

Chapter 1 - Vectors & Scalars

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1 Introduction to Vectors & Scalars

1.1 Vectors & Scalars

Scalar = Magnitude

Vector = Magnitude + Direction

Vectors are represented by arrows – tail and tip



Triangle addition:

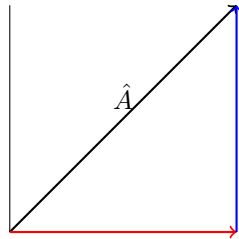
$$\hat{A} = 20 \text{ @ } 40^\circ \text{ above } +\hat{x}$$

$\hat{x} \rightarrow$ Unit vector gives direction with Magnitude 1 "the x direction"

Vector B is 45 @ 60 ° Right of $+\hat{y}$

1.2 Algebra as an alternative to Euclidean Geometry

In order to add vectors, first break vectors into components

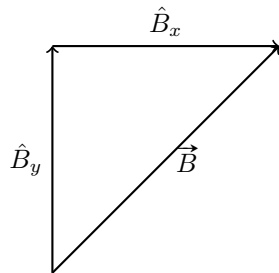


Any vector is the sum of its components

$$\hat{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

(1)

$$\hat{A} = A \cos(\theta) \hat{x} + A \sin(\theta) \hat{y} (+0\hat{z})$$



Write \vec{B} in component form:

$$\vec{B} = B \sin(\phi) \hat{x} + B \cos(\phi) \hat{y}$$

$$= (45) \sin(60^\circ) \hat{x} + (45) \cos(60^\circ) \hat{y}$$

$$\boxed{\vec{B} = 39\hat{x} + 23\hat{y}}$$

To add two vectors, add each direction separately:

$$\begin{aligned} &15\hat{x} + 13\hat{y} \\ &39\hat{x} + 23\hat{y} \\ \vec{A} + \vec{B} &= 54\hat{x} + 36\hat{y} \end{aligned}$$

To subtract two vectors, subtract each direction separately:

$$\begin{aligned} &15\hat{x} + 13\hat{y} \\ &39\hat{x} + 23\hat{y} \\ \vec{A} - \vec{B} &= -24\hat{x} - 10\hat{y} \end{aligned}$$

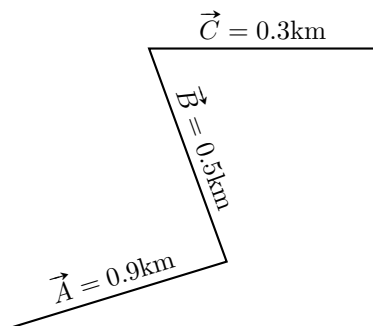
To compose (get magnitude and direction) of a vector, use:

$$\begin{aligned} |\vec{A}| &= \sqrt{A_x^2 + A_y^2 + A_z^2} \\ \theta_{\vec{A}} &= \arctan\left(\frac{A_i}{A_j}\right) \\ |\vec{A} + \vec{B}| &= \sqrt{(A+B)_x^2 + (A+B)_y^2} \\ &= \sqrt{54^2 + 36^2} = 65 \\ \theta &= \arctan\left(\frac{(A+B)_y}{(A+B)_x}\right) = \arctan\left(\frac{36}{54}\right) = 34^\circ \end{aligned}$$

1.3 Scalar Multiplication

$$\begin{aligned} \gamma \vec{A} &= \gamma A_x \hat{x} + \gamma A_y \hat{y} + \gamma A_z \hat{z} \\ -\vec{A} &= (-A_x) \hat{x} + (-A_y) \hat{y} + (-A_z) \hat{z} \end{aligned}$$

Given three vectors connected from head to tail, find the distance from the tail of \vec{A} to the head of \vec{C} :



$$\begin{aligned}
\vec{C} &= 0.3\text{km}\hat{x} \\
\vec{A} &= +A \cos(\theta)\hat{x} + A \sin(\theta)\hat{y} \\
&= (0.9\text{km}) \cos(17^\circ)\hat{x} + (0.9\text{km}) \sin(17^\circ)\hat{y} \\
\vec{A} &= 0.86\text{km}\hat{x} + 0.26\text{km}\hat{y} \\
\vec{B} &= -B \sin(\phi)\hat{x} + B \cos(\phi)\hat{y} \\
\vec{B} &= -0.17\text{km}\hat{x} + 0.47\text{km}\hat{y}
\end{aligned}$$

$$\begin{aligned}
\vec{A} &= 0.86\text{km}\hat{x} + 0.26\text{km}\hat{y} \\
\vec{B} &= -0.17\text{km}\hat{x} + 0.47\text{km}\hat{y} \\
\vec{C} &= 0.3\text{km}\hat{x} + 0\hat{y}
\end{aligned}$$

$$\boxed{\vec{A} + \vec{B} + \vec{C} = 0.99\text{km}\hat{x} + 0.73\text{km}\hat{y}}$$

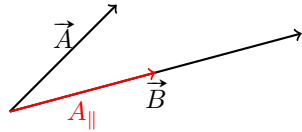
1.4 Multiplying Vectors

A dot product multiplies two vectors and returns a scalar.

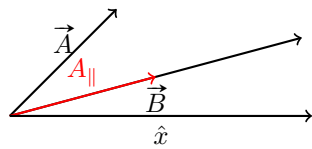
Notation:

$$\vec{A} \cdot \vec{B}$$

Essentially means: "What part of \vec{A} lies in the direction of \vec{B} ?"



Another example to show importance of angle between vectors:



$$\begin{aligned}
A_{\parallel} &= A \cos(\theta)_{AB} \\
\vec{A} \cdot \vec{B} &= (A \cos(\theta)_{AB}) (B) \\
\vec{A} \cdot \vec{B} &= (20 \cos(10^\circ)) (45)
\end{aligned}$$

$$\boxed{\vec{A} \cdot \vec{B} = 886}$$

* Where θ_{AB} is the angle between \vec{A} and \vec{B}

Dot product in three-dimensions:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} = 15\hat{x} + 13\hat{y}$$

$$\vec{B} = 39\hat{x} + 23\hat{y}$$

$$\begin{aligned}\vec{A} \cdot \vec{B} &= A_x B_x + A_y B_y \\ &= (15)(39) + (13)(23)\end{aligned}$$

$$\boxed{\vec{A} \cdot \vec{B} = 884}$$

$$\vec{A} \cdot \vec{B} = AB \cos(\theta)_{AB}$$

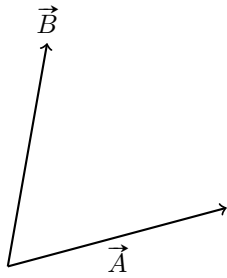
$$\cos(\theta)_{AB} = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$$

$$\theta_{AB} = \arccos\left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}\right)$$

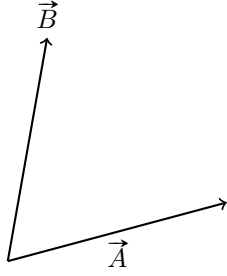
$$\theta_{AB} = \arccos\left(\frac{884}{(20)(45)}\right) = 11^\circ$$

1.5 Cross Product

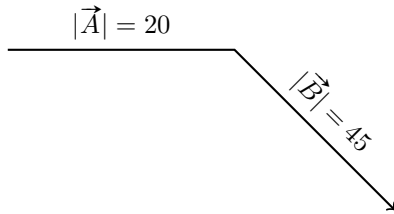
Counter-clockwise rotation - out:



Clockwise rotation - in:



$$|\vec{A} \times \vec{B}| = AB_{\perp}$$



$$\begin{aligned} |\vec{A} \times \vec{B}| &= AB_{\perp} \\ &= (20)(45) \sin(10^{\circ}) \\ \vec{A} \times \vec{B} &= -156\hat{z} \rightarrow (\text{clockwise}) \end{aligned}$$

Calculating mathematically (using determinant):

$$\begin{aligned} \vec{A} \times \vec{B} &= \hat{x}A_yB_z + \hat{y}A_zB_x + \hat{z}A_xB_y - \hat{x}A_zB_y - \hat{y}A_xB_z - \hat{z}A_yB_x \\ \vec{A} \times \vec{B} &= \hat{x}(A_yB_z - A_zB_y) + \hat{y}(A_zB_x - A_xB_z) + \hat{z}(A_xB_y - A_yB_x) \end{aligned}$$

1.6 Sphere Calculation

* *Three-dimensional graph omitted*

$$\begin{aligned} z &= r \cos(\theta) \\ \rho &= r \sin(\theta) \\ x &= \rho \cos(\phi) \\ y &= \rho \sin(\phi) \\ x &= r \sin(\theta) \cos(\phi) \\ y &= r \sin(\theta) \sin(\phi) \\ z &= r \cos(\theta) \end{aligned}$$