Contents

1 Section 5.2

1.1 5.2.1

Show directly that the given functions are linearly dependent on the real line. That is, find a nontrivial linear combination of the following functions that vanishes identically.

$$f(x) = 6x, g(x) = 4x^2, h(x) = 6x - 21x^2$$

Enter the non-trivial linear combination.

$$(24)6x + (-126)4x^2 + (-24)(6x - 21x^2) = 0$$

1.2 5.2.4

Show directly that the given functions are linearly dependent on the real line. That is, find a nontrivial linear combination of the following functions that vanishes identically.

$$f(x) = 23, g(x) = 3\sin^2(x), h(x) = 5\cos^2(x)$$

Enter the non-trivial linear combination.

$$(15)(23) + (-115)(3\sin^2(x)) + (-69)(5\cos^2(x)) = 0$$

1.3 5.2.7

Use the Wronskian to determine if the given functions are linearly independent on the indicated interval.

$$f(x) = 19, g(x) = 5x, h(x) = 5x^2; \text{ the real line}$$

The Wronskian $W(f, g, h) = 950$. As W is never 0 on the real line $f(x), g(x)$, and $h(x)$ are linearly independent.

1.4 5.2.13

A third-order homogeneous linear equation and three linearly independent solutions are given below. Find a particular solution satisfying the given initial conditions.

$$y^{(3)} + 2y'' - y' - 2y = 0; y(0) = 8, y'(0) = 12, y''(0) = 0;$$

$$y_1 = e^x, y_2 = e^{-x}, y_3 = e^{-2x}$$

$$\begin{aligned}
y(x) &= c_1e^x + c_2e^{-x} + c_3e^{-2x} \\
y(0) &= c_1e^0 + c_2e^0 + c_3e^0 = 8 \\
&= c_1 + c_2 + c_3 = 8 \\
c_1 &= 8 - c_2 - c_3 \\
y'(x) &= c_1e^x - c_2e^{-x} - 2c_3e^{-2x} \\
y'(0) &= c_1e^0 - c_2e^0 - 2c_3e^0 = 12 \\
&= c_1 - c_2 - 2c_3 = 12 \\
y''(x) &= c_1e^x + c_2e^{-x} + 4c_3e^{-2x} \\
y''(0) &= c_1e^0 + c_2e^0 + 4c_3e^0 = 0 \\
&= c_1 + c_2 + 4c_3 = 0 \\
(8 - c_2 - c_3) + c_2 + 4c_3 &= 0 \\
8 + 3c_3 &= 0 \\
3c_3 &= -8 \\
c_3 &= -\frac{8}{3} \\
c_2 &= -c_1 - c_3 + 8 \\
c_1 - (-c_1 - c_3 + 8) - 2c_3 &= 12 \\
2c_1 - c_3 &= 20 \\
2c_1 - \left(-\frac{8}{3}\right) &= 20 \\
c_1 &= \frac{20 - \frac{8}{3}}{2} \\
c_1 &= \frac{26}{3} \\
\frac{26}{3} + c_2 - \frac{8}{3} &= 8 \\
c_2 &= 2
\end{aligned}$$

$$\boxed{y(x) = \frac{26}{3}e^x + 2e^{-x} - \frac{8}{3}e^{-2x}}$$

1.5 5.2.19

A third-order homogeneous linear equation and three linearly independent solutions are given below. Find a particular solution satisfying the given initial conditions.

$$x^3y^{(3)} - 3x^2y'' + 6xy' - 6y = 0; y(1) = 9, y'(1) = 15, y''(1) = 28;$$

$$y_1 = x, y_2 = x^2, y_3 = x^3$$

$$\begin{aligned}
y(x) &= c_1x + c_2x^2 + c_3x^3 \\
y'(x) &= c_1 + 2c_2x + 3c_3x^2 \\
y''(x) &= 2c_2 + 6c_3x \\
y''(1) &= 2c_2 + 6c_3(1) = 28 \\
c_2 &= 14 - 3c_3 \\
y'(1) &= c_1 + 2(14 - 3c_3)(1) + 3c_3(1) = 15 \\
c_1 + 28 - 6c_3 + 3c_3 &= 15 \\
c_1 &= -13 + 3c_3 \\
y(1) &= (-13 + 3c_3)(1) + (14 - 3c_3)(1)^2 + c_3(1)^3 = 9 \\
c_3 &= 8 \\
2c_2 + 6(8)(1) &= 28 \\
c_2 &= -10 \\
c_1 + (-10) + (8) &= 9 \\
c_1 &= 11
\end{aligned}$$

$$\boxed{y(x) = 11x - 10x^2 + 8x^3}$$