Review of Direction of Arrival Estimation Using Music Algorithm

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Abstract

The concept of DOA estimation using the MUSIC algorithm was initially introduced to me during lecture for ECE 599: Matrix Analysis for SP and ML, but I learned more about it when completing the midterm. After I finished the midterm, I grew interested in trying to understand how changing certain initial conditions would change the accuracy of the estimation. This paper contains my understanding of the MUSIC algorithm for DOA estimation after reading *Direction of Arrival Estimation Using MUSIC Algorithm*. Also contained in this paper are experiments performed using the MUSIC algorithm. These experiments include reenactments of Revati Joshi and Ashwinikumar Dhande's data, where they manipulate the number of snapshots, the number array elements, and varying element separation. I also performed variations of the original tests with different variable values and manipulated the SNR for additive white Gaussian noise.

1. Introduction

Array signal processing has a wide variety of applications. These include sonar, radar, satellite, and communication systems. We use quite a lot of signal processing machines in everyday life without even thinking about it. The MUSIC (multiple signal classification) algorithm, proposed by Schmidt in 1979, is a spatial spectrum estimation, as the distribution of signals is presented in every possible direction in the space. The MUSIC algorithm is one of the more widely used methods of DOA estimation, but it is more difficult to use in real applications due to the large number of computations necessary to search for the spectral angle. (1).

The MUSIC algorithm uses the orthogonality between the covariance matrix and the array vectors that are associated with the true angles of the sources. The subspace of the noise is orthogonal to the subspace of the signal, this means that the array vectors that are associated with the true angles of the sources belong to the signal subspace of the covariance matrix. Since it is an estimation, this means that the true covariance matrix can't be used, so the eigenvectors of the sample covariance matrix are used (2).

Using an array of antennas, instead of a single one, can help improve the resolution of signal DOA. Using an array also increases the number of parameters, allowing for manipulation to enhance performance and create more accurate estimations (3). These parameters include the number of array elements, spacing between the elements, the number of samples (snapshots), and the SNR of the added noise.

2. Method

The first step for DOA estimation is to generate the signals. This requires three parts: the signal column vector, the noise vector, and the steering matrix. The steering matrix describes what spatial path each element of the antenna array should use. The signal column vector and the steering vector are multiplied together and noise is added. For this paper, the noise is AWGN (w(t)) and uncorrelated.

$x(t) = As(t) + w(t)$
$s(t) = [s_1(t), ..., s_K(t)]^T$ (Signal Column Vector)
$A = [a(\theta_1), ..., a(\theta_K)]$ (Steering Matrix)
$a(\theta) = [1, e^{-j\frac{2\pi d}{\lambda}sin(\theta)}, ..., e^{-j\frac{2\pi d(N-1)}{\lambda}sin(\theta)}]^T$ (Steering Column Vector)

The initial parameters needed for MUSIC include:

1. The number of sensors (N)
2. Spacing between sensors (d)
3. The number of sources (K)
4. The number of snapshots (T)
5. Incoming signal direction (θ)
6. SNR (R)

The input covariance matrix is

$$R_X = E[X(t)X(t)^H] = AE[s(t)s(t)^H]A^H + \sigma^2 I$$

The covariance matrix of the signal vector is

$$E[s(t)s(t)^H]$$

And I is the identity matrix (NxN).

When analyzing R_X, if $N > K$ then the covariance matrix is positive definite. To approximate $E[X(t)X(t)^H]$ the following equation is used:

$$R_x = \frac{1}{T}\sum_{t=1}^{T} x(t)x(t)^H$$

Eigendecomposition of R_x is then performed to split values up, and since noise is included, R_x is a full rank matrix and has N positive real eigenvalues. σ^2 will end up being the smallest eigenvalue of R, with N-K dimension. To summarize, the eigenvalues and eigenvectors belonging to R_X correspond to the signal and noise respectively. The noise eigenvectors:

$$V_n = [q_K, q_{K+1}, ..., q_{N-1}]$$

Since the noise subspace eigenvectors are orthogonal to the steering vectors, the DOAs of multiple signals can be estimated by using the peaks generated by the MUSIC spatial spectrum:

$$S_{MUSIC}(\theta) = \frac{1}{a^H(\theta)V_nV_n^Ha(\theta)}$$

3. Experiment

This section contains experiments performed by changing parameters.

3.1. From Original Paper

The experiments performed by Revati Joshi and Ashwinikumar Dhande include changing of the number of snapshots, the number of sensors, and the distance between sensors (3). The following will remain the same: The number of sources (3), incoming signal direction (-50°, 0°, 60°), and SNR (20dB).

Looking at Figure 2, the angles have a wider beam width compared to Figure 1. This means that as the number of snapshots decrease, the accuracy of the estimate also decreases. A similar relationship can be seen in Figure 3 and Figure 4, when the number of sensors or the distance between the sensors decreases, the estimate becomes less accurate.

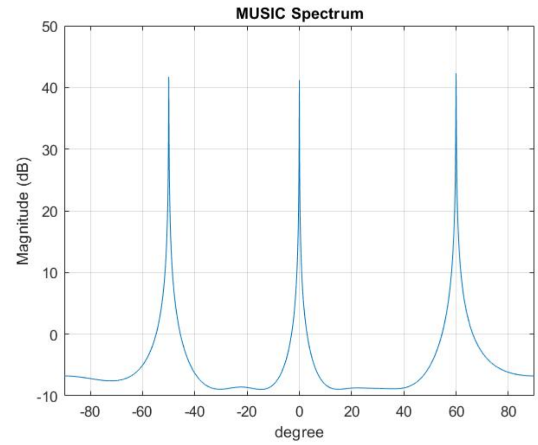

Figure 1. Control graph used to compare to other results. N=8, K=3, T=1000, R=20, θ=[-50°, 0°, 60°], d=0.5

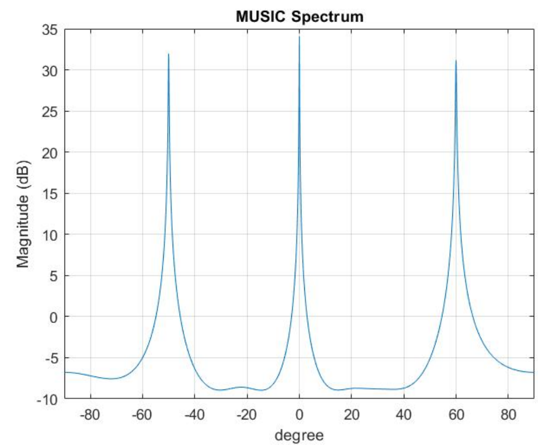

Figure 2. Decreasing the number of snapshots. T=100

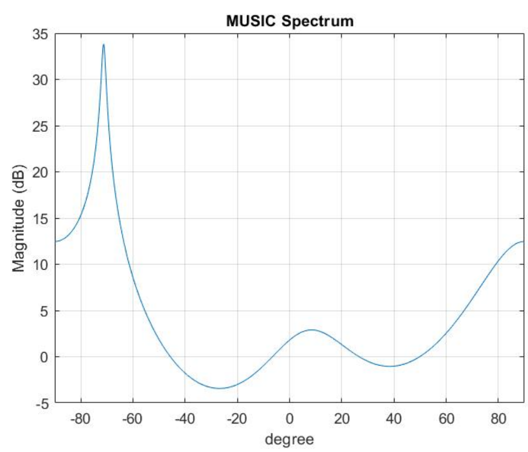

Figure 3. Decreasing the number of sensors. N=4

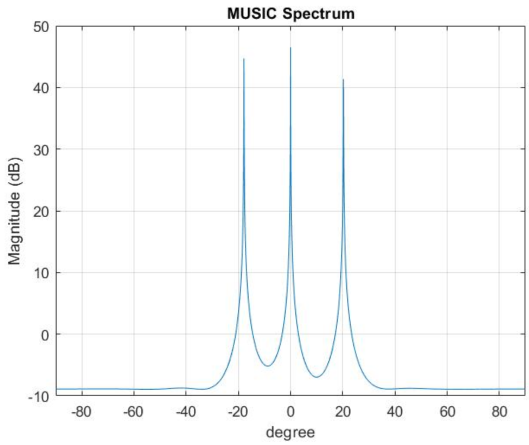

Figure 4. Decreasing the distance between sensors. d=0.2

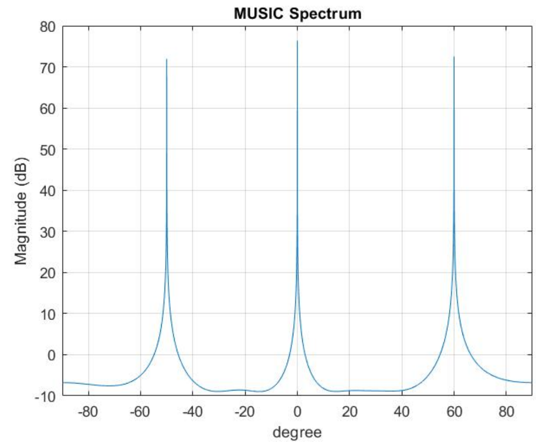

Figure 6. Increasing SNR. R=50

3.2. My Experiments

I decided to experiment with the SNR value and increasing the number of sensors to see how these changes would affect the estimate.

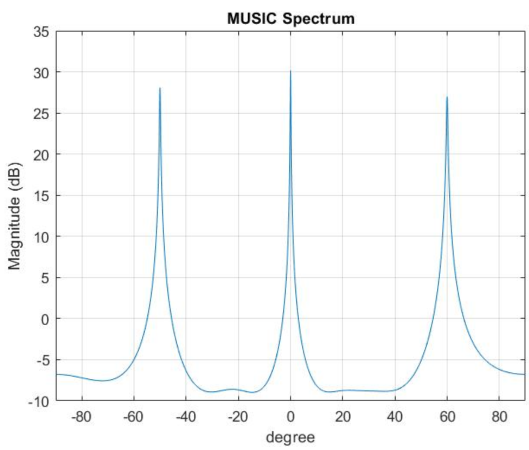

Figure 5. Decreasing SNR. R=5

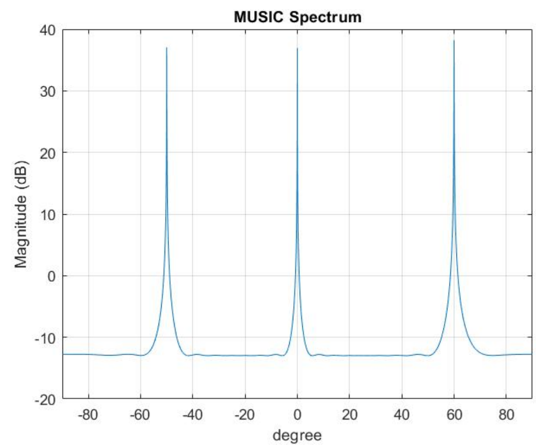

Figure 7. Increasing the number of sensors. N=20

Based off Figures 5 and 6, when SNR is decreased, the estimate accuracy will decrease. If SNR is increased, the accuracy will increase. The same is true for when the amount of sensors is increased, as shown in Figure 7.

4. Conclusion

Before I began to separately research the MUSIC algorithm, I was still a little confused on how it worked, even after seeing it on the midterm. In regards to the parameters, the only understanding I had was that in order for MUSIC to work, the number of sensors needed to be greater than the number of sources. After performing my own experiments, I learned that higher parameters tend to lead to a more accurate estimate. The purpose behind taking the eigendecomposition of R_x had been a point of question for me in the past, but now I understand that it is used to separate the noise from the signal. I also learned that the orthogonal relationship between the noise subspace eigenvectors and the steering vectors is what makes up the MUSIC spatial spectrum and generate the peaks used for estimating the DOA. Overall, I believe I gained a better understanding of how the MUSIC algorithm works with DOA estimation.

References

[1] Honghao Tang. *DOA estimation based on MUSIC algorithm*. B.S. Thesis, Department of Physics and Electrical Engineering, Linnaeus University, 2014.

[2] Byung-kwon Son So-Hee Jeong and Joon-Ho Lee. Asymptotic performance analysis of the music algorithm for direction-of-arrival estimation. *Applied Sciences*, 10(2063):1–25, 2020.

[3] Revati Joshi and Ashwinikumar Dhande. Direction of arrival estimation using music algorithm. *International Journal of Research in Engineering and Technology*, 3(3):633–636, 2014.