ROTATION AND TORQUE

Nov. 9, 2023

Instructor: Prof. Ashkar

Review

REVIEW

1. Break down the motion of an extended object into two parts
   - The translational motion of the center of mass
   - The rotational motion about the center of mass

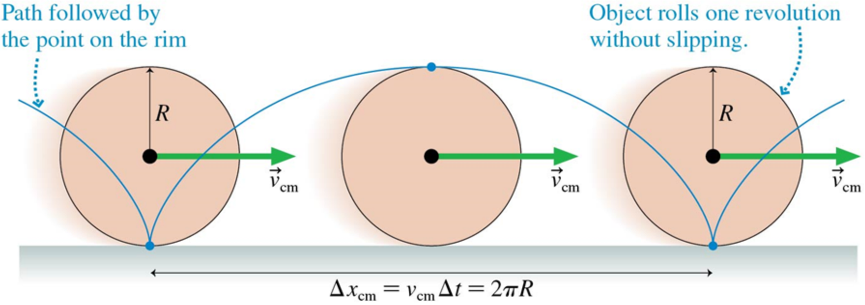
2. Kinetic Energy of an extended object is the sum of two kinetic energies:
   - Translational Kinetic Energy - from the translational motion of the center of mass
   - Rotational Kinetic Energy - from the rotational motion about an axis through the center of mass

$$\begin{aligned} K &= K_{trans} + K_{rot} \\ &= \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega_{cm}^2 \end{aligned}$$

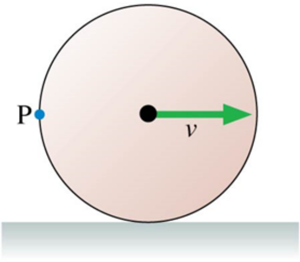
REVIEW

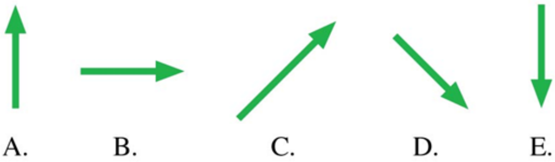
- Rolling is a combination of rotation and translation. For an object that rolls without slipping, the translation of the center of mass is related to the angular velocity by

$$v_{cm} = R\omega$$



A wheel rolls without slipping. Which is the correct velocity vector for point P on the wheel?





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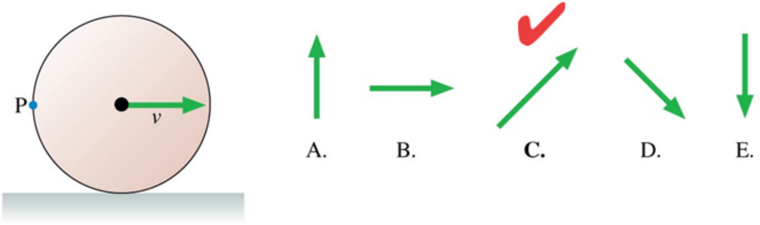
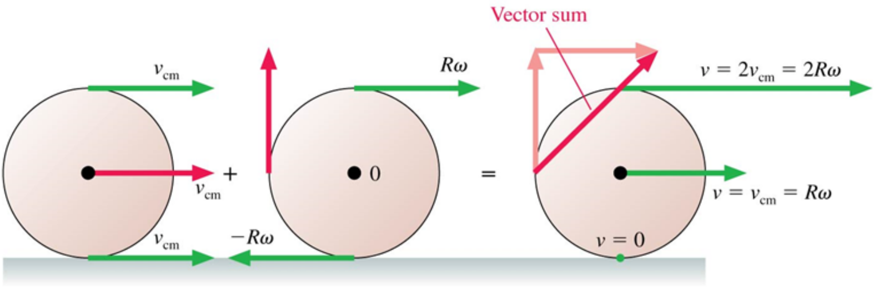




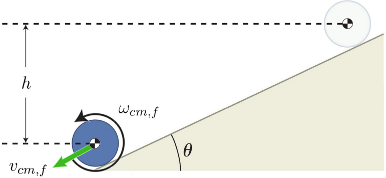
REVIEW

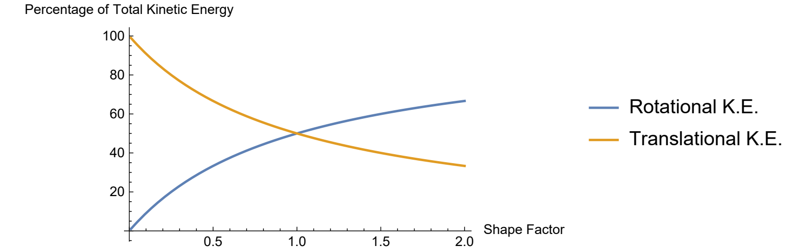
ROLLING MOTION

$$K_{tot} = \frac{1}{2} m v_{cm,f}^2 + \frac{1}{2} I_{cm} \omega_{cm}^2$$

$$\omega_{cm} = \frac{v_{cm}}{R}$$

$$v_{cm,f} = \sqrt{\frac{2gh}{1+c}}$$



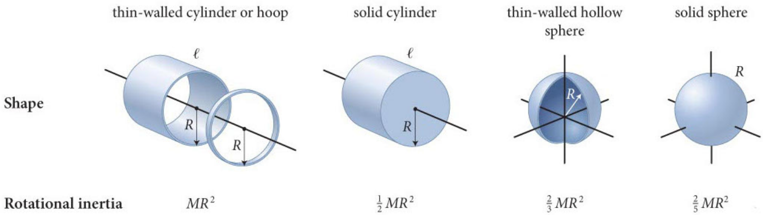


What about a small hoop and a large hoop, each with the same mass? Which one would reach the bottom of the ramp first?

A Large Hoop

B Small Hoop

C They finish at the same time

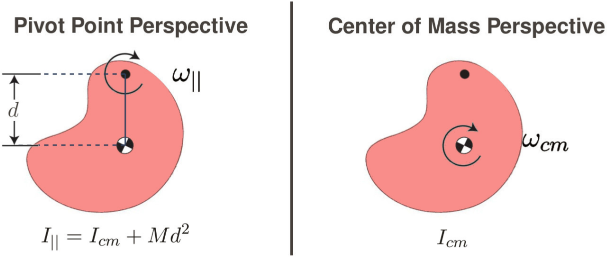


REVIEW

1. How does $I_{||}$ relate to I_{cm}?

Parallel-Axis Theorem: If the axis of rotation is distance d from a parallel axis through the center of mass, the moment of inertia is

$$\boxed{I_{||} = I_{cm} + Md^2}$$



Rotational Acceleration

ROTATIONAL MOTION REVIEW

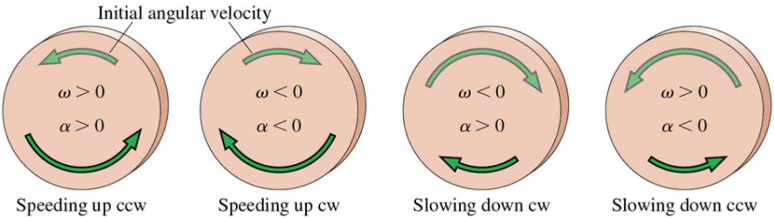
- Rotational kinematics for constant angular acceleration

$$\omega_f = \omega_i + \alpha\,\Delta t$$

$$\theta_f = \theta_i + \omega_i\,\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$$

- The signs of angular velocity and angular acceleration.



The rotation is speeding up if ω and α have the same sign, slowing down if they have opposite signs.

A DVD is initially at rest so that the line PQ on the disc's surface is along the +x-axis. The disc begins to turn with a constant α = 5.0 rad/s^2

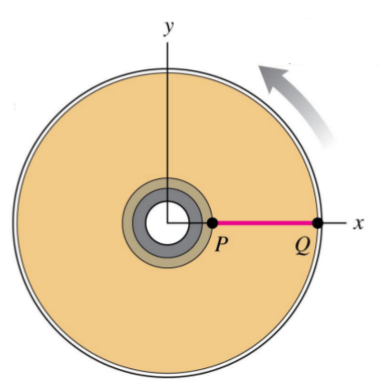
At t=0.40 s, what is the angle between the line PQ and the +x-axis?

A 0.40 rad

B 0.80 rad

C 1.0 rad

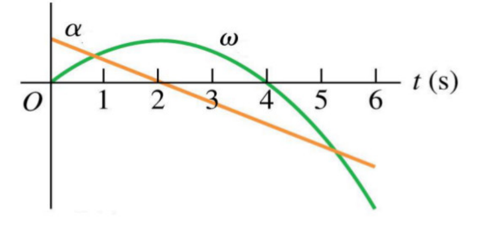
D 2.0 rad



The graph shows the angular velocity and angular acceleration versus time for a rotating body.

At which of the following times is the rotation speeding up at the greatest rate?

- **A** $t = 1s$
- **B** $t = 2s$
- **C** $t = 3s$
- **D** $t = 4s$
- **E** $t = 5s$



A ladybug sits at rest on a turntable that is rotating with constant rotational velocity. During this motion, the ladybug's translational acceleration vector points

A in the direction that the ladybug is traveling

B in the opposite direction that the ladybug is traveling

C towards the center of the turntable

D away from the center of the turntable

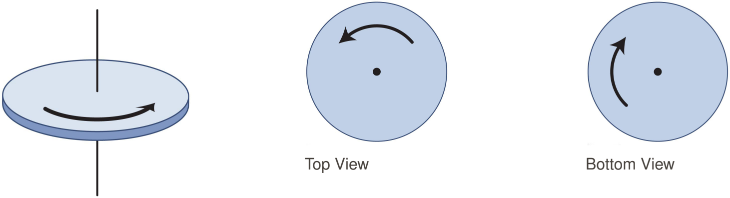
E in some other direction

A ladybug sits at rest on a turntable that is rotating with constant rotational velocity. The turntable is switched off and starts to slow down. As the ladybug is slowing down, the ladybug's translational acceleration vector points

- **A** in the direction that the ladybug was traveling before the disk was stopped
- **B** in the opposite direction that the ladybug was traveling before the disk was stopped
- **C** towards the center of the turntable
- **D** directly away from the center of the turntable
- **E** in some other direction

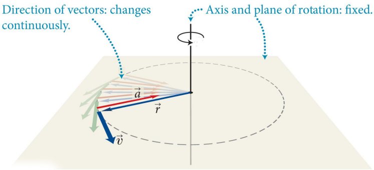
DIRECTION OF ROTATION IN 3D

1. In 2D (rotations in a single plane)
   - An algebraic sign is sufficient to indicate the direction of rotation
   - Counterclockwise $\omega > 0$; Clockwise $\omega < 0$
2. In 3D
   - Clockwise and counterclockwise are insufficient to specify the direction of rotation
   - Example: Top-view and Bottom-view of spinning disk



DIRECTION OF ROTATION IN 3D

1. Need an **unambiguous** direction for rotation
2. The direction of the $\vec{\omega}$ vector should not change as an object is observed from a different perspective.
   - Should the direction of rotation be along $\vec{r}$, $\vec{v}$, or $\vec{a}$?



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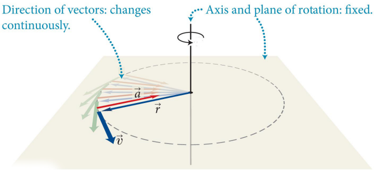

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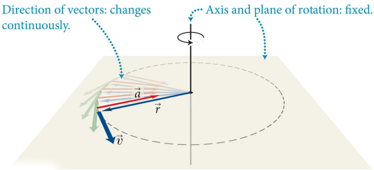
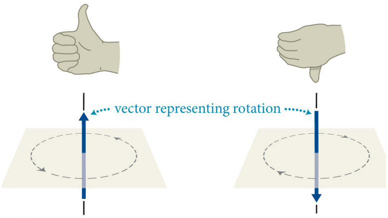
   - Should the direction of rotation be along $\vec{r}$, $\vec{v}$, or $\vec{a}$?
     NO. Their directions are always changing
   - What about along the axis of rotation?
     YES. But which direction along the axis
   - The direction along the rotation axis is determined by the right-hand rule

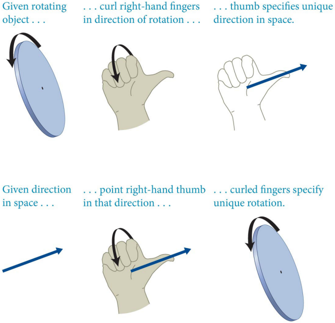


DIRECTION OF ROTATION IN 3D: RIGHT-HAND RULE

1 **Right-hand rule**:

- When you curl the fingers of your right hand along the direction of rotation, your thumb points in the direction of the vector representing that rotation ($\vec{\omega}$).





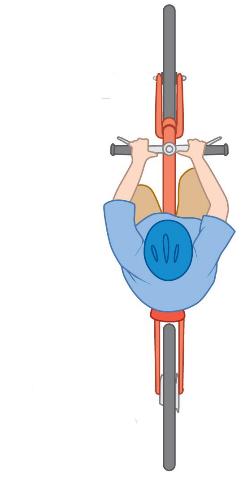
Suppose you're riding a bike at a constant speed. From your perspective, in which direction is the rotational velocity of the tires?

A In the forward direction

B In the backward direction

C In the rightward direction

D In the leftward direction



Now, suppose you apply the brakes and you start slowing down. From your perspective, in which direction is the rotational acceleration of the tires?

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B In the backward direction

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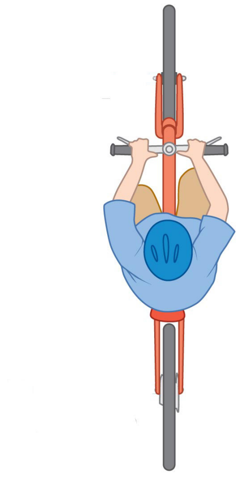
D In the leftward direction



Introduction to Torque

MOTION OF EXTENDED OBJECTS (SO FAR)

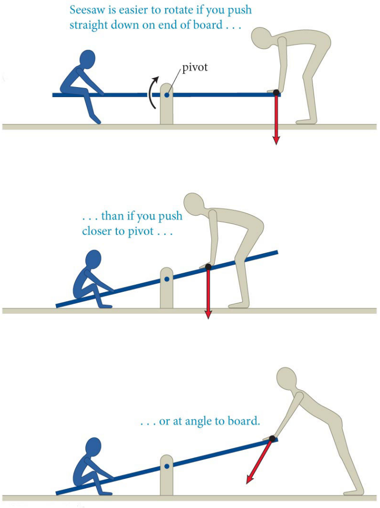
1. Motion of an extended object can be broken down into two parts
   - Translational Motion of the CM
   - Rotational Motion around the CM

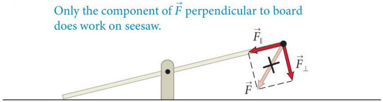
2. Translational Motion of the CM
   - Translational Kinematics: $\Delta x = v_{x0}\Delta t + \frac{1}{2}a_x\Delta t^2$
   - Translational Inertia: M_{tot}
   - Translational Dynamics: $\vec{a}_{cm} = \Sigma\vec{F}_{ext}/M_{tot}$

3. Rotational Motion around the CM
   - Rotational Kinematics: $\Delta\theta = \omega_0\Delta t + \frac{1}{2}\alpha\Delta t^2$
   - Rotational Inertia: $I = \Sigma\delta m r^2$
   - Rotational Dynamics

SEESAW AND CHILD

1. Forces have the ability to rotate objects
   - This ability depends on the point of application, magnitude and direction of the force





INTRODUCTION TO TORQUE

1. This ability of a force to rotate an object about an axis is called **torque**.

2. Torque depends on three factors:
   - The distance between the point of application and the pivot point
   - The magnitude of the force
   - The direction of the force

INTRODUCTION TO TORQUE

1. This ability of a force to rotate an object about an axis is called **torque**.
2. Torque depends on three factors:
   - The distance between the point of application and the pivot point
   - The magnitude of the force
   - The direction of the force
3. An unbalanced torque causes a change in rotational motion

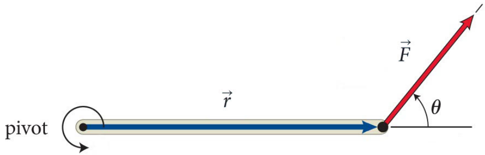
TORQUE MAGNITUDES

Two equivalent ways to determine the magnitude of the torque caused by a force

1. Perpendicular Component of Force:

$$\tau = rF_{\perp}$$

- Find part of the force that is perpendicular to $\vec{r}$



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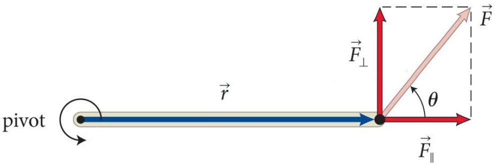

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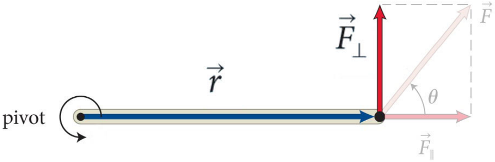

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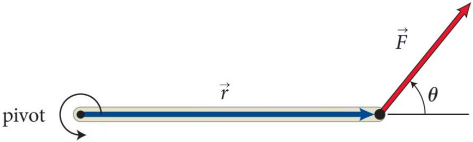
Two equivalent ways to determine the magnitude of the torque caused by a force

2. Lever arm ($r_\perp$):

$$\tau = r_\perp F$$

- Draw a line that runs through $\vec{F}$. This is the **line of action**.
- Draw a perpendicular line that runs from the pivot point to the line of action. This is the **lever arm**.



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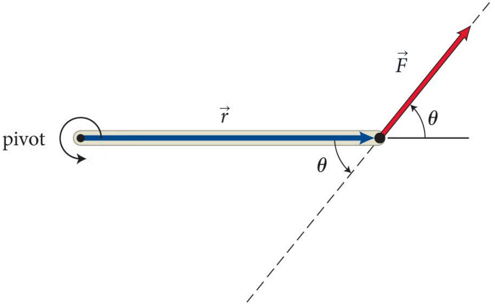

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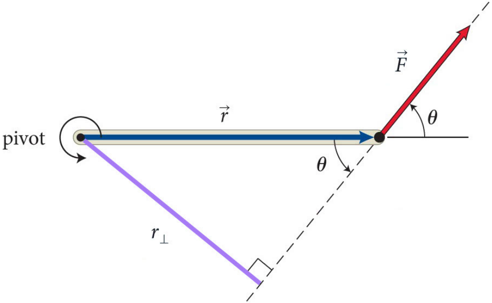

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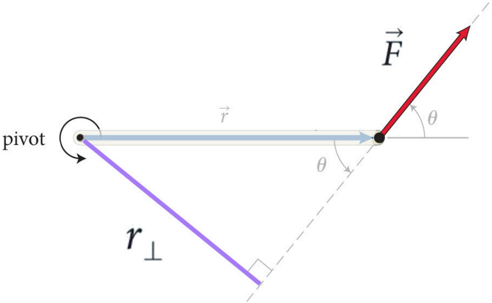


TORQUE MAGNITUDES

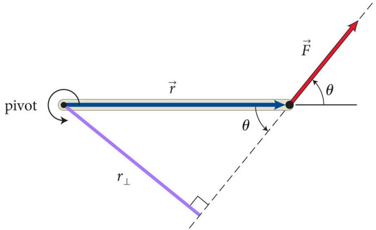
Two equivalent ways to determine the magnitude of the torque caused by a force

1. Perpendicular Component of Force:

$$\tau = rF_{\perp} = r(F\sin\theta)$$

2. Lever arm ($r_{\perp}$):

$$\tau = r_{\perp}F = (r\sin\theta)F$$



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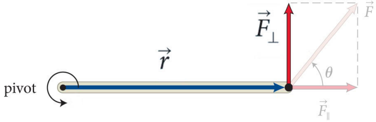

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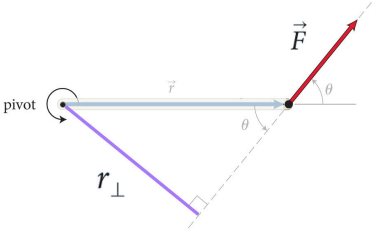


You try to rotate the L-shaped handle on a spigot (shown below) by pushing down on the end of the handle. What is the lever arm for the force shown below?

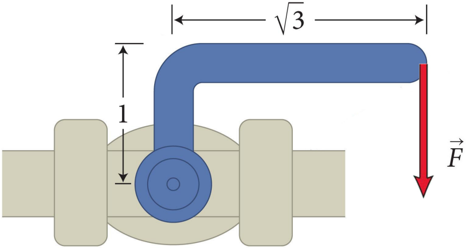
A 0 m

B $\sqrt{3}$ m

C 1 m

D $(1+\sqrt{3})$ m

E 2 m



You try to rotate the L-shaped handle on a spigot (shown below) by pushing down on the end of the handle. What is the lever arm for the force shown below? (The force is pointing directly at the pivot point.)

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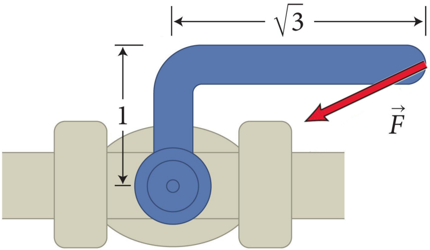

CROSS-PRODUCT

1. Torque is a vector
   - The magnitude of torque is determined by $\vec{r}$ and $\vec{F}$
   - What is the direction? Can we relate it to $\vec{r}$ and $\vec{F}$?
2. Need a product of vectors that generates another vector: **Cross Product**

$$|\vec{A} \times \vec{B}| = AB \sin\theta$$

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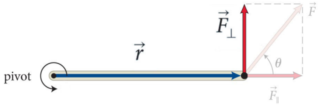
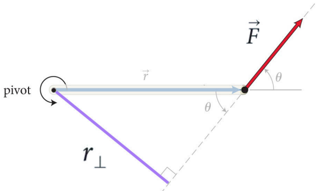


Magnitude of Torque

$$\begin{aligned} |\vec{\tau}| &= rF\sin\theta \\ &= rF_\perp \\ &= r_\perp F \end{aligned}$$



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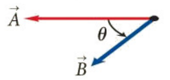
Direction of Torque

Use the Right-hand Rule

Finding the direction of a vector product



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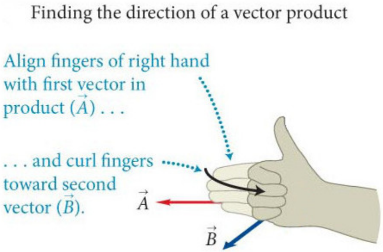

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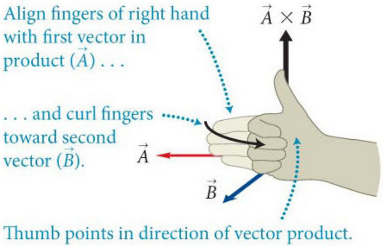

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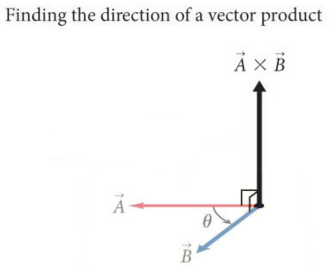


COMPONENT FORM OF THE CROSS PRODUCT

1. Component form of the cross product

$$\begin{aligned}\vec{A}\times\vec{B} &= (A_yB_z - A_zB_y)\hat{i} \\ &+ (A_zB_x - A_xB_z)\hat{j} \\ &+ (A_xB_y - A_yB_x)\hat{k}\end{aligned}$$

2. $\vec{\tau} = \vec{r}\times\vec{F}$
   - If $\vec{r}$ and $\vec{F}$ are in the XY plane, what is the direction of $\vec{\tau}$?

COMPONENT FORM OF THE CROSS PRODUCT

1. Component form of the cross product

$$\begin{aligned}\vec{r} \times \vec{F} &= (r_y F_z - r_z F_y)\hat{i} \\ &+ (r_z F_x - r_x F_z)\hat{j} \\ &+ (r_x F_y - r_y F_x)\hat{k}\end{aligned}$$

2. $\vec{\tau} = \vec{r} \times \vec{F}$
   - If $\vec{r}$ and $\vec{F}$ are in the XY plane, what is the direction of $\vec{\tau}$?

$$\begin{aligned}\vec{r} &= r_x\hat{i} + r_y\hat{j} \\ \vec{F} &= F_x\hat{i} + F_y\hat{j}\end{aligned}$$

1. Component form of the cross product

$$\begin{aligned} \vec{r} \times \vec{F} &= (r_y \cancel{F_z} - \cancel{r_z} F_y)\hat{i} \\ &+ (\cancel{r_z} F_x - r_x \cancel{F_z})\hat{j} \\ &+ (r_x F_y - r_y F_x)\hat{k} \end{aligned}$$

2. $\vec{\tau} = \vec{r} \times \vec{F}$
   - If $\vec{r}$ and $\vec{F}$ are in the XY plane, what is the direction of $\vec{\tau}$?

$$\begin{aligned} \vec{r} &= r_x \hat{i} + r_y \hat{j} \\ \vec{F} &= F_x \hat{i} + F_y \hat{j} \end{aligned}$$

COMPONENT FORM OF THE CROSS PRODUCT

1. Component form of the cross product

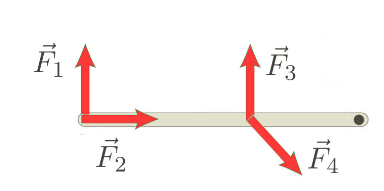
$$
\begin{aligned}
\vec{r} \times \vec{F} &= (0-0)\hat{i} \\
&+ (0-0)\hat{j} \\
&+ (r_x F_y - r_y F_x)\hat{k}
\end{aligned}
$$

2. $\vec{\tau} = \vec{r} \times \vec{F}$
   - If $\vec{r}$ and $\vec{F}$ are in the XY plane, what is the direction of $\vec{\tau}$?

$$
\begin{aligned}
\vec{r} &= r_x\hat{i} + r_y\hat{j} \\
\vec{F} &= F_x\hat{i} + F_y\hat{j}
\end{aligned}
$$

Which of the four forces shown here produces a torque on the bar about its pivot point (black dot) that is directed out of the plane of the drawing?

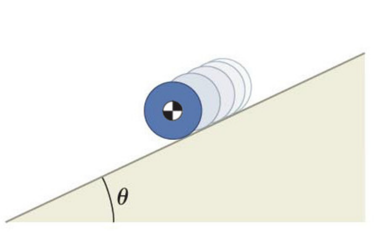
- **A** $\vec{F}_1$
- **B** $\vec{F}_2$
- **C** $\vec{F}_3$
- **D** $\vec{F}_4$
- **E** Two or more of these forces



Dynamics

Object Rolling Down a Ramp

A round object of mass m, rotational inertia I, and radius R is released from rest on a ramp, as shown below.

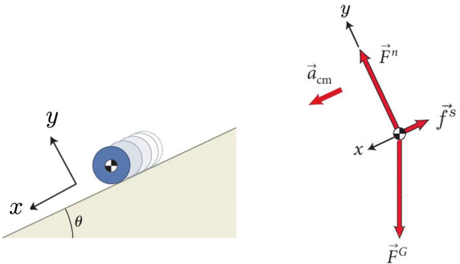


- Find the translational acceleration of the object's center of mass in terms of m, I, θ, and R.

OBJECT ROLLING DOWN RAMP - SOLUTION

Determine the net force in the x-direction (along the direction of translational motion)

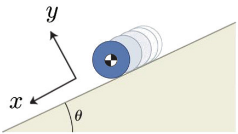
$$\Sigma F_x = ma_{(cm,x)} \tag{1}$$

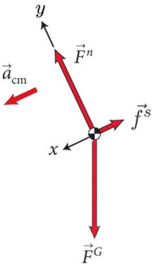


OBJECT ROLLING DOWN RAMP - SOLUTION

Determine the net force in the x-direction (along the direction of translational motion)

$$\Sigma F_x = ma_{(cm,x)} \quad (1)$$

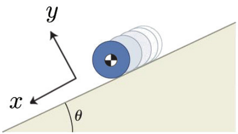


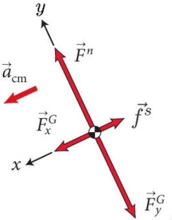


OBJECT ROLLING DOWN RAMP - SOLUTION

Determine the net force in the x-direction (along the direction of translational motion)

$$\Sigma F_x = ma_{(cm,x)} \tag{1}$$

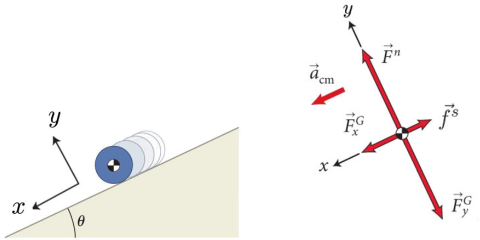




OBJECT ROLLING DOWN RAMP - SOLUTION

Determine the net force in the x-direction (along the direction of translational motion)

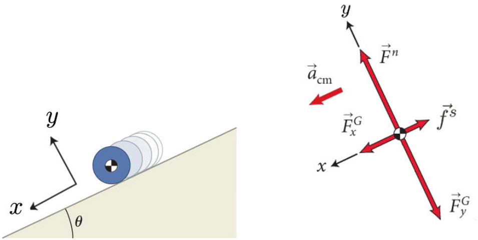
$$\begin{aligned} \Sigma F_x &= m a_{(cm,x)} \\ F_x^G - f^s &= m a_{(cm,x)} \end{aligned} \tag{1}$$



OBJECT ROLLING DOWN RAMP - SOLUTION

Determine the net force in the x-direction (along the direction of translational motion)

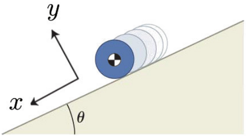
$$\begin{aligned}\Sigma F_x &= ma_{(cm,x)} \\ F_x^G - f^s &= ma_{(cm,x)} \\ mg\sin\theta - f^s &= ma_{(cm,x)} \end{aligned} \qquad (1)$$

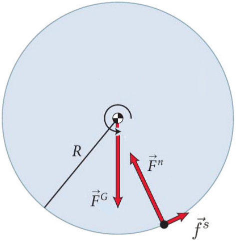


OBJECT ROLLING DOWN RAMP - SOLUTION

Determine the net torque (about the center of mass)

$$\Sigma\vec{\tau} = I\vec{\alpha}_{(cm)}$$



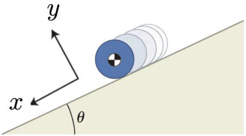


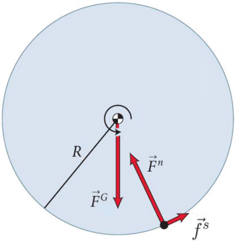
OBJECT ROLLING DOWN RAMP - SOLUTION

Determine the net torque (about the center of mass)

$$\Sigma\vec{\tau} = I\vec{\alpha}_{(cm)}$$

$$\vec{\tau}^{\,s} + \cancel{\vec{\tau}^{\,G}} + \cancel{\vec{\tau}^{\,N}} = I\vec{\alpha}_{(cm)}$$





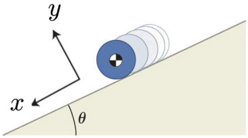
OBJECT ROLLING DOWN RAMP - SOLUTION

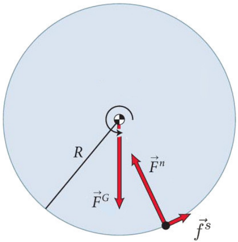
Determine the net torque (about the center of mass)

$$\Sigma\vec{\tau} = I\vec{\alpha}_{(cm)}$$

$$\vec{\tau}^{\,s} + \cancel{\vec{\tau}^{\,G}} + \cancel{\vec{\tau}^{\,N}} = I\vec{\alpha}_{(cm)}$$

$$\vec{\tau}^{\,s} = I\vec{\alpha}_{(cm)}$$





In which direction is the rotational acceleration of the spool around its center of mass?

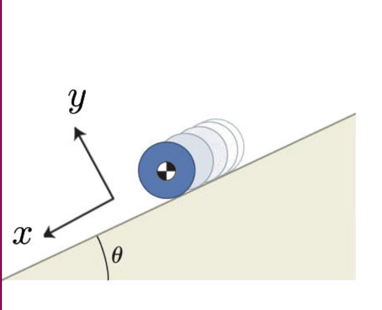
A + z-direction

B - z-direction

C + x-direction

D - x-direction

E The spool is not rotationally accelerating



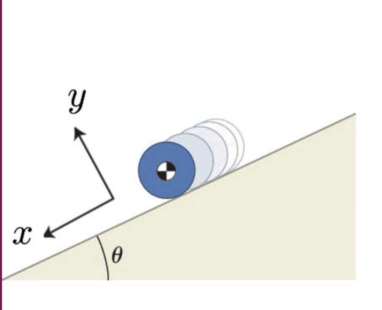
Which of the following is the correct equation that relates $a_{(cm,x)}$ to $\alpha_{(cm,z)}$?

A $a_{(cm,x)} = +R\alpha_{(cm,z)}$

B $a_{(cm,x)} = -R\alpha_{(cm,z)}$

C $a_{(cm,x)} = +\dfrac{\alpha_{(cm,z)}}{R}$

D $a_{(cm,x)} = -\dfrac{\alpha_{(cm,z)}}{R}$

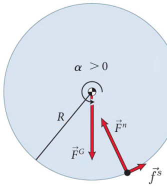


OBJECT ROLLING DOWN RAMP - SOLUTION

Determine the net torque in the z-direction (about the CoM)

Plug in the following conditions

- $a_{(cm,x)} = -R\alpha_{(cm,z)}$



$$\vec{\tau}^{\,s} = I\vec{\alpha}_{(cm)}$$

$$-\tau^{\,s} = I\alpha_{(cm,z)}$$

(z-components)

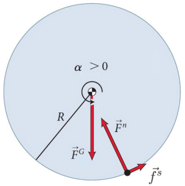
OBJECT ROLLING DOWN RAMP - SOLUTION

Determine the net torque in the z-direction (about the CoM)

Plug in the following conditions

- $a_{(cm,x)} = -R\alpha_{(cm,z)}$

$$\boxed{\alpha_{(cm,z)} = -\frac{a_{(cm,x)}}{R}}$$



$$\vec{\tau}^{\,s} = I\vec{\alpha}_{(cm)}$$

$$-\tau^{\,s} = I\alpha_{(cm,z)} \qquad \text{(z-components)}$$

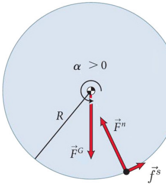
OBJECT ROLLING DOWN RAMP - SOLUTION

Determine the net torque in the z-direction (about the CoM)

Plug in the following conditions

- $a_{(cm,x)} = -R\alpha_{(cm,z)}$

$$\boxed{\alpha_{(cm,z)} = -\frac{a_{(cm,x)}}{R}}$$



$$\vec{\tau}^{\,s} = I\vec{\alpha}_{(cm)}$$

$$-\tau^{\,s} = I\alpha_{(cm,z)}$$ (z-components)

$$-f^s R = (I)(-\frac{a_{(cm,x)}}{R})$$

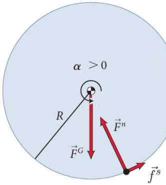
OBJECT ROLLING DOWN RAMP - SOLUTION

Determine the net torque in the z-direction (about the CoM)

Plug in the following conditions

- $a_{(cm,x)} = -R\alpha_{(cm,z)}$

$$\boxed{\alpha_{(cm,z)} = -\frac{a_{(cm,x)}}{R}}$$



$$\begin{aligned}
\vec{\tau}^{\,s} &= I\vec{\alpha}_{(cm)} \\
-\tau^{\,s} &= I\alpha_{(cm,z)} \qquad \text{(z-components)} \\
-f^s R &= (I)\left(-\frac{a_{(cm,x)}}{R}\right) \\
f^s &= \frac{I}{R^2} a_{(cm,x)}
\end{aligned} \tag{2}$$

OBJECT ROLLING DOWN RAMP - SOLUTION

Combine Equation 1 (from ΣF_x) and Equation 2 (from $\Sigma \tau_z$)...

$$mg\sin\theta - f^s = ma_{(cm,x)} \tag{1}$$

$$f^s = \frac{I}{R^2}a_{(cm,x)} \tag{2}$$

OBJECT ROLLING DOWN RAMP - SOLUTION

Combine Equation 1 (from ΣF_x) and Equation 2 (from $\Sigma \tau_z$)...

$$mg\sin\theta - f^s = ma_{(cm,x)} \tag{1}$$

$$f^s = \frac{I}{R^2}a_{(cm,x)} \tag{2}$$

...and solve for $a_{(cm,x)}$

$$a_{(cm,x)} = \frac{g\sin\theta}{1+\frac{I}{mR^2}}$$

OBJECT ROLLING DOWN RAMP - SOLUTION

Combine Equation 1 (from ΣF_x) and Equation 2 (from $\Sigma \tau_z$)...

$$mg\sin\theta - f^s = ma_{(cm,x)} \tag{1}$$

$$f^s = \frac{I}{R^2}a_{(cm,x)} \tag{2}$$

...and solve for $a_{(cm,x)}$

$$\begin{aligned} a_{(cm,x)} &= \frac{g\sin\theta}{1+\frac{I}{mR^2}} \\ &= \frac{g\sin\theta}{1+\frac{cmR^2}{mR^2}} \\ &= \frac{g\sin\theta}{1+c} \end{aligned}$$