Contents

1 Circular Motion

1.1 Roller coaster Example

$$R = 15\,\mathrm{m}$$
$$m = 2000\,\mathrm{kg}$$

Find v_B in terms of R:

$$\begin{aligned}\sum F_c &= ma_c \\ N_t + mg &= ma_c \\ g &= \frac{v_T^2}{R} \\ v_T &= \sqrt{gR}\end{aligned}$$

$$\begin{aligned}E_T &= E_B \\ \frac{1}{2}mv_T^2 + mgh_T &= \frac{1}{2}mv_B^2 \\ v_B^2 &= v_T^2 + 2gh_T \\ v_B &= \sqrt{(gR) + 2g(2R)} = \sqrt{5gR}\end{aligned}$$

Find N_B

$$\begin{aligned}\sum F &= ma_c \\ N_B - mg &= m\frac{v_B^2}{R} \\ N_B &= mg + m\frac{(5gR)}{R} \\ N_B &= 6mg\end{aligned}$$

2 Angular Derivations

$$S = R\theta$$
$$v = R\omega$$
$$a = R\alpha$$

$$\Delta x = v_0 t + \frac{1}{2}at^2$$
$$(R(\Delta\theta)) = (R\omega_0)t + \frac{1}{2}(R\alpha)t^2$$
$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

2.1 Angular Kinematics

$$\omega = \omega_0 + \alpha t$$
$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\Delta\theta = \frac{1}{2}(\omega + \omega_0)t$$
$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$
$$\Delta\theta = \omega t - \frac{1}{2}\alpha t^2$$

$$\Delta x \to \theta$$
$$v \to \omega$$
$$a \to \alpha$$
$$v = \frac{dx}{dt} \to \omega = \frac{d\theta}{dt}$$

$$F \to \tau$$
$$m^! = I^@$$

- ! - Resistance to change in v
- @ - Resistance to change in ω

$$KE_T = \frac{1}{2}mv^2 \to KE_R = \frac{1}{2}I\omega^2$$
$$\mathbf{P} = m\mathbf{v} \to \mathbf{L}^! = I\omega$$
$$\mathbf{P} = \frac{d\mathbf{p}}{dt} \to \tau = \frac{d\mathbf{L}}{dt}$$

- ! - Angular Momentum

$$
\begin{aligned}
\sum \mathbf{F}_i &= dm\mathbf{a} \\
\mathbf{r} \times \sum F_i &= dm(\mathbf{r} \times \mathbf{a}) \\
\sum \mathbf{r} \times \mathbf{F}_i &= ra_\perp dm \\
\tau_i &= r^2 \alpha dm \\
\sum \tau &= \int (r^2 dm)\alpha \\
\sum \tau_{ext} + \sum \tau_i &= I\alpha, \quad \sum \tau_i = 0 \\
\sum \tau_{ext} &= I\alpha
\end{aligned}
$$

3 Moments of Inertia

- Volume Density $\rightarrow \rho = \frac{dm}{dV}$
- Surface Density $\rightarrow \sigma = \frac{dm}{dA}, \quad A = \text{Area}$
- Linear Density $\rightarrow \lambda = \frac{dm}{dx}$

$$
\begin{aligned}
dm &= \rho dv = \rho(h(2\pi r)dr) \\
I_{cylinder} &= \int_0^R r^2 dm \\
I &= 2\pi\rho h \int_0^R r^3 dr \\
I &= \frac{1}{2}\pi\rho h r^4 \Big|_0^R \\
I &= \frac{\pi\rho h R^4}{2} \\
\rho &= \frac{M}{\pi R^2 h} \\
I &= \frac{1}{2}\pi \left(\frac{M}{\pi R^2 h}\right) R^4 \\
I_{cylinder} &= \frac{1}{2} M R^2
\end{aligned}
$$

$$I = \int x^2 dm$$

$$I = \int_0^L x^2 \lambda dx$$

$$I = \frac{1}{3} L^3 \lambda$$

$$\lambda = \frac{M}{L}$$

$$I = \frac{1}{3} L^3 \left(\frac{M}{L} \right)$$

$$I = \frac{1}{3} M L^2$$

$$I_{cylinder} = I_{disk} = \frac{1}{2} M R^2$$

$$I_{hoop} = M R^2$$

$$I_{sphere} = \frac{2}{5} M R^2$$

$$I_{spherialshell} = \frac{2}{3} M R^2$$