Contents

1 Lesson 4

1.1 Center of Mass

Center of Mass: average location of mass in a distribution

- Balance Point
- Natural Rotation axis
- Only line through center of mass on a 2D object always splits into 2 equal parts

A center of mass for point masses can be given by the equation

$$\vec{x}_{\text{CM}} = \frac{\vec{\sum} m_i \vec{x}_i}{\sum m_i}$$

In one-dimension

$$x_{\text{CM}} = \frac{\sum m_i x_i}{m_{\text{total}}}$$

$x_4 = 100$ cm

If you have a vector equation then find their components and handle them separately.

$$\begin{aligned}
P_1 &= (3, 3) \\
m_{P_1} &= 27\,\text{kg} \\
P_2 &= (0, 0) \\
m_{P_2} &= 10\,\text{kg} \\
P_3 &= (-1, 1) \\
m_{P_3} &= 9\,\text{kg} \\
P_4 &= (-4, -2) \\
m_{P_4} &= 15\,\text{kg}
\end{aligned}$$

First finding the x components

$$
\begin{aligned}
x_1 &= 3\,\mathrm{m} \\
m_1 &= 27\,\mathrm{kg} \\
x_2 &= 0\,\mathrm{m} \\
m_2 &= 10\,\mathrm{kg} \\
x_3 &= -1\,\mathrm{m} \\
m_3 &= 9\,\mathrm{kg} \\
x_4 &= -4\,\mathrm{m} \\
m_4 &= 15\,\mathrm{kg}
\end{aligned}
$$

Then the y components

$$
\begin{aligned}
y_1 &= 3\,\mathrm{m} \\
m_1 &= 27\,\mathrm{kg} \\
y_2 &= 0\,\mathrm{m} \\
m_2 &= 10\,\mathrm{kg} \\
y_3 &= 1\,\mathrm{m} \\
m_3 &= 9\,\mathrm{kg} \\
y_4 &= -2\,\mathrm{m} \\
m_4 &= 15\,\mathrm{kg}
\end{aligned}
$$

Then solving for x_{CM}

$$
\begin{aligned}
x_{CM} &= \frac{\sum m_i x_i}{m_{\text{total}}} \\
&= \frac{(3\,\mathrm{m})(27\,\mathrm{kg}) + (0\,\mathrm{m})(10\,\mathrm{kg}) + (-1\,\mathrm{m})(9\,\mathrm{kg}) + (-4\,\mathrm{m})(15\,\mathrm{kg})}{27\,\mathrm{kg} + 10\,\mathrm{kg} + 9\,\mathrm{kg} + 15\,\mathrm{kg}} \\
&= 0.2\,\mathrm{m}
\end{aligned}
$$

$$
\begin{aligned}
y_{CM} &= \frac{(3\,\mathrm{m}27\,\mathrm{kg}) + (0\,\mathrm{m}10\,\mathrm{kg}) + (1\,\mathrm{m}9\,\mathrm{kg}) + (-2\,\mathrm{m}15\,\mathrm{kg})}{27\,\mathrm{kg} + 10\,\mathrm{kg} + 9\,\mathrm{kg} + 15\,\mathrm{kg}} \\
&= 1\,\mathrm{m}
\end{aligned}
$$

1.2 Density

Density: $\frac{\text{amount}}{\text{space it takes up}}$

Volumetric mass density

$$
\rho \equiv \frac{dm}{dV}
$$

For continuous solids instead of $\sum m_i x_i$ we use

$$\int x dm$$

Because dm is hard to work with we say

$$dm = \rho dV$$

A **moment** is defined by

$$\int x \rho dV$$

Where ρdV shows integrating with respect to space

Counting moments is 0-indexed

Zeroth moment

$$\begin{aligned}
&\int x^0 \rho dV \\
&= \int \rho dV \\
&= \int_0^{M_{\text{total}}} dm \\
&= M_{\text{total}}
\end{aligned}$$

First moment

$$\begin{aligned}
&\int x^1 \rho dV \\
&= \int x \rho dV \text{, related to torque}
\end{aligned}$$

Center of Mass:

$$\vec{x}_{\text{CM}} = \frac{\int \vec{x} \rho dV}{\int \rho dr}$$

Given the general function for y of x

$$y = -\frac{h}{b}x + h$$

$$\begin{aligned}
M_{\text{total}}\vec{x}_{\text{CM}} &= \sum x \sigma dA \\
&= \int (x)\sigma(ydx) \\
&= \sigma \int_0^b x(-\frac{h}{b}x + h)dx \\
&= \sigma \int_0^b \left(-\frac{h}{b}x^2 + hx\right) dx \\
&= \sigma \left(-\frac{h}{3b}x^3 + \frac{h}{2}x^2\right)_0^b
\end{aligned}$$