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1 Section 3.1

1.1 3.1.3

Use the method of elimination to determine whether the given linear system is consistent or inconsistent. If the linear system is consistent, find the solution if it is unique; otherwise, describe the infinite solution set in terms of an arbitrary parameter t.

$$\begin{cases} 7x + 5y = -22 \\ 2x + 9y = 24 \end{cases}$$

$$\begin{cases} \quad x + \dfrac{5}{7}y = -\dfrac{22}{7} \\ -x - \dfrac{9}{2}y = -12 \end{cases}$$

$$\begin{aligned} -\frac{53}{14}y &= -\frac{106}{7} \\ y &= 4 \\ 7x + 5(4) &= -22 \\ x &= -6 \end{aligned}$$

$$\boxed{\text{Unique solution:} \quad x = -6, y = 4}$$

1.2 3.1.5

Use the method of elimination to determine whether the given linear system is consistent or inconsistent. If the linear system is consistent, find the solution if it is unique; otherwise, describe the infinite solution set in terms of an arbitrary parameter t.

$$\begin{cases} \quad x + 5y = 18 \\ 3x + 15y = 55 \end{cases}$$

$$\begin{cases} -3x - 15y = -54 \\ \quad 3x + 15y = 55 \end{cases}$$

$$0 + 0 = 1$$

$$\boxed{\text{Linear system is inconsistent and no solution exists}}$$

1.3 3.1.7

Use the method of elimination to determine whether the given linear system is consistent or inconsistent. If the linear system is consistent, find the solution if it is unique; otherwise, describe the infinite solution set in terms of an arbitrary parameter

$$\begin{cases} \quad x - 4y = -8 \\ -2x + 8y = 16 \end{cases}$$

$$\begin{cases} \quad 2x - 8y = -16 \\ -2x + 8y = 16 \end{cases}$$

$$0 = 0$$

$$x - 2t = -15$$

$$x = 2t - 15$$

$$\boxed{\text{Infinite many solutions}, x = 2t - 15}$$

1.4 3.1.9

Use the method of elimination to determine whether the given linear system is consistent or inconsistent. If the linear system is consistent, find the solution if it is unique; otherwise, describe the infinite solution set in terms of an arbitrary parameter t.

$$\begin{cases} \quad x + 4y + z = -3 \\ 2x + y - 4z = -16 \\ x + 6y + 2z = -3 \end{cases}$$

$$\begin{cases} x+4y+z=-3 \\ x+6y+2z=-3 \end{cases}$$

$$\begin{aligned} -2y-z&=0 \\ z&=-2y \end{aligned}$$

$$\begin{cases} 2x+8y+2z=-6 \\ 2x+y-4z=-16 \end{cases}$$

$$\begin{aligned} 7y+6z&=10 \\ 7y+6(-2y)&=10 \\ y&=-2 \\ z&=-2(-2) \\ z&=4 \\ x+4(-2)+4&=-3 \\ x&=1 \end{aligned}$$

$$\boxed{x=1, y=-2, z=4}$$

1.5 3.1.19

Use the method of elimination to determine whether the given linear system is consistent or inconsistent. If the linear system is consistent, find the solution if it is unique; otherwise, describe the infinite solution set in terms of an arbitrary parameter t.

$$\begin{cases} x-2y+2=-1 \\ 2x-y-8z=31 \\ x-y-2z=10 \end{cases}$$

$$\begin{aligned} x&=2y-2z-1 \\ 2(2y-2z-1)-y-8z&=31 \\ y&=4z+11 \\ x&=2(4z+11)-2z-1 \\ x&=6z+21 \\ (6z+21)-(4z+11)-2z&=10 \\ (6z-4z-2z)&=(-21+11+10) \\ 0&=0, \quad \text{Infinite many solutions} \end{aligned}$$

$$\begin{cases} x=6t+21 \\ y=4t+11 \end{cases}$$

$$\boxed{\begin{cases} x=6t+21 \\ y=4t+11 \end{cases}}$$

1.6 3.1.23

A second-order differential equation and its general solution $y(x)$ are given. Determine the constants A and B so as to find a solution of the differential equation that satisfies the given initial conditions involving $y(0)$ and $y'(0)$.

$$y'' + 9y = 0, y(x) = A\cos(3x) + B\sin(3x), y(0) = 3, y'(0) = 27$$

$$\begin{aligned} y(x) &= A\cos(3x) + B\sin(3x) \\ y'(x) &= -3A\sin(3x) + 3B\cos(3x) \\ y''(x) &= -9A\cos(3x) - 9B\sin(3x) \end{aligned}$$

$$\begin{aligned} y(0) &= A\cos(3(0)) + B\sin(3(0)) = 3 \\ A &= 3 \\ y'(0) &= -3(3)\sin(3(0)) + 3B\cos(3(0)) = 27 \\ B &= 9 \end{aligned}$$

$$\boxed{A = 3, B = 9}$$

1.7 3.1.25

A second-order differential equation and its general solution $y(x)$ are given. Determine the constants A and B so as to find a solution of the differential equation that satisfies the given initial conditions involving $y(0)$ and $y'(0)$.

$$y'' - 9y = 0, y(x) = Ae^{3x} + Be^{-3x}, y(0) = 12, y'(0) = 6$$

$$\begin{aligned} y(x) &= Ae^{3x} + Be^{-3x} \\ y'(x) &= 3Ae^{3x} - 3Be^{-3x} \\ y''(x) &= 9Ae^{3x} + 9Be^{-3x} \end{aligned}$$

$$\begin{aligned} y(0) &= Ae^{3(0)} + Be^{-3(0)} = 12 \\ A + B &= 12 \\ A &= 12 - B \\ y'(0) &= 3Ae^{3(0)} - 3Be^{-3(0)} = 6 \\ 3A - 3B &= 6 \\ 3(12 - B) - 3B &= 6 \\ B &= 5 \\ A + (5) &= 12 \\ A &= 7 \end{aligned}$$

$$\boxed{A = 7, B = 5}$$