

Chapter 1 - Vectors & Scalars

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1 Introduction to Vectors & Scalars

1.1 Vectors & Scalars

Scalar = Magnitude

Vector = Magnitude + Direction

Vectors are represented by arrows – tail and tip



Triangle addition:

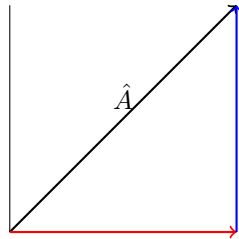
$$\hat{A} = 20 \text{ @ } 40^\circ \text{ above } +\hat{x}$$

$\hat{x} \rightarrow$ Unit vector gives direction with Magnitude 1 "the x direction"

Vector B is 45 @ 60 ° Right of $+\hat{y}$

1.2 Algebra as an alternative to Euclidean Geometry

In order to add vectors, first break vectors into components

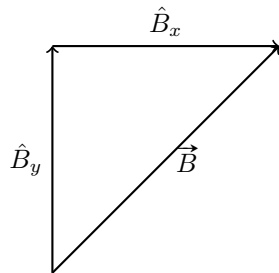


Any vector is the sum of its components

$$\hat{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

(1)

$$\hat{A} = A \cos(\theta) \hat{x} + A \sin(\theta) \hat{y} (+0\hat{z})$$



Write \vec{B} in component form:

$$\vec{B} = B \sin(\phi) \hat{x} + B \cos(\phi) \hat{y}$$

$$= (45) \sin(60^\circ) \hat{x} + (45) \cos(60^\circ) \hat{y}$$

$$\boxed{\vec{B} = 39\hat{x} + 23\hat{y}}$$

To add two vectors, add each direction separately:

$$\begin{aligned} 15\hat{x} + 13\hat{y} \\ 39\hat{x} + 23\hat{y} \\ \vec{A} + \vec{B} = 54\hat{x} + 36\hat{y} \end{aligned}$$

To subtract two vectors, subtract each direction separately:

$$\begin{aligned} 15\hat{x} + 13\hat{y} \\ 39\hat{x} + 23\hat{y} \\ \vec{A} - \vec{B} = -24\hat{x} - 10\hat{y} \end{aligned}$$

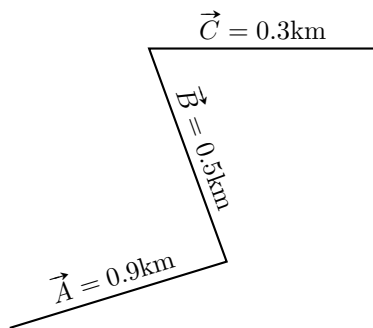
To compose (get magnitude and direction) of a vector, use:

$$\begin{aligned} |\vec{A}| &= \sqrt{A_x^2 + A_y^2 + A_z^2} \\ \theta_{\vec{A}} &= \arctan\left(\frac{A_i}{A_j}\right) \\ |\vec{A} + \vec{B}| &= \sqrt{(A+B)_x^2 + (A+B)_y^2} \\ &= \sqrt{54^2 + 36^2} = 65 \\ \theta &= \arctan\left(\frac{(A+B)_y}{(A+B)_x}\right) = \arctan\left(\frac{36}{54}\right) = 34^\circ \end{aligned}$$

1.3 Scalar Multiplication

$$\begin{aligned} \gamma \vec{A} &= \gamma A_x \hat{x} + \gamma A_y \hat{y} + \gamma A_z \hat{z} \\ -\vec{A} &= (-A_x) \hat{x} + (-A_y) \hat{y} + (-A_z) \hat{z} \end{aligned}$$

Given three vectors connected from head to tail, find the distance from the tail of \vec{A} to the head of \vec{C} :



$$\vec{C} = 0.3\text{km}\hat{x}$$

$$\begin{aligned}\vec{A} &= +A\cos(\theta)\hat{x} + A\sin(\theta)\hat{y} \\ &= (0.9\text{km})\cos(17^\circ)\hat{x} + (0.9\text{km})\sin(17^\circ)\hat{y}\end{aligned}$$

$$\vec{A} = 0.86\text{km}\hat{x} + 0.26\text{km}\hat{y}$$

$$\vec{B} = -B\sin(\phi)\hat{x} + B\cos(\phi)\hat{y}$$

$$\vec{B} = -0.17\text{km}\hat{x} + 0.47\text{km}\hat{y}$$

$$\vec{A} = 0.86\text{km}\hat{x} + 0.26\text{km}\hat{y}$$

$$\vec{B} = -0.17\text{km}\hat{x} + 0.47\text{km}\hat{y}$$

$$\vec{C} = 0.3\text{km}\hat{x} + 0\hat{y}$$

$$\boxed{\vec{A} + \vec{B} + \vec{C} = 0.99\text{km}\hat{x} + 0.73\text{km}\hat{y}}$$