Contents

1 Section 4.5

1.1 4.5.1

Find both a basis for the row space and a basis for the column space of the given matrix $\mathbf{A}$.

$$\begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 6 \\ 2 & 5 & 8 \end{bmatrix}$$

Find rref($\mathbf{A}$):

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 6 \\ 2 & 5 & 8 \end{bmatrix}$$

$$\begin{aligned} \mathbf{A}_2 &= \mathbf{A}_2 - \mathbf{A}_1 \\ \mathbf{A}_3 &= \mathbf{A}_3 - 2\mathbf{A}_1 \\ \mathbf{A} &= \begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 4 \\ 0 & -1 & 4 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \mathbf{A}_3 &= \mathbf{A}_3 + \mathbf{A}_2 \\ \mathbf{A}_2 &= -1\mathbf{A}_2 \\ \mathbf{A} &= \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

A basis for the row space is $\left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} \right\}$.

A basis for the column space is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix} \right\}$.

1.2 4.5.3

Find both a basis for the row space and a basis for the column space of the given matrix **A**.

$$\begin{bmatrix} 1 & -3 & -2 & -5 \\ 4 & -7 & -3 & -5 \\ 1 & 3 & 4 & 13 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 1 & -3 & -2 & -5 \\ 4 & -7 & -3 & -5 \\ 1 & 3 & 4 & 13 \end{bmatrix}$$

$$\begin{aligned} \mathbf{A}_2 &= \mathbf{A}_2 - 4\mathbf{A}_1 \\ \mathbf{A}_2 &= \frac{1}{5}\mathbf{A}_2 \\ \mathbf{A}_3 &= \mathbf{A}_3 - \mathbf{A}_1 \\ \mathbf{A}_3 &= \frac{1}{6}\mathbf{A}_3 \\ \mathbf{A} &= \begin{bmatrix} 1 & -3 & -2 & -5 \\ 0 & 1 & 1 & 3 \\ 0 & 1 & 1 & 3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \mathbf{A}_3 &= \mathbf{A}_3 - \mathbf{A}_2 \\ \mathbf{A}_1 &= \mathbf{A}_1 + 3\mathbf{A}_2 \\ \mathbf{A} &= \begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

A basis for the row space is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 3 \end{bmatrix} \right\}$

A basis for the column space is $\left\{ \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -7 \\ 3 \end{bmatrix} \right\}$

1.3 4.5.9

Find both a basis for the row space and a basis for the column space of the given matrix $\mathbf{A}$.

$$\begin{bmatrix} 5 & 1 & 2 & 9 \\ 10 & 7 & 5 & 7 \\ 5 & 16 & 3 & 13 \\ 15 & 28 & 9 & 9 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 5 & 1 & 2 & 9 \\ 10 & 7 & 5 & 7 \\ 5 & 16 & 3 & 13 \\ 15 & 28 & 9 & 9 \end{bmatrix}$$

$$\mathbf{A}_2 = \mathbf{A}_2 - 2\mathbf{A}_1$$
$$\mathbf{A}_3 = \mathbf{A}_3 - \mathbf{A}_1$$
$$\mathbf{A}_4 = \mathbf{A}_4 - 3\mathbf{A}_1$$
$$\mathbf{A} = \begin{bmatrix} 5 & 1 & 2 & 9 \\ 0 & 5 & 1 & -11 \\ 0 & 15 & 1 & 4 \\ 0 & 25 & 3 & -18 \end{bmatrix}$$

$$\mathbf{A}_3 = \mathbf{A}_3 - 3\mathbf{A}_2$$
$$\mathbf{A}_4 = \mathbf{A}_4 - 5\mathbf{A}_2$$
$$\mathbf{A} = \begin{bmatrix} 5 & 1 & 2 & 9 \\ 0 & 5 & 1 & -11 \\ 0 & 0 & -2 & 37 \\ 0 & 0 & -2 & 37 \end{bmatrix}$$

$$\mathbf{A}_4 = \mathbf{A}_4 - \mathbf{A}_3$$
$$\mathbf{A} = \begin{bmatrix} 5 & 1 & 2 & 9 \\ 0 & 5 & 1 & -11 \\ 0 & 0 & -2 & 37 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{A}_2 = 2\mathbf{A}_2 + \mathbf{A}_3$$
$$\mathbf{A}_2 = \frac{1}{5}\mathbf{A}_2$$
$$\mathbf{A} = \begin{bmatrix} 5 & 1 & 2 & 9 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & -2 & 37 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$
\begin{aligned}
\mathbf{A}_1 &= 2\mathbf{A}_1 - \mathbf{A}_2 \\
\mathbf{A}_1 &= \mathbf{A}_1 + 2\mathbf{A}_3 \\
\mathbf{A}_1 &= \frac{1}{10}A_1 \\
\mathbf{A}_2 &= \frac{1}{2}A_2 \\
\mathbf{A}_3 &= -\frac{1}{2}A_3 \\
\mathbf{A} &= \begin{bmatrix} 1 & 0 & 0 & \frac{89}{10} \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & -\frac{37}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}
\end{aligned}
$$

$$
\boxed{\begin{array}{l}
\text{A basis for the row space is } \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ \frac{89}{10} \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{3}{2} \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -\frac{37}{2} \end{bmatrix} \right\} \\
\text{A basis for the column space is } \left\{ \begin{bmatrix} 5 \\ 10 \\ 5 \\ 15 \end{bmatrix}, \begin{bmatrix} 1 \\ 7 \\ 16 \\ 28 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 3 \\ 9 \end{bmatrix} \right\}
\end{array}}
$$

1.4 4.5.13

A set S of vectors in $\mathbb{R}^4$ is given. Find a subset of S that forms a basis for the subspace of $\mathbb{R}^4$ spanned by S.

$$
\vec{v}_1 = \begin{bmatrix} 2 \\ 2 \\ -2 \\ 6 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ 23 \\ -44 \\ 30 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 5 \\ 26 \\ -47 \\ 39 \end{bmatrix}
$$

$$
\mathbf{A} = \begin{bmatrix} 2 & 2 & 5 \\ 2 & 23 & 26 \\ -2 & -44 & -47 \\ 6 & 30 & 39 \end{bmatrix}
$$

$$\begin{aligned}\mathbf{A}_2 &= \mathbf{A}_2 - \mathbf{A}_1\\ \mathbf{A}_3 &= \mathbf{A}_3 + \mathbf{A}_1\\ \mathbf{A}_4 &= \mathbf{A}_4 - 3\mathbf{A}_1\\ \mathbf{A} &= \begin{bmatrix} 2 & 2 & 5\\ 0 & 21 & 21\\ 0 & -42 & -42\\ 0 & 24 & 24 \end{bmatrix}\end{aligned}$$

$$\begin{aligned}\mathbf{A}_2 &= \frac{1}{21}\mathbf{A}_2\\ \mathbf{A}_3 &= -\frac{1}{42}\mathbf{A}_3\\ \mathbf{A}_4 &= \frac{1}{24}\mathbf{A}_4\\ \mathbf{A}_4 &= \mathbf{A}_4 - \mathbf{A}_3\\ \mathbf{A}_3 &= \mathbf{A}_3 - \mathbf{A}_2\\ \mathbf{A} &= \begin{bmatrix} 2 & 2 & 5\\ 0 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix}\end{aligned}$$

$$\begin{aligned}\mathbf{A}_1 &= \mathbf{A}_1 - 2\mathbf{A}_2\\ \mathbf{A}_1 &= \frac{1}{2}\mathbf{A}_1\\ \mathbf{A} &= \begin{bmatrix} 1 & 0 & \frac{3}{2}\\ 0 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix}\end{aligned}$$

A basis for the row space is $\left\{ \begin{bmatrix} 1\\ 0\\ \frac{3}{2} \end{bmatrix}, \begin{bmatrix} 0\\ 1\\ 1 \end{bmatrix} \right\}$

A basis for the column space is $\left\{ \begin{bmatrix} 2\\ 2\\ -2\\ 6 \end{bmatrix}, \begin{bmatrix} 2\\ 23\\ -44\\ 30 \end{bmatrix} \right\}$

A basis for the subspace is given by $\left\{ \begin{bmatrix} 2\\ 2\\ -2\\ 6 \end{bmatrix}, \begin{bmatrix} 2\\ 23\\ -44\\ 30 \end{bmatrix} \right\}$