Problem Set 1

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April 5, 2018 ________All calculations can be found at github.com/tagordon/stars/hw1.ipynb

Problem 1

Part a

Using the relationship $M_\lambda = m_\lambda + 5 - 5\log(d)$, I find the absolute magnitude of τ Sco to be $M_\lambda = -2.99$.

Part b

Using the relationship $M_{\text{bol}} = -2.5\log(L/L_\odot) + 4.74$ and $M_{\text{bol}} = M_{\text{v}} + BC$, I find the luminosity of τ Sco to be about $2.2 \times 10^4 L_\odot$.

Part c

The flux from the star is related to the effective temperature by $F = \sigma T^4$, and the luminosity is related to the star's radius by $L = 4\pi R^2 F$. Solving for radius, we have:

$$R = \sqrt{\frac{L}{4\pi\sigma T_{\text{eff}}^4}}$$

I find a radius of about $5.6 R_\odot$ for τ Sco.

Part d

Applying the mass luminosity relationship with the mean index $\alpha = 3.8$, I find that the mass of τ Sco is about $14 M_\odot$.

Part e

With the usual relations for surface gravity and escape velocity, I find $\log(g) = 4.09$ and $v_{\text{esc}} = 1 \times 10^6$ m/s.

Part f

The mean density of τ Sco is 0.11 g/cm^{-3}

Part g

The sun has mean density 1.4 g/cm^{-3}, which is much larger than that fo τ Sco. It's surface gravity is also larger ($\log(g) = 4.44$), as expected for a more dense star, and it's escape velocity is 6×10^5. The escape velocity is smaller because the sun is a less massive star that τ Sco.

Problem 2

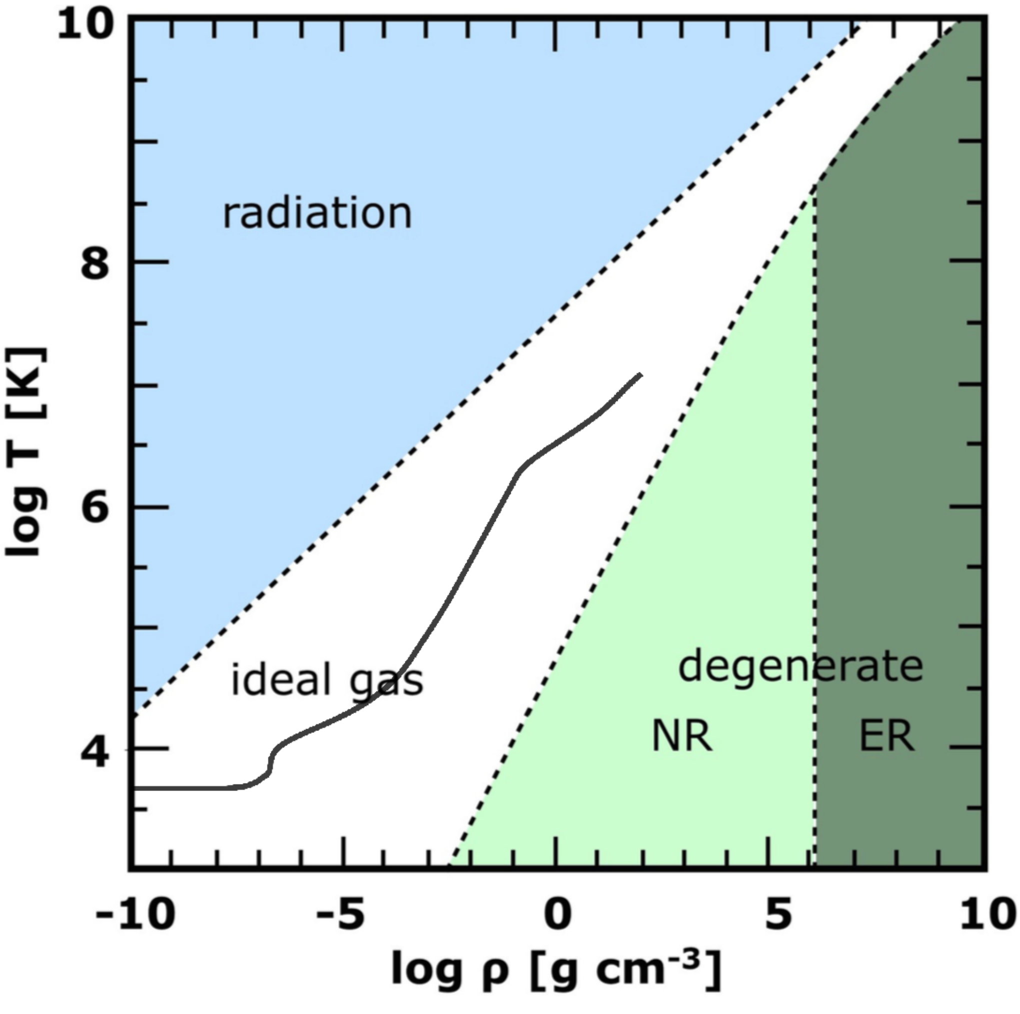
From the Virial Theorem, $K = -U/2$. For a self gravitating sphere of mass M and radius R, the potential energy is $U = -GM^2/R$. For an ideal gas, the total kinetic energy is $K = 3Nk\bar{T}/2$. Where N, the total number of particles, is $N = M/\mu m_H$. Plugging these expressions into the Virial Theorem:

$$\frac{3}{2}\frac{M}{\mu m_H}k\bar{T} = \frac{GM^2}{2R}$$

Solving for the mean temperature $\bar{T}$, we find

$$\bar{T} = \frac{3\mu m_H}{k}\frac{GM}{R} \sim \frac{\mu m_H}{k}\frac{GM}{R}$$

Problem 3



From this plot we can conclude that the material making up the sun will behave as an ideal gas throughout the entire star.

Problem 4

Part a

Solving for N, the photon will undergo about 5×10^{21} scatterings on it's way to the surface.

Part b

The total path length is thus 5×10^{21} cm, and the time it takes to arrive at the surface would be 5×10^{21} cm$/3 \times 10^{10}$ cm/s $= 1.7 \times 10^{11}$ s

Part c

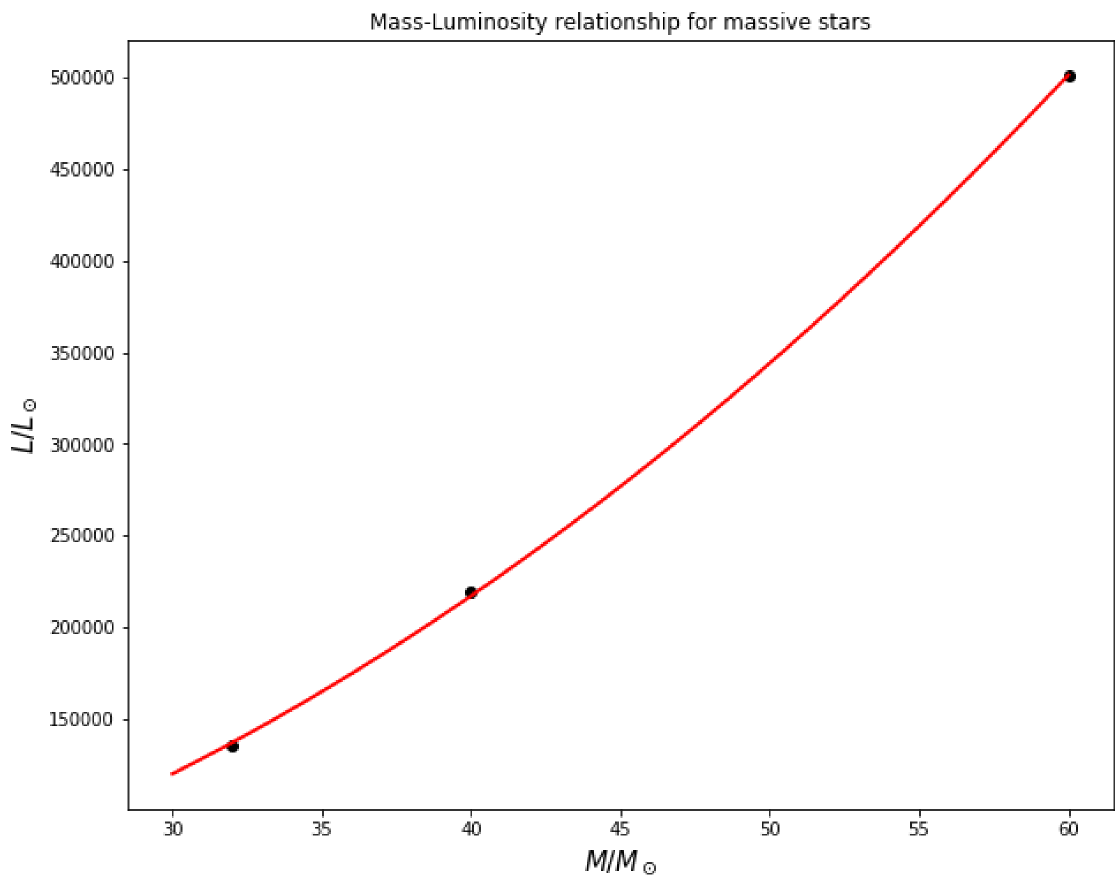
No, scattering events change the energy of the photon. Changes in energy equate to changes in the photon's wavelength. So it's a "different" photon by the time it reaches the surface.

Problem 5

Part a

Fitting a power-law to the data from the appendix, I find the relation:

$$\frac{L}{L_\odot} = 106 \left(\frac{M}{M_\odot}\right)^{2.07}$$



Part b

Setting the right hand side equal to the Eddington Luminosity, the corresponding mass is found to be about $M_{\max} \sim 244 M_\odot$.

Problem 6

Part a

The amount of energy that can be derived from the fuel in the core is given by:

$$E = \chi M_{\text{core}} c^2$$

where χ is the efficiency of hydrogen fusion at converting mass to energy. To derive a timescale for the main sequence evolution of the star we divide this by the star's luminosity. From figure 7.6 I estimate the core mass-fraction of a $4M_\odot$ star to be about $M_{\text{core}} = 0.2$, and for a $20M_\odot$ star, $M_{\text{core}} = 0.4$. The luminosities are given in the appendix as $log(L/L_\odot) = 2.37$ and 4.61 respectively. From these numbers (and adopting $\chi = 0.0071$ from section 8.1 of the book) I find an approximate main sequence lifetime of 71 Myr for the $4M_\odot$ star and about 8 Myr for the $20M_\odot$ star.

Part b

From appendix D, the $4M_\odot$ star should have a main-sequence lifetime of about 151 Myr, and the $20M_\odot$ star should have a main-sequence lifetime of 7.7 Myr. Both of these are about correct to an order of magnitude. I'm not sure why our assumptions give us a better estimate for the more massive star.