第一章 事件的概率

作者: 徐建军　向一波

中国科学技术大学

统计与金融系

E-mail:xjj1994@mail.ustc.edu.cn

1.

(1) $\Omega = \{0, 1, 2, \cdots, 10\}$

(2) $\Omega = \{0, 1, 2, \cdots, n\}$, n为总字数

(3) $\Omega = \{0, 1, 2, \cdots, n\}$

(4) $\Omega = \{(x, y) | x^2 + y^2 < 1\}$

2.

(1) $B \, \overline{A} \, \overline{C}$

(2) $A \, B \, \overline{C}$

(3) $A \bigcup B \bigcup C$

(4) $AB \bigcup BC \bigcup AC$

(5) $AB\overline{C} + BC\overline{A} + AC\overline{B}$

(6) $\overline{AB} \bigcup \overline{BC} \bigcup \overline{AC}$ 至多一个事件发生 $\Longleftrightarrow$ 至少两个事件不发生

(7) $\overline{A} \bigcup \overline{B} \bigcup \overline{C}$ 至多两个事件发生 $\Longleftrightarrow$ 至少一个事件不发生

3.

(1) $\overline{A}B = (\frac{1}{4}, \frac{1}{2}] \bigcup (1, \frac{3}{2}]$

(2) $\overline{A} \bigcup B = [0, 2]$

(3) $\overline{AB} = [0, \frac{1}{2}] \bigcup (1, 2]$

(4) $\overline{\overline{AB}} = (\frac{1}{4}, \frac{3}{2}]$

4.

Easy 略

5.

$A_1 \bigcup A_2 \bigcup \cdots \bigcup A_n = A_1 + A_2 \setminus A_1 + A_3 \setminus (A_1 \bigcup A_2) + \cdots + A_n \setminus (A_1 \bigcup A_2 \bigcup \cdots \bigcup A_{n-1})$

6.

有误　那些比例加起来等于101% > 100%

7.

解法一:开奖号固定

$$P_1 = \frac{1}{C_{33}^7} \qquad P_2 = \frac{C_7^6}{C_{33}^7} \qquad P_3 = \frac{C_7^6 \cdot 25}{C_{33}^7} \qquad P_4 = \frac{C_7^5 \cdot 25}{C_{33}^7}$$

解法二:彩票号固定

$$P_1 = \frac{26}{C_{33}^7 \cdot 26} \qquad P_2 = \frac{C_7^6 \cdot 26}{C_{33}^7 \cdot 26} \qquad P_3 = \frac{C_7^6 \cdot 26 \cdot 25}{C_{33}^7 \cdot 26} \qquad P_4 = \frac{C_7^5 \cdot C_{26}^2 \cdot C_2^1}{C_{33}^7 \cdot 26}$$

8.

Easy 略

9.

$$1 - \frac{A_{12}^6}{12^6}$$

10.

$$\frac{C_3^2 \cdot C_6^1 \cdot C_5^1}{6^3}$$

11.

(1)

有放回: $\dfrac{7}{10} \times \dfrac{7}{10} \times \dfrac{7}{10}$

无放回: $\dfrac{7}{10} \times \dfrac{6}{9} \times \dfrac{5}{8}$

(2)

有放回: $C_3^1 \dfrac{1}{10} \times (\dfrac{6}{10})^2 + C_3^2 (\dfrac{1}{10})^2 \times \dfrac{6}{10} + C_3^3 (\dfrac{1}{10})^3$

无放回: $\dfrac{3 \times 6 \times 5}{10 \times 9 \times 8}$

12.

略

13.

枚举法

14.

由对称性，$\dfrac{1}{2}$

17.

儿子赢父亲p_1，儿子赢教练p_2，则$p_1 > p_2$

父亲-教练-父亲：$p_1 p_2 + (1 - p_1)p_2 p_1 = p_1 p_2 (2 - p_1)$

教练-父亲-教练：$p_2 p_1 + (1 - p_1)p_1 p_2 = p_1 p_2 (2 - p_2)$

选后一种

19.

$\dfrac{A_{19}^8}{19^8}$

21.

在某一次游戏后：

赢的概率：$\dfrac{2}{9}$，输的概率：$\dfrac{1}{9}$，继续的概率：$\dfrac{2}{3}$

$\therefore$ 最终赢的概率：$\dfrac{2}{9}\left[(\dfrac{2}{3})^0 + (\dfrac{2}{3})^1 + \cdots + (\dfrac{2}{3})^n + \cdots\right] = \dfrac{2}{3}$

24.

$A = \{$至少一个是$6\}$，$P(A) = \dfrac{36 - 25}{36} = \dfrac{11}{36}$

$B = \{$两个面不一样$\}$，$P(B) = \dfrac{36 - 6}{36} = \dfrac{5}{6}$

$AB = \{$两个中有且只有一个是$6\}$，$P(AB) = \dfrac{11 - 1}{36} = \dfrac{5}{18}$

$P(A|B) = \dfrac{P(AB)}{P(B)} = \dfrac{1}{3}$

27.

$P(AC) = P(A|C)P(C) = 0.45 \qquad P(BC) = P(B|C)P(C) = 0.45$

$P(A\bar{C}) = P(A|\bar{C})P(\bar{C}) = 0.1 \qquad P(B\bar{C}) = P(B|\bar{C})P(\bar{C}) = 0.05$

$P(A) = P(AC) + P(A\bar{C}) = 0.55 \qquad P(B) = P(BC) + P(B\bar{C}) = 0.5$

$P(AB|C) = P(A|C)P(B|C) = 0.81 \qquad P(AB|\bar{C}) = P(A|\bar{C})P(B|\bar{C}) = 0.02 \,(条件独立)$

$P(ABC) = P(AB|C)P(C) = 0.405 \qquad P(AB\bar{C}) = P(AB|\bar{C})P(\bar{C}) = 0.01$

$P(AB) = P(ABC) + P(AB\bar{C}) = 0.415 \neq 0.275 = P(A)P(B) \implies A, B$不独立

30.
(1) $P_A P_B P_C$

(2) $1 - (1 - P_A)(1 - P_B)(1 - P_C)$

(3) $1 - (1 - P_A^2)(1 - P_B^2)(1 - P_C^2)$

(4) $P_D^2 \big[1 - (1 - P_A)(1 - P_B)(1 - P_C) \big]$

(5)
由全概率公式:

$$
\begin{aligned}
P(正常) &= P(正常|C不正常)P(C不正常) + P(正常|C正常)P(C正常) \\
&= (1 - P_C)\big[1 - (1 - P_A P_B)^2 \big] \\
&\quad + P_C \big[1 - (1 - P_A)^2 \big]\big[1 - (1 - P_B)^2 \big]
\end{aligned}
$$

33.
(1) $0.5 \times 0.4 \times 0.2 + 0.5 \times 0.6 \times 0.2 + 0.5 \times 0.4 \times 0.8 = 0.04 + 0.06 + 0.16 = 0.26$

(2) $1 - 0.5 \times 0.4 \times 0.2 = 1 - 0.04 = 0.96$

36.
A_i : 3件中有i件音色不纯, $i = 0, 1, 2, 3$

B : 乐器被接收

$$
\begin{aligned}
P(B) &= P(B|A_0)P(A_0) + P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3) \\
&= \frac{C_4^0 C_{96}^3}{C_{100}^3} \times (0.99)^3 + \frac{C_4^1 C_{96}^2}{C_{100}^3} \times 0.05 \times (0.99)^2 \\
&\quad + \frac{C_4^2 C_{96}^1}{C_{100}^3} \times (0.05)^2 \times 0.99 + \frac{C_4^3 C_{96}^0}{C_{100}^3} \times (0.05)^3
\end{aligned}
$$

38.
(1) 全概率公式: $\dfrac{1}{2} \times 1\% + \dfrac{1}{3} \times 1\% + \dfrac{1}{6} \times 2\% = \dfrac{7}{600}$

(2) A : 产品是次品 A_i : 产品由一号车间生产, $i = 1, 2, 3$

Bayes公式: $P(A_1|A) = \dfrac{P(A|A_1)P(A_1)}{P(A|A_1)P(A_1) + P(A|A_2)P(A_2) + P(A|A_3)P(A_3)} = \dfrac{3}{7}$

40.

(1) $P(\text{先抽女}) = \frac{1}{3} \times \frac{3}{10} + \frac{1}{3} \times \frac{7}{15} + \frac{1}{3} \times \frac{5}{25} = \frac{29}{90}$

(2) $P(\text{后抽男}) = \frac{1}{3} \times \left(\frac{3}{10} \times \frac{7}{9} + \frac{7}{10} \times \frac{6}{9}\right) + \frac{1}{3} \times \left(\frac{7}{15} \times \frac{8}{14} + \frac{8}{15} \times \frac{7}{14}\right) + \frac{1}{3} \times \left(\frac{5}{25} \times \frac{20}{24} + \frac{20}{25} \times \frac{19}{24}\right)$

$$= \frac{61}{90}$$

$P(\text{先抽女，后抽男}) = \frac{1}{3} \times \frac{3}{10} \times \frac{7}{9} + \frac{1}{3} \times \frac{7}{15} \times \frac{8}{14} + \frac{1}{3} \times \frac{5}{25} \times \frac{20}{24} = \frac{2}{9}$

$P(\text{先抽女|后抽男}) = \frac{20}{61}$

42.

A : 确有乙肝

B : 诊断有乙肝

Bayes公式:

$P(A|B) = \dfrac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})} = \dfrac{0.98 \times 0.05}{0.98 \times 0.05 + 0.05 \times 0.95}$

43.

$P(\text{双正面|正面}) = \dfrac{P(\text{正面|双正面})P(\text{双正面})}{P(\text{正面|双正面})P(\text{双正面}) + P(\text{正面|均匀})P(\text{均匀}) + P(\text{正面|不均匀})P(\text{不均匀})}$

$$= \dfrac{1 \times \dfrac{1}{3}}{1 \times \dfrac{1}{3} + \dfrac{1}{2} \times \dfrac{1}{3} + \dfrac{3}{4} \times \dfrac{1}{3}} = \dfrac{4}{9}$$

44.

(a)

$P(\text{带菌|阳性}) = \dfrac{P(\text{阳性|带菌})P(\text{带菌})}{P(\text{阳性|带菌})P(\text{带菌}) + P(\text{阳性|不带菌})P(\text{不带菌})} = \dfrac{0.95 \times 10\%}{0.95 \times 10\% + 0.01 \times 90\%}$

$$= \dfrac{95}{104}$$

(b)

$$\dfrac{0.95^2 \times 10\%}{0.95^2 \times 10\% + 0.01^2 \times 90\%} = \dfrac{9025}{9034}$$