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## 1 Chapter 02 - Basic Structures

### 1.1 Sets

$\in$ : belong to, is in

#### 1.1.1 Definition 2

Two sets are equal if and only if they have the same elements. Therefore, if  $A$  and  $B$  are sets, then  $A$  and  $B$  are equal if and only if  $\forall x (x \in A \iff x \in B)$ . We write  $A = B$  if  $A$  and  $B$  are equal sets.

#### 1.1.2 Definition 3

The set  $A$  is also a subset of  $B$ , and  $B$  is a superset of  $A$ , if and only if every element of  $A$  is also an element of  $B$ . We use the notation  $A \subseteq B$  to indicate that  $A$  is a subset of the set  $B$ . If, instead, we want to stress that  $B$  is a superset of  $A$ , we use the equivalent notation  $B \supseteq A$ . (So,  $A \subseteq B$  and  $B \supseteq A$  are equivalent statements.)

### 1.1.3 Definition 4

Let  $S$  be a set. If there are exactly  $n$  distinct elements in  $S$  where  $n$  is a nonnegative integer, we say that  $S$  is a finite set and that  $n$  is the cardinality of  $S$ . The cardinality of  $S$  is denoted by  $|S|$ .

### 1.1.4 Countable and Uncountable Sets

- Countable
  - $\mathbb{N}$
  - $\mathbb{Z}$
  - $\mathbb{Q}$
- Uncountable
  - $\mathbb{R}$
  - $\mathbb{C}$

Let  $S_0 = \{x\}$ , and  $S_1 = \{\{x\}\}$ .

$$S_0 \neq S_1 \tag{1}$$

### 1.1.5 Example

1. List the members of these sets.

a)  $\{x \mid x \text{ is a real number such that } x^2 = 1\}$

$$S = \{x \in \mathbb{R} \mid x^2 = 1\}$$

b)  $\{x \mid x \text{ is a positive integer less than } 12\}$

$$S = \{x \in \mathbb{R} \mid 0 \leq x < 12\}$$

### 1.1.6 Definition 6

Given a set  $S$ , the power set of  $S$  is the set of all subsets of the set  $S$ . The power set of  $S$  is denoted by  $\mathcal{P}(S)$ .

## 1.2 Set Operations

### 1.2.1 Definition 1

Let  $A$  and  $B$  be sets. The union of the sets  $A$  and  $B$ , denoted by  $A \cup B$ , is the set that contains those elements that are either in  $A$  or in  $B$ , or in both.

$$A \cup B = \{x \in U \mid (x \in A) \vee (x \in B)\} \tag{2}$$

### 1.2.2 Definition 2

Let  $A$  and  $B$  be sets. The intersection of the sets  $A$  and  $B$ , denoted by  $A \cap B$ , is the set containing those elements in both  $A$  and  $B$ .

$$A \cap B = \{x \in U \mid (x \in A) \wedge (x \in B)\} \quad (3)$$

### 1.2.3 Definition 3

Two sets are disjoint if their intersection is the empty set.

### 1.2.4 Definition 4

Let  $A$  and  $B$  be sets. The difference of  $A$  and  $B$ , denoted by  $A - B$ , is the set containing those elements that are in  $A$  but not in  $B$ . The difference of  $A$  and  $B$  is also called the complement of  $B$  with respect to  $A$ .

### 1.2.5 Definition 5

Let  $U$  be the universal set. The complement of the set  $A$ , denoted by  $\bar{A}$ , is the complement of  $A$  with respect to  $U$ . Therefore, the complement of the set  $A$  is  $U - A$ .

### 1.2.6 Proof

Let  $A, B$  be sets from  $U$ . Show that  $A \subseteq B$  if and only if  $\bar{B} \subseteq \bar{A}$ .

**Proof:**

- For “ $\implies$ ”

Given  $A \subseteq B$ , need to show  $\bar{B} \subseteq \bar{A}$ . Then  $\forall x \in \bar{B}$ , we have  $x \in \bar{A}$ .

By contrapositive we have

$$\begin{aligned} \neg(x \in B) &\implies \neg(x \in A) \\ x \notin B &\implies x \notin A, \quad \bar{B} \subseteq \bar{A} \end{aligned}$$

- For “ $\impliedby$ ”

Given  $\bar{B} \subseteq \bar{A}$ , we have  $\forall y \in \bar{B}, y \in \bar{A}$  then the contrapositive is

$$\begin{aligned} \neg(y \in \bar{A}) &\implies \neg(y \in \bar{B}) \\ y \in A &\implies y \in B \end{aligned}$$

### 1.2.7 Proof:

Use the identities to show that  $\overline{(A \cup B)} \cap \overline{(B \cup C)} \cap \overline{(A \cup C)} = \overline{A} \cap \overline{B} \cap \overline{C}$

**Proof:**

$$\begin{aligned}\overline{A} \cap \overline{B} \cap \overline{C} &= (\overline{A} \cap \overline{B}) \cap (\overline{B} \cap \overline{C}) \cap (\overline{A} \cap \overline{C}) \\ &= \overline{A} \cap (\overline{B} \cap \overline{B}) \cap (\overline{C} \cap \overline{C}) \cap \overline{A} \\ &= \overline{A} \cap \overline{B} \cap \overline{C}\end{aligned}$$

### 1.2.8 Union

The union of a collection of sets is the set that contains those elements that are members of at least one set in the collection.

$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^n A_i$$

### 1.2.9 Intersection

The intersection of a collection of sets is the set that contains those elements that are members of all the sets in the collection.

$$A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^n A_i$$

## 1.3 Functions

- Sets:  $A$ ,  $B$ ,  $C$ , domain, codomain, range
- Relations: functions  $(f, g, h)$
- Elements: image, preimage

### 1.3.1 Example

$$f_1 : \mathbb{R} \rightarrow \mathbb{R} \text{ as } y = f_1(x) = 3x - 2$$

$$f_2 : \mathbb{R} \rightarrow \mathbb{R} \text{ as } y = e^x$$

$$f_3 : \mathbb{R} \rightarrow \mathbb{R} \text{ as } y = \sqrt{x}$$

$$f : A \rightarrow B$$

$$\text{range}(f) = \{y \in B \mid \forall x \in S \subseteq A\}$$

### 1.3.2 Properties

1)  $f$  is injective – one-to-one

2)  $f$  is surjective –

3)  $f$  is bijective if 1) and 2)

Let  $f : A \rightarrow B$  be a function.

We say  $f$  is injective if

$$(x_1 \neq x_2 \implies f(x_1) \neq f(x_2)) \iff (f(x_1) = f(x_2) \implies x_1 = x_2)$$

We say  $f$  is surjective if  $\forall y \in B, \exists x \in A$  such that  $y = f(x) \iff \text{range}(f) = B$

We say  $f$  is bijective if  $f$  is injective and surjective.

**Monotonic Function:** We say  $f$  is increasing (strictly increasing) if  $x_1 > x_2 \implies f(x_1) > f(x_2)$

## 1.4 Sequences and Summations

## 1.5 Cardinality of Sets

## 1.6 Matrices