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1 One More 1D Motion Problem

1.1 Finding Drag

$$F_{\text{drag}} = -bv \tag{1}$$

Find the equation of motion if:

$$\begin{aligned}\sum F &= ma \\ -v &= ma\end{aligned}$$

$$a = -\frac{b}{m}v \tag{2}$$

$$\begin{aligned}\frac{dv}{dt} &= -\frac{b}{m}v \\ \int_{v_0}^{v} \frac{dv}{v} &= \int_0^t -\frac{b}{m}dt \\ \ln\left(\frac{v}{v_0}\right) &= -\frac{b}{m}(t-0) \\ \frac{v}{v_0} &= e^{-\frac{b}{m}t} \\ v &= v_0 e^{-\frac{b}{m}t}\end{aligned}$$

$$v = \frac{dx}{dt} \tag{3}$$

$$\frac{dx}{dt} = v_0 e^{-\frac{b}{m}t}$$

$$\int dx = \int v_0 e^{-\frac{b}{m}t} dt$$

$$x + C = v_0 \int e^{-\frac{b}{m}t} dt$$

$$x + C = -\frac{v_0 m}{b e^{\frac{b}{m}}}, \quad \text{at} \quad t = 0; x = 0$$

$$C = -\frac{m v_0}{b}$$

$$\begin{aligned}
x + C &= -\frac{m v_0}{b} e^{-\frac{b}{m}t} \\
x + \left(-\frac{m v_0}{b}\right) &= -\frac{m v_0}{b} e^{-\frac{b}{m}t} \\
x &= \frac{m v_0}{b} \left(1 - e^{-\frac{b}{m}t}\right)
\end{aligned}$$

$$x = \frac{m v_0}{b} \left(1 - e^{-\frac{b}{m}t}\right) \tag{4}$$

2 2D (Projectile) Motion

Previously Known Kinematic Equations:

1. $$v = v_0 + at$$

2. $$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

3. $$x = x_0 + \frac{1}{2}(v + v_0)t$$

4. $$v^2 = v_0^2 + 2a\Delta x$$

5. $$x = x_0 + vt - \frac{1}{2} a t^2$$

Converted to 2D Kinematics:

1. $$\vec{v} = \vec{v}_0 + \vec{a}t$$

2. $$\vec{x} = \vec{x_0} + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

3. $$\vec{x} = \vec{x_0} + \frac{1}{2}(\vec{v} + \vec{v}_0)t$$

4. $$\vec{v}^2 = \vec{v_0}^2 + 2\vec{a} \cdot \Delta\vec{x}$$

5. $$\vec{x} = \vec{x_0} + \vec{v}t - \frac{1}{2}\vec{a}t^2$$

2.1 Assumptions

1. An object in free fall is subject only to gravity
$$a_x = 0$$
$$a_y = \pm g(down)$$

2. Vector components act completely independently

x (horizontal)	**y (vertical)**
$v_x = v_{0_x} = C$	$v_y = v_{0_y} + a_y t$
$x = x_0 + v_{0_x} t$	$y = y_0 + v_{0_y} t + \frac{1}{2} a_y t^2$
	$y = y_0 + \frac{1}{2}(v_{0_y} + v_y)t$
	$v_y^2 = v_{0_y}^2 + 2a\Delta y$
	$y = y_0 + v_y t - \frac{1}{2} a_y t^2$

3. Any object in free fall is a projectile

2.2 Homework Sketches

Required Elements

1. Any variable
2. Any location of interest should be indexed
3. Path of motion
4. Any vectors need arrows
5. Origin & positive directions

2.3 Example One

$$\theta = 30.0°$$
$$v_0 = 45\,\mathrm{m\,s^{-1}}$$
$$a_y = -10\,\mathrm{m\,s^{-2}}$$
$$x_0 = 0$$
$$y_0 = 0$$

Find total time off the ground t

$$\begin{aligned}
\cos(\theta) &= \frac{v_{0_x}}{v_0} \\
v_{0_x} &= (45\,\mathrm{m\,s^{-1}})(\cos(30.0°)) \\
v_{0_x} &= 38.97\,\mathrm{m\,s^{-1}}
\end{aligned}$$

$$\begin{aligned}
\sin(\theta) &= \frac{x_{0_y}}{v_0} \\
v_{0_y} &= (45\,\mathrm{m\,s^{-1}})(\sin(30.0°)) \\
v_{0_y} &= 22.5\,\mathrm{m\,s^{-1}}
\end{aligned}$$

$$\begin{aligned}
v_y &= v_{0_y} + a_y t_1 \\
t_1 &= \frac{v_{1_y} - v_{0_y}}{a_y} \\
t_1 &= \frac{-22.5\,\mathrm{m\,s^{-1}} - 22.5\,\mathrm{m\,s^{-1}}}{-10\,\mathrm{m\,s^{-2}}} \\
t_1 &= 4.5\,\mathrm{s}
\end{aligned}$$

$$\boxed{t_1 = 4.5\,\mathrm{s}}$$

Find total distance traveled x_1

$$\begin{aligned}
x_1 &= x_0 + v_{0_x} t \\
x_1 &= (0) + (39.0\,\mathrm{m\,s^{-1}})(4.5\,\mathrm{s}) \\
x_1 &= 176\,\mathrm{m}
\end{aligned}$$

$$\boxed{x_1 = 176\,\mathrm{m}}$$

Find the maximum height y_2

$$\begin{aligned}
v_{2_y}^2 &= v_{0_y}^2 + 2a_y(y_2 - y_0) \\
y_2 &= -\frac{v_{0_y}^2}{2a_y} \\
y_2 &= -\frac{(22.5\,\mathrm{m\,s^{-1}})^2}{2(-10\,\mathrm{m\,s^{-2}})} \\
y_2 &= 25.3\,\mathrm{m}
\end{aligned}$$

$$\boxed{y_2 = 25.3\,\mathrm{m}}$$

2.4 Example Two

$$\begin{aligned}\Delta x &= 88\,\mathrm{m}\\ t &= 4.0\,\mathrm{s}\\ \theta_0 &= 30^\circ\end{aligned}$$

Find v_0, v_{0_x}, v_{0_y}

$$\begin{aligned}x &= x_0 + v_{0_x} t\\ v_{0_x} &= \frac{x - x_0}{t}\\ &= \frac{88\,\mathrm{m} - 0}{4.0\,\mathrm{s}}\\ v_{0_x} &= 22\,\mathrm{m\,s^{-1}}\end{aligned}$$

$$\begin{aligned}\tan(\theta) &= \frac{v_{0_y}}{v_{0_x}}\\ v_{0_y} &= (v_{0_x})(\tan(\theta))\\ &= (22\,\mathrm{m\,s^{-1}})(\tan(30^\circ))\\ v_{0_y} &= 12.70\,\mathrm{m\,s^{-1}}\end{aligned}$$

$$\begin{aligned}v_0 &= \sqrt{v_{0_x}^2 + v_{0_y}^2}\\ &= \sqrt{(22\,\mathrm{m\,s^{-1}})^2 + (12.70\,\mathrm{m\,s^{-1}})^2}\\ v_0 &= 25.40\,\mathrm{m\,s^{-1}}\end{aligned}$$

$$\boxed{v_{0_x} = 22\,\mathrm{m\,s^{-1}}, v_{0_y} = 12.70\,\mathrm{m\,s^{-1}}, v_0 = 25.40\,\mathrm{m\,s^{-1}}}$$

Find y_0, y

$$\begin{aligned}v_y &= v_{0_y} + a_y t\\ v_y &= (12.70\,\mathrm{m\,s^{-1}}) + (-10\,\mathrm{m\,s^{-2}})(4.0\,\mathrm{s})\\ v_y &= -27.3\,\mathrm{m\,s^{-1}}\end{aligned}$$

$$\begin{aligned}y &= y_0 + v_y t - \frac{1}{2} a_y t^2\\ y &= (0) + (-27.3\,\mathrm{m\,s^{-1}})(4.0\,\mathrm{s}) - \frac{1}{2}(-10\,\mathrm{m\,s^{-2}})(4.0\,\mathrm{s})^2\\ y &= -29.2\,\mathrm{m}\end{aligned}$$

$$\boxed{y_0 = 0, y = -29.2\,\mathrm{m}}$$

Find time at maximum height

$$\begin{aligned} v_y &= v_{0_y} + a_y t \\ (0) &= 12.70\,\mathrm{m\,s^{-1}} + (-10\,\mathrm{m\,s^{-2}})t \\ t &= 1.27\,\mathrm{s} \end{aligned}$$

$$\boxed{t = 1.27\,\mathrm{s}}$$

Find maximum height

$$\begin{aligned} y &= y_0 + v_{0_y} t + \frac{1}{2} a_y t^2 \\ y &= (0) + (12.70\,\mathrm{m\,s^{-1}})(1.27\,\mathrm{s}) + \frac{1}{2}(-10\,\mathrm{m\,s^{-2}})(1.27\,\mathrm{s})^2 \\ y &= 8.065\,\mathrm{m} \end{aligned}$$

$$\boxed{y = 8.065\,\mathrm{m}}$$