1 Week 15 and Week 16 Participation Assignment (1 of 2)

For the following equations in the form of $A(x)y'' + B(x)y' + C(x)y = 0$, classify each singular point (real or complex) of the given equation as regular or irregular by finding the functions $P(x), Q(x), p(x) = xP(x), q(x) = x^2Q(x)$ and then determine whether $p(x)$ and $q(x)$ are analytic or not.

1)

$$(x^2 - 1)y'' + xy' + 3y = 0$$

$$P(x) = \frac{x}{x^2 - 1}$$

$$p(x) = \frac{x}{x + 1}, \text{ analytic}$$

$$Q(x) = \frac{3}{x^2 - 1}$$

$$q(x) = \frac{3(x - 1)}{x + 1}, \text{ analytic}$$

2)

$$x^2y'' + 8xy' - 3xy = 0$$

$$P(x) = \frac{8}{x}$$

$$p(x) = 8, \text{ analytic}$$

$$Q(x) = -\frac{3}{x}$$

$$q(x) = -3x, \text{ analytic}$$

3)

$$(x^2 + 1)y'' + 7x^2y' - 3xy = 0$$

$$x = \pm i, \quad \therefore \text{ ordinary}$$

4)

$$(x^2 - x)y'' + xy' + 7y = 0$$

$$P(x) = \frac{1}{x - 1}$$

$$p(x) = 1, \text{ analytic}$$

$$Q(x) = \frac{7}{x(x - 1)}$$

$$q(x) = \frac{7(x - 1)}{x}, \text{ not analytic}$$