Contents

1 5.1 Weak Induction

2 5.2 Strong Induction

2.1 Example

Prove that any positive integer can be written as a sum of 2^k, where k's are distinct.

Proof:

1. Basis step:
$$1 = 2^0, 2 = 2^1, 3 = 2^0 + 2^1$$

2. Inductive step:

   We assume that k can be written as a sum of distinct powers of 2.

   Where $1 \leq k \leq n$, we want to show that $n + 1$ is a sum of distinct powers of 1.

   Consider the following cases

   (a) $n = \sum 2^r$, where r is some distinct positive integers. Then $n + 1 = 1 + \sum 2^r = 2^0 + \sum 2^r$ is a sum of distinct powers of 2.

   (b) $n = 1 + \sum 2^s$, where s is some distinct positive integers. Then $n + 1 = 1 + \sum 2^s + 1 = 2 + \sum 2^s = 2\left(1 + \sum 2^{s-1}\right)$. Since $n \geq 1$, $\frac{n+1}{2} = 1 + \sum 2^{s-1}$, then $\frac{n+1}{2}$ is a sum of distinct powers of 2.

3 5.3 Recursive Definitions

3.0.1 Example

Prove that $f(0) = 0, f(n+1) = f(n) + 2n + 1$ is equivalent to $f(n) = n^2$.

Proof By Induction:

- Basis Step:

  When $n = 0, f(0) = 0, f(0) = n^2 = 0^2 = 0$. Then the recursive is equivalent to the closed formula when $n = 1$. Then $f(1) = f(0+1) = f(0) + 2 \cdot 0 + 1 = 0 + 0 + 1 = 1$ and $f(1) = 1^2 = 1$.

- Inductive Step:

  Then for the closed formula,

  $$f(n+1) = (n+1)^2$$

  and for the recursive form

  $$f(n+1) = f(n) + 2n + 1 = n^2 + 2n + 1 = (n+1)^2$$

  Thus, the closed form and the recursive form are equivalent (for $n+1$). Therefore, the equivalency is proved.