Лабораторная работа №1

Интерполяция алгебраическими многочленами

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Вариант 2

2	$f_1(x) = x^3 \cos(3x-1), \quad f_2(x) = \lvert 5\cos 3x + 3 \rvert, \quad [a,b] = [-2,2]$.

Способы выбора узлов:

1) Равноотстоящие узлы (расстояние между любыми двумя соседними точками одинаково) выбирались следующим образом:

```
x[0] = a;
for (int i = 1; i <= n; i++)
{
    x[i] = i * (b - a) / n + a;
}
```

2) Чебышевские узлы вычислялись по формуле:

```
x[i] = (a + b) / 2 + (b - a) / 2 * cos((2 * i + 1) * M_PI / 2 / (n + 1));
```

Представление, использованное при построении интерполяции многочленов

При построении интерполяции многочленов использовались:

1) Функции получения вектора узлов описанными выше способами

```
vector<double> getEqualNodes(int n)
{
    vector<double> x(n + 1);
    x[0] = a;
    for (int i = 1; i <= n; i++)
    {
        x[i] = i * (b - a) / n + a;
    }

    return x;
}

vector<double> getChebyshevNodes(int n)
{
    vector<double> x(n + 1);
    for (int i = 0; i <= n; i++)
    {
        x[i] = (a + b) / 2 + (b - a) / 2 * cos((2 * i + 1) * M_PI / 2 / (n + 1));
    }
                const double a = (-2.0)
                Поиск в Интернете
    return x;
}
```

2) Функция получения всех α_i

```
vector<double> getConstants(vector<double>& x, double(*f)(double))
{
    vector<double>base(x.size());
    for (int i = 0; i < x.size(); i++)
    {
        base[i] = f(x[i]);
    }

    vector<double>res;
    res.push_back(base[0]);
    for (int cnt = x.size() - 1; cnt > 0; cnt--)
    {
        vector<double> temp(cnt);
        for (int i = 0; i < cnt; i++)
        {
            temp[i] = (base[i + 1] - base[i]) / (x[i + x.size() - cnt] - x[i]);
        }

        res.push_back(temp[0]);
        base.clear();
        base = temp;
        temp.clear();
    }

    return res;
}
```

3) Непосредственно сама функция вычисления полинома в точке

```
double NewtonPolynom(vector<double>& scalars, vector<double>& x, double xp)
{
    double res = 0;
    double mnoz = 1;
    for (int i = 0; i < x.size(); i++)
    {
        res += scalars[i] * mnoz;
        mnoz *= (xp - x[i]);
    }

    return res;
}
```

Таблица погрешностей для функции 1)

n	Равноотстоящие узлы	Чебышевские узлы
3	4.12512	3.27213
4	4.10145	2.46963
5	3.19041	1.60894
6	1.51405	0.54151
7	1.03857	0.391995
8	1.85339	0.313566
9	1.41301	0.199272
10	0.891228	0.0792641
11	0.489626	0.0354085

12	0.20678	0.0101618
13	0.0956468	0.00353947
14	0.0307953	0.000810207
15	0.0120534	0.000232827
16	0.00310756	4.41263e-05
17	0.00105985	1.10107e-05
18	0.000236755	1.76486e-06
19	7.5773e-05	3.92725e-07
20	1.48644e-05	5.43171e-08
21	4.29792e-06	1.09507e-08
22	7.38597e-07	1.32995e-09
23	1.94962e-07	2.47447e-10
24	2.97996e-08	4.55431e-11
25	7.23942e-09	4.18749e-11
26	9.94186e-10	2.60663e-11
27	2.23025e-10	4.73843e-11
28	3.4811e-11	2.41922e-11
29	2.50582e-11	2.26237e-11
30	2.24007e-11	4.56648e-11

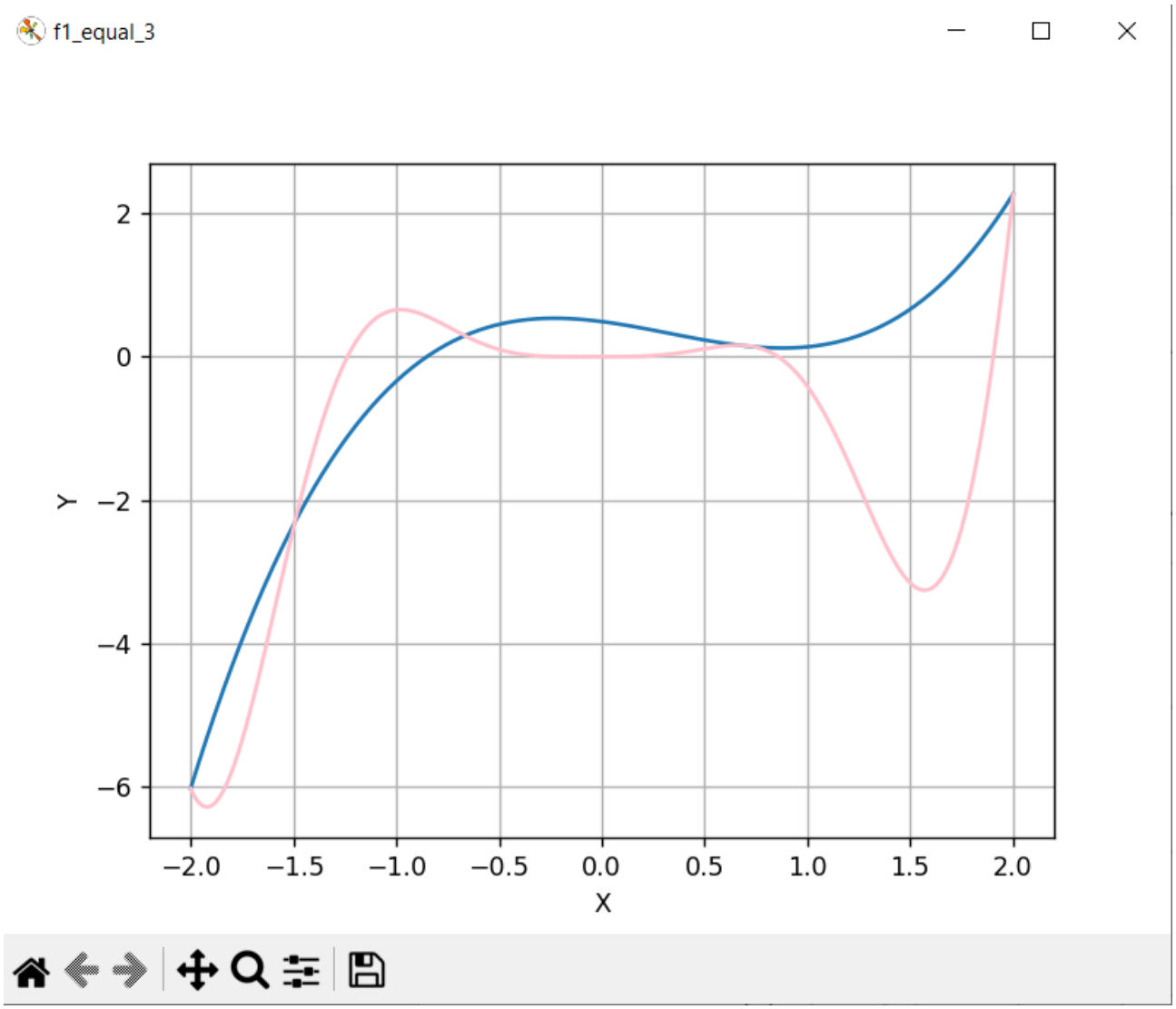
Таблица погрешностей для функции 2)

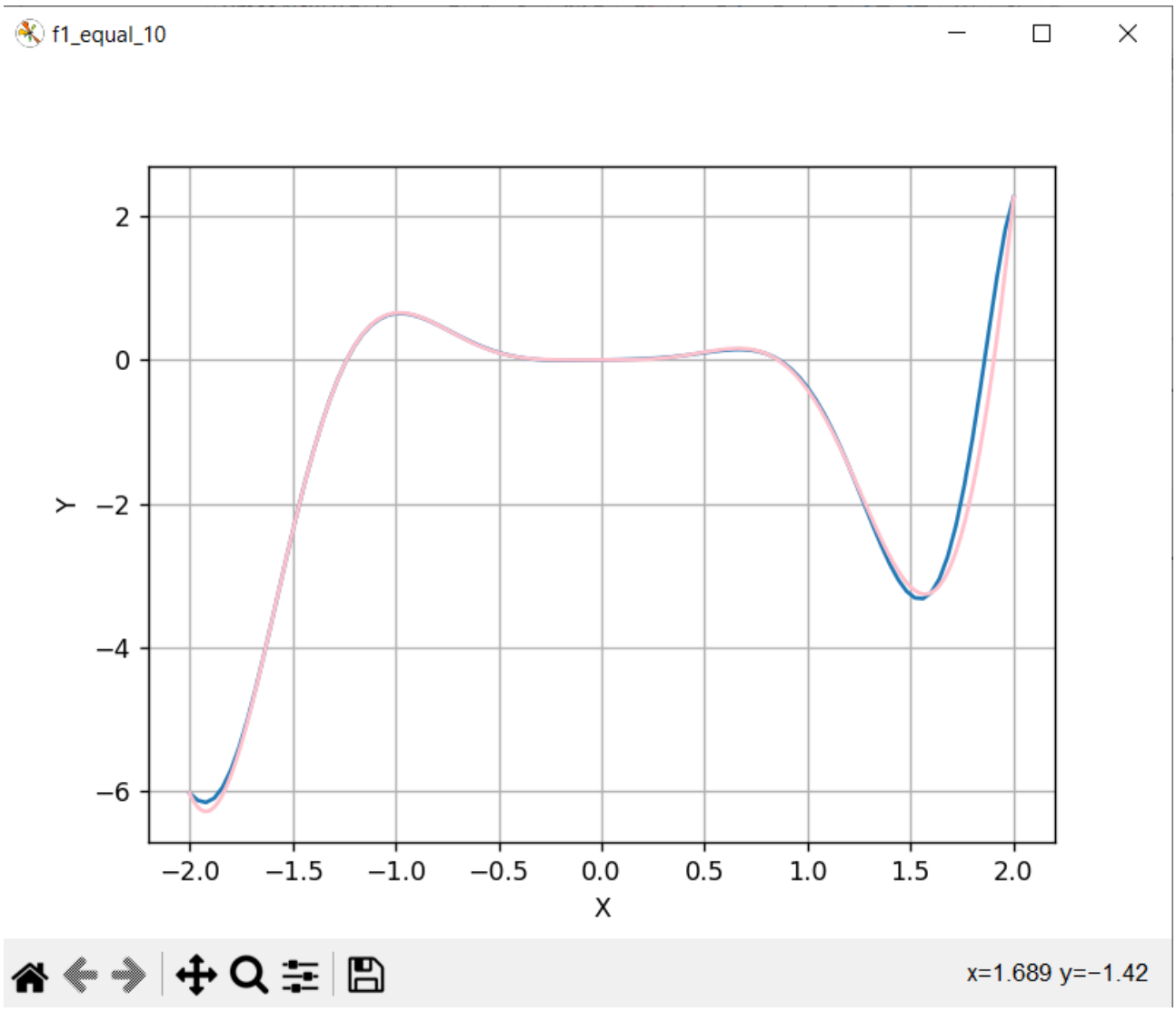
n	Равноотстоящие узлы	Чебышевские узлы
3	7.94093	9.00366
4	4.14281	4.5129
5	3.22255	3.73981
6	4.33285	2.47277
7	2.20164	3.0711
8	10.9413	3.17221
9	7.69212	2.9131
10	2.35104	2.25655
11	8.77435	2.04637
12	41.6615	1.82223
13	23.3406	1.10199
14	31.1349	1.55376
15	44.9435	1.43417
16	157.351	0.94878
17	9.61235	0.938441
18	313.262	0.736069
19	280.518	0.502749

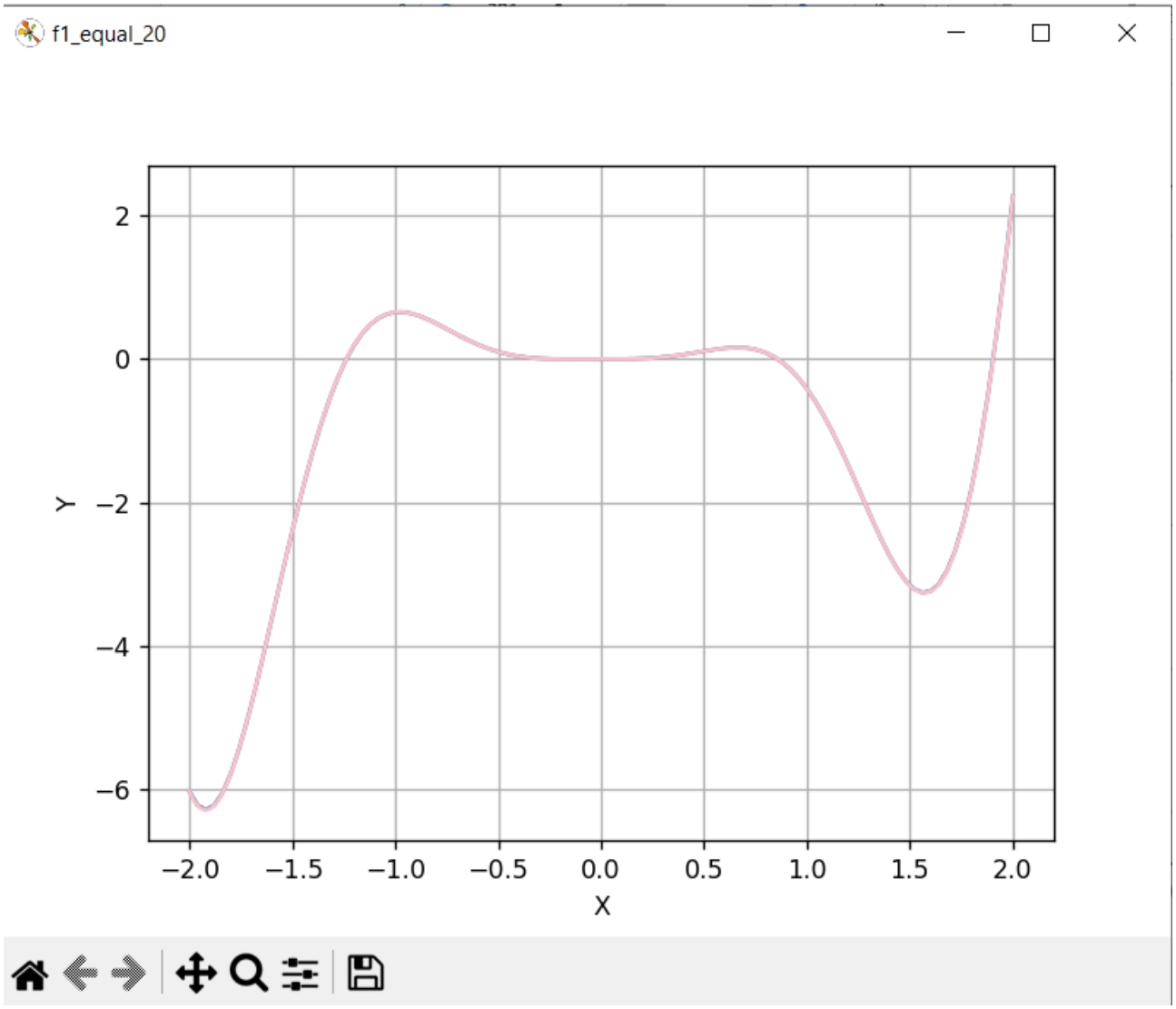
20	98.8846	0.795094
21	459.312	0.487587
22	3004.54	0.606728
23	364.695	0.595527
24	5490.81	0.723672
25	5636.46	0.696574
26	6192.74	0.697593
27	6733.91	0.472944
28	61005.6	0.606958
29	23873.7	0.560365
30	73721.6	0.539263

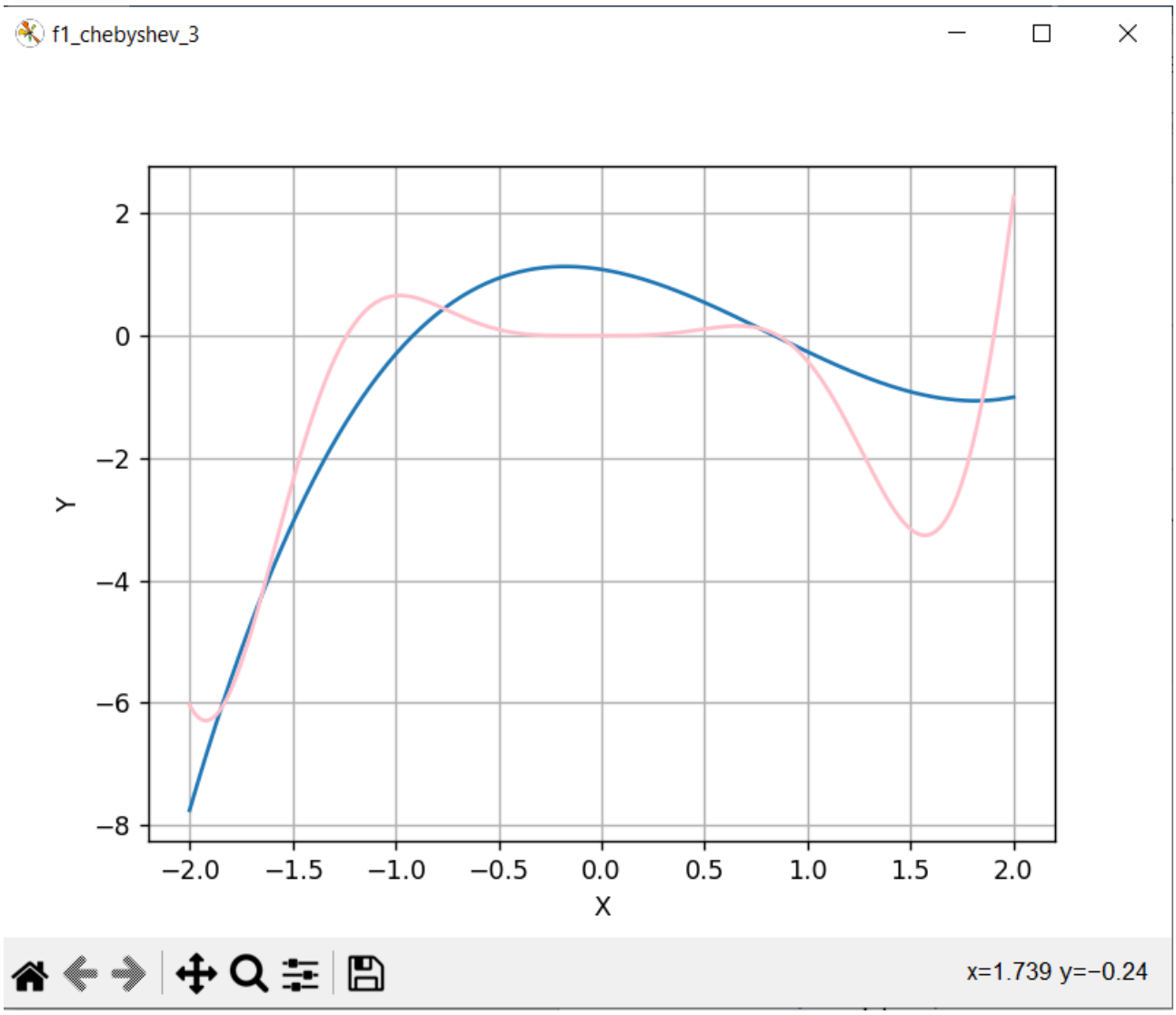
Получены следующие графики:

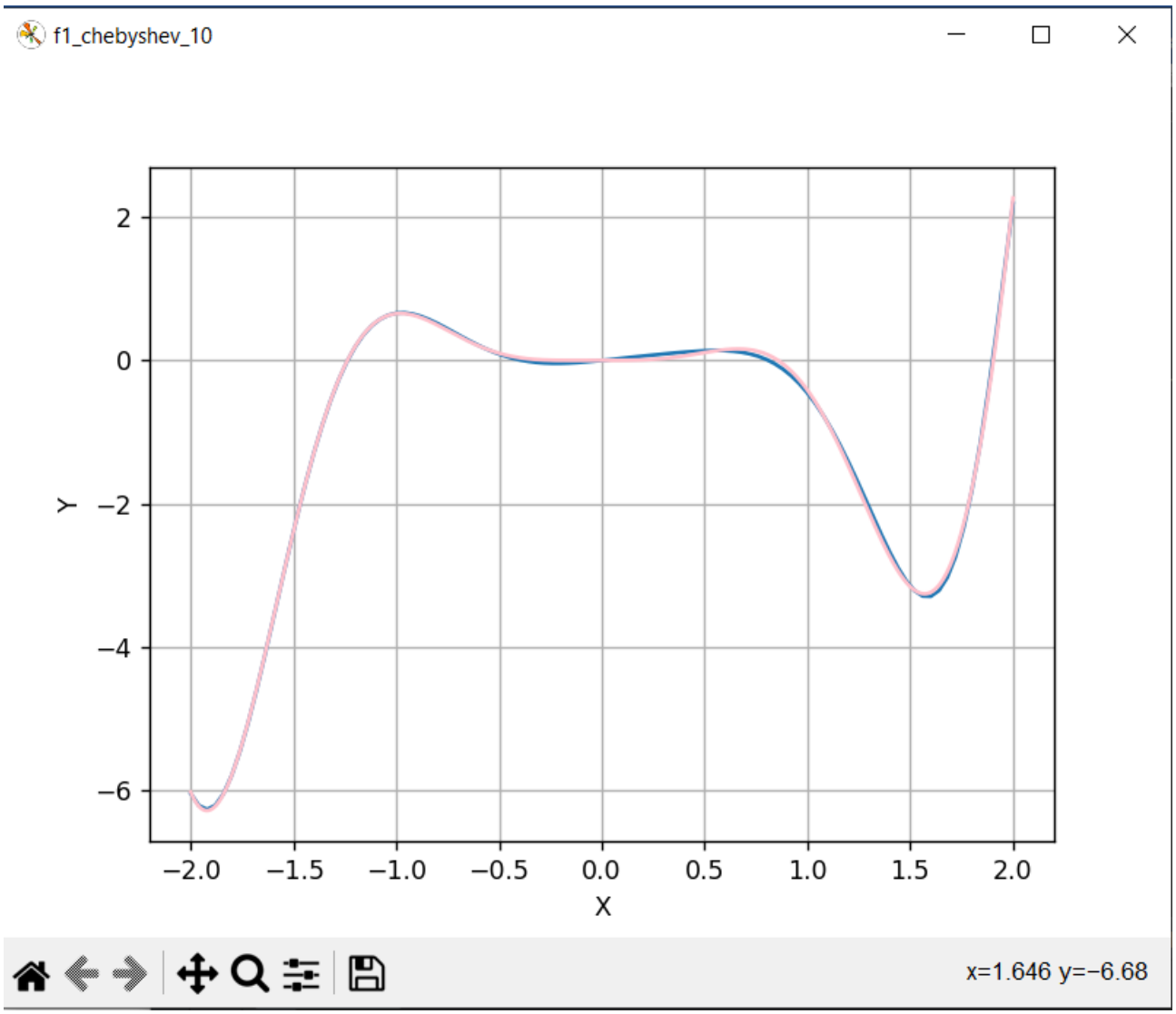
Розовый – график исходной функции, синий – график по полученным точкам

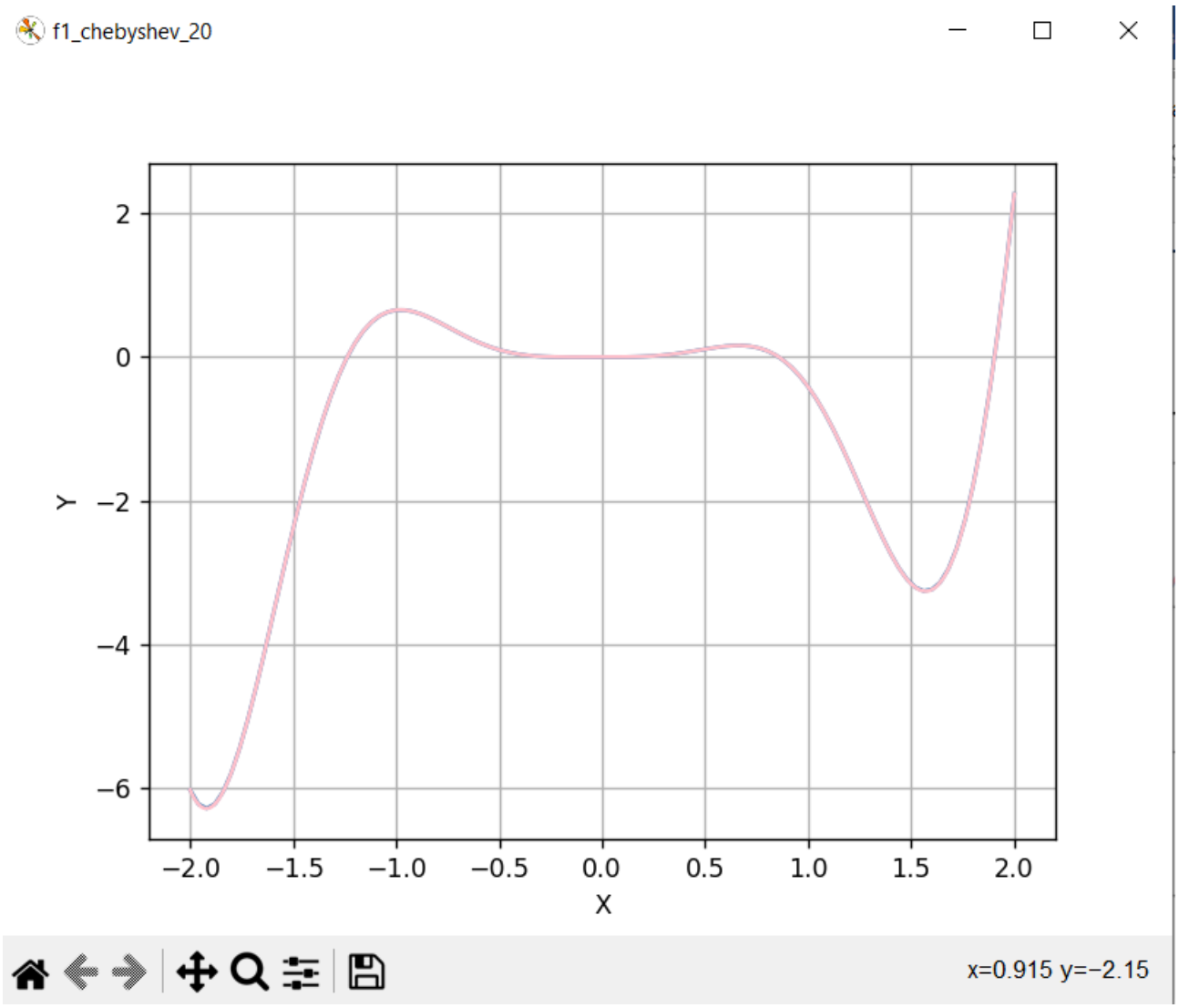


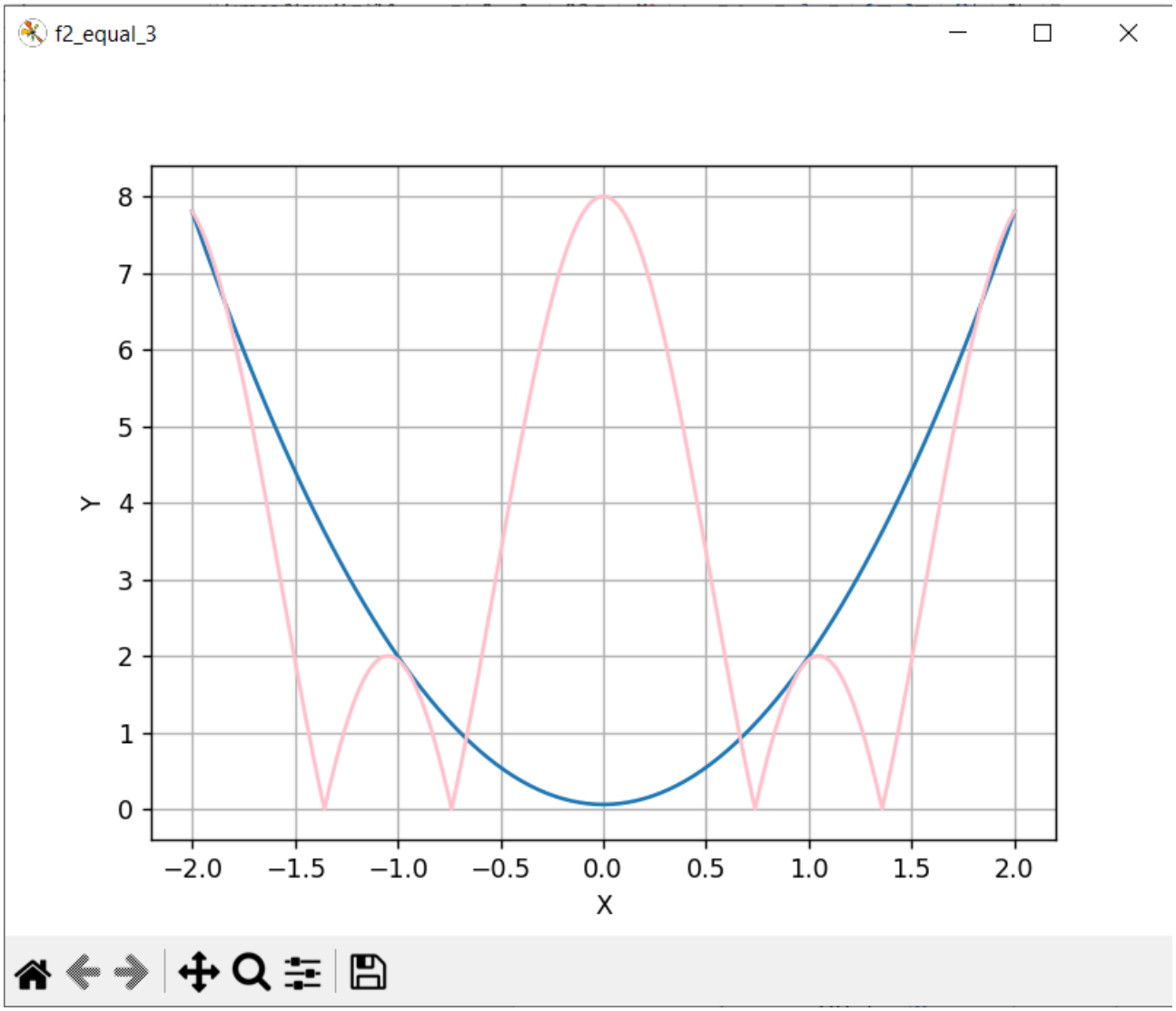

f1_equal_10
2
0
−2
−4
−6
Y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
X
x=1.689 y=−1.42

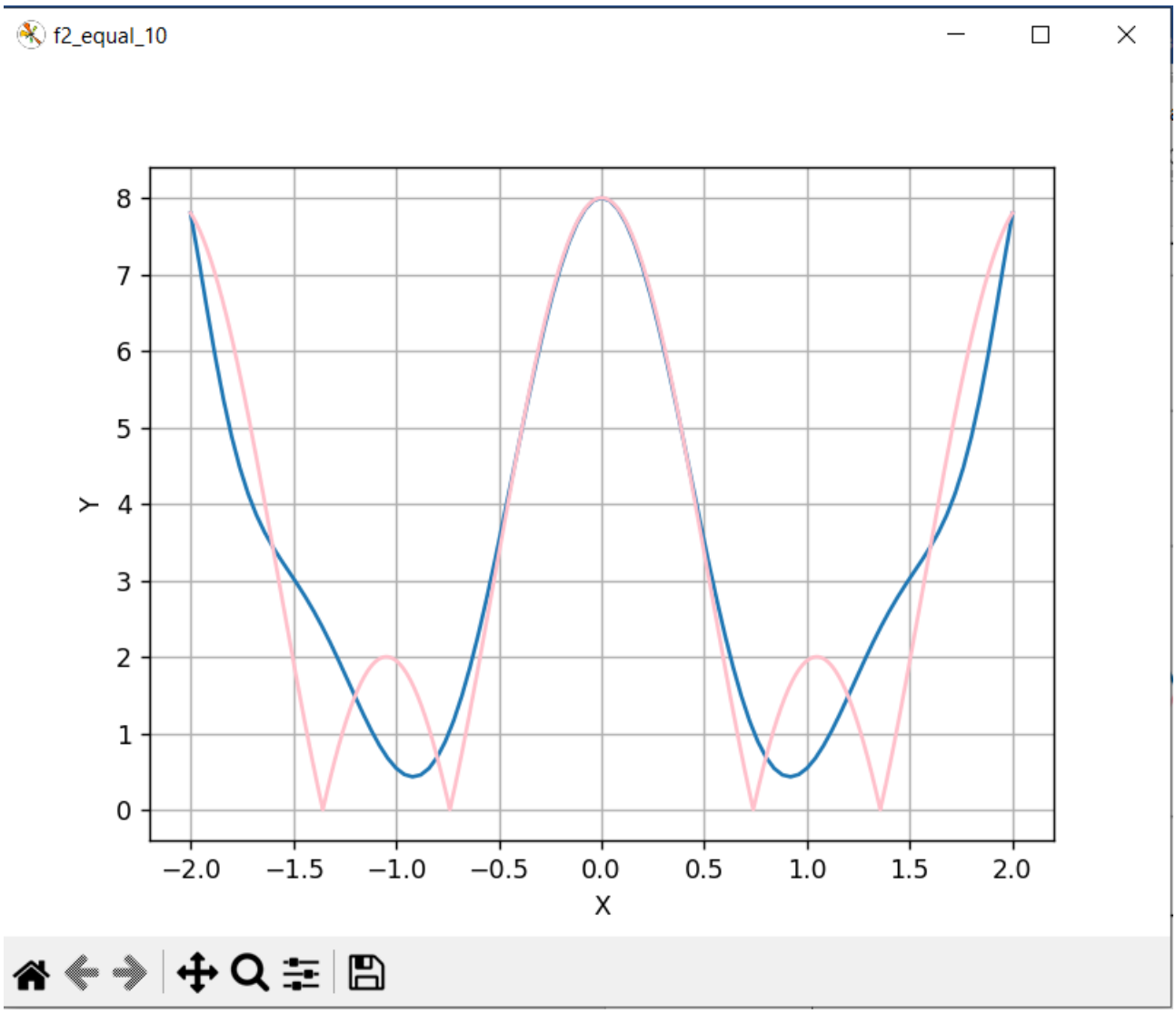

f1_equal_20
2
0
−2
−4
−6
Y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
X

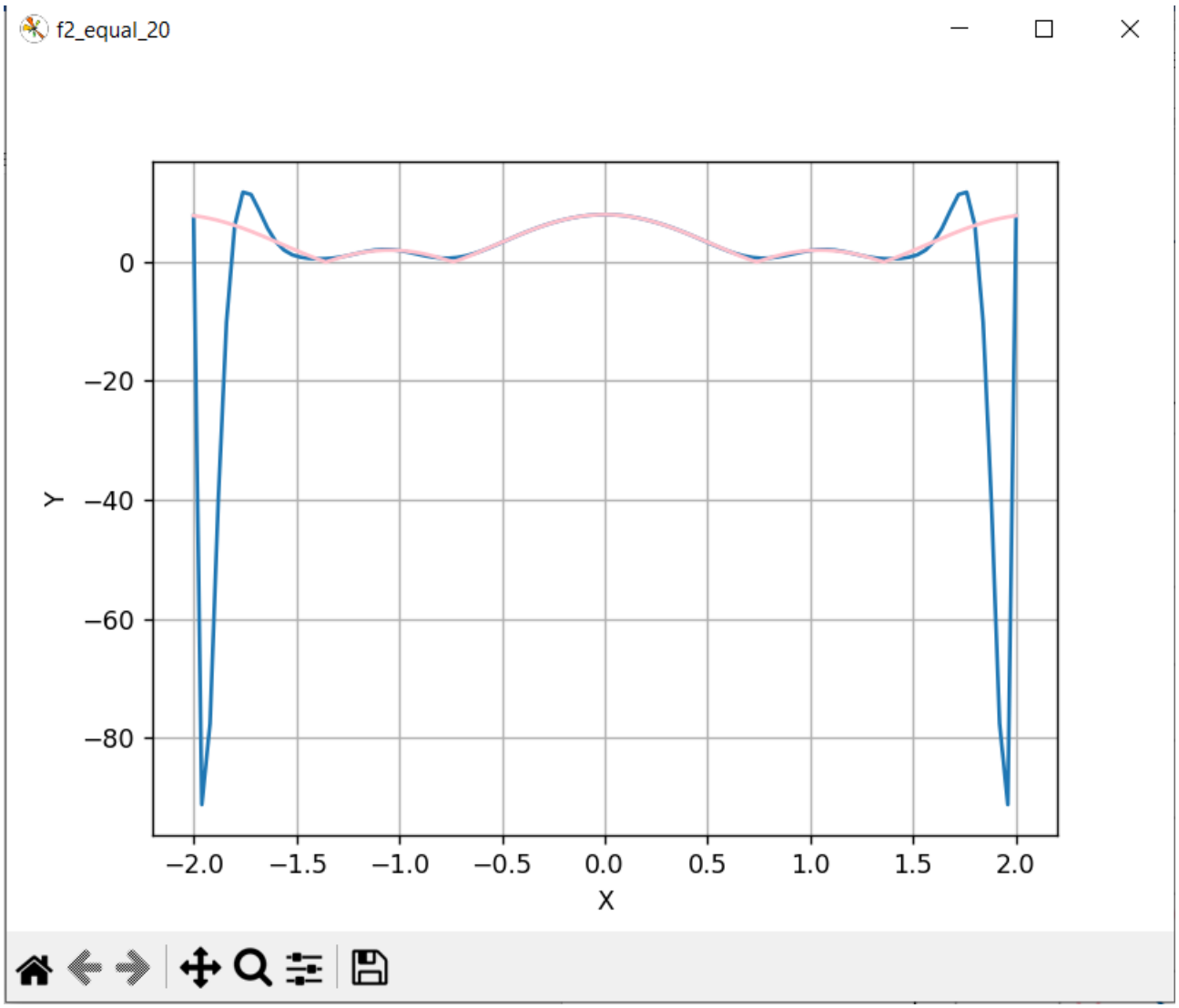

f1_chebyshev_3
2
0
−2
−4
−6
−8
Y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
X
x=1.739 y=−0.24

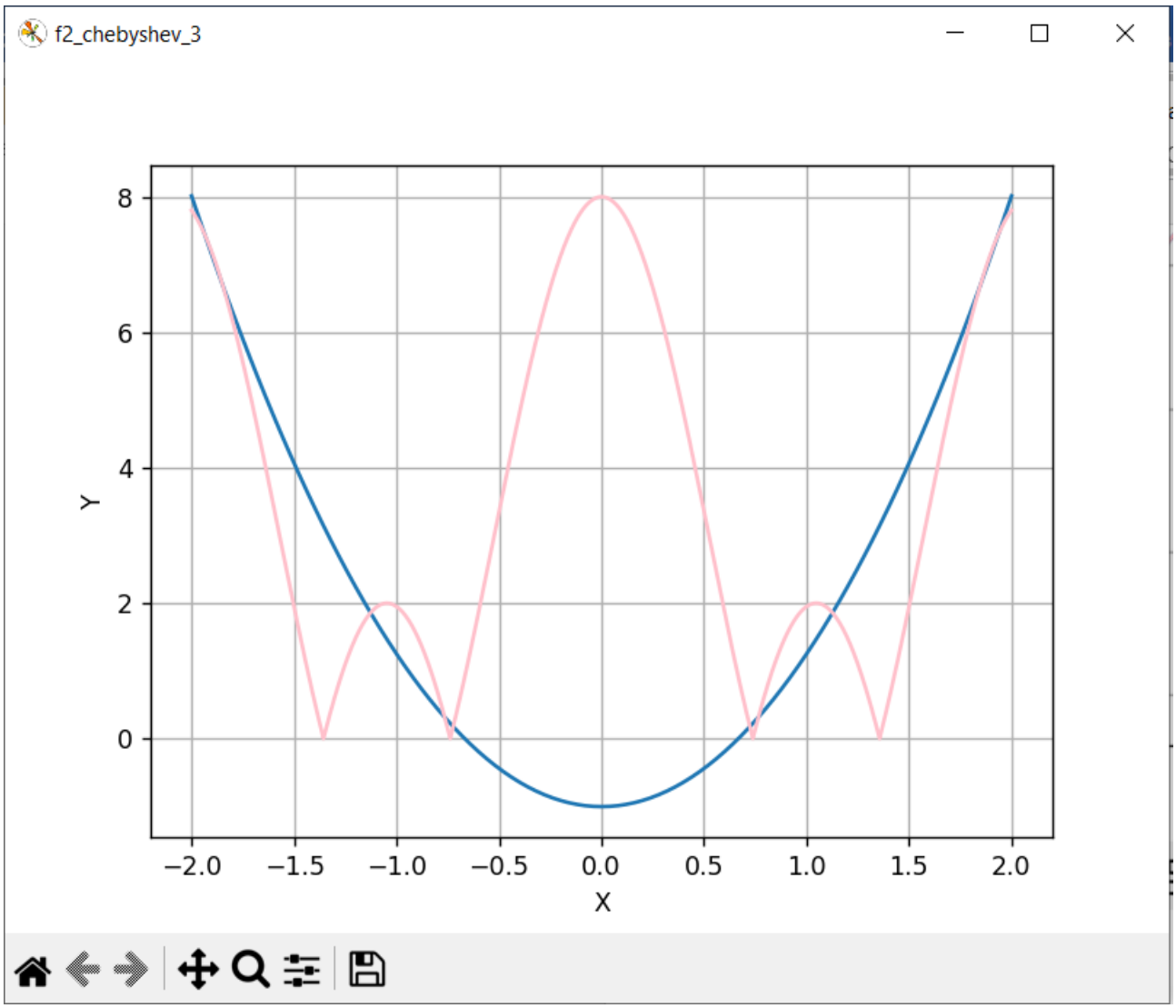

f1_chebyshev_10
2
0
−2
−4
−6
Y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
X
x=1.646 y=−6.68

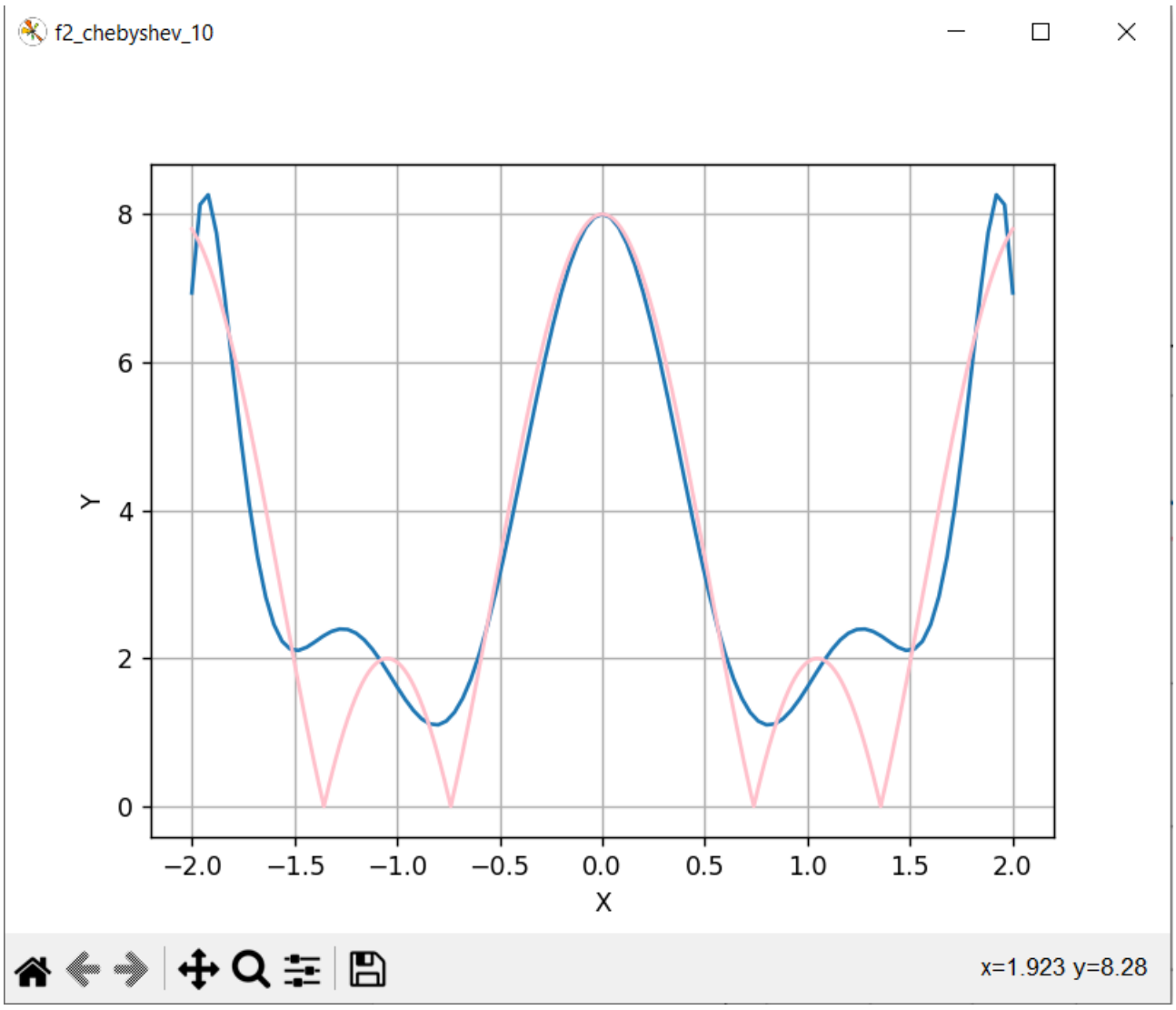

f1_chebyshev_20
2
0
−2
−4
−6
Y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
X
x=0.915 y=−2.15

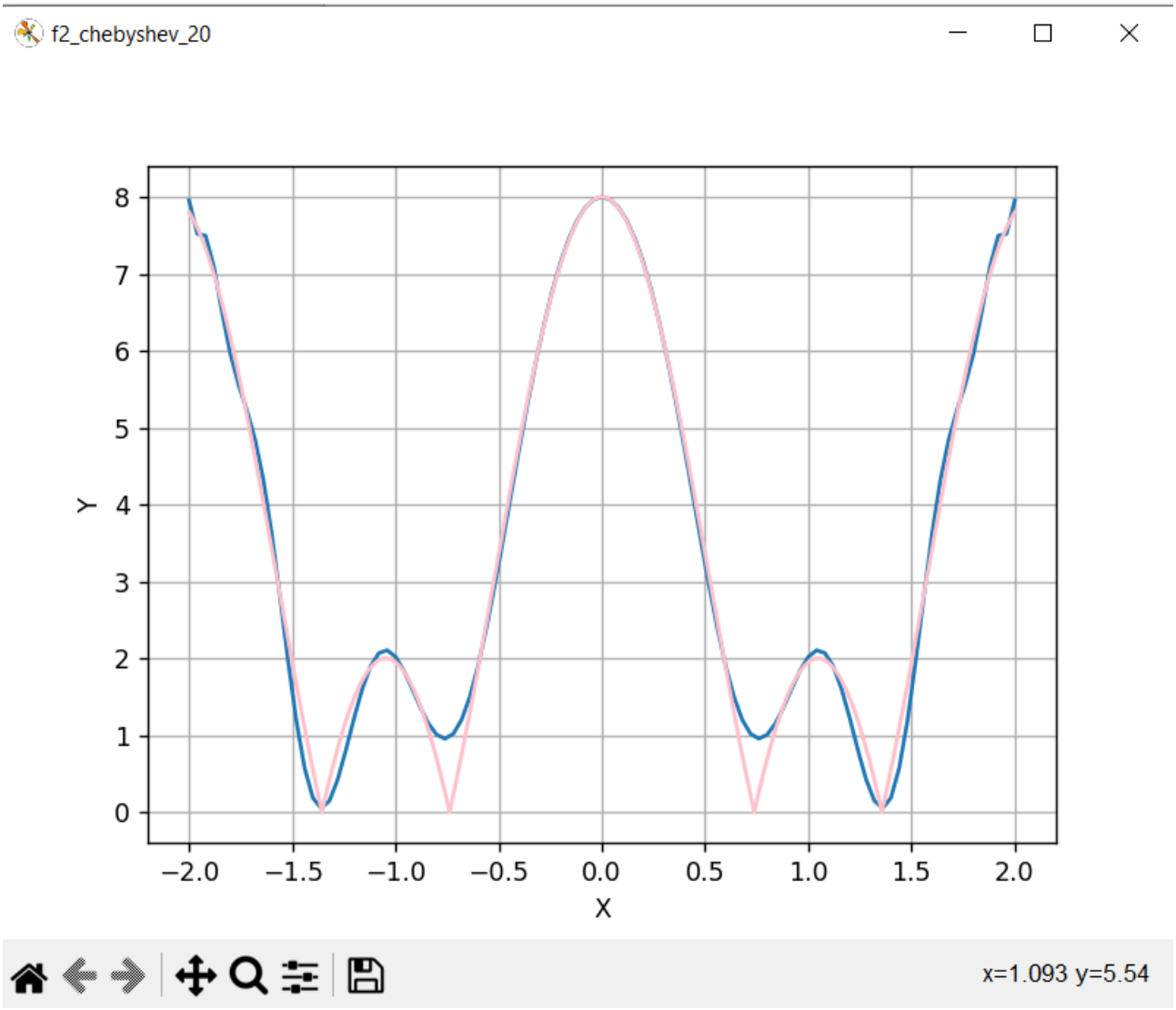

f2_equal_3
8
7
6
5
4
3
2
1
0
Y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
X


f2_equal_10
8
7
6
5
4
3
2
1
0
Y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
X


f2_equal_20
0
−20
−40
−60
−80
Y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
X


f2_chebyshev_3
8
6
4
2
0
Y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
X


f2_chebyshev_10
8
6
4
2
0
Y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
X
x=1.923 y=8.28



Исходный код программы:

```
#define _USE_MATH_DEFINES

#include <iostream>
#include <fstream>
#include <vector>
#include <cmath>
#include <math.h>
#include <string>
#include <map>

using namespace std;

const double a = -2;
const double b = 2;

double f1(double x)
{
	return x * x * x * cos(3 * x - 1);
}
```

```cpp
double f2(double x)
{
	return abs(5 * cos(3 * x) + 3);
}

vector<double> getEqualNodes(int n)
{
	vector<double> x(n + 1);
	x[0] = a;
	for (int i = 1; i <= n; i++)
	{
		x[i] = i * (b - a) / n + a;
	}

	return x;
}

vector<double> getChebyshevNodes(int n)
{
	vector<double> x(n + 1);
	for (int i = 0; i <= n; i++)
	{
		x[i] = (a + b) / 2 + (b - a) / 2 * cos((2 * i + 1) * M_PI / 2 / (n + 1));
	}

	return x;
}

vector<double> getConstants(vector<double>& x, double(*f)(double))
{
	vector<double>base(x.size());
	for (int i = 0; i < x.size(); i++)
	{
		base[i] = f(x[i]);
	}

	vector<double>res;
	res.push_back(base[0]);
	for (int cnt = x.size() - 1; cnt > 0; cnt--)
	{
		vector<double> temp(cnt);
		for (int i = 0; i < cnt; i++)
		{
			temp[i] = (base[i + 1] - base[i]) / (x[i + x.size() - cnt] - x[i]);
		}

		res.push_back(temp[0]);
		base.clear();
		base = temp;
		temp.clear();
	}

	return res;
}

double NewtonPolynom(vector<double>& scalars, vector<double>& x, double xp)
{
	double res = 0;
	double mnoz = 1;
	for (int i = 0; i < x.size(); i++)
	{
		res += scalars[i] * mnoz;
		mnoz *= (xp - x[i]);
	}
```

```
	return res;
}

void tableLine(int n, double(*f)(double))
{
	cout << n << '\t';
	double res1 = 0;
	double res2 = 0;
	vector<double>temp = getEqualNodes(n);
	vector<double>temp2 = getConstants(temp, f);

	vector<double>temp3 = getChebyshevNodes(n);
	vector<double>temp4 = getConstants(temp3, f);


	for (int i = 0; i <= 100; i++)
	{
		double xp = a + i * (b - a) / 100;
		res1 = max(res1, abs(NewtonPolynom(temp2, temp, xp) - f(xp)));
		res2 = max(res2, abs(NewtonPolynom(temp4, temp3, xp) - f(xp)));
	}

	cout << res1 << "\t\t\t\t" << res2 << '\n';
}

void outFile(ostream& out, vector<double>& scalars, vector<double>& x)
{
	for (int i = 0; i <= 100; i++)
	{
		double xp = a + i * (b - a) / 100;
		out << xp << ' ' << NewtonPolynom(scalars, x, xp) << '\n';
	}
}

const vector<int> N_DATA = { 3, 10, 20 };
const vector<double(*)(double)> FUNC = { f1, f2 };
const map<double(*)(double), string> FILE_IDENTITY_FUNC = { {f1, "f1"}, {f2, "f2"} };
const vector<vector<double>(*)(int)> NODES = { getEqualNodes, getChebyshevNodes };
const map<vector<double>(*)(int), string> FILE_IDENTITY_NODES = { {getEqualNodes,
"equal"}, {getChebyshevNodes, "chebyshev"} };

int main()
{
	for (auto n : N_DATA)
		for (auto f : FUNC)
			for (auto nodes : NODES)
			{
				vector<double> x = nodes(n);
				vector<double> scalars = getConstants(x, f);
				ofstream out(FILE_IDENTITY_FUNC.at(f) + '_' +
FILE_IDENTITY_NODES.at(nodes) + '_' + to_string(n) + ".txt");
				outFile(out, scalars, x);
				out.close();
			}



	for (auto f : FUNC)
	{
		cout << FILE_IDENTITY_FUNC.at(f);
		cout << "-----------TABLE-----------\n";
		cout << "n\tmax difference(equal nodes)\tmax difference(chebyshev
nodes)\n";
		for (int k = 3; k <= 30; k++)
		{
```

```
                tableLine(k, f);
            }

            cout << "-----------TABLE------------\n\n";
        }
}
```