Contents

1 Numerical Approximation: Euler's Method

Given the initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0,$$

Euler's method with step size h consists of applying the iterative formula

$$y_{n+1} = y_n + h \cdot f(x_n, y_n) \quad (m \geq 0)$$

1.1 Example 1

Apply Euler's method to approximate the solution of the initial value problem

(a) first with step size $h = 1$ on the interval $[0, 5]$,

(b) then with the step size $h = 0.2$ on the interval $[0, 1]$,

$$\frac{dy}{dx} = x + \frac{1}{5}y, \quad y(0) = -3$$

(a)

$$\begin{aligned} x_0 &= 0 \\ y_0 &= -3 \\ f(x, y) &= x + \frac{1}{5}y \\ h &= 1 \end{aligned}$$

$$\begin{aligned} y_1 = y_0 + h \cdot \left[x_0 + \frac{1}{5}y_0\right] &= (-3) + (1)\left[0 + \frac{1}{5}(-3)\right] = -3.6 \\ y_2 = y_1 + h \cdot \left[x_1 + \frac{1}{5}y_1\right] &= (-3.6) + (1)\left[1 + \frac{1}{5}(-3.6)\right] = -3.32 \\ y_3 &= (-3.32) + (1)\left[2 + \frac{1}{5}(-3.32)\right] = -1.984 \\ y_4 &= (-1.984) + (1)\left[3 + \frac{1}{5}(-1.984)\right] = 0.6192 \\ y_5 &= (0.6912) + (1)\left[4 + \frac{1}{5}(0.6912)\right] \approx 4.7430 \end{aligned}$$

(b)

$$
\begin{aligned}
x_0 &= 0 \\
y_0 &= -3 \\
f(x,y) &= x + \frac{1}{5}y \\
h &= 0.2
\end{aligned}
$$

$$
\begin{aligned}
y_1 = y_0 + h \cdot \left[x_0 + \frac{1}{5}y_0\right] &= (-3) + (0.2)\left[0 + \frac{1}{5}(-3)\right] = -3.12 \\
y_2 &= (-3.12) + (0.2)\left[0.2 + \frac{1}{5}(-3.12)\right] \approx -3.205 \\
y_3 &\approx (-3.205) + (0.2)\left[0.4 + \frac{1}{5}(-3.205)\right] \approx -3.253 \\
y_4 &\approx (-3.253) + (0.2)\left[0.6 + \frac{1}{5}(-3.253)\right] \approx -3.263 \\
y_5 &\approx (-3.263) + (0.2)\left[0.8 + \frac{1}{5}(-3.263)\right] \approx -3.234
\end{aligned}
$$