Contents

1 Section 10.2

Laplace transform of the second derivative of a function

$$\mathcal{L}(f'') = s^2\mathcal{L}(f) - sf(0) - f'(0) \tag{1}$$

1.1 10.2.1

Solve the following differential equation by Laplace transforms. The function is subject to the given conditions.

$$y'' + 64y = 0, y(0) = 0, y'(0) = 1$$

$$\begin{aligned}
\mathcal{L}(y'') + 64\mathcal{L}(y) &= \mathcal{L}(0) \\
\left[s^2\mathcal{L}(y) - sy(0) - y'(0)\right] + 64\mathcal{L}(y) &= \mathcal{L}(0) \\
s^2\mathcal{L}(y) - 0 - 1 + 64\mathcal{L}(y) &= \mathcal{L}(0) \\
\mathcal{L}(y)(s^2 + 64) &= 1 \\
\mathcal{L}(y) &= \frac{1}{s^2 + 64} \\
y &= \mathcal{L}^{-1}\left(\frac{1}{s^2 + 64}\right) \\
y &= \frac{1}{8}\sin(8t)
\end{aligned}$$

1.2 10.2.3

Solve the following differential equation by Laplace transforms. The function is subject to the given conditions.

$$y'' + 4y' - 5y = 0, y(0) = 5, y'(0) = 11$$

$$\begin{aligned}
\mathcal{L}(y'') + 4\mathcal{L}(y') - 5\mathcal{L}(y) &= \mathcal{L}(0) \\
\left[s^2\mathcal{L}(y) - sy(0) - y'(0)\right] + 4\left[s\mathcal{L}(y) - y(0)\right] - 5\mathcal{L}(y) &= 0 \\
s^2\mathcal{L}(y) - 5s - 11 + 4s\mathcal{L}(y) - 20 - 5\mathcal{L}(y) &= 0 \\
\mathcal{L}(y)(s^2 + 4s - 5) &= 5s + 31 \\
\mathcal{L}(y) &= \frac{5s + 31}{s^2 + 4s - 5} \\
\mathcal{L}(y) &= \frac{5s + 31}{(s-1)(s+5)}
\end{aligned}$$

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\begin{aligned}
\frac{5s+31}{(s-1)(s+5)} &= \frac{A}{s-1} + \frac{B}{s+5} \\
5s+31 &= A(s+5) + B(s-1) \\
5(1)+31 &= A(1+5) + B(1-1), \quad s=1 \\
A &= 6 \\
5s+31 &= 6(s+5) + B(s-1), \quad s=-5 \\
5(-5)+31 &= 6(-5+5) + B(-5-1) \\
B &= -1
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}(y) &= \frac{6}{s-1} - \frac{1}{s+5} \\
y &= \mathcal{L}^{-1}\left(\frac{6}{s-1}\right) - \mathcal{L}^{-1}\left(\frac{1}{s+5}\right) \\
y &= 6e^{t} - e^{-5t}
\end{aligned}
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