Homework 8 - Momentum

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Contents

1 Book

1.1 8.16

$$\begin{aligned} m_{(a)stronaut} &= 65.5\,\mathrm{kg} \\ m_{(t)ool} &= 2.50\,\mathrm{kg} \\ v_{t_1} &= 3.10\,\mathrm{m\,s^{-1}} \\ v_{a_1} &=? \end{aligned}$$

$$\begin{aligned} P_0 &= P_1 \\ m_a v_{a_0} + m_t v_{t_0} &= m_a v_{a_1} + m_t v_{t_1} \\ 0 + 0 &= m_a v_{a_1} + m_t v_{t_1} \\ v_{a_1} &= -\frac{m_t v_{t_1}}{m_a} \\ v_{a_1} &= -\frac{(2.50\,\mathrm{kg})(3.10\,\mathrm{m\,s^{-1}})}{65.5\,\mathrm{kg}} \\ v_{a_1} &= -0.118\,\mathrm{m\,s^{-1}} \end{aligned}$$

The astronaut will move at a speed of $0.118\,\mathrm{m\,s^{-1}}$ opposite of the tool's direction.

1.2 8.21

$$
\begin{aligned}
m_A &= 0.245\,\mathrm{kg} \\
m_B &= 0.360\,\mathrm{kg} \\
v_{B_0} &= 0 \\
v_{A_1} &= -0.118\,\mathrm{m\,s^{-1}} \\
v_{B_1} &= 0.660\,\mathrm{m\,s^{-1}} \\
v_{A_0} &= ?
\end{aligned}
$$

$-\hat{x} \longleftarrow \bullet \longrightarrow +\hat{x}$

(a) What was the speed of puck A before the collision?

$$
\begin{aligned}
P_0 &= P_1 \\
m_A v_{A_0} + m_B v_{B_0} &= m_A v_{A_1} + m_B v_{B_1} \\
m_A v_{A_0} + 0 &= m_A v_{A_1} + m_B v_{B_1} \\
v_{A_0} &= \frac{m_A v_{A_1} + m_B v_{B_1}}{m_A} \\
v_{A_0} &= \frac{(0.245\,\mathrm{kg})(-0.118\,\mathrm{m\,s^{-1}}) + (0.360\,\mathrm{kg})(0.660\,\mathrm{m\,s^{-1}})}{0.245\,\mathrm{kg}} \\
v_{A_0} &= 0.852\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$\boxed{v_{A_0} = 0.852\,\mathrm{m\,s^{-1}}}$$

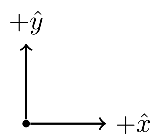
(b) Calculate the change in the total kinetic energy of the system that occurs during the collision.

$$
\begin{aligned}
\Delta PE &= E_{A_1} + E_{B_1} - E_{A_0} + E_{B_0} \\
\Delta PE &= \frac{1}{2} m_A v_{A_1}^2 + \frac{1}{2} m_B v_{B_1}^2 - \frac{1}{2} m_A v_{A_0}^2 + 0 \\
\Delta PE &= \frac{1}{2}(0.245\,\mathrm{kg})(-0.118\,\mathrm{m\,s^{-1}})^2 + \frac{1}{2}(0.360\,\mathrm{kg})(0.660\,\mathrm{m\,s^{-1}})^2 \\
&\quad - \frac{1}{2}(0.245\,\mathrm{kg})(0.852\,\mathrm{m\,s^{-1}})^2 \\
\Delta PE &= -0.008\,81\,\mathrm{J} = 8.81 \times 10^{-3}\,\mathrm{J}
\end{aligned}
$$

$$\boxed{\Delta PE = -0.008\,81\,\mathrm{J} = 8.81 \times 10^{-3}\,\mathrm{J}}$$

1.3 8.30

$$
\begin{aligned}
m_A = m_B &= ? \\
v_{A_0} &= 40.0\,\mathrm{m\,s^{-1}} \\
\theta_A &= 30.0^\circ \\
v_{B_0} &= 0 \\
\theta_B &= -45.0^\circ \\
v_{A_1} &= ? \\
v_{B_1} &= ?
\end{aligned}
$$



(a) Find the speed of each asteroid after the collision.
Speed of asteroid in $\hat{x}$ direction:

$$
\begin{aligned}
v_{A_0} &= 40.0\,\mathrm{m\,s^{-1}}\cos(0^\circ) = 40.0\,\mathrm{m\,s^{-1}} \\
v_{B_0} &= 0 \\
v_{A_1} &= v_{A_1}\cos(30.0^\circ) \\
v_{B_1} &= v_{B_1}\cos(-45.0^\circ)
\end{aligned}
$$

$$
\begin{aligned}
P_{0_x} &= P_{1_x} \\
m_A v_{A_0} + m_B v_{B_0} &= m_A v_{A_1} + m_B v_{B_1} \\
v_{A_0} &= v_{A_1} + v_{B_1} \\
40.0\,\mathrm{m\,s^{-1}} &= v_{A_1}\cos(30.0^\circ) + v_{B_1}\cos(-45.0^\circ)
\end{aligned}
$$

Speed of asteroid in $\hat{y}$ direction:

$$
\begin{aligned}
v_{A_0} &= 40.0\,\mathrm{m\,s^{-1}}\cos(90^\circ) = 0 \\
v_{B_0} &= 0 \\
v_{A_1} &= v_{A_1}\sin(30.0^\circ) \\
v_{B_1} &= v_{B_1}\sin(-45.0^\circ)
\end{aligned}
$$

$$
\begin{aligned}
P_{0_y} &= P_{1_y} \\
m_A v_{A_0} + m_B v_{B_0} &= m_A v_{A_1} + m_B v_{B_1} \\
0 &= v_{A_1} + v_{B_1} \\
v_{A_1}\sin(30.0^\circ) + v_{B_1}\sin(-45.0^\circ) &= 0
\end{aligned}
$$

$$[\boldsymbol{A}|\mathbf{v}] = \left[\begin{array}{cc|c} \cos(30.0°) & \cos(-45.0°) & 40.0\,\mathrm{m\,s^{-1}} \\ \sin(30.0°) & \sin(-45.0°) & 0 \end{array}\right]$$

$$\boldsymbol{A}_2 = \boldsymbol{A}_2 - \boldsymbol{A}_1\frac{\sqrt{3}}{3}$$
$$[\boldsymbol{A}|\mathbf{v}] = \left[\begin{array}{cc|c} \cos(30.0°) & \cos(-45.0°) & 40.0\,\mathrm{m\,s^{-1}} \\ 0 & -1.12 & -23.1\,\mathrm{m\,s^{-1}} \end{array}\right]$$

$$-1.12\mathbf{v}_B = -23.1\,\mathrm{m\,s^{-1}}$$
$$\mathbf{v}_B = 20.6\,\mathrm{m\,s^{-1}}$$

$$(\cos(30.0°))\mathbf{v}_A + (\cos(-45.0°))\mathbf{v}_B = 40.0\,\mathrm{m\,s^{-1}}$$
$$(\cos(30.0°))\mathbf{v}_A + (\cos(-45.0°))(20.6\,\mathrm{m\,s^{-1}}) = 40.0\,\mathrm{m\,s^{-1}}$$
$$\mathbf{v}_A = 29.4\,\mathrm{m\,s^{-1}}$$

Asteroid A moves $29.4\,\mathrm{m\,s^{-1}}$ at $30.0°$ above the horizontal while asteroid B moves $20.6\,\mathrm{m\,s^{-1}}$ at $-45.0°$ below the horizontal.

(b) What fraction of the original kinetic energy of asteroid A dissipates during this collision.

$$E_1 : E_0 = \frac{E_1}{E_0}$$
$$E_1 : E_0 = \frac{\frac{1}{2}m_A v_{A_1}^2 + \frac{1}{2}m_B v_{B_1}^2}{\frac{1}{2}m_A v_{A_0}^2 + \frac{1}{2}m_B v_{B_0}^2}$$
$$E_1 : E_0 = \frac{v_{A_1}^2 + v_{B_1}^2}{v_{A_0}^2}$$
$$E_1 : E_0 = \frac{(29.4\,\mathrm{m\,s^{-1}}) + (20.6\,\mathrm{m\,s^{-1}})}{40.0\,\mathrm{m\,s^{-1}}}$$
$$E_1 : E_0 = 0.805 = 80.5\,\%$$

$80.5\,\%$ of asteroid A's kinetic energy is conserved; therefore also meaning that $19.5\,\%$ is dissipated during collision.

1.4 8.34

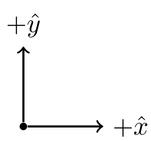
$$m_{(a)pple} = M$$
$$v_{a_0} = 0$$
$$m_{(d)art} = \frac{M}{4}$$
$$v_{d_0} = v_0$$

$$\begin{aligned}
P_0 &= P_1 \\
m_a v_{a_0} + m_d v_{d_0} &= v_1(m_a + m_d) \\
0 + m_d v_{d_0} &= v_1(m_a + m_d) \\
v_1 &= \frac{m_d v_{d_0}}{m_a + m_d}
\end{aligned}$$

$$\begin{aligned}
E_0 &= E_1 \\
\frac{1}{2}mv_0^2 + mgh_0 &= \frac{1}{2}mv_1^2 + mgh_1 \\
\frac{1}{2}(m_a + m_d)\left(\frac{m_d v_{d_0}}{m_a + m_d}\right)^2 + 0 &= 0 + (m_a + m_d)gh_1 \\
h_1 &= \frac{m_d^2 v_0^2}{2(m_a + m_d)^2 g} \\
h_1 &= \frac{\left(\frac{M}{4}\right)^2 v_0^2}{2\left(M + \frac{M}{4}\right)^2 g} \\
h_1 &= \frac{v_0^2}{50g}
\end{aligned}$$

$$\boxed{\text{The height of the collided apple and dart reach } h_1 = \tfrac{v_0^2}{50g}.}$$

1.5 8.41



$$\begin{aligned}
m_{(c)ar} &= 950\,\mathrm{kg} \\
m_{(t)ruck} &= 1900\,\mathrm{kg} \\
v_1 &= 16.0\,\mathrm{m\,s^{-1}} \\
\theta &= 24.0\,^\circ\,\text{East of North} \\
v_{c_0} &=? \\
v_{t_0} &=?
\end{aligned}$$

Find conservation of momentum in $\hat{x}$ direction:

$$\begin{aligned}
v_{c_0} &= v_{c_0}\cos(0^\circ) = v_{c_0} \\
v_{t_0} &= v_{t_0}\cos(90.0^\circ) = 0 \\
v_{c_1} &= 16.0\,\mathrm{m\,s^{-1}}\sin(24.0^\circ) = 6.51\,\mathrm{m\,s^{-1}} \\
v_{t_1} &= 16.0\,\mathrm{m\,s^{-1}}\sin(24.0^\circ) = 6.51\,\mathrm{m\,s^{-1}}
\end{aligned}$$

$$
\begin{aligned}
P_{0_x} &= P_{1_x} \\
m_c v_{c_0} + m_t v_{t_0} &= m_c v_{c_1} + m_t v_{t_1} \\
m_c v_{c_0} + 0 &= v_1 \left(m_c + m_t\right) \\
v_{c_0} &= \frac{v_1 \left(m_c + m_t\right)}{m_c} \\
v_{c_0} &= \frac{(6.51\,\mathrm{m\,s^{-1}})(950\,\mathrm{kg} + 1900\,\mathrm{kg})}{950\,\mathrm{kg}} \\
v_{c_0} &= 19.53\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

Find conservation of momentum in $\hat{y}$ direction:

$$
\begin{aligned}
v_{c_0} &= v_{c_0} \cos(90.0^\circ) = 0 \\
v_{t_0} &= v_{t_0} \cos(0^\circ) = v_{t_0} \\
v_{c_1} &= 16.0\,\mathrm{m\,s^{-1}} \cos(24.0^\circ) = 14.6\,\mathrm{m\,s^{-1}} \\
v_{t_1} &= 16.0\,\mathrm{m\,s^{-1}} \cos(24.0^\circ) = 14.6\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
m_c v_{c_0} + m_t v_{t_0} &= m_c v_{c_1} + m_t v_{t_1} \\
0 + m_t v_{t_0} &= v_1 (m_c + m_t) \\
v_{t_0} &= \frac{v_1 (m_c + m_t)}{m_t} \\
v_{t_0} &= \frac{(14.6\,\mathrm{m\,s^{-1}})(950\,\mathrm{kg} + 1900\,\mathrm{kg})}{1900\,\mathrm{kg}} \\
v_{t_0} &= 21.9\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

The speed of the car before collision was $v_{c_0} = 19.53\,\mathrm{m\,s^{-1}}$, and the speed of the truck before collision was $v_{t_0} = 21.9\,\mathrm{m\,s^{-1}}$.

1.6 8.44

$-\hat{x} \longleftarrow\!\bullet\!\longrightarrow +\hat{x}$

$$
\begin{aligned}
m_{(b)lock} &= 15.0\,\mathrm{kg} \\
k &= 575.0\,\mathrm{N\,m^{-1}} \\
v_{b_0} &= 0 \\
m_{(s)tone} &= 3.00\,\mathrm{kg} \\
v_{s_0} &= 8.00\,\mathrm{m\,s^{-1}} \\
v_{s_1} &= -2.00\,\mathrm{m\,s^{-1}} \\
\Delta x_{b,max} &=?
\end{aligned}
$$

Find the velocity of the upon collision v_1:

$$\begin{aligned}
P_0 &= P_1 \\
m_s v_{s_0} + m_b v_{b_0} &= m_s v_{s_1} + m_b v_{b_1} \\
m_s v_{s_0} + 0 &= m_s v_{s_1} + m_b v_{b_1} \\
v_{b_1} &= \frac{m_s v_{s_0} - m_s v_{s_1}}{m_b} \\
v_{b_1} &= \frac{(3.00\,\mathrm{kg})(8.00\,\mathrm{m\,s^{-1}}) - (3.00\,\mathrm{kg})(-2.00\,\mathrm{m\,s^{-1}})}{15.0\,\mathrm{kg}} \\
v_{b_1} &= 2.00\,\mathrm{m\,s^{-1}}
\end{aligned}$$

Find the max distance using conservation of energy x_{max}:

$$\begin{aligned}
E_0 &= E_1 \\
\frac{1}{2} m_b v_0^2 &= \frac{1}{2} k x_{max}^2 \\
x_{max} &= \sqrt{\frac{m_b v_0^2}{k}} \\
x_{max} &= \sqrt{\frac{(15.0\,\mathrm{kg})(2.00\,\mathrm{m\,s^{-1}})^2}{575.0\,\mathrm{N\,m^{-1}}}} \\
x_{max} &= 0.323\,\mathrm{m}
\end{aligned}$$

$$\boxed{\text{The steel ball moves the block to a maximum of } x_{max} = 0.323\,\mathrm{m}.}$$

1.7 8.48

$-\hat{x} \longleftarrow\!\bullet\!\longrightarrow +\hat{x}$

$$\begin{aligned}
m_{(s)mall} &= 10.0\,\mathrm{g} \\
v_{s_0} &= -0.400\,\mathrm{m\,s^{-1}} \\
m_{(l)arge} &= 30.0\,\mathrm{g} \\
v_{l_0} &= 0.200\,\mathrm{m\,s^{-1}}
\end{aligned}$$

(a) Find the velocity of each marble after the collision.
Since the collision is perfectly elastic, the conservation of momentum and energy both apply. Evaluating each equation will result in two unknowns (the final velocities of the respective marble) which forms a system of equations that can be solved for a single variable, and substituted in the other. * I attempted to use an augmented matrix to solve the system of equations, but was immediately stumped as the conservation of energy equation's unknowns were both squared values which I am unsure of how

to handle.

$$
\begin{aligned}
P_0 &= P_1 \\
m_s v_{s_0} + m_l v_{l_0} &= m_s v_{s_1} + m_l v_{l_1} \\
(10.0\,\mathrm{g})(-0.400\,\mathrm{m\,s^{-1}}) + (30.0\,\mathrm{g})(0.200\,\mathrm{m\,s^{-1}}) &= (10.0\,\mathrm{g})v_{s_1} + (30.0\,\mathrm{g})v_{l_1} \\
(10.0\,\mathrm{g})v_{s_1} + (30.0\,\mathrm{g})v_{l_1} &= 2.00\,\mathrm{g\,m\,s^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
E_0 &= E_1 \\
\frac{1}{2}m_s v_{s_0}^2 + \frac{1}{2}m_l v_{l_0}^2 &= \frac{1}{2}m_s v_{s_1}^2 + \frac{1}{2}m_l v_{l_1}^2 \\
(10.0\,\mathrm{g})(-0.400\,\mathrm{m\,s^{-1}})^2 + (30.0\,\mathrm{g})(0.200\,\mathrm{m\,s^{-1}})^2 &= (10.0\,\mathrm{g})v_{s_1}^2 + (30.0\,\mathrm{g})v_{l_1}^2 \\
(10.0\,\mathrm{g})v_{s_1}^2 + (30.0\,\mathrm{g})v_{l_1}^2 &= 2.8\,\mathrm{g\,m\,s^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
(10.0\,\mathrm{g})v_{s_1} + (30.0\,\mathrm{g})v_{l_1} &= 2.00\,\mathrm{g\,m\,s^{-1}} \\
v_{s_1} &= (-3.00)v_{l_1} + 0.200\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
(10.0\,\mathrm{g})v_{s_1}^2 + (30.0\,\mathrm{g})v_{l_1}^2 &= 2.8\,\mathrm{g\,m\,s^{-1}} \\
(10.0\,\mathrm{g})((-3.00)v_{l_1} + 0.200\,\mathrm{m\,s^{-1}})^2 + (30.0\,\mathrm{g})v_{l_1}^2 &= 2.8\,\mathrm{g\,m\,s^{-1}} \\
(120.0\,\mathrm{g\,m\,s^{-1}})v_{l_1}^2 - (12.0\,\mathrm{g\,m\,s^{-1}})v_{l_1} - 2.40\,\mathrm{g\,m\,s^{-1}} &= 0 \\
v_{l_1} &= -0.100\,\mathrm{m\,s^{-1}}, 0.200\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

Since the collision is perfectly elastic, the correct value to use for v_{l_1} would be $-0.100\,\mathrm{m\,s^{-1}}$ as the large marble would have to move in the opposite direction $(-\hat{x})$ of it's initial velocity.

$$
\begin{aligned}
v_{s_1} &= (-3.00)v_{l_1} + 0.200\,\mathrm{m\,s^{-1}} \\
v_{s_1} &= (-3.00)(-0.100\,\mathrm{m\,s^{-1}}) + 0.200\,\mathrm{m\,s^{-1}} \\
v_{s_1} &= 0.500\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

Upon collision, the large marble will move $0.100\,\mathrm{m\,s^{-1}}$ to the left, while the small marble will move $0.500\,\mathrm{m\,s^{-1}}$ to the right.

(b) Calculate the *change in momentum* for each marble.

$$
\begin{aligned}
\Delta P_l &= P_{l_1} - P_{l_0} \\
\Delta P_l &= m_l v_{l_1} - m_l v_{l_0} \\
\Delta P_l &= (30.0\,\mathrm{g})(-0.100\,\mathrm{m\,s^{-1}}) - (30.0\,\mathrm{g})(0.200\,\mathrm{m\,s^{-1}}) \\
\Delta P_l &= -9.00\,\mathrm{g\,m\,s^{-1}}
\end{aligned}
$$

$$\Delta P_s = P_{s_1} - P_{s_0}$$
$$\Delta P_s = m_s v_{s_1} - m_s v_{s_0}$$
$$\Delta P_s = (10.0\,\mathrm{g})(0.500\,\mathrm{m\,s^{-1}}) - (10.0\,\mathrm{g})(-0.400\,\mathrm{m\,s^{-1}})$$
$$\Delta P_s = 9.00\,\mathrm{g\,m\,s^{-1}}$$

> The change in momentum energy for the large marble is $-9.00\,\mathrm{g\,m\,s^{-1}}$ and $9.00\,\mathrm{g\,m\,s^{-1}}$ for the small marble.

(c) Calculate the *change in kinetic energy* for each marble.

$$\Delta E_l = E_{l_1} - E_{l_0}$$
$$\Delta E_l = \frac{1}{2} m_l v_{l_1}^2 - \frac{1}{2} m_l v_{l_0}^2$$
$$\Delta E_l = \frac{1}{2}(30.0\,\mathrm{g})(-0.100\,\mathrm{m\,s^{-1}})^2 - \frac{1}{2}(30.0\,\mathrm{g})(0.200\,\mathrm{m\,s^{-1}})$$
$$\Delta E_l = -0.450\,\mathrm{g\,m^2\,s^{-2}}$$

$$\Delta E_s = E_{s_1} - E_{s_0}$$
$$\Delta E_s = \frac{1}{2} m_s v_{s_1}^2 - \frac{1}{2} m_s v_{s_0}^2$$
$$\Delta E_s = \frac{1}{2}(10.0\,\mathrm{g})(0.500\,\mathrm{m\,s^{-1}})^2 - \frac{1}{2}(10.0\,\mathrm{g})(-0.400\,\mathrm{m\,s^{-1}})^2$$
$$\Delta E_s = 0.450\,\mathrm{g\,m^2\,s^{-2}}$$

> The change in kinetic energy for the large marble is $-0.450\,\mathrm{g\,m^2\,s^{-2}}$ and $0.450\,\mathrm{g\,m^2\,s^{-2}}$ for the small marble.

1.8 8.62

$$m_{(f)uel} = 5.40 \times 10^{-2}\,\mathrm{kg\,s^{-1}}$$
$$v = 1550\,\mathrm{m\,s^{-1}}$$

(a) What is the thrust of the rocket?

$$\sum F_{(t)hrust} = \dot{m}\dot{v}$$
$$\sum F_t = (5.40 \times 10^{-2}\,\mathrm{kg\,s^{-1}})(1550\,\mathrm{m\,s^{-1}})$$
$$\sum F_t = 83.7\,\mathrm{N}$$

The thrust of the rocket is $83.7\,\mathrm{N}$.

(b) Would the rocket operate in outer space where there is no atmosphere? If so, how would you steer it? Could you brake it?

Based on Newton's Third Law (for every force there is an equal and opposite reaction), in order to "brake" (decelerate) the rocket, all that would be needed is for the rocket to expel fuel in the opposing direction (create an opposing force). The difference that the lack of atmosphere introduces is that the force of gravity from the Earth is much less potent requiring a greater amount of opposing force to decelerate the rocket. Similarly, directing the rocket to a different direction of motion would require that the fuel would be expended at an angle nonparallel to the direction of motion.

1.9 8.87

$-\hat{x} \longleftarrow\!\bullet\!\longrightarrow +\hat{x}$

$$
\begin{aligned}
m_{(c)art} &= 50.0\,\mathrm{kg} \\
v_{c_0} &= -5.00\,\mathrm{m\,s^{-1}} \\
m_{(p)ackage} &= 15.0\,\mathrm{kg} \\
\theta &= 37^\circ \\
v_{p_0} &= 3.00\,\mathrm{m\,s^{-1}} \\
h_0 &= 4.00\,\mathrm{m} \\
h_1 &= 0
\end{aligned}
$$

(a) What is the speed of the package just before it lands in the cart?

$$
\begin{aligned}
E_{p_0} &= E_{p_1} \\
\frac{1}{2}m_p v_{p_0}^2 + m_p g h_0 &= \frac{1}{2}m_p v_{p_1}^2 + m_p g h_1 \\
v_{p_0}^2 + 2gh_0 &= v_{p_1}^2 + 0 \\
v_{p_1} &= \sqrt{v_{p_0}^2 + 2gh_0} \\
v_{p_1} &= \sqrt{(3.00\,\mathrm{m\,s^{-1}})^2 + 2(10.0\,\mathrm{m\,s^{-2}})(4.00\,\mathrm{m})} \\
v_{p_1} &= 9.43\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

The speed of the package before landing is $9.43\,\mathrm{m\,s^{-1}}$.

(b) What is the final speed of the cart?

$$\begin{aligned}
P_{0_x} &= P_{1_x} \\
m_c v_{c_0} + m_p v_{p_0} &= v_1(m_c + m_p) \\
v_1 &= \frac{m_c v_{c_0} + m_p v_{p_0}}{m_c + m_p} \\
v_1 &= \frac{(50.0\,\mathrm{kg})(-5.00\,\mathrm{m\,s^{-1}}) + (15.0\,\mathrm{kg})(3.00\,\mathrm{m\,s^{-1}}\cos(37.0°))}{50.0\,\mathrm{kg} + 15.0\,\mathrm{kg}} \\
v_1 &= -3.29\,\mathrm{m\,s^{-1}}
\end{aligned}$$

The final velocity of the collision results in the collective cart and package moving at $3.29\,\mathrm{m\,s^{-1}}$ in the $-\hat{x}$ direction (left).

2 Lab Manual

2.1 972

(a) Find the velocity of each body after the collision, in terms of the masses and the velocities given.

$$\begin{aligned}
P_0 &= P_1 \\
m_A v_{A_0} + m_B v_{B_0} &= m_A v_{A_1} + m_B v_{B_1} \\
v_{A_1} &= \frac{m_A v_{A_0} + m_B v_{B_0} - m_B v_{B_1}}{m_A} \\
v_{B_1} &= \frac{m_A v_{A_0} + m_B v_{B_0} - m_A v_{A_1}}{m_B}
\end{aligned}$$

$$\begin{aligned}
E_0 &= E_1 \\
\frac{1}{2}m_A v_{A_0}^2 + \frac{1}{2}m_B v_{B_0}^2 &= \frac{1}{2}m_A v_{A_1}^2 + \frac{1}{2}m_B v_{B_1}^2 \\
v_{A_1} &= \sqrt{\frac{m_A v_{A_0}^2 + m_B v_{B_0}^2 - m_B v_{B_1}^2}{m_A}} \\
v_{B_1} &= \sqrt{\frac{m_A v_{A_0}^2 + m_B v_{B_0}^2 - m_A v_{A_1}^2}{m_B}}
\end{aligned}$$

$$\begin{aligned}
v_{A_1} &= \frac{m_A v_{A_0} + m_B v_{B_0} - m_B v_{B_1}}{m_A} \\
v_{A_1} &= \frac{m_A v_{A_0} + m_B v_{B_0} - m_B \left(\sqrt{\frac{m_A v_{A_0}^2 + m_B v_{B_0}^2 - m_A v_{A_1}^2}{m_B}}\right)}{m_A} \\
v_{A_1} &= \frac{v_{A_0}(m_A - m_B) + 2m_B v_{B_0}}{m_A + m_B}
\end{aligned}$$

$$v_{B_1} = \frac{m_A v_{A_0} + m_B v_{B_0} - m_A v_{A_1}}{m_B}$$

$$v_{B_1} = \frac{m_A v_{A_0} + m_B v_{B_0} - m_A \left(\frac{v_{A_0}(m_A - m_B) + 2m_B v_{B_0}}{m_A + m_B}\right)}{m_B}$$

$$v_{B_1} = \frac{v_{B_0}(m_B - m_A) + 2m_A v_{A_0}}{m_A + m_B}$$

The final velocities are

$$v_{A_1} = \frac{v_{A_0}(m_A - m_B) + 2m_B v_{B_0}}{m_A + m_B}$$
$$v_{B_1} = \frac{v_{B_0}(m_B - m_A) + 2m_A v_{A_0}}{m_A + m_B}$$

(b) For the special case in which B is at rest before collision, find the ratio

$$K = \frac{\text{Kinetic Energy of } B \text{ after collision}}{\text{Kinetic Energy of } A \text{ before collision}}$$

, in terms of m_A/m_B.

$$K = \frac{\frac{1}{2} m_B v_{B_1}^2}{\frac{1}{2} m_A v_{A_0}^2}$$

$$K = \frac{m_B \left(\frac{v_{B_0}(m_B - m_A) + 2m_A v_{A_0}}{m_A + m_B}\right)^2}{m_A v_{A_0}^2}$$

$$K = \frac{m_B \left(\frac{(0)(m_B - m_A) + 2m_A v_{A_0}}{m_A + m_B}\right)^2}{m_A v_{A_0}^2}$$

$$K = \frac{4 m_A m_B}{(m_A + m_B)^2}$$

When B is at rest before collision, the ratio of kinetic energy is $K = \frac{4m_A m_B}{(m_A + m_B)^2}$.

(c) Let r stand for the ratio m_A/m_B. Find the value of r that makes $K(r)$ a maximum. What does m_B have to be (in terms of m_A) for the maximum transfer of kinetic energy in the collision?

$$
\begin{aligned}
K &= \frac{4m_A m_B}{m_A^2 + 2m_A m_B + m_B^2} \\
K &= \frac{4m_A m_B}{m_B^2\left(\frac{m_A^2}{m_B^2} + \frac{2m_A}{m_B} + 1\right)} \\
K(r) &= \frac{4r}{r^2 + 2r + 1} \\
K' &= -\frac{4(r-1)}{(r+1)^3}
\end{aligned}
$$

Find when $K' = 0$:

$$
\begin{aligned}
-\frac{4(r-1)}{(r+1)^3} &= 0 \\
-4r + 4 &= 0 \\
r &= 1
\end{aligned}
$$

When $r = 1$, the function $K(r)$ is at its maximum.

2.2 975

$$-\hat{x} \longleftarrow \bullet \longrightarrow +\hat{x}$$

$$
\begin{aligned}
m_A &= 100\,\mathrm{g} \\
v_{A/E} &= 10\,\mathrm{cm\,s^{-1}} \\
m_B &= 50\,\mathrm{g} \\
v_{B/E} &= -8\,\mathrm{cm\,s^{-1}}
\end{aligned}
$$

(a) Find the velocity of the coupled cars by taking momenta relative to the earth.

$$
\begin{aligned}
P_0 &= P_1 \\
m_A v_{A/E} + m_B v_{B/E} &= v_{AB/E}(m_A + m_B) \\
v_{AB/E} &= \frac{m_A v_{A/E} + m_B v_{B/E}}{m_A + m_B} \\
v_{AB/E} &= \frac{(100\,\mathrm{g})(10\,\mathrm{cm\,s^{-1}}) + (50\,\mathrm{g})(-8\,\mathrm{cm\,s^{-1}})}{100\,\mathrm{g} + 50\,\mathrm{g}} \\
v_{AB/E} &= 4.00\,\mathrm{cm\,s^{-1}}
\end{aligned}
$$

(b) Find the velocity of the coupled cars by taking momenta relative to B

(initially).

$$\begin{aligned}
v_{AB/B} &= v_{AB/E} - v_{B/E} \\
v_{AB/B} &= 4.00\,\mathrm{cm\,s^{-1}} - (-8\,\mathrm{cm\,s^{-1}}) \\
v_{AB/B} &= 12.0\,\mathrm{cm\,s^{-1}}
\end{aligned}$$

(c) Show that the velocity of the center of mass of the system initially, relative to earth, is $4\,\mathrm{cm\,s^{-1}}$. Then find the velocity of the coupled cars by taking the momenta relative to the center of mass of the system. Why does this result hold for any isolated system, regardless of v's and m's?

$$\begin{aligned}
v_{CM} &= \sum_{i=1}^{n} \frac{p_i}{m_i} \\
v_{CM} &= \frac{m_A v_{A/E} + m_B v_{B/E}}{m_A + m_B} \\
v_{CM} &= \frac{(100\,\mathrm{g})(10\,\mathrm{cm\,s^{-1}}) + (50\,\mathrm{g})(-8\,\mathrm{cm\,s^{-1}})}{100\,\mathrm{g} + 50\,\mathrm{g}} \\
v_{CM} &= 4.00\,\mathrm{cm\,s^{-1}}
\end{aligned}$$

$$\begin{aligned}
P_0 &= P_1 \\
m_A v_{A/CM} + m_B v_{B/CM} &= v_{AB/CM}(m_A + m_B) \\
v_{AB/CM} &= \frac{m_A v_{A/CM} + m_B v_{B/CM}}{m_A + m_B} \\
v_{AB/CM} &= \frac{(100\,\mathrm{g})(4.00\,\mathrm{cm\,s^{-1}} - 10\,\mathrm{cm\,s^{-1}}) + (50\,\mathrm{g})(4.00\,\mathrm{cm\,s^{-1}} - (-8\,\mathrm{cm\,s^{-1}}))}{100\,\mathrm{g} + 50\,\mathrm{g}} \\
v_{AB/CM} &= 0
\end{aligned}$$

As there are no external forces that impact the system, momentum is conserved and is constant. This means that for any velocity vector, there is an opposing vector that will cancel it out and produce a total velocity of zero.

(d) What is the total K.E. of cars A and B before and after the collision, and the heart lost in the collision, calculated relative to the center of the earth?

$$\begin{aligned}
E_0 &= E_1 \\
\frac{1}{2} m_A v_{A/E}^2 + \frac{1}{2} m_B v_{B/E}^2 &= \frac{1}{2} v_{AB/E}^2 (m_A + m_B) \\
(100\,\mathrm{g})(10\,\mathrm{cm\,s^{-1}})^2 + (50\,\mathrm{g})(-8\,\mathrm{cm\,s^{-1}})^2 &= (4.00\,\mathrm{cm\,s^{-1}})^2(100\,\mathrm{g} + 50\,\mathrm{g}) \\
13\,200\,\mathrm{erg} &= 2400\,\mathrm{erg} \\
2(6600\,\mathrm{erg}) &= 2(1200\,\mathrm{erg})
\end{aligned}$$

$$\Delta E = |E_1 - E_0|$$
$$\Delta E = |1200\,\mathrm{erg} - 6600\,\mathrm{erg}|$$
$$\Delta E = 5400\,\mathrm{erg}$$

(e) What is the total K.E. of cars A and B before and after the collision, and the heat lost, calculated relative to the center of mass of the system?

$$E_{CM_0} = \frac{1}{2}m_A v_{A/CM}^2 + \frac{1}{2}m_B v_{B/CM}^2$$
$$E_{CM_0} = \frac{1}{2}(100\,\mathrm{g})(4.00\,\mathrm{cm\,s^{-1}} - 10\,\mathrm{cm\,s^{-1}})^2 + \frac{1}{2}(50\,\mathrm{g})(4.00\,\mathrm{cm\,s^{-1}} - (-8\,\mathrm{cm\,s^{-1}}))^2$$
$$E_{CM_0} = 5400\,\mathrm{erg}$$

$$E_{CM_1} = \frac{1}{2}v_{AB/CM}(m_A + m_B)$$
$$E_{CM_1} = 0$$

Energy lost to heat would be the difference; 5400 erg.

2.3 986

(a)

$$\theta = 30.0^\circ$$
$$\lambda = 0.50\,\mathrm{kg\,m^{-1}}$$
$$v_1 = 4.0\,\mathrm{m\,s^{-1}}$$
$$m = 9.0\,\mathrm{kg}$$
$$a_1 = ?$$

$$\sum F_x = \frac{dp}{dt}$$
$$mg\sin(\theta) = \frac{(m+dm)(v+dv) - mv}{dt}$$
$$mg\sin(\theta) = \frac{mv + vdm + mdv + dmdv - mv}{dt}$$
$$mg\sin(\theta) = v\dot{m} + m\dot{v}$$
$$a_x = g\sin(\theta) - \frac{\dot{m}v}{m}$$
$$a_x = (10\,\mathrm{m\,s^{-2}})(\sin(30.0^\circ)) - \frac{(0.50\,\mathrm{kg\,m^{-1}})(4.0\,\mathrm{m\,s^{-1}})}{9.0\,\mathrm{kg}}$$
$$\boxed{a_x = 4.78\,\mathrm{m\,s^{-2}}}$$

(b) (1)

$$\begin{aligned} m &= 100\,\mathrm{kg} \\ v_0 &= 6\,\mathrm{m\,s^{-1}} \\ m_1 &= 150\,\mathrm{kg} \\ v_1 &= ? \end{aligned}$$

$$\begin{aligned} \sum F_x &= \frac{dp}{dt} \\ \frac{m(v+dv)+dmv-m_0v}{dt} &= 0 \\ m_0dv + vdm &= 0 \\ -vdm &= m_0dv \\ -\int_0^m \frac{dm}{m_0} &= \int_{v_0}^{v} \frac{dv}{v} \\ -\frac{m}{m_0} &= \ln(v) - \ln(v_0) \\ \ln(v) &= -\frac{m}{m_0} + \ln(v_0) \\ \ln(v) &= -\frac{150\,\mathrm{kg}}{100\,\mathrm{kg}} + \ln(6\,\mathrm{m\,s^{-1}}) \\ v &= \frac{6}{e^{3/2}} \end{aligned}$$

(2)

$$\begin{aligned} -\int_0^m \frac{dm}{m_0} &= \int_{v_0}^{v} \frac{dv}{v} \\ -\frac{m}{m_0} &= \ln(v) - \ln(v_0) \\ -\frac{100\,\mathrm{kg}}{50\,\mathrm{kg}} &= \ln(v) - \ln(6\,\mathrm{m\,s^{-1}}) \\ v &= \frac{6}{e^2} \end{aligned}$$

(3) Because in order to comply with Newton's Third Law, the force the rain temporarily imposes upon the car causes it to decrease in speed.

3 Problem B

Consider a Tsiolkovsky Rocket in a gravitational field, g. At time $t = 0$, the velocity of the rocket is $v = v_0$, and the mass is $m = m_0$. Let the mass loss rate of the rocket be constant in time: $\dot{m} = -km_0$ [recall that a variable with a dot on top is the time derivative: $\dot{m} = \frac{dm}{dt}$, $\dot{v} = \frac{dv}{dt}$, etc.]

1. Show that the acceleration of the rocket is

$$a = \dot{v} = -\frac{u_{rel}}{m}\dot{m} - g$$

$$\begin{aligned}
F_y &= \frac{dp}{dt} \\
-g(m + dm) &= m\frac{dv}{dt} + u_{rel}\frac{dm}{dt} \\
m\frac{dv}{dt} + u_{rel}\frac{dm}{dt} + gm + gdm &= 0 \\
mdv + u_{rel}dm + gmdt + gdmdt &= 0, \quad dmdt = 0 \\
mdv + u_{rel}dm + gmdt &= 0 \\
m\frac{dv}{dt} + u_{rel}\frac{dm}{dt} + gm &= 0 \\
m\frac{dv}{dt} &= -u_{rel}\frac{dm}{dt} - gm \\
\dot{v} &= -\frac{u_{rel}}{m}\dot{m} - g
\end{aligned}$$

$$\boxed{a = \dot{v} = -\frac{u_{rel}}{m}\dot{m} - g}$$

2. Show that the mass as a function of time is

$$m = m_0(1 - kt)$$

$$\begin{aligned}
\dot{m} &= -km_0 \\
\int \dot{m}dt &= \int -km_0 dt \\
m(t) &= -ktm_0 + C \\
m(0) &= -k(0)m_0 + C = C = m_0, \quad \text{Given from prompt} \\
m &= -ktm_0 + m_0 = m_0(1 - kt)
\end{aligned}$$

$$\boxed{m = m_0(1 - kt)}$$

3. Show that acceleration can also be written as

$$a = \dot{v} = \frac{ku_{rel}}{1 - kt} - g$$

$$\begin{aligned}
a &= -\frac{u_{rel}}{m}\dot{m} - g \\
a &= -\frac{u_{rel}}{m_0(1 - kt)}(-km_0) - g \\
a &= \frac{ku_{rel}}{1 - kt} - g
\end{aligned}$$

$$\boxed{a = \frac{ku_{rel}}{1-kt} - g}$$

4. Show that the ΔV for a constant mass loss rate rocket is given by:

$$\Delta V = u_{rel} \ln\left[\frac{1}{1-kt}\right] - gt$$

$$\begin{aligned}
\int_0^t (a)dt &= \int_0^t \left(\frac{ku_{rel}}{1-kt} - g\right) dt \\
V|_0^t &= ku_{rel} \int_0^t \left(\frac{1}{1-kt}\right) dt - gt|_0^t \\
\Delta V &= ku_{rel} \left(\frac{\ln((kt-1)^{-1})}{k}\Bigg|_0^t\right) - gt \\
\Delta V &= u_{rel} \ln\left[\frac{1}{1-kt}\right] - gt
\end{aligned}$$

$$\boxed{\Delta V = u_{rel} \ln\left[\frac{1}{1-kt}\right] - gt}$$