Contents

1 Energy Ext.

In 1D

$$\vec{F} = -\frac{\partial \mathrm{PE}}{\partial x}\hat{x} \tag{1}$$

$$\mathrm{PE}_g = mgy \to \vec{F} = -mg\hat{y} \tag{2}$$

Potential Energy is only based on position.

871:

- k = Kinetic Energy
- u = Potential Energy
- H = Total Energy

2 Hooke's Law

$$\vec{F}_s = -k\vec{x} \tag{3}$$

2.1 Example

$$\begin{aligned}
\sum F &= 0 \\
kx - mg &= 0 \\
k &= \frac{mg}{x} \\
k &= \frac{(17\,\mathrm{g})(1000\,\mathrm{cm\,s^{-2}})}{10\,\mathrm{cm}} \\
k &= 1700 \text{ dyn/cm}
\end{aligned}$$

$$\boxed{k = 1700 \text{ dyn/cm}}$$

2.2 Series and Parallel Springs

Series

$$\begin{aligned}
F_1 &= F_2 \\
\Delta x_1 &\neq \Delta x_2
\end{aligned}$$

Parallel:

$$\begin{aligned}
\Delta x_1 &= \Delta x_2 \\
F_1 \neq F_2
\end{aligned}$$

Replace Parallel Springs with k_{eq}

$$\begin{aligned}
\sum F_{eq} &= 0 \\
mg &= k_{eq}\Delta x
\end{aligned}$$

$$\begin{aligned}
\sum F_{1,2} = 0 \\
mg &= k_1\Delta x_1 + k_2\Delta x_2 \\
k_{eq}\Delta x_{eq} &= k_1\Delta x_1 + k_2\Delta x_2 \\
\Delta x_{eq} &= \Delta x_1 = \Delta x_2
\end{aligned}$$

$$\begin{aligned}
k_{\|} &= k_1 + k_2 \\
\frac{1}{k_s} &= \frac{1}{k_1} + \frac{1}{k_2}
\end{aligned}$$

Find out the work done against a spring

Recall:

$$W = \int \vec{F} \cdot d\vec{x}$$
$$\mathrm{PE} = -W$$

$$\mathrm{PE}_s = -\int_0^x (-kx)dx$$
$$\mathrm{PE}_s = \frac{1}{2}kx^2$$

Potential Energy:

$$\boxed{\mathrm{PE}_s = \frac{1}{2}kx^2}$$

$x = 0$ at equilibrium

The only time the final height depends on the shape of the path is when there is friction.

2.3 Example

Variables

$$k = 200\,\mathrm{N\,m^{-1}}$$
$$x_1 = 0.5\,\mathrm{m}$$
$$m = 1\,\mathrm{kg}$$
$$v_1 = 0$$
$$v_2 = ?$$
$$v_3 = 0$$
$$h_3 = ?$$

$$E_1 = E_3$$
$$\frac{1}{2}kx_1^2 = mgh_3$$
$$h_3 = \frac{kx^2}{2mg}$$
$$h_3 = \frac{(200\,\mathrm{N\,m^{-1}})(0.5\,\mathrm{m})^2}{2(10\,\mathrm{N})}$$
$$h_3 = 2.5\,\mathrm{m}$$

$$\boxed{h_3 = 2.5\,\mathrm{m}}$$

3 Power

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} \tag{4}$$

$$\boxed{P = \frac{dE}{dt}}$$

Definition of Power

$$P = \frac{d\left(\int \vec{F} \cdot d\vec{x}\right)}{dt}$$
$$P = \frac{d\left(\frac{1}{2}mv^2\right)}{dt}$$
$$P = \frac{1}{2}\frac{d}{dt}\left[mv^2\right]$$
$$P = \frac{1}{2}(\dot{m}v)v + \frac{1}{2}m(2v\dot{v})$$

In the case that m doesn't change ($\dot{m} = 0$)

$$P = (ma)v$$
$$P = \vec{F} \cdot \vec{v}$$

$$\boxed{P = \vec{F} \cdot \vec{v}}$$

4 Momentum

Recall:

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{5}$$
$$\vec{P} \equiv m\vec{v} \tag{6}$$

Momentum - "how hard is it to reduce $\vec{v}$ to zero?"

$\vec{P}$ is a vector that points in the same direction as $\vec{v}$. $\vec{P}$ is conserved.

$$KE = \frac{p^2}{2m}; F = \frac{d\vec{P}}{dt}$$

Collision $\rightarrow$Momentum

$$\sum \vec{P}_i = \sum \vec{P}_f \tag{7}$$
$$\sum \vec{P}_{i_x} = \sum \vec{P}_{f_x} \tag{8}$$
$$\sum \vec{P}_{i_y} = \sum \vec{P}_{f_y} \tag{9}$$
$$\sum \vec{P}_{i_z} = \sum \vec{P}_{f_z} \tag{10}$$

4.1 2D Momentum Conservation:

Situation - Two masses (vehicles) are colliding

$$
\begin{aligned}
M &= 3000\,\mathrm{kg} \\
u_i &= 90\,\mathrm{mi\,h^{-1}} \\
m &= 2500\,\mathrm{kg} \\
v_i &= 75\,\mathrm{mi\,h^{-1}} \\
v_f &= ? \\
\phi &= ? \\
\theta_f &= 30^\circ \\
u_f &= 60\,\mathrm{mi\,h^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
\sum \vec{P}_{i_x} &= \sum \vec{P}_{f_x} \\
Mu_i - mv_i &= Mu_f\cos(\theta_f) - mv_f\cos(\phi_f) \\
mv_f\cos(\phi_f) &= Mu_f\cos(\theta_f) - Mu_i + mv_i
\end{aligned}
$$

$$
\begin{aligned}
\sum \vec{P}_{i_y} &= \sum \vec{P}_{f_y} \\
0 &= mv_f\sin(\phi_f) - Mu_f\sin(\theta_f) \\
mv_f\sin(\phi_f) &= Mu_f\sin(\theta_f)
\end{aligned}
$$

$$
\begin{aligned}
\tan(\phi_f) &= \frac{Mu_f\sin(\theta_f)}{Mu_f\cos(\theta_f) - Mu_i + mv_i} \\
\phi_f &= \arctan\left[\frac{(3000\,\mathrm{kg})(60\,\mathrm{mi\,h^{-1}})(\sin(30^\circ))}{(3000\,\mathrm{kg})(60\,\mathrm{mi\,h^{-1}})(\cos(30^\circ)) - (3000\,\mathrm{kg})(90\,\mathrm{mi\,h^{-1}}) + (2500\,\mathrm{kg})(75\,\mathrm{mi\,h^{-1}})}\right] \\
\phi_f &= 51^\circ
\end{aligned}
$$

$$
\boxed{\phi_f = 51^\circ}
$$

$$
\begin{aligned}
mv_f\sin(\phi_f) &= Mu_f\sin(\theta_f) \\
v_f &= \left[\frac{M}{m}\frac{\sin(\theta_f)}{\sin(\phi_f)}\right]u_f \\
v_f &= \frac{(3000\,\mathrm{kg})(\sin(30^\circ))(60\,\mathrm{mi\,h^{-1}})}{(2500\,\mathrm{kg})(\sin(51^\circ))} \\
v_f &= 46\,\mathrm{mi\,h^{-1}}
\end{aligned}
$$

$$
\boxed{v_f = 46\,\mathrm{mi\,h^{-1}}}
$$

5 Collisions

Perfectly Elastic ($\epsilon = 1$):

Energy is conserved.

Partially Elastic ($0 < \epsilon < 1$):

Bounre; Energy not conserved.

Inelastic ($\epsilon = 0$):

Two objects stick together or explode.

Is the car collision elastic?

$$\frac{1}{2}mv_i^2 + \frac{v}{2}Mu_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}Mu_f^2$$

$$(2500\,\mathrm{kg})(75\,\mathrm{mi\,h^{-1}})^2 + (3000\,\mathrm{kg})(90\,\mathrm{mi\,h^{-1}})^2 = (2500\,\mathrm{kg})(46\,\mathrm{mi\,h^{-1}})^2 + (3000\,\mathrm{kg})(60\,\mathrm{mi\,h^{-1}})^2$$

$$3.8 \times 10^7\,\mathrm{kg\,mi\,h^{-1}} = 1.6 \times 10^7\,\mathrm{kg\,mi\,h^{-1}}$$

$$\boxed{3.8 \times 10^7\,\mathrm{kg\,mi\,h^{-1}} \neq 1.6 \times 10^7\,\mathrm{kg\,mi\,h^{-1}} \therefore \text{ Partially Elastic}}$$