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1 Section 5.5

1.1 5.5.2

Find a particular solution y_p of the following equation using the Method of Undetermined Coefficients. Primes denote the derivatives with respect to x.

$$y'' - y' - 2y = 4x + 6$$

$$\begin{aligned} r^2 - r - 2 &= 0 \\ r &= 2, -1 \\ y(x) &= c_1 e^{2x} + c_2 e^{-x} \\ y_p(x) &= Ax + B \\ y_p'(x) &= A \\ y_p''(x) &= 0 \end{aligned}$$

$$\begin{aligned} (0) - (A) - 2(Ax + B) &= 4x + 6 \\ -2Ax - A - 2B &= 4x + 6 \\ -2A &= 4 \\ A &= -2 \\ -A - 2B &= 6 \\ -(-2) - 2B &= 6 \\ B &= -2 \\ y(x) &= -2x - 2 \end{aligned}$$

$$\boxed{y(x) = -2x - 2}$$

1.2 5.5.3

Find a particular solution y_p of the following equation using the Method of Undetermined Coefficients. Primes denote the derivatives with respect to x.

$$y'' - y' - 6y = 20\sin(3x)$$

$$
\begin{aligned}
r^2 - r - 6 &= 0 \\
r &= 3, -2 \\
y(x) &= c_1 e^{3x} + c_2 e^{-2x} \\
y_p(x) &= A\cos(3x) + B\sin(3x) \\
y_p'(x) &= -3A\sin(3x) + 3B\cos(3x) \\
y_p''(x) &= -9A\cos(3x) + -9B\sin(3x) \\
20\sin(3x) &= (-9A\cos(3x) + -9B\sin(3x)) - (-3A\sin(3x) + 3B\cos(3x)) - 6(A\cos(3x) + B\sin(3x))
\end{aligned}
$$

$$
\begin{aligned}
\cos(3x)(-9A - 3B - 6A) + \sin(3x)(-9B + 3A - 6B) &= 20\sin(3x) \\
-9A - 3B - 6A &= 0 \\
-15A - 3B &= 0 \\
B &= -5A \\
-9B + 3A - 6B &= 20 \\
-15B + 3A &= 20 \\
-15(-5A) + 3A &= 20 \\
A &= \frac{10}{39} \\
B &= -5\left(\frac{10}{39}\right) \\
B &= -\frac{50}{39} \\
y(x) &= \frac{10}{39}\cos(3x) - \frac{50}{39}\sin(3x)
\end{aligned}
$$

$$
\boxed{y(x) = \frac{10}{39}\cos(3x) - \frac{50}{39}\sin(3x)}
$$

1.3 5.5.4

Find a particular solution y_p of the following equation using the Method of Undetermined Coefficients. Primes denote the derivatives with respect to x.

$$
y'' - 4y' + 5y = xe^x
$$

$$
\begin{aligned}
r^2 - 4r + 5 &= 0 \\
r &= 2 \pm 1i \\
y(x) &= c_1 e^{2x}\cos(x) + c_2 e^{2x}\sin(x) \\
y_p(x) &= (Ax + B)Ce^x \\
y_p(x) &= e^x(Ax + B) \\
y_p'(x) &= e^x(Ax + A + B) \\
y_p''(x) &= e^x(Ax + 2A + B) \\
xe^x &= (e^x(Ax + 2A + B)) - 4(e^x(Ax + A + B)) + 5(e^x(Ax + B))
\end{aligned}
$$

$$
\begin{aligned}
xe^x(A - 4A + 5A) + e^x(2A + B - 4A - 4B + 5B) &= xe^x \\
A - 4A + 5A &= 1 \\
A &= \frac{1}{2} \\
2A + B - 4A - 4B + 5B &= 0 \\
-2A + 2B &= 0 \\
-2\left(\frac{1}{2}\right) + 2B &= 0 \\
B &= \frac{1}{2} \\
y(x) &= e^x\left(\frac{x}{2} + \frac{1}{2}\right)
\end{aligned}
$$

$$\boxed{y(x) = e^x\left(\frac{x}{2} + \frac{1}{2}\right)}$$

1.4 5.5.10

Find a particular solution y_p of the following equation using the Method of Undetermined Coefficients. Primes denote the derivatives with respect to x.

$$y'' + 9y = 4\cos(3x) + 6\sin(3x)$$

$$
\begin{aligned}
r^2 + 9 &= 0 \\
r &= 0 \pm 3i \\
y(x) &= c_1\cos(3x) + c_2\sin(3x) \\
y_p(x) &= A\cos(3x) + B\sin(3x) \\
y_p'(x) &= -3A\sin(3x) + 3B\cos(3x) \\
y_p''(x) &= -9A\cos(3x) - 9B\sin(3x) \\
&(-9A\cos(3x) - 9B\sin(3x)) + 9(A\cos(3x) + B\sin(3x)) \\
&= 4\cos(3x) + 6\sin(3x)
\end{aligned}
$$

$$\cos(3x)(-9A + 9A) + \sin(3x)(-9B + 9B) = 4\cos(3x) + 6\sin(3x)$$