Week 12 Participation Assignment

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1 Week 12 Participation Assignment

Let's calculate the expected value of the random variable X, which represents the money that one ticket can win.

Complete the following table and then calculate the expectation of the game.

Random Variable X	$x = 4$	$x = 7$	$x = 100$	$x = 50000$	$x = 1000000$	$x =$ grand prize
Probability: $P(X = r)$	0.037	0.00316	0.000975	0.00000110	8.56×10^{-8}	3.42×10^{-9}

1.1 Sample Space Cardinality

$$\begin{aligned} n &= 69 && \text{Amount of white balls} \\ r &= 5 && \text{Number of white balls to choose} \\ m &= 26 && \text{Amount of red balls} \\ w &= 1 && \text{Amount of red balls to choose} \end{aligned}$$

$$\begin{aligned} |S| &= (\text{White Balls Combinations}) \cdot (\text{Red Ball Combinations}) \\ |S| &= \frac{69!}{5!(69-5)!} \cdot 26 \\ |S| &= 292201338 \end{aligned}$$

1.2 Probability

The equation I will use to derive the probability of winning the respective category is:

$$\begin{aligned} |E|_{no\ match\ red} &= \binom{r}{x} \cdot \binom{n-r}{r-x} \cdot \binom{w}{y} \cdot \binom{m-w}{w-y} \\ \mathbb{P}(X) &= \frac{|E|}{|S|} \end{aligned}$$

Where x represents the amount of white balls to match, and y red balls (in reality ball) to match.

$$\binom{r}{x} \cdot \binom{n-r}{r-x}$$

represents the amount of winning white balls you need to match multiplied against the amount of losing white balls you need to match.

$$\binom{w}{y} \cdot \binom{m-w}{w-y}$$

similarly represents matching the winning red ball multiplied against the remaining losses.

1.2.1 x = 4

Red:

$$\begin{aligned}
|E| &= \binom{5}{0} \cdot \binom{69-5}{5-0} \cdot \binom{1}{1} \cdot \binom{26-1}{1-1} \\
|E| &= 7624512 \\
\mathbb{P}(4_1) &= \frac{7624512}{292201338} \\
\mathbb{P}(4_1) &= 0.0261
\end{aligned}$$

One White & Red:

$$\begin{aligned}
|E| &= \binom{5}{1} \cdot \binom{69-5}{5-1} \cdot \binom{1}{1} \cdot \binom{26-1}{1-1} \\
|E| &= 3176880 \\
\mathbb{P}(4_2) &= \frac{3176880}{292201338} \\
\mathbb{P}(4_2) &= 0.0109
\end{aligned}$$

$$\mathbb{P}(4) = 0.0261 + 0.0109 = 0.037$$

1.2.2 x = 7

Two White & Red

$$\begin{aligned}
|E| &= \binom{5}{2} \cdot \binom{69-5}{5-2} \cdot \binom{1}{1} \cdot \binom{26-1}{1-1} \\
|E| &= 416640 \\
\mathbb{P}(7_1) &= \frac{416640}{292201338} \\
\mathbb{P}(7_1) &= 0.00143
\end{aligned}$$

Three White

$$
\begin{aligned}
|E| &= \binom{5}{3} \cdot \binom{69-5}{5-3} \cdot \binom{1}{0} \cdot \binom{26-1}{1-0} \\
|E| &= 504000 \\
\mathbb{P}(7_2) &= \frac{504000}{292201338} \\
\mathbb{P}(7_2) &= 0.00173
\end{aligned}
$$

$$\mathbb{P}(7) = 0.00143 + 0.00173 = 0.00316$$

1.2.3 x = 100

Three White & Red

$$
\begin{aligned}
|E| &= \binom{5}{3} \cdot \binom{69-5}{5-3} \cdot \binom{1}{1} \cdot \binom{26-1}{1-1} \\
|E| &= 20160 \\
\mathbb{P}(100_1) &= \frac{20160}{292201338} \\
\mathbb{P}(100_1) &= 0.0000690
\end{aligned}
$$

Four White

$$
\begin{aligned}
|E| &= \binom{5}{4} \cdot \binom{69-5}{5-4} \cdot \binom{1}{0} \cdot \binom{26-1}{1-0} \\
|E| &= 8320 \\
\mathbb{P}(100_2) &= \frac{8320}{292201338} \\
\mathbb{P}(100_2) &= 0.0000285
\end{aligned}
$$

$$\mathbb{P}(100) = 0.0000690 + 0.0000285 = 0.0000975$$

1.2.4 x = 50000

Four White & Red

$$
\begin{aligned}
|E| &= \binom{5}{4} \cdot \binom{69-5}{5-4} \cdot \binom{1}{1} \cdot \binom{26-1}{1-1} \\
|E| &= 320 \\
\mathbb{P}(50000) &= \frac{320}{292201338} \\
\mathbb{P}(50000) &= 0.00000110
\end{aligned}
$$

1.2.5 x = 1000000

Five White

$$
\begin{aligned}
|E| &= \binom{5}{5} \cdot \binom{69-5}{5-5} \cdot \binom{1}{0} \cdot \binom{26-1}{1-0} \\
|E| &= 25 \\
\mathbb{P}(1000000) &= \frac{25}{292201338} \\
\mathbb{P}(1000000) &= 8.56 \times 10^{-8}
\end{aligned}
$$

1.2.6 x = grand prize

Five White & Red

$$
\begin{aligned}
|E| &= \binom{5}{5} \cdot \binom{69-5}{5-5} \cdot \binom{1}{1} \cdot \binom{26-1}{1-1} \\
|E| &= 1 \\
\mathbb{P}(1000000) &= \frac{1}{292201338} \\
\mathbb{P}(1000000) &= 3.42 \times 10^{-9}
\end{aligned}
$$

1.3 Expected Value

Assume the grand prize is 200000000\$.

$$
\begin{aligned}
E(X) = {} & (0.037)(4\$) + (0.00316)(7\$) + (9.75 \times 10^{-5})(100\$) \\
& + (1.1 \times 10^{-6})(50000\$) + (8.56 \times 10^{-8})(1000000\$) + (3.42 \times 10^{-9})(200000000\$) \\
E(X) = {} & 1.00447\$ = 1\$
\end{aligned}
$$