bottom surface is $1133\,\mathrm{Pa}$. What is the density of the fluid?

$$\begin{aligned}h &= 5.00\,\mathrm{cm} = 0.05\,\mathrm{m}\\ p_0 &= 594\,\mathrm{Pa}\\ p_1 &= 1133\,\mathrm{Pa}\\ \rho &= ?\end{aligned}$$

$$
\begin{aligned}
p_1 &= p_0 + \rho g h \\
\rho &= \frac{p_1 - p_0}{gh} \\
\rho &= \frac{1133\,\mathrm{Pa} - 594\,\mathrm{Pa}}{(9.80\,\mathrm{m\,s^{-2}})(0.05\,\mathrm{m})} \\
\rho &= 1100\,\mathrm{kg\,m^{-3}}
\end{aligned}
$$

$$
\boxed{\rho = 1100\,\mathrm{kg\,m^{-3}}}
$$

0.3 Question 3

Pressure in a fluid: As shown in the figure, a container has a vertical tube, whose inner radius is 32.00 mm, connected to it at its side. An unknown liquid reaches level A in the container and level B in this tube - level A being 5.0 cm higher than level B. The liquid supports a 20.0 cm high column of oil, between levels B and C, whose density is $460\,\mathrm{kg\,m^{-3}}$. What is the density of the unknown liquid?

$$
\begin{aligned}
y_{A,B} &= 5.0\,\mathrm{cm} = 0.05\,\mathrm{m} \\
y_{B,C} &= 20.0\,\mathrm{cm} = 0.20\,\mathrm{m} \\
\rho_{oil} &= 460\,\mathrm{kg\,m^{-3}} \\
\rho_{unknown} &= ?
\end{aligned}
$$

$$
\begin{aligned}
\rho_{unk} g y_{A,B} &= \rho_{oil} g y_{B,C} \\
\rho_{unk} &= \frac{\rho_{oil} y_{B,C}}{y_{A,B}} \\
\rho_{unk} &= \frac{(460\,\mathrm{kg\,m^{-3}})(0.20\,\mathrm{m})}{0.05\,\mathrm{m}} \\
\rho_{unk} &= 1840\,\mathrm{kg\,m^{-3}}
\end{aligned}
$$

$$
\boxed{\rho_{unknown} = 1840\,\mathrm{kg\,m^{-3}} \approx 1800\,\mathrm{kg\,m^{-3}}}
$$

0.4 Question 4

Pressure in a fluid: In the figure, an open tank contains a layer of oil floating on top of a layer of water (of density $1000\,\mathrm{kg\,m^{-3}}$) that is 3.0 m thick, as shown. What must be the thickness of the oil layer if the gauge pressure at the bottom of the tank is to be 5.0×10^4 Pa? The density of the oil is $510\,\mathrm{kg\,m^{-3}}$.

$$
\begin{aligned}
\rho_{water} &= 1000\,\mathrm{kg\,m^{-3}} \\
y_{water} &= 3.0\,\mathrm{m} \\
p_0 &= 5.0 \times 10^4\,\mathrm{Pa} \\
\rho_{oil} &= 510\,\mathrm{kg\,m^{-3}} \\
y_{oil} &= ?
\end{aligned}
$$

$$\begin{aligned}
\rho_{oil} g y_{oil} &= p_0 + \rho_{water} g y_{water} \\
y_{oil} &= \frac{p_0 + \rho_{water} g y_{water}}{\rho_{oil} g} \\
y_{oil} &= \frac{5.0 \times 10^4\,\mathrm{Pa} + (1000\,\mathrm{kg\,m^{-3}})(9.80\,\mathrm{m\,s^{-2}})(3.0\,\mathrm{m})}{(510\,\mathrm{kg\,m^{-3}})(9.80\,\mathrm{m\,s^{-2}})} \\
y_{oil} &= 15.8864\,\mathrm{m} = 15.9\,\mathrm{m}
\end{aligned}$$

$$\boxed{y_{oil} = 15.9\,\mathrm{m} \approx 16.0\,\mathrm{m}}$$

0.5 Question 5

Pascal's principle: A 12 000 N car is raised using a hydraulic lift, which consists of a U-tube with arms of unequal areas, filled with incompressible oil and capped at both ends with tight-fitting pistons. The wider arm of the U-tube has a radius of 18.0 cm and the narrower arm has a radius of 5.00 cm. The car rests on the piston on the wider arm of the U-tube. The pistons are initially at the same level. What is the initial force that must be applied to the smaller piston in order to start lifting the car?

$$\begin{aligned}
w_{car} &= 12\,000\,\mathrm{N} \\
r_{wide} &= 18.0\,\mathrm{cm} = 0.18\,\mathrm{m} \\
r_{narrow} &= 5.00\,\mathrm{cm} = 0.05\,\mathrm{m} \\
F &= ?
\end{aligned}$$

$$\begin{aligned}
\frac{F}{r_{narrow}} &= \frac{w_{car}}{r_{wide}} \\
F &= \frac{w_{car} r_{narrow}}{r_{wide}} \\
F &= \frac{(12\,000\,\mathrm{N})(0.05\,\mathrm{m})}{0.18\,\mathrm{m}} \\
F &= 3333.33\,\mathrm{N} = 3.33 \times 10^3\,\mathrm{N}
\end{aligned}$$

$$\boxed{F = 3.33 \times 10^3\,\mathrm{N} \approx 3330\,\mathrm{N}}$$