Homework 10 Rotations

Corey Mostero - 2566652

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Contents

1 Book

1.1 10.22

$$
\begin{aligned}
r &= 8.00\,\mathrm{cm} \\
m &= 0.180\,\mathrm{kg} \\
v_0 &= 0 \\
\Delta y &= 75.0\,\mathrm{cm} \\
I &= mr^2
\end{aligned}
$$

(a)

$$
\begin{aligned}
E_{K_0} + E_{P_0} &= E_{K_1} + E_{P_1} \\
0 + mgh &= \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} m r^2 \omega^2 + 0 \\
mgh &= \omega^2 \left(\frac{1}{2} \left(m r^2 \right) + \frac{1}{2} m r^2 \right) \\
\omega &= \frac{\sqrt{gh}}{r} \\
\omega &= \frac{\sqrt{(10.0\,\mathrm{m\,s^{-2}})(0.75\,\mathrm{m})}}{0.08\,\mathrm{m}} \\
\omega &= 34.2\,\mathrm{rad\,s^{-1}}
\end{aligned}
$$

(b)

$$
\begin{aligned}
E_{k_0} + E_{p_0} &= E_{k_1} + E_{p_1} \\
0 + mgh &= \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} m v_{cm}^2 + 0 \\
mgh &= \frac{1}{2} (m r^2) \left(\frac{v_{cm}}{r} \right)^2 + \frac{1}{2} m v_{cm}^2 \\
v &= \sqrt{gh} \\
v &= \sqrt{(10.0\,\mathrm{m\,s^{-2}})(0.75\,\mathrm{m})} \\
v &= 2.74\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

1.2 10.26

$$
I_{cm} = \frac{2}{5} m r^2
$$

(a) Velocity for the first half of the bowl:

$$
\begin{aligned}
E_{K_0} + E_{P_0} &= E_{K_1} + E_{P_1} \\
0 + mgh &= \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} m v_{cm}^2 + 0 \\
mgh &= \frac{1}{2} \left(\frac{2}{5} m r^2 \right) \left(\frac{v_{cm}^2}{r^2} \right) + \frac{1}{2} m v_{cm}^2 \\
v_{cm} &= \sqrt{\frac{10gh}{7}}
\end{aligned}
$$

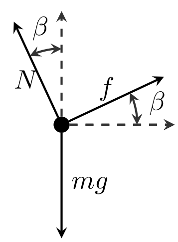
Since the ball only slides and doesn't rotate, the kinetic energy it experiences it purely linear velocity and *not* angular.

$$
\begin{aligned}
E_{K_0} + E_{P_0} &= E_{K_1} + E_{P_1} \\
\frac{1}{2}mv_{cm}^2 + 0 &= 0 + mgh_1 \\
\left(\sqrt{\frac{10gh_0}{7}}\right)^2 &= 2gh_1 \\
h_1 &= \frac{5}{7}h_0
\end{aligned}
$$

The ball reaches only $\frac{5}{7}$ of the height of the side of the bowl.

1.3 10.30

(a) Free-body diagram:



The angular velocity of the bowling ball is clockwise ↻ which the friction has to oppose resulting in the friction going upwards (up the incline).

(b) Normal force has no affect as it is directed towards the axis of rotation.

$$
\begin{aligned}
\sum F_x &= ma_{cm} \\
-f &= ma_{cm} + mg\sin(\beta) \\
-\frac{I_{cm}\alpha}{r} &= m(a_{cm} + g\sin(\beta)) \\
-\frac{\left(\frac{2}{5}mr^2\right)\left(\frac{a_{cm}}{r}\right)}{r} &= m(a_{cm} + g\sin(\beta)) \\
-\frac{2}{5}a_{cm} &= a_{cm} + g\sin(\beta) \\
a_{cm} &= \frac{5g\sin(\beta)}{7}
\end{aligned}
$$

(c)

$$
\begin{aligned}
\sum F_y &= 0 \\
N &= mg\cos(\beta)
\end{aligned}
$$

$$
\begin{aligned}
\sum F_x &= ma_{cm} \\
-f &= ma_{cm} + mg\sin(\beta) \\
-\mu(mg\cos(\beta)) &= m\left(\frac{5g\sin(\beta)}{7} + g\sin(\beta)\right) \\
\mu &= \frac{2\tan(\beta)}{7}
\end{aligned}
$$

1.4 10.79

$$
\begin{aligned}
I_{cylinder_{cm}} &= \frac{1}{2}m(2r)^2 \\
I_{disk_{cm}} &= \frac{1}{2}mr^2
\end{aligned}
$$

$$
\begin{aligned}
E_{K_0} + E_{P_0} &= E_{K_1} + E_{P_1} \\
\left(\frac{1}{2}I_{cylinder_{cm}}\left(\frac{v}{2r}\right)^2 + \frac{1}{2}mv^2\right) + \left(\frac{1}{2}I_{disk_{cm}}\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2\right) + 0 &= 0 + mgh \\
\left(\frac{1}{2}\left(\frac{1}{2}m(2r)^2\right)\left(\frac{v}{2r}\right)^2 + \frac{1}{2}mv^2\right) + \left(\frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2\right) &= mgh \\
v &= \sqrt{\frac{2gh}{3}}
\end{aligned}
$$

$$
\begin{aligned}
v_1^2 &= v_0^2 + 2ah \\
a &= \frac{v_1^2}{2h} \\
a &= \frac{\sqrt{\frac{2gh}{3}}^2}{2h} \\
a &= \frac{g}{3}
\end{aligned}
$$

1.5 9.30

$$
\begin{aligned}
m &= 0.200\,\mathrm{kg} \\
l &= 0.400\,\mathrm{m}
\end{aligned}
$$

(a)

$$
\begin{aligned}
r &= \sqrt{(l/2)^2 + (l/2)^2} \\
r &= 0.283\,\mathrm{m}
\end{aligned}
$$

$$
\begin{aligned}
I &= \sum_{i=1}^{4}(mr^2) \\
I &= 4(0.200\,\mathrm{kg})(0.283\,\mathrm{m})^2 \\
I &= 0.0641\,\mathrm{kg\,m^2} = 6.41\times10^{-2}\,\mathrm{kg\,m^2}
\end{aligned}
$$

(b)

$$r = 0.200\,\mathrm{m}$$

$$
\begin{aligned}
I &= \sum_{i=1}^{4}(mr^2) \\
I &= 4(0.200\,\mathrm{kg})(0.200\,\mathrm{m})^2 \\
I &= 0.320\,\mathrm{kg\,m^2}
\end{aligned}
$$

(c)

$$r = 0.283\,\mathrm{m}$$

$$
\begin{aligned}
I &= \sum_{i=0}^{2}(mr^2) \\
I &= 2(0.200\,\mathrm{kg})(0.283\,\mathrm{m})^2 \\
I &= 0.320\,\mathrm{kg\,m^2}
\end{aligned}
$$

1.6 9.49

$$
\begin{aligned}
r &= 0.280\,\mathrm{m} \\
m_{block} &= 4.20\,\mathrm{kg} \\
v_0 &= 0 \\
\Delta y &= 3.00\,\mathrm{m} \\
t &= 2.00\,\mathrm{s} \\
I_{wheel} &= \frac{1}{2}mr^2 \\
m_{wheel} &= ?
\end{aligned}
$$

Method 1: Using Force Equations + Torque

$$
\begin{aligned}
\sum F_y^{block} &= m_{block}a \\
m_{block}g &= T + m_{block}a \\
T &= m_{block}(g-a)
\end{aligned}
$$

$$
\begin{aligned}
\sum F^{wheel} &= m_{wheel}\alpha \\
\tau &= I_{wheel}\alpha \\
Tr &= \left(\frac{1}{2}m_{wheel}r^2\right)\left(\frac{a}{r}\right) \\
T &= \frac{m_{wheel}a}{2}
\end{aligned}
$$

$$
\begin{aligned}
a &= \frac{v_1 - v_0}{t_1 - t_0} \\
a &= \frac{3.00\,\mathrm{m\,s^{-1}} - 0}{2.00\,\mathrm{s} - 0} \\
a &= 1.50\,\mathrm{m\,s^{-2}}
\end{aligned}
$$

$$
\begin{aligned}
m_{block}(g-a) &= \frac{m_{wheel}a}{2} \\
m_{wheel} &= \frac{2m_{block}(g-a)}{a} \\
m_{wheel} &= \frac{2(4.20\,\mathrm{kg})(10.0\,\mathrm{m\,s^{-2}} - 1.50\,\mathrm{m\,s^{-2}})}{1.50\,\mathrm{m\,s^{-2}}} \\
m_{wheel} &= 47.6\,\mathrm{kg}
\end{aligned}
$$

Method 2: Using Energy

$$
\begin{aligned}
\Delta y &= \frac{1}{2}(v_f + v_o)t \\
v_f &= \frac{2\Delta y}{t} \\
v_f &= \frac{2(3.00\,\mathrm{m})}{2.00\,\mathrm{s}} \\
v_f &= 3.00\,\mathrm{m\,s^{-1}} \\
\omega &= \frac{v_f}{r} \\
\omega &= \frac{3.00\,\mathrm{m\,s^{-1}}}{0.280\,\mathrm{m}} \\
\omega &= 10.7\,\mathrm{rad\,s^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
E_{K_0} + E_{P_0} &= E_{K_1} + E_{P_1} \\
0 + E_{P_{block}} &= E_{R_{wheel}} + E_{K_{block}} \\
m_{block} g \Delta y &= \frac{1}{2} I_{wheel} \omega_{wheel}^2 + \frac{1}{2} m_{block} v_{block}^2 \\
m_{block} g \Delta y &= \frac{1}{2} \left(\frac{1}{2} m_{wheel} r^2 \right) \omega_{wheel}^2 + \frac{1}{2} m_{block} v_{block}^2 \\
m_{block} \left(4 g \Delta y - 2 v_{block}^2 \right) &= m_{wheel} r^2 \omega_{wheel}^2 \\
m_{wheel} &= \frac{m_{block} \left(4 g \Delta y - 2 v_{block}^2 \right)}{r^2 \omega_{wheel}^2} \\
m_{wheel} &= \frac{(4.20\,\mathrm{kg}) \left(4 (10.0\,\mathrm{m\,s^{-2}})(3.00\,\mathrm{m}) - 2 (3.00\,\mathrm{m\,s^{-1}})^2 \right)}{(0.280\,\mathrm{m})^2 (10.7\,\mathrm{rad\,s^{-1}})^2} \\
m_{wheel} &= 47.7\,\mathrm{kg}
\end{aligned}
$$

1.7 9.79

$$
\begin{aligned}
r_1 &= 2.55\,\mathrm{cm} = 0.0255\,\mathrm{m} \\
m_1 &= 0.85\,\mathrm{kg} \\
r_2 &= 5.02\,\mathrm{cm} = 0.0502\,\mathrm{m} \\
m_2 &= 1.58\,\mathrm{kg}
\end{aligned}
$$

(a)

$$
\begin{aligned}
I &= \sum_{i=1}^{2} I_i \\
I &= \frac{1}{2} (m_1 r_1^2 + m_2 r_2^2) \\
I &= \frac{1}{2} ((0.85\,\mathrm{kg})(0.0255\,\mathrm{m})^2 + (1.58\,\mathrm{kg})(0.0502\,\mathrm{m})^2) \\
I &= 0.002\,27\,\mathrm{kg\,m^2} = 2.27 \times 10^{-3}\,\mathrm{kg\,m^2}
\end{aligned}
$$

(b)

$$
\begin{aligned}
v_{block_0} &= 0 \\
y_1 &= 2.03\,\mathrm{m} \\
y_0 &= 0 \\
v_{block_1} &= ?
\end{aligned}
$$

$$\begin{aligned}
E_{K_0} + E_{P_0} &= E_{K_1} + E_{P_1} \\
m_{block} g \Delta y + 0 &= \frac{1}{2} I \left(\frac{v_{block_1}}{r_1} \right)^2 + \frac{1}{2} m_{block} v_{block_1}^2 + 0 \\
2g\Delta y &= v_{block_1}^2 \left(\frac{I}{m_{block}(r_1)^2} + 1 \right) \\
v_{block_1} &= \sqrt{\frac{2g\Delta y}{\frac{I}{m_{block}(r_1)^2} + 1}} \\
v_{block_1} &= \sqrt{\frac{2(10.0\,\mathrm{m\,s^{-2}})(2.03\,\mathrm{m})}{\frac{0.002\,77\,\mathrm{kg\,m^2}}{(1.50\,\mathrm{kg})(0.0255\,\mathrm{m})^2} + 1}} \\
v_{block_1} &= 3.25\,\mathrm{m\,s^{-1}}
\end{aligned}$$

(c)

$$\begin{aligned}
v_{block_1} &= \sqrt{\frac{2g\Delta y}{\frac{I}{m_{block}(r_2)^2} + 1}} \\
v_{block_1} &= \sqrt{\frac{2(10.0\,\mathrm{m\,s^{-2}})(2.03\,\mathrm{m})}{\frac{0.002\,77\,\mathrm{kg\,m^2}}{(1.50\,\mathrm{kg})(0.0502\,\mathrm{m})^2} + 1}} \\
v_{block_1} &= 4.84\,\mathrm{m\,s^{-1}}
\end{aligned}$$

The final velocity of the block is greater when the string is wrapped around the higher radius metal disk due to how linear velocity increases with longer lengths. Velocity can be calculated from radius and ω using the formula $v = r\omega$ proving that a greater radius will result in a greater velocity.

1.8 9.86

$$\begin{aligned}
P &= 5 \times 10^{31}\,\mathrm{W} \\
T &= 0.0331\,\mathrm{s} \\
\Delta t &= 4.22 \times 10^{-13}\,\mathrm{s}
\end{aligned}$$

(a)

$$\begin{aligned}
E &= \frac{1}{2}I\omega^2 \\
E &= \frac{1}{2}I\left(\frac{4\pi^2}{T^2}\right) \\
\frac{d}{dt}(E) &= \frac{d}{dt}\left(\frac{2I\pi^2}{T^2}\right) \\
P &= -\frac{4I\pi^2\Delta t}{T^3} \\
I &= -\frac{PT^3}{4\pi^2\Delta t} \\
I &= -\frac{(5\times10^{31}\,\mathrm{W})(0.0331\,\mathrm{s})^3}{4\pi^2(4.22\times10^{-13}\,\mathrm{s})} \\
I &= 1.09\times10^{38}\,\mathrm{kg\,m^2}
\end{aligned}$$

(b)

$$m_{sun} = 1.9891\times10^{30}\,\mathrm{kg}$$

$$\begin{aligned}
I &= \frac{2}{5}mr^2 \\
r &= \sqrt{\frac{5I}{2m}} \\
r &= \sqrt{\frac{5(1.09\times10^{38}\,\mathrm{kg\,m^2})}{2(1.4)(1.9891\times10^{30}\,\mathrm{kg})}} \\
r &= 9892.16\,\mathrm{m} = 9.89\times10^3\,\mathrm{m}
\end{aligned}$$

(c)

$$\begin{aligned}
v &= \frac{2\pi r}{T} \\
v &= \frac{2\pi(9892.16\,\mathrm{m})}{0.0331\,\mathrm{s}} \\
v &= 1.88\times10^6\,\mathrm{m\,s^{-1}} = 1\,877\,772\,\mathrm{m\,s^{-1}} < c = 299\,792\,458\,\mathrm{m\,s^{-1}}
\end{aligned}$$

(d)

$$\begin{aligned}
\rho_{rock} &= 3000\,\mathrm{kg\,m^{-3}} \\
\rho_{nucleus} &= 10\times10^{17}\,\mathrm{kg\,m^{-3}}
\end{aligned}$$

$$\rho_{\text{neutron star}} = \frac{m}{\frac{4\pi r^3}{3}}$$
$$\rho_{\text{neutron star}} = \frac{(1.4)(1.99 \times 10^{30}\,\text{kg})}{\frac{4\pi(9.89\times10^3\,\text{m})^3}{3}}$$
$$\rho_{\text{neutron star}} = 6.88 \times 10^{17}\,\text{kg}\,\text{m}^{-3}$$

The density of a neutron star is much closer to the density of an atomic nuclear when compared with the density of an ordinary rock.

2 Lab Manual

2.1 1170

$$I = \sum_{i=1}^{\infty}\left[\left(\frac{M}{n}\right)\left(i\frac{a}{n}\right)^2\right]$$
$$I = \frac{Ma^2}{n^3}\sum_{i=1}^{\infty}\left[i^2\right]$$
$$I = \frac{Ma^2}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$
$$I = \frac{Ma^2}{6}\left(\frac{n(n+1)(2n+1)}{n^3}\right)$$

$$I = \lim_{n\to\infty}(I)$$
$$I = \frac{Ma^2}{6}\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\cdots\right]$$
$$I = \frac{Ma^2}{6}(3)$$
$$I = \frac{Ma^2}{3}$$

2.2 1173

(a)

$$A_{small} = \pi R^2$$
$$A = \pi\left(\frac{R}{2}\right)^2$$
$$A = \frac{\pi R^2}{4}$$
$$m_{small} = \frac{m_{disk}}{4}$$

$$
\begin{aligned}
I_{small} &= \frac{1}{2} m_{small} \left(\frac{R}{2}\right)^2 + m_{small} \left(\frac{R}{2}\right)^2 \\
I_{small} &= \frac{3}{32} m_{disk} R^2
\end{aligned}
$$

$$
\begin{aligned}
I_{disk} - I_{small} &= I_{crescent} \\
\frac{1}{2} m_{disk} R^2 - \frac{3}{32} m_{disk} R^2 &= I_{crescent} \\
I_{crescent} &= \frac{13}{32} m_{disk} R^2
\end{aligned}
$$

(b)

$$
\begin{aligned}
I &= I_{disk} - I_{small} \\
I &= \frac{1}{2} m_{disk} r^2 - \left(\frac{1}{2} m_{small} \left(\frac{r}{2}\right)^2 + m_{small} \left(\frac{r}{2}\right)^2\right) \\
I &= \frac{1}{2} \left(\frac{4M}{3}\right) r^2 - \left(\frac{1}{2} \left(\frac{M}{3}\right) \left(\frac{r}{2}\right)^2 + \left(\frac{M}{3}\right) \left(\frac{r}{2}\right)^2\right) \\
I &= \frac{13Mr^2}{24}
\end{aligned}
$$

2.3 1175

$$
\begin{aligned}
r = r_{1,2} &= 10\,\mathrm{cm} = 0.100\,\mathrm{m} \\
m = m_{1,2} &= 1000\,\mathrm{g} = 1.00\,\mathrm{kg} \\
l &= 30\,\mathrm{cm} = 0.300\,\mathrm{m}
\end{aligned}
$$

(a)

$$
\begin{aligned}
I &= I_1 + I_{rod} + I_2 \\
I &= \left(\frac{2}{5} m r^2\right) + 0 + \left(\frac{2}{5} m r^2 + m l^2\right) \\
I &= \left(\frac{2}{5} (1.00\,\mathrm{kg})(0.100\,\mathrm{m})^2\right) + \left(\frac{2}{5} (1.00\,\mathrm{kg})(0.100\,\mathrm{m})^2 + (1.00\,\mathrm{kg})(0.300\,\mathrm{m})^2\right) \\
I &= 0.0980\,\mathrm{kg\,m^2}
\end{aligned}
$$

(b)

$$
\tau = 98\,000\,\mathrm{dyn\,cm} = 0.009\,80\,\mathrm{N\,m}
$$

$$\tau = I\alpha$$
$$\tau = I\left(\frac{\omega}{\frac{2\pi}{\omega}}\right)$$
$$\omega = \sqrt{\frac{2\tau\pi}{I}}$$
$$\omega = \sqrt{\frac{2(0.009\,80\,\mathrm{N\,m})\pi}{0.0980\,\mathrm{kg\,m^2}}}$$
$$\omega = 0.793\,\mathrm{rad\,s^{-1}}$$

2.4 1177

2.5 1181

2.6 1283

2.7 1284

3 Problem C: Spherical Symmetry Problem

Starting with $I = \int r^2 dm$, calculate the moment of inertial for an axis of rotation that goes through the center of a sphere with uniform mass density ρ, and radius R. As discussed in class, you may treat this problem like the integration of a series of concentric spherical shells with thickness dr.