Problem Set 3

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Problem 1

Part A

The entropy is found from

$$S = (U + PV - N\mu)/T$$

The pressure and number terms are given in the notes:

$$\frac{PV}{kT} = \frac{4(2s+1)V}{3\pi^{1/2}\lambda^3}F_{3/2}(z)$$

$$\frac{N\mu}{kT} = \frac{2(2s+1)V}{\pi^{1/2}\lambda^3}F_{1/2}(z)$$

with $\lambda = h/\sqrt{2\pi mkT}$ We can compute the internal energy by integrating the energy of each state weighted by the population in each state:

$$\frac{U}{kT} = \frac{(2s+1)V}{h^3kT}\int_0^\infty \epsilon N(\epsilon)d^3p$$

where the factor in front of the integral gives the multiplicity of each energy state. For a non-interacting Fermi gas we can make the substitution $\epsilon = p^2/2m$ and then introduce the variable $w = \epsilon/kT$:

$$\frac{U}{kT} = \frac{4\pi(2s+1)V}{h^3kT}\int_0^\infty N(w)\frac{(2mkT)^{5/2}}{10m}d(w^{5/2})$$

where

$$N(w) = \frac{1}{e^w/z+1}$$

$d(w^{5/2}) = (5/2)w^{3/2}dw$, so the integral becomes:

$$\int_0^\infty (mkT)^{5/2}\frac{\sqrt{2}}{m}\frac{w^{3/2}}{e^w/z+1}dw = (mkT)^{5/2}\frac{\sqrt{2}}{m}F_{3/2}(z)$$

And therefore the internal energy is:

$$\frac{U}{kT} = \frac{4\pi\sqrt{2}(2s+1)V(mkT)^{3/2}}{h^3}F_{3/2}(z) = \frac{2(2s+1)}{\sqrt{\pi}\lambda^3}F_{3/2}(z)$$

where $\lambda = (2\pi mkT)^{3/2}/h^3$. Comparing this to the pressure term, we find that $u = \frac{3}{2}P$. Putting this all together, we find the expression for entropy:

$$S = \frac{5k}{2}\frac{PV}{kT} - \frac{N\mu}{T} = \frac{(2s+1)V}{\sqrt{\pi}\lambda^3}\left(\frac{10}{3}F_{3/2}(z) + \frac{2\mu}{kT}F_{1/2}(z)\right)$$

Part B