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1 Section 6.1

1.1 6.1.1

Find the (real) eigenvalues and associated eigenvectors of the larger matrix $\boldsymbol{A}$. Find a basis of each eigenspace of dimension 2 or larger.

$$\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$$

$$\begin{aligned} |\boldsymbol{A} - \lambda \mathbf{I}| &= \begin{bmatrix} 4-\lambda & -1 \\ 2 & 1-\lambda \end{bmatrix} \\ \det(\boldsymbol{A}) &= (4-\lambda)(1-\lambda) - (-1)(2) \\ \det(\boldsymbol{A}) &= \lambda^2 - 5\lambda + 2 = (\lambda-2)(\lambda-3) \\ \lambda &= 2, 3 \end{aligned}$$

$$\begin{aligned} \lambda &= 2 \\ \boldsymbol{A} - \lambda \mathbf{I} &= \begin{bmatrix} 4-\lambda & -1 \\ 2 & 1-\lambda \end{bmatrix} \\ \boldsymbol{A} - \lambda \mathbf{I} &= \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} (\boldsymbol{A} - \lambda \mathbf{I})\mathbf{v} &= 0 \\ \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ 2x - y &= 0 \\ x &= \frac{1}{2}y \\ 2\left(\frac{1}{2}y\right) - y &= 0 \\ 0 &= 0 \end{aligned}$$

$$\lambda = 2 \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2}y \\ y \end{bmatrix} = y \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}$$

$$\begin{aligned}
\lambda &= 3 \\
\boldsymbol{A} - \lambda\mathbf{I} &= \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \\
(\boldsymbol{A} - \lambda\mathbf{I})\mathbf{v} &= 0 \\
\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
x &= y \\
2x &= 2y \\
2(y) &= 2y \\
0 &= 0
\end{aligned}$$

$$\lambda = 3 \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ y \end{bmatrix} = y \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

1.2 6.1.2

Find the (real) eigenvalues and associated eigenvectors of the given matrix $\boldsymbol{A}$. Find a basis of each eigenspace of dimension 2 or larger.

$$\begin{bmatrix} 6 & -7 \\ 4 & -5 \end{bmatrix}$$

$$\begin{aligned}
|\boldsymbol{A} - \lambda\mathbf{I}| &= \begin{bmatrix} 6-\lambda & -7 \\ 4 & -5-\lambda \end{bmatrix} \\
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= (6-\lambda)(-5-\lambda) - (-7)(4) \\
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= \lambda^2 - \lambda - 2 = (\lambda-2)(\lambda+1) \\
\lambda_{1,2} &= 2, -1
\end{aligned}$$

$$\begin{aligned}
\lambda_1 &= 2 \\
[\boldsymbol{A} - \lambda_1]\,\mathbf{x} &= 0 \\
\begin{bmatrix} 6-2 & -7 \\ 4 & -5-2 \end{bmatrix} \mathbf{x} &= 0 \\
\begin{bmatrix} 4 & -7 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= 0 \\
(4)x_1 + (-7)x_2 &= 0 \\
(4)x_1 + (-7)x_2 &= 0
\end{aligned}$$

$$\begin{aligned}
(4)x_1 &= (7)x_2 \\
x_1 &= \left(\frac{7}{4}\right) x_2
\end{aligned}$$

$$
\begin{aligned}
(4)x_1 + (-7)x_2 &= 0 \\
(4)\left(\frac{7}{4}\right)x_2 + (-7)x_2 &= 0 \\
0 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{x} &= \begin{bmatrix} \left(\frac{7}{4}\right)x_2 \\ x_2 \end{bmatrix} \\
\mathbf{x} &= x_2 \begin{bmatrix} \frac{7}{4} \\ 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\lambda_2 &= -1 \\
[\boldsymbol{A} - \lambda_2]\,\mathbf{x} &= 0 \\
\begin{bmatrix} 6-(-1) & -7 \\ 4 & -5-(-1) \end{bmatrix} \mathbf{x} &= 0 \\
\begin{bmatrix} 7 & -7 \\ 4 & -4 \end{bmatrix} \mathbf{x} &= 0 \\
\begin{bmatrix} 7 & -7 \\ 4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= 0 \\
(7)x_1 + (-7)x_2 &= 0 \\
(4)x_1 + (-4)x_2 &= 0
\end{aligned}
$$

$$
\begin{aligned}
(7)x_1 + (-7)x_2 &= 0 \\
x_1 &= x_2
\end{aligned}
$$

$$
\begin{aligned}
(4)x_1 + (-4)x_2 &= 0 \\
x_1 &= x_2
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{x} &= \begin{bmatrix} x_2 \\ x_2 \end{bmatrix} \\
\mathbf{x} &= x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\end{aligned}
$$

Eigenvalues: $\lambda_{1,2} = 2, -1$

Eigenvectors: $\mathbf{x}_{1,2} = \begin{bmatrix} \frac{7}{4} \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

1.3 6.1.13

Find the (real) eigenvalues and associated eigenvectors of the larger matrix $\boldsymbol{A}$. Find a basis of each eigenspace of dimension 2 or larger.

$$\begin{bmatrix} 3 & 0 & 0 \\ 6 & -2 & -2 \\ -3 & 4 & 4 \end{bmatrix}$$

$$\begin{aligned}
|\boldsymbol{A} - \lambda \mathbf{I}| &= \begin{bmatrix} 3-\lambda & 0 & 0 \\ 7 & -2-\lambda & -2 \\ -3 & 4 & 4-\lambda \end{bmatrix} \\
\det(\boldsymbol{A} - \lambda \mathbf{I}) &= (3-\lambda)((-2-\lambda)(4-\lambda) - (-2)(4)) - 0 - 0 \\
\det(\boldsymbol{A} - \lambda \mathbf{I}) &= -\lambda(\lambda-3)(\lambda-2) \\
\lambda_{1,2,3} &= 0, 3, 2
\end{aligned}$$