Week 07 Participation Assignment (2 of 3)

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Contents

1 Part 2

Consider the following matrices:

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 5 & 4 & -1 \\ -3 & 2 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} 7 & 1 & -4 \\ 2 & -3 & 2 \\ -1 & -3 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 6 & -6 \\ 6 & 7 \end{bmatrix}$$

$$D = \begin{bmatrix} \lambda - 5 & -1 & -1 \\ -1 & \lambda - 5 & -1 \\ -1 & -1 & \lambda - 5 \end{bmatrix}$$

$$E = \begin{bmatrix} 3 & 2 & -4 & 1 \\ -2 & 1 & 0 & 3 \\ 5 & 3 & 2 & -1 \\ 2 & -5 & 3 & 6 \end{bmatrix}$$

Perform the following calculations:

1). $\det(C)$ using the formula for two by two matrix.

2). $\det(D)$ using the co-factor expansion.

3). $\det(E)$ using row/column operations.

Find $\det(E)$ using elementary row/column operations

1.1 $\det(C)$

$$\det(C) = (6)(7) - (-6)(6)$$
$$\det(C) = 78$$

$$\boxed{\det(C) = 78}$$

1.2 $\det(D)$

$$\begin{aligned}
\det(D) &= (\lambda-5)\det\begin{bmatrix}\lambda-5 & -1\\ -1 & \lambda-5\end{bmatrix}\\
&\quad -(-1)\det\begin{bmatrix}-1 & -1\\ -1 & \lambda-5\end{bmatrix}\\
&\quad +(-1)\det\begin{bmatrix}-1 & \lambda-5\\ -1 & -1\end{bmatrix}\\
\det(D) &= (\lambda-5)(\lambda^2-10\lambda+24)+(1)(-\lambda+4)+(-1)(\lambda-4)\\
\det(D) &= \lambda^3-15\lambda^2+72\lambda-112
\end{aligned}$$

$$\boxed{\det(D)=\lambda^3-15\lambda^2+72\lambda-112}$$

1.3 $\det(E)$

$$\begin{aligned}
E_2 &= E_2+\frac{2}{3}E_1\\
E_3 &= E_3-\frac{5}{3}E_1\\
E_4 &= E_4-\frac{2}{3}E_1\\
E &= \begin{bmatrix}3 & 2 & -4 & 1\\ 0 & \frac{7}{3} & -\frac{8}{3} & \frac{11}{3}\\ 0 & -\frac{1}{3} & \frac{26}{3} & -\frac{8}{3}\\ 0 & -\frac{19}{3} & \frac{17}{3} & \frac{16}{3}\end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
E_3 &= E_3+\frac{1}{7}E_2\\
E_4 &= E_4+\frac{19}{7}E_2\\
E &= \begin{bmatrix}3 & 2 & -4 & 1\\ 0 & \frac{7}{3} & -\frac{8}{3} & \frac{11}{3}\\ 0 & 0 & \frac{58}{7} & -\frac{15}{7}\\ 0 & 0 & -\frac{11}{7} & \frac{107}{7}\end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
E_4 &= E_4+\frac{11}{58}E_3\\
E &= \begin{bmatrix}3 & 2 & -4 & 1\\ 0 & \frac{7}{3} & -\frac{8}{3} & \frac{11}{3}\\ 0 & 0 & \frac{58}{7} & -\frac{15}{7}\\ 0 & 0 & 0 & \frac{863}{58}\end{bmatrix}
\end{aligned}$$

$$\det(E) = (3)\left(\frac{7}{3}\right)\left(\frac{58}{7}\right)\left(\frac{863}{58}\right)$$
$$\det(E) = 863$$

$$\boxed{\det(E) = 863}$$