Homework 2

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Force Statics

1 Book

1.1 5.2

(a)

$$\begin{aligned}
\sum F_y &= 0 \\
T_{\text{wall, b}} - w_b &= 0 \\
T_{\text{wall, b}} &= w_b
\end{aligned}$$

$$\boxed{T_{\text{wall, b}} = w_b}$$

(b)

$$\begin{aligned}
\sum F_y^{(b_1)} &= 0 \\
T_{b_2,b_1} - w_{b_1} &= 0 \\
T_{b_2,b_1} &= w_{b_1} \\
\sum F_y^{(b_2)} &= 0 \\
T_{b_1,b_2} - w_{b_2} &= 0 \\
T_{b_1,b_2} &= w_{b_2} \\
T_{b_2,b_1} + T_{b_1,b_2} &= w_{b_1} + w_{b_2}
\end{aligned}$$

where

$$T_{b_1,b_2} = T_{b_2,b_1} \;\;\&\;\; w_{b_1} = w_{b_2}$$

$$\begin{aligned}
T + T &= w + w \\
2T &= 2w \\
T &= w
\end{aligned}$$

$$\boxed{T = w}$$

(c)

$$\begin{aligned}
\sum F_y^{(b_1)} &= 0 \\
T_{b_2,b_1} - w &= 0 \\
T_{b_2,b_1} &= w \\
\sum F_y^{(b_2)} &= 0 \\
T_{b_1,b_2} - w &= 0 \\
T_{b_1,b_2} &= w
\end{aligned}$$

where

$$T_{b_1,b_2} = T_{b_2,b_1}$$

$$\begin{aligned}
T + T &= w + w \\
2T &= 2w \\
T &= w
\end{aligned}$$

$$\boxed{T = w}$$

1.2 5.6

$$\begin{aligned}
b &= \text{ball} \\
m &= 3620\,\mathrm{kg} \\
\theta_{T_B,\hat{y}} &= 40^\circ
\end{aligned}$$

(a)

$$\begin{aligned}
T_B &= ? \\
\cos(\theta) &= \frac{m_b g}{T_B} \\
T_B &= \frac{m_b g}{\cos(\theta)} \\
&= \frac{3620\,\mathrm{kg} \cdot 10\,\mathrm{m\,s^{-2}}}{\cos(40^\circ)} \\
T_B &= 47\,255.7\,\mathrm{N}
\end{aligned}$$

$$\boxed{T_B = 47.3 \times 10^3\,\mathrm{N}}$$

(b)

$$\begin{aligned}
T_A &= ? \\
\theta_{T_B,\hat{x}} &= ? \\
\theta_{T_B,\hat{x}} &= 90^\circ - \theta_{T_B,\hat{y}} \\
&= 90^\circ - 40^\circ \\
\theta_{T_B,\hat{x}} &= 50^\circ \\
\cos(\theta_{T_B,\hat{x}}) &= \frac{T_{B_x}}{T_B} \\
T_{B_x} &= (T_B)\cos\left(\theta_{T_B,\hat{x}}\right) \\
&= \left(47.3 \times 10^3\,\mathrm{N}\right)\cos\left(50^\circ\right) \\
T_{B_x} &= 30\,403.9\,\mathrm{N} \\
\sum F_x^{(b)} &= 0 \\
T_{B_x} - T_A &= 0 \\
T_A &= T_{B_x} \\
T_A &= 30\,403.9\,\mathrm{N}
\end{aligned}$$

$$\boxed{T_A = 30.4 \times 10^3\,\mathrm{N}}$$

1.3 5.62

$$\begin{aligned}
T_{r,p_1} &= ? \\
T_{w,p_1} &= ? \\
w &= m_w g \\
T_{p_2,p_1} &= ? \\
T_{r,p_2} &= ? \\
\vec{F} &= ?
\end{aligned}$$

Based on the free body diagrams, it can be concluded that

$$T_{r,p_1} = T_{p_2,p_1} = \vec{F} \tag{1}$$

as they share a common rope.

Therefore the forces of p_1 in the $\hat{y}$ direction can be found as

$$\begin{aligned}
\sum F_y^{(p_1)} &= 0 \\
T_{r,p_1} + T_{p_2,p_1} - T_{w,p_1} &= 0 \\
T_{w,p_1} &= 2T
\end{aligned}$$

Finding $T_{p_1,w}$ from the free body diagram of the weight

$$\begin{aligned}\sum F_y^{(\text{weight})} &= 0 \\ T_{p_1,w} - w &= 0 \\ T_{p_1,w} &= w\end{aligned}$$

In order to withhold Newton's third law, the combined tension of T_{r,p_1} and T_{p_2,p_1} must equal T_{w,p_1} (as shown in Equation 1)

$$\begin{aligned}2T &= T_{w,p_1} \\ &= w \\ T &= \frac{w}{2}\end{aligned}$$

It can therefore be concluded that (according to *(1)*) $\vec{F}$ must equal T, finding the magnitude in terms of w

$$\boxed{\vec{F} = T = \frac{w}{2}}$$

1.4 5.64

(a)

(b)

$$\begin{aligned}m_{\text{ball}} &=? \\ \theta_{\hat{x},\text{ramp}} &= 35.0° \\ T_{\text{ramp,ball}} &=?\end{aligned}$$

Determine the normal force

$$\begin{aligned}\cos(\theta) &= \frac{N_{\text{ball}y}}{N_{\text{ball}}} \\ N_{\text{ball}} &= \frac{N_{\text{ball}y}}{\cos(\theta)} \\ \text{as well as: } N_{\text{ball}y} &= N_{\text{ball}}\cos(\theta)\end{aligned}$$

To find $N_{\text{ball}y}$, utilize the forces in the $\hat{y}$ direction

$$\begin{aligned}
\sum F_y^{(\text{ball})} &= 0 \\
N_{\text{ball}y} - m_{\text{ball}} g &= 0 \\
N_{\text{ball}y} &= m_{\text{ball}} g \\
N_{\text{ball}} \cos(\theta) &= m_{\text{ball}} g \\
N_{\text{ball}} &= \frac{m_{\text{ball}} g}{\cos(\theta)} \\
&= \frac{m_{\text{ball}} 10\,\mathrm{m\,s^{-2}}}{\cos(35.0°)} \\
N_{\text{ball}} &= (m_{\text{ball}})(12.2\,\mathrm{m\,s^{-2}})
\end{aligned}$$

$$\boxed{N_{\text{ball}} = (m_{\text{ball}})(12.2\,\mathrm{m\,s^{-2}})}$$

(c) Finding the tension in the wire requires finding the forces in $\hat{x}$ direction

$$\begin{aligned}
\sum F_x^{(\text{ball})} &= 0 \\
T_{\text{ramp,ball}} - N_{\text{ball}x} &= 0
\end{aligned}$$

Finding $N_{\text{ball}x}$

$$\begin{aligned}
\sin(\theta) &= \frac{N_{\text{ball}x}}{N_{\text{ball}}} \\
N_{\text{ball}x} &= N_{\text{ball}} \sin(\theta)
\end{aligned}$$

And using the value in the force equation above

$$\begin{aligned}
T_{\text{ramp,ball}} &= N_{\text{ball}} \sin(\theta) \\
&= (m_{\text{ball}})(12.2\,\mathrm{m\,s^{-2}}) \sin(35.0°) \\
T_{\text{ramp,ball}} &= (m_{\text{ball}})(7.00\,\mathrm{m\,s^{-2}})
\end{aligned}$$

$$\boxed{T_{\text{ramp,ball}} = (m_{\text{ball}})(7.00\,\mathrm{m\,s^{-2}})}$$

1.5 5.79

(a)

$$
\begin{aligned}
N_A &= ? \\
N_B &= ? \\
N_A &= N_B, \text{ Newton's Third Law} \\
N &= ? \\
m_A g &= 1.20\,\mathrm{N} \\
m_B g &= 3.60\,\mathrm{N} \\
\mu_k &= 0.300 \\
f &= \mu_k N \\
\vec{F} &= ?
\end{aligned}
$$

In order to find $\vec{F}$, the normal force is needed which can be found by observing the forces in the $\hat{y}$ direction

$$
\begin{aligned}
\sum \vec{F}_{\hat{y}}^{(\mathrm{A})} &= 0 \\
N_A - m_A g &= 0 \\
N_A &= m_A g
\end{aligned}
$$

$$
\begin{aligned}
\sum \vec{F}_{\hat{y}}^{(\mathrm{B})} &= 0 \\
N - m_B g - N_B &= ? \\
N &= m_B g + N_B \\
&= m_B g + m_A g \\
&= 1.20\,\mathrm{N} + 3.60\,\mathrm{N} \\
N &= 4.80\,\mathrm{N}
\end{aligned}
$$

$$
f = \mu_k N = (0.300)(4.80\,\mathrm{N}) = 1.44\,\mathrm{N}
$$

Now finding $\vec{F}$

$$
\begin{aligned}
\sum \vec{F}_{\hat{x}}^{(B)} &= 0 \\
-\vec{F} + f &= 0 \\
\vec{F} &= f \\
&= 1.44\,\mathrm{N} \\
\vec{F} &= 1.44\,\mathrm{N}
\end{aligned}
$$

$$
\boxed{\vec{F} = 1.44\,\mathrm{N}}
$$

(b)

$$\begin{aligned}
N_A &= ? \\
N_B &= ? \\
N_A &= N_B \\
f_A &= ? \\
f_B &= ? \\
f_A &= f_B, \text{ Newton's Third Law} \\
T_{\text{wall},A} &= ? \\
m_A g &= 1.20\,\mathrm{N} \\
N &= ? \\
m_B g &= 3.60\,\mathrm{N} \\
\vec{F} &= ?
\end{aligned}$$

First determine the forces in the $\hat{x}$ direction of block A to find tension

$$\begin{aligned}
\sum \vec{F}_{\hat{x}}^{(A)} &= 0 \\
T_{\text{wall},A} - f_A &= 0 \\
f_A &= T_{\text{wall},A}
\end{aligned}$$

Similarly to part **(a)**, find N_A using the $\hat{y}$ forces of block A

$$\begin{aligned}
\sum \vec{F}_{\hat{y}}^{(A)} &= 0 \\
N_A - m_A g &= 0 \\
N_A &= m_A g \\
N_A &= 1.20\,\mathrm{N}
\end{aligned}$$

The friction of block A upon block B can now be calculated, and further utilized through f_B due to Newton's Third Law

$$f_A = \mu_k N_A = (0.300)(1.20\,\mathrm{N}) = 0.360\,\mathrm{N}$$

Find N to aid in finding the friction between the ground and block B

$$\begin{aligned}
\sum \vec{F}_{\hat{y}}^{(B)} &= 0 \\
N - m_B g - N_B &= 0 \\
N &= m_B g + N_B \\
&= 3.60\,\mathrm{N} + 1.20\,\mathrm{N} \\
N &= 4.80\,\mathrm{N}
\end{aligned}$$

Solve for the forces in the $\hat{x}$ direction of block B to finally compute the pulling force

$$\begin{aligned}
\sum \vec{F}_{\hat{x}}^{(B)} &= 0 \\
f + f_B - \vec{F} &= 0 \\
\vec{F} &= f + f_B \\
&= \mu_k N + 0.360\,\mathrm{N} \\
&= (0.300)(4.80\,\mathrm{N}) + 0.360\,\mathrm{N} \\
\vec{F} &= 1.80\,\mathrm{N}
\end{aligned}$$

$$\boxed{\vec{F} = 1.80\,\mathrm{N}}$$

2 Lab Manual

2.1 270

(a)

$$\begin{aligned}
r_A &=? \\
r_B &=? \\
r_C &=? \\
r_A &= r_B = r_C \\
R &=? \\
\theta &=?
\end{aligned}$$

The center of mass of all three identical cylinders created 60° angles at each corner. This makes the angle of the top corner of A, $\frac{60^\circ}{2} = 30^\circ$ when focusing on only one half of the illustration.

$$\begin{aligned}
\cos(30^\circ) &= \frac{m_A g}{2r_A} \\
r_A &= \frac{m_A g}{2\cos(30^\circ)} \\
r_A &= \frac{m_A g}{\sqrt{3}} \\
m_A g &= \sqrt{3} r_A
\end{aligned}$$

It can be observed that the opposite side of both θ and the angle between

$m_A g$ and $2r_A$ are the same

$$\begin{aligned}
\sin(\theta) &= \frac{\frac{r_B}{2}}{R} \\
\sin(30°) &= \frac{\frac{r_B}{2}}{r_A} \\
r_A \sin(30°) &= R\sin(\theta) \\
\frac{m_A g}{\sqrt{3}} \sin(30°) &= R\sin(\theta) \\
R\sin(\theta) &= \frac{m_A g}{2\sqrt{3}}
\end{aligned}$$

Half of the opposite side of θ has been found, now the adjacent is required. The adjacent can be observed as the combined forces in the $\hat{y}$ direction of the illustration

$$\begin{aligned}
&\sum F_y = \cos(30°)(N_{B,A} - N_{A,B} + N_{C,A} - N_{A,C}) - m_A g - m_B g - m_C g \\
&\sum F_y = 3mg\text{, divide by 2 to get the half's weight} \\
&\sum F_y = \frac{3mg}{2}
\end{aligned}$$

$$\begin{aligned}
\cos(\theta) &= \frac{F_y}{R} \\
&= \frac{\frac{3mg}{2}}{R} \\
R\cos(\theta) &= \frac{3mg}{2}
\end{aligned}$$

Therefore solving for $\tan(\theta)$

$$\begin{aligned}
\tan(\theta) &= \frac{\sin(\theta)}{\cos(\theta)} \\
&= \frac{\frac{mg}{2R\sqrt{3}}}{\frac{3mg}{2R}} \\
&= \frac{(mg)(2R)}{(2R\sqrt{3})(3mg)} \\
\tan(\theta) &= \frac{1}{3\sqrt{3}}
\end{aligned}$$

$$\boxed{\tan(\theta) = \frac{1}{3\sqrt{3}}}$$

(b)

2.2 273

2.3 274

2.4 287

2.5 290