Week 01 Participation Assignment

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Contents

1 Problem 1

Use a truth table to verify the first De Morgan law: $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

The truth values for the compound propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are the same, and are therefore logically equivalent thereby verifying the first De Morgan law.

2 Problem 2

Verify by showing either both sides are true, or both sides are false, for exactly the same combinations of truth values of the propositional variables in these expressions (except for 18).

2.1 16

Show that $p \iff q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

2.1.1 Truth Table

p	q	$p \iff q$	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
T	T	T	T	F	F	F	T
T	F	F	F	F	T	F	F
F	T	F	F	T	F	F	F
F	F	T	F	T	T	T	T

2.1.2 Proof

Start by observing that $p \iff q$ is true only if $(p = T \wedge q = T) \vee (p = F \wedge q = F)$
Looking at the second compound proposition,

2.2 17

Show that $\neg(p \iff q)$ and $p \iff \neg q$ are logically equivalent.

p	q	$p \iff q$	$\neg(p \iff q)$	$\neg q$	$p \iff \neg q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	T
F	F	T	F	T	F

2.3 18

Show that $p \implies q$ and $\neg q \implies \neg p$ are logically equivalent.

p	q	$p \implies q$	$\neg q$	$\neg p$	$\neg q \implies \neg p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

2.4 19

Show that $\neg p \iff q$ and $p \iff \neg q$ are logically equivalent.

p	$\neg p$	q	$\neg p \iff q$	$\neg q$	$p \iff \neg q$
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	T	F	T
F	T	F	F	T	F

2.5 20

Show that $\neg(p \oplus q)$ and $p \iff q$ are logically equivalent.

p	q	$p \oplus q$	$\neg(p \oplus q)$	$p \iff q$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

2.6 21

Show that $\neg(p \iff q)$ and $\neg p \iff q$ are logically equivalent.

p	q	$p \iff q$	$\neg(p \iff q)$	$\neg p$	$\neg p \iff q$
T	T	T	F	F	F
T	F	F	T	F	T
F	T	F	T	T	T
F	F	T	F	T	F

2.7 22

Show that $(p \implies q) \wedge (p \implies r)$ and $p \implies (q \wedge r)$ are logically equivalent.

p	q	r	$p \implies q$	$p \implies r$	$(p \implies q) \wedge (p \implies r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

p	q	r	$q \wedge r$	$p \implies (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

2.8 23

Show that $(p \implies r) \wedge (q \implies r)$ and $(p \vee q) \implies r$ are logically equivalent.

p	q	r	$p \implies r$	$q \implies r$	$(p \implies r) \land (q \implies r)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

p	q	r	$p \lor q$	$(p \lor q) \implies r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

2.9 24

Show that $(p \implies q) \lor (p \implies r)$ and $p \implies (q \lor r)$ are logically equivalent.

p	q	r	$p \implies q$	$p \implies r$	$(p \implies q) \lor (p \implies r)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

p	q	r	$q \lor r$	$p \implies (q \lor r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

2.10 25

Show that $(p \implies r) \lor (q \implies r)$ and $(p \land q) \implies r$ are logically equivalent.

p	q	r	$p \implies r$	$q \implies r$	$(p \implies r) \vee (q \implies r)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

p	q	r	$p \wedge q$	$(p \wedge q) \implies r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

2.11 26

Show that $\neg p \implies (q \implies r)$ and $q \implies (p \vee r)$ are logically equivalent.

p	q	r	$\neg p$	$q \implies r$	$\neg p \implies (q \implies r)$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

p	q	r	$p \vee r$	$q \implies (p \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	F	T

2.12 27

Show that $p \iff q$ and $(p \implies q) \wedge (q \implies p)$ are logically equivalent.

p	q	$p \iff q$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	$p \implies q$	$q \implies p$	$(p \implies q) \wedge (q \implies p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

2.13 28

Show that $p \iff q$ and $\neg p \iff \neg q$ are logically equivalent.

p	q	$p \iff q$	$\neg p$	$\neg q$	$\neg p \iff \neg q$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T