Week 10 Participation Assignment (2 of 2)

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1 Part 2

$$A = \begin{bmatrix} 3 & -1 & 3 & 7 & 2 & 2 & 15 \\ -4 & 3 & 11 & 4 & 2 & 3 & -17 \\ -3 & 2 & 6 & 1 & 1 & 1 & -16 \\ 1 & 4 & 40 & 37 & 12 & 17 & 24 \\ -5 & 3 & 7 & -1 & 0 & 1 & -22 \end{bmatrix} \xrightarrow{rref} R = \begin{bmatrix} 1 & 0 & 4 & 5 & 0 & 0 & 3 \\ 0 & 1 & 9 & 8 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find a basis for

1). $W_1 = \text{colspace}(A)$

2). $W_2 = \text{rowspace}(A)$

3). $W_3 = \text{nullspace}(A)$

4). Next, we can define $W_1^{\perp}$, $W_2^{\perp}$, $W_3^{\perp}$. Last time, we knew the ambient space of the orthogonal complements. Then let's use the definition of the orthogonal complements as well as the basis you found above to set up system of linear equations so that we can find a basis for the complement.

1.1 1).

$$\left\{ \begin{bmatrix} 3 \\ -4 \\ -3 \\ 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \\ 12 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \\ 17 \\ 1 \end{bmatrix} \right\}$$

1.2 2).

$$\left\{ \begin{bmatrix} 3 \\ -1 \\ 3 \\ 7 \\ 2 \\ 2 \\ 15 \end{bmatrix}, \begin{bmatrix} -4 \\ 3 \\ 11 \\ 4 \\ 2 \\ 3 \\ -17 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 6 \\ 1 \\ 1 \\ 1 \\ -16 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 40 \\ 37 \\ 12 \\ 17 \\ 24 \end{bmatrix} \right\}$$

1.3 3).

$$
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}
= \begin{bmatrix} -4a - 3c - 3d \\ -4a - 8c + 4d \\ a \\ c \\ 4d \\ -5d \\ d \end{bmatrix}
= a \begin{bmatrix} -4 \\ -9 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} -5 \\ -8 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + d \begin{bmatrix} -3 \\ 4 \\ 0 \\ 0 \\ 4 \\ -5 \\ 1 \end{bmatrix}
= \left\{ \begin{bmatrix} -4 \\ -9 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -8 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 0 \\ 0 \\ 4 \\ -5 \\ 1 \end{bmatrix} \right\}
$$

1.4 4).

$$A^T = \begin{bmatrix} 3 & -4 & -3 & 1 & -5 \\ -1 & 3 & 2 & 4 & 3 \\ 3 & 11 & 6 & 40 & 7 \\ 7 & 4 & 1 & 37 & -1 \\ 2 & 2 & 1 & 12 & 0 \\ 2 & 3 & 1 & 17 & 1 \\ 15 & -17 & -16 & 24 & -22 \end{bmatrix}$$

$$\text{rref}(A^T) = \begin{bmatrix} 1 & 0 & 0 & 3 & 0 \\ 0 & 1 & 0 & 5 & 0 \\ 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

basis $W_1^\perp$:

$$\left\{ \begin{bmatrix} 3 \\ -4 \\ -3 \\ 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 11 \\ 6 \\ 40 \\ 7 \end{bmatrix}, \begin{bmatrix} 7 \\ 4 \\ 1 \\ 37 \\ -1 \end{bmatrix} \right\}$$

basis $W_2^\perp$:

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 4 \\ 5 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 9 \\ 8 \\ 0 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 5 \end{bmatrix} \right\}$$

basis $W_3^\perp$:

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 4 \\ 5 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 9 \\ 8 \\ 0 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 5 \end{bmatrix} \right\}$$