Contents

1 Chapter 18 - Thermal Properties of Matter

Avogadro's number

$$N_A = 6.02 \times 10^{23}\,\text{mol} \tag{1}$$

1.1 The Ideal Gas Law

Ideal gas: a collection of atoms or molecules that move randomly and exert no long-range forces on each other.

Number of moles

$$n = \frac{N}{N_A} = \frac{m_{particle}N}{m_{particle}N_A} = \frac{m}{M} \tag{2}$$

The **molar mass** M **(molecular weight)** is the mass per mole. The total mass of n moles is $m_{total} = nM$.

Ideal-gas equation

$$pV = nRT \tag{3}$$

Universal gas constant

$$R = 8.31\,\mathrm{J\,mol^{-1}\,K^{-1}} = 0.0821\,\mathrm{L\,atm\,mol^{-1}\,K^{-1}} \tag{4}$$

The volume occupied by 1 mol of any ideal gas at atmospheric pressure and at 0 °C is 22.4 L.

1.1.1 Question

$$\begin{aligned}
V &= 22.4 \times 10^{-3}\,\mathrm{L} \\
T &= 273.15\,\mathrm{K} \\
p &= 1.013 \times 10^{5}\,\mathrm{Pa} = 1.0\,\mathrm{atm} \\
n &=?
\end{aligned}$$

$$\begin{aligned}
pV &= nRT \\
n &= \frac{pV}{RT} \\
n &= \frac{(1.0\ \mathrm{atm})(22.4\,\mathrm{L})}{(0.0821\,\mathrm{L\,atm\,mol^{-1}\,K^{-1}})(273.15\,\mathrm{K})} \\
n &= 1.000\,\mathrm{mol}
\end{aligned}$$

1.1.2 18.3

$$\begin{aligned}
V_0 &= 0.110\,\mathrm{m^3} \\
p_0 &= 0.355\,\mathrm{atm} \\
V_1 &= 0.390\,\mathrm{m^3} \\
T &= \text{constant} \\
p_1 &=?
\end{aligned}$$

$$\begin{aligned}
p_0V_0 &= p_1V_1 \\
p_1 &= \frac{p_0V_0}{V_1} \\
p_1 &= \frac{(0.355\,\mathrm{atm})(0.110\,\mathrm{m^3})}{0.390\,\mathrm{m^3}} \\
p_1 &= 0.1001\,\mathrm{atm}
\end{aligned}$$

1.1.3 18.4

$$
\begin{aligned}
V_0 &= 3.00\,\mathrm{L} \\
p_0 &= 3.00\,\mathrm{atm} \\
T_0 &= 20.0\,^\circ\mathrm{C} = 293\,\mathrm{K} \\
p_1 &= 1.00\,\mathrm{atm}
\end{aligned}
$$

(a)

$$
\begin{aligned}
pV &= nRT \\
\frac{p}{T} &= \frac{nR}{V} \\
\frac{p_0}{T_0} &= \frac{p_1}{T_1} \\
T_1 &= \frac{p_1 T_0}{p_0} \\
T_1 &= \frac{(1.00\,\mathrm{atm})(293\,\mathrm{K})}{3.00\,\mathrm{atm}} \\
T_1 &= 97.7\,\mathrm{K} = -175.3\,^\circ\mathrm{C}
\end{aligned}
$$

1.1.4 18.7

$$
\begin{aligned}
V_0 &= 499\,\mathrm{cm}^3 = 499 \times 10^{-6}\,\mathrm{m}^3 \\
p_0 &= 1.01 \times 10^5\,\mathrm{Pa} \\
T_0 &= 27.0\,^\circ\mathrm{C} = 300\,\mathrm{K} \\
V_1 &= 46.2\,\mathrm{cm}^3 = 46.2 \times 10^{-6}\,\mathrm{m}^3 \\
p_1 &= 2.72 \times 10^6\,\mathrm{Pa} + 1\,\mathrm{atm} = 2.821 \times 10^6\,\mathrm{Pa} \\
T_1 &= ?
\end{aligned}
$$

$$
\begin{aligned}
pV &= nR\Delta T \\
\frac{p_0 V_0}{T_0} &= \frac{p_1 V_1}{T_1} \\
T_1 &= \frac{T_0 p_1 V_1}{p_0 V_0} \\
T_1 &= \frac{(300\,\mathrm{K})(2.821 \times 10^6\,\mathrm{Pa})(46.2 \times 10^{-6}\,\mathrm{m}^3)}{(1.01 \times 10^5\,\mathrm{Pa})(499 \times 10^{-6}\,\mathrm{m}^3)} \\
T_1 &= 755.79\,\mathrm{K}
\end{aligned}
$$

1.1.5 18.13

$$
\begin{aligned}
p_0 &= 1\,\mathrm{atm} V_0 && = V_{earth} \\
V_1 &= V_{venus} \\
T_1 &= 1003\,^{\circ}\mathrm{C} = 1276\,\mathrm{K} \\
p_1 &= 92\,\mathrm{atm} \\
T_0 &= 273\,\mathrm{K}
\end{aligned}
$$

$$
\begin{aligned}
pV &= nR\Delta T \\
\frac{p_0 V_0}{T_0} &= \frac{p_1 V_1}{T_1} \\
V_1 &= \frac{T_1 p_0}{T_0 p_1} V_0 \\
V_1 &= \frac{(1276\,\mathrm{K})(1\,\mathrm{atm})}{(273\,\mathrm{K})(92\,\mathrm{atm}} \\
V_1 &= (0.051) V_0
\end{aligned}
$$

1.1.6 18.16

$$
\begin{aligned}
n &= 3\,\mathrm{mol} \\
l &= 0.300\,\mathrm{m}
\end{aligned}
$$

(a)

$$
T = 20.0\,^{\circ}\mathrm{C} = 293\,\mathrm{K}
$$

$$
\begin{aligned}
F &= pA \\
F &= \frac{nRTA}{V} \\
F &= \frac{(3\,\mathrm{mol})(8.31\,\mathrm{J\,mol^{-1}\,K^{-1}})(293\,\mathrm{K})(0.300\,\mathrm{m})^2}{(0.300\,\mathrm{m})^3} \\
F &= 24\,348.3\,\mathrm{N} = 2.43 \times 10^4\,\mathrm{N}
\end{aligned}
$$

(b)

$$
T = 100.0\,^{\circ}\mathrm{C} = 373\,\mathrm{K}
$$

$$
\begin{aligned}
F &= \frac{nRTA}{V} \\
F &= \frac{(3\,\mathrm{mol})(8.31\,\mathrm{J\,mol^{-1}\,K^{-1}})(373\,\mathrm{K})(0.300\,\mathrm{m})^2}{(0.300\,\mathrm{m})^3} \\
F &= 30\,996.3\,\mathrm{N} = 3.10 \times 10^4\,\mathrm{N}
\end{aligned}
$$

1.1.7 18.18

$$
\begin{aligned}
\Delta y &= 11\,000\,\mathrm{m} \\
T &= -56.5\,^{\circ}\mathrm{C} = 216.5\,\mathrm{K} \\
\rho &= 0.364\,\mathrm{kg\,m^{-3}} \\
p &= ?
\end{aligned}
$$

$$
\begin{aligned}
\rho &= \frac{m}{V} \\
m &= \rho V \\
n &= \frac{m}{M} \\
n &= \frac{\rho V}{M}
\end{aligned}
$$

$$
\begin{aligned}
pV &= nRT \\
pV &= \left(\frac{\rho V}{M}\right) RT \\
p &= \frac{\rho R T}{M} \\
p &= \frac{(0.364\,\mathrm{kg\,m^{-3}})(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})(216.5\,\mathrm{K})}{28.8\times 10^{-3}\,\mathrm{kg\,mol^{-1}}} \\
p &= 22\,749.8\,\mathrm{Pa} = 2.27\times 10^{4}\,\mathrm{Pa}
\end{aligned}
$$

1.1.8 Question

$$
\begin{aligned}
T &= 0.00\,^{\circ}\mathrm{C} = 273\,\mathrm{K} \\
g &= 9.80\,\mathrm{m\,s^{-2}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{dp}{dy} &= -\rho g \\
p &= \rho RT \\
\rho &= \frac{p}{RT} \\
\frac{dp}{dy} &= -\left(\frac{p}{RT}\right) g \\
\frac{dp}{dy} &= -\frac{pg}{RT} \\
p' &= -p \cdot \frac{g}{RT} \\
\mathcal{L}\{p'\} + \mathcal{L}\{p\} &= \frac{g}{RT} \\
sF(s) - f(0) + F(s) &= \frac{g}{RT} \\
F(s)(s-1) &= \frac{g}{RT} \\
F(s) &= \frac{g}{RT} \cdot \frac{1}{s-1} \\
s &= \frac{ge^t}{RT}
\end{aligned}
$$

2 Molecules and Intermolecular Forces

2.1 The Van Der Waals Equation

The model used for the ideal-gas equation ignores the volumes of molecules and the attractive forces between them.

$$\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT \tag{5}$$

2.2 Kinetic-Molecular Model of an Ideal Gas

1. Molecules are in constant motion and undergo perfectly elastic collisions.

2.3 Collisions and Gas Pressure

$$
\begin{aligned}
\Delta\mathbf{P_y} &= m\Delta\mathbf{v_y} = 0 \\
\Delta\mathbf{P_x} &= m\Delta\mathbf{v_x} = 2m\mathbf{v_x}
\end{aligned}
$$

The amount of molecules per volume that collide with a given wall area A in a time interval dt:

$$
\begin{aligned}
V &= Ah \\
\Delta\mathbf{x} &= v_{0_y}t + \frac{1}{2}a_y\Delta t^2 \\
\Delta\mathbf{x} &= v_{0_y}t \\
V &= A|v_x|dt
\end{aligned}
$$

$$\frac{1}{2}\left(\frac{N}{V}\right)(A|v_x|dt)$$

For all molecules in the gas, the total momentum change dP_x during dt is the number of collisions multiplied by the momentum change.

$$
\begin{aligned}
dP_x &= \frac{1}{2}\left(\frac{N}{V}\right)(A|v_x|dt)(2m|v_x|) \\
&= \frac{NAmv_x^2dt}{V}
\end{aligned}
$$

The rate of change of momentum component P_x:

$$\frac{dP_x}{dt} = \frac{NAmv_x^2}{V}$$

Pressure:

$$p = \frac{F}{A} = \frac{m\frac{\Delta\mathbf{p}}{\Delta t}}{A} = \frac{\frac{NAmv_x^2}{V}}{A} = \frac{Nmv_x^2}{V} \tag{6}$$

2.3.1 Pressure and Molecular Kinetic Energies

The speed v of a molecule is related to the velocity components by

$$v^2 = v_x^2 + v_y^2 + v_z^2 \tag{7}$$

$$(v^2)_{av} = (v_x^2)_{av} + (v_y^2)_{av} + (v_z^2)_{av} \tag{8}$$

As there is no real difference in our model between directions:

$$(v_x^2)_{av} = \frac{1}{3}(v^2)_{av} \tag{9}$$

$$
\begin{aligned}
p &= \frac{Nmv_x^2}{V} \\
pV &= nRT \\
\left[\frac{Nmv_x^2}{V}\right]V &= nRT \\
\frac{N}{3}(2K) &= nRT, \quad KE = \frac{1}{2}mv^2, n = \frac{m}{M} = \frac{N}{N_A} \\
K &= \frac{3}{2}nRT
\end{aligned}
$$

$$
\begin{aligned}
pV &= nRT \\
nV &= \frac{N}{N_A}RT \\
pV &= Nk_BT
\end{aligned}
$$

2.3.2 18.24

$$
\begin{aligned}
v_{rmt} &= 2\sqrt{\frac{3RT}{m}} \\
v_{rmt} &= \sqrt{\frac{3R}{m}\cdot 4T}
\end{aligned}
$$

$$
\begin{aligned}
p_1V &= nRT_1 \\
p_1 &= 2p_2 \\
2p_2V &= n_2R(4T_1) \\
n_2 &= \frac{2}{4}\cdot\frac{p_1V}{RT_1} \\
n_2 &= \frac{1}{2}n_1
\end{aligned}
$$

2.3.3 18.28

$$
\begin{aligned}
V &= 1.64\,\mathrm{L} \\
m &= 0.226\,\mathrm{kg} \\
v_{rms} &= 182\,\mathrm{m\,s^{-1}} \\
p &=?
\end{aligned}
$$

$$
\begin{aligned}
v_{rms} &= \sqrt{\frac{3RT}{M}} \\
v_{rms}^2 &= \frac{3RT}{M} \\
RT &= \frac{v_{rms}^2M}{3}
\end{aligned}
$$

$$
\begin{aligned}
pV &= nRT \\
pV &= n\left(\frac{v_{rms}^2 M}{3}\right) \\
p &= \frac{\frac{m}{M}\left(\frac{v_{rms}^2 M}{3}\right)}{V} \\
p &= \frac{m v_{rms}^2}{3V} \\
p &= \frac{(0.226\,\mathrm{kg})(182\,\mathrm{m\,s^{-1}})^2}{3(1.64\,\mathrm{L})} \\
p &= 1521.55\,\mathrm{Pa}
\end{aligned}
$$

2.3.4 18.30

$$
\begin{aligned}
M_{mars} &= 44.0\,\mathrm{g\,mol^{-1}} = 0.044\,\mathrm{kg\,mol^{-1}} \\
P_{mars} &= 650\,\mathrm{Pa} \\
T_0 &= 0.0\,{}^\circ\mathrm{C} = 273\,\mathrm{K} \\
T_1 &= -100.0\,{}^\circ\mathrm{C} = 173\,\mathrm{K}
\end{aligned}
$$

(a)

$$
\begin{aligned}
v_{rms} &= \sqrt{\frac{3RT}{M}} \\
v_{rms} &= \sqrt{\frac{3(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})(273\,\mathrm{K})}{0.044\,\mathrm{kg\,mol^{-1}}}} \\
v_{rms} &= 393.4\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

$$
\begin{aligned}
v_{rms} &= \sqrt{\frac{3RT}{M}} \\
v_{rms} &= \sqrt{\frac{3(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})(173\,\mathrm{K})}{0.044\,\mathrm{kg\,mol^{-1}}}} \\
v_{rms} &= 313.2\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

(b)

$$
\begin{aligned}
pV &= nRT \\
pV &= \left(\frac{m}{M}\right) RT \\
m &= \frac{pVM}{RT} \\
\rho &= \frac{m}{V} \\
\rho &= \frac{\frac{pVM}{RT}}{V} \\
\rho &= \frac{pM}{RT}
\end{aligned}
$$

$$
\begin{aligned}
\rho_0 &= \frac{(650\,\mathrm{Pa})(0.044\,\mathrm{kg\,mol^{-1}})}{(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})(273\,\mathrm{K})} \\
\rho_0 &= 0.0126\,\mathrm{kg\,m^{-3}} = 12.6\,\mathrm{g\,m^{-3}} \cdot \frac{1}{44.0\,\mathrm{g\,mol^{-1}}} = 0.286\,\mathrm{mol\,m^{-3}}
\end{aligned}
$$

$$
\begin{aligned}
\rho_1 &= \frac{(650\,\mathrm{Pa})(0.044\,\mathrm{kg\,mol^{-1}})}{(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})(173\,\mathrm{K})} \\
\rho_1 &= 0.0199\,\mathrm{kg\,m^{-3}} = 19.9\,\mathrm{g\,m^{-3}} \cdot \frac{1}{44.0\,\mathrm{g\,mol^{-1}}} = 0.452\,\mathrm{mol\,m^{-3}}
\end{aligned}
$$

3 Collisions Between Molecules

The volume of the cylinder is

$$V = \pi 4r^2 v dt \tag{10}$$

There are $\frac{N}{V}$ molecules per unit volume, so the number dN with centers in this cylinder is

$$
\begin{aligned}
dN &= 4\pi r^2 v dt \frac{N}{V} \\
\frac{dN}{dt} &= \frac{4\pi r^2 v N}{V} \\
\frac{dN}{dt} &= \frac{4\pi \sqrt{2} r^2 v N}{V}
\end{aligned}
$$

$$\lambda = \frac{V}{4\pi \sqrt{2} r^2 N} \tag{11}$$

3.0.1 18.32

$$\begin{aligned} \lambda &= ? \\ p &= 3.50 \times 10^{-13}\,\mathrm{atm} \\ T &= 300\,\mathrm{K} \end{aligned}$$

$$\begin{aligned} \lambda &= \frac{V}{4\pi\sqrt{2}r^2N} \\ \frac{V}{N} &= \frac{kT}{p} \\ \lambda &= \frac{kT}{p} \cdot \frac{1}{4\pi\sqrt{2}r^2} \\ \lambda &= \frac{(1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}})(300\,\mathrm{K})}{(3.50 \times 10^{-13}\,\mathrm{atm})(1.01 \times 10^5\,\mathrm{Pa\,atm^{-1}})} \cdot \frac{1}{4\pi\sqrt{2}(2.0 \times 10^{-10}\,\mathrm{m})^2}\lambda &&= 1.65 \times 10^5\,\mathrm{m} \end{aligned}$$

3.0.2 18.35

$$\begin{aligned} n &= 3\,\mathrm{mol} \\ M &= 4.00\,\mathrm{g\,mol^{-1}} \\ V &= \text{constant} \\ v_{rms_0} &= 900\,\mathrm{m\,s^{-1}} \\ v_{rms_1} &= ? \\ Q &= 2400\,\mathrm{J} \end{aligned}$$

$$\begin{aligned} v_{rms} &= \sqrt{\frac{3RT}{M}} \\ T &= \frac{v_{rms}^2 M}{3R} \\ T &= \frac{(900\,\mathrm{m\,s^{-1}})^2(0.004\,\mathrm{kg\,mol^{-1}})}{3(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})} \\ T_0 &= 129.9\,\mathrm{K} \end{aligned}$$

$$\begin{aligned} Q &= nC_V\Delta T \\ Q &= nC_V(T_1 - T_0) \\ Q &= nC_VT_1 - nC_VT_0 \\ T_1 &= \frac{Q + nC_VT_1}{nC_v} \\ T_1 &= \frac{2400\,\mathrm{J} + (3\,\mathrm{mol})\left(\frac{3}{2}(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})\right)(129.9\,\mathrm{K})}{(3\,\mathrm{mol})\left(\frac{3}{2}(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})\right)} \\ T_1 &= 194.0\,\mathrm{K} \end{aligned}$$

$$v_{rms} = \sqrt{\frac{3RT_0}{M}}$$
$$v_{rms} = \sqrt{\frac{3(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})(194.0\,\mathrm{K})}{0.004\,\mathrm{kg\,mol^{-1}}}}$$
$$v_{rms} = 1100\,\mathrm{m\,s^{-1}}$$

3.0.3 18.38

$$n = n_0 = n_1$$
$$Q = 300\,\mathrm{J}$$
$$\Delta T_{hydrogen} = 2.50\,^{\circ}\mathrm{C}$$
$$\Delta T_{neon} = ?$$

$$Q = nC_V\Delta T$$
$$C_V\Delta T = \frac{Q}{n}$$
$$C_{V_0}\Delta T_0 = C_{V_1}\Delta T_1$$
$$\Delta T_1 = \frac{C_{V_0}\Delta T_0}{C_{V_1}}$$
$$\Delta T_1 = \frac{\frac{5}{2}R(2.5\,\mathrm{K})}{\frac{3}{2}R}$$
$$\Delta T_1 = 4.16\,^{\circ}\mathrm{C}$$

3.0.4 18.40

(a)

$$M_{water} = 18.0\,\mathrm{g\,mol^{-1}}$$

$$C_V = Mc$$
$$c = \frac{C_V}{M}$$
$$c = \frac{6\left(\frac{1}{2}(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})\right)}{18.0\,\mathrm{kg\,mol^{-1}}}$$
$$c = 1386.0\,\mathrm{J\,kg^{-1}\,K^{-1}}$$

4 Molecular Speeds

The function $f(v)$ describing the actual distribution of molecular speeds is called the **Maxwell-Boltzmann distribution**.

$$f(v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{\frac{-mv^2}{2kT}} \tag{12}$$

We can also express this function in terms of the translational kinetic energy of a molecule, which we denote by ϵ; that is $\epsilon = \frac{1}{2}mv^2$

$$f(\epsilon) = \frac{8\pi}{m}\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \epsilon e^{\frac{-\epsilon}{kT}} \tag{13}$$

$$v_{av} = \int_0^{\infty} v f(v) dv$$

4.0.1 18.41

$$\begin{aligned} M_{CO_2} &= 44.0\,\mathrm{g\,mol^{-1}} = 0.044\,\mathrm{kg\,mol^{-1}} \\ T &= 300\,\mathrm{K} \end{aligned}$$

$$\begin{aligned} pV &= nRT = NkT \\ nR &= Nk \\ \frac{N}{N_A}R &= \frac{m}{M}k \end{aligned}$$

(a)

$$\begin{aligned} v_{mp} &= \sqrt{\frac{2kT}{m}} \\ v_{mp} &= \sqrt{\frac{2RT}{M}} \\ v_{mp} &= \sqrt{\frac{2(8.314\,\mathrm{J\,mol^{-1}\,K^{-1}})(300\,\mathrm{K})}{0.044\,\mathrm{kg\,mol^{-1}}}} \\ v_{mp} &= 336.7\,\mathrm{m\,s^{-1}} \end{aligned}$$

(b)

$$\begin{aligned} v_{av} &= \sqrt{\frac{8kT}{\pi m}} \\ v_{av} &= \sqrt{\frac{8RT}{\pi M}} \end{aligned}$$

(c)

$$\begin{aligned} v_{rms} &= \sqrt{\frac{3kT}{m}} \\ v_{rms} &= \sqrt{\frac{3RT}{M}} \end{aligned}$$