Contents

1 Energy

$$KE = \frac{1}{2}mv^2 \tag{1}$$

$$W = \Delta KE \tag{2}$$

When you do work against a conservative force (gravity, springs, electro magnetism (1C)) that energy is stored by the force and can be released later.

If energy is conserved the change in total energy for a given process is zero!

1.1 Conservation of Energy

$$E_i = E_f \tag{3}$$

$$KE_i + PE_i = KE_f + PE_f \tag{4}$$

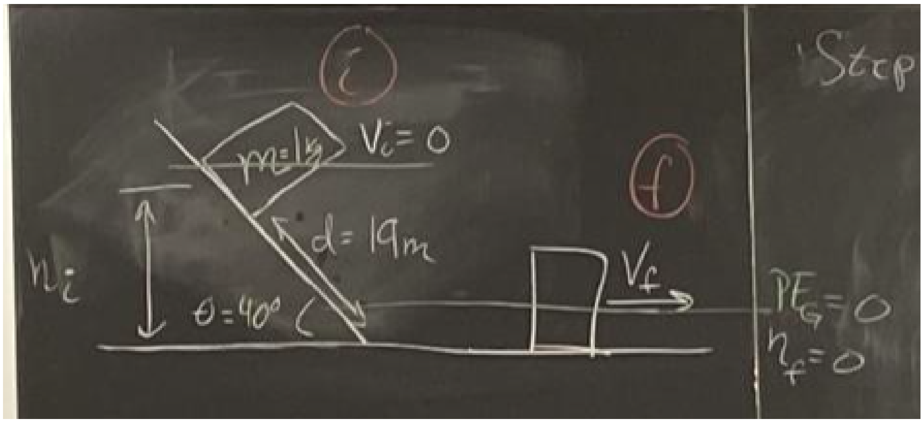
If dealing only in gravity:

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f \tag{5}$$

1.2 Example - Energy

Step 1: Sketch that includes:

- every position
- every speed
- index
- zero potential energy



$$
\begin{aligned}
m &= 1\,\mathrm{kg}\\
d &= 19\,\mathrm{m}\\
\theta &= 40^\circ\\
v_i &= 0\\
h_i &= ?\\
v_f &= ?\\
PE_G &= 0\\
h_f &= 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{2}mv_i^2 + mgh_i &= \frac{1}{2}mv_f^2 + mgh_f\\
\frac{1}{2}m(0) + mgh_i &= \frac{1}{2}mv_f^2 + mg(0)\\
gh_i &= \frac{1}{2}v_f^2\\
v_f &= \sqrt{2gh_i}\\
v_f &= \sqrt{2gd\sin(\theta)}\\
v_f &= \sqrt{2(10\,\mathrm{m\,s^{-2}})(19\,\mathrm{m})\sin(90^\circ)}\\
v_f &= 15.6\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

Supposed we do the experiment and we find $v_f = 11.6\,\mathrm{m\,s^{-1}}$. How much energy was lost to friction? How much energy was lost to friction? What is μ?

1) Kinematics
Find $a, f, N \rightarrow f = \mu N$

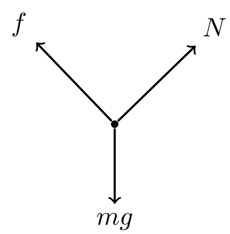
2)

$$
\begin{aligned}
W &= \Delta KE\\
-fd &= \Delta KE
\end{aligned}
$$

find $N \rightarrow f = \mu N$

If you have a non-conservative force:

$$E_i + W_{nc} = E_f \tag{6}$$

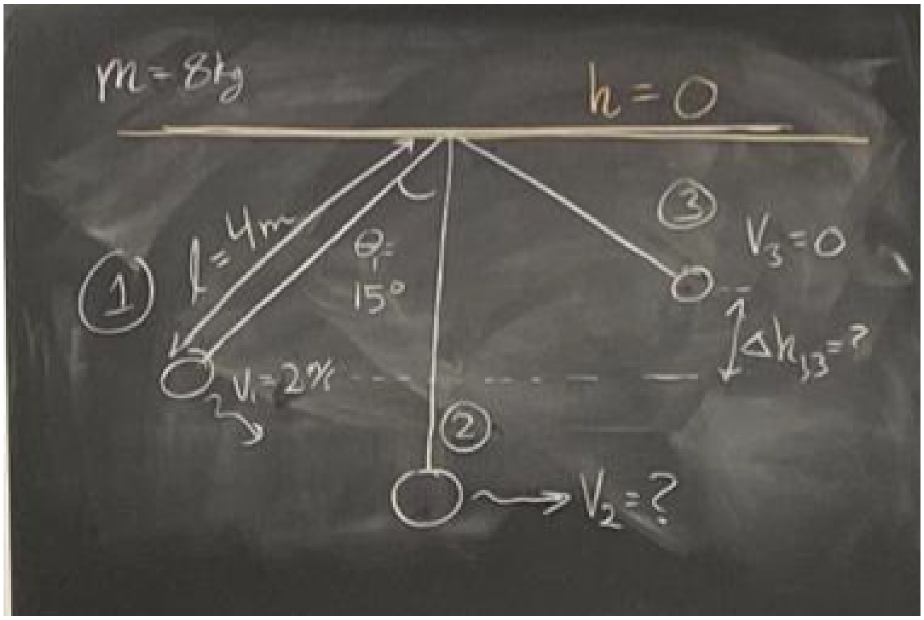


$$\begin{aligned}
\sum F_y &= 0 \\
N &= mg\cos(\theta) \\
f &= \mu mg\cos(\theta)
\end{aligned}$$

$$\begin{aligned}
W_f &= -fd \\
W_f &= -\mu mgd\cos(\theta)
\end{aligned}$$

$$\begin{aligned}
E_i + W_f &= E_f \\
mgd\sin(\theta) - \mu mgd\cos(\theta) &= \frac{1}{2}mv_f^2 \\
gd\sin(\theta) - \mu gd\cos(\theta) &= \frac{1}{2}v_f^2 \\
\mu &= \frac{2gd\sin(\theta) - v_f^2}{2gd\cos(\theta)}
\end{aligned}$$

1.3 Example - Pendulum



$$
\begin{aligned}
h_2 &= -4\,\mathrm{m} \\
\cos(\theta_1) &= \frac{-h_1}{l} \\
h_1 &= -l\cos(\theta_1) \\
h_1 &= -(4\,\mathrm{m})\cos(15^\circ) \\
h_1 &= -3.86\,\mathrm{m}
\end{aligned}
$$

$$
\begin{aligned}
E_1 &= E_2 \\
mgh_1 + \frac{1}{2}mv_1^2 &= mgh_2 + \frac{1}{2}mv_2^2 \\
v_2^2 &= v_1^2 + 2g(h_1 - h_2) \\
v_2 &= \sqrt{v_1^2 + 2g(h_1 - h_2)} \\
v_2 &= \sqrt{(2\,\mathrm{m\,s^{-1}})^2 + 2(10\,\mathrm{m\,s^{-2}})(-3.86\,\mathrm{m} - (-4\,\mathrm{m}))} \\
v_2 &= 2.6\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
E_1 &= E_3 \\
mgh_1 + \frac{1}{2}mv_1^2 &= mgh_3 \\
gh_1 + \frac{1}{2}v_1^2 &= gh_3 \\
\frac{1}{2}v_1^2 &= g(h_3 - h_1) \\
\Delta h_{1,3} &= \frac{v_1^2}{2g} \\
\Delta h_{1,3} &= \frac{(2\,\mathrm{m\,s^{-1}})^2}{20\,\mathrm{m\,s^{-2}}} \\
\Delta h_{1,3} &= 0.2\,\mathrm{m}
\end{aligned}
$$

2 Log Scale Graphs

Log-log graph - x and y axes are log scale
Log scales are in sets of 10

2.1 Power Law:

log-log

$$
\begin{aligned}
y &= kx^n \\
\log(y) &= \log(kx^n) \\
\log(y) &= \log(k) + \log(x^n) \\
\log(y) &= n \cdot \log(x) + \log(k) \\
Y &= nX + K
\end{aligned}
$$

semi log

$$
\begin{aligned}
y &= ae^{kx} \\
\ln(y) &= \ln(ae^{kx}) \\
\ln(y) &= \ln(a) + \ln(e^{kx}) \\
\ln(y) &= \ln(a) + kx \\
Y &= kx + \ln(a)
\end{aligned}
$$

Slope

$$
\text{Slope} = \frac{\log(y_2) - \log(y_1)}{\log(x_2) - \log(x_1)}
$$