# ASTR 507: Problem Set 3

Due Feb. 14, 2018

1. (a) (2 pt) Derive the entropy for an ideal, non-relativistic Fermi gas in terms of $V, z$ and $T$. You may leave your answer in terms of Fermi-Dirac integrals.

   (b) (3 pt) Expanding $P/(nkT)$ as a series in $z$, and keeping first order terms in $z$, show that the electron degeneracy increases the pressure of the gas.

2. (10 pts) The center of a low mass star has a density $\rho_c = 325$ g cm$^{-3}$ and a temperature of $6 \times 10^6$ K. Assuming that the helium atom to hydrogen atom number density ratio is 0.1 throughout the star (and ignoring heavy elements), estimate:

   (a) the electron fugacity (suggestion: either plot the Fermi-Dirac function versus $z$ and match to the central conditions of the star, or solve numerically),

   (b) the electron gas pressure in the center of the star using the partially degenerate, non-relativistic Fermi equation of state for electrons.

   (c) Are the electrons in fact non-relativistic?

   (d) Demonstrate that degeneracy contributes significantly to the pressure by computing the pressure as if the gas were a classical ideal gas.

   (e) Estimate the mass and radius of the star using the polytrope relations $\rho_c = 8.44 M/R^3$ g cm$^{-3}$ and $RM^{1/3} = 0.13/(\mu_e F_{1/2}(z))^{2/3}$, where $M$ and $R$ are the mass and radius of the star in solar units, and $\mu_e$ is the mass per electron in units of the proton mass.

See `https://arxiv.org/abs/0811.0116` for suggestions on approximating the Fermi-Dirac integrals, but feel free to use other resources. E.g., Numerical Recipes has a C++ routine in Ch. 6.10 (third edition). Be aware of definition changes by a factor of $1/\Gamma(\nu + 1)$.