

# 1 Week 15 and Week 16 Participation Assignment (2 of 2)

- 1) Find a general formula for the coefficient  $a_n$  in a power series expansion about  $x = 0$  for a general solution to the equation  $(1-x^2)y'' + xy' + 3y = 0$ .

$$\begin{aligned} (1-x^2) \sum_{n=0}^{\infty} (n)(n-1)a_n x^{n-2} + x \sum_{n=0}^{\infty} n a_n x^{n-1} + 3 \sum_{n=0}^{\infty} a_n x^n &= 0 \\ \sum_{n=0}^{\infty} (n+1)(n+2)a_{n+2} x^n - \sum_{n=2}^{\infty} n(n-1)a_n x^n + \sum_{n=1}^{\infty} n a_n x^n + 3 \sum_{n=0}^{\infty} a_n x^n &= 0 \\ (2a_2 + 3a_0) + 16a_3 + a_1 + 3a_1)x + \sum_{n=2}^{\infty} [(n+1)(n+2)a_n - n(n-1)a_n + n a_n + 3a_n] x^n &= 0 \end{aligned}$$

$$y(x) = c_0 \left( \frac{3x^{2n}(2n-1)}{2n} \right)$$

- 2) Use the method of Frobenius to find at least first four non-zero terms in the series expansion about  $x = 0$  for a solution to the equation  $x^2 y'' + (x^2 + 2x)y' - 2y = 0$  for  $x > 0$ .

$$x^2 \sum_{n=0}^{\infty} (n+r)(n+r-1)a_n x^{n+r-2} + (x^2 + 2x) \sum_{n=0}^{\infty} (n+r)a_n x^{n+r-1} - 2 \sum_{n=0}^{\infty} a_n x^{n+r} = 0$$