Contents

1 Section 6.1

1.1 6.1.1

Find the (real) eigenvalues and associated eigenvectors of the larger matrix $\boldsymbol{A}$. Find a basis of each eigenspace of dimension 2 or larger.

$$\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$$

$$\begin{aligned} |\boldsymbol{A} - \lambda \mathbf{I}| &= \begin{bmatrix} 4-\lambda & -1 \\ 2 & 1-\lambda \end{bmatrix} \\ \det(\boldsymbol{A}) &= (4-\lambda)(1-\lambda) - (-1)(2) \\ \det(\boldsymbol{A}) &= \lambda^2 - 5\lambda + 2 = (\lambda - 2)(\lambda - 3) \\ \lambda &= 2, 3 \end{aligned}$$

$$\begin{aligned} \lambda &= 2 \\ \boldsymbol{A} - \lambda \mathbf{I} &= \begin{bmatrix} 4-\lambda & -1 \\ 2 & 1-\lambda \end{bmatrix} \\ \boldsymbol{A} - \lambda \mathbf{I} &= \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} (\boldsymbol{A} - \lambda \mathbf{I})\mathbf{v} &= 0 \\ \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ 2x - y &= 0 \\ x &= \frac{1}{2}y \\ 2\left(\frac{1}{2}y\right) - y &= 0 \\ 0 &= 0 \end{aligned}$$

$$\lambda = 2 \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2}y \\ y \end{bmatrix} = y \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}$$

$$
\begin{aligned}
\lambda &= 3 \\
\boldsymbol{A} - \lambda\mathbf{I} &= \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \\
(\boldsymbol{A} - \lambda\mathbf{I})\mathbf{v} &= 0 \\
\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
x &= y \\
2x &= 2y \\
2(y) &= 2y \\
0 &= 0
\end{aligned}
$$

$$
\lambda = 3 \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ y \end{bmatrix} = y \begin{bmatrix} 1 \\ 1 \end{bmatrix}
$$