1 Part 3

After finding the complex eigenvectors from the previous assignment, it's time to solve the system of first order linear differential equations $\vec{x}' = A\vec{x}$ completely, where A is given as followed:

1) $\begin{bmatrix} -1 & -1 & 0 \\ 2 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

2) $\begin{bmatrix} 5 & -5 & -5 \\ -1 & 4 & 2 \\ 3 & -5 & -3 \end{bmatrix}$

1.1 1)

$$\mathbf{x} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + i \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\mathbf{x}(t) = e^{(-1+i)t} \begin{bmatrix} -1 \\ i \\ 1 \end{bmatrix}$$

$$\mathbf{x}(t) = e^{-t}e^{it} \begin{bmatrix} -1 \\ i \\ 1 \end{bmatrix}$$

$$\mathbf{x}(t) = e^{-t}(\cos(t) + i\sin(t)) \begin{bmatrix} -1 \\ i \\ 1 \end{bmatrix}$$

$$\mathbf{x}(t) = e^{-t} \begin{bmatrix} -\cos(t) - i\sin(t) \\ i\cos(t) - \sin(t) \\ \cos(t) + i\sin(t) \end{bmatrix}$$

$$\mathbf{x}(t) = e^{-t} \begin{bmatrix} -\cos(t) \\ -\sin(t) \\ \cos(t) \end{bmatrix} + ie^{-t} \begin{bmatrix} -\sin(t) \\ \cos(t) \\ \sin(t) \end{bmatrix}$$

General Solution:

$$\boxed{\mathbf{x}(t) = C_0 e^{-t} \begin{bmatrix} -\cos(t) \\ -\sin(t) \\ \cos(t) \end{bmatrix} + C_1 ie^{-t} \begin{bmatrix} -\sin(t) \\ \cos(t) \\ \sin(t) \end{bmatrix}}$$

1.2 2)

$$\mathbf{x} = \begin{bmatrix} 1 \\ -\frac{2}{5} \\ 1 \end{bmatrix} - i \begin{bmatrix} 0 \\ \frac{1}{5} \\ 0 \end{bmatrix}$$

$$\mathbf{x}(t) = e^{(2+i)t}\begin{bmatrix} 1 \\ -\frac{2}{5} - \frac{i}{5} \\ 1 \end{bmatrix}$$

$$\mathbf{x}(t) = e^{2t}e^{it}\begin{bmatrix} 1 \\ -\frac{2}{5} - \frac{i}{5} \\ 1 \end{bmatrix}$$

$$\mathbf{x}(t) = e^{2t}(\cos(t) + i\sin(t))\begin{bmatrix} 1 \\ -\frac{2}{5} - \frac{i}{5} \\ 1 \end{bmatrix}$$

$$\mathbf{x}(t) = e^{2t}\begin{bmatrix} \cos(t) + i\sin(t) \\ -\frac{2\cos(t)}{5} - \frac{i\cos(t)}{5} - \frac{2i\sin(t)}{5} + \frac{\sin(t)}{5} \\ \cos(t) + i\sin(t) \end{bmatrix}$$

$$\mathbf{x}(t) = e^{2t}\begin{bmatrix} \cos(t) \\ \frac{\sin(t)-2\cos(t)}{5} \\ \cos(t) \end{bmatrix} + ie^{2t}\begin{bmatrix} \sin(t) \\ -\frac{\cos(t)+2\sin(t)}{5} \\ \sin(t) \end{bmatrix}$$

General Solution:

$$\boxed{\mathbf{x}(t) = C_0e^{2t}\begin{bmatrix} \cos(t) \\ \frac{\sin(t)-2\cos(t)}{5} \\ \cos(t) \end{bmatrix} + C_1ie^{2t}\begin{bmatrix} \sin(t) \\ -\frac{\cos(t)+2\sin(t)}{5} \\ \sin(t) \end{bmatrix}}$$