第五章　假设检验

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1.

(1)

功效函数$\beta_\Phi(\mu) = P(\overline{X} \geq 2.6) = P\left(\sqrt{n}(\overline{X}-\mu) \geq \sqrt{n}(2.6-\mu)\right)$

$= 1-\Phi\left(\sqrt{n}(2.6-\mu)\right) = \begin{cases} 1-\Phi(0.6\sqrt{n}) & , \quad \mu = 2 \\ \\ \Phi(0.4\sqrt{n}) & , \quad \mu = 3 \end{cases}$

$\therefore$犯第一类错误概率$\alpha = 1-\Phi(0.6\sqrt{n})$

犯第二类错误概率$\beta = 1-\Phi(0.4\sqrt{n})$

当$n = 20$时，

犯第一类错误概率$\alpha = 1-\Phi(2.68) = 0.00368$

犯第二类错误概率$\beta = 1-\Phi(1.79) = 0.0367$

(2)

$\beta = 1-\Phi(0.4\sqrt{n}) \leq 0.01$

$\therefore \Phi(0.4\sqrt{n}) \geq 0.99$

$\therefore 0.4\sqrt{n} \geq 2.33$

$\therefore n \geq 33.9$

$\therefore n$最小取34

(3)

当$n \to +\infty$时，$\Phi(0.6\sqrt{n}) \to 1$，$\Phi(0.4\sqrt{n}) \to 1$

$\therefore \alpha = 1-\Phi(0.6\sqrt{n}) \to 0$，$\beta = 1-\Phi(0.4\sqrt{n}) \to 0$

2.

(1)

功效函数$\beta_\Phi(\theta) = P(X_{(n)} \leq 2.5) = \left[P(X_1 \leq 2.5)\right]^n = \begin{cases} 1, & , \quad \theta \leq 2.5 \\ \\ \left(\dfrac{2.5}{\theta}\right)^n & , \quad \theta > 2.5 \end{cases}$

$\because \beta_{\Phi}(\theta)$关于$\theta$递减

$\therefore$检验水平为$\left(\frac{2.5}{3}\right)^n$

(3)

$\left(\frac{2.5}{3}\right)^n \leq 0.05$

$\therefore n \geq 16.43$

$\therefore n$至少为17

4.

$H_0 : \mu = 52 \leftrightarrow H_1 : \mu \neq 52$

拒绝域为$\left\{(X_1, X_2, \cdots, X_n) : |T| = \left|\frac{\sqrt{n}(\bar{X} - \mu_0)}{S}\right| > t_{n-1}\left(\frac{\alpha}{2}\right)\right\}$

此处$n = 6$, $\mu_0 = 52$, $\alpha = 0.05$，计算得

$|T| = 0.41 < 2.571 = t_5(0.025)$

$\therefore$接受原假设

5.

$H_0 : \mu \geq 1000 \leftrightarrow H_1 : \mu < 1000$

拒绝域为$\left\{(X_1, X_2, \cdots, X_n) : U = \frac{\sqrt{n}(\bar{X} - \mu_0)}{\sigma} < -u_\alpha\right\}$

此处$n = 25$, $\mu_0 = 1000$, $\alpha = 0.05$, $\sigma = 100$, $\bar{X} = 950$，计算得

$U = -2.5 < -1.6449 = -u_{0.05}$

$\therefore$拒绝原假设，即不合格

7.

$H_0 : \mu \leq 19 \leftrightarrow H_1 : \mu > 19$

拒绝域为$\left\{(X_1, X_2, \cdots, X_n) : T = \frac{\sqrt{n}(\bar{X} - \mu_0)}{S} > t_{n-1}(\alpha)\right\}$

此处$n = 16$, $\mu_0 = 19$, $\alpha = 0.01$，计算得

$T = 4.45 > 2.602 = t_{15}(0.01)$

$\therefore$拒绝原假设，可认为有所提高

注:这里主观上希望工艺有所提高，故$\mu > 19$应放在对立假设

9.

$H_0: \mu = 5 \leftrightarrow H_1: \mu \neq 5$

拒绝域为$\left\{(X_1, X_2, \cdots, X_n): |T| = \left|\frac{\sqrt{n}(\overline{X} - \mu_0)}{S}\right| > t_{n-1}\left(\frac{\alpha}{2}\right)\right\}$

此处$n = 10$, $\mu_0 = 5$, $\alpha_1 = 0.05$, $\alpha_2 = 0.01$, $\overline{X} = 5.3$, $S = 0.3$，计算得

$|T| = 3.16 > 2.262 = t_9(0.025)$

$|T| = 3.16 < 3.250 = t_9(0.005)$

$\therefore$在水平0.05下拒绝原假设，认为机器工作不良好。

在水平0.01下接受原假设，没有充分理由认为机器工作不良好。

10.

记第i个男孩试穿鞋子的磨损情况为(X_i, Y_i) , $i = 1, 2, \ldots, n$

令$Z_i = X_i - Y_i$，可假定$Z_1, \cdots, Z_n \sim N(\mu, \sigma^2)$

于是可归结为检验如下假设

$$H_0: \mu = 0 \longleftrightarrow H_1: \mu \neq 0$$

拒绝域为$\left\{(Z_1, \cdots, Z_n): |T| = \left|\frac{\sqrt{n}(\overline{Z} - \mu_0)}{S}\right| > t_{n-1}\left(\frac{\alpha}{2}\right)\right\}$

此处$n = 10$, $\mu_0 = 0$, $\alpha = 0.05$, $\overline{Z} = -0.41$, $S = 0.3872$，计算得

$|T| = 3.348 > 2.262 = t_9(0.025)$

$\therefore$拒绝原假设，不可认为耐磨性无显著差异

11.

记第i个人训练前后的体重为(X_i, Y_i) , $i = 1, 2, \ldots, n$

令$Z_i = X_i - Y_i$，可假定$Z_1, \cdots, Z_n \sim N(\mu, \sigma^2)$

于是可归结为检验如下假设

$$H_0: \mu \leq 8 \longleftrightarrow H_1: \mu > 8$$

拒绝域为$\left\{(Z_1, \cdots, Z_n): T = \frac{\sqrt{n}(\overline{Z} - \mu_0)}{S} > t_{n-1}(\alpha)\right\}$

此处$n = 9$, $\mu_0 = 8$, $\alpha = 0.05$, $\overline{Z} = 8.09$, $S = 1.83$，计算得

$T = 0.149 < 1.86 = t_8(0.05)$

$\therefore$接受原假设，不可认为俱乐部宣传可信

12.

$H_0:\mu_1-\mu_2\le 8 \longleftrightarrow H_1:\mu_1-\mu_2>8$

拒绝域为$\left\{(X_1,\cdots,X_{n_1},Y_1,\cdots,Y_{n_2}): T=\dfrac{\overline{X}-\overline{Y}-\mu_0}{S_w}\sqrt{\dfrac{n_1n_2}{n_1+n_2}}>t_{n_1+n_2-2}(\alpha)\right\}$

其中$S_w^2=\dfrac{1}{n_1+n_2-2}\left[\sum_{i=1}^{n_1}(X_i-\overline{X})^2+\sum_{j=1}^{n_2}(Y_j-\overline{Y})^2\right]$

此处$n_1=n_2=9$, $\mu_0=8$, $\alpha=0.05$，计算得

$T=0.0275<1.746=t_{16}(0.05)$

$\therefore$ 接受原假设，不可认为俱乐部宣传可信

14.

$H_0:\sigma^2=16 \longleftrightarrow H_1:\sigma^2\neq 16$

拒绝域为$\left\{(X_1,\cdots,X_n):\dfrac{(n-1)S^2}{\sigma_0^2}<\chi^2_{n-1}(1-\dfrac{\alpha}{2})\text{或}\dfrac{(n-1)S^2}{\sigma_0^2}>\chi^2_{n-1}(\dfrac{\alpha}{2})\right\}$

此处$n=10$, $\sigma_0^2=16$, $\alpha=0.05$, $S^2=32.652$，$\chi_9^2(0.025)=19.023$，$\chi_9^2(0.975)=2.7$

计算得$\dfrac{(n-1)S^2}{\sigma_0^2}=18.367$不在拒绝域内

$\therefore$接受原假设，可认为方差是16

16.

①:

$H_0:\dfrac{\sigma_2^2}{\sigma_1^2}=1 \longleftrightarrow H_1:\dfrac{\sigma_2^2}{\sigma_1^2}\neq 1$

拒绝域为$\left\{(X_1,\cdots,X_{n_1},Y_1,\cdots,Y_{n_2}):\dfrac{S_2^2}{S_1^2}<F_{n_2-1,n_1-1}(1-\dfrac{\alpha}{2})\text{或}\dfrac{S_2^2}{S_1^2}>F_{n_2-1,n_1-1}(\dfrac{\alpha}{2})\right\}$

此处$n_1=7$, $n_2=8$, $F_{7,6}(0.025)=5.70$, $F_{7,6}(0.975)=\dfrac{1}{F_{6,7}(0.025)}=0.195$,

计算得$\dfrac{S_2^2}{S_1^2}=1.398$不在拒绝域内

$\therefore$接受原假设，认为方差相等

②: $H_0':\mu_1-\mu_2\ge 0 \longleftrightarrow H_1':\mu_1-\mu_2<0$

拒绝域为$\left\{(X_1,\cdots,X_{n_1},Y_1,\cdots,Y_{n_2}): T=\dfrac{\overline{X}-\overline{Y}-\mu_0}{S_w}\sqrt{\dfrac{n_1n_2}{n_1+n_2}}<-t_{n_1+n_2-2}(\alpha)\right\}$

计算得$T = -1.83 < -1.7709 = -t_{13}(0.05)$

$\therefore$拒绝原假设，认为甲企业平均工资低于乙

19.

拒绝域为$\left\{(X_1,\cdots,X_{n_1},Y_1,\cdots,Y_{n_2}) : \dfrac{S_2^2}{S_1^2} > F_{n_2-1,n_1-1}(\alpha)\right\}$

此处$n_1 = n_2 = 9$，$\alpha = 0.05$，$F_{8,8}(0.05) = 3.44$，$S_1^2 = 1.536 \times 10^{-3}$，$S_2^2 = 8.644 \times 10^{-3}$

计算得$\dfrac{S_2^2}{S_1^2} = 5.628 > 3.44 = F_{8,8}(0.05)$

$\therefore$拒绝原假设，认为$\sigma_1^2 < \sigma_2^2$

22.

①:

$H_0 : \dfrac{\sigma_2^2}{\sigma_1^2} = 1 \longleftrightarrow H_1 : \dfrac{\sigma_2^2}{\sigma_1^2} \neq 1$

拒绝域为$\left\{(X_1,\cdots,X_{n_1},Y_1,\cdots,Y_{n_2}) : \dfrac{S_2^2}{S_1^2} < F_{n_2-1,n_1-1}(1-\dfrac{\alpha}{2})\text{或}\dfrac{S_2^2}{S_1^2} > F_{n_2-1,n_1-1}(\dfrac{\alpha}{2})\right\}$

此处$n_1 = 6$，$n_2 = 6$，$F_{5,5}(0.025) = 7.15$，$F_{5,5}(0.975) = \dfrac{1}{F_{5,5}(0.025)} = 0.140$，

$S_1^2 = 7.867 \times 10^{-6}$，$S_2^2 = 7.1 \times 10^{-6}$

计算得$\dfrac{S_2^2}{S_1^2} = 0.903$不在拒绝域内

$\therefore$接受原假设，认为方差相等

②:

$H_0' : \mu_1 - \mu_2 = 0 \longleftrightarrow H_1' : \mu_1 - \mu_2 \neq 0$

拒绝域为$\left\{(X_1,\cdots,X_{n_1},Y_1,\cdots,Y_{n_2}) : |T| = \left|\dfrac{\overline{X} - \overline{Y} - \mu_0}{S_w}\sqrt{\dfrac{n_1 n_2}{n_1 + n_2}}\right| > t_{n_1+n_2-2}(\alpha/2)\right\}$

计算得$|T| = 1.372 < 2.208 = t_{10}(0.05)$

$\therefore$接受原假设，认为这两组电子原件无差异

24.

$$\overline{X}-2\overline{Y}\sim N\left(\mu_1-2\mu_2,\frac{\sigma_1^2}{n}+\frac{4\sigma_2^2}{m}\right)$$

$\therefore$否定域为$\left\{(X_1,\cdots,X_n,Y_1,\cdots,Y_m):T=(\overline{X}-2\overline{Y}-\mu_0)\middle/\sqrt{\frac{\sigma_1^2}{n}+\frac{4\sigma_2^2}{m}}>u(\alpha)\right\}$

26.

记$X=\begin{cases}2\ ,\ \text{年龄在65岁以上}\\ 1\ ,\ \text{否则}\end{cases}$

则检验问题为$H_0:P(X=1)=86.45\%$，$P(X=2)=13.55\%$

H_0成立时，χ^2统计量为$Z=\sum_{i=1}^{k}\frac{(v_i-np_i)^2}{np_i}$近似服从$\chi^2_{k-1}$

此处$v_1=343$，$v_2=57$，$k=2$，$n=400$，计算得

$$Z_0=\frac{(343-400\times 86.45\%)^2}{400\times 86.45\%}+\frac{(57-400\times 13.55\%)^2}{400\times 13.55\%}=0.167<3.841=\chi^2_1(0.05)$$

$\therefore$接受原假设，认为没有变化

29.

红球个数为5时，

$p_0=P(X=0)=\frac{C_5^0C_3^3}{C_8^3}=\frac{1}{56}$，$p_1=P(X=1)=\frac{C_5^1C_3^2}{C_8^3}=\frac{15}{56}$

$p_2=P(X=2)=\frac{C_5^2C_3^1}{C_8^3}=\frac{15}{28}$，$p_3=P(X=3)=\frac{C_5^3C_3^0}{C_8^3}=\frac{5}{28}$

检验问题可转化为$H_0:P(X=i)=p_i$，$i=0,1,2,3$

$v_0=1$，$v_1=31$，$v_2=55$，$v_3=25$，$n=112$，$k=4$

χ^2统计量为$Z_0=\frac{(1-2)^2}{2}+\frac{(31-30)^2}{30}+\frac{(55-60)^2}{60}+\frac{(25-20)^2}{20}=2.2<7.815=\chi^2_3(0.05)$

$\therefore$接受原假设，认为红球个数为5

31.

H_0:甲、乙、丙三个工厂质量一致

检验统计量:

$K=n\sum_{i=1}^{r}\sum_{j=1}^{s}\frac{(n_{ij}-n_{i\cdot}n_{\cdot j}/n)^2}{n_{i\cdot}n_{\cdot j}}$，当$H_0$成立时，近似服从$\chi^2_{(r-1)(s-1)}$

$r=3$, $s=3$, $n=300$, $n_{1\cdot}=126$, $n_{2\cdot}=119$, $n_{3\cdot}=55$, $n_{\cdot 1}=109$, $n_{\cdot 2}=100$, $n_{\cdot 3}=91$

计算得χ^2统计量为

$K=15.41>9.488=\chi^2_4(0.05)$

$\therefore$拒绝H_0，认为各工厂质量不一致，甲厂较优，丙厂较劣

32.

将表中错误个数大于3的合并得到:

错误个数f_i	0	1	2	≥ 3
含f_i个错误的页数	86	40	19	5

Poisson分布参数λ的MLE为$\hat{\lambda}=\overline{X}=\dfrac{2}{3}$

令错误个数为$r.v.X$，则检验问题可视为

$$H_0: r.v.X \sim Poi\left(\frac{2}{3}\right)$$

此处$n=150$，理论值

$np_0=nP(X=0)=150\times\dfrac{(2/3)^0e^{-2/3}}{0!}=77.01$

$np_1=nP(X=1)=150\times\dfrac{(2/3)^1e^{-2/3}}{1!}=51.34$

$np_2=nP(X=2)=150\times\dfrac{(2/3)^2e^{-2/3}}{2!}=17.11$

$np_3=nP(X\geq 3)=150-77.01-51.34-17.11=4.54$

$\therefore\chi^2$统计量为

$Z_0=\dfrac{(86-77.01)^2}{77.01}+\dfrac{(40-51.34)^2}{51.34}+\dfrac{(19-17.11)^2}{17.11}+\dfrac{(5-4.54)^2}{4.54}=3.81<7.815=\chi^2_3(0.05)$

$\therefore$接受原假设，认为印刷错误个数服从Poisson分布

33.

i	区间	v_i	p_i	np_i	$(v_i-np_i)^2$	$\dfrac{(v_i-np_i)^2}{np_i}$
1	$(-\infty,30]$	5	0.0228	4.56	0.1936	0.0425
2	(30,40]	15	0.0690	13.8	1.44	0.1043
3	(40,50]	30	0.1596	31.92	3.6864	0.1155
4	(50,60]	51	0.2486	49.72	1.6348	0.0329
5	(60,70]	60	0.2486	49.72	105.6784	2.1255
6	(70,80]	23	0.1596	31.92	79.5664	2.4927
7	(80,90]	10	0.0690	13.8	14.44	1.0464
8	$(90,+\infty]$	6	0.0228	4.56	2.0736	0.4547
$\sum$	-	200	1	200	-	6.4145

χ^2统计量为$Z_0 = 6.4145 < 14.067 = \chi_7^2(0.05)$
$\therefore$接受原假设，认为成绩服从正态$N(60, 15^2)$