Homework 10 Rotations

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6 June 2023

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1 Book

1.1 10.22

$$
\begin{aligned}
r &= 8.00\,\mathrm{cm} \\
m &= 0.180\,\mathrm{kg} \\
v_0 &= 0 \\
\Delta y &= 75.0\,\mathrm{cm} \\
I &= mr^2
\end{aligned}
$$

(a)

$$\begin{aligned}
E_{K_0} + E_{P_0} &= E_{K_1} + E_{P_1} \\
0 + mgh &= \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} mr^2\omega^2 + 0 \\
mgh &= \omega^2 \left(\frac{1}{2}\left(mr^2\right) + \frac{1}{2} mr^2 \right) \\
\omega &= \frac{\sqrt{gh}}{r} \\
\omega &= \frac{\sqrt{(10.0\,\mathrm{m\,s^{-2}})(0.75\,\mathrm{m})}}{0.08\,\mathrm{m}} \\
\omega &= 34.2\,\mathrm{rad\,s^{-1}}
\end{aligned}$$

(b)

$$\begin{aligned}
E_{k_0} + E_{p_0} &= E_{k_1} + E_{p_1} \\
0 + mgh &= \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} mv_{cm}^2 + 0 \\
mgh &= \frac{1}{2}(mr^2)\left(\frac{v_{cm}}{r}\right)^2 + \frac{1}{2} mv_{cm}^2 \\
v &= \sqrt{gh} \\
v &= \sqrt{(10.0\,\mathrm{m\,s^{-2}})(0.75\,\mathrm{m})} \\
v &= 2.74\,\mathrm{m\,s^{-1}}
\end{aligned}$$

1.2 10.26

$$I_{cm} = \frac{2}{5} mr^2$$

(a) Velocity for the first half of the bowl:

$$\begin{aligned}
E_{K_0} + E_{P_0} &= E_{K_1} + E_{P_1} \\
0 + mgh &= \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} mv_{cm}^2 + 0 \\
mgh &= \frac{1}{2}\left(\frac{2}{5} mr^2\right)\left(\frac{v_{cm}^2}{r^2}\right) + \frac{1}{2} mv_{cm}^2 \\
v_{cm} &= \sqrt{\frac{10gh}{7}}
\end{aligned}$$

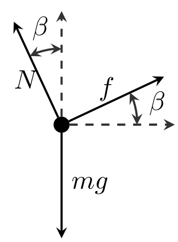
Since the ball only slides and doesn't rotate, the kinetic energy it experiences it purely linear velocity and *not* angular.

$$\begin{aligned}
E_{K_0} + E_{P_0} &= E_{K_1} + E_{P_1} \\
\frac{1}{2} m v_{cm}^2 + 0 &= 0 + mgh_1 \\
\left(\sqrt{\frac{10gh_0}{7}}\right)^2 &= 2gh_1 \\
h_1 &= \frac{5}{7} h_0
\end{aligned}$$

The ball reaches only $\frac{5}{7}$ of the height of the side of the bowl.

1.3 10.30

(a) Free-body diagram:



The angular velocity of the bowling ball is clockwise ↻ which the friction has to oppose resulting in the friction going upwards (up the incline).

(b) Normal force has no affect as it is directed towards the axis of rotation.

$$\begin{aligned}
\sum F_x &= ma_{cm} \\
-f &= ma_{cm} + mg\sin(\beta) \\
-\frac{I_{cm}\alpha}{r} &= m(a_{cm} + g\sin(\beta)) \\
-\frac{\left(\frac{2}{5}mr^2\right)\left(\frac{a_{cm}}{r}\right)}{r} &= m(a_{cm} + g\sin(\beta)) \\
-\frac{2}{5}a_{cm} &= a_{cm} + g\sin(\beta) \\
a_{cm} &= \frac{5g\sin(\beta)}{7}
\end{aligned}$$

(c)

$$\begin{aligned}
\sum F_y &= 0 \\
N &= mg\cos(\beta)
\end{aligned}$$

$$\begin{aligned}
\sum F_x &= ma_{cm} \\
-f &= ma_{cm} + mg\sin(\beta) \\
-\mu(mg\cos(\beta)) &= m\left(\frac{5g\sin(\beta)}{7} + g\sin(\beta)\right) \\
\mu &= \frac{2\tan(\beta)}{7}
\end{aligned}$$

1.4 10.79

$$\begin{aligned}
I_{cylinder_{cm}} &= \frac{1}{2}m(2r)^2 \\
I_{disk_{cm}} &= \frac{1}{2}mr^2
\end{aligned}$$

$$\begin{aligned}
E_{K_0} + E_{P_0} &= E_{K_1} + E_{P_1} \\
\left(\frac{1}{2}I_{cylinder_{cm}}\left(\frac{v}{2r}\right)^2 + \frac{1}{2}mv^2\right) + \left(\frac{1}{2}I_{disk_{cm}}\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2\right) + 0 &= 0 + mgh \\
\left(\frac{1}{2}\left(\frac{1}{2}m(2r)^2\right)\left(\frac{v}{2r}\right)^2 + \frac{1}{2}mv^2\right) + \left(\frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2\right) &= mgh \\
v &= \sqrt{\frac{2gh}{3}}
\end{aligned}$$

$$\begin{aligned}
v_1^2 &= v_0^2 + 2ah \\
a &= \frac{v_1^2}{2h} \\
a &= \frac{\sqrt{\frac{2gh}{3}}^2}{2h} \\
a &= \frac{g}{3}
\end{aligned}$$

1.5 9.30

1.6 9.49

1.7 9.79

1.8 9.86

2 Lab Manual

2.1 1170

2.2 1173

2.3 1175

2.4 1177

2.5 1181

2.6 1283

2.7 1284

3 Problem C: Spherical Symmetry Problem

Starting with $I = \int r^2 dm$, calculate the moment of inertial for an axis of rotation that goes through the center of a sphere with uniform mass density ρ, and radius R. As discussed in class, you may treat this problem like the integration of a series of concentric spherical shells with thickness dr.