Week 11 Participation Assignment (1 of 2)

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Contents

1 Part 1

Given matrix $\boldsymbol{A} = \begin{bmatrix} 29 & 64 & -32 \\ -16 & -35 & 16 \\ -8 & -16 & 5 \end{bmatrix}$; let's try to form the matrix $\boldsymbol{A} - \lambda\mathbf{I}$ and find the rref so that we can determine whether the given value is the eigenvalue or not.

1) $\lambda = 1$

2) $\lambda = 3$

3) $\lambda = 5$

4) $\lambda = -1$

5) $\lambda = -3$

$$\begin{aligned}
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= \begin{bmatrix} 29-\lambda & 64 & -32 \\ -16 & -35-\lambda & 16 \\ -8 & -16 & 5-\lambda \end{bmatrix} \\
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= (29-\lambda)((-35-\lambda)(5-\lambda) - (16)(-16)) \\
&\quad + (64)((16)(-8) - (-16)(5-\lambda)) \\
&\quad + (-32)((-16)(-16) - (-35-\lambda)(-8)) \\
\det(\boldsymbol{A} - \lambda\mathbf{I}) &= -\lambda^3 - \lambda^2 + 21\lambda + 45 = -(\lambda-5)(\lambda+3)^2 \\
\lambda_{1,2} &= 5, -3
\end{aligned}$$

$$\begin{aligned}
[\boldsymbol{A} - \lambda_1]\,\mathbf{x} &= 0 \\
\begin{bmatrix} 24 & 64 & -32 \\ -16 & -40 & 16 \\ -8 & -16 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} &= 0 \\
\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} &= 0
\end{aligned}$$

$$\begin{aligned}
(1)\mathbf{x}_1 + (0)\mathbf{x}_2 + (4)\mathbf{x}_3 &= 0 \\
\mathbf{x}_1 &= (-4)\mathbf{x}_3
\end{aligned}$$

$$\begin{aligned}
(0)\mathbf{x}_1 + (1)\mathbf{x}_2 + (-2)\mathbf{x}_3 &= 0 \\
\mathbf{x}_2 &= (2)\mathbf{x}_3
\end{aligned}$$

$$
\begin{aligned}
(0)\mathbf{x}_1 + (0)\mathbf{x}_2 + (0)\mathbf{x}_3 &= 0 \\
0 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{x} &= \begin{bmatrix} (-4)\mathbf{x}_3 \\ (2)\mathbf{x}_3 \\ \mathbf{x}_3 \end{bmatrix} \\
\mathbf{x} &= \mathbf{x}_3 \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
[\boldsymbol{A} - \lambda_2]\,\mathbf{x} &= 0 \\
\begin{bmatrix} 32 & 64 & -32 \\ -16 & -32 & 16 \\ -8 & -16 & 8 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} &= 0 \\
\begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} &= 0
\end{aligned}
$$

For both $[\boldsymbol{A} - \lambda_2]_{1,2}$:

$$
\begin{aligned}
(0)\mathbf{x}_1 + (0)\mathbf{x}_2 + (0)\mathbf{x}_3 &= 0 \\
0 &= 0
\end{aligned}
$$

$$
\begin{aligned}
(1)\mathbf{x}_1 + (2)\mathbf{x}_2 + (-1)\mathbf{x}_3 &= 0 \\
\mathbf{x}_1 &= (-2)\mathbf{x}_2 + \mathbf{x}_3
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{x} &= \begin{bmatrix} (-2)\mathbf{x}_2 \\ \mathbf{x}_2 \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{x}_3 \\ 0 \\ \mathbf{x}_3 \end{bmatrix} \\
\mathbf{x} &= \mathbf{x}_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + \mathbf{x}_3 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}
\end{aligned}
$$

Where $\boldsymbol{P}$ is the modal matrix and $\boldsymbol{D}$ is the diagonal matrix:

$$
\begin{aligned}
\boldsymbol{P} &= \begin{bmatrix} -4 & -2 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \\
\boldsymbol{D} &= \begin{bmatrix} 5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}
\end{aligned}
$$