Week 09 Participation Assignment (1 of 1)

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Contents

1 Part 1

In this participation assignment, we would like to apply the term "linear combination" to two different types of question and observe the difference.

1) Let $S = \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4 \subseteq \mathbb{R}^6$, where $\vec{v}_1 = \langle 3, 4, 1, -6, -1, 9\rangle, \vec{v}_2 = \langle 1, -3, -2, 1, 1, 2\rangle, \vec{v}_3 = \langle -4, -5, -1, 9, -3, 18\rangle, \vec{v}_4 = \langle 0, 2, 1, -2, 2, -18\rangle$.

To show that the set S is linearly independent, we must begin with the dependence test equation (set the linear combination of the vectors equal to the zero vector). Here are what you need to work on

a) Write the dependence test equation as a **vector equation**.

b) Rewrite the dependence test equation as a **matrix equation**.

c) Begin with your vector equation and combine the linear combination as one vector (with variables), then write a system of linear equations by equating the vector on the left hand side and the right hand side.

d) With your matrix equation, write the coefficient matrix and augmented matrix separately.

2) Let $S = \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4 \subseteq \mathbb{R}^6$, where $\vec{v}_1 = \langle 3, 4, 1, -6, -1, 9\rangle, \vec{v}_2 = \langle 1, -3, -2, 1, 1, 2\rangle, \vec{v}_3 = \langle -4, -5, -1, 9, -3, 18\rangle, \vec{v}_4 = \langle 0, 2, 1, -2, 2, -18\rangle$.

Now given another vector $\vec{b} = \langle 1, 8, 4, -6, -1, -5\rangle$, In order to determine if vector $\vec{b}$ is in the span(S), we must begin with the vector equation $\vec{b} = x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 + x_4\vec{v}_4$. Here is what you need to work on

a) Write the **vector equation** explicitly. (This means substitute the numbers into the vector equation).

b) Rewrite the dependence test equation as a **matrix equation**.

c) Begin with your vector equation and combine the linear combination as one vector (with variables), then write a system of linear equation by equating the vector on the right hand side and the right hand side.

d) With your matrix equation, write the coefficient matrix and augmented matrix separately.

1.1 1)

a)

$$\begin{aligned}
&a_1\langle 3,4,1,-6,-1,9\rangle\\
&+a_2\langle 1,-3,-2,1,1,2\rangle\\
&+a_3\langle -4,-5,-1,9,-3,18\rangle\\
&+a_4\langle 0,2,1,-2,2,-18\rangle=\langle 0,0,0,0,0,0\rangle
\end{aligned}$$

b)

$$a_1\begin{bmatrix}3\\4\\1\\-6\\-1\\9\end{bmatrix}+a_2\begin{bmatrix}1\\-3\\-2\\1\\1\\2\end{bmatrix}+a_3\begin{bmatrix}-4\\-5\\-1\\9\\-3\\18\end{bmatrix}+a_4\begin{bmatrix}0\\2\\1\\-2\\2\\-18\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}$$

c)

$$\begin{cases}
3a_1+a_2-4a_3=0\\
4a_1-3a_2-5a_3+2a_4=0\\
a_1-2a_2-a_3+a_4=0\\
-6a_1+a_2+9a_3-2a_4=0\\
-a_1+a_2-3a_3+2a_4=0\\
9a_1+2a_2+18a_3-18a_4=0
\end{cases}$$

d) Coefficient Matrix:

$$\begin{bmatrix}
3&1&-4&0\\
4&-3&-5&2\\
1&-2&-1&1\\
-6&1&9&-2\\
-1&1&-3&2\\
9&2&18&-18
\end{bmatrix}$$

Augmented Matrix:

$$\left[\begin{array}{cccc|c}
3&1&-4&0&0\\
4&-3&-5&2&0\\
1&-2&-1&1&0\\
-6&1&9&-2&0\\
-1&1&-3&2&0\\
9&2&18&-18&0
\end{array}\right]$$

1.2 2)

a)

$$\begin{aligned}&a_1\langle 3,4,1,-6,-1,9\rangle\\&+a_2\langle 1,-3,-2,1,1,2\rangle\\&+a_3\langle -4,-5,-1,9,-3,18\rangle\\&+a_4\langle 0,2,1,-2,2,-18\rangle=\langle 1,8,4,-6,-1,-5\rangle\end{aligned}$$

b)

$$a_1\begin{bmatrix}3\\4\\1\\-6\\-1\\9\end{bmatrix}+a_2\begin{bmatrix}1\\-3\\-2\\1\\1\\2\end{bmatrix}+a_3\begin{bmatrix}-4\\-5\\-1\\9\\-3\\18\end{bmatrix}+a_4\begin{bmatrix}0\\2\\1\\-2\\2\\-18\end{bmatrix}=\begin{bmatrix}1\\8\\4\\-6\\-1\\-5\end{bmatrix}$$

c)

$$\begin{cases}3a_1+a_2-4a_3=1\\4a_1-3a_2-5a_3+2a_4=8\\a_1-2a_2-a_3+a_4=4\\-6a_1+a_2+9a_3-2a_4=-6\\-a_1+a_2-3a_3+2a_4=-1\\9a_1+2a_2+18a_3-18a_4=-5\end{cases}$$

d) Coefficient Matrix:

$$\begin{bmatrix}3&1&-4&0\\4&-3&-5&2\\1&-2&-1&1\\-6&1&9&-2\\-1&1&-3&2\\9&2&18&-18\end{bmatrix}$$

Augmented Matrix:

$$\left[\begin{array}{cccc|c}3&1&-4&0&1\\4&-3&-5&2&8\\1&-2&-1&1&4\\-6&1&9&-2&-6\\-1&1&-3&2&-1\\9&2&18&-18&-5\end{array}\right]$$