Week 01 Participation Assignment

Corey Mostero - 2566652

8 September 2023

Contents

1	Pro	bler	n	1																	
2	Pro	bler	n	2																	
	2.1	16																			
	2.2	17																			
	2.3	18																			
	2.4	19																			
	2.5	20																			
	2.6	21																			
	2.7	22																			
	2.8	23																			
	2.9	24																			
	2.10	25																			
	2.11	26																			
	2.12	27																			
	2.13	28																			

1 Problem 1

Use a truth table to verify the first De Morgan law: $\neg(p \land q) \equiv \neg p \lor \neg q$

p	q	$p \wedge q$	$\neg (p \land q)$	$\neg p$	$\neg q$	$\neg p \lor \neg q$
Т	Т	Т	F	F	F	\mathbf{F}
T	F	F	Т	F	T	${ m T}$
F	T	F	T	T	F	${ m T}$
F	F	F	T	T	T	${ m T}$

The truth values for the compound propositions $\neg(p \land q)$ and $\neg p \lor \neg q$ are the same, and are therefore logically equivalent thereby verifying the first De Morgan law.

2 Problem 2

Verify by showing either both sides are true, or both sides are false, for exactly the same combinations of truth values of the propositional variables in these expressions (except for 18).

2.1 16

Show that $p \iff q$ and $(p \land q) \lor (\neg p \land \neg q)$ are logically equivalent.

p	q	$p \iff q$	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \land \neg q$	$ (p \land q) \lor (\neg p \land \neg q) $
Т	Т	Т	Т	F	F	F	T
Τ	F	\mathbf{F}	F	F	Τ	F	F
F	Τ	\mathbf{F}	F	Τ	\mathbf{F}	\mathbf{F}	F
F	F	${ m T}$	F	Τ	Τ	Т	m T

2.2 17

Show that $\neg(p \iff q)$ and $p \iff \neg q$ are logically equivalent.

p	q	$p \iff q$	$\neg (p \iff q)$	$\neg q$	$p \iff \neg q$
Τ	Τ	T	F	F	F
\mathbf{T}	F	\mathbf{F}	${ m T}$	Τ	T
\mathbf{F}	Τ	\mathbf{F}	${ m T}$	\mathbf{F}	T
\mathbf{F}	F	${ m T}$	F	Τ	F

2.3 18

Show that $p \implies q$ and $\neg q \implies \neg p$ are logically equivalent.

p	q	$p \implies q$	$\neg q$	$\neg p$	$\neg q \implies \neg p$
Τ	Τ	Τ	F	F	Τ
\mathbf{T}	F	\mathbf{F}	Т	F	\mathbf{F}
F	Τ	${ m T}$	F	Τ	${ m T}$
\mathbf{F}	\mathbf{F}	${ m T}$	T	Τ	${ m T}$

2.4 19

Show that $\neg p \iff q$ and $p \iff \neg q$ are logically equivalent.

p	$\neg p$	q	$\neg p \iff q$	$\neg q$	$p \iff \neg q$
Т	F	Τ	F	F	F
T	F	F	${ m T}$	Τ	${ m T}$
F	Т	T	T	F	T
F	Т	F	F	Т	F

2.5 20

Show that $\neg(p \oplus q)$ and $p \iff q$ are logically equivalent.

p	q	$p\oplus q$	$\neg (p \oplus q)$	$p \iff q$
Т	Т	F	Т	T
T	F	Τ	F	F
F	T	T	F	F
F	F	F	T	T

2.6 21

Show that $\neg(p \iff q)$ and $\neg p \iff q$ are logically equivalent.

p	q	$p \iff q$	$\neg (p \iff q)$	$\neg p$	$\neg p \iff q$
Т	Т	Т	F	F	F
T	F	\mathbf{F}	${ m T}$	F	T
F	Т	\mathbf{F}	${ m T}$	Τ	T
F	F	${ m T}$	\mathbf{F}	Τ	F

2.7 22

Show that $(p \implies q) \land (p \implies r)$ and $p \implies (q \land r)$ are logically equivalent.

p	q	r	$p \implies q$	$p \implies r$	$(p \implies q) \land (p \implies r)$
Т	Τ	Τ	${ m T}$	T	T
T	T	F	${ m T}$	F	F
T	F	Т	\mathbf{F}	T	\mathbf{F}
T	F	F	\mathbf{F}	F	\mathbf{F}
F	Т	Т	${ m T}$	T	${ m T}$
F	T	F	${ m T}$	T	${ m T}$
F	F	Τ	${ m T}$	T	${ m T}$
F	F	F	Τ	Γ	T

p	q	r	$q \wedge r$	$p \implies (q \wedge r)$
Т	Т	Т	Т	Т
Т	Τ	F	F	F
Т	F	Τ	F	\mathbf{F}
Т	F	F	F	F
F	Τ	Т	Τ	m T
F	Τ	F	F	${ m T}$
F	F	Т	F	${ m T}$
F	F	F	F	${ m T}$

2.8 23

Show that $(p \implies r) \land (q \implies r)$ and $(p \lor q) \implies r$ are logically equivalent.

p	q	r	$p \implies r$	$q \implies r$	$\mid (p \implies r) \land (q \implies r) \mid$
Τ	Т	Τ	${ m T}$	T	T
Т	T	F	\mathbf{F}	F	F
T	F	T	${ m T}$	$^{\rm T}$	${ m T}$
T	F	F	\mathbf{F}	$^{\rm T}$	\mathbf{F}
F	T	Т	${ m T}$	$^{\rm T}$	${ m T}$
F	Т	F	${ m T}$	F	\mathbf{F}
F	F	Т	${ m T}$	${ m T}$	${f T}$
F	F	F	T	ight]	T

p	q	r	$p \lor q$	$(p \lor q) \implies r$
Т	Т	Т	Т	Т
T	T	F	T	F
T	F	Т	T	T
Т	F	F	Т	F
F	Т	Τ	T	T
F	T	F	T	F
F	F	Т	F	T
F	F	F	F	T

2.9 24

Show that $(p \implies q) \lor (p \implies r)$ and $p \implies (q \lor r)$ are logically equivalent.

$\mid p$	q	r	$p \implies q$	$p \implies r$	$(p \implies q) \lor (p \implies r)$
Т	Т	Т	Т	Т	Т
T	T	F	T	F	${ m T}$
T	F	T	F	$^{\rm T}$	${f T}$
T	F	F	F	F	\mathbf{F}
F	T	T	\mathbf{T}	Γ	${ m T}$
F	T	F	T	$^{\rm T}$	${f T}$
F	F	Τ	T	$^{\mathrm{T}}$	${f T}$
F	F	F	${f T}$	T	${f T}$

p	q	r	$q \vee r$	$p \implies (q \lor r)$
T	Τ	Τ	Т	T
T	Τ	F	T	T
T	F	Τ	T	T
T	F	F	F	F
F	Τ	Τ	Т	T
F	Τ	F	Т	T
F	\mathbf{F}	Τ	Т	T
F	\mathbf{F}	F	F	T

2.10 25

Show that $(p \implies r) \lor (q \implies r)$ and $(p \land q) \implies r$ are logically equivalent.

p	q	r	$p \implies r$	$ q \implies r$	$(p \implies r) \lor (q \implies r)$
Т	Т	Т	Т	T	T
T	T	F	F	F	${ m F}$
T	F	Τ	${ m T}$	$^{\mathrm{T}}$	${ m T}$
T	F	F	\mathbf{F}	$^{\mathrm{T}}$	${ m T}$
F	T	Т	${ m T}$	$^{\rm T}$	${ m T}$
F	Т	F	${ m T}$	F	${ m T}$
F	F	Т	${ m T}$	$^{\mathrm{T}}$	${ m T}$
F	F	F	T	Γ	T

p	q	r	$p \wedge q$	$(p \land q) \implies r$
Т	Т	Т	Т	T
T	Τ	\mathbf{F}	Τ	F
T	F	Τ	\mathbf{F}	T
T	F	F	\mathbf{F}	T
F	Τ	Τ	\mathbf{F}	T
F	Τ	\mathbf{F}	\mathbf{F}	T
F	F	Τ	\mathbf{F}	T
F	F	F	F	T

2.11 **26**

Show that $\neg p \implies (q \implies r)$ and $q \implies (p \lor r)$ are logically equivalent.

p	q	r	$\neg p$	$q \implies r$	$\neg p \implies (q \implies r)$
T	Т	Т	F	${ m T}$	T
T	T	F	F	\mathbf{F}	T
T	F	Τ	\mathbf{F}	${ m T}$	T
T	F	F	F	T	T
F	Т	Т	Τ	${ m T}$	T
F	T	F	Τ	\mathbf{F}	F
F	F	Т	T	${ m T}$	T
F	F	F	Τ	T	m T

p	q	r	$p \vee r$	$q \implies (p \lor r)$
T	Τ	Τ	Т	T
T	Τ	F	Т	T
T	F	Τ	T	T
T	F	F	T	T
F	Τ	Τ	Т	ightharpoons T
F	Τ	F	F	F
F	\mathbf{F}	Τ	Т	ightharpoons T
F	\mathbf{F}	F	F	${ m T}$

2.12 27

Shov	v tha	at $p \iff q$	and $(p \implies q) \land (q \implies p)$ are logically equivalent.
p	q	$p \iff q$	
Т	Т	Т	
Τ	F	F	
\mathbf{F}	Τ	\mathbf{F}	
\mathbf{F}	F	${ m T}$	

p	q	$p \implies q$	$q \implies p$	$(p \implies q) \land (q \implies p)$
Τ	Τ	T	T	T
Τ	F	F	$^{\rm T}$	F
F	Т	${ m T}$	\mathbf{F}	\mathbf{F}
F	F	${ m T}$	Т	${f T}$

2.13 28

Show that $p \iff q$ and $\neg p \iff \neg q$ are logically equivalent.

$\mid p \mid$	q	$p \iff q$	$\neg p$	$\neg q$	$ \neg p \iff \neg q $
T	Т	T	F	F	Т
T	F	\mathbf{F}	F	Γ	F
F	Т	\mathbf{F}	Τ	F	F
F	F	${ m T}$	Τ	Т	T