Homework 10 Rotations

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Contents

1 Book

1.1 10.22

$$r = 8.00\,\mathrm{cm}$$
$$m = 0.180\,\mathrm{kg}$$
$$v_0 = 0$$
$$\Delta y = 75.0\,\mathrm{cm}$$
$$I = mr^2$$

(a)

$$E_{K_0} + E_{P_0} = E_{K_1} + E_{P_1}$$
$$0 + mgh = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mr^2\omega^2 + 0$$
$$mgh = \omega^2 \left(\frac{1}{2}\left(mr^2\right) + \frac{1}{2}mr^2 \right)$$
$$\omega = \frac{\sqrt{gh}}{r}$$
$$\omega = \frac{\sqrt{(10.0\,\mathrm{m\,s^{-2}})(0.75\,\mathrm{m})}}{0.08\,\mathrm{m}}$$
$$\omega = 34.2\,\mathrm{rad\,s^{-1}}$$

(b)

$$E_{k_0} + E_{p_0} = E_{k_1} + E_{p_1}$$
$$0 + mgh = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv_{cm}^2 + 0$$
$$mgh = \frac{1}{2}(mr^2)\left(\frac{v_{cm}}{r}\right)^2 + \frac{1}{2}mv_{cm}^2$$
$$v = \sqrt{gh}$$
$$v = \sqrt{(10.0\,\mathrm{m\,s^{-2}})(0.75\,\mathrm{m})}$$
$$v = 2.74\,\mathrm{m\,s^{-1}}$$

1.2 10.26

$$I_{cm} = \frac{2}{5}mr^2$$

(a) Velocity for the first half of the bowl:

$$E_{K_0} + E_{P_0} = E_{K_1} + E_{P_1}$$
$$0 + mgh = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv_{cm}^2 + 0$$
$$mgh = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_{cm}^2}{r^2}\right) + \frac{1}{2}mv_{cm}^2$$
$$v_{cm} = \sqrt{\frac{10gh}{7}}$$

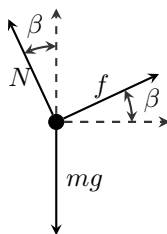Since the ball only slides and doesn't rotate, the kinetic energy it experi-

ences it purely linear velocity and *not* angular.

$$E_{K_0} + E_{P_0} = E_{K_1} + E_{P_1}$$

$$\frac{1}{2}mv_{cm}^2 + 0 = 0 + mgh_1$$

$$\left(\sqrt{\frac{10gh_0}{7}}\right)^2 = 2gh_1$$

$$h_1 = \frac{5}{7}h_0$$

The ball reaches only $\frac{5}{7}$ of the height of the side of the bowl.

1.3 10.30

(a) Free-body diagram:

The angular velocity of the bowling ball is clockwise ↻ which the friction has to oppose resulting in the friction going upwards (up the incline).

(b) Normal force has no affect as it is directed towards the axis of rotation.

$$\sum F_x = ma_{cm}$$

$$-f = ma_{cm} + mg\sin(\beta)$$

$$-\frac{I_{cm}\alpha}{r} = m(a_{cm} + g\sin(\beta))$$

$$-\frac{\left(\frac{2}{5}mr^2\right)\left(\frac{a_{cm}}{r}\right)}{r} = m(a_{cm} + g\sin(\beta))$$

$$-\frac{2}{5}a_{cm} = a_{cm} + g\sin(\beta)$$

$$a_{cm} = \frac{5g\sin(\beta)}{7}$$

(c)

$$\sum F_y = 0$$

$$N = mg\cos(\beta)$$

4

$$\sum F_x = ma_{cm}$$
$$-f = ma_{cm} + mg\sin(\beta)$$
$$-\mu(mg\cos(\beta)) = m\left(\frac{5g\sin(\beta)}{7} + g\sin(\beta)\right)$$
$$\mu = \frac{2\tan(\beta)}{7}$$

1.4 10.79

$$I_{cylinder_{cm}} = \frac{1}{2}m(2r)^2$$
$$I_{disk_{cm}} = \frac{1}{2}mr^2$$

$$E_{K_0} + E_{P_0} = E_{K_1} + E_{P_1}$$
$$\left(\frac{1}{2}I_{cylinder_{cm}}\left(\frac{v}{2r}\right)^2 + \frac{1}{2}mv^2\right) + \left(\frac{1}{2}I_{disk_{cm}}\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2\right) + 0 = 0 + mgh$$
$$\left(\frac{1}{2}\left(\frac{1}{2}m(2r)^2\right)\left(\frac{v}{2r}\right)^2 + \frac{1}{2}mv^2\right) + \left(\frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2\right) = mgh$$
$$v = \sqrt{\frac{2gh}{3}}$$

$$v_1^2 = v_0^2 + 2ah$$
$$a = \frac{v_1^2}{2h}$$
$$a = \frac{\sqrt{\frac{2gh}{3}}^2}{2h}$$
$$a = \frac{g}{3}$$

1.5 9.30

$$m = 0.200\,\text{kg}$$
$$l = 0.400\,\text{m}$$

(a)

$$r = \sqrt{(^l/_2)^2 + (^l/_2)^2}$$
$$r = 0.283\,\text{m}$$

$$I = \sum_{i=1}^{4}(mr^2)$$
$$I = 4(0.200\,\text{kg})(0.283\,\text{m})^2$$
$$I = 0.0641\,\text{kg}\,\text{m}^2 = 6.41 \times 10^{-2}\,\text{kg}\,\text{m}^2$$

(b)

$$r = 0.200\,\text{m}$$

$$I = \sum_{i=1}^{4}(mr^2)$$
$$I = 4(0.200\,\text{kg})(0.200\,\text{m})^2$$
$$I = 0.320\,\text{kg}\,\text{m}^2$$

(c)

$$r = 0.283\,\text{m}$$

$$I = \sum_{i=0}^{2}(mr^2)$$
$$I = 2(0.200\,\text{kg})(0.283\,\text{m})^2$$
$$I = 0.320\,\text{kg}\,\text{m}^2$$

1.6   9.49

$$r = 0.280\,\text{m}$$
$$m_{block} = 4.20\,\text{kg}$$
$$v_0 = 0$$
$$\Delta y = 3.00\,\text{m}$$
$$t = 2.00\,\text{s}$$
$$I_{wheel} = \frac{1}{2}mr^2$$
$$m_{wheel} = ?$$

Method 1:   Using Force Equations + Torque

$$\sum F_y^{block} = m_{block}a$$
$$m_{block}g = T + m_{block}a$$
$$T = m_{block}(g - a)$$

$$\sum F^{wheel} = m_{wheel}\alpha$$

$$\tau = I_{wheel}\alpha$$

$$Tr = \left(\frac{1}{2}m_{wheel}r^2\right)\left(\frac{a}{r}\right)$$

$$T = \frac{m_{wheel}a}{2}$$

$$a = \frac{v_1 - v_0}{t_1 - t_0}$$

$$a = \frac{3.00\,\mathrm{m\,s^{-1}} - 0}{2.00\,\mathrm{s} - 0}$$

$$a = 1.50\,\mathrm{m\,s^{-2}}$$

$$m_{block}(g - a) = \frac{m_{wheel}a}{2}$$

$$m_{wheel} = \frac{2m_{block}(g - a)}{a}$$

$$m_{wheel} = \frac{2(4.20\,\mathrm{kg})(10.0\,\mathrm{m\,s^{-2}} - 1.50\,\mathrm{m\,s^{-2}})}{1.50\,\mathrm{m\,s^{-2}}}$$

$$m_{wheel} = 47.6\,\mathrm{kg}$$

Method 2: Using Energy

$$\Delta y = \frac{1}{2}(v_f + v_o)t$$

$$v_f = \frac{2\Delta y}{t}$$

$$v_f = \frac{2(3.00\,\mathrm{m})}{2.00\,\mathrm{s}}$$

$$v_f = 3.00\,\mathrm{m\,s^{-1}}$$

$$\omega = \frac{v_f}{r}$$

$$\omega = \frac{3.00\,\mathrm{m\,s^{-1}}}{0.280\,\mathrm{m}}$$

$$\omega = 10.7\,\mathrm{rad\,s^{-1}}$$

$$E_{K_0} + E_{P_0} = E_{K_1} + E_{P_1}$$

$$0 + E_{P_{block}} = E_{R_{wheel}} + E_{K_{block}}$$

$$m_{block}g\Delta y = \frac{1}{2}I_{wheel}\omega_{wheel}^2 + \frac{1}{2}m_{block}v_{block}^2$$

$$m_{block}g\Delta y = \frac{1}{2}\left(\frac{1}{2}m_{wheel}r^2\right)\omega_{wheel}^2 + \frac{1}{2}m_{block}v_{block}^2$$

$$m_{block}\left(4g\Delta y - 2v_{block}^2\right) = m_{wheel}r^2\omega_{wheel}^2$$

$$m_{wheel} = \frac{m_{block}\left(4g\Delta y - 2v_{block}^2\right)}{r^2\omega_{wheel}^2}$$

$$m_{wheel} = \frac{(4.20\,\text{kg})\left(4(10.0\,\text{m s}^{-2})(3.00\,\text{m}) - 2(3.00\,\text{m s}^{-1})^2\right)}{(0.280\,\text{m})^2(10.7\,\text{rad s}^{-1})^2}$$

$$m_{wheel} = 47.7\,\text{kg}$$

1.7   9.79

$$r_1 = 2.55\,\text{cm} = 0.0255\,\text{m}$$
$$m_1 = 0.85\,\text{kg}$$
$$r_2 = 5.02\,\text{cm} = 0.0502\,\text{m}$$
$$m_2 = 1.58\,\text{kg}$$

(a)

$$I = \sum_{i=1}^{2} I_i$$

$$I = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2)$$

$$I = \frac{1}{2}\left((0.85\,\text{kg})(0.0255\,\text{m})^2 + (1.58\,\text{kg})(0.0502\,\text{m})^2\right)$$

$$I = 0.002\,27\,\text{kg m}^2 = 2.27 \times 10^{-3}\,\text{kg m}^2$$

(b)

$$v_{block_0} = 0$$
$$y_1 = 2.03\,\text{m}$$
$$y_0 = 0$$
$$v_{block_1} = ?$$

$$E_{K_0} + E_{P_0} = E_{K_1} + E_{P_1}$$

$$m_{block} g \Delta y + 0 = \frac{1}{2} I \left(\frac{v_{block_1}}{r_1} \right)^2 + \frac{1}{2} m_{block} v_{block_1}^2 + 0$$

$$2g\Delta y = v_{block_1}^2 \left(\frac{I}{m_{block}(r_1)^2} + 1 \right)$$

$$v_{block_1} = \sqrt{\frac{2g\Delta y}{\frac{I}{m_{block}(r_1)^2} + 1}}$$

$$v_{block_1} = \sqrt{\frac{2(10.0\,\mathrm{m\,s^{-2}})(2.03\,\mathrm{m})}{\frac{0.002\,77\,\mathrm{kg\,m^2}}{(1.50\,\mathrm{kg})(0.0255\,\mathrm{m})^2} + 1}}$$

$$v_{block_1} = 3.25\,\mathrm{m\,s^{-1}}$$

(c)

$$v_{block_1} = \sqrt{\frac{2g\Delta y}{\frac{I}{m_{block}(r_2)^2} + 1}}$$

$$v_{block_1} = \sqrt{\frac{2(10.0\,\mathrm{m\,s^{-2}})(2.03\,\mathrm{m})}{\frac{0.002\,77\,\mathrm{kg\,m^2}}{(1.50\,\mathrm{kg})(0.0502\,\mathrm{m})^2} + 1}}$$

$$v_{block_1} = 4.84\,\mathrm{m\,s^{-1}}$$

The final velocity of the block is greater when the string is wrapped around the higher radius metal disk due to how linear velocity increases with longer lengths. Velocity can be calculated from radius and ω using the formula $v = r\omega$ proving that a greater radius will result in a greater velocity.

1.8   9.86

$$P = 5 \times 10^{31}\,\mathrm{W}$$

$$T = 0.0331\,\mathrm{s}$$

$$\Delta t = 4.22 \times 10^{-13}\,\mathrm{s}$$

(a)

$$E = \frac{1}{2}I\omega^2$$

$$E = \frac{1}{2}I\left(\frac{4\pi^2}{T^2}\right)$$

$$\frac{d}{dt}(E) = \frac{d}{dt}\left(\frac{2I\pi^2}{T^2}\right)$$

$$P = -\frac{4I\pi^2\Delta t}{T^3}$$

$$I = -\frac{PT^3}{4\pi^2\Delta t}$$

$$I = -\frac{(5\times 10^{31}\,\mathrm{W})(0.0331\,\mathrm{s})^3}{4\pi^2(4.22\times 10^{-13}\,\mathrm{s})}$$

$$I = 1.09\times 10^{38}\,\mathrm{kg\,m^2}$$

(b)

$$m_{sun} = 1.9891\times 10^{30}\,\mathrm{kg}$$

$$I = \frac{2}{5}mr^2$$

$$r = \sqrt{\frac{5I}{2m}}$$

$$r = \sqrt{\frac{5(1.09\times 10^{38}\,\mathrm{kg\,m^2})}{2(1.4)(1.9891\times 10^{30}\,\mathrm{kg})}}$$

$$r = 9892.16\,\mathrm{m} = 9.89\times 10^3\,\mathrm{m}$$

(c)

$$v = \frac{2\pi r}{T}$$

$$v = \frac{2\pi(9892.16\,\mathrm{m})}{0.0331\,\mathrm{s}}$$

$$v = 1.88\times 10^6\,\mathrm{m\,s^{-1}} = 1\,877\,772\,\mathrm{m\,s^{-1}} < c = 299\,792\,458\,\mathrm{m\,s^{-1}}$$

(d)

$$\rho_{rock} = 3000\,\mathrm{kg\,m^{-3}}$$

$$\rho_{nucleus} = 10\times 10^{17}\,\mathrm{kg\,m^{-3}}$$

$$\rho_{\text{neutron star}} = \frac{m}{\frac{4\pi r^3}{3}}$$

$$\rho_{\text{neutron star}} = \frac{(1.4)(1.99 \times 10^{30}\,\text{kg})}{\frac{4\pi(9.89 \times 10^3\,\text{m})^3}{3}}$$

$$\rho_{\text{neutron star}} = 6.88 \times 10^{17}\,\text{kg}\,\text{m}^{-3}$$

The density of a neutron star is much closer to the density of an atomic nuclear when compared with the density of an ordinary rock.

2 Lab Manual

2.1 1170

$$I = \sum_{i=1}^{\infty} \left[\left(\frac{M}{n} \right) \left(i\frac{a}{n} \right)^2 \right]$$

$$I = \frac{Ma^2}{n^3} \sum_{i=1}^{\infty} \left[i^2 \right]$$

$$I = \frac{Ma^2}{n^3} \left(\frac{n(n+1)(2n+1)}{6} \right)$$

$$I = \frac{Ma^2}{6} \left(\frac{n(n+1)(2n+1)}{n^3} \right)$$

$$I = \lim_{n \to \infty} (I)$$

$$I = \frac{Ma^2}{6} \lim_{n \to \infty} \left[\left(1 + \frac{1}{n} \right) \left(2 + \frac{1}{n} \right) \cdots \right]$$

$$I = \frac{Ma^2}{6}(3)$$

$$I = \frac{Ma^2}{3}$$

2.2 1173

(a)

$$A_{small} = \pi R^2$$

$$A = \pi \left(\frac{R}{2} \right)^2$$

$$A = \frac{\pi R^2}{4}$$

$$m_{small} = \frac{m_{disk}}{4}$$

$$I_{small} = \frac{1}{2}m_{small}\left(\frac{R}{2}\right)^2 + m_{small}\left(\frac{R}{2}\right)^2$$

$$I_{small} = \frac{3}{32}m_{disk}R^2$$

$$I_{disk} - I_{small} = I_{crescent}$$

$$\frac{1}{2}m_{disk}R^2 - \frac{3}{32}m_{disk}R^2 = I_{crescent}$$

$$I_{crescent} = \frac{13}{32}m_{disk}R^2$$

(b)

$$I = I_{disk} - I_{small}$$

$$I = \frac{1}{2}m_{disk}r^2 - \left(\frac{1}{2}m_{small}\left(\frac{r}{2}\right)^2 + m_{small}\left(\frac{r}{2}\right)^2\right)$$

$$I = \frac{1}{2}\left(\frac{4M}{3}\right)r^2 - \left(\frac{1}{2}\left(\frac{M}{3}\right)\left(\frac{r}{2}\right)^2 + \left(\frac{M}{3}\right)\left(\frac{r}{2}\right)^2\right)$$

$$I = \frac{13Mr^2}{24}$$

2.3   1175

$$r = r_{1,2} = 10\,\text{cm} = 0.100\,\text{m}$$

$$m = m_{1,2} = 1000\,\text{g} = 1.00\,\text{kg}$$

$$l = 30\,\text{cm} = 0.300\,\text{m}$$

(a)

$$I = I_1 + I_{rod} + I_2$$

$$I = \left(\frac{2}{5}mr^2\right) + 0 + \left(\frac{2}{5}mr^2 + ml^2\right)$$

$$I = \left(\frac{2}{5}(1.00\,\text{kg})(0.100\,\text{m})^2\right) + \left(\frac{2}{5}(1.00\,\text{kg})(0.100\,\text{m})^2 + (1.00\,\text{kg})(0.300\,\text{m})^2\right)$$

$$I = 0.0980\,\text{kg}\,\text{m}^2$$

(b)

$$\tau = 98\,000\,\text{dyn}\,\text{cm} = 0.009\,80\,\text{N}\,\text{m}$$

$$\tau = I\alpha$$

$$\omega = \sqrt{\frac{4\tau\pi}{I}}$$

$$\omega = \sqrt{\frac{4(0.009\,80\,\text{N m})\pi}{0.0980\,\text{kg m}^2}}$$

$$\omega = 1.12\,\text{rad s}^{-1}$$

2.4   1177

$$m_{pulley} = 4\,\text{slug}$$
$$m_1 = 5\,\text{slug}$$
$$m_2 = 3\,\text{slug}$$
$$R = 0.5\,\text{feet}$$

(a)

$$\sum F_y^{m_1} = m_1 a$$
$$m_1 g = m_1 a + T_1$$
$$T_1 = m_1(g - a)$$

$$\sum F_y^{m_2} = m_2 a$$
$$T_2 = m_2 a + m_2 g = m_2(a + g)$$

$$\tau = I\alpha$$

$$RT_1 - RT_2 = \left(\frac{1}{2}m_{pulley}R^2\right)\left(\frac{a}{R}\right)$$

$$2R(m_1 g - m_1 a - m_2 a - m_2 g) = Ram_{pulley}$$
$$2a(-m_1 - m_2) + 2g(m_1 - m_2) = am_{pulley}$$
$$2a(-m_1 - m_2) - am_{pulley} = -2g(m_1 - m_2)$$
$$a(-2(m_1 + m_2) - m_{pulley}) = -2g(m_1 - m_2)$$

$$a = -\frac{2g(m_1 - m_2)}{-2(m_1 + m_2) - m_{pulley}}$$

$$a = -\frac{2(32.17\,\text{feet s}^{-2})(5\,\text{slug} - 3\,\text{slug})}{-2(5\,\text{slug} + 3\,\text{slug}) - 4\,\text{slug}}$$

$$a = 6.54\,\text{feet s}^{-2}$$

$$T_1 = m_1(g - a)$$
$$T_1 = (5\,\text{slug})(32.17\,\text{feet s}^{-2} - 6.54\,\text{feet s}^{-2})$$
$$T_1 = 128.15\,\text{slug feet s}^{-2} = 1.28 \times 10^2\,\text{slug feet s}^{-2}$$

$$T_2 = m_2(a + g)$$
$$T_2 = (3\,\text{slug})(6.54\,\text{feet s}^{-2} + 32.17\,\text{feet s}^{-2})$$
$$T_2 = 116.13\,\text{slug feet s}^{-2} = 1.16 \times 10^2\,\text{slug feet s}^{-2}$$

(b)

$$\sum F_y^{pulley} = 0$$
$$T_{top} = T_1 + T_2$$
$$T_{top} = 128.15\,\text{slug feet s}^{-2} + 116.13\,\text{slug feet s}^{-2}$$
$$T_{top} = 244.28\,\text{slug feet s}^{-2} = 2.44 \times 10^2\,\text{slug feet s}^{-2}$$

The reason the tension in the cord support the pulley may not be equal to the sum of the tensions may be due to the cord itself *having mass*. It could also stem from how the cord holding the pulley isn't exactly R feet from the pulley's center of mass.

2.5   1181

$$I_{ring} = \frac{1}{2}m(r_{outer}^2 + r_{inner}^2)$$
$$m = 2.4\,\text{kg}$$
$$r_{inner} = 6.0\,\text{cm} = 0.06\,\text{m}$$
$$r_{outer} = 8.0\,\text{cm} = 0.08\,\text{m}$$
$$v_{y=2.0\,m} = 2.8\,\text{m s}^{-1}$$
$$y_{max} = ?$$
$$\theta = 37.0°$$

$$\omega_{ring_{y=2.0\,m}} = \frac{v}{r_{outer}} = \frac{2.8\,\text{m s}^{-1}}{0.08\,\text{m}} = 35.0\,\text{rad s}^{-1}$$

$$E_{K_0} + E_{P_0} = E_{K_1} + E_{P_1}$$
$$\left(\frac{1}{2}I_{ring}\omega_{ring}^2 + \frac{1}{2}mv^2\right) + 0 = 0 + mgy_1\sin(\theta)$$
$$\frac{1}{2}(r_{outer}^2 + r_{inner}^2)\omega_{ring}^2 + v^2 = 2gy_1\sin(\theta)$$
$$y_1 = \frac{\omega_{ring}^2(r_{outer}^2 + r_{inner}^2) + 2v^2}{4g\sin(\theta)}$$
$$y_1 = \frac{(35.0\,\text{rad s}^{-1})^2\left[(0.08\,\text{m})^2 + (0.06\,\text{m})^2\right] + 2(2.8\,\text{m s}^{-1})^2}{4(10.0\,\text{m s}^{-2})\sin(37.0°)}$$
$$y_1 = 1.16\,\text{m}$$

$$y_{max} = y_0 + y_1$$
$$y_{max} = 2.0\,\text{m} + 1.16\,\text{m}$$
$$y_{max} = 3.16\,\text{m}$$

2.6 1283

$$I_{cm} = \frac{2}{5}mr^2$$
$$v_0 = 0$$
$$\theta = 30.0°$$
$$d_{1_{slope}} = 70\,\text{cm} = 0.70\,\text{m}$$
$$d_2 = 140\,\text{cm} = 1.40\,\text{m}$$
$$\Delta y = 245\,\text{cm} = 2.45\,\text{m}$$
$$d_3 = ?$$

(a) Normal force, weight, inertia

(b)

$$\sin(\theta) = \frac{d_{1_y}}{d_{1_{slope}}}$$
$$d_{1_y} = d_{1_{slope}}\sin(\theta)$$
$$d_{1_y} = (0.70\,\text{m})\sin(30.0°)$$
$$d_{1_y} = 0.35\,\text{m}$$

$$E_{K_0} + E_{P_0} = E_{K_1} + E_{P_1}$$
$$0 + mgd_{1_y} = \left[\frac{1}{2}I_{cm}\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2\right] + 0$$
$$gd_{1_y} = \frac{1}{2}\left[\frac{2}{5}v^2 + v^2\right]$$
$$v = \sqrt{\frac{10gd_{1_y}}{7}}$$
$$v = 2.24\,\text{m}\,\text{s}^{-1}$$

$$v_1^2 = v_0^2 + 2ad_1$$
$$a = \frac{(2.24\,\text{m}\,\text{s}^{-1})^2}{2(0.70\,\text{m})}$$
$$a = 3.584\,\text{m}\,\text{s}^{-2}$$

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$$d_1 = v_o^2 t_1 + \frac{1}{2} a t_1^2$$

$$t_1 = \sqrt{\frac{2d_1}{a}}$$

$$t_1 = \sqrt{\frac{2(0.70\,\mathrm{m})}{3.584\,\mathrm{m\,s^{-2}}}}$$

$$t_1 = 0.625\,\mathrm{s}$$

$$d_2 = v t_2$$

$$t_2 = \frac{d_2}{v}$$

$$t_2 = \frac{1.40\,\mathrm{m}}{2.24\,\mathrm{m\,s^{-1}}}$$

$$t_2 = 0.625\,\mathrm{s}$$

$$\Delta y = v_{o_y}^2 t_3 + \frac{1}{2} g t_3^2$$

$$t_3 = \sqrt{\frac{2\Delta y}{g}}$$

$$t_3 = \sqrt{\frac{2(2.45\,\mathrm{m})}{10.0\,\mathrm{m\,s^{-2}}}}$$

$$t_3 = 0.7\,\mathrm{s}$$

$$\Delta t = \sum_{i=1}^{3} t_i$$

$$\Delta t = 2(0.625\,\mathrm{s}) + 0.70\,\mathrm{s}$$

$$\Delta t = 1.95\,\mathrm{s}$$

(c)

$$R = v_0 t_3 + \frac{1}{2} a_x t_3^2$$

$$R = (2.24\,\mathrm{m\,s^{-1}})(0.7\,\mathrm{s})$$

$$R = 1.568\,\mathrm{m} = 1.57\,\mathrm{m}$$

2.7   1284

$$E_{K_0} + E_{P_0} = E_{K_1} + E_{P_1}$$

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\omega^2\right) + 0 = 0 + 2mgR$$

$$v^2 = \frac{20gR}{7}$$

$$\frac{v^2}{R} = \frac{20g}{7}$$

$$\sum \tau = I\alpha$$

$$\sum \tau = \left(\frac{2}{5}mR^2\right)\frac{a}{R}$$

$$mg + \frac{2}{5}ma = ma$$

$$a = \frac{5g}{r}$$

3   Problem C: Spherical Symmetry Problem

Starting with $I = \int r^2 dm$, calculate the moment of inertial for an axis of rotation that goes through the center of a sphere with uniform mass density ρ, and radius R. As discussed in class, you may treat this problem like the integration of a

series of concentric spherical shells with thickness dr.

$$I_{disk} = \frac{1}{2}MR^2$$

$$dI = \frac{1}{2}r^2 dm$$

$$dm = \rho dv$$

$$dv = \pi r^2 dr$$

$$dm = \rho \pi r^2 dr$$

$$dI = \frac{1}{2}\rho \pi r^4 dr$$

$$r^2 = R^2 - x^2$$

$$dI = \frac{1}{2}\rho \pi (R^2 - x^2)^2 dr$$

$$I = \frac{1}{2}\rho \pi \int_{-R}^{R} (R^2 - x^2)^2 dx$$

$$I = \rho \pi \left(\frac{8}{15}\right) R^5$$

$$I = \frac{8}{15}\frac{3M\pi R^3}{4}\pi R^5$$

$$I = \frac{2}{5}MR^2$$