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Course name: 3413 - Advanced Simulation Technologies

CALCULATION REPORT

Finite Element Study of a Conical Bumper Assembly
Material: Al 3003-O

Load Cases:

- Load Case A (Crash limit): Maximum acceleration in the initial crash phase $< 30\,g$.

- Load Case B (Force limit): Maximum total force acting on the train $< 50\,\text{kN}$.

- Load Case C (Fatigue): Fatigue life $N > 10^6$ for a vertical acceleration of $\pm 1\,g$.

Tuesday 30$^{\text{th}}$ September, 2025

Abstract

TASK:

Determine the minimum number of identical cones required so that the bumper assembly satisfies all assignment load-case criteria, and then verify the fatigue life for the achieved assembly.

REALIZATION:

To balance accuracy and computational cost, the analysis was partitioned by load cases. Load Cases A and B were solved with a 2-D axisymmetric method, which is adequate because the geometry and loading are uniform through the width. Load Case C cannot be represented in 2-D owing to the eccentric, out-of-plane excitation from the bumper mass. A 3-D half-model with a symmetry plane was therefore used. For fatigue verification, unaveraged principal stresses at the hotspot were extracted from the Finite Element solution at the last substep and imported into the FKM guideline.

EXAMINED SYSTEM

Sytem Name
Conical bumper support

Drawing
Created on Design Modeler based on the provided sketches

Constituent parts
One solid body representing the cone with top and bottom plates

METHOD OF EVALUATION

Finite-Element analysis was performed in student version of ANSYS Mechanical 2025. Fatigue verification followed the FKM Guideline.

RESULTS

Comparing the reaction force with 50kN limit, the required number of cones was decided to be 24. With this number of cones the requirement of load case A is also satisfied. The fatigue check at 10^6 cycles gives a safe degree of utilization.

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Symbol	Description
M	Bending moment at a distance x along the beam
I	Moment of inertia of the beam's cross-section
σ	Bending stress at a distance y from the neutral axis
y	Distance from the neutral axis to the point of interest
h	Height of the rectangular cross-section
w	Width of the rectangular cross-section
E	Young's modulus of the material
ϵ	Strain experienced at a distance y
P	Load applied at the free end of the cantilever beam
d	Deflection at the free end of the cantilever beam
L	Length of the cantilever beam
x	Distance from the gauge end to a given point
GF	Strain gauge factor
δ	Small change in resistance due to strain
R	Nominal resistance of the strain gauge
R_f	Feedback resistor in the operational amplifier
V_{out}	Output voltage from the operational amplifier
V_{ref}	Reference voltage applied to the Wheatstone bridge
σ_{vM}	von-Mises stress

1 Object of Examination

1.1 Parts

The CAD model of the examined part was designed in Design Modeler by ANSYS 2025 according to the provided sketches.[2]

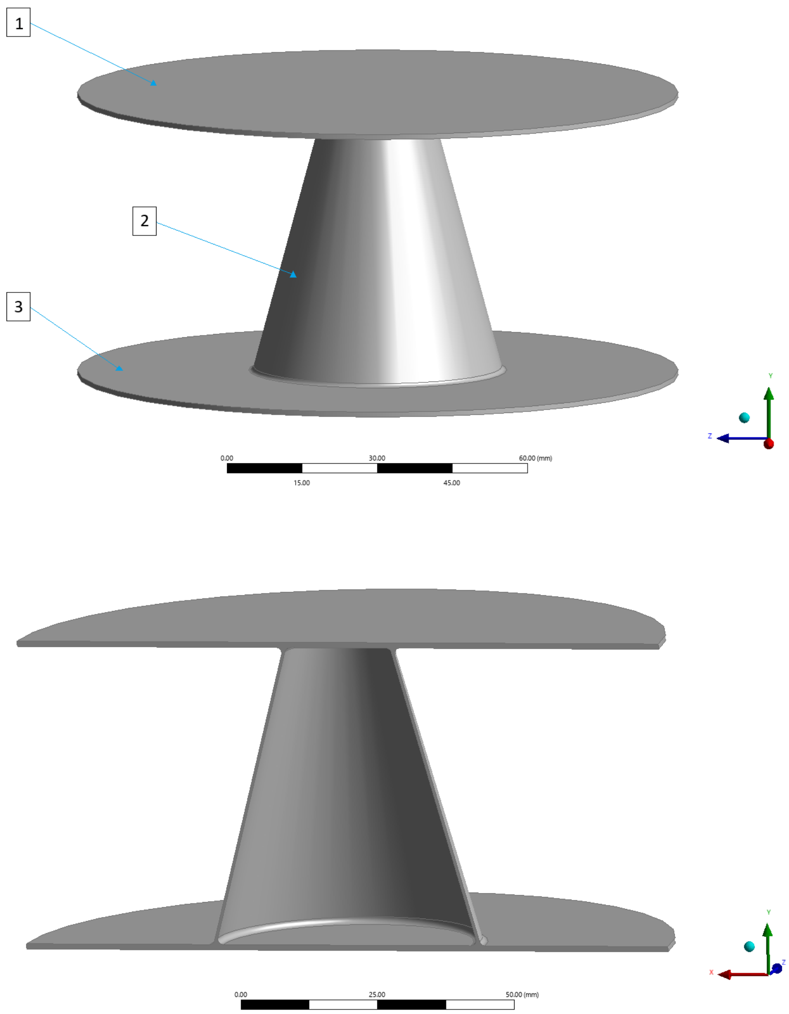

Figure 1: CAD Model

The table below preseents components shown in Figure 1.

No	Part name
1	Upper Plate (Facing Bumper)
2	Conical Absorber
3	Lower Plate (Facing the vehicle

Table 1: List of parts

1.2 Environment

The set of cones sits between the top plate attached to the plastic bumper (load input side) and the floor plate that is attached to the vehicle body (support side). The bumper bar, fasteners, and car-body structure are represented in the analysis by boundary conditions and idealized loads. Therefore, they are not designed as CAD models. Below is a rough illustration to better visualize the set of conical absorbers placement.

Figure 2: Part placement

2 Simulation Environment

2.1 Software

Below is a detailed information about versions of used softwares.

Task	Software details
CAD design	ANSYS Design Modeller 2025 R1 v25.1.0.0.
Meshing	ANSYS Mechanical 2025 R1 v25.1.0.0. Student License
Solution	ANSYS Solver v25.1.0.0. Student License
Fatigue Assessment	FKM Guideline 6th Edition

Table 2: List of Softwares

2.2 System of Units

The table providing a list of SI units used during the simulation is given below.

Dimension	Unit used	Abbreviation
Length/Displacement	Millimeter	mm
Time	Second	s
Mass	Kilogramms	kg
Force	Newton	N
Strain	-(dimensionless)	-
Stress	Megapascals	MPa
Young's modulus	Megapascals	MPa
Poisson's ratio	-(dimensionless)	-
Surface roughness	Micrometer	μm

Table 3: List of Units

2.3 Co-ordinate System

In this report the global, right-handed Cartesian system is used for all loads and boundary conditions. Y is the longitudinal/impact direction along the vehicle motion and cone axis. Impact loads for load cases of A and B act in –Y axis. The body acceleration for load case C is ± 1 g applied along $\pm$Z. The model is halved by the symmetry plane X = 0 (YZ-plane).

3 FEM Model Setup

3.1 Geometry Simplifications

The finite elements model contains only those parts that carry the load path between the bumper plate and the base plate. Those are the upper circular plate, the conical absorber and the lower base plate. Welds are represented by the existing fillet as designed initially. The geometry and loading are rotationally symmetric about the cone axis and impact acts through the centre, circumferentially uniform. Therefore the 3-D solid body was reduced to a surface body representing the meridional mid-surface of the part. This reduction preserves the through-thickness stress state while cutting the computational effort by orders of magnitude, allowing very fine local refinement in the areas of interest. The illustration of the 2d surface body is provided in Figure 3.

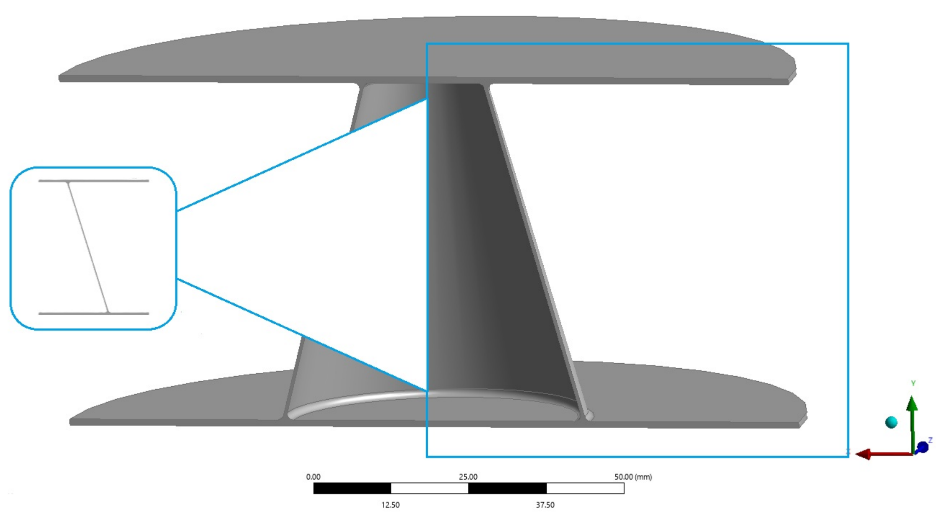

Figure 3: 2D surface body

The ±1 g vertical body acceleration produces bending of the cone and breaks rotational symmetry. To capture this non-axisymmetric response, a 3D half-model was used with a symmetry plane at X = 0 (YZ-plane). The external bumper mass is represented numerically as remote point and point mass., while the other attached bodies is not modeled explicitly, their effect is taken into account via boundary conditions.

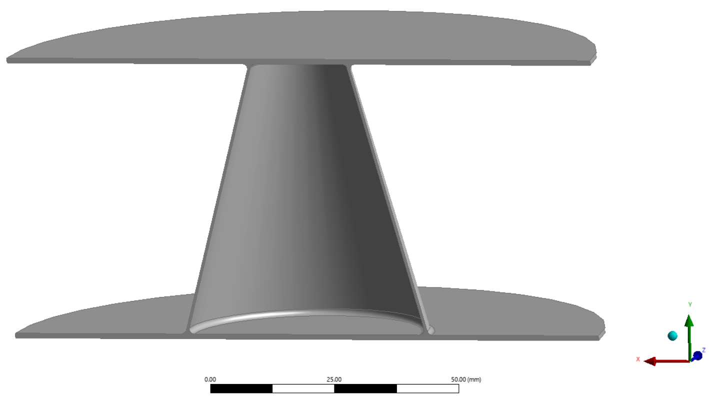

Figure 4: 3D Half body of conical support

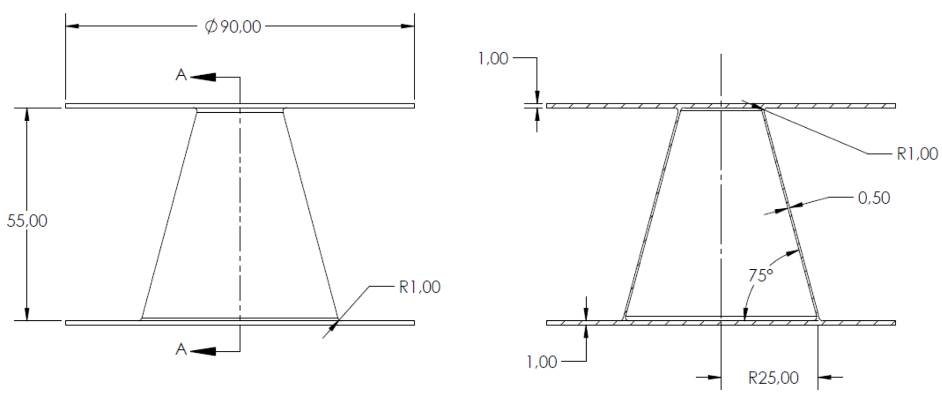

Figure 5: Technical drawing of the part

3.2 Contacts

All structural interfaces are modeled as frictional, no-penetration contacts with options of Augmented Lagrange, finite sliding. Since no detailed knowledge was available about contacts, 0.1 friction coefficient was taken to avoid the interface being unrealistically rigid while allowing slip in a conservative and numerically stable way. Only interfaces that can physically engage under the specified load cases were modeled as contact pairs. The figure below represents the list of all those pairs inserted into the simulation. Limiting contacts to kinematically admissible interfaces avoids unnecessary contact searches and iterations, reduces run time and memory, and prevents spurious over-constraint from redundant pairs.

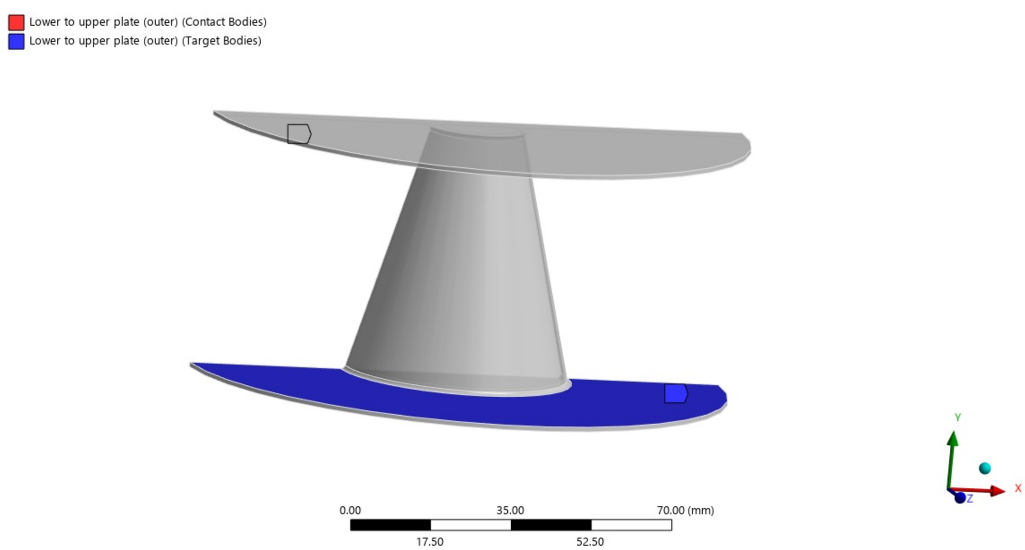

Figure 6: Contact between upper and lower plates (outer)

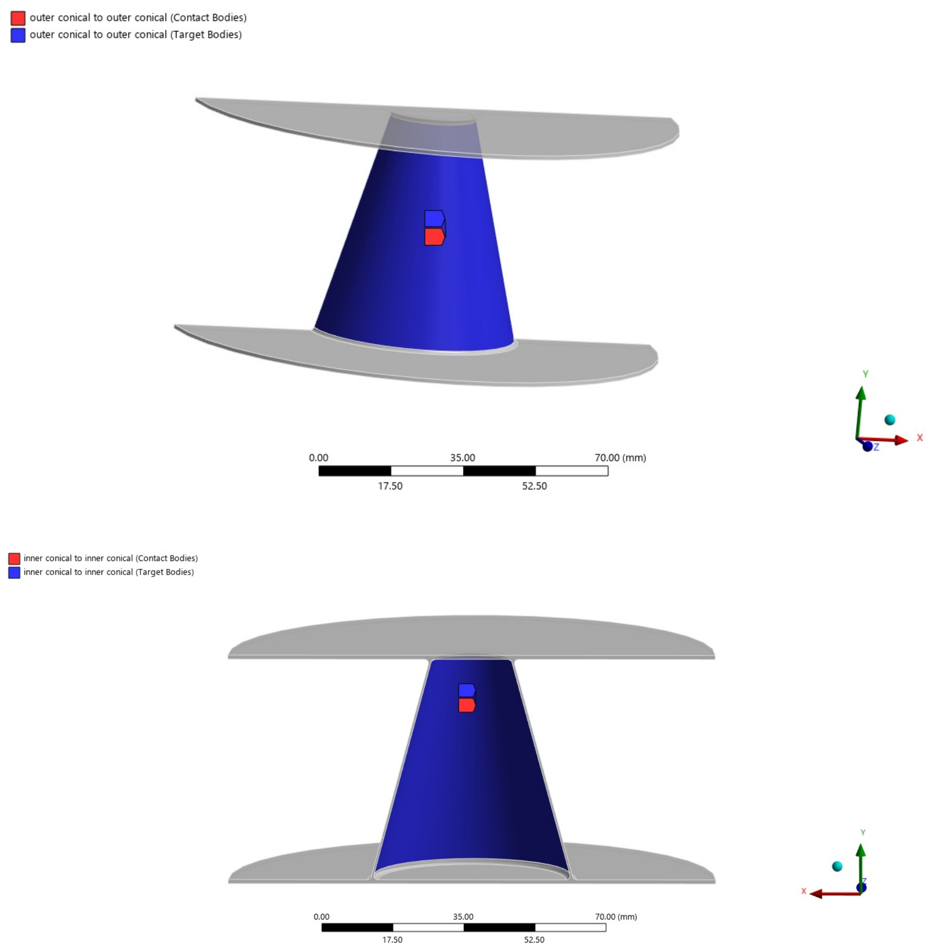

Figure 7: Set of contacts in outer and inner surfaces of the conical support

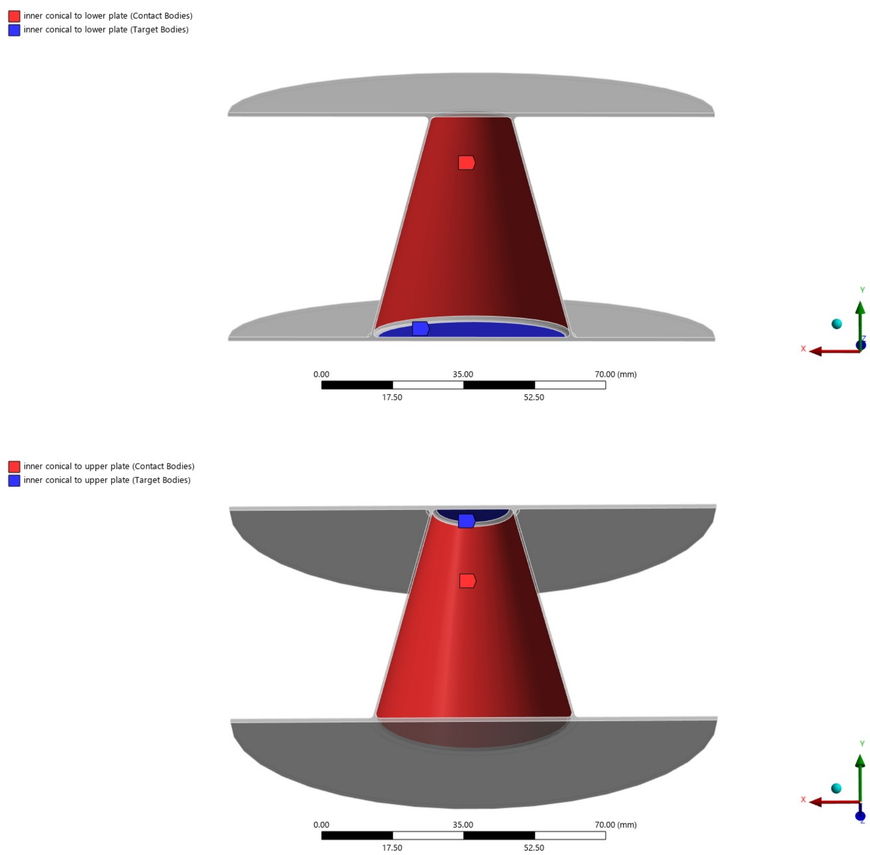

Figure 8: Additional contacts of inner surface

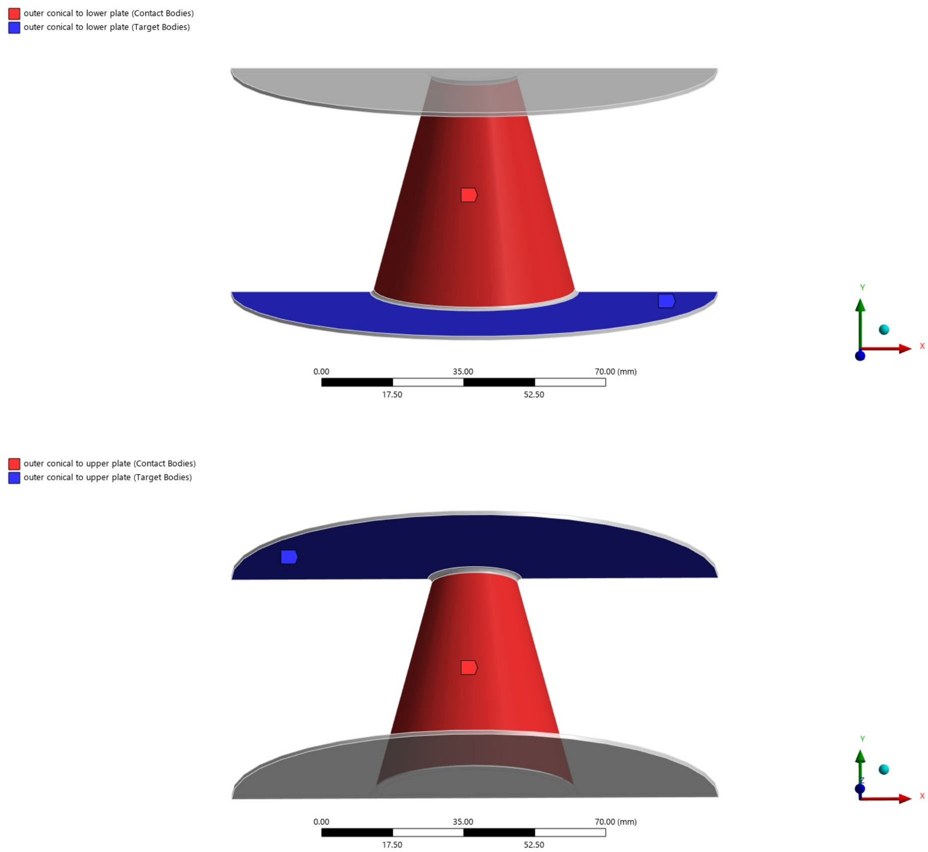

Figure 9: Additional contacts of outer surface

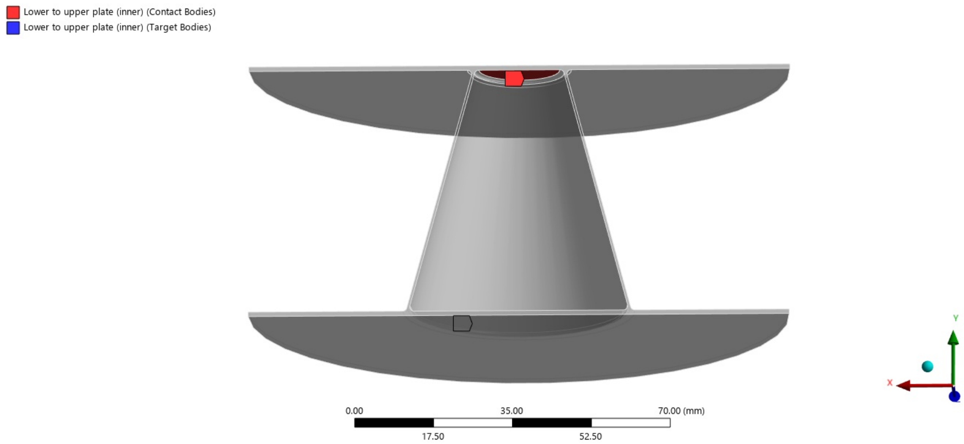

Figure 10: Contact between upper and lower plates (inner)

3.3 Meshing

All models were discretized with quadratic second order elements to capture bending and steep stress gradients accurately. Meshes were tailored to each load case: the 2D models for load case A and B used local edge sizing control on the symmetry cut / bending edge to guarantee minimum 4 elements along that cut (and around the adjacent fillet) and controlled growth rate of 1.4, ensuring the bending curvature and stress gradient are properly resolved. An iterative refinement was done for each model and assessed at the hot spot using the **maximum principal stress** as the convergence metric. The finest meshes of the refinement process achieved by edge sizing of 0.1 mm.

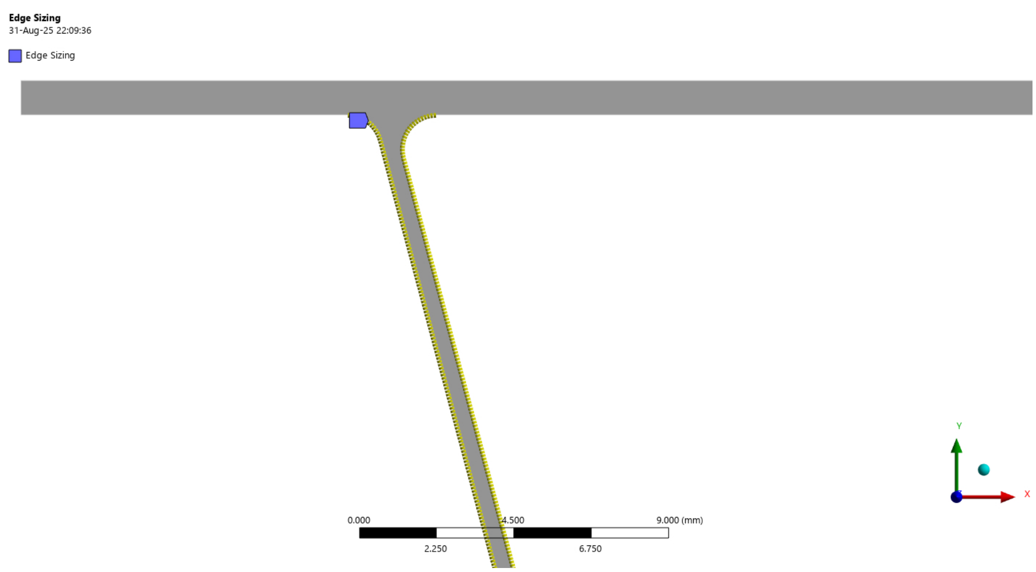

Figure 11: Edge sizing of 2D surface body

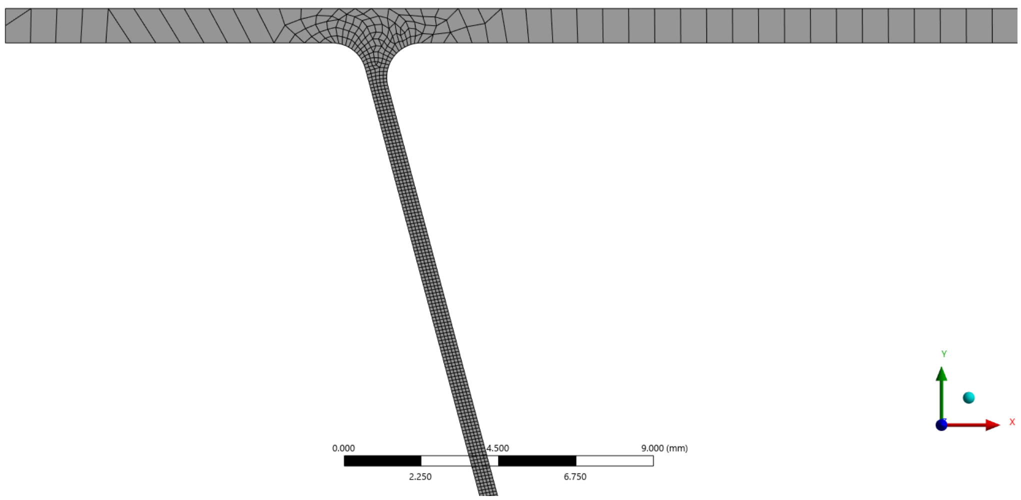

Figure 12: End result of the mesh refinement

Table provides detailed information on reaction force and maximum principal stress changes due to the mesh refinement

Mesh size	№Nodes	№Elements	Max. principal stress	Change in %
0.4 mm	1691	408	103.49 Mpa	
0.2 mm	3877	1038	106.46 Mpa	2.86
0.1 mm	10789	3148	108.4 Mpa	1.8

Table 4: Mesh refinement results

Convergence was accepted when the change in σ_1 at the hot spot between the last two meshes was $\leq 1{-}2\%$, which was achieved for both the 2D and 3D analysis. Mesh quality remained within acceptable limits of student license throughout, and all meshes were organized in ANSYS Workbench for quick comparison and traceability.

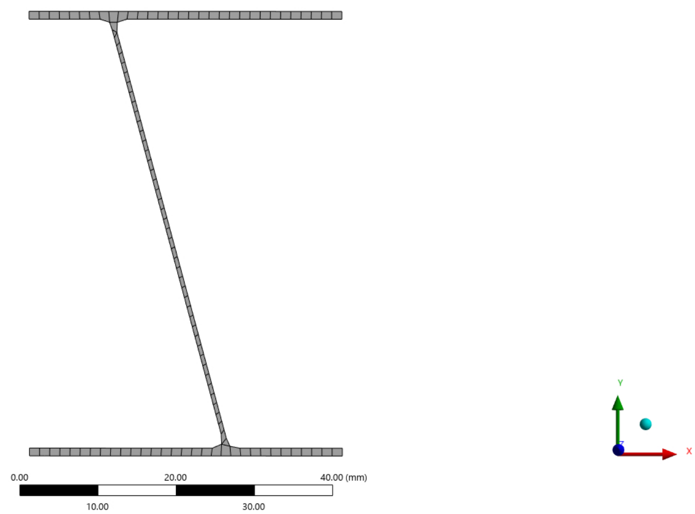

Figure 13: Depiction of the default mesh for comparison

The 3D half-model for load case C received targeted refinement in the hotspot region and along the upper fillet edge. Element sizes were reduced only where needed with controlled growth rate of 1.3, yielding very fine meshes in the regions of interest and a coarser background elsewhere to keep run time reasonable.

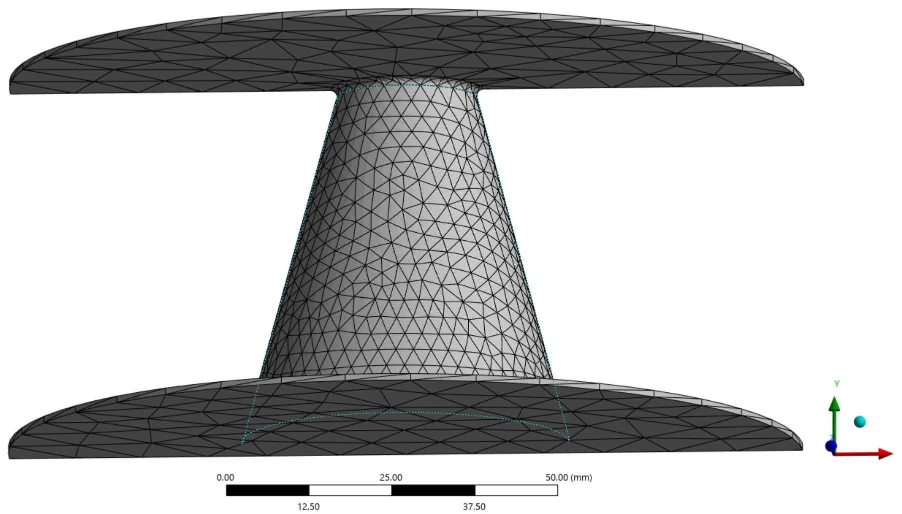

Figure 14: The default mesh of 3D half-model

$$
\begin{aligned}
D = &\frac{1}{18}(0.3 + 4.8 + 6.5 - 1.5 + 2.3 + 1.9 - 1.8 + 0 - 1.8 \\
&+ 2 + 2.9 + 3 - 4 + 2 + 6 - 2.9 + 2 - 0.8) \\
= &\frac{20.9}{18} \\
\approx &\ 1.16
\end{aligned} \tag{1}
$$

The value of "D" is our mean difference for which we got the value of 1.16 microns.

To find the degrees of freedom we will use following formula:

$$df = n - 1 \tag{2}$$

where "n" is the number of plates used in the experiment.

$$df = 18 - 1 = 17 \tag{3}$$

Then we need to calculate the "**SE** (Standard Error)", which can be found using the formula:

$$SE = \frac{SD}{\sqrt{n}} \tag{4}$$

In this equation, the value of "**SD** (Standard Deviation)" is found using the formula:

$$SD = \sqrt{\frac{\sum(D_i - D)^2}{n - 1}} \tag{5}$$

where "D_i" is the difference for each exposed and non-exposed pair.

plate	non-ex	exp.	differenc	D_i
1	1.4	1.1	0.3	0.7396
2	7	2.2	4.8	13.2496
3	8	1.5	6.5	28.5156
4	6.6	8.1	-1.5	7.0756
5	4.3	2	2.3	1.2996
6	5.1	3.2	1.9	0.5476
7	3.2	5	-1.8	8.7616
8	4	4	0	1.3456
9	5.2	7	-1.8	8.7616
10	2	0	2	0.7056
11	4	1.1	2.9	3.0276
12	6	3	3	3.3856
13	8	12	-4	26.6256
14	5	3	2	0.7056
15	8	2	6	23.4256
16	6.1	9	-2.9	16.4836
17	4	2	2	0.7056
18	5.2	6	-0.8	3.8416
				149.2028

Figure 15: Excel table of calculation

The value of "149.2028" in the table is squared differences given in the numerator of the equation. So,

$$SD = \sqrt{\frac{149.2028}{17}} \approx \sqrt{8.77766} \approx 2.9625 \text{ microns} \tag{6}$$

Now, we can calculate the "Standard Error".

$$SE = \frac{SD}{\sqrt{n}} = \frac{2.9625}{\sqrt{18}} \approx 0.698 \tag{7}$$

The next step is calculating the "t-value". This can be calculated using the following formula:

$$t = \frac{D}{SE} = \frac{1.16}{0.698} \approx 1.662 \tag{8}$$

After that, the critical "t-value" is found using the "t-distribution" table at a 5% significance level with 17 degrees of freedom.

Table A-3 *t* Distribution

Degrees of freedom	.005 (one tail) .01 (two tails)	.01 (one tail) .02 (two tails)	.025 (one tail) .05 (two tails)	.05 (one tail) .10 (two tails)	.10 (one tail) .20 (two tails)	.25 (one tail) .50 (two tails)
1	63.657	31.821	12.706	6.314	3.078	1.000
2	9.925	6.965	4.303	2.920	1.886	.816
3	5.841	4.541	3.182	2.353	1.638	.765
4	4.604	3.747	2.776	2.132	1.533	.741
5	4.032	3.365	2.571	2.015	1.476	.727
6	3.707	3.143	2.447	1.943	1.440	.718
7	3.500	2.998	2.365	1.895	1.415	.711
8	3.355	2.896	2.306	1.860	1.397	.706
9	3.250	2.821	2.262	1.833	1.383	.703
10	3.169	2.764	2.228	1.812	1.372	.700
11	3.106	2.718	2.201	1.796	1.363	.697
12	3.054	2.681	2.179	1.782	1.356	.696
13	3.012	2.650	2.160	1.771	1.350	.694
14	2.977	2.625	2.145	1.761	1.345	.692
15	2.947	2.602	2.132	1.753	1.341	.691
16	2.921	2.584	2.120	1.746	1.337	.690
17	2.898	2.567	2.110	1.740	1.333	.689
18	2.878	2.552	2.101	1.734	1.330	.688
19	2.861	2.540	2.093	1.729	1.328	.688
20	2.845	2.528	2.086	1.725	1.325	.687
21	2.831	2.518	2.080	1.721	1.323	.686
22	2.819	2.508	2.074	1.717	1.321	.686
23	2.807	2.500	2.069	1.714	1.320	.685
24	2.797	2.492	2.064	1.711	1.318	.685
25	2.787	2.485	2.060	1.708	1.316	.684
26	2.779	2.479	2.056	1.706	1.315	.684
27	2.771	2.473	2.052	1.703	1.314	.684
28	2.763	2.467	2.048	1.701	1.313	.683
29	2.756	2.462	2.045	1.699	1.311	.683
Large (z)	2.575	2.327	1.960	1.645	1.282	.675

23

Figure 16: t-distribution table. Source: [2]

As marked in the table, we will get the critical t-value according to this table for our case as "2.110".

3.4 Conclusion

The next step is comparing the t-value that we got from our calculations with the critical value we obtained from the table. Therefore, if the calculated t-value is larger than the critical value, we can reject the null hypothesis, which means the statistical difference between the exposed and non-exposed film thicknesses is significant. However, in our case, the calculated t-value is less than the critical value. This, on the other hand, means that the difference between exposed and non-exposed film thicknesses is not significant at a 5% significance level. The inference that can be made based on these results is that we do not have enough statistical evidence to state that exposure to combustion by-products significantly affects the thickness of lubricant film.

4 Question 2: Fuel consumption

4.1 Introduction

In this task, our company is responsible for investigating the effectiveness of newly suggested devices to a low-cost airline company to reduce their planes' fuel consumption. We aim to provide the company with valuable information based on which the company will decide whether these devices will help them to cut fuel costs on a reasonable scale. If yes, it is necessary to find out which device will be more suitable for our case. Below is the table which provides us with the data of "Standard 737", "Device 1" and "Device 2" fuel consumption in "L/km/passenger". To do so, the "Analysis of Variance (ANOVA)" method will be implemented.

Standard 737	Device 1	Device 2
3.5	3.2	3.1
2.8	3	2.7
3	2.5	3.2
3.15	2.65	2.8
3.2	2.7	2.6
3.4	2.6	2.9

Table 5: Data presenting fuel consumptions

4.2 ANOVA

"ANOVA" will help us to determine if there is any statistically significant difference among these three independent options. To perform "ANOVA" test, we will calculate the **mean** for each group and **total mean** as follows:

Mean for "Standard 737":

$$X_{\text{Standard 737}} = \frac{3.5 + 2.8 + 3 + 3.15 + 3.2 + 3.4}{6} \approx 3.175 \tag{9}$$

Mean for "Device 1":

$$X_{\text{Device 1}} = \frac{3.2 + 3 + 2.5 + 2.65 + 2.7 + 2.6}{6} \approx 2.775 \tag{10}$$

Mean for "Device 2":

$$X_{\text{Device 2}} = \frac{3.1 + 2.7 + 3.2 + 2.8 + 2.6 + 2.9}{6} \approx 2.883 \tag{11}$$

Overall Mean:

$$\begin{aligned}\bar{X} = &\frac{3.5 + 2.8 + 3 + 3.15 + 3.2 + 3.4 + 3.2 + 3 + 2.5}{18} \\ &+ \frac{2.65 + 2.7 + 2.6 + 3.1 + 2.7 + 3.2 + 2.8 + 2.6 + 2.9}{18} \\ &\approx 2.944\end{aligned} \tag{12}$$

Next step is calculating **the total sum of squares (SST)**:

$$SST = \sum (X_i - \bar{X})^2 \tag{13}$$

Where "X_i" represents each data point and "$\overline{X}$" represents the overall mean of all the data points. Therefore, our calculation will be:

$$SST = (3.5 - 2.944)^2 + (2.8 - 2.944)^2 + \cdots + (2.9 - 2.944)^2 \approx 1.4688 \tag{14}$$

Between-group sum of squares (SSB):

$$SSB = \sum n_i (X_i - \bar{X})^2 \tag{15}$$

Where "n_i" is the number of observations in each group which is "6" for our case. "X_i" are the group means which are "$X_{\text{Standard 737}}$", "$X_{\text{Device 1}}$", and "$X_{\text{Device 2}}$" in our task. Therefore,

$$SSB = 6 \times ((3.175 - 2.944)^2 + (2.775 - 2.944)^2 + (2.883 - 2.944)^2) \approx 0.5142 \tag{16}$$

Sum of squares within-groups (SSW):

$$SSW = SST - SSB = 1.4688 - 0.5142 = 0.9546 \tag{17}$$

In this part, degrees of freedom will be calculated. Below are the calculations that show calculations of degrees of freedom between and within groups.

Degrees of freedom between groups (df_b):

$$df_b = k - 1 = 3 - 1 = 2 \tag{18}$$

where "k" is the number of groups.

Degrees of freedom within groups (df_W):

$$df_W = N - k = 18 - 3 = 15 \tag{19}$$

where N is the total number of observations.

The next step will be calculating the "mean square between groups (MS_B)" and the "mean square within groups (MS_W)" which will be later used for calculating the "F-Statistic (F)" as follows:

$$MS_b = \frac{SSB}{df_b} \tag{20}$$

$$MS_w = \frac{SSW}{df_W} \tag{21}$$

$$F = \frac{MS_B}{MS_W} \tag{22}$$

Let's implement these formulas to our task:

$$MS_B = \frac{0.5142}{2} \approx 0.2571 \tag{23}$$

$$MS_W = \frac{0.9546}{15} \approx 0.0636 \tag{24}$$

$$F = \frac{0.2571}{0.0636} \approx 4.0425 \tag{25}$$

which results with "p-value (p)" equal to "0.0394" [?]

Based on these results we can already make some interpretations about the level of significance among the three groups. The “F-value” of “4.0425” suggests that there is a significant difference at least between two groups out of these three. On the other hand, the “p-value” of “0.0394” is slightly below the threshold level of “0.05” which indicates that there is enough evidence to reject the null hypothesis at this level and there is a significant difference at least between two groups out of given three. As a result, we will continue our investigation by implementing the “Tukey’s honestly significant difference” test to find out between which group the significant difference is observed.

4.3 Tukey’s HSD

This test will help us to identify which exact pair among these three groups has a significant difference. To implement this test we will begin with calculating the “**Standard Error (SE)**” as described in the formula below:

$$SE = \sqrt{\frac{MS_W}{n}} \tag{26}$$

where “n” is the number of observations per group. Therefore,

$$SE = \sqrt{\frac{0.0636}{6}} \approx 0.103 \tag{27}$$

Then **“q-statistics”** is calculated:

$$q = \frac{|\text{mean}_i - \text{mean}_j|}{SE} \tag{28}$$

where “$|\text{mean}_i - \text{mean}_j|$” stands for the absolute differences between each pair of group means.

Now, let’s proceed with our pairwise comparison:

For “Standard 737” vs “Device 1”:

$$|X_{\text{Standard 737}} - X_{\text{Device 1}}| = |3.175 - 2.775| = 0.4 \tag{29}$$

$$q = \frac{0.4}{0.103} \approx 3.8835 \tag{30}$$

For “Standard 737” vs “Device 2”:

$$|X_{\text{Standard 737}} - X_{\text{Device 2}}| = |3.175 - 2.883| = 0.292 \tag{31}$$

$$q = \frac{0.292}{0.103} \approx 2.835 \tag{32}$$

For “Device 1” vs “Device 2”:

$$|X_{\text{Device 2}} - X_{\text{Device 1}}| = |2.883 - 2.775| = 0.108 \tag{33}$$

$$q = \frac{0.108}{0.103} \approx 1.0485 \tag{34}$$

4.4 Conclusion

In this part, the results of calculations will be summarized and final conclusion will be provided to explain the findings. The results of our statistical tests and calculations show that “Device 1” ended up having more significant results while “Device 2” was less significant in comparison with its opponent. All in all, we can conclude that the fuel consumption has changed significantly when it was measured with the suggested “Device 1”.

5 Question 3: Mars Exploration

5.1 Introduction

While performing the "Beagle 2" mission on Mars, due to some unknown failures, goods were not successfully delivered to the surface of the planet, and it crashed while landing. After experiencing this failure, a new solution was suggested by a European group to increase the chances of delivering goods to this planet. The suggestion was to send them with the help of a stack of paper planes. The idea behind this suggestion is to design these paper planes in a way that they will glide above the surface of Mars as long as possible in order to gather information about the environment they are surrounded by and the ground they are facing.

In this part of the report, our task is to design three different models of paper planes and prepare 5 examples for each design option. After that, we will make test flights to gather data about their traveling distances. This data will be used for performing the statistical analysis discussed before.

Let's take a look at different design options for paper planes. It should be noted that all three groups are made of simple A4 papers using different folding techniques. Below you see a group of paper planes which are labeled as Option A, and named as A_1, A_2, A_3, A_4, and A_5.

5.1.1 Design A

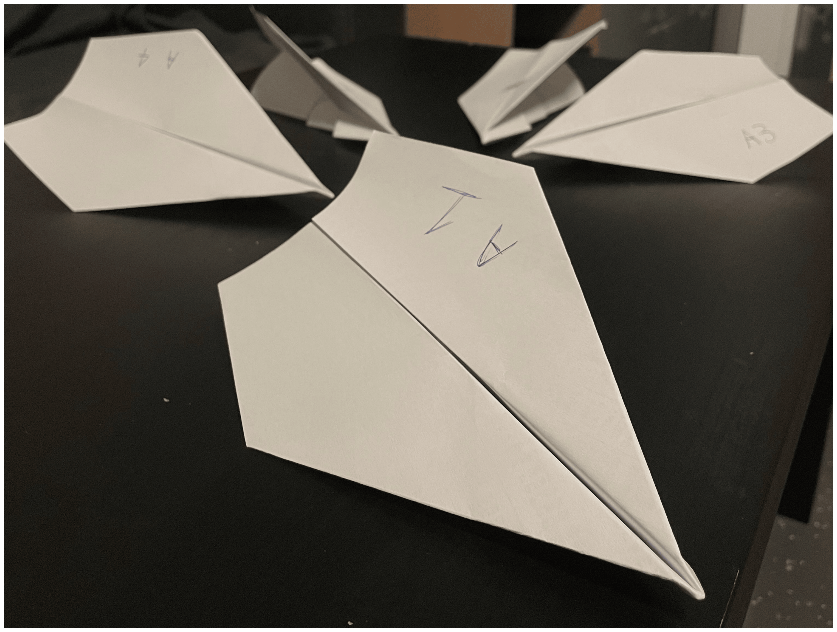

Figure 17: Design Group “A”

In this design, paper planes have more basic designs in comparison to the other two groups. Their backside is slightly curved to reduce air resistance and help them glide as long as possible. The center of mass of these planes is approximated to be in the middle of the body of the planes. In the table below you can see the results of test flights performed by group “A”.

Name of the plane	Travelled distance in meters
A_1	4.95
A_2	8.35
A_3	4.55
A_4	7.10
A_5	5.35

Table 6: Data of group “A”

5.1.2 Design B

Group “B ”also consists of 5 planes which have a more complex design, having the center of mass dragged to the front side of the plane. Wide wings of these planes help them to perform a consistent flight and smooth landing. Their mass distribution allows them to have a more aggressive start at the beginning to glide for longer distances. Below you can visually see paper planes labeled under group “B” and make yourself familiarize with the results of their test flights.

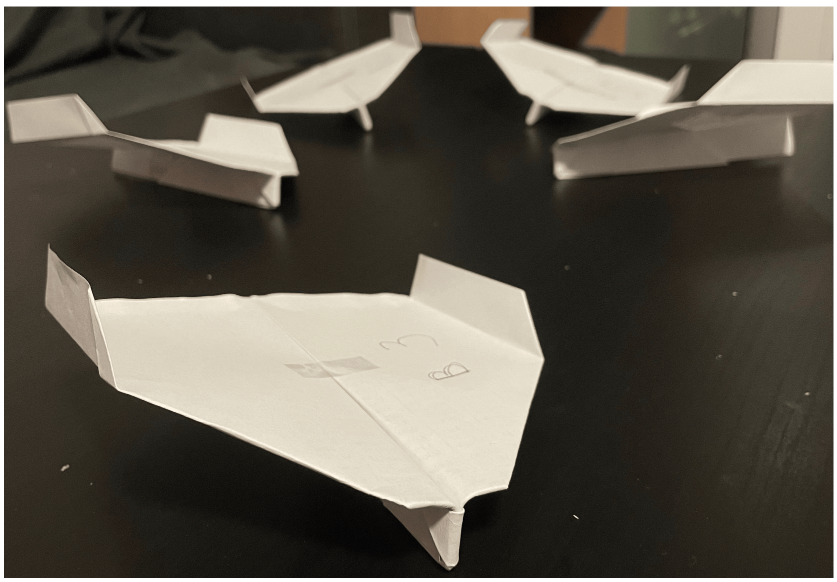

Figure 18: Design Group “B”

Name of the plane	Travelled distance in meters
B_1	3.40
B_2	4.32
B_3	6.60
B_4	8.91
B_5	5.20

Table 7: Data of group “B”

5.1.3 Design C

Planes included in group “C” had the most promising design option since they were sharper in shape and lighter in weight. These features gave them an advantage in speed and reliable self-control. Special cuts in their wings helped to reduce air resistance to a minimum and stay in the air as long as possible. Below the image is provided to make an impression about the design of planes in “group C”.

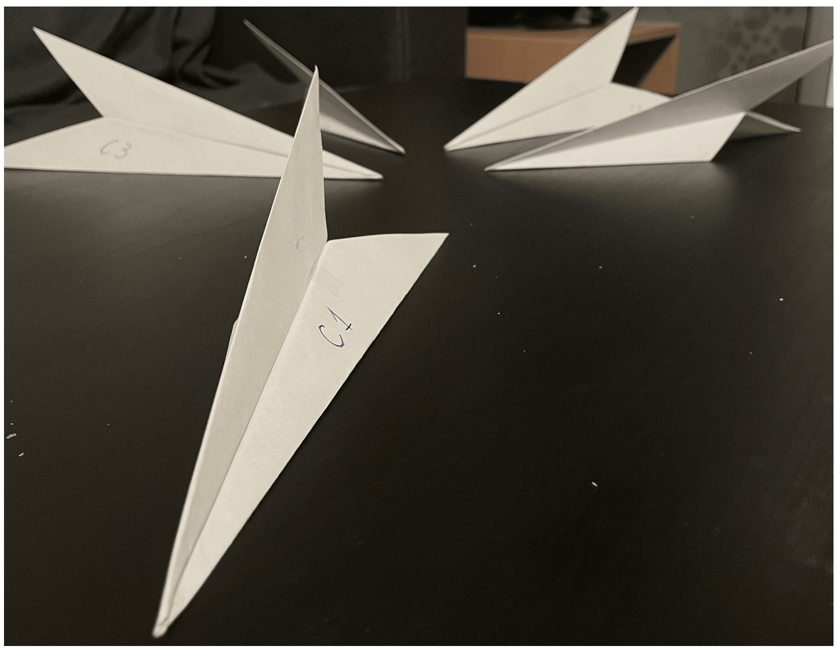

Figure 19: Design Group “C”

Name of the plane	Travelled distance in meters
C_1	4.39
C_2	4.86
C_3	4.55
C_4	4.17
C_5	4.42

Table 8: Data of group “C”

5.2 Calculation

Once data is provided for all three designs, mean, median and standard deviation will be calculated for each design based on their results. Let's calculate these values using the appropriate values:

For the Group A:

Mean is calculated using the formula of:

$$\mu = \frac{\sum x_i}{n} \tag{35}$$

where "μ" is the mean, "x_i" represents individual observations, and "n" is the number of observations. For our case:

$$\mu_A = \frac{A_1 + A_2 + A_3 + A_4 + A_5}{5} = \frac{30.3}{5} = 6.06 \tag{36}$$

For calculation of the "**median**" the results of the group need to be sorted in an ascending order and the middle value is considered to be the median of the group. In case an even number of observations is indicated, the median will be calculated as an average of the two middle values. For the group "A", "5.35" will be taken as the median:

Ascending order of values in group "A":

$$\{4.55, 4.95, \underline{5.35}, 7.10, 8.35\}$$

The next step is calculating **standard deviation** of the group. The standard deviation is a value that indicates how well values are spread out in a specific range of the group. In other words, while a low standard deviation value stands for the groups where values are closely distributed to the mean value, a high standard deviation is a sign of widely spread out values in the group. Standard deviation will be calculated using the formula:

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{n}} \tag{37}$$

where "x_i" represents individual observations, "μ" is the mean, and "n" is the number of observations.

Now let's calculate the **standard deviation** for the group "A":

$$\sigma_A = \sqrt{\frac{(4.95-6.06)^2+(8.35-6.06)^2+(4.55-6.06)^2+(7.10-6.06)^2+(5.35-6.06)^2}{5}}$$
$$\approx \sqrt{\frac{1.23+5.24+2.28+1.08+0.5}{5}} \approx \sqrt{2.066} \approx 1.44 \tag{38}$$

For the group "B": **Mean** will be calculated as follows:

$$\mu_B = \frac{B_1+B_2+B_3+B_4+B_5}{5} = \frac{28.43}{5} \approx 5.686 \tag{39}$$

Median distance:

$$\{3.40, 4.32, \underline{5.20}, 6.60, 8.91\} \tag{40}$$

Standard deviation for the group "B":

$$\sigma_B = \sqrt{\frac{(3.40-5.686)^2+(4.32-5.686)^2+(6.60-5.686)^2+(8.91-5.686)^2+(5.20-5.686)^2}{5}}$$
$$= \sqrt{\frac{5.23+1.87+0.84+0.84+10.39+0.24}{5}} \approx \sqrt{3.71} \approx 1.93 \tag{41}$$

For the Group C:
Calculation of **mean:**

$$\mu_C = \frac{C_1+C_2+C_3+C_4+C_5}{5} = \frac{22.39}{5} \approx 4.478 \tag{42}$$

Median distance:

$$\{4.17, 4.39, \underline{4.42}, 4.55, 4.86\} \tag{43}$$

Standard deviation of the group:

$$\sigma_C = \sqrt{\frac{(4.39-4.478)^2+(4.86-4.478)^2+(4.55-4.478)^2+(4.17-4.478)^2+(4.42-4.478)^2}{5}}$$
$$\approx \sqrt{\frac{0.008+0.15+0.005+0.09+0.003}{5}} \approx \sqrt{0.05} \approx 0.23 \tag{44}$$

5.3 Analysis

In this part of the report, a graphical representation of a set of data will be provided. To do so, the following Python code is plotted to get an illustration of the necessary information:

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Figure 20: Python code for graphical illustration

Based on these graphs, we can analyze the data and highlight some significant points. Therefore, according to the charts, design "A" shows a wide range for the set of data and also the highest median. On the other hand, design "B" has a wide range of traveling distance from 3.4 meters up to 8.9 meters which can be an indicator of inconsistency in performance. The third group which is named as design group of "C" displays the smallest range of data and the shortest overall distances at the same time. In this part of the report, we can already begin with implementing the "Shapiro-Wilk" test to check each group for normality. If the results show that the data is normally distributed, the next step will be conducting the "ANOVA" test to compare the means of groups. To do so, we will use the Python method with "SciPy" for the calculation p-value and w-value of each group. Below is the Python code used for group "A".

The result shows us the “p-value” of “0.39” and “w-value” of “0.896” for the design group of “A”. Inputs are changed for groups “B” and “C” to calculate these values for other design groups of planes. Calculations for group “B” resulted in a “p-value” of “0.78” and “w-value of 0.956”. Lastly, the “p” and “w-values” of group “C” appeared to be “0.82” and “0.96” accordingly. The “w-values” are the “Shapiro-Wilk” statistic which represents how closely is the data aligned with a normal distribution. The value itself ranges from “0” to “1” where a closer value to “1” stands for a more aligned set of data with a normal distribution which is the case in our analysis. In addition to this, since the “p-values” of all three groups are higher than “0.05”, we fail to reject the null hypothesis and the data is consistent with a normal distribution.

5.4 ANOVA

For implementing "**ANOVA**" we need to calculate the overall mean for our groups. This will be calculated as follows:

$$\mu_{\text{total}} = \frac{4.95 + 8.35 + 4.55 + 7.10 + 5.35 + 3.40 + 4.32 + 6.60 + 8.91 + 5.20}{15} + \frac{4.39 + 4.86 + 4.55 + 4.17 + 4.42}{15} \approx 5.41 \tag{45}$$

We have already calculated the means of each group:

$$\mu_A = 6.06$$
$$\mu_B = 5.686$$
$$\mu_C = 4.478$$

Now let's calculate the sum of the squares between groups (SSB):

$$SSB = 5 \times (\mu_A - \mu_{\text{total}})^2 + 5 \times (\mu_B - \mu_{\text{total}})^2 + 5 \times (\mu_C - \mu_{\text{total}})^2 \tag{46}$$

$$SSB \approx 2.113 + 0.38 + 4.34 \approx 6.84 \tag{47}$$

The next step is calculating the sum of squares within groups for each group (SSW):

$$SSW_A = (A_1 - \mu_A)^2 + (A_2 - \mu_A)^2 + (A_3 - \mu_A)^2 + (A_4 - \mu_A)^2 + (A_5 - \mu_A)^2 \tag{48}$$

$$SSW_A \approx 1.23 + 5.24 + 2.28 + 1.08 + 0.5 \approx 10.33 \tag{49}$$

$$\begin{aligned} SSW_B &= (B_1 - \mu_B)^2 + (B_2 - \mu_B)^2 + (B_3 - \mu_B)^2 + (B_4 - \mu_B)^2 + (B_5 - \mu_B)^2 \\ &= 5.23 + 1.87 + 0.84 + 10.39 + 0.24 = 18.57 \end{aligned} \tag{50}$$

$$SSW_C = (C_1 - \mu_C)^2 + (C_2 - \mu_C)^2 + (C_3 - \mu_C)^2 + (C_4 - \mu_C)^2 + (C_5 - \mu_C)^2 \tag{51}$$

$$= 0.007 + 0.15 + 0.005 + 0.09 + 0.003 = 0.255$$

$$SSW_{\text{total}} = SSW_A + SSW_B + SSW_C = 10.33 + 18.57 + 0.255 = 29.155 \tag{52}$$

In this part of the report, we will continue with the calculation of degrees of freedom for our analysis. Let's begin with calculating the degrees of freedom between groups:

$$df_{\text{between}} = K - 1 \tag{53}$$

Where "K" stands for the number of groups which is "3" in our case. So the equation will be as follows:

$$df_{\text{between}} = 3 - 1 = 2 \tag{54}$$

Degrees of freedom within groups:

$$df_{\text{within}} = N - K \tag{55}$$

where "N" stands for the total number of observations including all groups. Therefore, for our case, the value of "N" will be "15".

$$df_{\text{within}} = 15 - 3 = 12 \tag{56}$$

Total degrees of freedom:

$$df_{\text{total}} = N - 1 = 15 - 1 = 14 \tag{57}$$

The next step will be the calculation of the mean squares:

$$MS_{\text{between}} = \frac{SSB}{df_{\text{between}}} = \frac{6.84}{2} = 3.42 \tag{58}$$

$$MS_{\text{within}} = \frac{SSW_{\text{total}}}{df_{\text{within}}} = \frac{29.155}{12} = 2.43 \tag{59}$$

The value of the "F-statistic" will be:

$$F_{\text{calculated}} = \frac{MS_{\text{between}}}{MS_{\text{within}}} = \frac{3.42}{2.43} = 1.4 \tag{60}$$

According to the "F-value" distribution table for "$\alpha = 0.05$", we get the "F_{critical}" value of "3.89" as shown in the figure below.

5.5 Conclusion

At the end of the statistical analysis of three design groups of paper planes, the result showed that the calculated "F-value" is higher than the critical "F-value", we failed to reject the null hypothesis. In detail, it means that we do not have enough evidence to prove that there is a significant difference among these three design groups. [?]

References

[1] Forschungskuratorium Maschinenbau (FKM). https://woe-eu-com.ezproxy2.hsrw.eu/pageview/pageview$_5 48C5.html$.

[2] Task Description. https://moodle.hochschule-rhein-waal.de/course/view.php?id=11255.

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Figure 21: Graphical illustration of the given data. Source: [1]

```python
import numpy as np
from scipy.stats import shapiro

# Data
data = np.array([4.95, 8.35, 4.55, 7.10, 5.35])

# Shapiro-Wilk test
statistic, p_value = shapiro(data)
print(f"Shapiro-Wilk Statistic: {statistic}, P-value:
{p_value}")
```

Figure 22: Python code for “Shapiro-Wilk” test for group “A”

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Figure 23: F-distribution table. Source: [?]