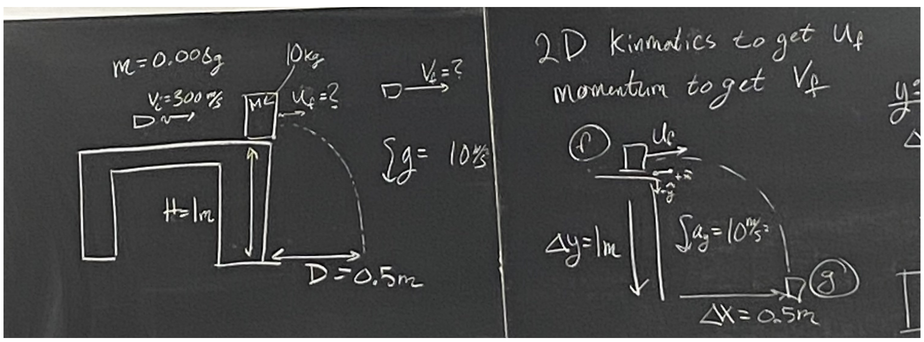
Contents

1 Energy

1.1 Example



$\hat{y}$ Direction

$$\Delta y = v_{f_y} t + \frac{1}{2} a_y t^2$$
$$t = \sqrt{\frac{2\Delta y}{a_y}}$$
$$t = \sqrt{\frac{2(1\,\mathrm{m})}{10\,\mathrm{m\,s^{-2}}}}$$
$$t = 0.45\,\mathrm{s}$$

$\hat{x}$ Direction

$$\Delta x = u_{f_x} t$$
$$u_{f_x} = \frac{\Delta x}{t}$$
$$u_{f_x} = \frac{0.5\,\mathrm{m}}{0.45\,\mathrm{s}}$$
$$u_f = 1.11\,\mathrm{m\,s^{-1}}$$

$$\begin{aligned}
\sum p_i &= \sum p_f \\
mv_i &= Mu_f + Mv_f \\
v_f &= \frac{mv_i - Mu_f}{m} \\
v_f &= \frac{(8\times10^{-6}\,\mathrm{kg})(3\times10^{2}\,\mathrm{kg}) - (10\,\mathrm{kg})(1.1\,\mathrm{m\,s^{-1}})}{8\times10^{-6}\,\mathrm{kg}} \\
v_f &= -1\times10^{6}\,\mathrm{m\,s^{-1}}
\end{aligned}$$

1.2 Example

$$\begin{aligned}
m &= 3\,\mathrm{kg} \\
v_i &= 12\,\mathrm{m\,s^{-1}} \\
M &= 5\,\mathrm{kg} \\
u_i &= 4\,\mathrm{m\,s^{-1}}
\end{aligned}$$

1) What is the total momentum of the system?

$$\begin{aligned}
\vec{P}_i &= mv_i\hat{x} - Mu_i\hat{x} \\
\vec{P}_i &= (36\,\mathrm{kg\,m\,s^{-1}})\hat{x} - (20\,\mathrm{kg\,m\,s^{-1}})\hat{x} \\
\vec{P}_i &= 16\,\mathrm{kg\,m\,s^{-1}}
\end{aligned}$$

2) Inelastic:

$$\begin{aligned}
v_f &= ? \\
u_f &= ?
\end{aligned}$$

$$\begin{aligned}
P_i &= P_f \\
mv_i - Mu_i &= (m+M)v_f \\
v_f &= \frac{mv_i - Mu_i}{m+M} = v_{CM} \\
v_f &= \frac{16\,\mathrm{kg\,m\,s^{-1}}}{8\,\mathrm{kg}} \\
v_f &= 2\,\mathrm{m\,s^{-1}} = u_f
\end{aligned}$$

3) Elastic:

$$\begin{aligned}
v_f &= ? \\
u_f &= ?
\end{aligned}$$

$$\begin{aligned}
\sum P_i &= \sum P_f \\
mv_i - Mu_i &= -mv_f + Mu_f
\end{aligned}$$

Find two unknowns (v_f, u_f):

$$(3\,\mathrm{kg})(12\,\mathrm{m\,s^{-1}}) - (5\,\mathrm{kg})(4\,\mathrm{m\,s^{-1}}) = -(3\,\mathrm{kg})v_f + (5\,\mathrm{kg})u_f$$
$$-(3\,\mathrm{kg})v_f + (5\,\mathrm{kg})u_f = 16\,\mathrm{kg\,m\,s^{-1}}$$

$$\begin{aligned}
E_i &= E_f \\
\frac{1}{2}mv_i^2 + \frac{1}{2}Mu_i^2 &= \frac{1}{2}mv_f^2 + \frac{1}{2}Mu_f^2 \\
mv_i^2 + Mu_i^2 &= mv_f^2 + Mu_f^2 \\
(3\,\mathrm{kg})v_f^2 + (5\,\mathrm{kg})u_f^2 &= (3\,\mathrm{kg})(12\,\mathrm{m\,s^{-1}})^2 + (5\,\mathrm{kg})(4\,\mathrm{m\,s^{-1}})^2 \\
(3\,\mathrm{kg})v_f^2 + (5\,\mathrm{kg})u_f^2 &= 512\,\mathrm{J}
\end{aligned}$$

$$\begin{aligned}
-(3\,\mathrm{kg})v_f + (5\,\mathrm{kg})u_f &= 16\,\mathrm{kg\,m\,s^{-1}} \\
u_f &= 3.2\,\mathrm{m\,s^{-1}} + (0.6)v_f \\
(3\,\mathrm{kg})v_f^2 + (5\,\mathrm{kg})(3.2\,\mathrm{m\,s^{-1}} + (0.6)v_f)^2 &= 512\,\mathrm{J} \\
(4.8)v_f^2 + 19.2v_f + 51.2 &= 512\,\mathrm{J} \\
v_f &= 8\,\mathrm{m\,s^{-1}}
\end{aligned}$$

4) Finding v_{CM}

$$\begin{aligned}
v_{CM} &= \frac{mv_i - Mu_i}{m+M}\hat{x} \\
v_{CM} &= \frac{36\,\mathrm{kg\,m\,s^{-1}} - 20\,\mathrm{kg\,m\,s^{-1}}}{8\,\mathrm{kg}}\hat{x} \\
v_{CM} &= 2\,\mathrm{m\,s^{-1}}\hat{x}
\end{aligned}$$

To boost into the ZMF, subtract the vector v_{CM} from all velocities.

initial	LAB	$-v_{CM}$	ZMF
$\vec{v}$	$12\,\mathrm{m\,s^{-1}}$	-($2\,\mathrm{m\,s^{-1}}$)	$10\,\mathrm{m\,s^{-1}}$
$\vec{u}$	-$4\,\mathrm{m\,s^{-1}}$	-($2\,\mathrm{m\,s^{-1}}$)	-($6\,\mathrm{m\,s^{-1}}$)

5) Restitution

$$\begin{aligned}
m\vec{v}_f^{ZMF} &= -\epsilon m\vec{v}_i^{ZMF} \\
\vec{v}_f^{ZMF} &= -\epsilon\vec{v}_i^{ZMF} \\
\epsilon &= 1 \therefore \\
\vec{v}_f^{ZMF} &= -\vec{v}_i^{ZMF} = -10\,\mathrm{m\,s^{-1}}\hat{x} \\
\vec{u}_f^{ZMF} &= -\vec{u}_i^{ZMF} = 6\,\mathrm{m\,s^{-1}}\hat{x}
\end{aligned}$$

find	ZMF	v_{CM}	LAB
$\vec{v}$	-$10\,\mathrm{m\,s^{-1}}$	$2\,\mathrm{m\,s^{-1}}$	-$8\,\mathrm{m\,s^{-1}}$
$\vec{u}$	$6\,\mathrm{m\,s^{-1}}$	$2\,\mathrm{m\,s^{-1}}$	$8\,\mathrm{m\,s^{-1}}$

2 Collision

In a free space collision which conserves momentum:

$$\sum F = \frac{dp}{dt} = 0ma_{CM} = 0 \rightarrow a_{CM} = 0$$
$$\frac{dv_{CM}}{dt} = 0 \rightarrow v_{CM} = \text{constant}$$

The problems for momentum are usually given in the LAB frame (lit. in the laboratory), however if we co-more with the center of mass momentum problems become trivial. This frame is the <u>Zero Momentum Frame</u> or <u>ZMF</u>.

To find the ZMF,

$$v_{CM} = \frac{\sum \vec{P}}{\sum m} \tag{1}$$

$$\vec{P}_f^{ZMF} = -\epsilon \vec{P}_i^{ZMF} \tag{2}$$

<u>Coefficient of Restitution</u> - a description of how much the relative speed changes in a collision.

- Means relative speed is fully conserved.

$$\epsilon = 1$$

- Means $v_{ref_f} = 0$

$$\epsilon = 0$$

- Reflects some speed loss.

$$0 < \epsilon < 1$$

2.1 Example

Two masses collide head-on

$$m = 3\,\mathrm{kg}$$
$$v_i = 5\,\mathrm{m\,s^{-1}}$$
$$M = 5\,\mathrm{kg}$$
$$u_i = 8\,\mathrm{m\,s^{-1}}$$

Find final speeds, elastic, inelastic, $\epsilon = 0.4$.

1) Final Speeds

$$\begin{aligned}
\sum P_i &= \sum P_f \\
mv_i - Mu_i &= -mv_f + Mu_f \\
(3\,\mathrm{kg})(5\,\mathrm{m\,s^{-1}}) - (5\,\mathrm{kg})(8\,\mathrm{m\,s^{-1}}) &= -(3\,\mathrm{kg})v_f + (5\,\mathrm{kg})u_f \\
-(3\,\mathrm{kg})v_f + (5\,\mathrm{kg})u_f &= -25\,\mathrm{kg\,m\,s^{-1}} \\
u_f &= -5\,\mathrm{m\,s^{-1}} + (0.6)v_f
\end{aligned}$$

$$\begin{aligned}
E_i &= E_f \\
mv_i^2 + Mu_i^2 &= mv_f^2 + Mu_f^2 \\
(3\,\mathrm{kg})(5\,\mathrm{m\,s^{-1}})^2 + (5\,\mathrm{kg})(8\,\mathrm{m\,s^{-1}})^2 &= (3\,\mathrm{kg})v_f^2 + (5\,\mathrm{kg})u_f^2 \\
(3\,\mathrm{kg})v_f^2 + (5\,\mathrm{kg})u_f^2 &= 395\,\mathrm{J}
\end{aligned}$$

$$\begin{aligned}
(3\,\mathrm{kg})v_f^2 + (5\,\mathrm{kg})(-5\,\mathrm{m\,s^{-1}} + (0.6)v_f)^2 &= 395\,\mathrm{J} \\
(4.8)v_f^2 - (30)v_f + 125 &= 395\,\mathrm{J} \\
v_f &= 11.25\,\mathrm{m\,s^{-1}}, -5.0\,\mathrm{m\,s^{-1}}
\end{aligned}$$