Contents

1 Section 3.5

1.1 3.5.1

First use the formula $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ for the general 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ to find A^{-1}. Then use A^{-1} to solve the system $Ax = b$.

$$A = \begin{bmatrix} 7 & 6 \\ 8 & 7 \end{bmatrix}; b = \begin{bmatrix} 17 \\ 19 \end{bmatrix}$$

$$\begin{aligned} \det(A) &= (7 \cdot 7) - (6 \cdot 8) = 1 \\ \operatorname{cof}(A) &= \begin{bmatrix} 7 & 6 \\ 8 & 7 \end{bmatrix} \\ \operatorname{adj} A &= \begin{bmatrix} 7 & -6 \\ -8 & 7 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} A^{-1} &= \frac{1}{\det(A)} \operatorname{adj}(A) \\ A^{-1} &= \frac{1}{1}\begin{bmatrix} 7 & -6 \\ -8 & 7 \end{bmatrix} \\ A^{-1} &= \begin{bmatrix} 7 & -6 \\ -8 & 7 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} Ax &= b \\ x &= bA^{-1} \\ x &= \begin{bmatrix} 7 & -6 \\ -8 & 7 \end{bmatrix}\begin{bmatrix} 17 \\ 19 \end{bmatrix} \\ x &= \begin{bmatrix} 5 \\ -3 \end{bmatrix} \end{aligned}$$

$$\boxed{x = \begin{bmatrix} 5 \\ -3 \end{bmatrix}}$$

1.2 3.5.5

Solve $Ax = b$

$$A = \begin{bmatrix} 4 & 3 \\ 6 & 5 \end{bmatrix}; b = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$

$$\det(A) = 4 \cdot 5 - 3 \cdot 6 = 2$$

$$\operatorname{cof}(A) = \begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix}$$

$$\operatorname{adj}(A) = \begin{bmatrix} 5 & -3 \\ -6 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 5 & -3 \\ -6 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{5}{2} & -\frac{3}{2} \\ -3 & 2 \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{5}{2} & -\frac{3}{2} \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$x = \begin{bmatrix} (\frac{5}{2} \cdot 6) - (-\frac{3}{2} \cdot 4) \\ (-3 \cdot 6) - (2 \cdot 4) \end{bmatrix}$$

$$x = \begin{bmatrix} 9 \\ -10 \end{bmatrix}$$

$$\boxed{x = \begin{bmatrix} 9 \\ -10 \end{bmatrix}}$$

1.3 3.5.8

Solve $Ax = b$

$$A = \begin{bmatrix} 7 & 13 \\ 5 & 10 \end{bmatrix}; b = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

$$\det(A) = (7 \cdot 10) - (13 \cdot 5) = 5$$

$$\operatorname{cof}(A) = \begin{bmatrix} 10 & 5 \\ 13 & 7 \end{bmatrix}$$

$$\operatorname{adj}(A) = \begin{bmatrix} 10 & -13 \\ -5 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 10 & -13 \\ -5 & 7 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & -\frac{13}{5} \\ -1 & \frac{7}{5} \end{bmatrix}$$

$$x = \begin{bmatrix} 2 & -\frac{13}{5} \\ -1 & \frac{7}{5} \end{bmatrix} \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

$$x = \begin{bmatrix} (2 \cdot 6) + (-\frac{13}{5} \cdot 2) \\ (-1 \cdot 6) + (\frac{7}{5} \cdot 2) \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{34}{5} \\ -\frac{16}{5} \end{bmatrix}$$

$$\boxed{x = \begin{bmatrix} \frac{34}{5} \\ -\frac{16}{5} \end{bmatrix}}$$

1.4 3.5.11

Adjoin I on the right of A, then use row operations to find the inverse A^{-1} of the given matrix A.

$$\begin{bmatrix} 1 & 15 & 1 \\ 2 & 15 & 0 \\ 2 & 22 & 1 \end{bmatrix}$$

$$[I|A^{-1}] = \left[\begin{array}{ccc|ccc} 1 & 15 & 1 & 1 & 0 & 0 \\ 2 & 15 & 0 & 0 & 1 & 0 \\ 2 & 22 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$I_2 = I_2 - 2I_1$$
$$I_3 = I_3 - 2I_1$$

$$[I|A^{-1}] = \left[\begin{array}{ccc|ccc} 1 & 15 & 1 & 1 & 0 & 0 \\ 0 & -15 & -2 & -2 & 1 & 0 \\ 0 & -8 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$I_3 = I_3 - \frac{8}{15}I_2$$

$$[I|A^{-1}] = \left[\begin{array}{ccc|ccc} 1 & 15 & 1 & 1 & 0 & 0 \\ 0 & -15 & -2 & -2 & 1 & 0 \\ 0 & 0 & \frac{1}{15} & -\frac{14}{15} & -\frac{8}{15} & 1 \end{array}\right]$$

$$I_1 = I_1 + I_2$$

$$[I|A^{-1}] = \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 1 & 0 \\ 0 & -15 & -2 & -2 & 1 & 0 \\ 0 & 0 & \frac{1}{15} & -\frac{14}{15} & -\frac{8}{15} & 1 \end{array}\right]$$

$$I_1 = I_1 + 15I_3$$

$$[I|A^{-1}] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -15 & -7 & 15 \\ 0 & -15 & -2 & -2 & 1 & 0 \\ 0 & 0 & \frac{1}{15} & -\frac{14}{15} & -\frac{8}{15} & 1 \end{array}\right]$$

$$I_2 = I_2 + 30I_3$$

$$[I|A^{-1}] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -15 & -7 & 15 \\ 0 & -15 & 0 & -30 & -15 & 30 \\ 0 & 0 & \frac{1}{15} & -\frac{14}{15} & -\frac{8}{15} & 1 \end{array}\right]$$

$$I_2 = -\frac{1}{15}I_2$$
$$I_3 = 15I_3$$

$$[I|A^{-1}] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -15 & -7 & 15 \\ 0 & 1 & 0 & 2 & 1 & -2 \\ 0 & 0 & 1 & -14 & -8 & 15 \end{array}\right]$$

$$\boxed{[I|A^{-1}] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -15 & -7 & 15 \\ 0 & 1 & 0 & 2 & 1 & -2 \\ 0 & 0 & 1 & -14 & -8 & 15 \end{array}\right]}$$

1.5 3.5.23

Find a matrix X such that $AX = B$

$$A = \begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & -4 \\ -1 & -2 & 4 \end{bmatrix}$$

Find A^{-1}

$$\det(A) = 4 \cdot 4 - 3 \cdot 5 = 1$$
$$\text{adj}(A) = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$$
$$A^{-1} = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$$

Find $A^{-1} \cdot B$

$$X = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 & -4 \\ -1 & -2 & 4 \end{bmatrix}$$
$$X = \begin{bmatrix} 4 \cdot 1 + -3 \cdot -1 & 4 \cdot 3 + -3 \cdot -2 & 4 \cdot -4 + -3 \cdot 4 \\ -5 \cdot 1 + 4 \cdot -1 & -5 \cdot 3 + 4 \cdot -2 & -5 \cdot -4 + 4 \cdot 4 \end{bmatrix}$$
$$X = \begin{bmatrix} 7 & 18 & -28 \\ -9 & -23 & 36 \end{bmatrix}$$

$$\boxed{X = \begin{bmatrix} 7 & 18 & -28 \\ -9 & -23 & 36 \end{bmatrix}}$$