Week 01 and Week 02 Participation Assignment (3 of 4)

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1 Part 3

Directions: We say a differential equation is separable if we can write the equation into the form of $\frac{dy}{dx} = f(x) \cdot g(y)$. For the following differential equation, determine whether the given equation is separable or not. If it is separable, identify f and g (we may encounter other variables other than x and y).

1.1 Problem 1

$$\frac{dy}{dx} - \sin(x+y) = 0$$

$$\begin{aligned}
\frac{dy}{dx} - \sin(x+y) &= 0 \\
\frac{dy}{dx} &= \sin(x+y) \\
\frac{dy}{dx} &= \sin(x)\cos(y) + \cos(x)\sin(y)
\end{aligned}$$

Cannot be turned into separable form.

1.2 Problem 2

$$\frac{dy}{dx} = 4y^2 - 3y + 1$$

$$\begin{aligned}
\frac{dy}{dx} &= 4y^2 - 3y + 1 \\
\left(\frac{dx}{4y^2 - 3y + 1}\right)\frac{dy}{dx} &= \left(4y^2 - 3y + 1\right)\left(\frac{dx}{4y^2 - 3y + 1}\right) \\
\left(\frac{1}{4y^2 - 3y + 1}\right)dy &= (1)dx
\end{aligned}$$

The differential equation is separable.
$f(x) = 1$, $g(y) = 4y^2 - 3y + 1$

1.3 Problem 3

$$\frac{ds}{dt} = t\ln\left(s^{2t}\right) + 8t^2$$

$$\begin{aligned}\frac{ds}{dt} &= t\ln\left(s^{2t}\right) + 8t^2 \\ &= t(2t)\ln(s) + 8t^2 \\ &= 2t^2\ln(s) + 8t^2 \\ &= (2t^2)(\ln(s)+4) \\ \left(\frac{1}{\ln(s)+4}\right)ds &= (2t^2)dt\end{aligned}$$

The differential equation is separable.
$f(s) = \ln(s)+4,\ g(t) = 2t^2$

1.4 Problem 4

$$\frac{dy}{dx} = \frac{ye^{x+y}}{x^2+2}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{ye^{x+y}}{x^2+2} \\ &= \frac{ye^xe^y}{x^2+2} \\ \frac{dy}{dx} &= (ye^y)\left(\frac{e^x}{x^2+2}\right)\end{aligned}$$

The differential equation is separable.
$f(x) = \left(\frac{e^x}{x^2+2}\right),\ g(y) = ye^y$

1.5 Problem 5

$$\left(xy^2+3y^2\right)dy - 2xdx = 0$$

$$
\begin{aligned}
\left(xy^2+3y^2\right)dy - 2xdx &= 0 \\
\left(xy^2+3y^2\right)dy &= (2x)dx \\
\frac{dy}{dx}\left(xy^2+3y^2\right) &= 2x \\
\frac{dy}{dx} &= \frac{2x}{xy^2+3y^2} \\
&= \frac{2x}{y^2(x+3)} \\
\frac{dy}{dx} &= \left(\frac{1}{y^2}\right)\left(\frac{2x}{x+3}\right)
\end{aligned}
$$

The differential equation is separable.

$f(x) = \left(\frac{2x}{x+3}\right)$, $g(y) = \left(\frac{1}{y^2}\right)$

1.6 Problem 6

$$s^2 + \frac{ds}{dt} = \frac{s+1}{st}$$

$$
\begin{aligned}
s^2 + \frac{ds}{dt} &= \frac{s+1}{st} \\
\frac{ds}{dt} &= \frac{s+1}{st} - s^2 \\
&= \frac{s}{st} + \frac{1}{st} - s^2 \\
&= \frac{1}{t} + \frac{1}{st} - s^2 \\
&= \frac{1}{t}\left(1 + \frac{1}{s}\right) - s^2
\end{aligned}
$$

Cannot be turned into separable form.