Contents

1 Section 5.3

1.1 5.3.1

Find the general solution of the differential equation.

$$y'' - 289y = 0$$

$$\begin{aligned} r^2 - 289 &= 0 \\ r &= 17, -17 \\ y(x) &= c_1 e^{17x} + c_2^{-17x} \end{aligned}$$

1.2 5.3.3

Find the general solution of the differential equation.

$$y'' + y' - 56y = 0$$

$$\begin{aligned} r^2 + r - 56 &= 0 \\ r &= 7, -8 \\ y(t) &= c_1 e^{7t} + c_2 e^{-8t} \end{aligned}$$

1.3 5.3.4

Find a general solution.

$$4y'' + 7y' - 2y = 0$$

$$\begin{aligned} 4r^2 + 7r - 2 &= 0 \\ r &= \frac{1}{4}, -2 \\ y(t) &= c_1 e^{t/4} + c_2 e^{-2t} \end{aligned}$$

1.4 5.3.5

Find a general solution to the given differential equation.

$$4w'' + 12w' + 9w = 0$$

$$\begin{aligned} 4r^2 + 12r + 9 &= 0 \\ r &= -\frac{3}{2} \\ w(t) &= c_1 e^{-3t/2} + c_2 t e^{-3t/2} \end{aligned}$$

1.5 5.3.7

Find the general solution of the differential equation.

$$36y'' - 84y' + 49y = 0$$

$$\begin{aligned} 36r^2 - 84r + 49 &= 0 \\ r &= \frac{7}{6} \\ y(x) &= c_1 e^{7x/6} + x c_2 e^{7x/6} \end{aligned}$$

1.6 5.3.9

The auxiliary equation for the given differential equation has complex roots. Find a general solution.

$$y'' - 10y' + 29y = 0$$

$$\begin{aligned} r^2 - 10r + 29 &= 0 \\ r &= 5 \pm 2\mathrm{i} \\ y(t) &= c_1 e^{5t} \cos(2t) + c_2 e^{5t} \sin(2t) \end{aligned}$$

1.7 5.3.11

Find the general solution of the differential equation.

$$y^{(4)} - 32y^{(3)} + 256y'' = 0$$

$$\begin{aligned} r^4 - 32r^3 + 256r^2 &= 0 \\ r &= 0, 0, 16, 16 \\ y(x) &= c_1 + c_2 x + c_3 e^{16x} + c_4 x e^{16x} \end{aligned}$$

1.8 5.3.13

Find the general solution of the differential equation.

$$9y^{(3)} + 12y'' + 4y' = 0$$

$$\begin{aligned} 9r^3 + 12r^2 + 4r = 0 \\ r = 0, -\frac{2}{3}, -\frac{2}{3} \\ y(x) = c_1 + c_2 e^{-2x/3} + c_3 x e^{-2x/3} \end{aligned}$$

1.9 5.3.18

Find the general solution of the differential equation.

$$256y^{(4)} = y$$

$$\begin{aligned} 256y^{(4)} - y = 0 \\ 256r^4 - 1 = 0 \\ r = 0 \pm \frac{1}{4}, 0 \pm \frac{1}{4}i \\ y(x) = c_1 e^{\frac{x}{4}} + c_2 e^{-\frac{x}{4}} + c_3 \cos\left(\frac{x}{4}\right) + c_4 \sin\left(\frac{x}{4}\right) \end{aligned}$$

1.10 5.3.19

Find three linearly independent solutions of the given third-order differential equation and write a general solution as an arbitrary linear combination of them.

$$z''' + 7z'' - 5z' - 75z = 0$$

$$\begin{aligned} r^3 + 7r^2 - 5r - 75 = 0 \\ r = -5, -5, 3 \\ z(t) = c_1 e^{-5t} + c_2 t e^{-5t} + c_3 e^{3t} \end{aligned}$$

1.11 5.3.21

Find the unique solution of the second-order initial value problem.

$$y'' + 2y' - 15y = 0, y(0) = -1, y'(0) = -11$$

$$\begin{aligned} r^2 + 2r - 15 = 0 \\ r = -5, 3 \\ y(x) = c_1 e^{-5x} + c_2 e^{3x} \\ y'(x) = -5c_1 e^{-5x} + 3c_2 e^{3x} \\ y''(x) = 25c_1 e^{-5x} + 9c_2 e^{3x} \end{aligned}$$

$$
\begin{aligned}
y(0) &= c_1e^{-5(0)} + c_2e^{3(0)} = -1\\
c_1 + c_2 &= -1\\
c_1 &= -1 - c_2\\
y'(0) &= -5c_1e^{-5(0)} + 3c_2e^{3(0)} = -11\\
-5c_1 + 3c_2 &= -11\\
-5(-1 - c_2) + 3c_2 &= -11\\
c_2 &= -2\\
c_1 &= -1 - (-2)\\
c_1 &= 1\\
y(x) &= e^{-5x} + -2e^{3x}
\end{aligned}
$$

$$\boxed{y(x) = e^{-5x} + -2e^{3x}}$$

1.12 5.3.23

Solve the given initial value problem.

$$y'' + 4y' + 29y = 0; y(0) = 3, y'(0) = -3$$

$$
\begin{aligned}
r^2 + 4r + 29 = 0&\\
r &= -2 \pm 5i\\
y(t) &= c_1e^{-2t}\cos(5t) + c_2e^{-2t}\sin(5t)
\end{aligned}
$$

$$
\begin{aligned}
y(t) &= c_1e^{-2t}\cos(5t) + c_2e^{-2t}\sin(5t)\\
y(0) &= c_1e^{-2(0)}\cos(5(0)) + c_2e^{-2t(0)}\sin(5(0)) = 3\\
c_1 &= 3\\
y'(t) &= c_1(-e^{-2t}(5\sin(5t) + 2\cos(5t))) + c_2(-e^{-2t}(2\sin(5t) - 5\cos(5t)))\\
y'(0) &= c_1(-e^{2(0)}(5\sin(5(0)) + 2\cos(5(0)))) + c_2(-e^{-2(0)}(2\sin(5(0)) - 5\cos(5(0)))) = -3\\
-2c_1 + 5c_2 &= -3\\
-2(3) + 5c_2 &= -3\\
c_2 &= \frac{3}{5}\\
y(t) &= 3e^{-2t}\cos(5t) + \frac{3}{5}e^{-2t}\sin(5t)
\end{aligned}
$$

$$\boxed{y(t) = 3e^{-2t}\cos(5t) + \frac{3}{5}e^{-2t}\sin(5t)}$$

1.13 5.3.39

Find a linear homogeneous constant-coefficient equation with the given general solution.

$$(A + Bx + Cx^2 + Dx^3)e^{4x}$$

$$\boxed{y^{(4)} - 16y^{(3)} + 96y'' - 256y' + 256y = 0}$$

1.14 5.3.54

A third-order Euler equation is one of the form $ax^3y''' + bx^2y'' + cxy' + ky = 0$, where a, b, c, and k are constants. If $x > 0$, then the substitution $v = \ln(x)$ transforms the equation into the constant coefficient linear equation below, with independent variable v.

$$a\frac{d^3y}{dv^3} + (b - 3a)\frac{d^2y}{dv^2} + (c - b + 2a)\frac{dy}{dv} + ky = 0$$

Make the substitution $v = \ln(x)$ to find the general solution of $2x^3y''' + 12x^2y'' + 8xy' = 0$ for $x > 0$.

$$\begin{aligned}
2y''' + (12 - 3(2))y'' + (8 - 12 + 2(2))y' &= 0 \\
2y''' + 6y'' &= 0 \\
2r^3 + 6y^2 &= 0 \\
r &= 0, 0, -3 \\
y(v) &= c_1 + c_2v + c_3e^{-3v} \\
y(x) &= c_1 + c_2\ln(x) + \frac{c_3}{x^3}
\end{aligned}$$