Textbook: pp. 445-478

Chapter 12: Waiting Lines and Queuing Theory Models

Learning Objectives (1 of 2)

After completing this chapter, students will be able to:

- Describe the **trade-off curves for cost-of-waiting time** and **cost of service**.
- Describe the **basic queuing system configurations** and the **three parts of a queuing system**: the calling population, the queue itself, and the service facility.
- Analyse a variety of operating characteristics of waiting lines for **single-channel models** with exponential service times and infinite calling populations.
- Analyse a variety of operating characteristics of waiting lines for **multichannel models** with exponential service times and infinite calling populations.

Learning Objectives (2 of 2)

After completing this chapter, students will be able to:

- Analyse a variety of operating characteristics of waiting lines for single-channel models with deterministic service times and infinite calling populations.
- Analyse a variety of operating characteristics of waiting lines for single-channel models with exponential service times and finite calling populations.
- Understand **Little’s Flow Equations**.
- Understand the need for **simulation** for more complex waiting line models.

Introduction

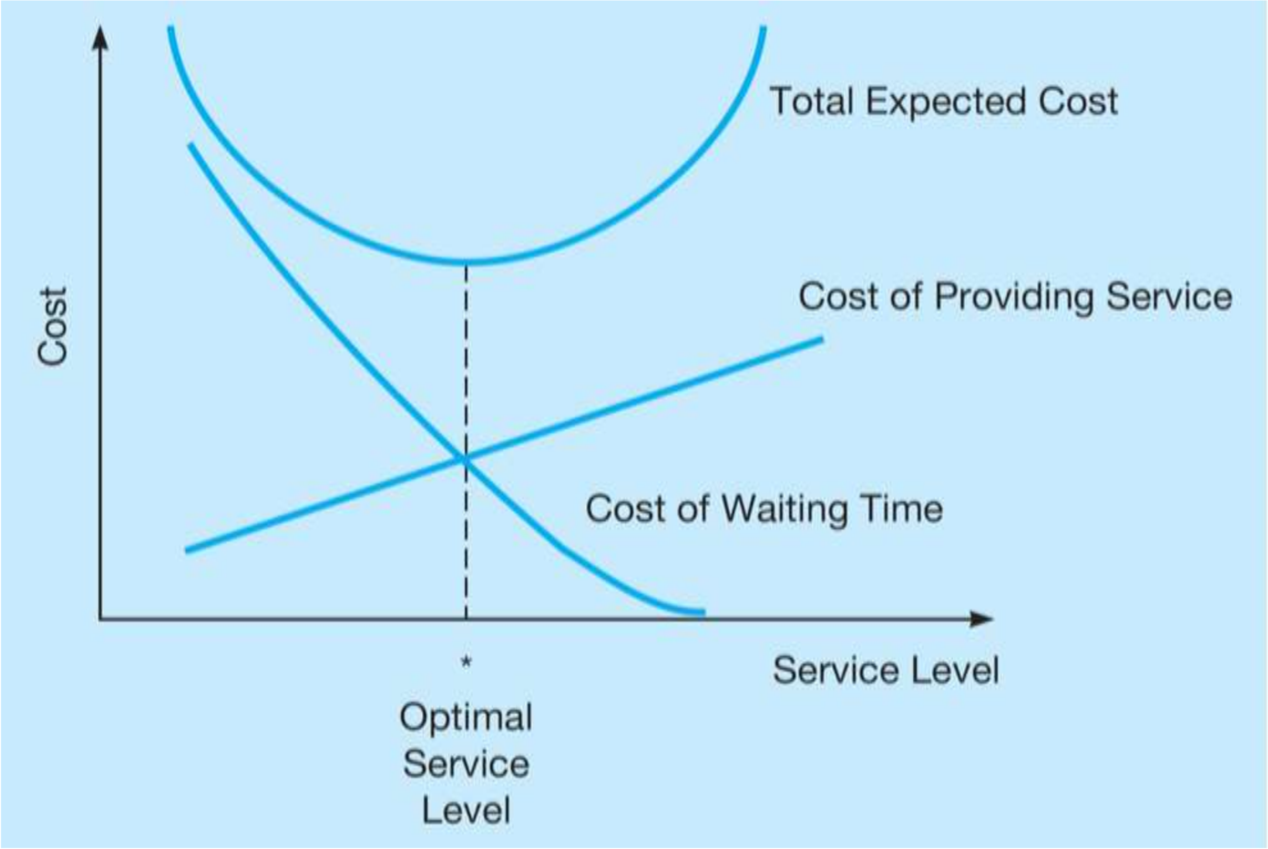
- **Queuing theory** is the study of waiting lines
  - One of the oldest and most widely used quantitative analysis techniques
- The three basic components of a queuing process
  - Arrivals
  - Service facilities
  - The actual waiting line
- Analytical models of waiting lines can help managers evaluate the cost and effectiveness of service systems

Waiting Line Costs (1 of 2)

- Most **waiting line** problems are focused on finding the ideal level of service a firm should provide
- Generally service level is something management can control
- Often try to find the balance between two extremes
  - A large staff and *many* service facilities
    - High levels of service but high costs
  - The *minimum* number of service facilities
    - **Service cost** is lower but may result in dissatisfied customers
- Service facilities are evaluated on their total expected cost which is the sum of service costs and waiting costs
  - Find the service level that minimises the total expected cost

Waiting Line Costs (2 of 2)

Queuing Costs and Service Levels:



Three Rivers Shipping Company (1 of 2)

- Three Rivers Shipping operates a docking facility on the Ohio River
    - An average of 5 ships arrive to unload their cargos each shift
    - Idle ships are expensive
    - More staff can be hired to unload the ships, but that is expensive as well
- Three Rivers Shipping Company wants to determine the optimal number of teams of stevedores to employ each shift to obtain the minimum total expected cost

Three Rivers Shipping Company (2 of 2)

Three Rivers Shipping Company Waiting Line Cost Analysis:

	NUMBER OF TEAMS OF STEVEDORES WORKING			
	1	2	3	4
(a) Average number of ships arriving per shift	5	5	5	5
(b) Average time each ship waits to be unloaded (hours)	7	4	3	2
(c) Total ship hours lost per shift (*a* × *b*)	35	20	15	10
(d) Estimated cost per hour of idle ship time	$1,000	$1,000	$1,000	$1,000
(e) Value of ship's lost time or waiting cost (*c* × *d*)	$35,000	$20,000	$15,000	$10,000
(f) Stevedore team salary or service cost	$6,000	$12,000	$18,000	$24,000
(g) Total expected cost (*e* + *f*)	$41,000	$32,000	$33,000	$34,000
		Optimal cost		

Characteristics of a Queuing System (1 of 10)

- There are **three parts to a queuing system**
  1. Arrivals or inputs to the system (sometimes referred to as the calling population)
  2. The queue or waiting line itself
  3. The service facility
- These components have certain characteristics that must be examined before mathematical queuing models can be developed

Characteristics of a Queuing System (2 of 10)

- Arrival Characteristics have three major characteristics
  - Size of the calling population
    - **Unlimited** (essentially infinite) or **limited** (finite)

- Pattern of arrivals
  - Arrive according to a known pattern or can arrive randomly
  - Random arrivals generally follow a Poisson distribution

Characteristics of a Queuing System (3 of 10)

Arrival Characteristics have three major characteristics:

- Behaviour of arrivals
  - Most queuing models assume customers are patient and will wait in the queue until they are served and do not switch lines
  - **Baulking** refers to customers who refuse to join the queue
  - **Reneging** customers enter the queue but become impatient and leave without receiving their service

Characteristics of a Queuing System (4 of 10)

Waiting Line Characteristics:

- Can be either **limited** or **unlimited**
- **Queue discipline** refers to the rule by which customers in the line receive service
- Most common **rule is first-in, first-out (FIFO)**
- Other rules can be applied to select which customers enter which queue, but may apply FIFO once they are in the queue

Characteristics of a Queuing System (5 of 10)

Service Facility Characteristics:

1. Configuration of the queuing system
   - **Single-channel system**
     - One server
   - **Multichannel systems**
     - Multiple servers fed by one common waiting line

Characteristics of a Queuing System (6 of 10)

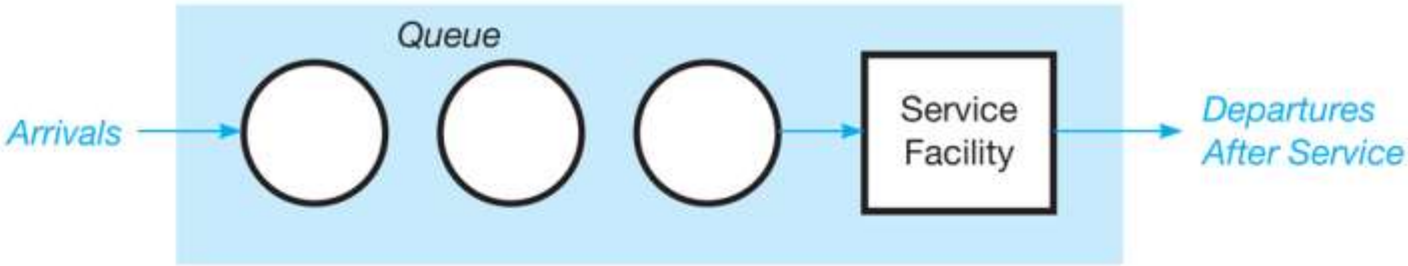
Service Facility Characteristics:

2. Pattern of service times

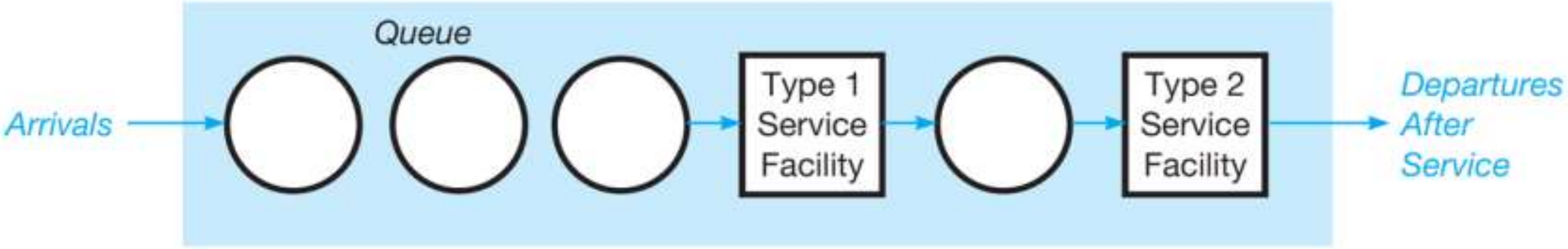
- **Single-phase system**
  - Customer receives service from just one server
- **Multiphase system**
  - Customer goes through more than one server

Characteristics of a Queuing System (7 of 10)

Four Basic Queuing System Configurations:



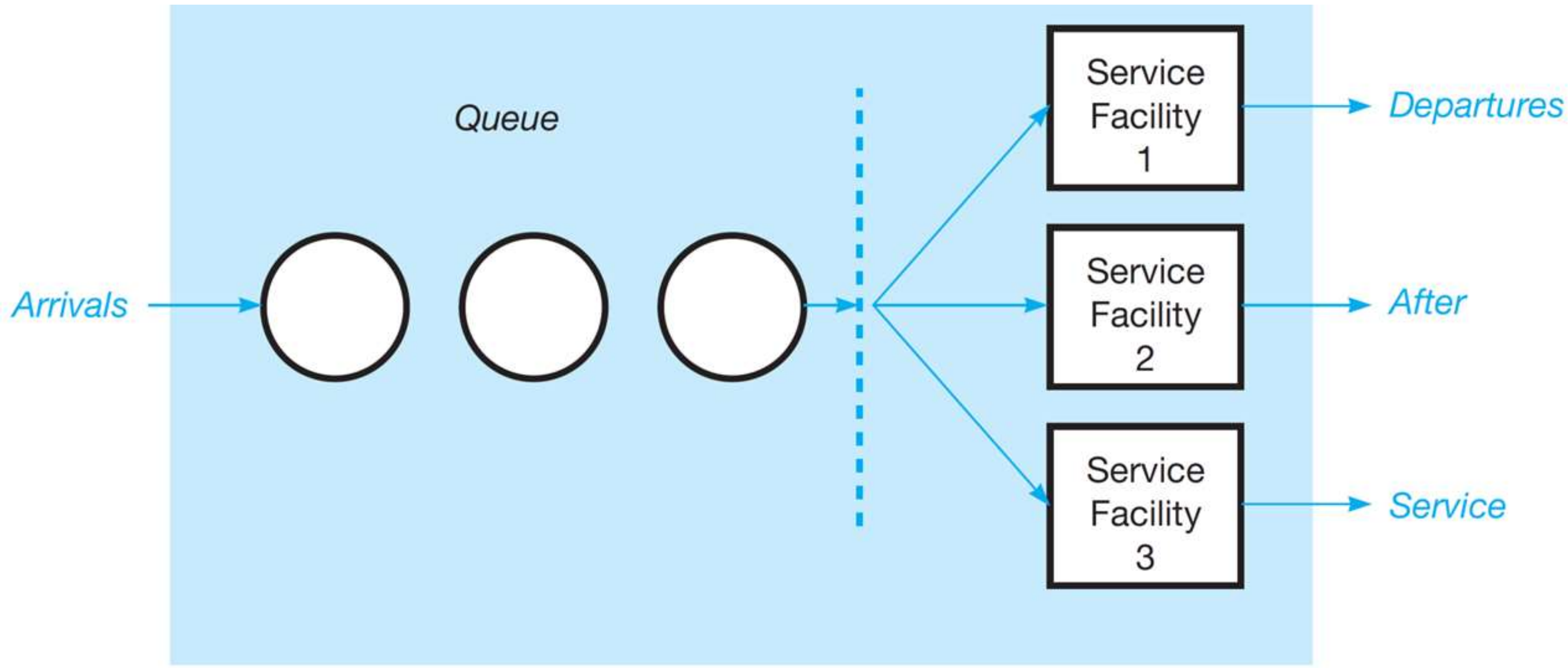
Single-Channel, Single-Phase System



Single-Channel, Multiphase System

Characteristics of a Queuing System (8 of 10)

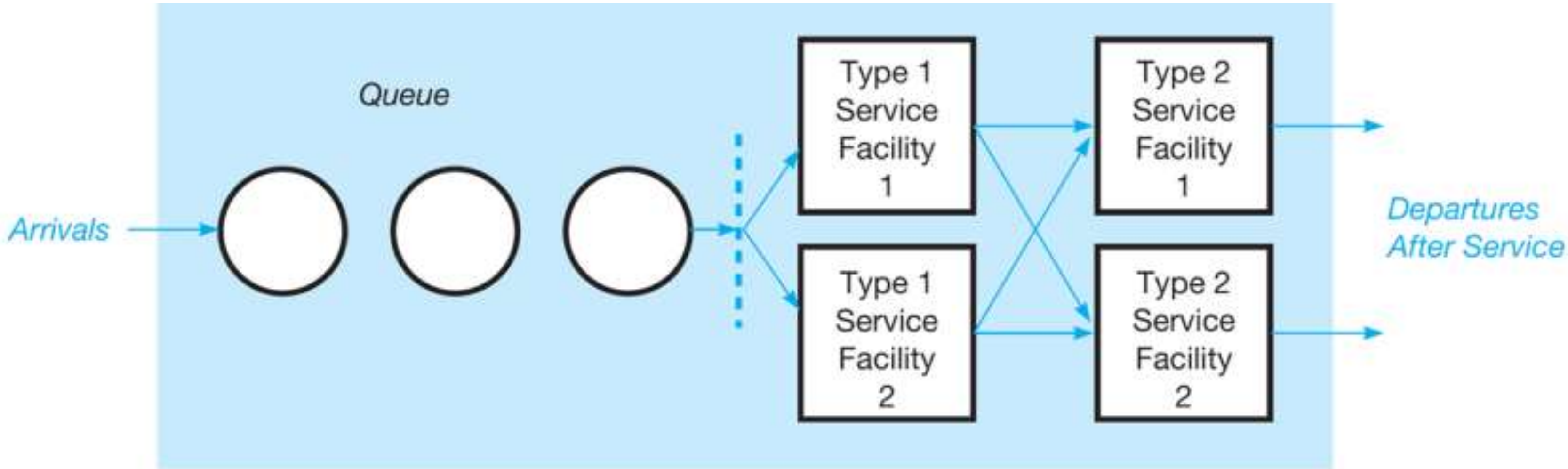
Four Basic Queuing System Configurations:



Multichannel, Single-Phase System

Characteristics of a Queuing System (9 of 10)

Four Basic Queuing System Configurations:



Multichannel, Multiphase System

Characteristics of a Queuing System (10 of 10)

Service Time Distribution:

- Service patterns can be either constant or random
- Constant service times are often machine controlled
- Generally service times are randomly distributed according to a **negative exponential probability distribution**
- Analysts should observe, collect, and plot service time data to ensure that the observations fit the assumed distributions when applying these models

Identifying Models Using Kendall Notation (1 of 2)

- A notation for queuing models that specifies the pattern of arrival, the service time distribution, and the number of channels
- Basic three-symbol Kendall notation has the form

  Arrival distribution / Service time distribution / Number of service channels open

- Specific letters used to represent probability distributions

  M = Poisson distribution for number of occurrences

  D = constant (deterministic) rate

  G = general distribution with mean and variance known

Identifying Models Using Kendall Notation (2 of 2)

- A single-channel model with Poisson arrivals and exponential service times would be represented by

$$M/M/1$$

- When a second channel is added

$$M/M/2$$

- A three-channel system with Poisson arrivals and constant service time would be

$$M/D/3$$

- A four-channel system with Poisson arrivals and normally distributed service times would be

$$M/G/4$$

Single-Channel Model, Poisson Arrivals and Exponential Service Times (*M*/*M*/*1*) (1 of 6)

Assumptions of the model:

1. Arrivals are served on a FIFO basis
2. There is no baulking or reneging
3. Arrivals are independent of each other but the arrival rate is constant over time
4. Arrivals follow a Poisson distribution
5. Service times are variable and independent but the average is known
6. Service times follow a negative exponential distribution
7. Average service rate is greater than the average arrival rate

Single-Channel Model, Poisson Arrivals and Exponential Service Times (*M*/*M*/*1*) (2 of 6)

When these assumptions are met, we can develop a series of equations that define the queue's **operating characteristics**

Single-Channel Model, Poisson Arrivals and Exponential Service Times (*M/M/1*) (3 of 6)

Queuing Equations:

Let

λ = mean number of arrivals per time period

μ = mean number of customers or units served per time period

- The same time period must be used for the arrival rate and service rate

Single-Channel Model, Poisson Arrivals and Exponential Service Times (*M*/*M*/*1*) (4 of 6)

1. The average number of customers or units in the system, *L*

$$L = \frac{\lambda}{\mu - \lambda}$$

2. The average time a customer spends in the system, *W*

$$W = \frac{1}{\mu - \lambda}$$

3. The average number of customers in the queue, L_q

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

Single-Channel Model, Poisson Arrivals and Exponential Service Times (*M*/*M*/*1*) (5 of 6)

4. The average time a customer spends waiting in the queue, W_q

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

5. The utilisation factor for the system, ρ, the probability the service facility is being used

$$\rho = \frac{\lambda}{\mu}$$

Single-Channel Model, Poisson Arrivals and Exponential Service Times (*M*/*M*/*1*) (6 of 6)

6. The percent idle time, P_0, or the probability no one is in the system

$$P_0 = 1 - \frac{\lambda}{\mu}$$

7. The probability that the number of customers in the system is greater than k, $P_{n>k}$

$$P_{n>k} = \left(\frac{\lambda}{\mu}\right)^{k+1}$$

Arnold's Muffler Shop (1 of 12)

- Arnold's mechanic can install mufflers at a rate of 3 per hour
  - Customers arrive at a rate of 2 per hour

So

λ = 2 cars arriving per hour

μ = 3 cars serviced per hour

$$L = \frac{\lambda}{\mu - \lambda} = \frac{2}{3 - 2} = \frac{2}{1} = 2 \text{ cars in the system on average}$$

$$W = \frac{1}{\mu - \lambda} = \frac{1}{3 - 2} = 1 \text{ hour that an average car spends in the system}$$

Arnold’s Muffler Shop (2 of 12)

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{2^2}{3(3-2)} = \frac{4}{3(1)}$$ = 1.33 cars waiting in line on average

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{2}{3} \text{ hour}$$ = 40 minutes average waiting time per car

$$\rho = \frac{\lambda}{\mu} = \frac{2}{3} = 0.67$$ = percentage of time mechanic is busy

$$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{2}{3} = 0.33$$ = probability that there are 0 cars in the system

Arnold's Muffler Shop (3 of 12)

- Probability of more than *k* cars in the system

k	$P_{n>k} = (^2/_3)^{k+1}$	
0	0.667	*Note that this is equal to* 1 − P_0 = 1 − 0.33 = 0.667
1	0.444	
2	0.296	
3	0.198	*This implies that there is a 19.8% chance that more than 3 cars are in the system*
4	0.132	
5	0.088	
6	0.058	
7	0.039	

Arnold's Muffler Shop (4 of 12)

Introducing costs into the model:

- Arnold wants to do an economic analysis of the queuing system and determine the waiting cost and service cost
- The total service cost is:

Total service cost = (Number of channels) × (Cost per channel)

Total service cost = mC_s

where

m = number of channels

C_s = service cost (labour cost) of each channel

Arnold's Muffler Shop (5 of 12)

- Waiting cost when the cost is based on time in the system

Total waiting cost = (Total time spent waiting by all arrivals)
× (Cost of waiting)
= (Number of arrivals) × (Average wait per arrival)C_w

Total waiting cost = $(\lambda W)C_w$

- If waiting time cost is based on time in the queue

Total waiting cost = $(\lambda W_q)C_w$

Arnold's Muffler Shop (6 of 12)

- So the total cost of the queuing system when based on time in the system is

  Total cost = Total service cost + Total waiting cost

  Total cost = $mC_s + \lambda WC_w$

  - And when based on time in the queue

  Total cost = $mC_s + \lambda W_qC_w$

Arnold's Muffler Shop (7 of 12)

- Arnold estimates the cost of customer waiting time in line is $50 per hour:

Total daily waiting cost = (8 hours per day) $\lambda W_q C_w$

$= (8)(2)(^2/_3)(\$50) = \$533.33/\text{day}$

- The mechanic's wage is $15 per hour:

Total daily service cost = (8 hours per day)mC_s

$= (8)(1)(\$15) = \$120/\text{day}$

- Total cost of the system is:

Total daily cost of the queuing system = $533.33 + $120 = $653.33

Arnold's Muffler Shop (8 of 12)

- Arnold is thinking about hiring a different mechanic who can install mufflers at a faster rate
  - The new operating characteristics would be

λ = 2 cars arriving per hour

μ = 4 cars serviced per hour

$$L = \frac{\lambda}{\mu - \lambda} = \frac{2}{4-2} = \frac{2}{2} = 1 \text{ car in the system on the average}$$

$$W = \frac{1}{\mu - \lambda} = \frac{1}{4-2} = 1/2 \text{ hour that an average car spends in the system}$$

Arnold's Muffler Shop (9 of 12)

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{2^2}{4(4-2)} = \frac{4}{8(1)}$$ $= 1/2$ car waiting in line on the average

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{1}{4} \text{ hour}$$ $=$ 15 minutes average waiting time per car

$$\rho = \frac{\lambda}{\mu} = \frac{2}{4} = 0.5$$ $=$ percentage of time mechanic is busy

$$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{2}{4} = 0.5$$ $=$ probability that there are 0 cars in the system

Arnold’s Muffler Shop (10 of 12)

Probability of more than *k* cars in the system:

k	$P_{n>k} = (^2/_4)^{k+1}$
0	0.500
1	0.250
2	0.125
3	0.062
4	0.031
5	0.016
6	0.008
7	0.004

Arnold's Muffler Shop (11 of 12)

- The customer waiting cost is the same \$50 per hour

  Total daily waiting cost = (8 hours per day) $\lambda W_q C_w$

  = (8)(2)(1/4)(\$50) = \$200.00/day

- The new mechanic is more expensive at \$20 per hour:

  Total daily service cost = (8 hours per day)mC_s

  = (8)(1)(\$20) = \$160/day

- So the total cost of the system is:

  Total daily cost of the queuing system = \$200 + \$160 = \$360

Arnold's Muffler Shop (12 of 12)

- The total time spent waiting for the 16 customers per day was formerly

  (16 cars per day) × (2/3 hour per car) = 10.67 hours

It is now:

(16 cars per day) × (1/4 hour per car) = 4 hours

The total daily system costs are less with the new mechanic resulting in significant savings:

$653.33 − $360 = $293.33

Enhancing the Queuing Environment

- Reducing waiting time is not the only way to reduce waiting cost

- Reducing the unit waiting cost (C_w) will also reduce total waiting cost

- This might be less expensive to achieve than reducing either W or W_q

Multichannel Model, Poisson Arrivals, Exponential Service Times (*M*/*M*/*m*) (1 of 4)

- Equations for the multichannel queuing model

  Let

  m = number of channels open

  λ = average arrival rate

  μ = average service rate at each channel

Multichannel Model, Poisson Arrivals, Exponential Service Times (*M/M/m*) (2 of 4)

- Equations for the multichannel queuing model

Let

m = number of channels open

λ = average arrival rate

μ = average service rate at each channel

1. The probability that there are zero customers in the system

$$P_0 = \frac{1}{\left[\sum_{n=0}^{n=m-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{m!}\left(\frac{\lambda}{\mu}\right)^m \frac{m\mu}{m\mu - \lambda}} \quad \text{for } m\mu > \lambda$$

Multichannel Model, Poisson Arrivals, Exponential Service Times (*M*/*M*/*m*) (3 of 4)

Same basic assumptions as in
the single-channel model!

Multichannel Model, Poisson Arrivals, Exponential Service Times (*M*/*M*/*m*) (4 of 4)

2. The average number of customers or units in the system

$$L = \frac{\lambda\mu(\lambda / \mu)^m}{(m-1)!(m\mu - \lambda)^2}P_0 + \frac{\lambda}{\mu}$$

3. The average time a unit spends in the waiting line or being served, in the system

$$W = \frac{\mu(\lambda / \mu)^m}{(m-1)!(m\mu - \lambda)^2}P_0 + \frac{1}{\mu} = \frac{L}{\lambda}$$

Single-Channel Model, Poisson Arrivals and Exponential Service Times (*M*/*M*/1)

- The average number of customers or units in line waiting for service

$$L_q = L - \frac{\lambda}{\mu}$$

- The average number of customers or units in line waiting for service

$$W_q = W - \frac{1}{\mu} = \frac{L_q}{\lambda}$$

- The average number of customers or units in line waiting for service

$$\rho = \frac{\lambda}{m\mu}$$

Arnold's Muffler Shop Revisited (1 of 4)

- Arnold wants to investigate opening a second garage bay
  - Hire a second worker who works at the same rate as his first worker
  - Customer arrival rate remains the same

$$P_0 = \frac{1}{\left[\sum_{n=0}^{n=m-1} \frac{1}{n!}\left(\frac{2}{3}\right)^n\right] + \frac{1}{2!}\left(\frac{2}{3}\right)^2 \left(\frac{2(3)}{2(3)-2}\right)}$$

$$P_0 = \frac{1}{1 + \frac{2}{3} + \frac{1}{2}\left(\frac{4}{9}\right)\left(\frac{6}{6-2}\right)} = \frac{1}{1 + \frac{2}{3} + \frac{1}{3}} = \frac{1}{2} = 0.5$$

$=$ probability of 0 cars in the system

Arnold's Muffler Shop Revisited (2 of 4)

$$L = \frac{(2)(3)(2/3)^2}{(1)![2(3) - 2]^2}\left(\frac{1}{2}\right) + \frac{2}{3} = \frac{8/3}{16}\left(\frac{1}{2}\right) + \frac{2}{3} = \frac{3}{4} = 0.75$$

$= \text{Average number of cars in the system}$

$$W = \frac{L}{\lambda} = \frac{3}{8} \text{ hour} = 22\,1/2 \text{ minutes}$$

$= \text{average time a car spends in the system}$

$$L_q = L - \frac{\lambda}{\mu} = \frac{3}{4} - \frac{2}{3} = \frac{1}{12} = 0.083$$

$= \text{ average number of cars in the queue}$

$$W_q = \frac{L_q}{\lambda} = \frac{0.083}{2} = 0.0417 \text{ hour} = 2.5 \text{ minutes}$$

$= \text{ average time a car spends in the queue}$

Arnold's Muffler Shop Revisited (3 of 4)

Effect of Service Level on Arnold's Operating Characteristics :

OPERATING CHARACTERISTIC	LEVEL OF SERVICE ONE MECHANIC (REID BLANK) $\mu = 3$	TWO MECHANICS $\mu = 3$ FOR BOTH	ONE FAST MECHANIC (JIMMY SMITH) $\mu = 4$
Probability that the system is empty (P_0)	0.33	0.50	0.50
Average number of cars in the system (L)	2 cars	0.75 cars	1 car
Average time spent in the system (W)	60 minutes	22.5 minutes	30 minutes
Average number of cars in the queue (L_q)	1.33 cars	0.083 car	0.50 car
Average time spent in the queue (W_q)	40 minutes	2.5 minutes	15 minutes

Arnold's Muffler Shop Revisited (4 of 4)

- Adding the second service bay reduces the waiting time in line but will increase the service cost as a second mechanic needs to be hired

Total daily waiting cost	= (8 hours per day) $\lambda W_q C_w$
	= (8)(2)(0.0417)($50) = $33.36
Total daily service cost	= (8 hours per day)mC_s
	= (8)2($15) = $240
Total daily cost of the system	= $33.36 + $240 = $273.36

Constant Service Time Model (*M/D/1*) (1 of 3)

- Constant service times are used when customers or units are processed according to a fixed cycle
- The values for L_q, W_q, L, and W are always less than they would be for models with variable service time
  - Both average queue length and average waiting time are halved in constant service rate models

Constant Service Time Model (*M/D/1*) (2 of 3)

- Equations for the Constant Service Time Model
  1. Average length of the queue

$$L_q = \frac{\lambda^2}{2\mu(\mu - \lambda)}$$

  2. Average waiting time in the queue

$$W_q = \frac{\lambda}{2\mu(\mu - \lambda)}$$

Constant Service Time Model (*M*/*D*/*1*) (3 of 3)

3. Average number of customers in the system

$$L = L_q + \frac{\lambda}{\mu}$$

4. Average time in the system

$$W = W_q + \frac{1}{\mu}$$

Garcia-Golding Recycling, Inc. (1 of 2)

- The company collects and compacts aluminum cans and glass bottles
- Trucks arrive at an average rate of 8 per hour (Poisson distribution)
- Truck drivers wait about 15 minutes before they empty their load
- Drivers and trucks cost $60 per hour
- A new automated machine can process truckloads at a constant rate of 12 per hour
- A new compactor would be amortised at $3 per truck unloaded

Garcia-Golding Recycling, Inc. (2 of 2)

- Analysis of cost versus benefit of the purchase

Current waiting cost/trip = ($^1/_4$ hour waiting time)($60/hour cost)
= $15/trip

New system: λ = 8 trucks/hour arriving

μ = 12 trucks/hour served

Average waiting time in queue = $W_q = {}^1/_{12}$ hour

Waiting cost/trip with new compactor = ($^1/_{12}$ hour wait)($60/hour cost) = $5/trip

Savings with new equipment = $15 (current system) − $5 (new system)
= $10 per trip

Cost of new equipment amortised = $3/trip

Net savings = $7/trip

Using Excel QM

Excel QM Solution for Constant Service Time Model for Garcia-Golding Recycling Example:

	A	B	C	D	E
1	**Garcia-Golding Recycling**				
2					
3	**Waiting Lines**		**M/D/1 (Constant Service Times)**		
4	The arrival RATE and service RATE both must be rates and use the same time unit. Given a time such as 10 minutes, convert it to a rate such as 6 per hour.				
5					
6	**Data**			**Results**	
7	Arrival rate (λ)	8		Average server utilization(ρ)	0.66667
8	Service rate (μ)	12		Average number of customers in the queue(L_q)	0.66667
9				Average number of customers in the system(L_s)	1.33333
10				Average waiting time in the queue(W_q)	0.08333
11				Average time in the system(W_s)	0.16667
12				Probability (% of time) system is empty (P_0)	0.33333

Finite Population Model (*M/M/1* with Finite Source) (1 of 4)

- Models are different when the population of potential customers is limited
- A dependent relationship between the length of the queue and the arrival rate
- The model has the following assumptions
  1. There is only one server
  2. The population of units seeking service is finite
  3. Arrivals follow a Poisson distribution and service times are exponentially distributed
  4. Customers are served on a first-come, first-served basis

Finite Population Model (*M/M/1* with Finite Source) (2 of 4)

- Equations for the finite population model

λ = mean arrival rate
μ = mean service rate
N = size of the population

1. Probability that the system is empty

$$P_0 = \frac{1}{\sum_{n=0}^{N} \frac{N!}{(N-n)!}\left(\frac{\lambda}{\mu}\right)^n}$$

Finite Population Model (*M/M/1* with Finite Source) (3 of 4)

2. Average length of the queue

$$L_q = N - \left(\frac{\lambda + \mu}{\lambda}\right)(1 - P_0)$$

3. Average number of customers (units) in the system

$$L = L_q + (1 - P_0)$$

4. Average waiting time in the queue

$$W_q = \frac{L_q}{(N - L)\lambda}$$

Finite Population Model (*M/M/1* with Finite Source) (4 of 4)

5. Average time in the system

$$W = W_q + \frac{1}{\mu}$$

6. Probability of n units in the system:

$$P_n = \frac{N!}{(N-n)!}\left(\frac{\lambda}{\mu}\right)^n P_0 \text{ for } n = 0,1,\ldots,N$$

Department of Commerce (1 of 3)

- The Department of Commerce has five printers that each need repair after about 20 hours of work
- Breakdowns follow a Poisson distribution
- The technician can service a printer in an average of about 2 hours following an exponential distribution

$$\lambda = 1/20 = 0.05 \text{ printer/hour}$$
$$\mu = 1/2 = 0.50 \text{ printer/hour}$$

Department of Commerce (2 of 3)

1. $$P_0 = \frac{1}{\sum_{n=0}^{5} \frac{5!}{(5-n)!}\left(\frac{0.05}{0.5}\right)^n} = 0.564$$

2. $$L_q = 5 - \left(\frac{0.05 + 0.5}{0.05}\right)(1 - P_0) = 0.2 \text{ printer}$$

3. $$L = 0.2 + (1 - 0.564) = 0.64 \text{ printer}$$

Department of Commerce (2 of 3)

4. $$W_q = \frac{0.2}{(5 - 0.64)(0.05)} = \frac{0.2}{0.22} = 0.91 \text{ hour}$$

5. $$W = 0.91 + \frac{1}{0.50} = 2.91 \text{ hours}$$

If printer downtime costs \$120 per hour and the technician is paid \$25 per hour, the total cost is

Total hourly cost = (Average number of printers down)
(Cost per downtime hour) + Cost per technician hour
= (0.64)(\$120) + \$25 = \$101.80

Some General Operating Characteristic Relationships

- Certain relationships exist for any queuing system in a **steady state**
  - Steady state condition
    - System is in its normal stabilized condition usually after an initial **transient state**
  - **Little's Flow Equations**

$$L = \lambda W \text{ (or } W = L/\lambda\text{)}$$

$$L_q = \lambda W_q \text{ (or } W_q = L_q/\lambda\text{)}$$

  - A third condition that must always be met

$$W = W_q + 1/\mu$$

More Complex Queuing Models and the Use of Simulation

- Often *variations* from basic queuing models

- Computer simulation can be used to solve these more complex problems
  - Simulation allows the analysis of controllable factors
  - Should be used when standard queuing models provide only a poor approximation of the actual service system

Homework --- Chapter 12

- End of chapter self-test 1-14 (pp. 469-470)

  Compile all answers into one document and submit at the beginning of the next lecture! On the top of the document, write your Pinyin-Name and Student ID.

- Please read Chapter 7!