Contents

1 Chapter 02 - Basic Structures

1.1 Sets

$\in$: belong to, is in

1.1.1 Definition 2

Two sets are equal if and only if they have the same elements. Therefore, if A and B are sets, then A and B are equal if and only if $\forall x\,(x \in A \iff x \in B)$. We write $A = B$ if A and B are equal sets.

1.1.2 Definition 3

The set A is also a subset of B, and B is a superset of A, if and only if every element of A is also an element of B. We use the notation $A \subseteq B$ to indicate that A is a subset of the set B. If, instead, we want to stress that B is a superset of A, we use the equivalent notation $B \supseteq A$. (So, $A \subseteq B$ and $B \supseteq A$ are equivalent statements.)

1.1.3 Definition 4

Let S be a set. If there are exactly n distinct elements in S where n is a nonnegative integer, we say that S is a finite set and that n is the cardinality of S. The cardinality of S is denoted by $|S|$.

1.1.4 Countable and Uncountable Sets

- Countable
  - $\mathbb{N}$
  - $\mathbb{Z}$
  - $\mathbb{Q}$
- Uncountable
  - $\mathbb{R}$
  - $\mathbb{C}$

Let $S_0 = \{x\}$, and $S_1 = \{\{x\}\}$.

$$S_0 \neq S_1 \tag{1}$$

1.1.5 Example

1. List the members of these sets.

   a) $\{x \mid x \text{ is a real number such that } x^2 = 1\}$

   $$S = \{x \in \mathbb{R} \mid x^2 = 1\}$$

   b) $\{x \mid x \text{ is a positive integer less than } 12\}$

   $$S = \{x \in \mathbb{R} \mid 0 \leq x < 12\}$$

1.1.6 Definition 6

Given a set S, the power set of S is the set of all subsets of the set S. The power set of S is denoted by $\mathcal{P}(S)$.

1.2 Set Operations

1.2.1 Definition 1

Let A and B be sets. The union of the sets A and B, denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.

$$A \cup B = \{x \in U \mid (x \in A) \vee (x \in B)\} \tag{2}$$

1.2.2 Definition 2

Let A and B be sets. The intersection of the sets A and B, denoted by $A \cap B$, is the set containing those elements in both A and B.

$$A \cap B = \{x \in U \mid (x \in A) \wedge (x \in B)\} \tag{3}$$

1.2.3 Definition 3

Two sets are disjoint if their intersection is the empty set.

1.2.4 Definition 4

Let A and B be sets. The difference of A and B, denoted by $A - B$, is the set containing those elements that are in A but not in B. The difference of A and B is also called the complement of B with respect to A.

1.2.5 Definition 5

Let U be the universal set. The complement of the set A, denoted by $\bar{A}$, is the complement of A with respect to U. Therefore, the complement of the set A is $U - A$.

1.3 Functions

1.4 Sequences and Summations

1.5 Cardinality of Sets

1.6 Matrices